TRANSMATH®

Making Sense of Rational Numbers

Teacher Guide Volume 1

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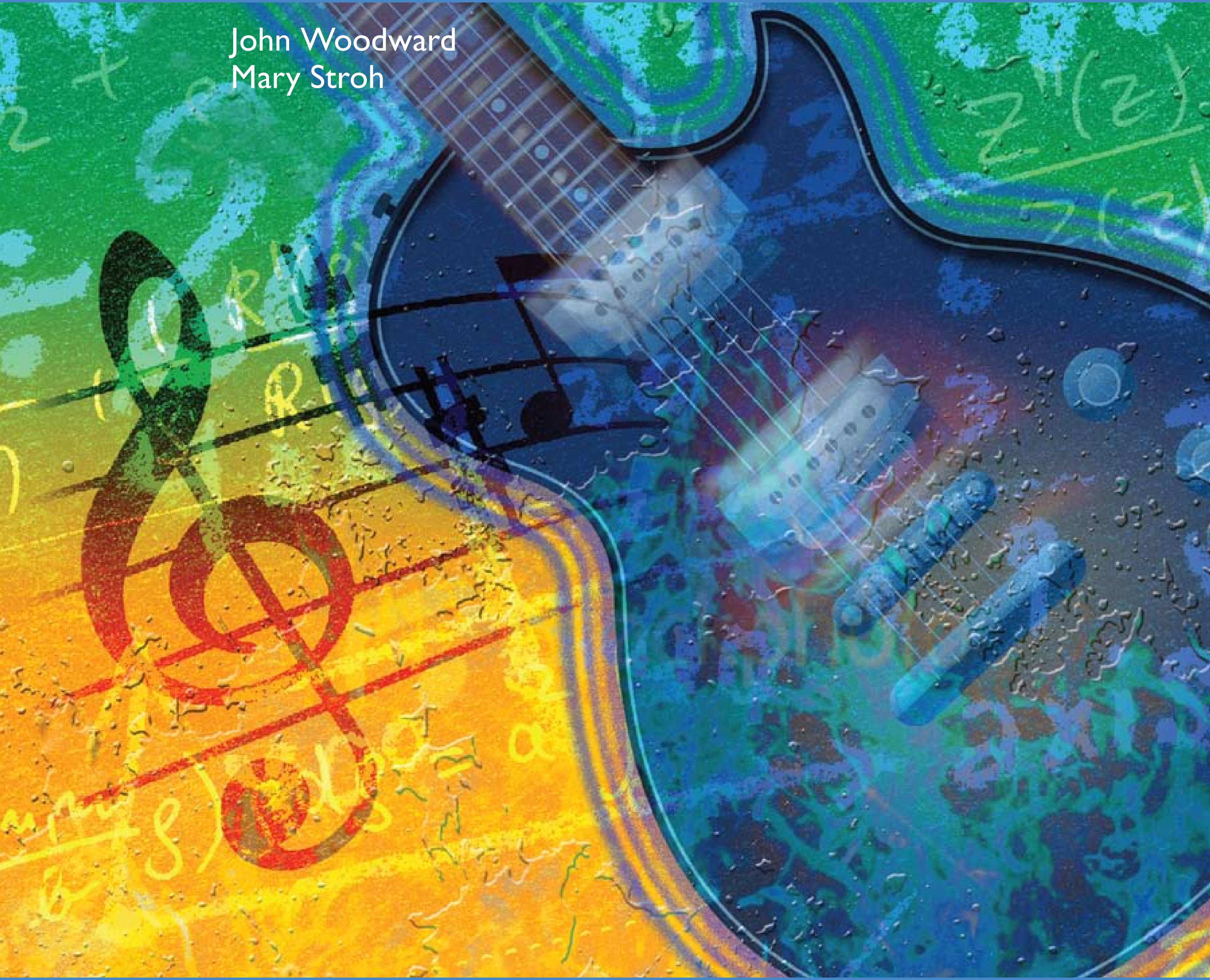


Cambium Learning® Group | Voyager Learning

Unit 2

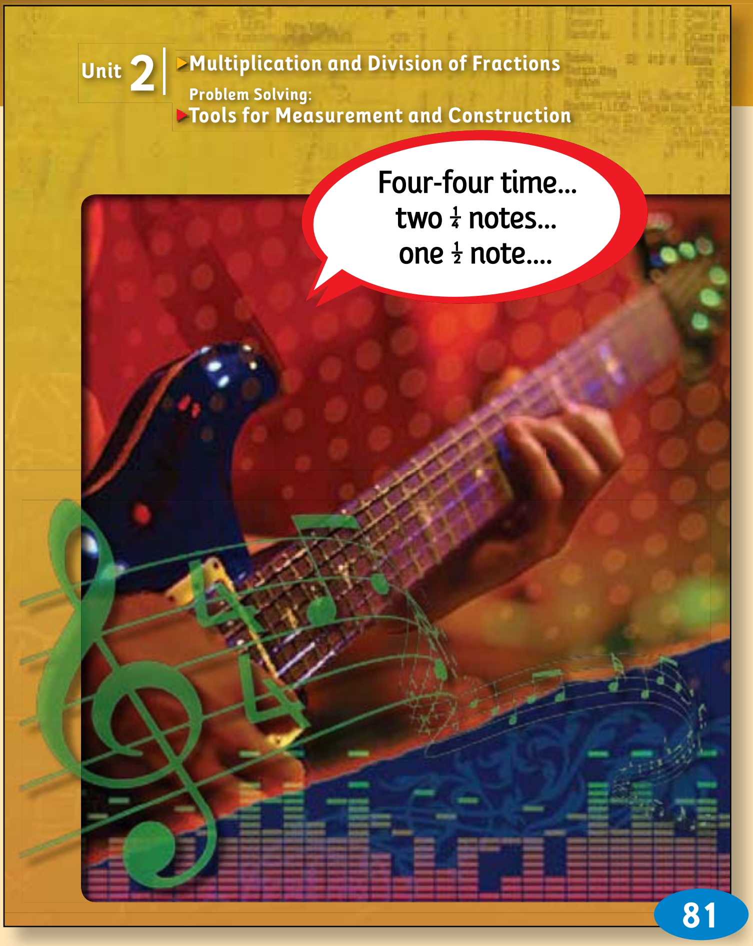
Multiplication and Division of Fractions

Problem Solving: Tools for Measurement and Construction

Unit Opener: Background Information

With students, read through the Unit Opener of *Student Text*, pages 81–82. Then share some of these musical notes.

- Classical musician Wolfgang Mozart wrote his first piece at age five or six.
- J. S. Bach composed more than 1,100 works in his lifetime.
- The longest song ever performed is "Marathon 2" by Mark Mallman, which lasts 52 hours.
- Math and music have important connections. For example, the rhythm of music is measured in beats, usually four beats to a measure. Whatever combination of notes used, the total beats must add up to the measure. A measure of four beats could be two half notes (two beats each) or one half note plus two quarter notes.
- Each note on the scale emits a certain sound frequency measured in hertz, or cycles per second. The note A has a frequency of 440 hertz. Doubling the frequency produces an A one octave higher. Halving it yields an A one octave lower.
- Music has patterns, just as numbers have patterns. Musicians can transpose a song from one key to another by following the logical pattern that keeps the spacing between the notes the same.
- Gottfried Wilhem von Leibniz, a mathematician, once said this about music: "Music is the pleasure the human mind experiences from counting without being aware that it is counting."



Unit 2

Building Number Concepts: Multiplication and Division of Fractions

Problem Solving: Tools for Measurement and Construction

OBJECTIVES

Building Number Concepts	Problem Solving
• Use models to show multiplication and division of fractions • Understand how multiplication and division of fractions is different from whole numbers • Use the traditional methods to multiply and divide fractions	• Develop an understanding of basic geometric terms • Measure lengths and angles using a variety of tools and units • Use a compass to complete basic geometric constructions

Overview

Learning how to multiply and divide fractions is often a turning point for many students because common denominators are unnecessary. This change in operations with fractions can lead students to believe that math has arbitrary rules and procedures. To combat such misunderstandings, this unit focuses on visual representations and a logical comprehension of multiplication and division of fractions.

Unit Vocabulary
area model **area** **factors** **common factors** **greatest common factor** **commute** **traditional method** **multiplicand**

The Problem Solving strand of this unit focuses on rulers, protractors, and compasses. These tools play a critical role in measurement and geometry and are fundamental to many occupations. We build on the use of these tools throughout the next three units.

Unit Vocabulary
point **line segment** **line** **ray** **metric ruler** **millimeters** **centimeters** **angle** **vertex** **right angle** **acute angle** **obtuse angle** **protractor** **bisect** **equilateral triangle**

Building Number Concepts:

Multiplication and Division of Fractions

Key Questions That Guide Student Understanding

- *Why do we generally get answers that are smaller when we multiply fractions?*
- *Why do we generally get answers that are larger when we divide fractions?*

Enduring Understandings for Multiplication and Division of Fractions

When we multiply two fractions, the product is generally smaller than the multiplicands. This is the opposite of what happens when we multiply two whole numbers. To show this concept, choose your numbers carefully, and talk students through the multiplication process. When we multiply $\frac{1}{2}$ by $\frac{6}{8}$, we take $\frac{1}{2}$ of $\frac{6}{8}$. The product is a fractional portion of the quantity $\frac{6}{8}$.

Dividing fractions is based on the observation that the quotient is generally bigger than the quantity being divided (or dividend). When we divide $\frac{3}{4}$ by $\frac{1}{2}$, we use $\frac{1}{2}$ to partition $\frac{3}{4}$ one and one-half times. Again, this is the opposite of what happens with whole-number division.

Tools for Understanding Multiplication and Division of Fractions

Using Models
Rectangular area models and fraction bars are essential tools for helping students think about multiplying and dividing fractions. Area models help students see how we take a portion of another fraction. Fraction bars help students see how in the division process, one fraction partitions another.

Using Strategies for Fraction Operations
Students easily confuse what method to use for operations with fractions. We help students understand the rationale behind multiplication of fractions by stressing the idea that we take a fraction of a fraction (e.g., we take $\frac{1}{3}$ of $\frac{3}{4}$). When we divide fractions, we break up, or partition, one fraction by another.

Problem Solving:
Tools for Measurement and Construction

Key Questions That Guide Student Understanding

- *What are important tools and benchmarks for measuring angles?*
- *How can we construct angles and shapes using a variety of tools?*

Enduring Understandings for Measurement and Construction

Understanding angles and how to measure them is important for geometry and secondary subjects like trigonometry. Knowing basic types of angles provides a foundation for understanding properties of triangles and other polygons. Students need to understand angles in relationship to benchmarks (e.g., 45 degrees, 90 degrees). Protractors, scales, and rulers provide a high level of precision in measurement. Compasses are not only precise in creating arcs, semicircles, and circles, but they allow students to create and investigate angles without the use of protractors. Finally, it is important for students to use benchmarks and approximations as ways of putting exact measurements in context. For example, an 87-degree angle is almost a right angle.

Tools for Understanding Measurement and Construction

Using a Variety of Tools

To students, protractors are impractical tools. They present degrees in both directions, making the tool hard to read. Creating protractors out of paper helps students see the logic of the tool and to focus on important benchmarks, such as 45 and 90 degrees. Protractors are also helpful in showing how to construct benchmark measurements. Such practice makes the transition to actual protractors more sensible to students.

We use the same logic for metric and U.S. Customary rulers. We have used the metric ruler as a way of reinforcing the base 10 system and because it is easier to read. In this unit, we contrast the metric ruler with the US customary ruler and then measure objects using benchmarks such as $\frac{1}{4}$ and $\frac{1}{2}$ inch.

Building Number Concepts:
Multiplication and Division of Fractions

Lesson	Lesson Objectives—Students will:
1	• Multiply whole numbers by fractions using number lines and fraction bars.
2	• Use fraction bars to multiply two fractions together.
3	• Multiply fractions using area models.
4	• Multiply fractions by multiplying across numerator and denominator.
5	• Simplify fractions.
6	
7	• Multiply fractions by multiplying across, then simplify the answers.
8	• Divide fractions.
9	• Use fraction bars to divide one fraction by another fraction.
10	• Divide fractions using the traditional method.
11	
12	• Multiply three fractions, and use the GCF to simplify.
13	• Compare the multiplication and division of fractions.
14	• Analyze common errors in dividing and multiplying fractions.
15	• Review Multiplication and Division of Fractions concepts.
Unit Assessments	End-of-Unit Assessment Performance Assessment

Problem Solving:
Tools for Measurement and Construction

Lesson Objectives—Students will:	Assessment
• Identify lines, line segments, points, and rays using a real-world context.	
• Use a metric ruler to measure line segments.	
• Measure distances on a map to the nearest centimeter.	
• Name angles, and identify their parts.	
	Quiz 1
• Make a protractor out of waxed paper, and use it to measure angles.	
• Measure angles with a protractor.	
• Design a city using a ruler and a protractor.	
• Measure angles in the real-world context of motorbike track design.	
	Quiz 2
• Students will measure using a U.S. customary ruler.	
• Use a compass to draw right angles.	
• Create perpendicular lines with arcs of a compass and bisections of angles.	
• Draw equilateral triangles with a compass.	
• Review Tools for Measurement and Construction concepts.	Unit Review
	End-of-Unit Assessment Performance Assessment

Lesson 1 | ▶ The Concept of Multiplication

Problem Solving: ▶ Points, Line Segments, Lines, and Rays

Lesson Planner

Vocabulary Development

point
line segment
line
ray

Skills Maintenance

Multiplication Facts

Building Number Concepts: ▶ The Concept of Multiplication

We review the concept of multiplication using a number line. We also multiply fractions by whole numbers using fraction bars.

Objective

Students will multiply whole numbers by fractions using number lines and fraction bars.

Problem Solving: ▶ Points, Line Segments, Lines, and Rays

We introduce important geometry terms: points, line segments, lines, and rays. To explain these terms, we use the context of a surveyor's job.

Objective

Students will identify lines, line segments, points, and rays using a real-world context.

Homework

Students multiply whole numbers by fractions using number lines and fraction bars. In Distributed Practice, students practice operations with whole numbers so they can continue to improve their skills.

Lesson 1 | Skills Maintenance

Name ______________________ Date ______________________

Skills Maintenance
Multiplication Facts

Activity 1

Solve the multiplication facts.

1. 3 • 7 = 21	2. 4 • 8 = 32	3. 5 • 6 = 30
4. 2 • 9 = 18	5. 7 • 7 = 49	6. 5 • 7 = 35
7. 8 • 7 = 56	8. 9 • 9 = 81	9. 4 • 4 = 16
10. 6 • 3 = 18	11. 6 • 9 = 54	12. 7 • 5 = 35
13. 8 • 8 = 64	14. 4 • 3 = 12	15. 6 • 2 = 12

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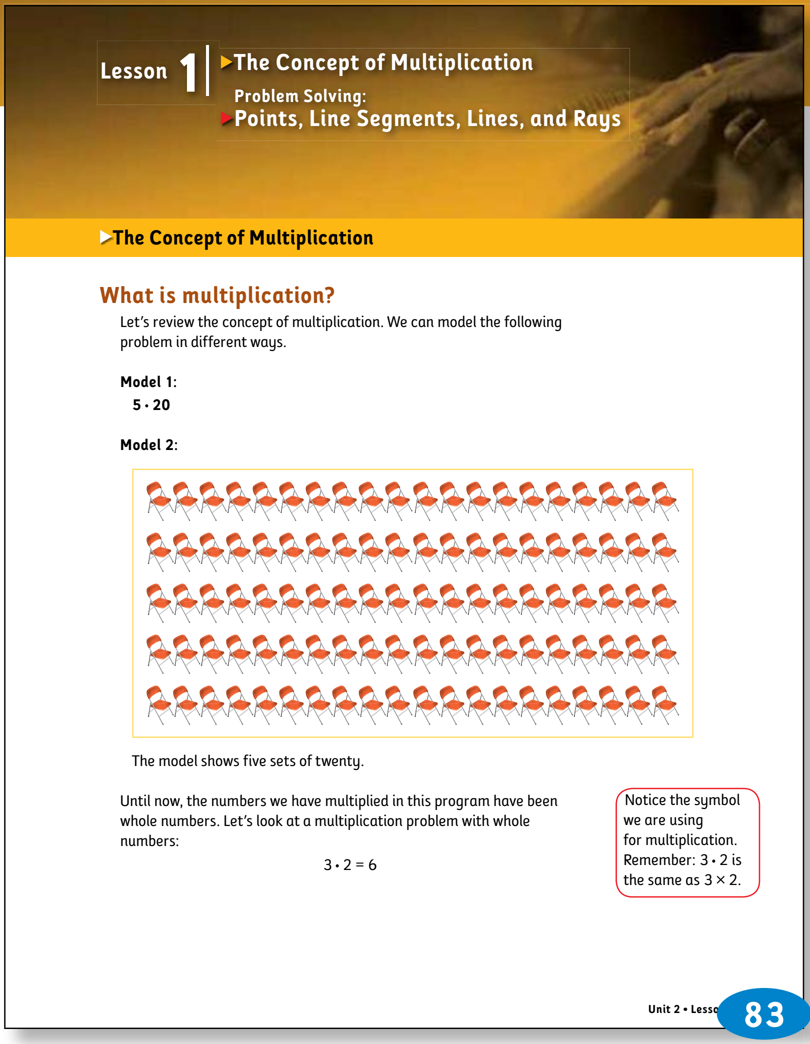
Skills Maintenance

Multiplication Facts

(*Interactive Text*, page 46)

Activity 1

Students solve multiplication facts.


Lesson 1 | ▶The Concept of Multiplication
Problem Solving:
▶Points, Line Segments, Lines, and Rays

▶The Concept of Multiplication

What is multiplication?

Let's review the concept of multiplication. We can model the following problem in different ways.

Model 1:
5 • 20

Model 2:

The model shows five sets of twenty.

Until now, the numbers we have multiplied in this program have been whole numbers. Let's look at a multiplication problem with whole numbers:

3 • 2 = 6

Notice the symbol we are using for multiplication. Remember: 3 • 2 is the same as 3 × 2.

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Building Number Concepts:
▶ The Concept of Multiplication

What is multiplication?

(*Student Text*, pages 83–86)

Connect to Prior Knowledge

Begin by reminding students about multiplication with whole numbers. Ask students to think of different situations that might require us to use multiplication.

Link to Today's Concept

Tell students that in today's lesson, we look at multiplication as a number of sets of another number.

Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate the concept of multiplication in one of the following ways:

mBook: Use the *mBook Teacher Edition* for *Student Text*, pages 83–84. m

Overhead Projector: Reproduce *Student Text*, pages 83–84, on a transparency.

- Display **5 • 20**. Ask students to think about situations in everyday life that require multiplication. Ask them where we would use a problem like 5 • 20 in the real world.
- Guide students to the illustration of the rows of chairs. In this example, we set up five rows of chairs for a concert with 20 chairs in each row. If we multiply 5 by 20, the product will be the total number of chairs for the concert. m
- Now ask for volunteers to give more examples of when we might need to use multiplication. Prompt students to explain why multiplication was needed in the example.

Listen for:

- Examples that represent multiplication.
- Key words such as of, sets of, and groups of. For example, the chairs are five sets of 20.

- Remind students that they already know how to multiply whole numbers. Display the problem **3 • 2 = 6.** Point out that 3 • 2 = 6 is the same as 3 × 2 = 6. Tell students that in higher level math, we often use x as a variable, so we need to use a different symbol for multiplication to avoid confusion. m

What is multiplication? *(continued)*

Demonstrate

Multiply Using a Number Line

- Discuss multiplication using a number line. Show students how to represent whole-number multiplication on the number line using the problem **3 • 2 = 6**.
- Display the number line, and count out three sets of two. Point out to students that when we take a unit of 2 three times, you get 6. Another way to say this is 3 sets of 2.

Check for Understanding

Engagement Strategy: Think Tank

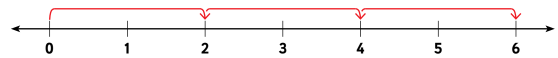
Distribute pieces of paper to students, and have them write their names on the papers. Then write the problem **2 • 5 = 10** on the board. Have students draw a number line to represent the multiplication problem. When students finish, collect their papers, and put them into a container. Draw a paper out, and share the answer with the class. If it is correct, congratulate the student. Invite the student to explain the answer with the class.

Demonstrate

- Next have students look at **Example 1**. We demonstrate a fraction multiplied by a whole number using a number line and fraction bars. The problem is $\mathbf{\frac{1}{2} \cdot 2 = 1}$.
- Tell students that we start by representing the second number in the problem. We start with the whole number 2 on the number line and take half of it to get an answer of 1.
- Look at **Example 2**. This example illustrates a fraction times a whole number using a number line and fraction bars. The problem is $\mathbf{\frac{2}{3} \cdot 3 = 2}$.

Multiply Using a Number Line

This is what 3 • 2 = 6 looks like on a number line.



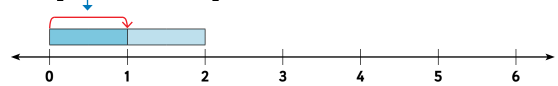
We see that when we multiply a unit of 2 three times, we get 6. This is the same as 3 sets of 2.

Now let's look at some examples where a whole number is multiplied by a fraction.

Example 1

Multiply using a number line.

$\frac{1}{2}$ of 2 = 1 $\frac{1}{2} \cdot 2 = 1$

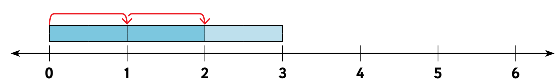


We start with the second number in the problem. In this case, we take the number 2 and place it on the number line. When we take $\frac{1}{2}$ of 2, we get 1. That is the same as saying we will take $\frac{1}{2}$ of a set of 2.

Example 2

Multiply using a number line.

$\frac{2}{3} \cdot 3$



We start with the number 3 on the number line. When we take $\frac{2}{3}$ of 3, we get 2. We are taking $\frac{2}{3}$ of a set of 3.

$\frac{2}{3} \cdot 3 = 2$

Remember that the multiplication sign means "of."

- Refer to the fraction bars for each step. We start with the number 3 on the number line. Then we take $\frac{2}{3}$ of it. Count out two of the rectangles.
- Remind students that the multiplication sign means "of." For example, $\mathbf{\frac{2}{3} \cdot 3}$ is the same as saying "$\frac{2}{3}$ of a set of 3s."

Demonstrate

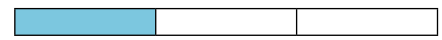
Multiplication Using Fraction Bars

- Next explain how we use fraction bars to represent the multiplication of a whole number by a fraction. The problem is $\mathbf{2 \cdot \frac{1}{3}}$. This means we need two sets of $\frac{1}{3}$. We shade one fraction bar to represent $\frac{1}{3}$. We need two sets of these fraction bars.
- Explain that we shade another fraction bar to represent $\frac{1}{3}$. We have two fraction bars that are $\frac{1}{3}$. We combine them and get $\frac{2}{3}$.

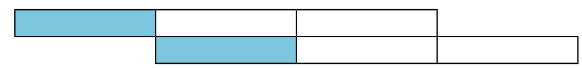
Multiply Using Fraction Bars

Let's look at multiplying a whole number by a fraction using fraction bars. Multiply $2 \cdot \frac{1}{3}$.

The fraction bar for $\frac{1}{3}$ looks like this:

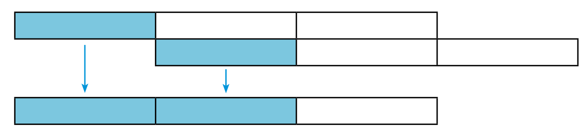


The problem $2 \cdot \frac{1}{3}$ means we are taking 2 sets of $\frac{1}{3}$, or two of the fraction bars above.



Now that we have the problem $2 \cdot \frac{1}{3}$ set up with fraction bars, let's look at the answer. Again, we show the two fraction bars of $\frac{1}{3}$.

Notice that each fraction bar has 1 part shaded. We combine the shaded parts to get our answer.



Now we have a fraction bar with two parts shaded. It represents $\frac{2}{3}$.

The answer is $2 \cdot \frac{1}{3} = \frac{2}{3}$.

What is multiplication? *(continued)*

Demonstrate

- Next have students look at page 86 of the *Student Text*. **Example 3** is another problem that uses fraction bars. In the problem $\mathbf{3 \cdot \frac{1}{4}}$, we shade a fraction bar to represent $\frac{1}{4}$. We need three of these.
- Point out the three fraction bars to represent the three sets of $\frac{1}{4}$. We combine them and get $\frac{3}{4}$: $\mathbf{3 \cdot \frac{1}{4} = \frac{3}{4}}$.
- Ask students to summarize multiplication of a fraction by a whole number.

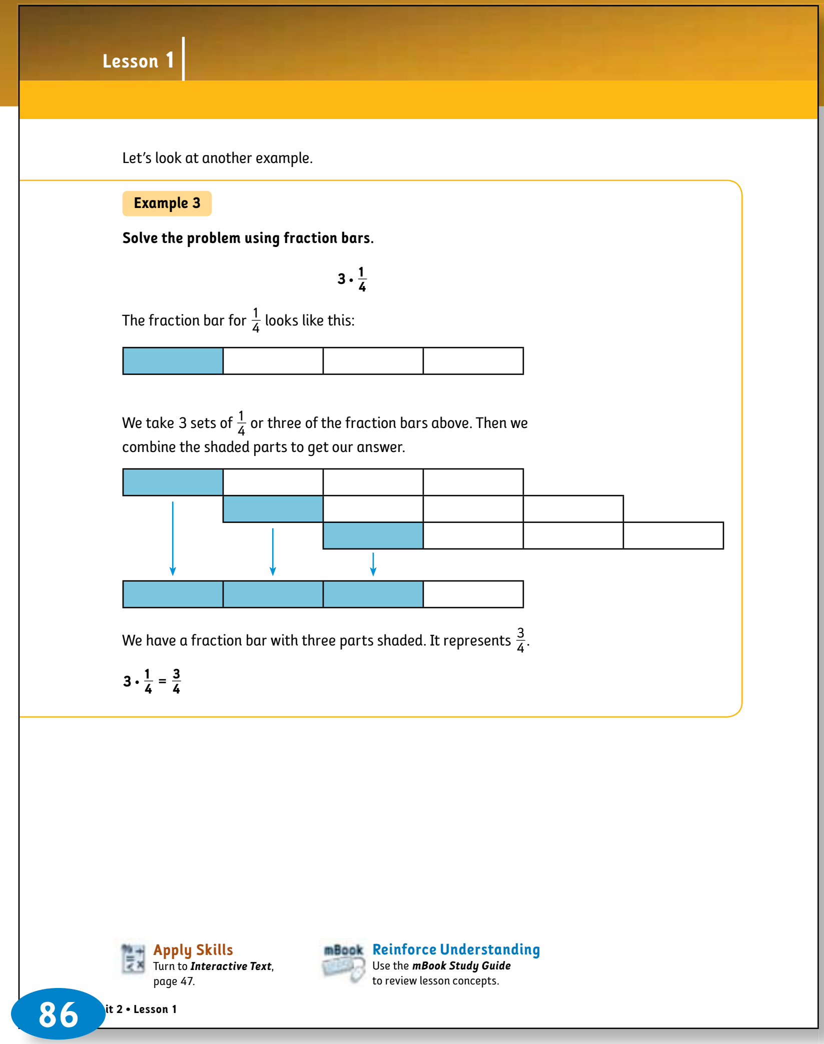
Listen for:

- *You think of it as a fraction of a number. For example, $\frac{1}{2} \cdot 4$ means half of four.*
- *You can use a number line to represent the multiplication. You show the whole number and then show how to take a fraction of it.*
- *You can use fraction bars to represent the multiplication. You shade a fraction bar to represent the fraction and then you see how many you need.*

Check for Understanding

Engagement Strategy: Look About

Write the problem $\mathbf{2 \cdot \frac{2}{5}}$ on the board. Tell students that they will use fraction bars to solve the problem with the help of the whole class ($2 \cdot \frac{2}{5} = \frac{4}{5}$). Students draw fraction bars and write their solutions in large writing on a piece of paper or a dry erase board. When students finish their work, they should hold up their answers for everyone to see.


Lesson 1

Let's look at another example.

Example 3

Solve the problem using fraction bars.

$3 \cdot \frac{1}{4}$

The fraction bar for $\frac{1}{4}$ looks like this:

We take 3 sets of $\frac{1}{4}$ or three of the fraction bars above. Then we combine the shaded parts to get our answer.

We have a fraction bar with three parts shaded. It represents $\frac{3}{4}$.

$3 \cdot \frac{1}{4} = \frac{3}{4}$

Apply Skills Turn to ***Interactive Text***, page 47.

Reinforce Understanding Use the ***mBook Study Guide*** to review lesson concepts.

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If students are not sure about the answer, prompt them to look about at other students' solutions to help with their thinking. Review the answers after all students have held up their solutions.

