H

Vmath®

Third Edition

VOYAGER SOPRIS
LEARNING

Lesson 10

Objective

To determine whether two quantities are in a proportional relationship and write an equation for the relationship

Preskills	Transparency
Coordinate Planes in Four Quadrants	H.1.9
Ratios	H.5.PL1
Rates	H.5.PL2
Proportional Relationships	H.5.3

Academic Vocabulary

Before the lesson, introduce and discuss the Academic Vocabulary. Refer to the Academic Vocabulary as needed during the lesson.

- **constant of proportionality**
  the ratio of y to x when x and y form a proportional relationship

GET STARTED

Model the following skills for students.

REVIEW PRESKILLS

Problem 1

To write a ratio in simplest form, divide the numerator and the denominator by their greatest common factor. What is the GCF of 36 and 4? (4) What is the simplest form of the ratio $\frac{36}{4}$? $(\frac{9}{1})$

Problem 2

What is the x-coordinate for point A? (4) Is point A to the left or right of the y-axis? (right) What is the y-coordinate for point A? (2) Is point A above or below the x-axis? (above) Graph (4, 2).

MODEL NEW SKILLS

Problem 3

Look at the table. One way to determine whether there is a relationship between the variables x and y is to write each pair of values as a ratio in simplest form. What is the first pair of values? ($x = 2$ and $y = 6$) What is the ratio of y to x for this pair of
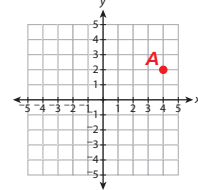

Proportional vs. Nonproportional Relationships

Name _____ Class _____ Date _____

GET STARTED

1 $\frac{36}{4} = \frac{9}{1}$

2 Graph $A(4, 2)$.

3

x	2	5	8	11	15
y	6	15	24	33	45

Ratio of y to x:	$\frac{6}{2}$	$\frac{15}{5}$	$\frac{24}{8}$	$\frac{33}{11}$	$\frac{45}{15}$
Ratio in simplest form:	$\frac{3}{1}$	$\frac{3}{1}$	$\frac{3}{1}$	$\frac{3}{1}$	$\frac{3}{1}$

Do x and y form a proportional relationship? **yes**

The equation that relates x and y is $y = $ **3** x.

BUILD THE CONCEPT

Use a graph to determine whether x and y are in a proportional relationship.

x	1	3	5	7	9
y	2	3	4	5	6

Do the points form a straight line? **yes**

Does the line pass through the origin? **no**

Do x and y form a proportional relationship? **no**

---
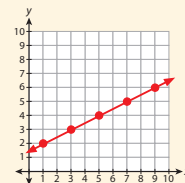
values written as a fraction? $(\frac{6}{2})$ **What is the simplest form of the ratio $\frac{6}{2}$?** $(\frac{3}{1})$

What is the second pair of values? ($x = 5$ and $y = 15$) **What is the ratio of y to x for this pair of values written as a fraction?** $(\frac{15}{5})$ **What is the simplest form of the ratio $\frac{15}{5}$?** $(\frac{3}{1})$ **What is the ratio of y to x for the third pair of values?** $(\frac{24}{8})$ **What is the simplest form of the ratio $\frac{24}{8}$?** $(\frac{3}{1})$ **What is the ratio of y to x for the fourth pair of values?** $(\frac{33}{11})$ **What is the simplest form of the ratio $\frac{33}{11}$?** $(\frac{3}{1})$ **What is the ratio of y to x for the last pair of values?** $(\frac{45}{15})$ **What is the simplest form of the ratio $\frac{45}{15}$?** $(\frac{3}{1})$

What do you notice about the ratios written in simplest form? (They are all the same.) **The relationship between x and y is proportional because all the ratios of y to x are equivalent. For each pair of values in the table, what number can you multiply x by to get y?** (3) **What equation represents this relationship?** ($y = 3x$) **In the proportional relationship $y = 3x$, 3 is the constant of proportionality. The constant of proportionality is the ratio of y to x, or $\frac{3}{1}$.**

Lesson 10

TRY IT TOGETHER

Determine whether x and y are in a proportional relationship. If x and y form a proportional relationship, write the equation.

4

x	4	10	12	15	16
y	6	15	18	20	24
Ratio of y to x:	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{3}{2}$

Do x and y form a proportional relationship? __no__

5

x	2	5	10	20	25
y	$\frac{4}{5}$	2	4	8	10
Ratio of y to x:	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Do x and y form a proportional relationship? __yes__

The equation that relates x and y is $y = \frac{2}{5}x$.

WORK ON YOUR OWN

HOW TO

Determine Whether a Relationship Is Proportional

Using Symbols

1.

x	2	4	6	8
y	1	2	3	4
Ratio of y to x:	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

2. Each ratio is $\frac{1}{2}$, so x and y are in a proportional relationship.

3. $y = \frac{1}{2}x$

Using Words

For each pair of values, write the ratio of y to x as a fraction in simplest form.