Reinforce Understanding

If students need further practice, have them draw fraction bars to solve the following problems:

$\mathbf{5 \cdot \frac{1}{6}}$ ($5 \cdot \frac{1}{6} = \frac{5}{6}$)

$\mathbf{3 \cdot \frac{1}{3}}$ ($3 \cdot \frac{1}{3} = \frac{3}{3}$)

Apply Skills

(*Interactive Text*, page 47)

Have students turn to page 47 in the *Interactive Text* and complete the activities.

Activity 1

Students rewrite whole numbers as fractions. Ask students to think about what we learned about the meaning of numerator and denominator. Be sure students see that we are able to rewrite any whole number as a fraction with 1 in the denominator. They use this idea extensively in upcoming lessons.

Activity 2

Students look at shaded fraction bars that represent a multiplication problem involving a fraction and a whole number and tell what problem is represented.

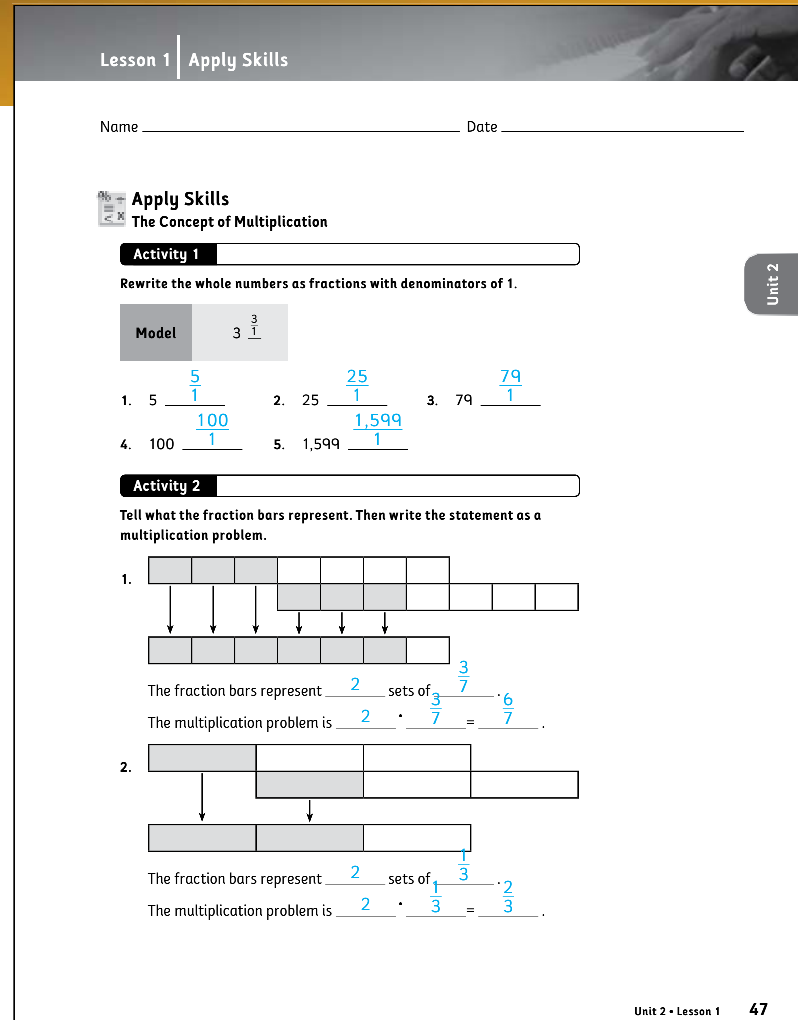
Monitor students' work as they complete the activities.

Watch for:

- Can students rewrite a whole number as a fraction?
- Can students recognize the problem represented by the fraction bars?

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.



Lesson 1 | Apply Skills

Unit 2

Name ____________ Date ____________

Apply Skills

The Concept of Multiplication

Activity 1

Rewrite the whole numbers as fractions with denominators of 1.

Model	$3 \quad \frac{3}{1}$

1. $5 \quad \frac{5}{1}$ 2. $25 \quad \frac{25}{1}$ 3. $79 \quad \frac{79}{1}$

4. $100 \quad \frac{100}{1}$ 5. $1{,}599 \quad \frac{1{,}599}{1}$

Activity 2

Tell what the fraction bars represent. Then write the statement as a multiplication problem.

1. The fraction bars represent 2 sets of $\frac{3}{7}$.
The multiplication problem is $2 \cdot \frac{3}{7} = \frac{6}{7}$.

2. The fraction bars represent 2 sets of $\frac{1}{3}$.
The multiplication problem is $2 \cdot \frac{1}{3} = \frac{2}{3}$.

Unit 2 • Lesson 1 47

Problem Solving: Points, Line Segments, Lines, and Rays

What are points and line segments?

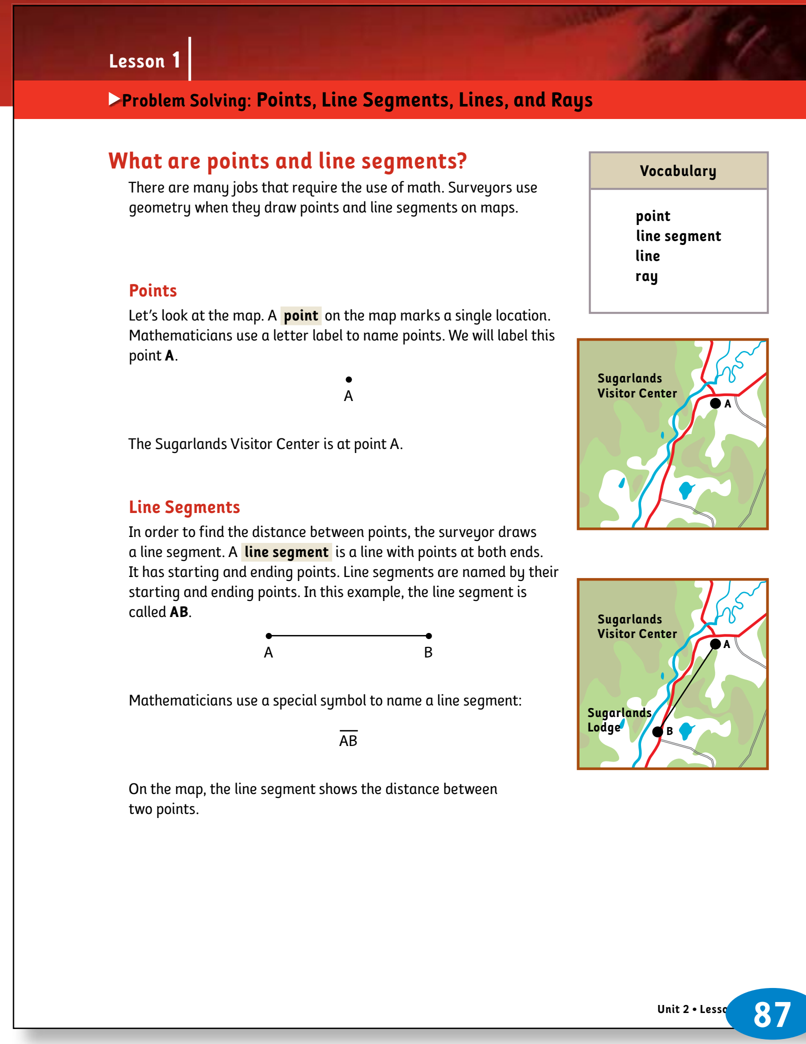
(*Student Text*, page 87)

Build Vocabulary

Start by asking students if they know what surveyors do. Explain that surveyors use math in their jobs. Explain that a **point** marks a single location, and a **line segment** is a line with points at both ends. Surveyors draw points and line segments on maps.

Demonstrate

- Show students the map on page 87 of the *Student Text*. Explain that single locations are labeled by points on a map. We use a letter to label the point. Refer to the map, and explain that the Sugarlands Visitor Center is at Point *A* on the map.
- Next explain that a line segment is a line with two points at the end. Walk across the room and explain to students that you started at Point *A* and walked in a line and stopped at Point *B*. The line in between the two points is the line segment.
- Point out that the same is true on the map. The line segment from Point *A* to Point *B* on the map is the distance between the Sugarlands Visitors Center and the Sugarlands Lodge.
- Explain that we use a symbol to name a line segment: $\overline{AB}$.



Lesson 1

Problem Solving: Points, Line Segments, Lines, and Rays

What are points and line segments?

There are many jobs that require the use of math. Surveyors use geometry when they draw points and line segments on maps.

Vocabulary: point, line segment, line, ray

Points

Let's look at the map. A **point** on the map marks a single location. Mathematicians use a letter label to name points. We will label this point **A**.

A

The Sugarlands Visitor Center is at point A.

Line Segments

In order to find the distance between points, the surveyor draws a line segment. A **line segment** is a line with points at both ends. It has starting and ending points. Line segments are named by their starting and ending points. In this example, the line segment is called **AB**.

A B

Mathematicians use a special symbol to name a line segment:

$\overline{AB}$

On the map, the line segment shows the distance between two points.

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What are lines and rays?

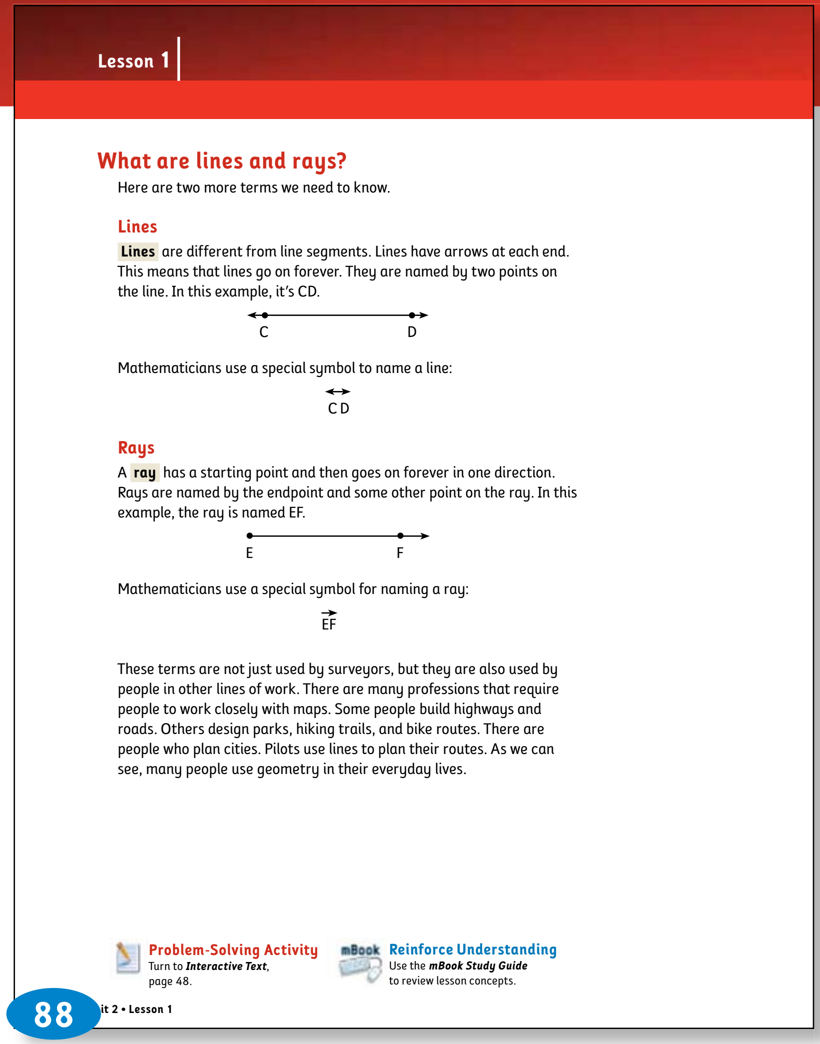
(*Student Text*, page 88)

Demonstrate

- Discuss the terms **lines** and **ray**. Explain that lines and rays are both named by points, but they are not line segments.
- Read page 88 of the *Student Text* with students. Explain that lines are different from line segments. Line segments have a starting point and an ending point. Lines go on forever, and are represented by an arrow at each end.
- Show students the line CD and point out the symbol: $\overleftrightarrow{CD}$.
- Explain the definition of a ray. A ray is like a line segment because it has a starting point, but it does not have an ending point. A ray is similar to a line because it goes on forever, but only in one direction.
- Show students the ray EF, and point out the symbol used to write it: $\overrightarrow{EF}$.
- Summarize by explaining that mathematicians use this standard notation for naming and writing about lines and rays. Finally, tell students that these terms and notations are used in many different professions in addition to surveying.
- Ask students to summarize the vocabulary they learned.

Listen for:

- *A point is a dot that represents a location.*
- *A line segment has a beginning and an end. You call the line segment by the name of the beginning and ending points.*
- *A line goes on forever. It has arrows at each end. You name the line segment by two points on the line.*



Lesson 1

What are lines and rays?

Here are two more terms we need to know.

Lines

Lines are different from line segments. Lines have arrows at each end. This means that lines go on forever. They are named by two points on the line. In this example, it's CD.

Mathematicians use a special symbol to name a line:

$\overleftrightarrow{CD}$

Rays

A **ray** has a starting point and then goes on forever in one direction. Rays are named by the endpoint and some other point on the ray. In this example, the ray is named EF.

Mathematicians use a special symbol for naming a ray:

$\overrightarrow{EF}$

These terms are not just used by surveyors, but they are also used by people in other lines of work. There are many professions that require people to work closely with maps. Some people build highways and roads. Others design parks, hiking trails, and bike routes. There are people who plan cities. Pilots use lines to plan their routes. As we can see, many people use geometry in their everyday lives.

Problem-Solving Activity Turn to *Interactive Text*, page 48.

mBook **Reinforce Understanding** Use the ***mBook Study Guide*** to review lesson concepts.

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- *A ray goes on forever in one direction but has a point at the other end. You call the ray by the end point and another point on the ray.*
- Be sure students have a basic understanding of these terms.

Problem-Solving Activity

(*Interactive Text,* page 48)

Have students turn to *Interactive Text*, page 48, and complete the activity.

Have students take out a blank sheet of lined paper and refer to the map and table as they answer the questions. Once students complete the questions, discuss them in class. Be sure students use the new vocabulary—points and line segments—as they discuss their work.

Monitor students' work as they complete the activity.

Watch for:

- Can students find distances in the table?
- Can students compute the length of the whole trail based on the map and the table of data?
- Can students determine the length of part of the trail?
- Can students identify the need for subtraction?

mBook Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

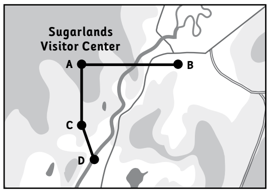
Lesson 1 | Problem-Solving Activity

Name ______________________ Date ______________________

Problem-Solving Activity

Points, Line Segments, Lines, and Rays

On the map, line segments show different sections of the hiking trail. The distances and labels for each section are given in the table. Answer the questions using the map and table.



From	To	Label	Distance
Visitor Center	Gatlinburg	$\overline{AB}$	$\frac{3}{4}$ mile
Visitor Center	Cove Mountain Point	$\overline{AC}$	$\frac{1}{2}$ mile
Cove Mountain Point	Fighting Creek Gap	$\overline{CD}$	$\frac{1}{4}$ mile

1. How far is it from the visitor center to Gatlinburg? $\frac{3}{4}$ mile
2. If you were to hike the whole trail, how far would you hike? $1\frac{1}{2}$ mile
3. If you were to just hike the trail from the visitor center to Fighting Creek Gap, how far would you hike? $\frac{3}{4}$ mile
4. How much longer is the trail from the visitor center to Gatlinburg than from the visitor center to Cove Mountain Point? $\frac{1}{4}$ mile

mBook **Reinforce Understanding**
Use the **mBook *Study Guide*** to review lesson concepts.

48 Unit 2 • Lesson 1

Homework

Go over the instructions for each activity on page 89 of the *Student Text*.

Activity 1

Students solve simple multiplication problems with a fraction and a whole number using number lines.

Activity 2

Students show each multiplication problem with a fraction bar.

Activity 3 • Distributed Practice

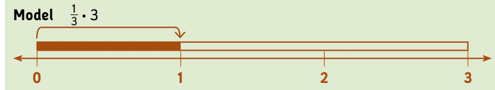
Students practice operations with whole numbers so they can continue to improve their skills.

Homework

Activity 1

Draw a number line to help you multiply by a fraction.

Model $\frac{1}{3} \cdot 3$



1. $\frac{1}{2} \cdot 4$
2. $\frac{1}{3} \cdot 6$
3. $\frac{1}{4} \cdot 8$
4. $\frac{1}{2} \cdot 12$
5. $\frac{2}{3} \cdot 10$
6. $\frac{3}{4} \cdot 12$

See Additional Answers below.

Activity 2

Show each problem using fraction bars.

1. $2 \cdot \frac{1}{3}$
2. $3 \cdot \frac{1}{5}$
3. $2 \cdot \frac{1}{4}$
4. $4 \cdot \frac{1}{8}$
5. $4 \cdot \frac{1}{6}$
6. $3 \cdot \frac{1}{8}$

See Additional Answers below.

Activity 3 • Distributed Practice

Solve.

1. $\begin{array}{r} 352 \\ +\ 19 \\ \hline 371 \end{array}$
2. $\begin{array}{r} 512 \\ -\ 101 \\ \hline 411 \end{array}$
3. $\begin{array}{r} 417 \\ -\ 238 \\ \hline 179 \end{array}$
4. $\begin{array}{r} 31 \\ \times\ 7 \\ \hline 217 \end{array}$

(*Additional Answers continue on Appendix, page A2.*)

Lesson 2 ▶ Finding a Fraction of a Fraction

Problem Solving: ▶ Measuring With a Metric Ruler

Lesson Planner

Vocabulary Development

metric ruler
millimeters
centimeters

Skills Maintenance

Fractions, Rays, Line Segments, Points, and Lines

Building Number Concepts: ▶ Finding a Fraction of a Fraction

In this lesson, we look at multiplying two fractions using fraction bars. Fraction bars help us make sense of multiplication with fractions. They show how we take fractional parts of fractional parts. When we multiply fractions, the product is usually smaller than either of the fractions we start with. This differs from whole numbers where the product is larger.

Objective

Students will use fraction bars to multiply two fractions together.

Problem Solving: ▶ Measuring With a Metric Ruler

We reintroduce the metric ruler and have students measure line segments in millimeters.

Objective

Students will use a metric ruler to measure line segments.

Homework

Students use a fraction bar to solve multiplication problems and draw points, line segments, rays, and lines following the instructions in the activity. In Distributed Practice, students practice whole-number operations.

Name ______________________ Date ______________

Skills Maintenance
Fractions

Activity 1

Write the whole numbers as fractions.

Model	2 $\frac{2}{1}$

1. 5 $\frac{5}{1}$
2. 17 $\frac{17}{1}$
3. 100 $\frac{100}{1}$
4. 50 $\frac{50}{1}$
5. 27 $\frac{27}{1}$

Rays, Line Segments, Points, and Lines

Activity 2

Write the correct term.

1. A B — Ray Line Segment Point Line — Ray
2. D E — Ray Line Segment Point Line — Line
3. X Y — Ray Line Segment Point Line — Line Segment
4. A — Ray Line Segment Point Line — Point
5. C D — Ray Line Segment Point Line — Ray

Unit 2

Skills Maintenance

Fractions, Rays, Line Segments, Points, and Lines
(*Interactive Text*, page 49)

Activity 1

Students rewrite whole numbers as fractions.

Activity 2

Students identify the correct description for each item shown. The choices are ray, line segment, point, and line.

Building Number Concepts:
▶ Finding a Fraction of a Fraction

How do we use fraction bars to find a fraction of a fraction?

(*Student Text*, pages 90–91)

Connect to Prior Knowledge

Begin by writing the following two problems on the board:

$\frac{1}{2} \cdot 6$

$2 \cdot 6$

Work through the problems with students. Ask them to comment about how the answers differ for these two multiplication problems.

Listen for:

- *When you multiply $\frac{1}{2} \cdot 6$, you take half of 6. The answer is 3.*
- *When you multiply $2 \cdot 6$, you double 6. The answer is 12.*
- *When you multiply 6 by a fraction, the answer is smaller than 6.*
- *When you multiply 6 by a whole number, the answer is bigger than 6.*

Link to Today's Concept

Tell students that in today's lesson, we look at the answers to fraction problems and compare them to whole-number answers. Have students think about the size of the product as they work through the problems in this lesson.

Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate how to multiply two fractions using fraction bars in one of the following ways:

mBook: Use the *mBook Teacher Edition* for *Student Text*, page 90.

Lesson 2 ▶ Finding a Fraction of a Fraction

Problem Solving:

▶ Measuring With a Metric Ruler

▶ Finding a Fraction of a Fraction

How do we use fraction bars to find a fraction of a fraction?

In the last lesson, we looked at the multiplication of whole numbers by fractions. When we multiply $\frac{1}{2} \cdot 2$, we find $\frac{1}{2}$ of 2. When we multiply $\frac{2}{3} \cdot 3$, we find $\frac{2}{3}$ of 3.

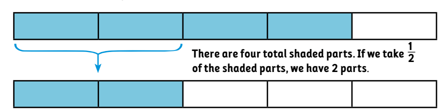
Now let's multiply a fraction by another fraction. When we multiply $\frac{1}{2} \cdot \frac{4}{5}$, we find $\frac{1}{2}$ of $\frac{4}{5}$.

Let's complete this problem using fraction bars.

$$\frac{1}{2} \cdot \frac{4}{5}$$

This fraction bar shows $\frac{4}{5}$.

Now we focus on just the shaded part and we take half of this.



We know that the 2 parts are fifths, so the answer to the problem is

$$\frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}$$

The answer is smaller than the starting number, $\frac{4}{5}$. It's smaller because we are only finding a part of the fraction $\frac{4}{5}$. We are taking $\frac{1}{2}$ of $\frac{4}{5}$.

90 it 2 • Lesson 2

Overhead Projector: Reproduce the fraction bars on a transparency, and modify as discussed.

Board: Copy the fraction bars on the board, and modify as discussed.

- Display the problem $\frac{1}{2} \cdot \frac{4}{5}$.
- Display the fraction bar for $\frac{4}{5}$.
- Explain that when we take half of it, we take away two of the shaded sections. That leaves us with two parts, or $\frac{2}{5}$.
- Show the answer $\frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}$. Point out how the answer compares in size with the two starting numbers, $\frac{1}{2}$ and $\frac{4}{5}$. The fraction $\frac{2}{5}$ is smaller than both $\frac{1}{2}$ and $\frac{4}{5}$.

How do we use fraction bars to find a fraction of a fraction? *(continued)*

Demonstrate

- Direct students' attention to **Example 1** on page 91 of the *Student Text*. Walk through the example as outlined. Here students take $\frac{1}{5}$ of $\frac{5}{8}$. Again we start with the second fraction, $\frac{5}{8}$.
- Point out that we represent it using fraction bars by shading five parts out of eight. Then we need to take $\frac{1}{5}$ of it. There are five parts, so one part is $\frac{1}{5}$. The answer to $\frac{1}{5} \cdot \frac{5}{8}$ is $\frac{1}{8}$. Again notice the size of the product is smaller with respect to the other two numbers.
- Ask students to describe in their own words why the product is smaller when we multiply a fraction by another fraction.

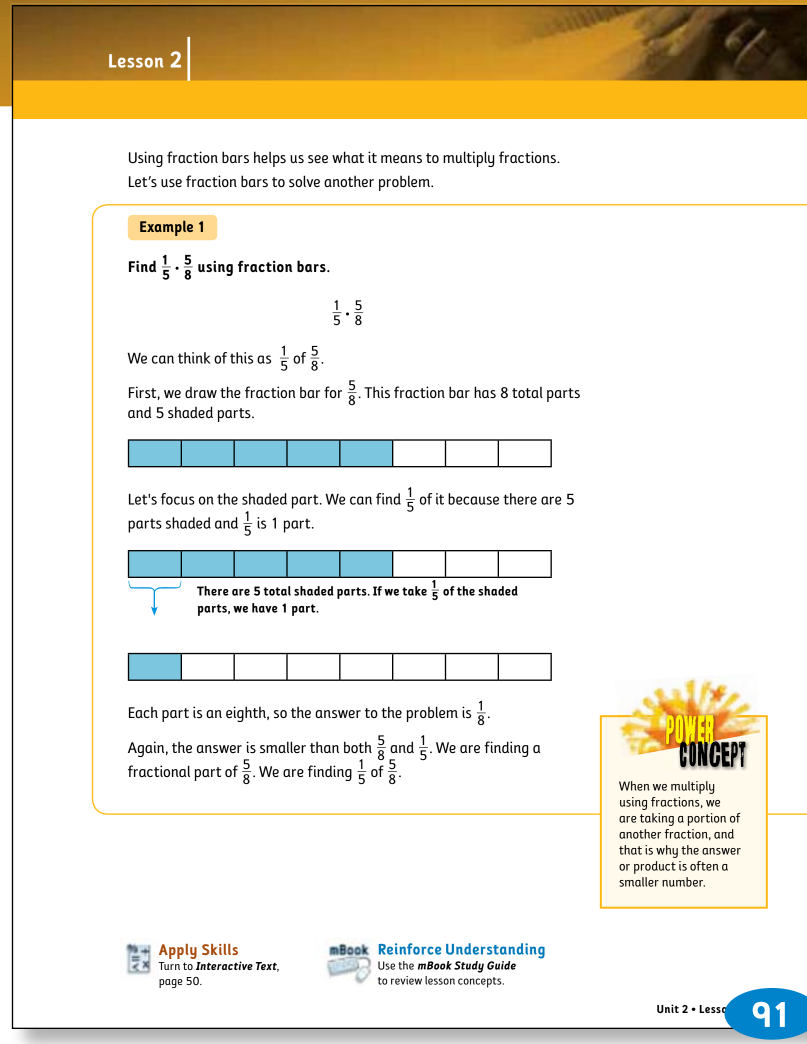
Listen for:

- *You have to phrase it differently. Instead of a number times another number, you need to think of it as a fraction of a fraction.*

Check for Understanding

Engagement Strategy: Think, Think

Write the problem $\frac{2}{3} \cdot \frac{3}{4}$ on the board. Have students solve the problem using fraction bars $\left(\frac{2}{3} \cdot \frac{3}{4} = \frac{6}{12}\right)$. Tell students that you will call on one of them for an answer. Give students time to think of their answer. Then call on a student to answer. If that student is correct, invite that student to draw the fraction bars on the board and explain the answer.


Lesson 2

Using fraction bars helps us see what it means to multiply fractions. Let's use fraction bars to solve another problem.

Example 1

Find $\frac{1}{5} \cdot \frac{5}{8}$ using fraction bars.

$$\frac{1}{5} \cdot \frac{5}{8}$$

We can think of this as $\frac{1}{5}$ of $\frac{5}{8}$.

First, we draw the fraction bar for $\frac{5}{8}$. This fraction bar has 8 total parts and 5 shaded parts.

Let's focus on the shaded part. We can find $\frac{1}{5}$ of it because there are 5 parts shaded and $\frac{1}{5}$ is 1 part.

There are 5 total shaded parts. If we take $\frac{1}{5}$ of the shaded parts, we have 1 part.

Each part is an eighth, so the answer to the problem is $\frac{1}{8}$.

Again, the answer is smaller than both $\frac{5}{8}$ and $\frac{1}{5}$. We are finding a fractional part of $\frac{5}{8}$. We are finding $\frac{1}{5}$ of $\frac{5}{8}$.

POWER CONCEPT

When we multiply using fractions, we are taking a portion of another fraction, and that is why the answer or product is often a smaller number.

Apply Skills Turn to ***Interactive Text***, page 50.

Reinforce Understanding Use the ***mBook Study Guide*** to review lesson concepts.

Unit 2 • Lesso 91

Apply Skills

(*Interactive Text*, page 50)

Have students turn to *Interactive Text*, page 50, and complete the activity.

Activity 1

Students multiply fractions using fraction bars. Have students use *Student Text*, page 91, if they need a model. Monitor students' work as they complete the activity.