If the ratios are all the same, x and y are in a proportional relationship.

Use the ratio to write an equation that describes the relationship.

BUILD THE CONCEPT

Model how to use a graph to determine whether a relationship is proportional.

Another way to determine whether a relationship between two quantities is proportional is by using a graph. Look at the table. What ordered pair can be written from the first pair of values? ((1, 2)) Graph (1, 2). What ordered pair can be written from the second pair of values? ((3, 3)) Graph (3, 3). What ordered pair should be graphed next? ((5, 4)) Graph (5, 4). What ordered pair should be graphed next? ((7, 5)) Graph (7, 5). What ordered pair should be graphed last? ((9, 6)) Graph (9, 6).

If the points form a straight line that passes through the origin, x and y are proportional. Do the points appear to form a straight line? (yes) Use a straightedge to draw the line. Does the graph of the line pass through the origin? (no) Is the relationship between x and y proportional? (no)

Check the answer by comparing the ratios of y to x for the first two pairs of values in the table. What are the ratios in simplest form? ($\frac{2}{1}$ and $\frac{1}{1}$) Are the ratios the same? (no) The answer checks.

TRY IT TOGETHER

Work with students to complete these skills.

SCAFFOLD INSTRUCTION

Problem 4

Look at the table. How do you determine whether x and y form a proportional relationship by using ratios? (Write the ratio of y to x for each pair of values as a fraction in simplest form.)

What is the ratio of y to x for the first pair of values written as a fraction? ($\frac{6}{4}$) What is the simplest form of the ratio $\frac{6}{4}$? ($\frac{3}{2}$) What is the ratio of y to x for the second pair of values? ($\frac{15}{10}$) What is the simplest form of the ratio $\frac{15}{10}$? ($\frac{3}{2}$) What is the simplest form of the ratio $\frac{18}{12}$? ($\frac{3}{2}$) What is the simplest form of the ratio $\frac{20}{15}$? ($\frac{4}{3}$) What is the simplest form of the ratio $\frac{24}{16}$? ($\frac{3}{2}$)

Are all of the ratios the same? (no) Can the relationship between x and y be proportional? (no)

Problem 5

Look at the table. What is the ratio of y to x for the first pair of values written as a fraction? ($\frac{4/5}{2}$) What is the simplest form of the ratio $\frac{4/5}{2}$? ($\frac{2}{5}$) What is the ratio of y to x for the second pair of values? ($\frac{2}{5}$) What is the simplest form of the ratio $\frac{4}{10}$? ($\frac{2}{5}$) What is the simplest form of the ratio $\frac{8}{20}$? ($\frac{2}{5}$) What is the simplest form of the ratio $\frac{10}{25}$? ($\frac{2}{5}$)

Do x and y form a proportional relationship? (yes) Why? (the ratios for all pairs of values are equivalent) What is the constant of proportionality? ($\frac{2}{5}$) What equation relates x and y? ($y = \frac{2}{5}x$)

WORK ON YOUR OWN

MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING This problem illustrates the **Using a Graph** strategy. Students are shown how the four-step problem-solving process is used to solve a word problem involving a graph. The instruction is immediately followed by application of the strategy in problem 11.

This problem gives data in a table and asks students to determine whether there is a proportional relationship between the data values in the table.

Have students look at the coordinate plane and have them read the labels. Ask students to name the first ordered pair from the table. Then have students explain how to graph the point. For the ordered pair (0.5, 1), make sure students understand that the x-coordinate is halfway between 0 and 1 on the x-axis. Have students graph the remaining points.

Next, ask students what they should do next to determine whether the relationship is proportional by using the graph. Students should recognize that the points have to lie in a straight line and that this line must pass through the origin. Have students draw the line and analyze it to answer the question.

Finally, have students write the equation. Ask them how to determine the constant of proportionality from the table. They should remember that the constant of proportionality is the ratio of y to x written as a fraction in simplest form. Have students verify their answer by calculating the ratio for each pair of values in the table.

Problems 6–12

Have students work independently. Check work and have students total the number correct. Use Additional Resources as needed.

SKILL BUILDING: NEW AND REVIEW

Determine whether x and y are in a proportional relationship. If x and y form a proportional relationship, write the equation.

6

x	3	4	7	9	11
y	18	24	42	54	66

Ratio of y to x: $\frac{18}{3} = \frac{6}{1}, \frac{24}{4} = \frac{6}{1}, \frac{42}{7} = \frac{6}{1}, \frac{54}{9} = \frac{6}{1}, \frac{66}{11} = \frac{6}{1}$; proportional relationship; equation: $y = 6x$

7

x	3	6	12	15	24
y	9	2	4	5	8

Ratio of y to x: $\frac{9}{3} = \frac{3}{1}, \frac{2}{6} = \frac{1}{3}, \frac{4}{12} = \frac{1}{3}, \frac{5}{15} = \frac{1}{3}, \frac{8}{24} = \frac{1}{3}$; not a proportional relationship

Write each ratio in simplest form.