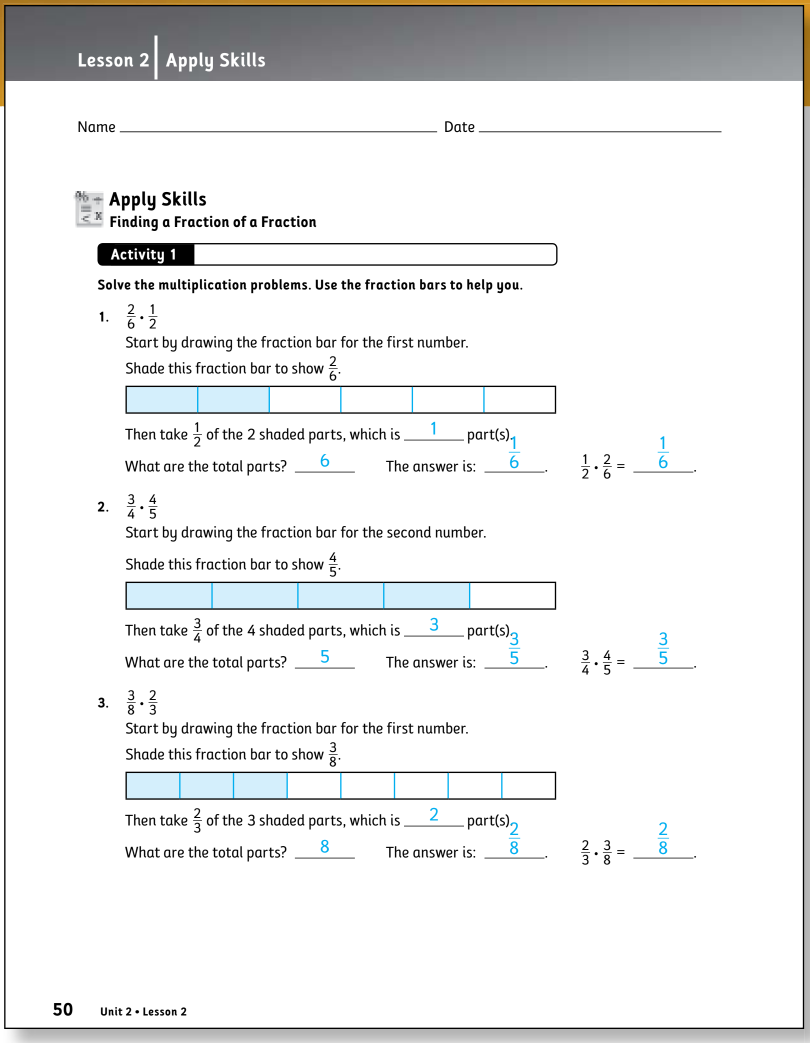
Watch for:

- Can students shade each fraction bar to represent the appropriate fraction?
- Do students understand how to take a fraction of the part that is shaded?
- Do students recognize the total parts, or denominator, of the answer?

mBook

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.



Name ______________________ Date ______________

Apply Skills
Finding a Fraction of a Fraction

Activity 1

Solve the multiplication problems. Use the fraction bars to help you.

1. $\frac{2}{6} \cdot \frac{1}{2}$
Start by drawing the fraction bar for the first number.
Shade this fraction bar to show $\frac{2}{6}$.
Then take $\frac{1}{2}$ of the 2 shaded parts, which is 1 part(s).
What are the total parts? 6 The answer is: $\frac{1}{6}$. $\frac{1}{2} \cdot \frac{2}{6} = \frac{1}{6}$.

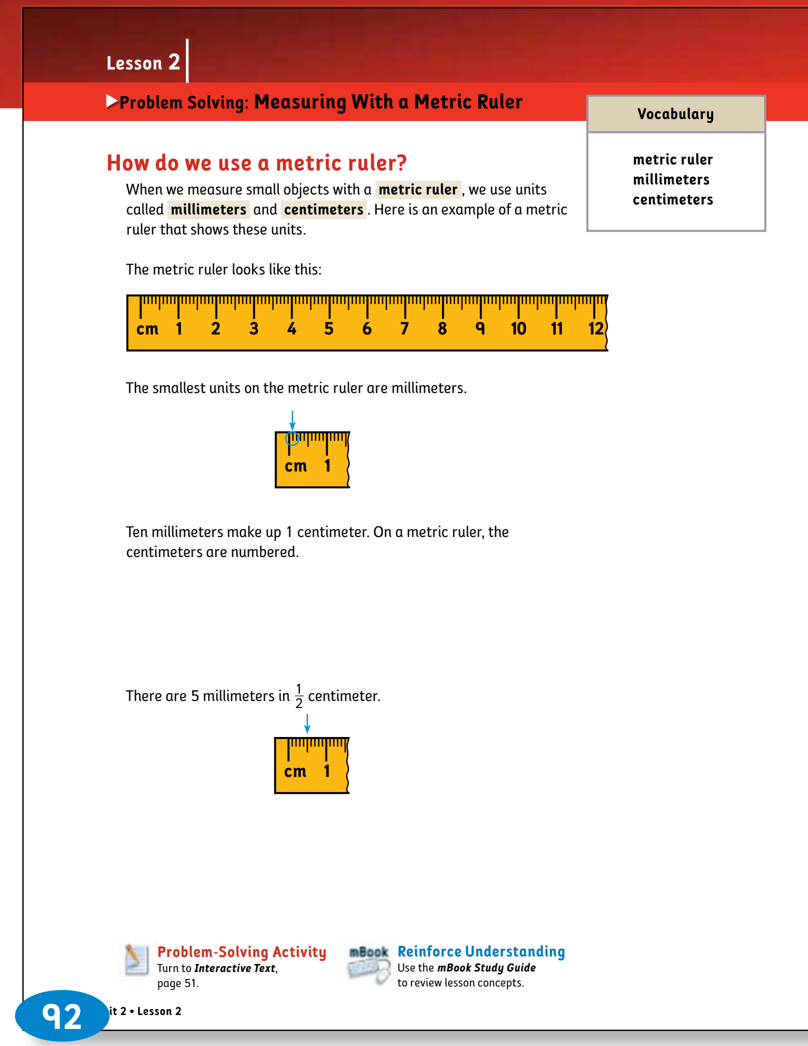
2. $\frac{3}{4} \cdot \frac{4}{5}$
Start by drawing the fraction bar for the second number.
Shade this fraction bar to show $\frac{4}{5}$.
Then take $\frac{3}{4}$ of the 4 shaded parts, which is 3 part(s).
What are the total parts? 5 The answer is: $\frac{3}{5}$. $\frac{3}{4} \cdot \frac{4}{5} = \frac{3}{5}$.

3. $\frac{3}{8} \cdot \frac{2}{3}$
Start by drawing the fraction bar for the first number.
Shade this fraction bar to show $\frac{3}{8}$.
Then take $\frac{2}{3}$ of the 3 shaded parts, which is 2 part(s).
What are the total parts? 8 The answer is: $\frac{2}{8}$. $\frac{2}{3} \cdot \frac{3}{8} = \frac{2}{8}$.

Problem Solving:
▶ Measuring With a Metric Ruler

How do we use a metric ruler?

(*Student Text*, page 92)



▶Problem Solving: Measuring With a Metric Ruler

Vocabulary
metric ruler **millimeters** **centimeters**

How do we use a metric ruler?

When we measure small objects with a **metric ruler**, we use units called **millimeters** and **centimeters**. Here is an example of a metric ruler that shows these units.

The metric ruler looks like this:

The smallest units on the metric ruler are millimeters.

Ten millimeters make up 1 centimeter. On a metric ruler, the centimeters are numbered.

There are 5 millimeters in $\frac{1}{2}$ centimeter.

Problem-Solving Activity
Turn to *Interactive Text*, page 51.

mBook Reinforce Understanding
Use the *mBook Study Guide* to review lesson concepts.

Build Vocabulary

Explain that we can measure line segments with a **metric ruler**. We use metric rulers to measure in **millimeters** and **centimeters** because they are based on powers of 10. Using a metric ruler makes the numbers easier to work with, and measurements of smaller objects result in whole numbers. When we measure small objects in inches, we end up with fractions (e.g., $\frac{1}{4}$ inches, $\frac{1}{2}$ inches) or mixed numbers (e.g., $1\frac{1}{4}$ inches, $2\frac{1}{4}$ inches).

Demonstrate

- Have students look at the metric ruler on page 92 of the *Student Text*.
- Ask students to make observations about the metric ruler.

Listen for:

- *There are really small lines.*
- *Every five units, there's a longer line.*
- *Every 10 units, there's a longer line with a number.*
- *I've worked with a metric ruler before, and I know the little lines are millimeters. The bigger lines with numbers are centimeters.*

- Read through the text, and make sure students understand which lines represent millimeters and which represent centimeters.
- Be sure students do not confuse the longer line that appears every five millimeters with centimeters that appear every 10 millimeters. The line for centimeters is the longest line. The other line is just a helpful tool for counting millimeters by fives.
- You might want to show students a meter stick, if you have one available, and make comparisons between millimeters, centimeters, decimeters (100 millimeters and 10 centimeters), and meters (1,000 millimeters, 100 centimeters, and 10 decimeters).

Problem-Solving Activity

(*Interactive Text*, page 51)

Have students turn to *Interactive Text,* page 51, to complete the activity.

Students need a metric ruler to answer the questions about the subway map.

Monitor students' work as they complete the activity.

Watch for:

- Do students line up the ruler correctly to measure in millimeters?
- Can students locate the landmarks and identify the line segments they are to measure on the map?
- Can students add the measurements to determine the total length of something?
- Can students subtract the measurements to determine the difference in the lengths of two line segments (e.g., how much bigger is one than the other)?

Once students complete the activity, discuss their answers. If they are off by a small amount, discuss the imprecision of measurement. Every measurement is an estimate of sorts; some are more precise than others.

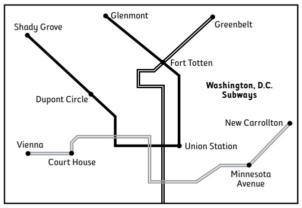
Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide.*

Name ______________________ Date ______________

Problem-Solving Activity
Measuring Line Segments

Look at the map of the subway system in Washington, D.C. Then answer the questions measuring in millimeters with a metric ruler.



1. What is the length of the line segment between Shady Grove and Dupont Circle?

   23 millimeters

2. What is the length of the line segment between Fort Totten and Union Station?

   24 millimeters

3. Measure and add up the lengths of the line segments between Union Station and Dupont Circle. What is the total length?

   35 millimeters

4. How much bigger is the line segment from Shady Grove to Dupont Circle than the line segment from the Courthouse to Vienna?

   11–12 millimeters

Reinforce Understanding
Use the **mBook** ***Study Guide*** to review lesson concepts.

Homework

Go over the instructions on page 93 of the *Student Text* for each part of the homework.

Activity 1

Students look at the fraction bar to determine the product of the multiplication problems involving fractions.

Activity 2

Students look at another fraction bar to determine the product of the multiplication problems involving fractions.

Activity 3

Students draw points, line segments, rays, and lines and label them appropriately.

Activity 4 • Distributed Practice

Students solve whole-number operations.

Homework

Activity 1

Use the fraction bar to help you solve the problems.

1. $\frac{1}{2} \cdot \frac{4}{6}$ $\frac{2}{6}$
2. $\frac{1}{3} \cdot \frac{3}{6}$ $\frac{1}{6}$
3. $2 \cdot \frac{1}{6}$ $\frac{2}{6}$

Activity 2

Use the fraction bar to help you solve the problems.

1. $\frac{1}{2} \cdot \frac{2}{4}$ $\frac{1}{4}$
2. $\frac{1}{3} \cdot \frac{3}{4}$ $\frac{1}{4}$
3. $2 \cdot \frac{1}{4}$ $\frac{2}{4}$

Activity 3

Draw the points, line segments, rays, and lines. Follow the instructions for labeling.

1. Draw a ray. Label it AB. A B
2. Draw a line segment. Label it CD. C D
3. Draw a point. Label it X. X
4. Draw a line. Label it EF. E F
5. Draw a line segment. Label it MN. M N

Activity 4 • Distributed Practice

Solve.

1. $253 + 28 = 281$
2. $125 - 11 = 114$
3. $173 - 88 = 85$
4. $17 \times 3 = 51$

▶ Multiplying Fractions Using an Area Model

Problem Solving:

▶ Measuring Line Segments

Lesson Planner

Vocabulary Development

area model
area

Skills Maintenance

Whole Numbers and Fractions

Building Number Concepts:

▶ Multiplying Fractions Using an Area Model

We introduce the area model as another representation to help students understand multiplication of fractions. When we use an area model to demonstrate multiplication of fractions, we shade one fraction horizontally and the other fraction vertically; the place where they overlap is the answer.

Objective

Students will multiply fractions using area models.

Problem Solving:

▶ Measuring Line Segments

Students use their metric rulers again to measure distances on a map. Students measure to the nearest centimeter.

Objective

Students will measure distances on a map to the nearest centimeter.

Homework

Students use area models to find products of fractions. In Distributed Practice, students add and subtract fractions and whole-numbers.

Lesson 3 | Skills Maintenance

Name ______________ Date ______________

Skills Maintenance
Whole Numbers and Fractions

Activity 1

Rewrite the whole numbers as fractions.

1. 17 $\frac{17}{1}$
2. 29 $\frac{29}{1}$
3. 42 $\frac{42}{1}$
4. 67 $\frac{67}{1}$
5. 125 $\frac{125}{1}$

52 Unit 2 • Lesson 3

Skills Maintenance

Whole Numbers and Fractions

(*Interactive Text*, page 52)

Activity 1

Students rewrite whole numbers as fractions. Remind students that we rewrite any whole number as a fraction with a denominator of 1.

Building Number Concepts: Multiplying Fractions Using an Area Model

What is an area model?

(*Student Text*, pages 94–96)

Connect to Prior Knowledge

Begin by reminding students of the different methods we have used to multiply. Prompt students to explain methods like using number lines and using fraction bars.

Link to Today's Concept

In today's lesson, students learn a technique for multiplying fractions that involves the **area model** for a rectangle. With this technique, we divide the height of the rectangle by one of the fractional parts and divide the width by the other fractional part. We shade each **area** that we have. The overlapping area is the product.

Build Vocabulary

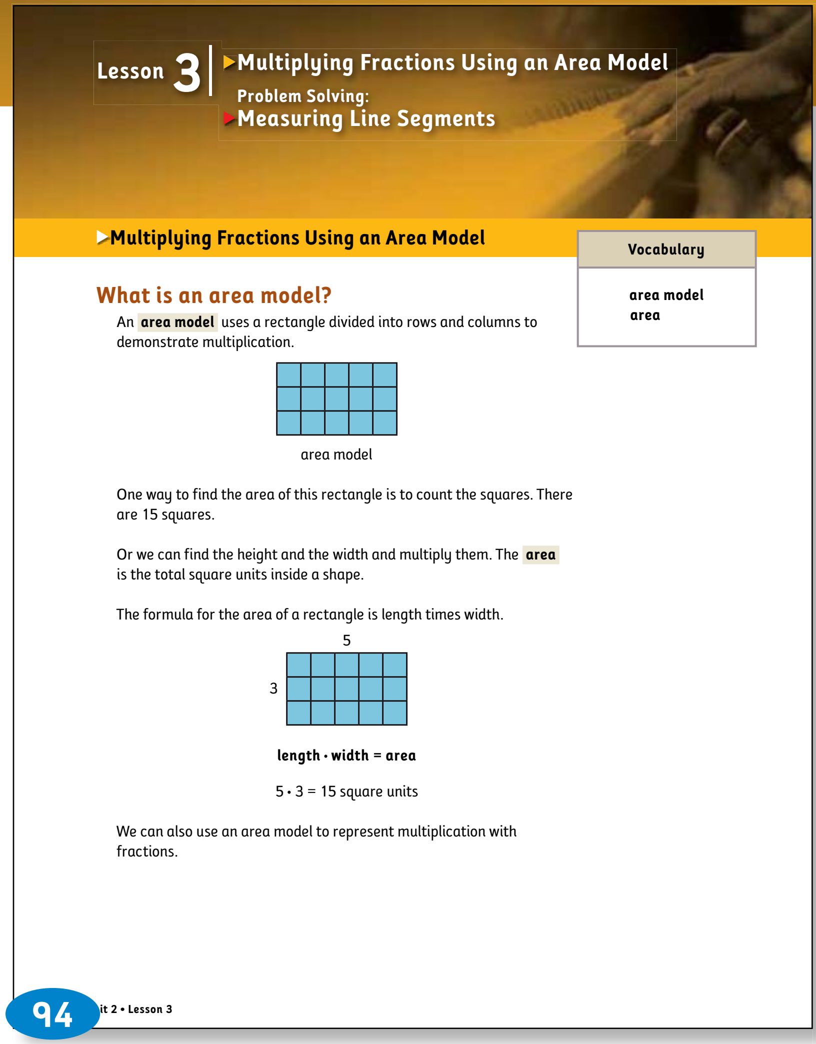
Tell students how an area model can help us multiply. We can find the area of a rectangle by multiplying length times width.

Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate finding the area of a rectangle in one of the following ways:

- **mBook**: Use the *mBook Teacher Edition* for *Student Text*, page 94.
- **Overhead Projector**: Reproduce the rectangle on a transparency, and modify as discussed.
- **Board**: Copy the rectangle on the board, and modify as discussed.



Lesson 3 ▶Multiplying Fractions Using an Area Model

Problem Solving:

▶Measuring Line Segments

▶Multiplying Fractions Using an Area Model

Vocabulary
area model area

What is an area model?

An **area model** uses a rectangle divided into rows and columns to demonstrate multiplication.

area model

One way to find the area of this rectangle is to count the squares. There are 15 squares.

Or we can find the height and the width and multiply them. The **area** is the total square units inside a shape.

The formula for the area of a rectangle is length times width.

5

3

length • width = area

5 • 3 = 15 square units

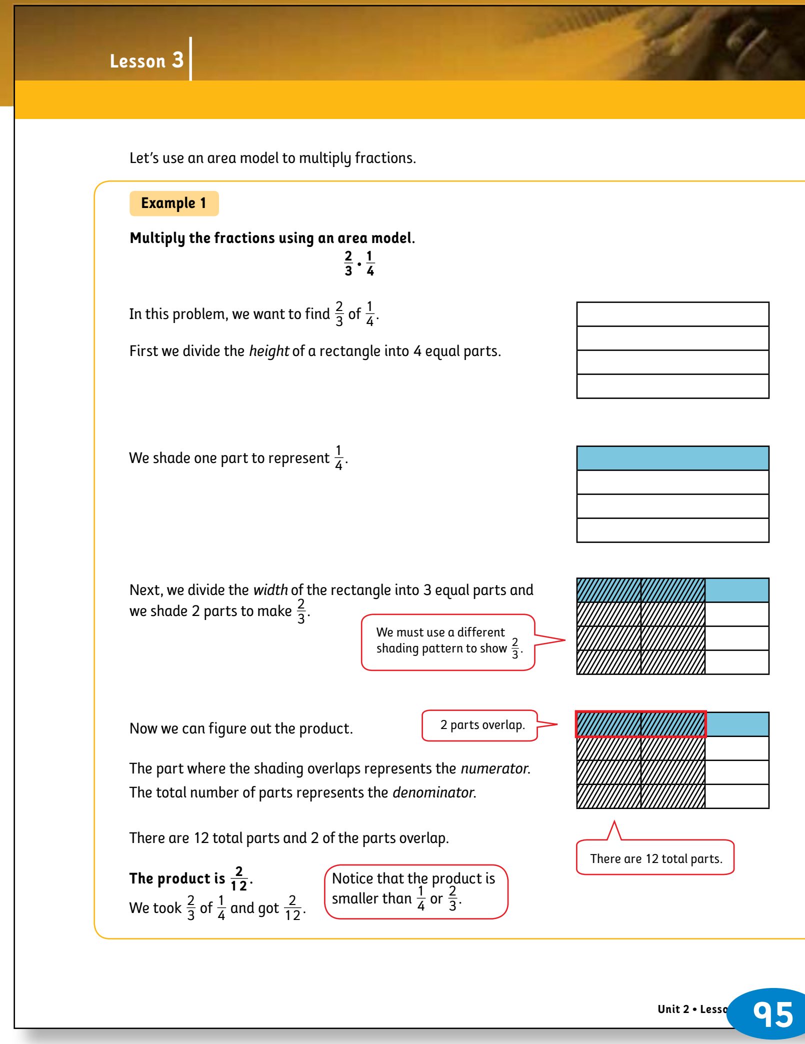
We can also use an area model to represent multiplication with fractions.

94 it 2 • Lesson 3

- Show a **5 × 3** rectangle. Remind students that to find the area, we count the squares. Count the squares to get **15 squares**.
- Remind students that another strategy for finding area is to find the length of the rectangle and the width.
- Then we multiply the length by the width, which is the formula for area.
- The area of this rectangle is **5 • 3 = 15 square units**.

Demonstrate

- Call attention to **Example 1** on page 95 of the *Student Text*. In this example, we multiply $\frac{2}{3} \cdot \frac{1}{4}$. Walk through the example as outlined.
- Tell students we use an area model to multiply the fractions. We shade a rectangle to represent the product of two fractions. We first shade $\frac{1}{4}$ of the length of the rectangle. The length of the rectangle is already divided into fourths, so we shade one of these parts to represent $\frac{1}{4}$. Point out to students that this is the teal portion of the area model.
- Point out that next we divide the width of the rectangle into thirds because we are multiplying by $\frac{2}{3}$. Then we shade two of these parts to represent $\frac{2}{3}$. Point out to students this is the portion of the area model with black diagonal lines.
- Explain that we now have a representation for the product of $\frac{2}{3} \cdot \frac{1}{4}$. The numerator is the section where the two fractions overlap. Point out to students that this is the section with both the teal shading and the black diagonal lines.
- Remind students that the denominator is the total parts. Ask students to count the total parts now that we have divided the length and the width of the area model. There are **12 total parts**. This means the denominator is 12. The answer is $\frac{2}{12}$.
- Make note of the fact that the product is smaller than either $\frac{1}{4}$ or $\frac{2}{3}$.



Lesson 3

Let's use an area model to multiply fractions.

Example 1

Multiply the fractions using an area model.

$$\frac{2}{3} \cdot \frac{1}{4}$$

In this problem, we want to find $\frac{2}{3}$ of $\frac{1}{4}$.

First we divide the *height* of a rectangle into 4 equal parts.

We shade one part to represent $\frac{1}{4}$.

Next, we divide the *width* of the rectangle into 3 equal parts and we shade 2 parts to make $\frac{2}{3}$.

We must use a different shading pattern to show $\frac{2}{3}$.

Now we can figure out the product.

2 parts overlap.

The part where the shading overlaps represents the *numerator*. The total number of parts represents the *denominator*.

There are 12 total parts and 2 of the parts overlap.

There are 12 total parts.

The product is $\frac{2}{12}$.

We took $\frac{2}{3}$ of $\frac{1}{4}$ and got $\frac{2}{12}$.

Notice that the product is smaller than $\frac{1}{4}$ or $\frac{2}{3}$.

Unit 2 • Lesso 95

What is an area model? *(continued)*

Demonstrate

- Point out **Example 2** on page 96, which illustrates another problem using the area model. Walk through the steps to solving the problem $\frac{1}{2} \cdot \frac{1}{3}$.
- Draw the area models on the board as you go through the problem. First divide the height of a rectangle into thirds. Shade one of the thirds. Then divide the width into halves. Shade one of the halves.
- Point out that the numerator is the shaded part that overlaps, **1**. The denominator is the total number of parts, **6**. So the answer is $\frac{1}{6}$. Explain that we took $\frac{1}{2}$ of $\frac{2}{3}$, so the product is smaller than $\frac{1}{2}$ or $\frac{1}{3}$.

Check for Understanding

Engagement Strategy: Pair/Share

Write the following problems on the board:

$\frac{1}{3} \cdot \frac{1}{6} \left(\frac{1}{18}\right)$

$\frac{1}{2} \cdot \frac{1}{4} \left(\frac{1}{8}\right)$

Divide students into pairs. Have one partner solve one problem, and the other partner solve the second problem. Have students use area models. Then have partners check each other's work. Invite pairs to explain their answers to the class.

Reinforce Understanding

If students need a few more examples together before they try this strategy on their own, use these problems:

$\frac{2}{3} \cdot \frac{3}{5} \left(\frac{6}{15}\right)$

$\frac{1}{2} \cdot \frac{2}{7} \left(\frac{2}{14}\right)$

Let's look at another example. This time the fractional parts are thirds and halves.

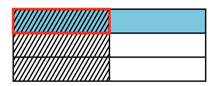
Example 2

Multiply using an area model.

$$\frac{1}{2} \cdot \frac{1}{3}$$

We divide a rectangle into 3 equal parts. We shade one of those parts to make $\frac{1}{3}$.

Next, we divide the width of the rectangle into two equal parts and shade one of the parts to make $\frac{1}{2}$.



Remember to use a different shading pattern.

Now we can find the product.
The numerator is where the shading overlaps, or 1 unit.
The denominator is the total number of parts, or 6 units.

The product is $\frac{1}{6}$.

$$\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

We took $\frac{1}{2}$ of $\frac{1}{3}$ and got $\frac{1}{6}$.

The product is smaller than $\frac{1}{2}$ or $\frac{1}{3}$.

The area model gives us a good picture of what is happening when we multiply two fractions. We can see the problem in two dimensions. We represent the height using one fraction and the width using another fraction. It is important to see these kinds of connections in mathematics.

Apply Skills
Turn to *Interactive Text*, page 53.

Reinforce Understanding
Use the *mBook Study Guide* to review lesson concepts.

Apply Skills

(*Interactive Text*, page 53)

Have students turn to *Interactive Text*, page 53, and complete the activity.

Activity 1

Students solve multiplication problems with fractions using area models. Monitor students' work as they complete the activity.

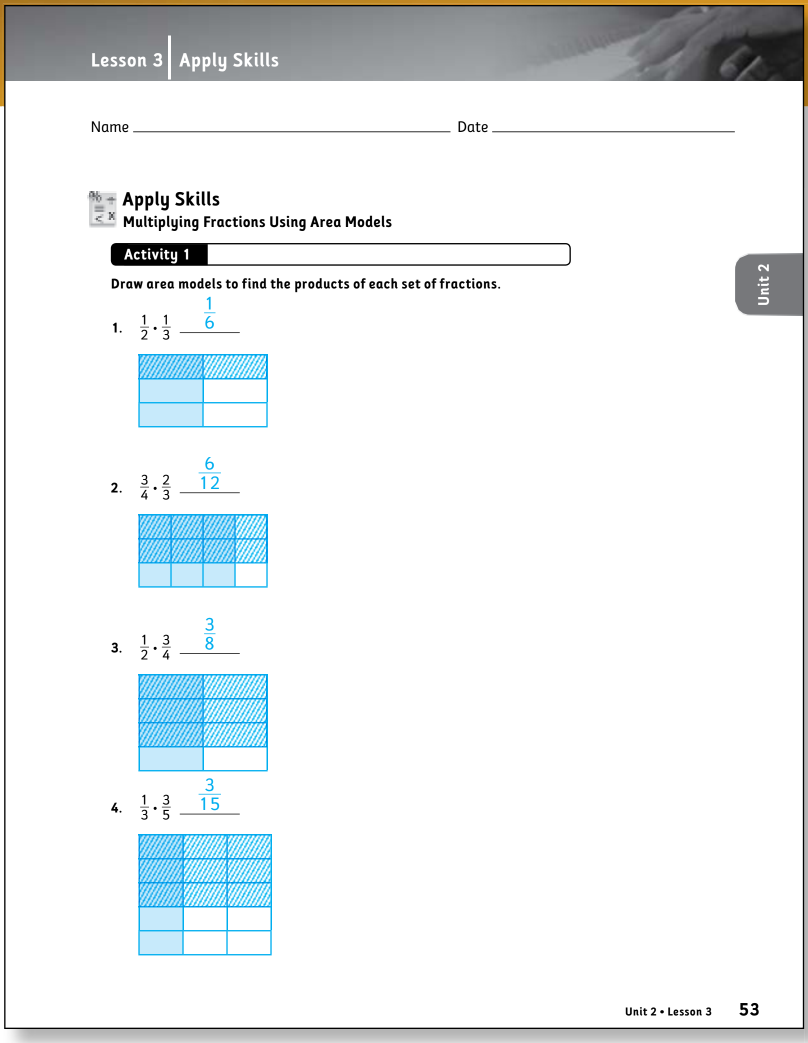
Watch for:

- Can students shade the fractions correctly on the area model?
- Can students identify the numerator of the product as the overlapping area?
- Do students know the total parts, or the denominator, of the answer?
- Can students write a fraction that represents the product?

Reinforce Understanding

mBook

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.



Lesson 3 | Apply Skills

Name ______ Date ______

Unit 2

Apply Skills

Multiplying Fractions Using Area Models

Activity 1

Draw area models to find the products of each set of fractions.

1. $\frac{1}{2} \cdot \frac{1}{3}$ $\frac{1}{6}$

2. $\frac{3}{4} \cdot \frac{2}{3}$ $\frac{6}{12}$

3. $\frac{1}{2} \cdot \frac{3}{4}$ $\frac{3}{8}$

4. $\frac{1}{3} \cdot \frac{3}{5}$ $\frac{3}{15}$

Unit 2 • Lesson 3 53

Problem Solving:
▶ Measuring Line Segments

How do we measure using centimeters?

(*Student Text*, page 97)

Connect to Prior Knowledge

Ask students if they have ever mapped out a vacation or a road trip using a map. Ask students to list some things they looked at on the map.

Listen for:

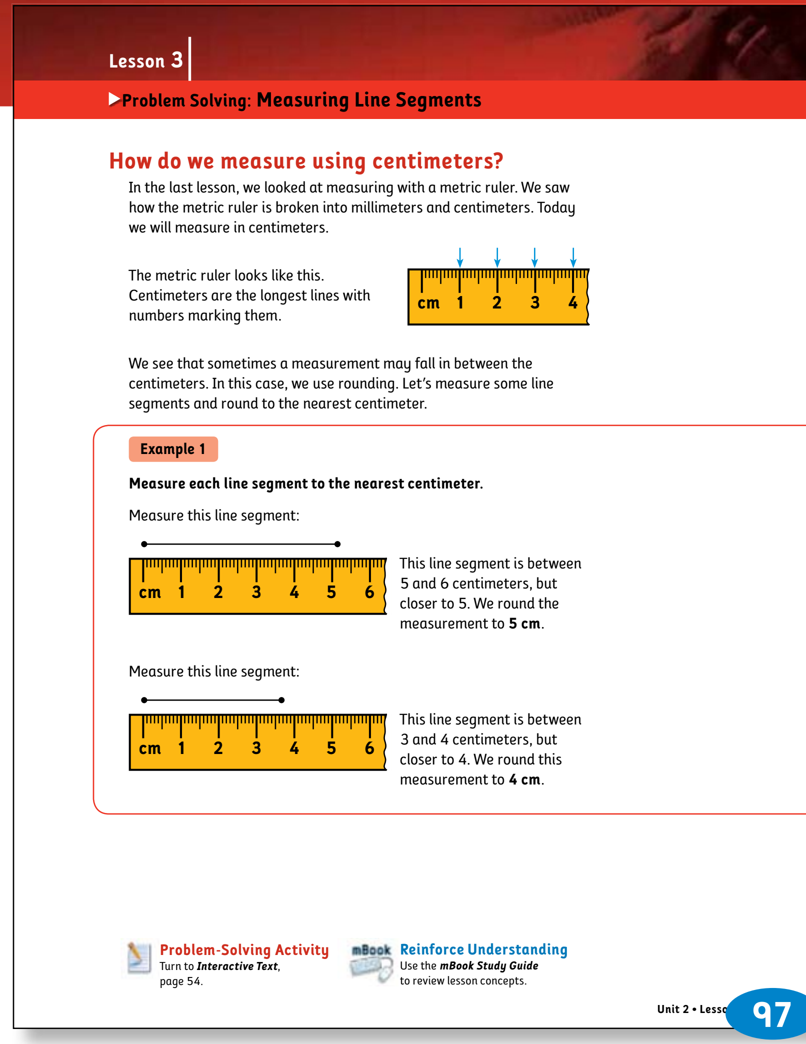
- *The city name where they started*
- *The city name where they were going*
- *The distance between the two cities on the map*
- *The key to the map to help figure out distances*

Link to Today's Concept

In today's lesson we measure line segments in centimeters.

Demonstrate

- Turn to page 97 of the *Student Text*. Show students the ruler and the lines that mark centimeters. State that if a measurement falls between centimeters, we round up or down.
- Tell students that **Example 1** shows how to measure the line segments to the nearest centimeter. Have students measure along with you in their books. The first line segment is between 5 and 6 cm, but it is closer to 5. So we round the measurement to **5 cm**. The second line segment is between 3 and 4 centimeters, but closer to 4. We round to **4 cm**.


Lesson 3

▶Problem Solving: Measuring Line Segments

How do we measure using centimeters?

In the last lesson, we looked at measuring with a metric ruler. We saw how the metric ruler is broken into millimeters and centimeters. Today we will measure in centimeters.

The metric ruler looks like this. Centimeters are the longest lines with numbers marking them.

We see that sometimes a measurement may fall in between the centimeters. In this case, we use rounding. Let's measure some line segments and round to the nearest centimeter.

Example 1

Measure each line segment to the nearest centimeter.

Measure this line segment:

This line segment is between 5 and 6 centimeters, but closer to 5. We round the measurement to **5 cm**.

Measure this line segment:

This line segment is between 3 and 4 centimeters, but closer to 4. We round this measurement to **4 cm**.

Problem-Solving Activity
Turn to ***Interactive Text***, page 54.

Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

Unit 2 • Lesson 97

Problem-Solving Activity

(*Interactive Text*, page 54)

Have students turn to page 54 in the *Interactive Text* and complete the activity. Students use a ruler to measure distances on a map to the nearest centimeter.

Monitor students' work as they complete this activity.

Watch for:

- Can students line up the ruler to make accurate measurements?
- Do students understand the phrase round to the nearest centimeter?
- Can students identify the operations needed in the problem?

Once students complete the Problem-Solving Activity, go over their answers, and discuss any difficulties they had when answering the problems.

mBook Reinforce Understanding

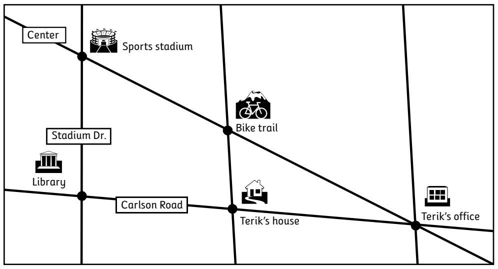
Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Lesson 3 | Problem-Solving Activity

Name ______________________ Date ______________________

Problem-Solving Activity
Measuring of Line Segments

Use a metric ruler to answer the questions. Round to the nearest centimeter.



1. Measure the line segment between Terik's house and the library. About how many centimeters long is it?
   4
2. Measure the line segment between Terik's house and his office building. About how many centimeters long is it?
   5
3. How much longer is the line segment from Terik's office building to the sports center than it is from Terik's office building to his house?
   5
4. If Terik travels from his house to his office building, then to the sports stadium, and then to the library, what is the total length of all of those line segments added together? Round each section and add them together.
   19

mBook Reinforce Understanding
Use the mBook *Study Guide* to review lesson concepts.

54 Unit 2 • Lesson 3

Homework

Go over the instructions on page 98 of the *Student Text* for each part of the homework.

Activity 1

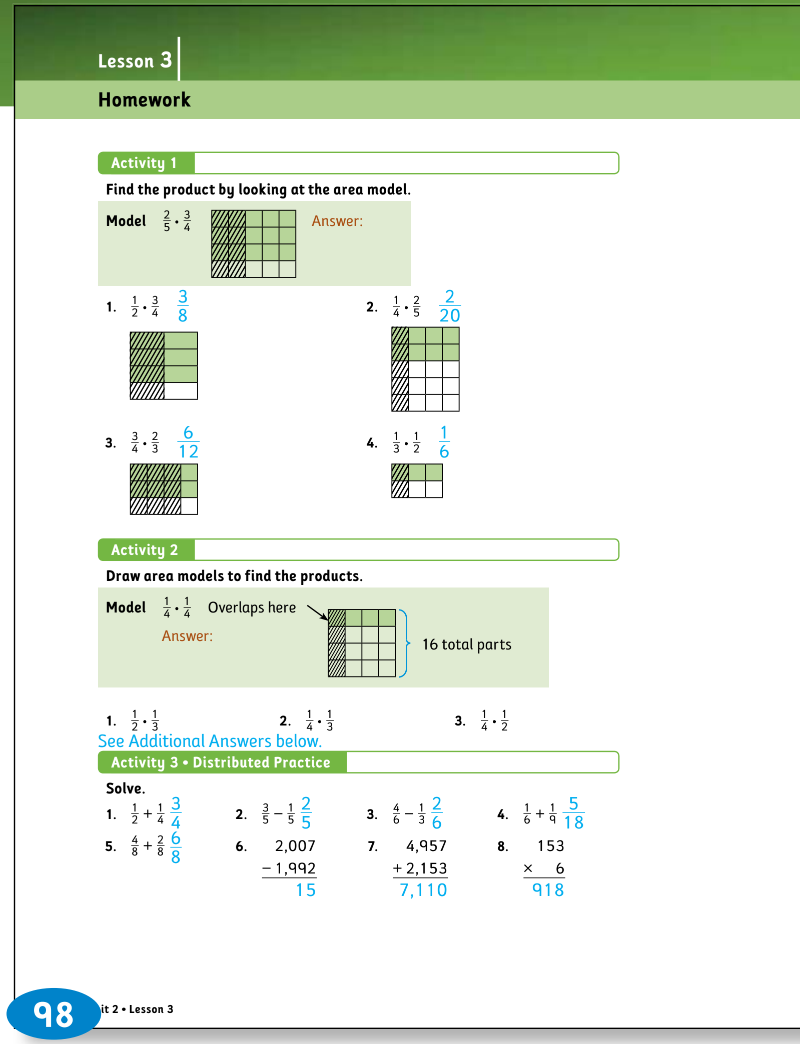
Students look at the area models that are already shaded and find the product.

Activity 2

Students draw and shade area models to find the product.

Activity 3 • Distributed Practice

Students practice basic computational skills. Five of the problems involve addition and subtraction of fractions, and three of the problems are whole-number operations.


Lesson 3

Homework

Activity 1

Find the product by looking at the area model.

Model $\frac{2}{5} \cdot \frac{3}{4}$ Answer:

1. $\frac{1}{2} \cdot \frac{3}{4}$ $\frac{3}{8}$
2. $\frac{1}{4} \cdot \frac{2}{5}$ $\frac{2}{20}$
3. $\frac{3}{4} \cdot \frac{2}{3}$ $\frac{6}{12}$
4. $\frac{1}{3} \cdot \frac{1}{2}$ $\frac{1}{6}$

Activity 2

Draw area models to find the products.

Model $\frac{1}{4} \cdot \frac{1}{4}$ Overlaps here

Answer:

16 total parts

1. $\frac{1}{2} \cdot \frac{1}{3}$
2. $\frac{1}{4} \cdot \frac{1}{3}$
3. $\frac{1}{4} \cdot \frac{1}{2}$

See Additional Answers below.

Activity 3 • Distributed Practice

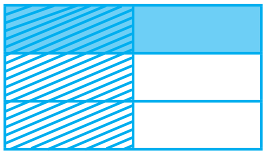
Solve.

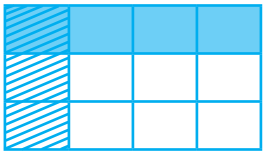
1. $\frac{1}{2} + \frac{1}{4}$ $\frac{3}{4}$
2. $\frac{3}{5} - \frac{1}{5}$ $\frac{2}{5}$
3. $\frac{4}{6} - \frac{1}{3}$ $\frac{2}{6}$
4. $\frac{1}{6} + \frac{1}{9}$ $\frac{5}{18}$
5. $\frac{4}{8} + \frac{2}{8}$ $\frac{6}{8}$
6. $2{,}007 - 1{,}992 = 15$
7. $4{,}957 + 2{,}153 = 7{,}110$
8. $153 \times 6 = 918$

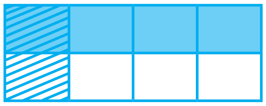
98 it 2 • Lesson 3

Additional Answers

Activity 2

1.  $\frac{1}{6}$

2.  $\frac{1}{12}$

3.  $\frac{1}{8}$

Lesson 4 ▸ Multiplying Fractions the Traditional Way

Problem Solving:

▸ Angles

Lesson Planner

Vocabulary Development

angle
vertex

Skills Maintenance

Multiplication Facts

Building Number Concepts:

▸ Multiplying Fractions the Traditional Way

We introduce the traditional way of multiplying fractions: multiplying across the numerator and the denominator. The area model gives a clearer picture of what is happening, but the traditional algorithm is more efficient. It is good to compare alternative algorithms. However, fluency with traditional algorithms is our main goal.

Objective

Students will multiply fractions by multiplying across the numerator and the denominator.

Problem Solving:

▸ Angles

In this lesson, we introduce angles. Students learn the parts of an angle and how angles are named.

Objective

Students will name angles and identify their parts.

Homework

Students multiply fractions using the traditional algorithm and area models, draw a line, line segment, and ray, and name the angles shown. In Distributed Practice, students add and subtract fractions and multiply and divide whole numbers.

Lesson 4 | Skills Maintenance

Name ______________________ Date ______________

Skills Maintenance
Multiplication Facts

Activity 1

Solve the multiplication facts.

1. $3 \cdot 8$ 24
2. $4 \cdot 5$ 20
3. $9 \cdot 7$ 63
4. $8 \cdot 7$ 56
5. $6 \cdot 9$ 54
6. $4 \cdot 4$ 16
7. $10 \cdot 8$ 80
8. $3 \cdot 9$ 27
9. $7 \cdot 6$ 42

Unit 2

Unit 2 • Lesson 4 55

Skills Maintenance

Multiplication Facts

(*Interactive Text*, page 55)

Activity 1

Students solve basic multiplication facts.

Building Number Concepts:
▶ Multiplying Fractions the Traditional Way

What is multiplying across?

(*Student Text*, pages 99–100)

Connect to Prior Knowledge

Review how we multiply fractions using the area model. Demonstrate the area model method for the problem $\frac{1}{2} \cdot \frac{4}{5}$. Ask students to tell you the steps as you go through the process on the board.

Write the following problems on the board. Write the fractions this way (rather than with slashes):

$$\frac{1}{4} \cdot \frac{2}{3} = \frac{2}{12} \qquad \frac{2}{5} \cdot \frac{2}{3} = \frac{4}{15} \qquad \frac{2}{4} \cdot \frac{3}{6} = \frac{6}{24}$$

Next ask students if they notice a pattern in the examples.

Listen for:

- *I notice that the numerator of the product is the numerator times the numerator in the problem.*
- *I notice that the denominator of the product is the denominator times the denominator in the problem.*

Link to Today's Concept

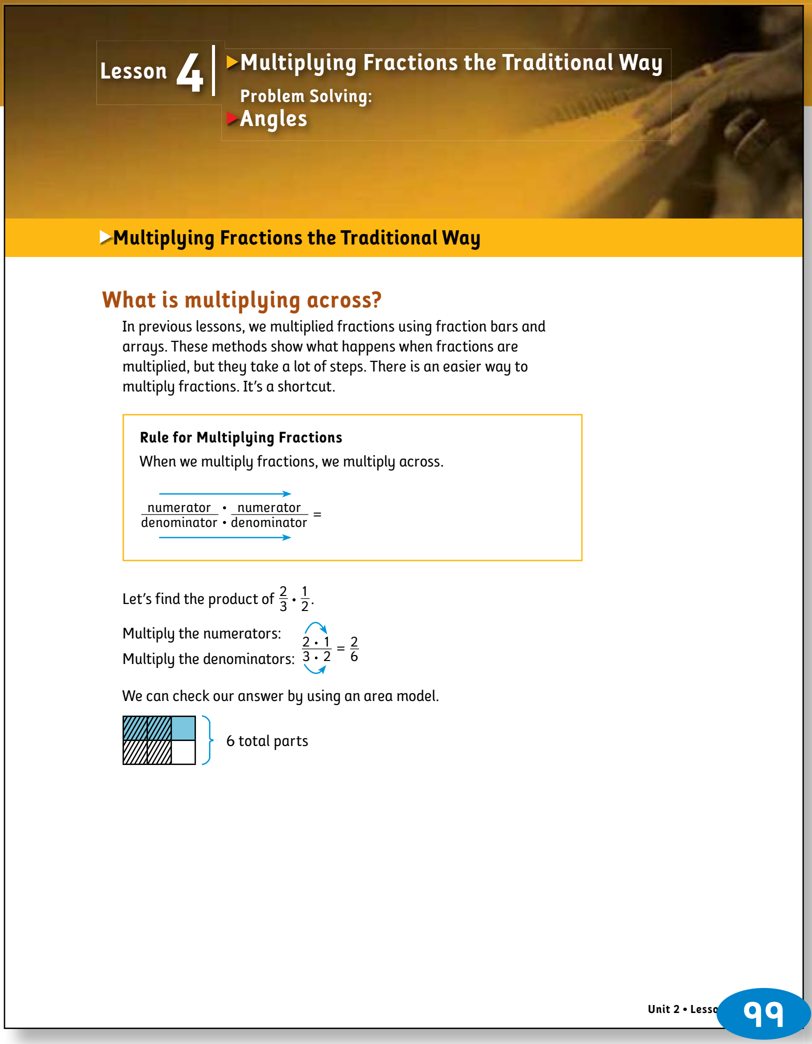
In today's lesson, students learn how to multiply fractions using the traditional algorithm of multiplying across the numerators and the denominators.

Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate the traditional algorithm for multiplying fractions by multiplying across, in one of the following ways:

mBook: Use the *mBook Teacher Edition* for *Student Text*, page 99. m


Lesson 4 ▶Multiplying Fractions the Traditional Way
Problem Solving:
▶Angles

▶Multiplying Fractions the Traditional Way

What is multiplying across?

In previous lessons, we multiplied fractions using fraction bars and arrays. These methods show what happens when fractions are multiplied, but they take a lot of steps. There is an easier way to multiply fractions. It's a shortcut.

Rule for Multiplying Fractions
When we multiply fractions, we multiply across.

$$\frac{\text{numerator}}{\text{denominator}} \cdot \frac{\text{numerator}}{\text{denominator}} =$$

Let's find the product of $\frac{2}{3} \cdot \frac{1}{2}$.

Multiply the numerators:
Multiply the denominators: $\frac{2 \cdot 1}{3 \cdot 2} = \frac{2}{6}$

We can check our answer by using an area model.

6 total parts

Unit 2 • Lesson 4 99

Overhead Projector: Reproduce the rule and problem on a transparency, and modify as discussed.

Board: Copy the rule and problem onto the board, and modify as discussed.

- Tell students the rule for multiplying across. We multiply the numerator by the numerator, and the denominator by the denominator to get the product. m
- Show students the example $\frac{2}{3} \cdot \frac{1}{2}$. Show the intermediate step of $\frac{2 \cdot 1}{3 \cdot 2}$. m
- Complete the multiplication and simplify the answer $\frac{2}{6} = \frac{1}{3}$. Display the area model to confirm the answer. m

Demonstrate

- Call attention to **Example 1** on page 100 of the *Student Text*. In this example we use the traditional method to multiply $\frac{1}{3} \cdot \frac{1}{4}$. Point out the process to students.
- Tell students to write the intermediate step (i.e., $\frac{1 \cdot 1}{3 \cdot 4}$) that is shown in the example if it helps them remember the process. Then have them check the answer by making sure the area model has the same answer of $\frac{1}{12}$.
- Next have students look at **Example 2**. This time we are using the traditional method to multiply a whole number by a fraction in the problem $7 \cdot \frac{2}{5}$.
- Remind students that whole numbers are fractions with the denominator of 1. So the first step would be to convert 7 to $\frac{7}{1}$. Write the intermediate step of the multiplication process: $\frac{7 \cdot 2}{1 \cdot 5}$. Then complete the multiplication to get the product, $\frac{14}{5}$.
- Draw the area model to confirm that the answer is correct.

Lesson 4

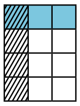
Let's look at other examples.

Example 1

Multiply across to solve the problem $\frac{1}{3} \cdot \frac{1}{4}$.

$\frac{1}{3} \cdot \frac{1}{4} = \frac{1 \cdot 1}{3 \cdot 4} = \frac{1}{12}$

We can confirm this answer by looking at an area model:



1 part overlaps out of 12 total parts

When we multiply by a whole number, the area model is not as easy to use. We need to convert the whole number to a fraction and multiply across.

Example 2

Multiply across to solve the problem $7 \cdot \frac{2}{5}$.

First we must convert the whole number to a fraction.

We do this by putting the whole number over 1 because any number divided by 1 equals itself.

$7 = \frac{7}{1}$

Then we multiply.

$\frac{7}{1} \cdot \frac{2}{5} = \frac{7 \cdot 2}{1 \cdot 5} = \frac{14}{5}$

It is important to remember to work carefully and think about each step when we are multiplying fractions.

Apply Skills Turn to *Interactive Text*, page 56.

Reinforce Understanding Use the ***mBook Study Guide*** to review lesson concepts.

100 Unit 2 • Lesson 4

Check for Understanding

Engagement Strategy: Pair/Share

Divide the class into pairs. For the first problem, have one partner multiply the traditional way, and have the other partner draw the area model to solve the problem.

For the second problem, have partners switch roles; the first partner solves the problem using an area model, and the second partner solves the problem by multiplying across.

Have partners compare their answers to each problem to make sure they are correct. Invite volunteers to explain their answers for both methods. Use the following problems:

$\frac{1}{3} \cdot \frac{1}{5} \left(\frac{1}{15}\right)$

$\frac{4}{5} \cdot \frac{1}{2} \left(\frac{4}{10}\right)$

Reinforce Understanding

For additional practice, have students solve the following problems using the traditional method and verifying with the area model:

$\frac{3}{7} \cdot \frac{1}{3} \left(\frac{3}{21}\right)$

$\frac{3}{4} \cdot \frac{2}{3} \left(\frac{6}{12}\right)$

$\frac{1}{2} \cdot \frac{6}{8} \left(\frac{6}{16}\right)$

Apply Skills

(*Interactive Text*, page 56)

Have students turn to *Interactive Text*, page 56, and complete the activity.

Activity 1

Students solve the problems using the traditional algorithm for multiplying fractions. Monitor students' work as they complete the activity.

Watch for:

- Can students use the algorithm to solve the problems?
- Do students remember to multiply across both the numerator and the denominator?
- Are students struggling with fact fluency?

mBook

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Name ______________________ Date ______________________

Apply Skills

Multiplying Fractions the Traditional Way

Activity 1

Solve the multiplication problems by multiplying across. You do not need to simplify your answers. Remember that a whole number may be rewritten as a fraction with a denominator of 1.

1. $\frac{3}{4} \cdot \frac{4}{5}$ $\underline{\frac{12}{20}}$
2. $\frac{5}{7} \cdot \frac{1}{2}$ $\underline{\frac{5}{14}}$
3. $\frac{2}{3} \cdot \frac{1}{2}$ $\underline{\frac{2}{6}}$
4. $\frac{3}{5} \cdot \frac{2}{7}$ $\underline{\frac{6}{35}}$
5. $\frac{2}{9} \cdot \frac{3}{4}$ $\underline{\frac{6}{36}}$
6. $\frac{1}{4} \cdot \frac{5}{9}$ $\underline{\frac{5}{36}}$
7. $\frac{4}{5} \cdot \frac{5}{6}$ $\underline{\frac{20}{30}}$
8. $\frac{6}{8} \cdot \frac{2}{3}$ $\underline{\frac{12}{24}}$
9. $\frac{3}{4} \cdot \frac{2}{5}$ $\underline{\frac{6}{20}}$
10. $\frac{1}{4} \cdot 5$ $\underline{\frac{5}{4}}$
11. $\frac{2}{10} \cdot \frac{3}{4}$ $\underline{\frac{6}{40}}$
12. $\frac{5}{9} \cdot 4$ $\underline{\frac{20}{9}}$

Problem Solving:
▶ Angles

What are angles?

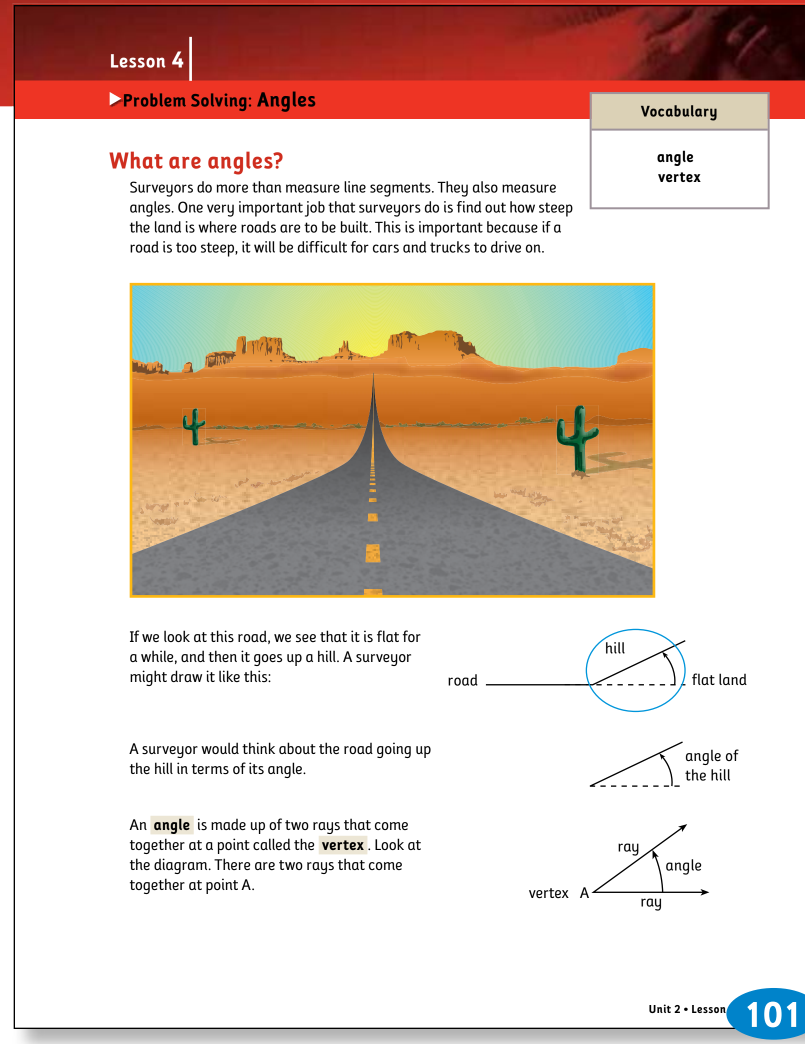
(*Student Text*, pages 101–102)

Build Vocabulary

Tell students that in today's lesson, they learn that an **angle** is made up of two rays that come together at a **vertex**, or point.

Demonstrate

- Read page 101 of the *Student Text* with students. Remind students of the discussion about surveyors and their use of math in measuring line segments.
- Tell students that surveyors also use and measure angles in their jobs. This is important because angles tell surveyors if a road is too steep for cars and trucks to drive on.
- Show students the graphic of the angles depicting the road and the angle of the hill. Then show students the different parts of the angle: the two rays and the vertex.
- Ask students if they can think of any other examples from real life in which angles are used. For example, tennis players also use angles as a strategy for where to hit the ball.


Lesson 4

▶Problem Solving: Angles

Vocabulary
angle
vertex

What are angles?

Surveyors do more than measure line segments. They also measure angles. One very important job that surveyors do is find out how steep the land is where roads are to be built. This is important because if a road is too steep, it will be difficult for cars and trucks to drive on.

If we look at this road, we see that it is flat for a while, and then it goes up a hill. A surveyor might draw it like this:

A surveyor would think about the road going up the hill in terms of its angle.

An **angle** is made up of two rays that come together at a point called the **vertex**. Look at the diagram. There are two rays that come together at point A.

Unit 2 • Lesson 4 101

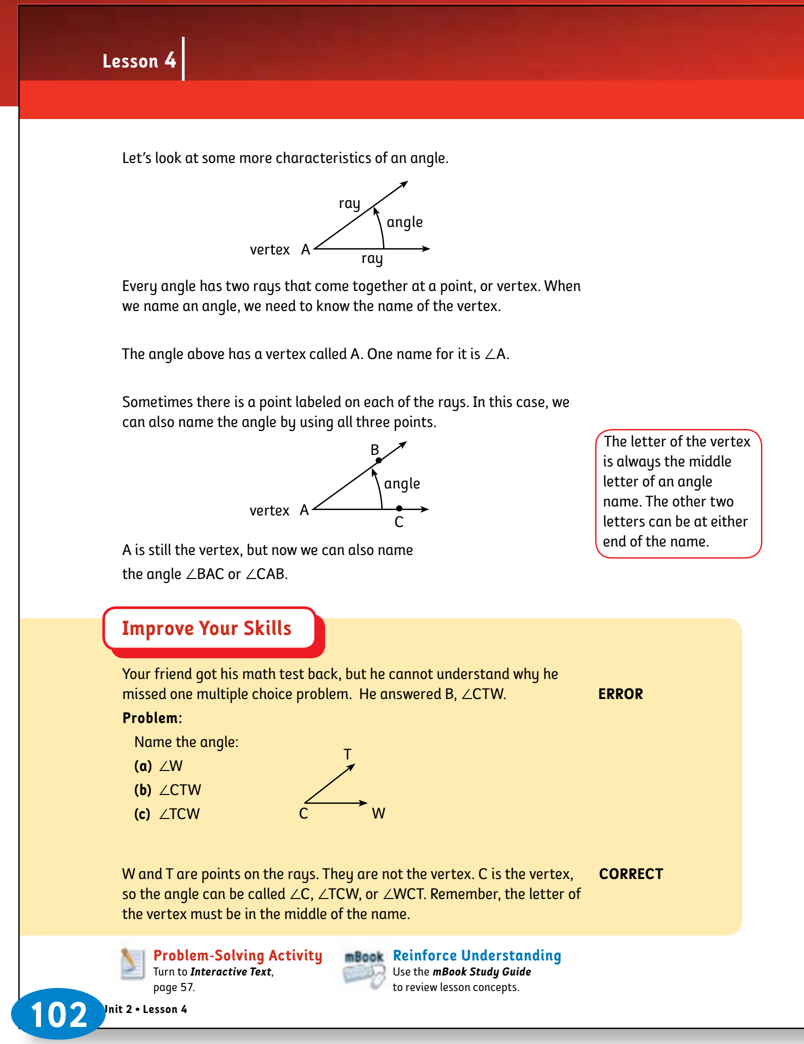
What are angles? *(continued)*

Demonstrate

- Have students look at the diagram on page 102. Show them that an angle is made up of two rays that come together at a point we call the vertex. Mathematicians use the symbol ∠ to refer to angles.
- Explain there is another convention mathematicians use to refer to angles. They identify three points to refer to the angles: one point on a ray, the point at the vertex, and another point on the other ray.
- Point out that in the diagram, the vertex is A, and the points on the rays are B and C. We refer to this angle as ∠BAC or ∠CAB. In this case, A always goes in the middle because it is the vertex. We always put the label for the vertex in the middle.
- Conclude by reminding students that it is important to use naming conventions in the world of surveying or any real-life application. This way, everyone knows exactly what we are referencing when we draw, label, construct, or analyze maps or technical drawings involving lines, rays, line segments, points, and angles.
- If students need extra practice remembering the naming convention, draw a series of angles on the board or overhead with labels and ask students to tell the names of each of the angles.