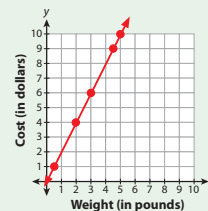**8** $\frac{18}{30}$ $\frac{3}{5}$ **9** $\frac{35}{14}$ $\frac{5}{2}$ **10** $\frac{36}{96}$ $\frac{3}{8}$

PROBLEM-SOLVING

Using a Graph
The weights and costs of baskets of apples at a fruit stand are shown in the table. Is there a proportional relationship between the weight and the cost? If so, write an equation for the relationship.

Weight (in pounds), x	0.5	2	3	4.5	5
Cost (in dollars), y	1	4	6	9	10

a. **Find:** the relationship between the weight and the cost

b. **How?** Use a graph to show the relationship.

c. **Solve the problem.**

Graph the points from the table.

Do x and y form a proportional relationship? __yes__

The equation that relates weight and cost is $y = 2x$

d. **Is the answer reasonable? Explain.** Yes; the rate of the cost to the weight is $2 for 1 pound for each pair in the table, so the relationship is proportional.

CHECK UP

ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a: The student chose the constant of proportionality instead of the value that correctly completes the table. Use Additional Resources in H.5.10 to review how to use the constant of proportionality.

If student answered 1c: The student found and used a pattern in the x-values in the table. Use Additional Resources in H.5.10 to review the meaning of a proportional relationship.

If student answered 1d: The student multiplied by the constant of proportionality instead of dividing. Use Additional Resources in H.5.10 to review how to use the constant of proportionality.

If student answered problem 2 incorrectly: The student confused the x- and y-values or calculated the ratio of y to x incorrectly. Use Additional Resources in H.5.10 to review how to use a ratio to determine a proportional relationship.
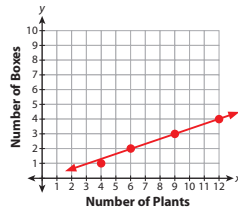
Lesson 10

PROBLEM-SOLVING: NEW AND REVIEW

Solve each problem.

11 A plant nursery mails its plants in boxes. The table shows the relationship between the number of plants and the number of boxes. Use a graph to determine whether the relationship is proportional.

Number of Plants, x	4	6	9	12
Number of Boxes, y	1	2	3	4

A straight line does not pass through all the points or the origin, so the relationship is not proportional.

12 Sean volunteers 8 hours every 2 weeks. How many hours does Sean volunteer in 6 weeks?

$\dfrac{\text{hours}}{\text{weeks}}$, $\dfrac{8}{2} = \dfrac{h}{6}$; $48 = 2h$; $24 = h$; 24 h

CHECK UP

Answer each question.

1 The table shows a proportional relationship between x and y. What is the missing value?

x	2	6	9	
y	8	24	36	40

a. $x = 4$ **b.** $x = 10$

c. $x = 12$ d. $x = 160$

2 The table shows a proportional relationship between x and y. Which pair of values belongs in the table?

x	6	15	24
y	4	10	16

a. $x = 2; y = 3$ **b.** $x = \dfrac{9}{2}; y = 3$

c. $x = 8; y = 6$ d. $x = 15; y = 20$

3 Which answer choice in problem 1 is the least reasonable? Explain.
a; the y-values in the table are increasing as the x-values are increasing, so the value of x must be greater than 9.

4 The graph shows a proportional relationship between the number of bags of balloons bought, x, and the total number of balloons, y. What does the point (1, 25) represent? How is this point related to the constant of proportionality? The point (1, 25) represents that there are 25 balloons in 1 bag. The constant of proportionality is the rate of y to x, or $\dfrac{25}{1} = 25$.

WRITE MATH

CRITICAL THINKING

40   Lesson 10 • Proportional vs. Nonproportional Relationships

---

WRITE MATH
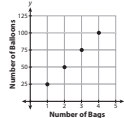
If student answered problem 3 incorrectly: While students work together, review with the student the procedure for using ratios to identify a proportional relationship. Use Additional Resources in H.5.10 to reteach this concept.

DIFFERENTIATION

Additional Resources

Vmath Reteach
Reteach Student Book Module 5 Lesson 10
Reteach Teacher Edition Module 5 Lesson 10

Extra Practice
Student Book page 43

English Language Learners

Use the VmathLive Animated Glossary to review the terms *equivalent fractions, ratio, rate*, and *proportion*. Demonstrate the vocabulary at the beginning of the lesson as students gather around the computer screen or through a projection system if possible.

Review the different forms of a ratio with students. Remind them that the ratio y to x is the same as $\dfrac{y}{x}$. When forming the ratios from a table, this means to write the value from the second row as the numerator and the value from the first row as the denominator.

Students with Special Needs

If students are having difficulty writing the ratios properly, have them use colored pencils or markers to draw a square around each x-value from the table in the same color. Have them repeat this process for the y-values in a different color. Then when students write the ratio, have them draw a box for the numerator using the color for the y-values and draw a box for the denominator using the color for the x-values.