Improve Your Skills

- Read the problem with students. Have them look at the angle and explain why a student made an error in naming the angle. Elicit the correct name of the angle, ∠TCW.



Lesson 4

Let's look at some more characteristics of an angle.

Every angle has two rays that come together at a point, or vertex. When we name an angle, we need to know the name of the vertex.

The angle above has a vertex called A. One name for it is ∠A.

Sometimes there is a point labeled on each of the rays. In this case, we can also name the angle by using all three points.

A is still the vertex, but now we can also name the angle ∠BAC or ∠CAB.

The letter of the vertex is always the middle letter of an angle name. The other two letters can be at either end of the name.

Improve Your Skills

Your friend got his math test back, but he cannot understand why he missed one multiple choice problem. He answered B, ∠CTW. **ERROR**

Problem:

Name the angle:

(a) ∠W
(b) ∠CTW
(c) ∠TCW

W and T are points on the rays. They are not the vertex. C is the vertex, so the angle can be called ∠C, ∠TCW, or ∠WCT. Remember, the letter of the vertex must be in the middle of the name. **CORRECT**

Problem-Solving Activity Turn to ***Interactive Text***, page 57.

Reinforce Understanding Use the ***mBook Study Guide*** to review lesson concepts.

102 Unit 2 • Lesson 4

- Reinforce the concept that the vertex is always in the middle of the name of an angle.

Check for Understanding

Ask:

What are the parts of an angle? (*two rays that come together at a vertex*)

In the angle ∠CAB, which letter represents the vertex? (*A*)

Which letters are points on the rays? (*C and B*)

Problem-Solving Activity
(*Interactive Text*, page 57)

Have students turn to page 57 in the *Interactive Text* and complete the activity. Instruct students to answer the questions by naming the angles that identify certain area locations on the map. Be sure students understand that they are to look at the icon for the landmark and not the words.

Have students take out a blank sheet of lined paper and begin the activity.

Monitor students' work as they complete this activity.

Watch for:

- Can students identify the three points that name the angle?
- Do students remember to put the vertex in the middle?
- Can students locate all the landmarks?

Once students complete the activity, be sure to discuss their answers. Some of the answers might be slightly different because there are two ways to write each of the angle names. For instance, the ski resort is at the angle named *∠IGJ* or *∠JGI*. As long as the vertex, G, is in the middle, we can reference the name either way.

mBook Reinforce Understanding

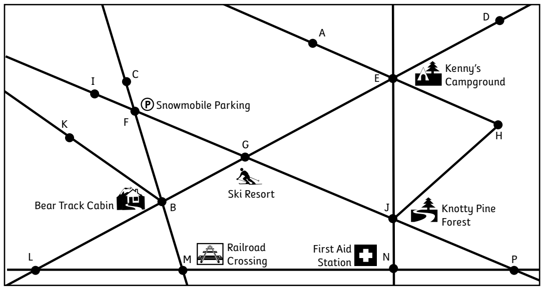
Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Name ____________________ Date ____________

Problem-Solving Activity
Angles

Look at the map of the snowmobile trails and landmarks in a park. Write the name of the angle that describes the location of each of the landmarks.

Model	Name the angle that describes the location of the ski resort. ∠IGJ



1. Name the angle that describes the location of the snowmobile parking. ∠CFG
2. Name the angle that describes the location of the first aid station. ∠MNJ
3. Name the angle that describes the location of the railroad crossing. ∠IMN
4. Name the angle that describes the location of Knotty Pine Forest. ∠HJP
5. Name the angle that describes the location of Bear Track Cabin. ∠KBL
6. Name the angle that describes the location of Kenny's Campground. ∠DEH

Answers will vary.

mBook Reinforce Understanding
Use the **mBook** ***Study Guide*** to review lesson concepts.

Unit 2

Homework

Go over the instructions on page 103 of the *Student Text* for each part of the homework.

Activity 1

Students solve multiplication problems using the traditional algorithm for multiplying fractions.

Activity 2

Students draw area models to multiply fractions.

Activity 3

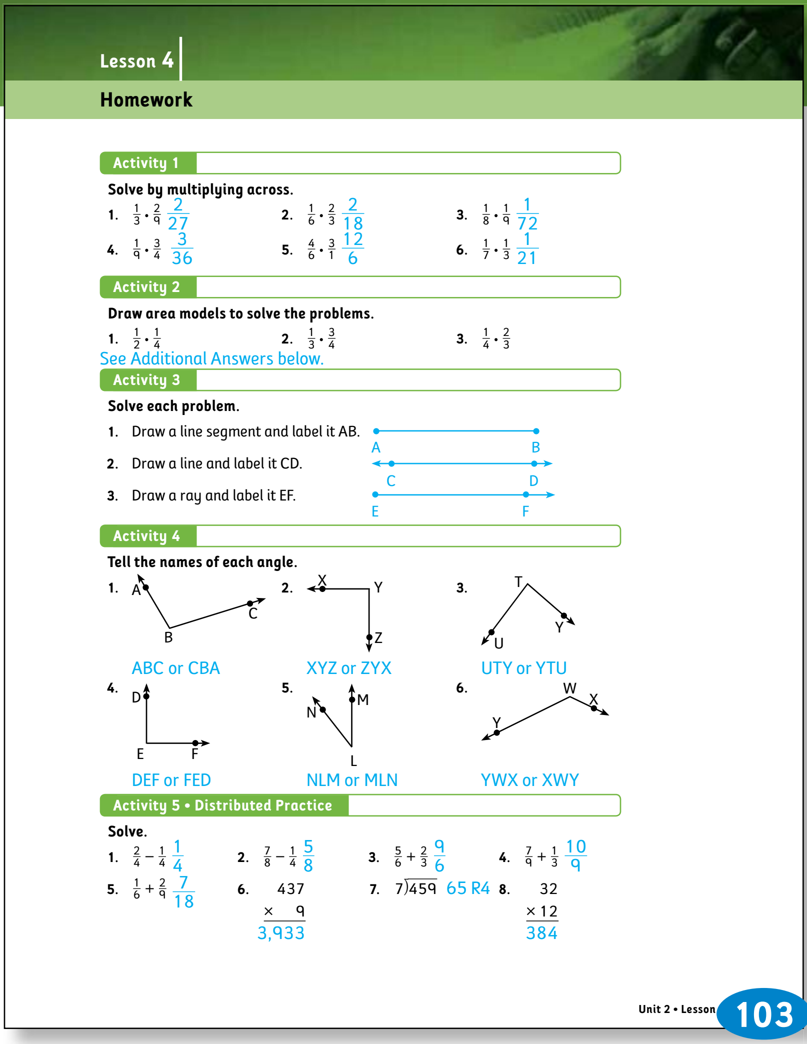
Students draw and label a line segment, a line, and a ray.

Activity 4

Students give the names of the angles.

Activity 5 • Distributed Practice

Students practice basic computational skills with fractions and whole numbers.



Lesson 4

Homework

Activity 1

Solve by multiplying across.

1. $\frac{1}{3} \cdot \frac{2}{9}$ $\frac{2}{27}$
2. $\frac{1}{6} \cdot \frac{2}{3}$ $\frac{2}{18}$
3. $\frac{1}{8} \cdot \frac{1}{9}$ $\frac{1}{72}$
4. $\frac{1}{9} \cdot \frac{3}{4}$ $\frac{3}{36}$
5. $\frac{4}{6} \cdot \frac{3}{1}$ $\frac{12}{6}$
6. $\frac{1}{7} \cdot \frac{1}{3}$ $\frac{1}{21}$

Activity 2

Draw area models to solve the problems.

1. $\frac{1}{2} \cdot \frac{1}{4}$
2. $\frac{1}{3} \cdot \frac{3}{4}$
3. $\frac{1}{4} \cdot \frac{2}{3}$

See Additional Answers below.

Activity 3

Solve each problem.

1. Draw a line segment and label it AB.
2. Draw a line and label it CD.
3. Draw a ray and label it EF.

Activity 4

Tell the names of each angle.

1. ABC or CBA
2. XYZ or ZYX
3. UTY or YTU
4. DEF or FED
5. NLM or MLN
6. YWX or XWY

Activity 5 • Distributed Practice

Solve.

1. $\frac{2}{4} - \frac{1}{4}$ $\frac{1}{4}$
2. $\frac{7}{8} - \frac{1}{4}$ $\frac{5}{8}$
3. $\frac{5}{6} + \frac{2}{3}$ $\frac{9}{6}$
4. $\frac{7}{9} + \frac{1}{3}$ $\frac{10}{9}$
5. $\frac{1}{6} + \frac{2}{9}$ $\frac{7}{18}$
6. 437×9 = 3,933
7. $7\overline{)459}$ 65 R4
8. 32×12 = 384

Unit 2 • Lesson 4 103

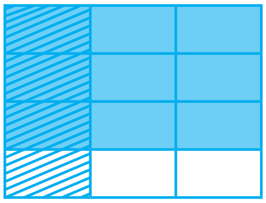
Additional Answers

Activity 2

1.

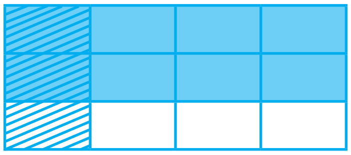
$\frac{1}{8}$

2.



$\frac{3}{12} \left(\frac{1}{4}\right)$

3.



$\frac{2}{12} \left(\frac{1}{6}\right)$

Lesson 5 | Simplifying Fractions

Monitoring Progress: Quiz 1

Lesson Planner

Vocabulary Development

factors
common factors
greatest common factor

Skills Maintenance

Multiplication With Fractions, Angles

Building Number Concepts:
Simplifying Fractions

Students learn to simplify fractions to lowest terms. This is also called reducing fractions to lowest terms. When the numerator and denominator have a common factor, the factor is pulled out of each, leaving a fraction in lowest terms. Students gain number sense by showing numbers in equivalent forms. In higher-level math courses, students are asked to give their answers in lowest terms.

Objective

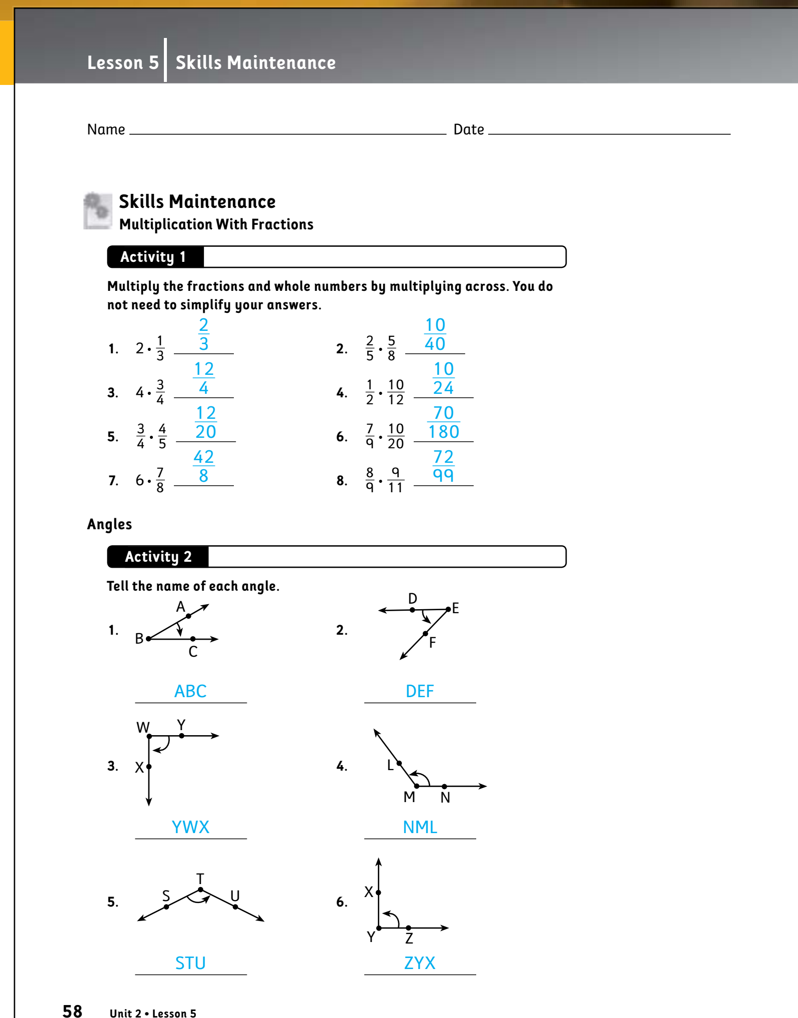
Students will simplify fractions.

Monitoring Progress:
Quiz 1

Distribute the quiz, and remind students that the questions involve material covered over the previous lessons in the unit.

Homework

Students multiply fractions using the traditional algorithm, simplify fractions, and draw area models to find the product of two fractions. In Distributed Practice, students add, subtract, and multiply fractions and whole numbers.



Lesson 5 | Skills Maintenance

Name ______________________ Date ______________

Skills Maintenance
Multiplication With Fractions

Activity 1

Multiply the fractions and whole numbers by multiplying across. You do not need to simplify your answers.

1. $2 \cdot \frac{1}{3}$ $\frac{2}{3}$
2. $\frac{2}{5} \cdot \frac{5}{8}$ $\frac{10}{40}$
3. $4 \cdot \frac{3}{4}$ $\frac{12}{4}$
4. $\frac{1}{2} \cdot \frac{10}{12}$ $\frac{10}{24}$
5. $\frac{3}{4} \cdot \frac{4}{5}$ $\frac{12}{20}$
6. $\frac{7}{9} \cdot \frac{10}{20}$ $\frac{70}{180}$
7. $6 \cdot \frac{7}{8}$ $\frac{42}{8}$
8. $\frac{8}{9} \cdot \frac{9}{11}$ $\frac{72}{99}$

Angles

Activity 2

Tell the name of each angle.

1. ABC
2. DEF
3. YWX
4. NML
5. STU
6. ZYX

58 Unit 2 • Lesson 5

Skills Maintenance

Multiplication With Fractions, Angles
(*Interactive Text*, page 58)

Activity 1

Students use the traditional algorithm to multiply fractions and whole numbers.

Activity 2

Students look at angles and give the angle names.

Building Number Concepts:
▶ Simplifying Fractions

What are factors?
(*Student Text*, page 104)

Connect to Prior Knowledge
Begin by reviewing **factors**. Ask students to share what they know about factors.

Listen for:

- *Factors are the small numbers in multiplication facts for a number.*

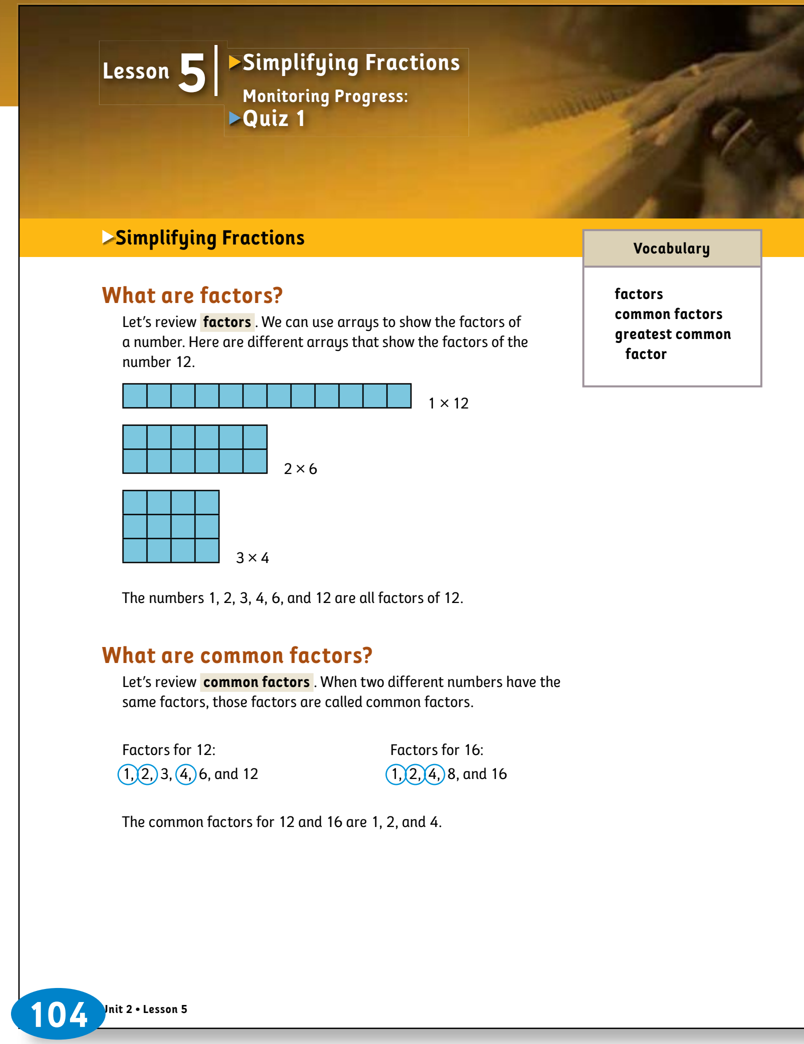
Link to Today's Concept
In today's lesson, we look at how factors help us simplify fractions.

Demonstrate
Engagement Strategy: Teacher Modeling

Demonstrate how factors and **common factors** help us simplify fractions. Factoring can be done in one of the following ways:

- **mBook**: Use the *mBook Teacher Edition* for *Student Text*, page 104.
- **Overhead Projector**: Draw the arrays and write the numbers on a transparency. Modify as discussed.
- **Board**: Draw the arrays and write the numbers on the board. Modify as discussed.

- Review factors with arrays. Remind students that when we draw the arrays for a number, we get its factors.
- Show the arrays for the number 12. Point out that the first array is the number 12 multiplied by 1. The next array shows 2 • 6, and the third array shows 3 • 4. Tell students that the factors of **12** are **1**, **2**, **3**, **4**, **6**, and **12**.


Lesson 5 ▶Simplifying Fractions
Monitoring Progress:
▶Quiz 1

▶Simplifying Fractions

Vocabulary
factors
common factors
greatest common factor

What are factors?

Let's review **factors**. We can use arrays to show the factors of a number. Here are different arrays that show the factors of the number 12.

1 × 12

2 × 6

3 × 4

The numbers 1, 2, 3, 4, 6, and 12 are all factors of 12.

What are common factors?

Let's review **common factors**. When two different numbers have the same factors, those factors are called common factors.

Factors for 12: 1, 2, 3, 4, 6, and 12

Factors for 16: 1, 2, 4, 8, and 16

The common factors for 12 and 16 are 1, 2, and 4.

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What are common factors?
(*Student Text*, page 104)

Demonstrate
- Now remind students about common factors. Display the factor lists for 12 and 16:

  Factors of 12: 1, 2, 3, 4, 6, and 12

  Factors of 16: 1, 2, 4, 8, and 16

- Point out the common factors **1**, **2**, and **4**.

How do common factors help us simplify fractions?

(*Student Text*, page 105)

Demonstrate

- Turn to page 105 of the *Student Text*, and discuss common factors. The illustration shows how to pull out common factors from the numerator and denominator of the fraction $\frac{4}{6}$. We can pull out a common factor of 2 from each.
- Point out **2 • ? = 4** and **2 • ? = 6**. Explain that our knowledge of basic facts helps us complete the problem to help us simplify. Be sure students see why $\frac{2}{3}$ is the simplest form of $\frac{4}{6}$.
- Point out $\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3}$. Explain that because $\frac{2}{2} = 1$, the problem can be written as $\frac{4}{6} = 1 \cdot \frac{2}{3}$. Because any number multiplied by 1 is itself, the answer is $\frac{2}{3}$.
- Point out how we check this with a fraction bar, as shown at the end of the illustration. We see the fraction bar for $\frac{4}{6}$ and the fraction bar for $\frac{2}{3}$. Point out how $\frac{2}{3}$ lines up with $\frac{4}{6}$. In other words, $\frac{2}{3}$ is an equivalent fraction of $\frac{4}{6}$.

Lesson 5

How do common factors help us simplify fractions?

We can easily find or "pull out" factors of whole numbers. For example, we can "pull" 10 out of 40 this way:

$$40 = 4 \cdot 10$$

The equation $4 \cdot 10$ is just another way of writing the number 40.

Pulling out numbers is also important when we work with fractions. By pulling out the same number from the numerator and the denominator, we can simplify a fraction.

Let's simplify $\frac{4}{6}$ by pulling out a common factor.

The numerator and denominator have a common factor of 2. We find the simplified fraction by thinking about our basic facts.

Pulling out the 2 gives us a simpler fraction.

$$2 \cdot ? = 4 \qquad 2 \cdot ? = 6$$
$$\downarrow \qquad\qquad \downarrow$$
$$2 \cdot 2 = 4 \qquad 2 \cdot 3 = 6$$

Basic facts help us make a connection between the original numerator and denominator and the common factor. When we know our basic facts, we can complete our problem by simplifying.

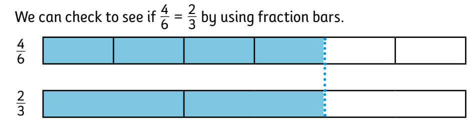
$\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3}$ is the same as $\frac{4}{6} = \frac{2}{2} \cdot \frac{2}{3}$

Remember, a number multiplied by 1 will always equal itself.

Because $\frac{2}{2} = 1$, the problem can be written as: $\frac{4}{6} = 1 \cdot \frac{2}{3}$.

Because $1 \cdot \frac{2}{3} = \frac{2}{3}$, we can rewrite the problem as $\frac{4}{6} = \frac{2}{3}$.

We can check to see if $\frac{4}{6} = \frac{2}{3}$ by using fraction bars.



How do we simplify fractions by using the greatest common factor?

(*Student Text*, pages 106–107)

Build Vocabulary

Remind students about the definition of **greatest common factor** (GCF). The largest of all the common factors for two or more numbers is the greatest common factor.

Demonstrate

- Next have students look at **Example 1** on page 106 of the *Student Text*. Tell students that we want to use the GCF to simplify the fraction $\frac{8}{12}$. Elicit the factors of 8 from students. Then elicit the factors of 12. Circle the greatest common factor (GCF), **4**.
- Point out that we use the GCF for 8 and 12 to simplify $\frac{8}{12}$. The GCF, 4, can be factored from both 8 and 12. When we factor 4 out of 8, we are left with **2**. When we factor 4 out of 12, we are left with **3**. The reduced fraction is $\frac{2}{3}$.

Check for Understanding

Engagement Strategy: Look About

Tell students that they are going to solve a problem with the help of the whole class. Write the fraction $\frac{5}{10}$ $(GCF = 2, \frac{5}{10} = \frac{1}{2})$ on the board. Tell students to simplify the fraction by pulling out the greatest common factor. Students should write their answers in large writing on a piece of paper or a dry erase board. When students finish their work, they should hold up their answer for everyone to see.

If students are not sure about the answer, prompt them to look about at other students' solutions to help with their thinking. Review the answers after all students have held up their solutions.

How do we simplify fractions by using the greatest common factor?

It is easy to pull out the same number from the numerator and denominator when the numerator and denominator are small numbers. It's a little harder when they are bigger numbers. Let's look at how to simplify fractions with bigger numbers. We can use what we've learned about greatest common factors.

We know the **greatest common factor** (GCF) is the biggest factor for two or more numbers.

Example 1

Find the GCF of 8 and 12. Use the GCF to simplify $\frac{8}{12}$.

Factors of 8: 1, 2, 4, 8

Factors of 12: 1, 2, 3, 4, 6, 12

The GCF of 8 and 12 is 4.

So, to simplify $\frac{8}{12}$, we "pull out" the GCF of 4 from both the numerator and denominator. We do this by thinking about the basic facts.

$$4 \cdot ? = 8 \quad\quad 4 \cdot ? = 12$$
$$\downarrow \quad\quad\quad\quad \downarrow$$
$$4 \cdot 2 = 8 \quad\quad 4 \cdot 3 = 12$$

We find the missing factor when we think about basic facts. Then we can simplify.

$$\frac{8}{12} = \frac{4}{4} \cdot \frac{2}{3} \text{ is the same as } \frac{8}{12} = 1 \cdot \frac{2}{3}$$

We also know from basic multiplication facts that multiplying any number by 1 will equal itself. So now we have our simplified fraction.

The GCF is $\frac{8}{12} = \frac{2}{3}$.

Reinforce Understanding

If students need more practice, have them simplify these problems by pulling out the GCF:

$\frac{8}{18}$ $(GCF = 2, \frac{8}{18} = \frac{4}{9})$

$\frac{12}{32}$ $(GCF = 4, \frac{12}{32} = \frac{3}{8})$

$\frac{25}{50}$ $(GCF = 25, \frac{25}{50} = \frac{1}{2})$

$\frac{24}{48}$ $(GCF = 24, \frac{24}{48} = \frac{1}{2})$

Demonstrate

- Direct students' attention to **Example 2** on page 107. We want to simplify $\frac{6}{12}$ using the GCF. Walk through each part of the process.
- Point out the factor lists. Show students that **6** is the GCF. We can factor it out of both the numerator and the denominator. The fraction $\frac{6}{6}$ is equal to 1 because anything times 1 is itself.
- Look at the problem as $1 \cdot \frac{1}{2} = \frac{1}{2}$. Point out to students that this result makes sense if we think about it carefully because 6 is half of 12.
- Finally draw the fraction bars for $\frac{6}{12}$ and $\frac{1}{2}$, and point out that the two fractions are equivalent.

Check for Understanding

Engagement Strategy: Think, Think

Ask students to summarize the procedure for simplifying fractions. Tell them that you will call on one of them to answer. Ask them to listen for their names. Allow time for students to think of the answer. Then call on a student.

Listen for:

- *You first find the factors for the number in the numerator and then the number in the denominator.*
- *Then you circle the common factors and find the greatest common factor (GCF).*
- *Then you pull out, or factor, the GCF from both the numerator and the denominator. You divide both the numerator and the denominator by the GCF.*
- *Now you have a simplified fraction. You can use fraction bars to prove the fractions are equal.*

We call this process simplifying fractions. Let's look at another example.

Example 2

Simplify $\frac{6}{12}$ using the GCF.

Factors of 6: 1, 2, 3, 6

Factors of 12: 1, 2, 3, 4, 6, 12

The GCF of 6 and 12 is 6.

Now we "pull out" the GCF of 6 from both the numerator and denominator.

$$\frac{6}{12} = \frac{6}{6} \cdot \frac{1}{2}$$

We have the fraction $\frac{6}{6}$, which is equal to 1. We replace $\frac{6}{6}$ with the number 1.

$$\frac{6}{12} = 1 \cdot \frac{1}{2}$$

We know from our basic multiplication facts that anything times 1 is itself.

$$1 \cdot \frac{1}{2} = \frac{1}{2}$$

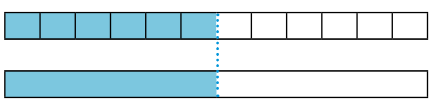
$$\frac{6}{12} = \frac{1}{2}$$

How can we be sure these two fractions are equivalent?

Let's look at fraction bars.

$\frac{6}{12}$

$\frac{1}{2}$



They are the same size.

Apply Skills Turn to *Interactive Text*, page 59.

Monitoring Progress Quiz 1

mBook Reinforce Understanding Use the *mBook Study Guide* to review lesson concepts.

Apply Skills
(*Interactive Text*, page 59)

Have students turn to *Interactive Text*, page 59, and complete the activity.

Activity 1

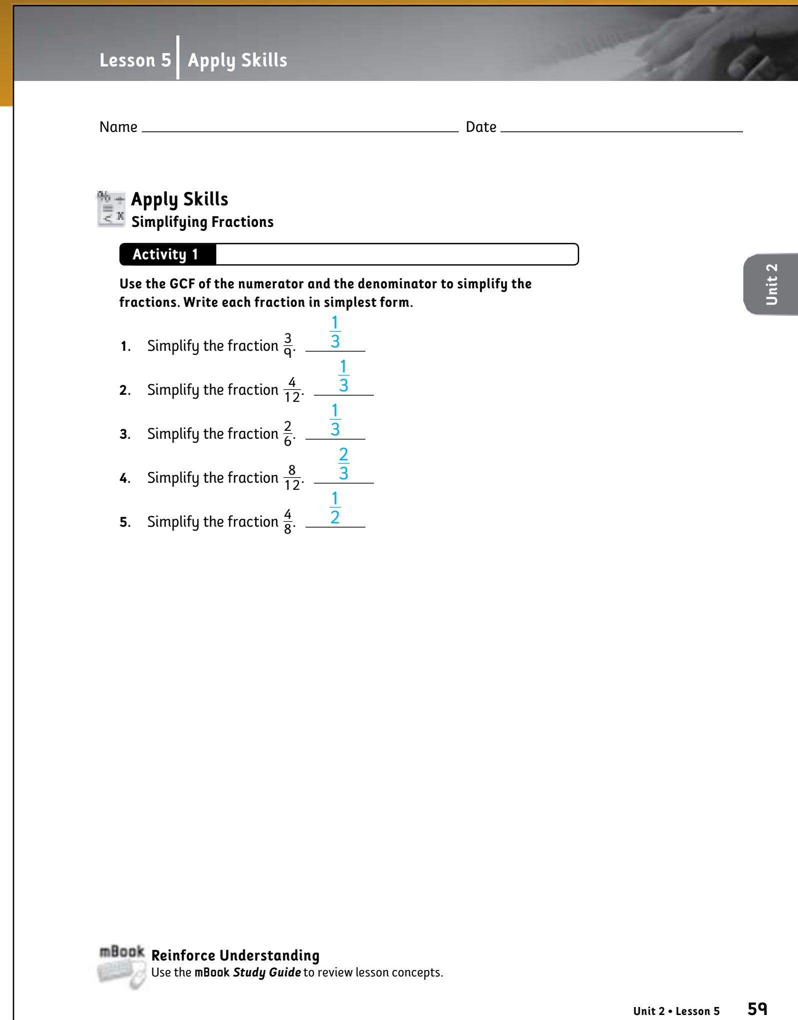
Students simplify the fractions by identifying and pulling out the GCF from the numerator and the denominator. Monitor students' work as they complete the activity.

Watch for:

- Can students identify the factors for the two numbers (numerator and denominator)?
- Can students identify common factors and the GCF?
- Can students factor out the GCF from the numerator and the denominator?

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.



Lesson 5 | Apply Skills

Name ______ Date ______

Apply Skills
Simplifying Fractions

Activity 1

Use the GCF of the numerator and the denominator to simplify the fractions. Write each fraction in simplest form.

1. Simplify the fraction $\frac{3}{9}$. $\frac{1}{3}$
2. Simplify the fraction $\frac{4}{12}$. $\frac{1}{3}$
3. Simplify the fraction $\frac{2}{6}$. $\frac{1}{3}$
4. Simplify the fraction $\frac{8}{12}$. $\frac{2}{3}$
5. Simplify the fraction $\frac{4}{8}$. $\frac{1}{2}$

Unit 2

Reinforce Understanding
Use the mBook *Study Guide* to review lesson concepts.

Unit 2 • Lesson 5 59

Monitoring Progress:
▶ Quiz 1

Assess
Quiz 1

- Administer Quiz 1 Form A in the *Assessment Book*, pages 23–24. (If necessary, retest students with Quiz 1 Form B from the *mBook Teacher Edition* following differentiation.)

Students	Assess	Differentiate
	Day 1	Day 2
All	Quiz 1 *Form A*	
Scored 80% or above		Extension
Scored Below 80%		Reinforcement

Differentiate

- Review Quiz 1 Form A with class.
- Identify students for Extension or Reinforcement.

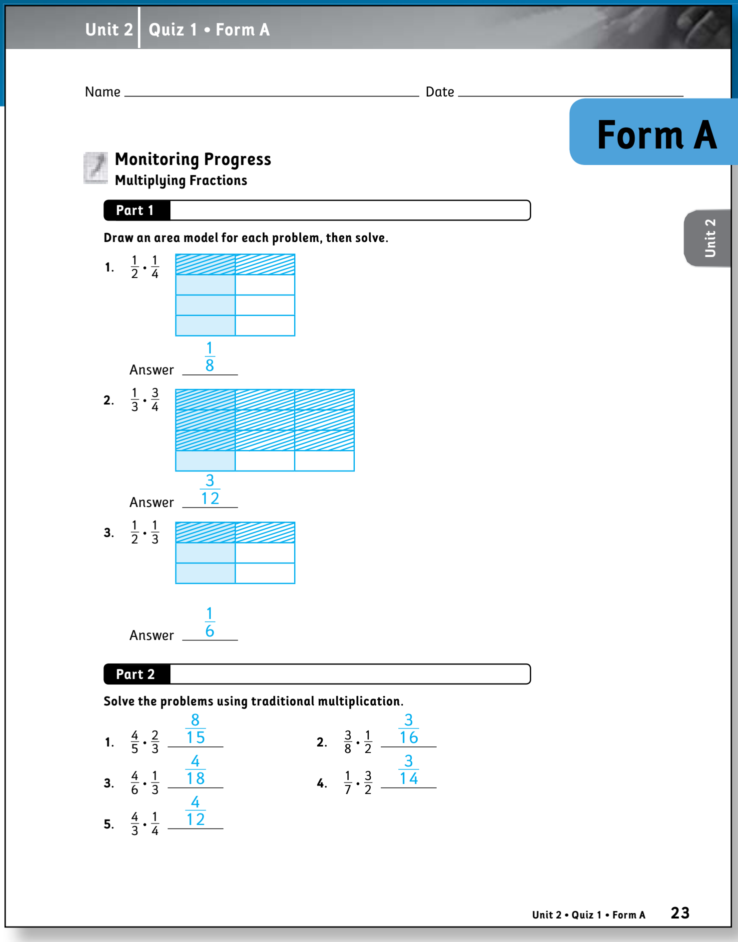
Extension

For those students who score 80 percent or better, provide the On Track! Activities from Unit 2, Lessons 1–5, from the *mBook Teacher Edition.*

Reinforcement

For those students who score below 80 percent, provide additional support in one of the following ways:

- Have students access the online tutorial provided in the *mBook Study Guide.*
- Have students complete the Interactive Reinforcement Exercises for Unit 2, Lessons 1–4, in the *mBook Study Guide.*
- Provide teacher-directed reteaching of unit concepts.



Unit 2 | Quiz 1 • Form A

Name ______ Date ______

Form A

Unit 2

Monitoring Progress
Multiplying Fractions

Part 1

Draw an area model for each problem, then solve.

1. $\frac{1}{2} \cdot \frac{1}{4}$

Answer $\frac{1}{8}$

2. $\frac{1}{3} \cdot \frac{3}{4}$

Answer $\frac{3}{12}$

3. $\frac{1}{2} \cdot \frac{1}{3}$

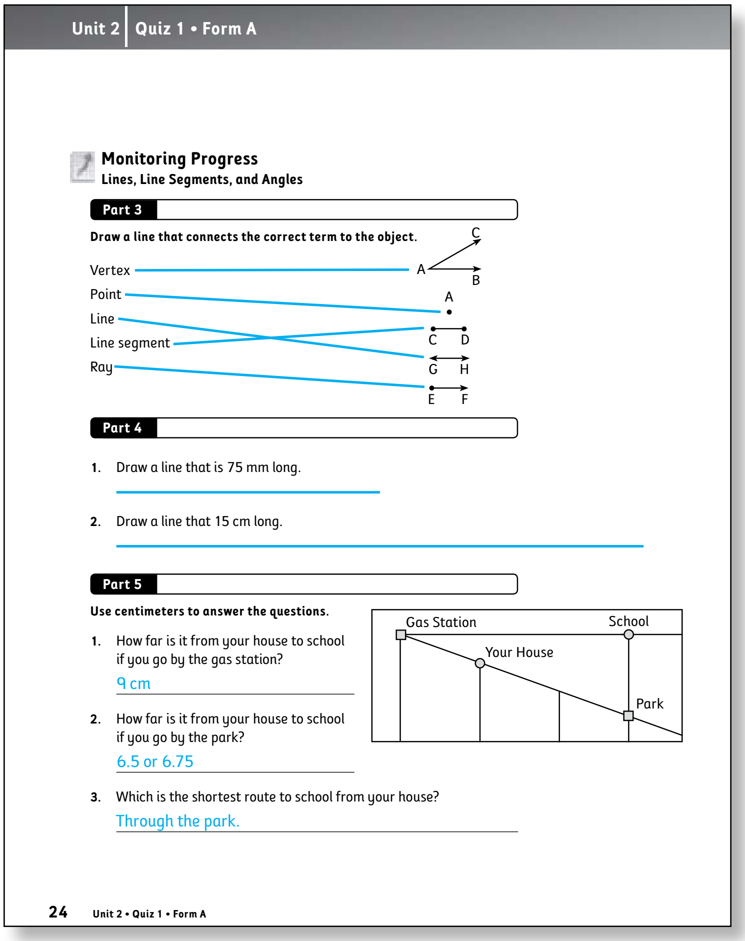
Answer $\frac{1}{6}$

Part 2

Solve the problems using traditional multiplication.

1. $\frac{4}{5} \cdot \frac{2}{3}$ $\frac{8}{15}$
2. $\frac{3}{8} \cdot \frac{1}{2}$ $\frac{3}{16}$
3. $\frac{4}{6} \cdot \frac{1}{3}$ $\frac{4}{18}$
4. $\frac{1}{7} \cdot \frac{3}{2}$ $\frac{3}{14}$
5. $\frac{4}{3} \cdot \frac{1}{4}$ $\frac{4}{12}$

Unit 2 • Quiz 1 • Form A 23



Unit 2 | Quiz 1 • Form A

Monitoring Progress
Lines, Line Segments, and Angles

Part 3

Draw a line that connects the correct term to the object.

Vertex
Point
Line
Line segment
Ray

Part 4

1. Draw a line that is 75 mm long.
2. Draw a line that 15 cm long.

Part 5

Use centimeters to answer the questions.

1. How far is it from your house to school if you go by the gas station?
   9 cm
2. How far is it from your house to school if you go by the park?
   6.5 or 6.75
3. Which is the shortest route to school from your house?
   Through the park.

24 Unit 2 • Quiz 1 • Form A

Name ______________________ Date ______________________

Form B

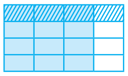
mBook

Monitoring Progress
Multiplying Fractions

Part 1

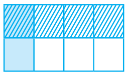
Draw an area model for each problem, then solve.

1. $\frac{3}{4} \cdot \frac{1}{4}$



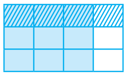
Answer $\frac{3}{16}$

2. $\frac{1}{4} \cdot \frac{1}{2}$



Answer $\frac{1}{8}$

3. $\frac{3}{4} \cdot \frac{1}{3}$



Answer $\frac{3}{12}$

Part 2

Solve the problems using traditional multiplication.

1. $\frac{5}{6} \cdot \frac{1}{3}$ $\frac{5}{18}$
2. $\frac{1}{8} \cdot \frac{1}{2}$ $\frac{1}{16}$
3. $\frac{2}{6} \cdot \frac{1}{3}$ $\frac{2}{18}$
4. $\frac{2}{7} \cdot \frac{3}{2}$ $\frac{6}{14}$
5. $\frac{4}{3} \cdot \frac{3}{4}$ $\frac{12}{12}$

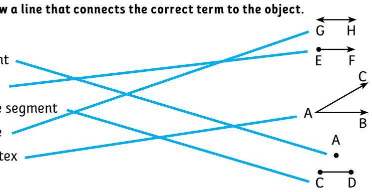
Name ______________________ Date ______________________

Monitoring Progress
Lines, Line Segments, and Angles

Part 3

Draw a line that connects the correct term to the object.

Point
Ray
Line segment
Line
Vertex



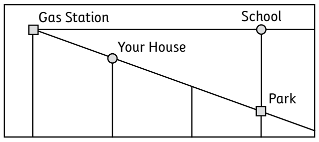
Part 4

1. Draw a line that is 14 cm long.

2. Draw a line that 65 mm long.

Part 5

Use centimeters to answer the questions.



1. How far is it from your house to school if you go by the park?
   6.75 or 7 cm
2. How far is it from the gas station to the park if you go by the school?
   8.75 or 9 cm
3. Which is the shortest route to school from your house?
   Through the park.

Homework

Go over the instructions on page 108 of the *Student Text* for each part of the homework.

Activity 1

Students multiply fractions using the traditional algorithm.

Activity 2

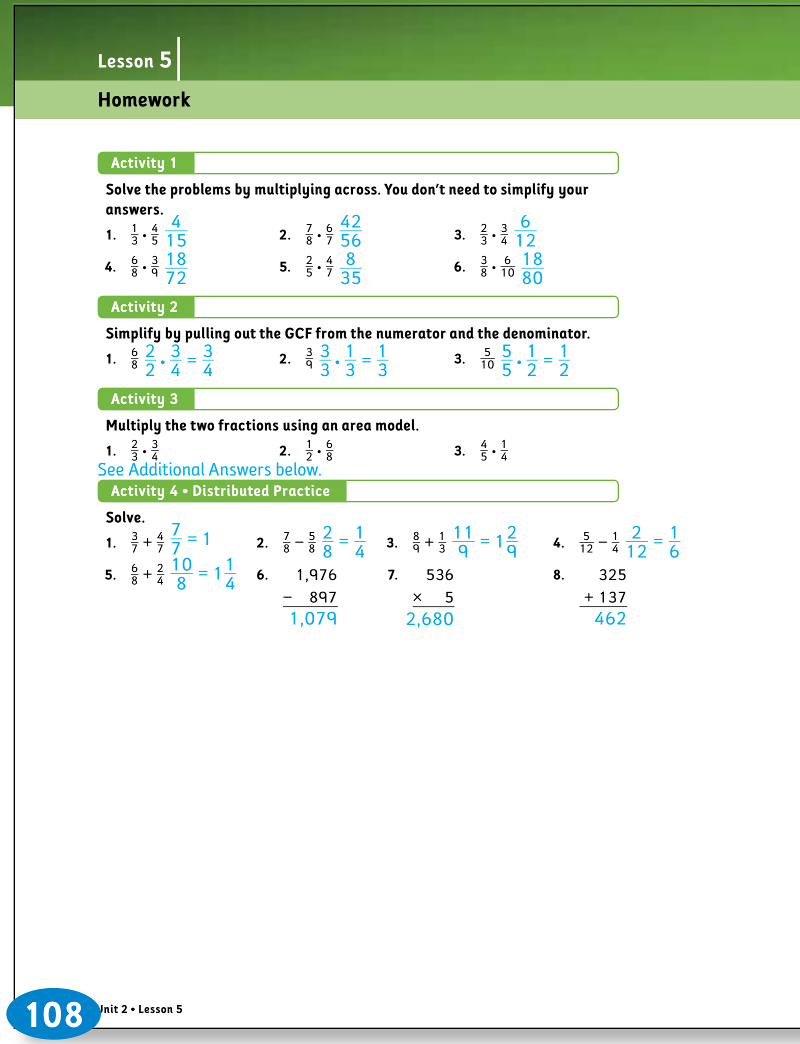
Students simplify fractions by identifying the GCF and factoring it from the numerator and denominator.

Activity 3

Students draw area models to find the product of two fractions.

Activity 4 • Distributed Practice

Students solve five problems involving addition and subtraction of fractions and three problems with whole-number operations.



Lesson 5

Homework

Activity 1

Solve the problems by multiplying across. You don't need to simplify your answers.

1. $\frac{1}{3} \cdot \frac{4}{5}$ $\frac{4}{15}$
2. $\frac{7}{8} \cdot \frac{6}{7}$ $\frac{42}{56}$
3. $\frac{2}{3} \cdot \frac{3}{4}$ $\frac{6}{12}$
4. $\frac{6}{8} \cdot \frac{3}{9}$ $\frac{18}{72}$
5. $\frac{2}{5} \cdot \frac{4}{7}$ $\frac{8}{35}$
6. $\frac{3}{8} \cdot \frac{6}{10}$ $\frac{18}{80}$

Activity 2

Simplify by pulling out the GCF from the numerator and the denominator.

1. $\frac{6}{8}$ $\frac{2}{2} \cdot \frac{3}{4} = \frac{3}{4}$
2. $\frac{3}{9}$ $\frac{3}{3} \cdot \frac{1}{3} = \frac{1}{3}$
3. $\frac{5}{10}$ $\frac{5}{5} \cdot \frac{1}{2} = \frac{1}{2}$

Activity 3

Multiply the two fractions using an area model.

1. $\frac{2}{3} \cdot \frac{3}{4}$
2. $\frac{1}{2} \cdot \frac{6}{8}$
3. $\frac{4}{5} \cdot \frac{1}{4}$

See Additional Answers below.

Activity 4 • Distributed Practice

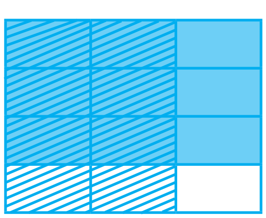
Solve.

1. $\frac{3}{7} + \frac{4}{7}$ $\frac{7}{7} = 1$
2. $\frac{7}{8} - \frac{5}{8}$ $\frac{2}{8} = \frac{1}{4}$
3. $\frac{8}{9} + \frac{1}{3}$ $\frac{11}{9} = 1\frac{2}{9}$
4. $\frac{5}{12} - \frac{1}{4}$ $\frac{2}{12} = \frac{1}{6}$
5. $\frac{6}{8} + \frac{2}{4}$ $\frac{10}{8} = 1\frac{1}{4}$
6. $1{,}976 - 897$ = 1,079
7. 536×5 = 2,680
8. $325 + 137$ = 462

Additional Answers

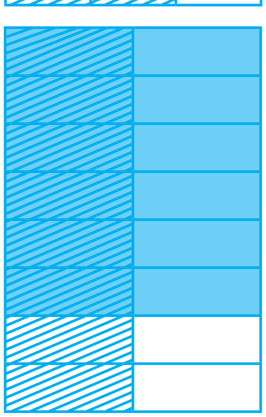
Activity 3

1.



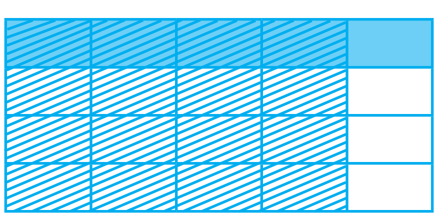
$\frac{6}{12}$

2.



$\frac{6}{16}$

3.



$\frac{4}{20}$

Lesson 6 | Problem Solving: Measuring Angles

Lesson Planner

Vocabulary Development

right angle
acute angle
obtuse angle
protractor

Skills Maintenance

Multiplying Fractions the Traditional Way

Problem Solving: Measuring Angles

In this lesson, students make an informal protractor out of waxed paper and use it to estimate based on benchmark angles.

Objective

Students will make a protractor out of waxed paper and use it to measure angles.

Homework

Students use the traditional algorithm to multiply fractions and select the type of angle shown. In Distributed Practice, students solve problems involving addition and subtraction of fractions.

Lesson 6 | Skills Maintenance

Name ______________________ Date ______________

Skills Maintenance
Multiplying Fractions the Traditional Way

Activity 1

Solve the problems by multiplying across. You do not need to simplify your answers.

1. $\frac{1}{2} \cdot \frac{3}{4}$ $\frac{3}{8}$
2. $\frac{2}{3} \cdot \frac{1}{4}$ $\frac{2}{12}$
3. $\frac{1}{5} \cdot \frac{2}{1}$ $\frac{2}{5}$
4. $\frac{7}{9} \cdot \frac{1}{5}$ $\frac{7}{45}$
5. $\frac{4}{5} \cdot \frac{5}{6}$ $\frac{20}{30}$
6. $\frac{3}{8} \cdot \frac{1}{4}$ $\frac{3}{32}$
7. $\frac{3}{7} \cdot \frac{2}{3}$ $\frac{6}{21}$
8. $\frac{1}{8} \cdot \frac{3}{1}$ $\frac{3}{8}$

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Skills Maintenance

Multiplying Fractions the Traditional Way
(*Interactive Text*, page 60)

Activity 1

Students solve multiplication problems using the traditional algorithm (i.e., multiplying across) for multiplying fractions.

Problem Solving:
▶ Measuring Angles

How do we classify angles?
(*Student Text*, page 109)

Demonstrate

Engagement Strategy: Teacher Modeling

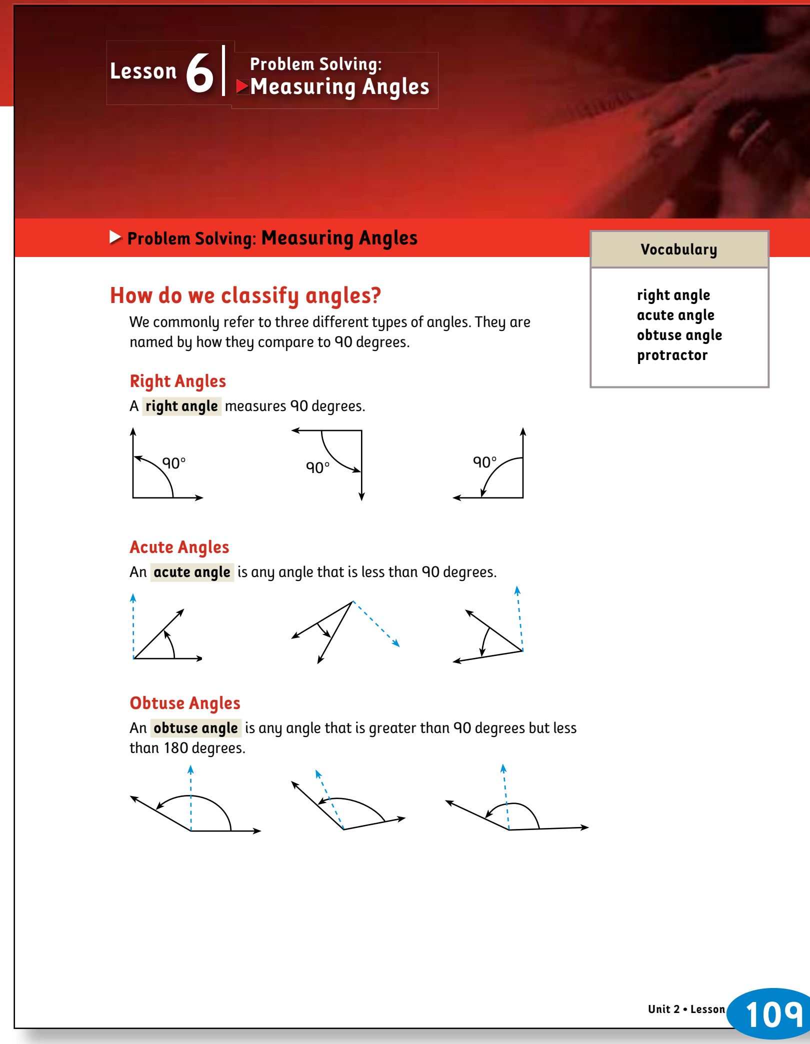
Demonstrate the definition of an angle and the naming convention for angles. This can be done in one of the following ways:

mBook: Use the *mBook Teacher Edition* for *Student Text*, page 109.

Overhead Projector: Copy the angles onto a transparency, and modify as discussed.

Board: Copy the angles onto the board, and modify as discussed.

- Be sure to explain to students that the unit of measurement we use to discuss angles is degrees. We write the degree symbol as a small circle to the upper right of the measurement (e.g., we write 45 degrees as 45°).
- Show students the first set of angles. Tell them these are all **right angles**. They measure 90 degrees. Point out that they look like corners of a rectangle. The first one looks like the lower left corner, the second looks like the upper right corner, and the third looks like the lower right corner. Explain that even though they have been rotated, they are all still right angles.
- Show students the second set of angles. Note they are all smaller than 90 degrees. Tell students these are all **acute angles**. Even though the vertex of each angle is in a different position, each is still an acute angle.


Lesson 6 | Problem Solving: ▶ Measuring Angles

▶ Problem Solving: Measuring Angles

Vocabulary
right angle
acute angle
obtuse angle
protractor

How do we classify angles?
We commonly refer to three different types of angles. They are named by how they compare to 90 degrees.

Right Angles
A **right angle** measures 90 degrees.

90° 90° 90°

Acute Angles
An **acute angle** is any angle that is less than 90 degrees.

Obtuse Angles
An **obtuse angle** is any angle that is greater than 90 degrees but less than 180 degrees.

Unit 2 • Lesson 109

- Show students the third set of angles. Tell them that all these angles are greater than 90 degrees but less than 180 degrees. These are called **obtuse angles**. Even though the vertex of each angle is in a different position, each is still an obtuse angle.
- Ask students to make observations about the different types of angles and the different representations for each type.

Listen for:

- *Right angles are all shaped like the corners of a rectangle.*
- *Acute angles are small angles. They are smaller than right angles.*
- *Obtuse angles are bigger angles. They are bigger than right angles.*

What is a protractor?

(*Student Text*, page 110)

Discuss

Have students look at page 110 of the *Student Text*. We discuss how to make an informal **protractor**. Ask students if they know what a protractor is used for.

Listen for:

- *You use it to measure angles.*
- *It's a math tool that helps you find the size of angles.*

How do we make a measuring tool?

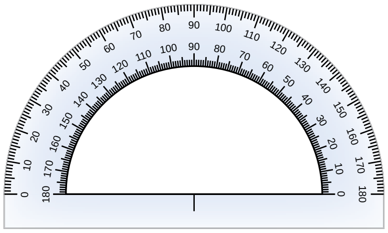
(*Student Text*, pages 110–113)

Explain

Explain that we make an informal protractor out of waxed paper before we use a real protractor. This approach helps us better understand the formal protractor and angles.

What is a protractor?

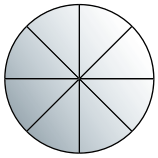
Surveyors use a number of different tools to measure distances and angles. One of the tools for measuring angles is a **protractor**.



Architects, engineers, and surveyors use a protractor to measure angles in small units called degrees. Remember, the symbol for degrees is °.

How do we make a measuring tool?

Before we practice using an actual protractor, we are going to make one that will help us understand angles. We will make it out of waxed paper. It will look like this.



Demonstrate

- Prepare waxed paper squares ahead of time, and distribute them to students. Then walk through the steps of making a protractor out of waxed paper, as outlined on page 111 of the *Student Text*. Check to make sure students are following along as you complete each step.

STEP 1

- Fold a square sheet of wax paper in half.

STEP 2

- Fold the paper in half again. Be sure to follow the direction of the fold, as shown.

STEP 3

- With the fold at the bottom, fold the left corner to the right corner to make a triangle.

STEP 4

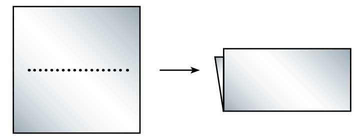
- Make sure the final fold is slanting to the right, with the open ends down. Then cut an arc with scissors. When students open up their waxed paper, it should be a circle. If it turns out flower-shaped, have them trim the edges.

- You might need to help some students who don't have the motor skills to fold, cut, etc.

Steps for Making Measuring Tools

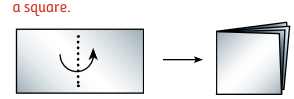
STEP 1

Start with a square sheet of waxed paper, and fold it in half. The dotted line shows where to fold it.



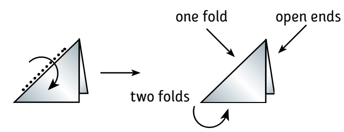
STEP 2

With the fold at the bottom, fold the paper in half again to make a square.



STEP 3

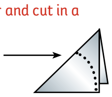
The first fold should still be at the bottom. The second fold should be on the left. Now fold it into a triangle by matching the upper left corner to the lower right corner.



STEP 4

The final fold should be slanting to the right, with the open ends down and to the right. With a pair of scissors, hold it together and cut in a slight arc. Start cutting from the bottom right corner.

The arc should end about $\frac{3}{4}$ of the way up the diagonal line made from the final fold.



Unfold the paper to see a circle divided into eighths.

If it comes out flower shaped, trim the edges until it looks more like a circle. Since it's an informal measuring device, it does not need to be a perfect circle.

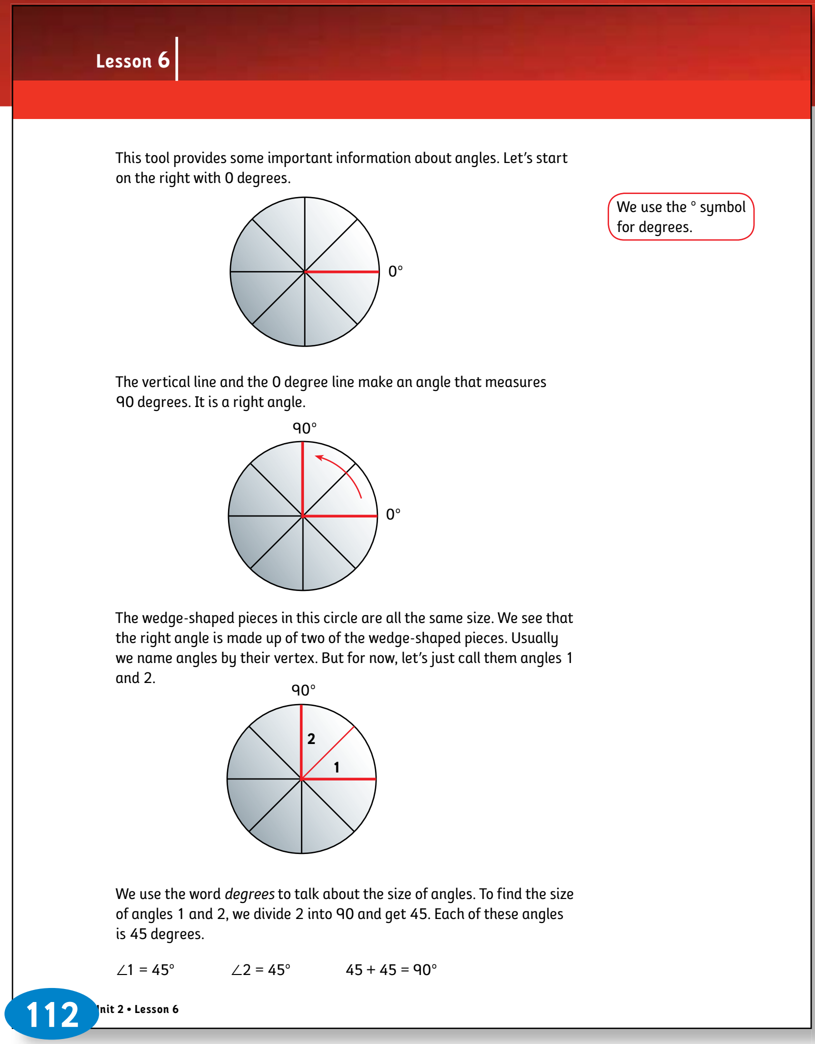
How do we make a measuring tool?
(*continued*)

Demonstrate

- Have students open their waxed-paper circle. Point out the different angles on the circle, as shown on page 112 of the *Student Text.*
- Once students complete their protractors, have them write the benchmarks on them. Show them where to place 0 degrees and 90 degrees. Show them that 45 degrees is halfway in between.
- Show the 90-degree angle made by the vertical line and the zero-degree line. Have students locate a right angle on their tool.
- Next show the two angles that make up the right angle (angles 1 and 2 in the *Student Text*). Explain that these angles are equal and make up a right angle. They each measure 45 degrees: 45 + 45 = 90 degrees.
- Ask students to show the other right angles in their tools.

Watch for:

- Students show $\frac{1}{4}$ of the circle in any of the four quadrants of the circle.


Lesson 6

This tool provides some important information about angles. Let's start on the right with 0 degrees.

We use the ° symbol for degrees.

0°

The vertical line and the 0 degree line make an angle that measures 90 degrees. It is a right angle.

90°

0°

The wedge-shaped pieces in this circle are all the same size. We see that the right angle is made up of two of the wedge-shaped pieces. Usually we name angles by their vertex. But for now, let's just call them angles 1 and 2.

90°

2

1

We use the word *degrees* to talk about the size of angles. To find the size of angles 1 and 2, we divide 2 into 90 and get 45. Each of these angles is 45 degrees.

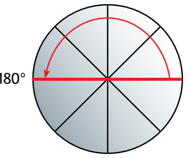
$\angle 1 = 45°$ $\angle 2 = 45°$ $45 + 45 = 90°$

112 Unit 2 • Lesson 6

Demonstrate

- Next turn to page 113 of the *Student Text*. Explain that there is a special symbol to represent degrees. Students might be familiar with it because it is also used when talking about temperature. The symbol is a small circle placed to the upper right of the number.
- Write out a few examples: **45°**, **90°**, and **180°**.
- Refer to the tool. Explain that all horizontal lines measure 180°. Point out all the horizontal lines, which are creases, in the tool. Show students that the horizontal line is made up of two right angles together: **90°+ 90° = 180°**.
- Tell students that this tool is a good introduction to using a real protractor. For now, the important thing is to understand angles. We measure more precisely later.
- If students need more time to familiarize themselves with the benchmarks, ask them to think about the following angles and their place on the protractor:
  - 30 degrees
  - 60 degrees
  - 135 degrees
  - 179 degrees
  - 2 degrees
- Make sure students understand that this tool provides us with an estimate of the angles, which explains the variation in measurements.

Let's look at the circle measuring tool again. The horizontal line is straight, and it measures 180 degrees. All straight lines have a measurement of 180 degrees.



This tool is good for finding the measure of angles. Some of the measurements made with it will be exact. Other measurements will be approximations. For now, we do not have to worry about being exact.

Problem-Solving Activity
Turn to *Interactive Text*, page 61.

Reinforce Understanding
Use the *mBook Study Guide* to review lesson concepts.

Problem-Solving Activity

(*Interactive Text,* pages 61–62)

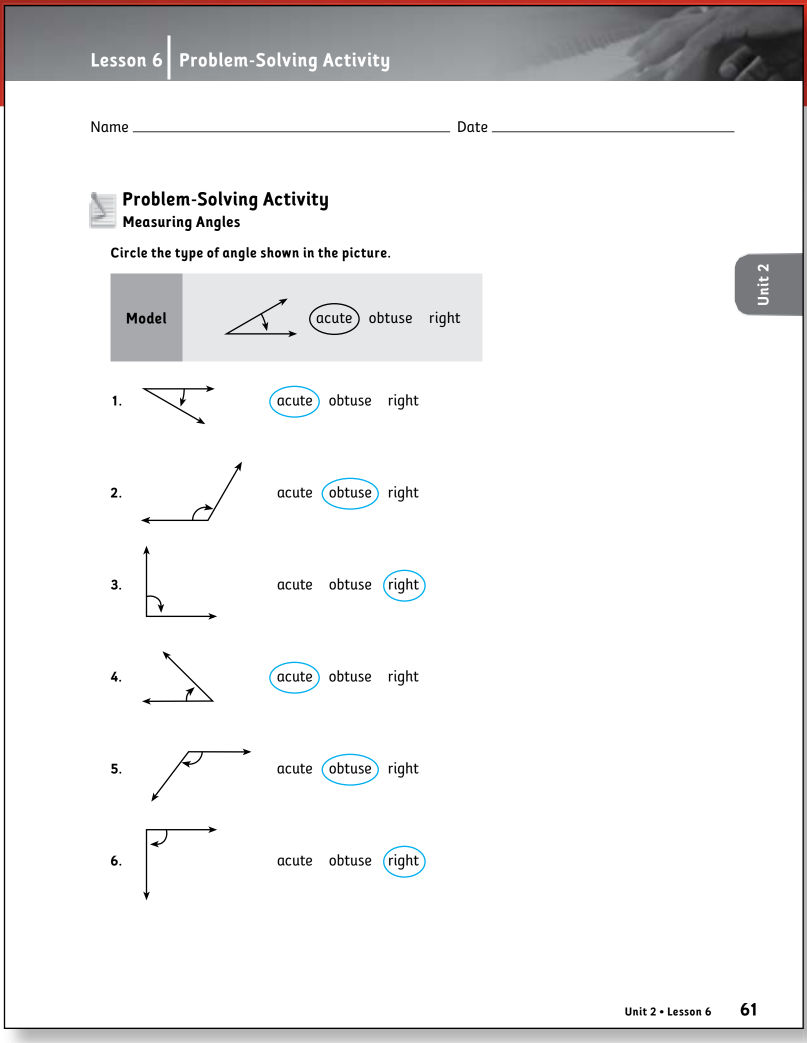
Have students turn to page 61 in the *Interactive Text*, and complete the activity.

Students identify the type of angle by circling one of the following: acute, obtuse, or right.

Monitor students' work as they complete the activity.

Watch for:

- Can students identify the different types of angles and associate the correct formal name with an angle based on its size?
- Do students have difficulty remembering the formal names? If so, you might want to try mnemonic cues (e.g., acute is a "cute little angle").



Lesson 6 | Problem-Solving Activity

Name ______________________ Date ______________

Unit 2

Problem-Solving Activity
Measuring Angles

Circle the type of angle shown in the picture.

Model acute obtuse right

1. acute obtuse right
2. acute obtuse right
3. acute obtuse right
4. acute obtuse right
5. acute obtuse right
6. acute obtuse right

Unit 2 • Lesson 6 61

Direct students to *Interactive Text*, page 62, and read through the instructions together.

Students use their waxed-paper measuring tools to measure the angles.

Monitor students' work as they complete this activity.

Watch for:

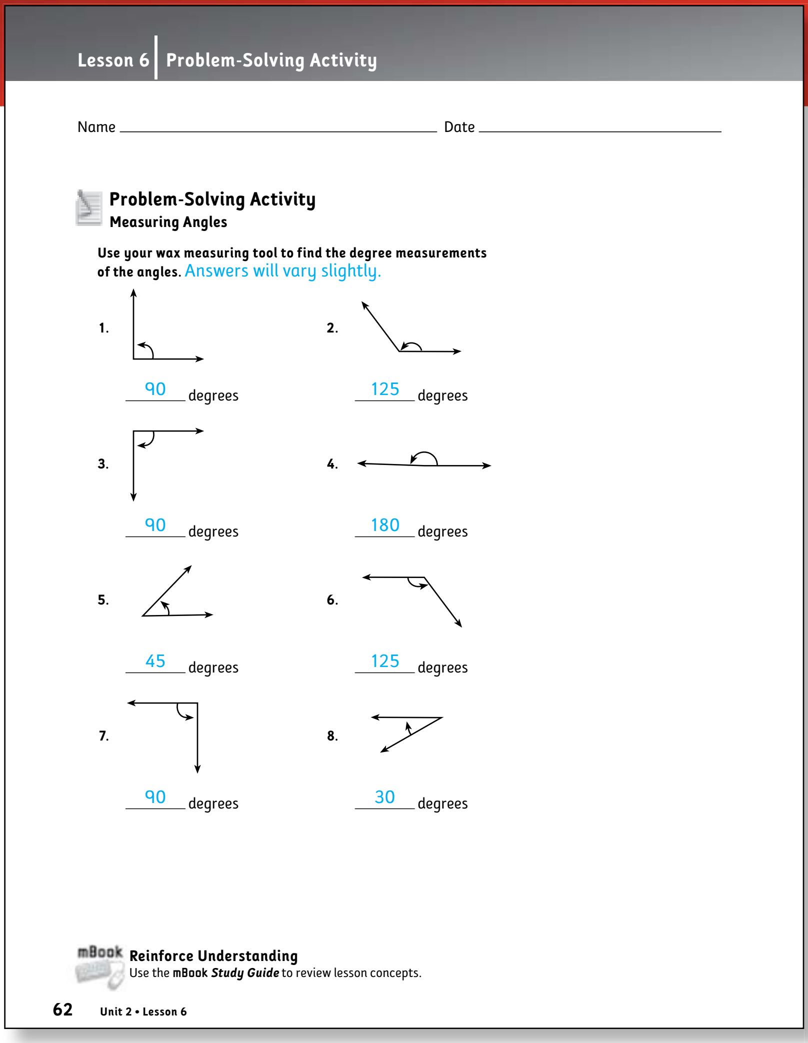
- Can students make reasonable estimates using the informal protractor and the benchmarks?
- Do students use their knowledge of types of angles to help them make reasonable estimates?
- Can students identify the various types of angles even if they are positioned with a different orientation? (e.g., an upside down right angle or an acute angle facing downward.)

Once students complete the activity, go over their answers. Student answers will vary. Ask students to explain why their answers might not all be the same.

Listen for:

- *The measuring tool is not a formal protractor.*
- *Measurements are estimates. It's hard to be precise when measuring angles because it's hard to line things up perfectly.*
- *The informal protractor might have some of the benchmarks off by a little.*

mBook **Reinforce Understanding**
Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.


Lesson 6 | Problem-Solving Activity

Name ______________________ Date ______________

Problem-Solving Activity
Measuring Angles

Use your wax measuring tool to find the degree measurements of the angles. Answers will vary slightly.

1. 90 degrees
2. 125 degrees
3. 90 degrees
4. 180 degrees
5. 45 degrees
6. 125 degrees
7. 90 degrees
8. 30 degrees

mBook **Reinforce Understanding**
Use the mBook *Study Guide* to review lesson concepts.

62 Unit 2 • Lesson 6

Homework

Go over the instructions on page 114 of the *Student Text* for each part of the homework.

Activity 1

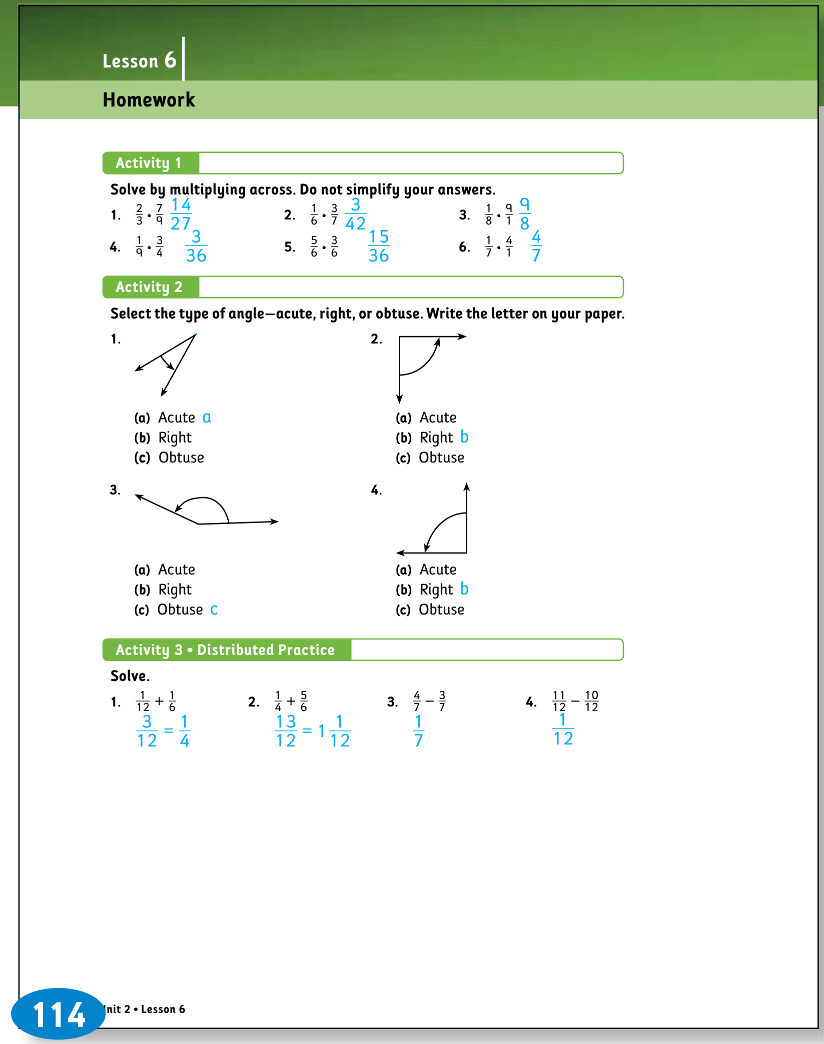
Students use the traditional algorithm to multiply fractions.

Activity 2

Students select the type of angle shown.

Activity 3 • Distributed Practice

Students practice addition and subtraction of fractions.



Lesson 6

Homework

Activity 1

Solve by multiplying across. Do not simplify your answers.

1. $\frac{2}{3} \cdot \frac{7}{9}$ $\frac{14}{27}$
2. $\frac{1}{6} \cdot \frac{3}{7}$ $\frac{3}{42}$
3. $\frac{1}{8} \cdot \frac{9}{1}$ $\frac{9}{8}$
4. $\frac{1}{9} \cdot \frac{3}{4}$ $\frac{3}{36}$
5. $\frac{5}{6} \cdot \frac{3}{6}$ $\frac{15}{36}$
6. $\frac{1}{7} \cdot \frac{4}{1}$ $\frac{4}{7}$

Activity 2

Select the type of angle—acute, right, or obtuse. Write the letter on your paper.

1. (a) Acute a
(b) Right
(c) Obtuse

2. (a) Acute
(b) Right b
(c) Obtuse

3. (a) Acute
(b) Right
(c) Obtuse c

4. (a) Acute
(b) Right b
(c) Obtuse

Activity 3 • Distributed Practice

Solve.

1. $\frac{1}{12} + \frac{1}{6}$ $\frac{3}{12} = \frac{1}{4}$
2. $\frac{1}{4} + \frac{5}{6}$ $\frac{13}{12} = 1\frac{1}{12}$
3. $\frac{4}{7} - \frac{3}{7}$ $\frac{1}{7}$
4. $\frac{11}{12} - \frac{10}{12}$ $\frac{1}{12}$

114 Unit 2 • Lesson 6

Lesson 7 ▶Multiplying Fractions and Simplifying Answers

Problem Solving:
▶Measuring and Drawing Angles

Lesson Planner

Build Vocabulary

commute

Skills Maintenance

Multiplication With Fractions

Building Number Concepts:

▶ Multiplying Fractions and Simplifying Answers

We combine two skills we have been working on: multiplying across to solve fraction multiplication and simplifying the answer. Procedural tasks are not as difficult for students if you break the tasks into smaller, more manageable parts.

Objective

Students will multiply fractions by multiplying across, then simplify the answers.

Problem Solving:

▶ Measuring and Drawing Angles

We introduce the formal protractor for measuring angles.

Objective

Students will measure angles with a protractor.

Homework

Students multiply fractions and simplify the answers, then identify the types of angles. In Distributed Practice, students solve five problems involving addition and subtraction of fractions and one problem involving whole-number operations.

Name ______________________ Date ______________

Skills Maintenance
Multiplication With Fractions

Activity 1

Multiply the fractions by multiplying across. You do not need to simplify your answers.

1. $\frac{2}{4} \cdot \frac{2}{9}$ $\frac{4}{36}$
2. $\frac{3}{8} \cdot \frac{5}{9}$ $\frac{15}{72}$
3. $\frac{4}{6} \cdot 3$ $\frac{12}{6}$
4. $\frac{8}{9} \cdot \frac{5}{6}$ $\frac{40}{54}$

Unit 2

Skills Maintenance

Multiplication With Fractions
(*Interactive Text*, page 63)

Activity 1

Students multiply fractions using the traditional algorithm.

Building Number Concepts:
▶ Multiplying Fractions and Simplifying Answers

How do we multiply a fraction and simplify the answer?

(*Student Text*, page 115)

Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate how to multiply fractions using the traditional algorithm in one of the following ways:

mBook: Use the *mBook Teacher Edition* for *Student Text*, page 115.

Overhead Projector: Copy the fractions onto a transparency, and modify as discussed.

Board: Copy the fractions onto the board, and modify as discussed.

- Show the problem $\frac{2}{3} \cdot \frac{1}{4}$.
- Ask students to tell you the steps to solve this problem using the traditional algorithm.

Listen for:

- *First you multiply across the numerators: $2 \cdot 1 = 2$. The numerator is 2.*
- *Next you multiply across the denominators: $3 \cdot 4 = 12$. The denominator is 12.*
- *The answer is $\frac{2}{12}$.*

- Walk through multiplying across the numerator. Show the numerator as $\mathbf{2 \cdot 1}$. Multiply across the denominator. Show the denominator as $\mathbf{3 \cdot 4}$.
- After you multiply across, you get $\mathbf{\frac{2}{12}}$.
- Next demonstrate how we simplify fractions. Point out that $\frac{2}{12}$ is not simplified. Tell students that we have to simplify most products of traditional multiplication.
- Remind students that to simplify, we pull out the GCF, which is 2, from both the numerator and the denominator: $\mathbf{\frac{2}{2} \cdot \frac{1}{6} = 1 \cdot \frac{1}{6}}$. The answer in its simplest form is $\mathbf{\frac{1}{6}}$.
- Make sure students see the step for doing this. Remind them about fractions equal to 1. Note that all the statements are equivalent even though they look different. For example, $\frac{2}{2} \cdot \frac{1}{6}$ is the same as $\frac{2}{12}$. We write it this way so we can pull out $\frac{2}{2}$, which is equal to 1. Anything times 1 is itself.

Lesson 7 ▶Multiplying Fractions and Simplifying Answers
Problem Solving:
▶Measuring and Drawing Angles

▶Multiplying Fractions and Simplifying Answers

Vocabulary
commute

How do we multiply a fraction and simplify the answer?

We know how to multiply fractions. We also know how to simplify fractions. Many times when we multiply fractions, we will not get the simplest form for the answer. So now we will combine these two tasks.

Let's look at the steps for multiplying fractions and simplifying the product.

$$\frac{2}{3} \cdot \frac{1}{4}$$

We begin by multiplying the two fractions. We use the traditional method, or multiply across.

$$\frac{2}{3} \cdot \frac{1}{4} = \frac{2 \cdot 1}{3 \cdot 4} = \frac{2}{12}$$

The answer is $\frac{2}{12}$. But this fraction is not in simplest form. To simplify the fraction we pull the GCF out of both the numerator and the denominator.

The GCF is 2.

$$\frac{2}{12} = \frac{2}{2} \cdot \frac{1}{6}$$
$$\frac{2}{12} = 1 \cdot \frac{1}{6}$$

The answer in simplest form is $\frac{1}{6}$.

Unit 2 • Lesson 115

How do we commute?

(*Student Text*, pages 116–117)

Build Vocabulary

Explain to students that sometimes we can **commute**, or create a fraction equal to 1, and avoid simplifying.

Demonstrate

- Direct students' attention to **Example 1** on page 116 of the *Student Text*, which shows how to commute the numbers. Remind students that the commutative property allows us to change the order of the numbers we are adding or multiplying.
- Point out that this property only works for addition or multiplication. In this example, we are multiplying $\frac{2}{3} \cdot \frac{3}{5}$. We multiply across and get $\frac{6}{15}$. Then we simplify by pulling out the GCF, 3: $\frac{6}{15} = \frac{3}{3} \cdot \frac{2}{5} = 1 \cdot \frac{2}{5}$. The answer is $\frac{2}{5}$.
- Tell students that we rewrite the problem so that we see the $\frac{3}{3}$ while we solve the multiplication.

How do we commute?

Sometimes when we multiply fractions, the numerator of one of the fractions is the same as the denominator of the other fraction. In that case, we can **commute** to create a fraction equal to 1 and avoid the simplification step.

When we commute in an addition or multiplication problem, we simplify the problem by changing the order of the numbers. Let's look at an example.

Example 1

Multiply the fractions.

$$\frac{2}{3} \cdot \frac{3}{5}$$

We begin by multiplying the two fractions. We use the traditional method, or multiply across.

$$\frac{2}{3} \cdot \frac{3}{5} = \frac{2 \cdot 3}{3 \cdot 5}$$
$$= \frac{6}{15}$$

The answer is $\frac{6}{15}$. But this fraction is not in simplest form. We can pull the GCF, 3, out of both the numerator and the denominator. It looks like this:

$$\frac{6}{15} = \frac{3}{3} \cdot \frac{2}{5}$$
$$= 1 \cdot \frac{2}{5}$$

The answer is $\frac{2}{5}$.

We would simplify this problem by commuting, or changing the order of the 3s in the numerator and denominator.

How do we commute? *(continued)*

Demonstrate

- Turn to page 117 of the *Student Text,* and explain how to avoid simplifying by commuting the numbers. Explain that in this situation, because the numerator of one of the fractions is the same as the denominator of the other fraction, we can commute the numbers so that the threes are on top of each other: $\frac{\mathbf{3 \cdot 2}}{\mathbf{3 \cdot 5}}$.
- Then point out that we get a fraction equal to 1: $\frac{\mathbf{3}}{\mathbf{3}} \cdot \frac{\mathbf{2}}{\mathbf{5}}$. Finally, we multiply by 1: $\mathbf{1} \cdot \frac{\mathbf{2}}{\mathbf{5}} = \frac{\mathbf{2}}{\mathbf{5}}$. The fraction $\frac{2}{5}$ is the answer, $\frac{6}{15}$, already simplified.
- Ask students to summarize the steps for solving and simplifying.

Listen for:

- *First you multiply across and get an answer.*
- *If you notice that one of the numerators is the same as one of the denominators, you commute so that you have a number over itself, or 1. That saves you the simplification step.*
- *You look for the GCF of the numerator and the denominator and you pull it out of both.*

Check for Understanding

Engagement Strategy: Think Tank

Distribute strips of paper to students, and have them write their names on the papers. Then have students solve the following multiplication problem by commuting the numbers and simplifying: $\frac{\mathbf{4}}{\mathbf{7}} \cdot \frac{\mathbf{7}}{\mathbf{8}}$. Collect the strips of paper, and put them in a container. Draw a strip of paper, and read the answer out loud. If correct, congratulate that student. If incorrect, invite a student volunteer to explain the solution $\left(\frac{1}{2}\right)$.

Let's go back to the step where we multiply across. We see that there is a 3 on top and a 3 on the bottom. We can commute so that the 3s are one on top of the other.

$$\frac{2}{3} \cdot \frac{3}{5} = \frac{2 \cdot 3}{3 \cdot 5}$$
$$= \frac{3 \cdot 2}{3 \cdot 5} \leftarrow \text{Commute}$$
$$= \frac{3}{3} \cdot \frac{2}{5} \leftarrow \text{Get a fraction equal to 1}$$
$$= 1 \cdot \frac{2}{5} \leftarrow \text{Multiply by 1}$$

The answer is $\frac{2}{5}$.

It is a simplified fraction. We do not need to perform the simplification step. We already have the simplified answer for $\frac{6}{15}$.
It is $\frac{2}{5}$.

Apply Skills
Turn to ***Interactive Text***, page 64.

Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

Reinforce Understanding

If students need additional practice, use these problems:

$\frac{\mathbf{6}}{\mathbf{7}} \cdot \frac{\mathbf{2}}{\mathbf{6}} \left(\frac{2}{7}\right)$

$\frac{\mathbf{1}}{\mathbf{2}} \cdot \frac{\mathbf{2}}{\mathbf{9}} \left(\frac{1}{9}\right)$

Apply Skills
(*Interactive Text*, page 64)

Have students turn to *Interactive Text*, page 64, and complete the activity.

Activity 1

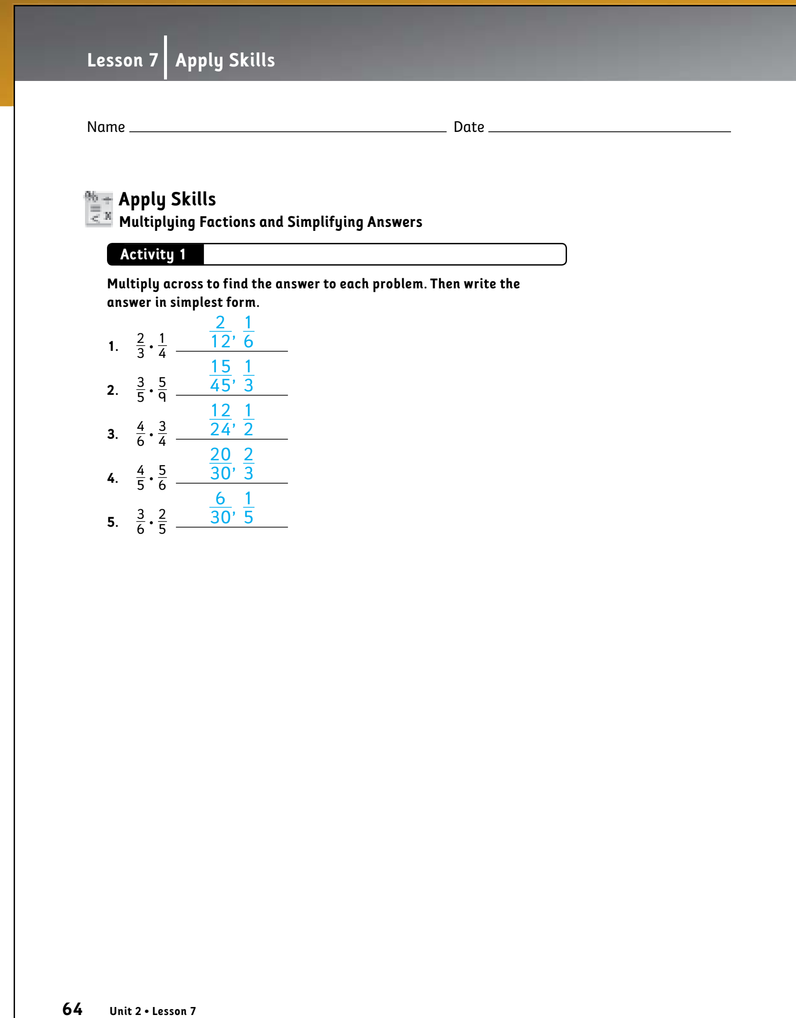
Students multiply and simplify fractions. Monitor students' work as they complete the activity.

Watch for:

- Do students remember the traditional algorithm for multiplication and perform it with efficiency?
- Do students recognize when they can commute and get a fraction equal to one to avoid the extra simplification step?
- Can students find the GCF for the numerator and the denominator when simplifying?

mBook

Reinforce Understanding
Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.


Lesson 7 | Apply Skills

Name ______________________ Date ______________

Apply Skills
Multiplying Factions and Simplifying Answers

Activity 1

Multiply across to find the answer to each problem. Then write the answer in simplest form.

1. $\frac{2}{3} \cdot \frac{1}{4}$ $\frac{2}{12}, \frac{1}{6}$
2. $\frac{3}{5} \cdot \frac{5}{9}$ $\frac{15}{45}, \frac{1}{3}$
3. $\frac{4}{6} \cdot \frac{3}{4}$ $\frac{12}{24}, \frac{1}{2}$
4. $\frac{4}{5} \cdot \frac{5}{6}$ $\frac{20}{30}, \frac{2}{3}$
5. $\frac{3}{6} \cdot \frac{2}{5}$ $\frac{6}{30}, \frac{1}{5}$

64 Unit 2 • Lesson 7

Problem Solving:
▶ Measuring and Drawing Angles

How do we compare a formal protractor with the one we made?
(*Student Text*, page 118)

Connect to Prior Knowledge
Remind students of the informal protractor they made in the previous lesson.

Link to Today's Concept
In today's lesson students learn to measure and draw angles using a formal protractor.

Demonstrate
- Direct students' attention to page 118 of the *Student Text*. We discuss the steps for measuring with a protractor. Read the text, and have students compare the formal protractor with the informal protractor they made.
- Ask students to make observations about the similarities and differences.

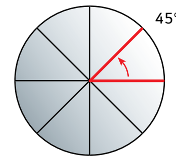
Listen for:
- *The homemade protractor is a circle. The formal one is a half circle.*
- *There are lots of numbers on the formal protractor. There are just a few on the homemade one.*
- *You can see some of the same numbers. There are 45, 90, 180, and 0 on both the measuring tools.*

Lesson 7

▶Problem Solving: **Measuring and Drawing Angles**

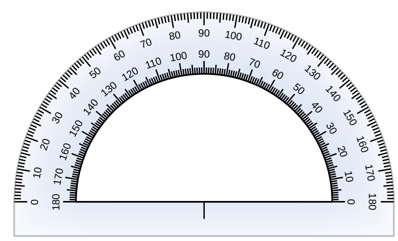
How do we compare a formal protractor with the one we made?

Let's look again at the measurement tool we made in a previous lesson. Notice that the tool has eight wedges, or triangle-like shapes, and each one measures 45 degrees.



If we multiply 45 × 8, we find that a circle has 360 degrees. This tool allows us to estimate angles up to 360 degrees based on the nearest 45 degrees. The tool works well for estimating angles, but there are times when exact measurements are needed. A surveyor, for instance, needs exact measurements in his or her work.

Architects, engineers, and surveyors find these exact measurements using a protractor. Let's take a moment to compare the two tools—the one we made and a formal protractor.



118 Unit 2 • Lesson 7

How do we measure angles using a formal protractor?

(*Student Text*, page 119)

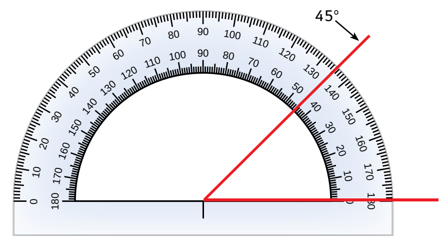
Demonstrate

- Read the text on page 119 of the *Student Text*. Remind students that angles are measured in degrees. There are **360 degrees** in a circle. There are **180 degrees** in a half circle.
- Draw an angle (about 45 degrees) on the board or overhead projector. Demonstrate how to measure this angle.
- Remind students to line up the base at one of the rays of the angle. Then see where the other ray lines up on the protractor.
- Point out that it lines up at 135, or 45. The biggest confusion for most students when using a protractor is what number they are supposed to line up with.
- Remind students of the types of angles we discussed. Ask what type of angle the angle looks like.
- Remind students that acute angles are less than 90 degrees. Then elicit from students which number is less than 90: 135 or 45. When we think about it this way, it is very clearly 45.
- You might want to place several different types of angles on the board and practice measuring them, reminding students each time to use their benchmarks and types of angles to determine which number to read on the protractor.

Lesson 7

How do we measure angles using a formal protractor?

The protractor is a semicircle, and it has small lines along its curved edges. Each one of these is a degree. We can measure angles very closely using this tool. When using a protractor, we look for the nearest large unit first and then count carefully to measure the exact degree for an angle.



When we read a protractor, we always start at zero. In this case, we start from zero and stop at 45 degrees. Remembering to start from 0 degrees and not 180 degrees will help us avoid mistakes.

Unit 2 • Lesson 119

How do we use a protractor to help us draw angles?

(*Student Text*, pages 120–121)

Demonstrate

- Next have students look at page 120 of the *Student Text*, where we demonstrate how to draw an angle. Go through all the steps carefully. Have students take out a sheet of paper to complete the steps as you discuss them.

STEP 1

- Tell students to draw a line segment.

STEP 2

- Line up the center of the protractor with the left end point of the line segment.

STEP 3

- Look for the angle on the curved part of the protractor. Read through the text, and remind students about the types of angles and the benchmarks.
- Review the two rows of numbers with students. Again, many students get confused at this step because they don't know which number to choose.

Lesson 7

How do we use a protractor to help us draw angles?

Protractors can also be used to draw angles. Here are the steps.

Steps for Drawing an Angle Using a Protractor

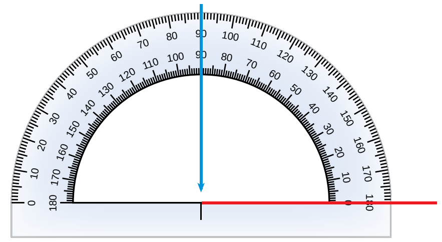
STEP 1

Draw a line segment using a straightedge. We can use a ruler or the straight part of our protractor.

STEP 2

Line up the center of the protractor with the left endpoint of the line segment.

In the diagram, an arrow points to the center of the protractor. It lines up with the left endpoint of the line segment.



STEP 3

Look for the angle we want to draw on the curved edge of the protractor. One important thing to notice is there are two rows of numbers. One row of numbers counts by tens from 0 to 180, and the other row counts by tens from 180 to 0. The rows meet at 90, which is the center.

Our knowledge of the types of angles will help us. Suppose we want to draw a 60-degree angle. We look at the numbers and see that there are two places on the protractor labeled with 60. We need to think about what type of angle 60 degrees is. It's an acute angle because it's less than 90 degrees. So we look for the 60 on the protractor that makes an acute angle with the line segment we have previously drawn.

120 Unit 2 • Lesson 7

Demonstrate

- Continue walking through the steps of using a protractor to draw angles on page 121 of the *Student Text*.

STEP 4

- After locating the number on the protractor, mark it with a pencil.

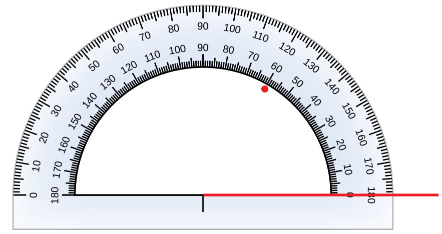
STEP 5

- Draw the angle with a straightedge, and add an arrow so that the lines become rays.
- Finally remind students one more time to use good number sense by thinking about what type of angle they are measuring or drawing. Tell them they will not choose the wrong number on the protractor if they use good number sense.

STEP 4

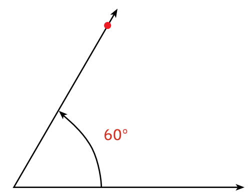
Once we have found the number on the protractor for the angle we want to draw, we put a pencil mark by it.

The picture shows a pencil mark for a 60-degree angle.



STEP 5

Use a straightedge to connect the left endpoint of the line segment with the pencil mark that we made on the paper. Add an arrow at the end of each line segment to make the line segments rays. We also draw the curved arrow to show the angle.



Problem-Solving Activity
Turn to *Interactive Text*, page 65.

Reinforce Understanding
Use the *mBook Study Guide* to review lesson concepts.

Problem-Solving Activity

(*Interactive Text*, pages 65–66)

Turn to the activity on *Interactive Text* pages 65–66, and read the instructions together.

Tell students they are drawing and measuring angles using their protractors. You might want to select a few out of the six problems to focus on if students seem unfamiliar with the protractor. They might need more time for each problem. The last problems are included as bonus or challenge problems; you need to determine if your group is ready for these problems.

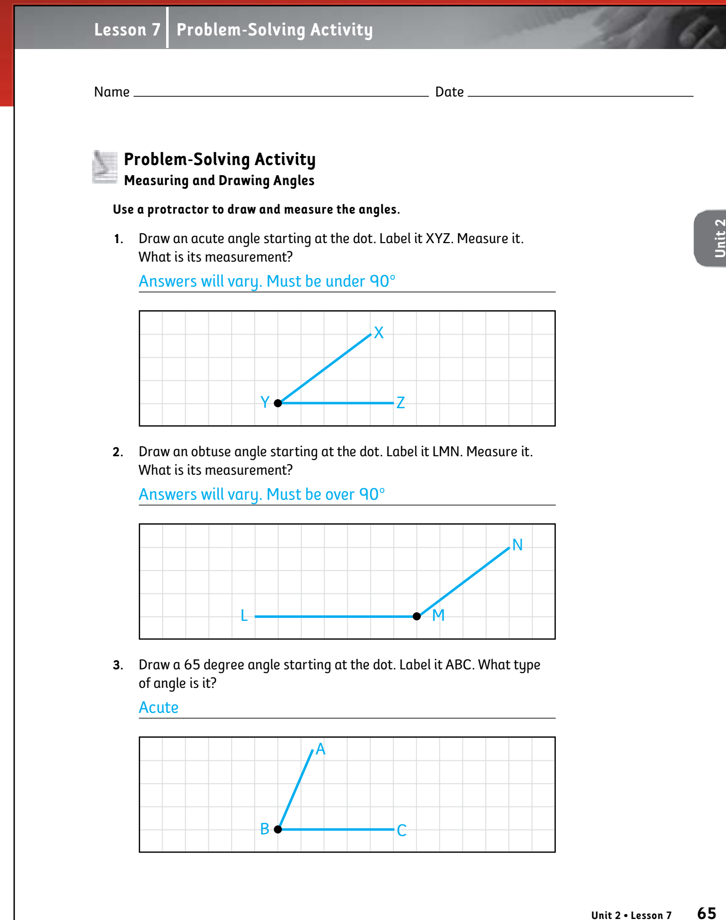
Monitor students' work as they complete this activity.

Watch for:

- Can students line up the protractor correctly?
- Can students draw a specific angle?
- Can students measure angles?

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.


Lesson 7 | Problem-Solving Activity

Name ______ Date ______

Problem-Solving Activity
Measuring and Drawing Angles

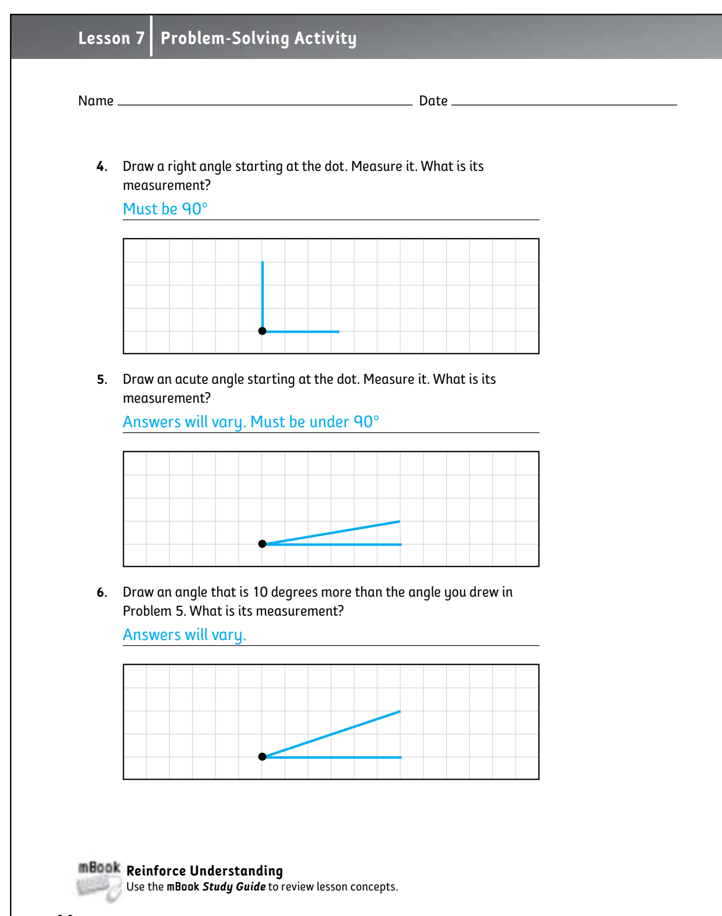
Use a protractor to draw and measure the angles.

1. Draw an acute angle starting at the dot. Label it XYZ. Measure it. What is its measurement?
Answers will vary. Must be under 90°

2. Draw an obtuse angle starting at the dot. Label it LMN. Measure it. What is its measurement?
Answers will vary. Must be over 90°

3. Draw a 65 degree angle starting at the dot. Label it ABC. What type of angle is it?
Acute

Unit 2
Unit 2 • Lesson 7 65


Lesson 7 | Problem-Solving Activity

Name ______ Date ______

4. Draw a right angle starting at the dot. Measure it. What is its measurement?
Must be 90°

5. Draw an acute angle starting at the dot. Measure it. What is its measurement?
Answers will vary. Must be under 90°

6. Draw an angle that is 10 degrees more than the angle you drew in Problem 5. What is its measurement?
Answers will vary.

mBook **Reinforce Understanding**
Use the **mBook *Study Guide*** to review lesson concepts.

66 Unit 2 • Lesson 7

Homework

Go over the instructions on page 122 of the *Student Text* for each part of the homework.

Activity 1

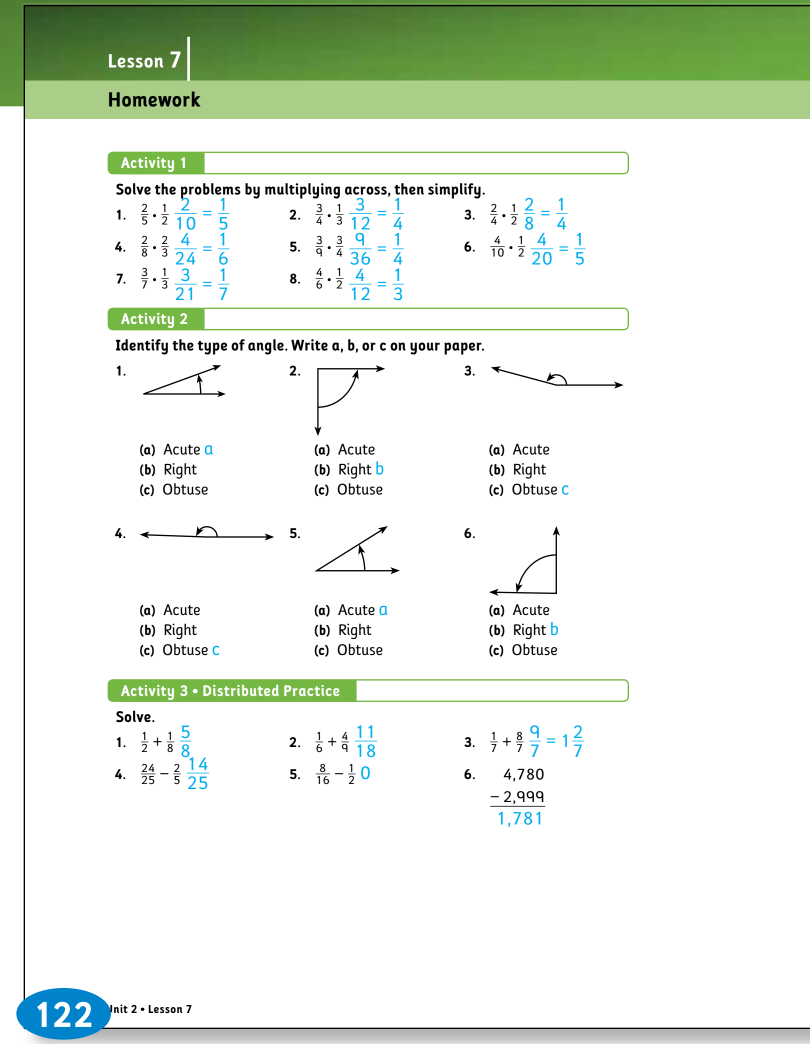
Students solve and simplify multiplication problems with fractions.

Activity 2

Students identify each type of angle shown.

Activity 3 • Distributed Practice

Students solve five problems involving addition and subtraction of fractions and one problem with whole-number operations.



Lesson 8

▶Dividing Fractions

Problem Solving: ▶Using Measurement Tools With a Map

Lesson Planner

Skills Maintenance

Dividing With Whole Numbers

Building Number Concepts: ▶ Dividing Fractions

Students learn to divide fractions. It is important students understand division conceptually as partitioning. For example, 12 ÷ 6 = 2 means we are partitioning 12 by the unit of 6; we find there are 2 units of 6 in 12. The problem $12 \div \frac{1}{2} = 24$ means that we are partitioning 12 by the unit of $\frac{1}{2}$; there are 24 units of $\frac{1}{2}$ in 12.

When dividing a whole number by another whole number, the quotient is usually smaller than either of the numbers. When dividing a whole number by a fraction or a fraction by a fraction, the quotient is usually bigger.

Objective

Students will divide fractions.

Problem Solving: ▶ Using Measurement Tools With a Map

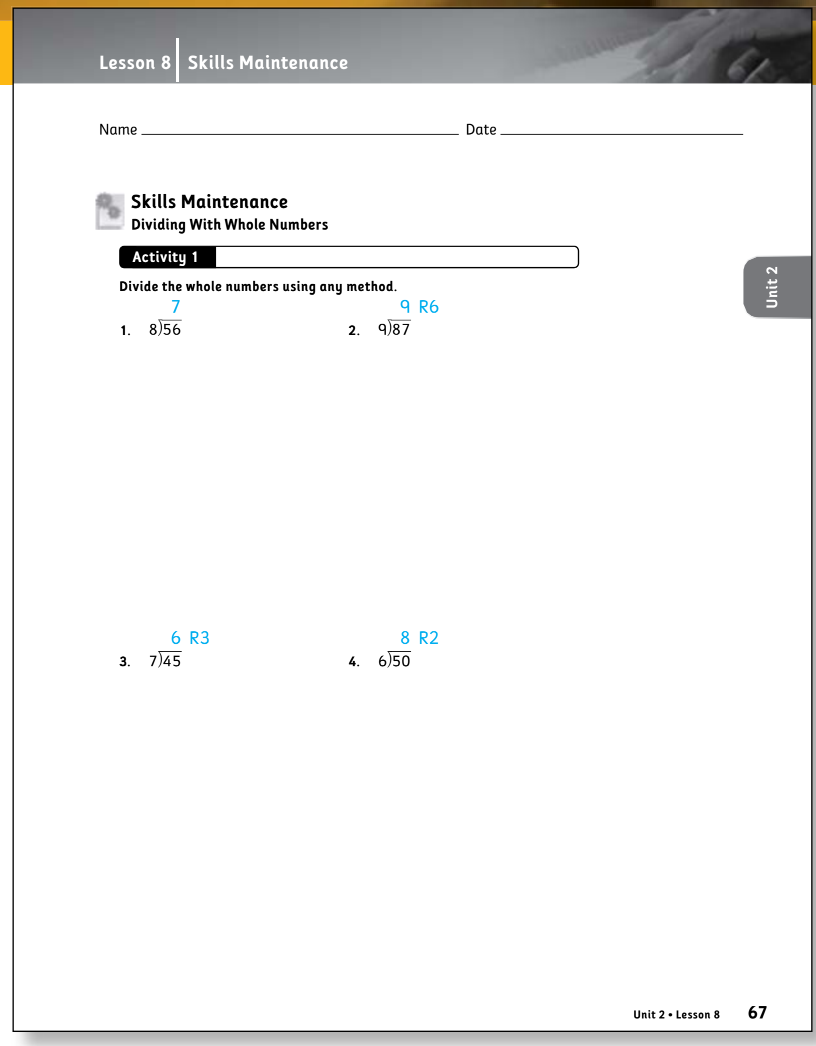
In this lesson, students use their rulers and protractors to design their own city.

Objective

Students will design a city using a ruler and a protractor.

Homework

Students multiply fractions and simplify the answers, use fraction bars to solve division problems, and tell if the angles are acute, right, or obtuse. In Distributed Practice, students add and subtract fractions.


Lesson 8 | Skills Maintenance

Name ______ Date ______

Skills Maintenance
Dividing With Whole Numbers

Activity 1

Divide the whole numbers using any method.

1. $8\overline{)56}$ — 7
2. $9\overline{)87}$ — 9 R6
3. $7\overline{)45}$ — 6 R3
4. $6\overline{)50}$ — 8 R2

Unit 2

Unit 2 • Lesson 8 67

Skills Maintenance

Dividing With Whole Numbers

(*Interactive Text*, page 67)

Activity 1

Students divide whole numbers.

Building Number Concepts:
▶ Dividing Fractions

What happens when we divide fractions?

(*Student Text*, page 123)

Connect to Prior Knowledge

Begin by reminding students what division using whole numbers means. Ask students to think about what division is. Use the problem 12 ÷ 6 to help them think about it. The focus is to get students to remember that division is about partitioning.

Link to Today's Concept

Tell students that in today's lesson, we begin to divide fractions.

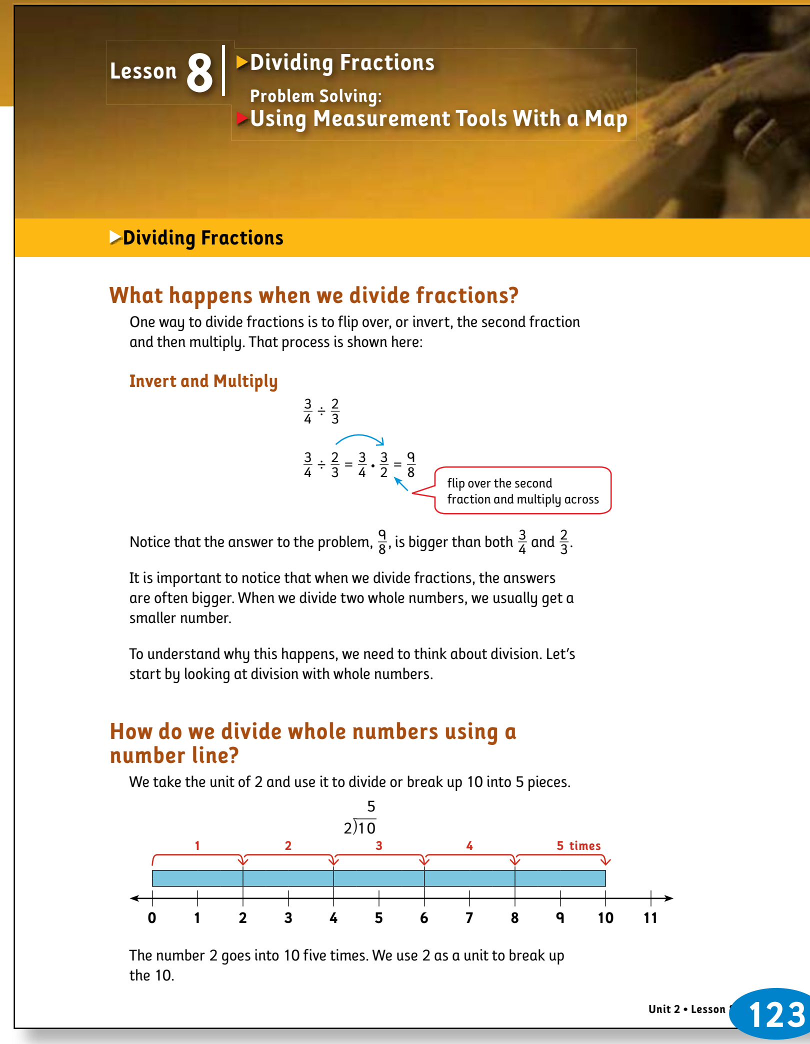
Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate how to divide fractions in one of the following ways:

- **mBook**: Use the *mBook Teacher Edition* for *Student Text*, pages 123–124. m
- **Overhead Projector**: Display the fractions and number line, and modify as discussed.
- **Board**: Copy the fractions and number line on the board, and modify as discussed.

- Show students the traditional method of invert and multiply. Make sure they understand which fraction to invert. Display $\frac{3}{4} \div \frac{2}{3}$. m
- Show the problem as a multiplication problem, inverting the second fraction: $\frac{3}{4} \cdot \frac{3}{2}$. Multiply across: $\frac{9}{8}$. m


Lesson 8 ▶Dividing Fractions
Problem Solving:
▶Using Measurement Tools With a Map

▶Dividing Fractions

What happens when we divide fractions?

One way to divide fractions is to flip over, or invert, the second fraction and then multiply. That process is shown here:

Invert and Multiply

$\frac{3}{4} \div \frac{2}{3}$

$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \cdot \frac{3}{2} = \frac{9}{8}$

flip over the second fraction and multiply across

Notice that the answer to the problem, $\frac{9}{8}$, is bigger than both $\frac{3}{4}$ and $\frac{2}{3}$.

It is important to notice that when we divide fractions, the answers are often bigger. When we divide two whole numbers, we usually get a smaller number.

To understand why this happens, we need to think about division. Let's start by looking at division with whole numbers.

How do we divide whole numbers using a number line?

We take the unit of 2 and use it to divide or break up 10 into 5 pieces.

$2\overline{)10}$ with quotient 5

The number 2 goes into 10 five times. We use 2 as a unit to break up the 10.

Unit 2 • Lesson 8 123

- Help students notice that $\frac{9}{8}$ is bigger than both $\frac{3}{4}$ and $\frac{2}{3}$.

How do we divide whole numbers?

(*Student Text*, page 123)

Demonstrate

- Point out that one confusing issue in the division of fractions is the size of the quotient. When dividing whole numbers, the quotient is always smaller than the dividend.
- Point out the number line showing 10 ÷ 2 = 5. There are five units of two in the number 10. With fractions, the unit size is so small that you need a lot of them, so the quotient is bigger, as we saw in the problem $\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$.

How do we divide fractions using a number line?

(*Student Text*, page 124)

Demonstrate

- Explain to students that the invert-and-multiply rule is efficient, but it does not explain this issue of the size of the dividend.
- Show students the problem $\frac{1}{2}\overline{)3}$ and the number line from **0** to $3\frac{1}{2}$. It clearly shows 6 units of $\frac{1}{2}$ in the number 3. It is much easier to understand this using a number line than using the invert-and-multiply method.
- Label the intervals by units of $\frac{1}{2}$. Point out that we divide the unit of $\frac{1}{2}$ into the number 3. So each of the three bars above the number line is broken into $\frac{1}{2}$. There are six halves in the three units.
- Help students notice that **6** is bigger than both 3 and $\frac{1}{2}$. Point out that we divided by a fraction, so the quotient is bigger. Remind students that fractions are less than one, and whole numbers have a 1 in the denominator. So the problem $\mathbf{3 \div \frac{1}{2} = 6}$ can be rewritten as $\mathbf{\frac{3}{1} \div \frac{1}{2} = 6}$.

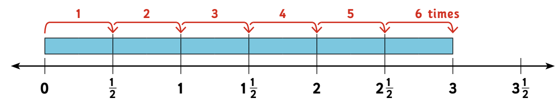
Now let's look at how we divide using a fraction.

How do we divide fractions using a number line?

Let's use a number line to solve the problem:

$\frac{1}{2}\overline{)3}$

This number line is stretched out so that we can see what we are doing when we divide $\frac{1}{2}$ into 3.



We take the unit $\frac{1}{2}$ and divide it into 3. We can divide, or break up, 3 into 6 parts when we divide by $\frac{1}{2}$.

We can also think of it as finding the number of halves in 3. There are 6 halves in 3. It is important for us to notice that the answer, 6, is bigger than both $\frac{1}{2}$ and 3.

The number line helps us see why the quotient is bigger than either fraction. When we divide by $\frac{1}{2}$, we are breaking 3 into units that are smaller than 1. Each unit is $\frac{1}{2}$ of the whole in length. There are 6 of these units. That is why we get a bigger number when we divide with fractions.

One important thing to remember when we are working with fractions and whole numbers is that we rewrite the whole numbers as fractions with 1 in the denominator. So, $3 = \frac{3}{1}$.

Here is how we write a problem like the one above using fractions.

$$\frac{3}{1} \div \frac{1}{2} = 6$$

Check for Understanding

Engagement Strategy: Look About

Tell students they are going to solve a problem with the help of the whole class. Write the problem $\mathbf{2 \div \frac{1}{4}}$ on the board. Tell students to solve the problem and draw the number line. Students should write their answers in large writing on a piece of paper or a dry erase board. When students finish their work, they should hold up their answer for everyone to see.

If students are not sure about their answer, prompt them to look about at other students' solutions to help with their thinking. Review the answer after all students have held up their solutions. Point out the size of the quotient (*8*) is larger than both 2 and $\frac{1}{4}$.

How do we use fraction bars to show division?

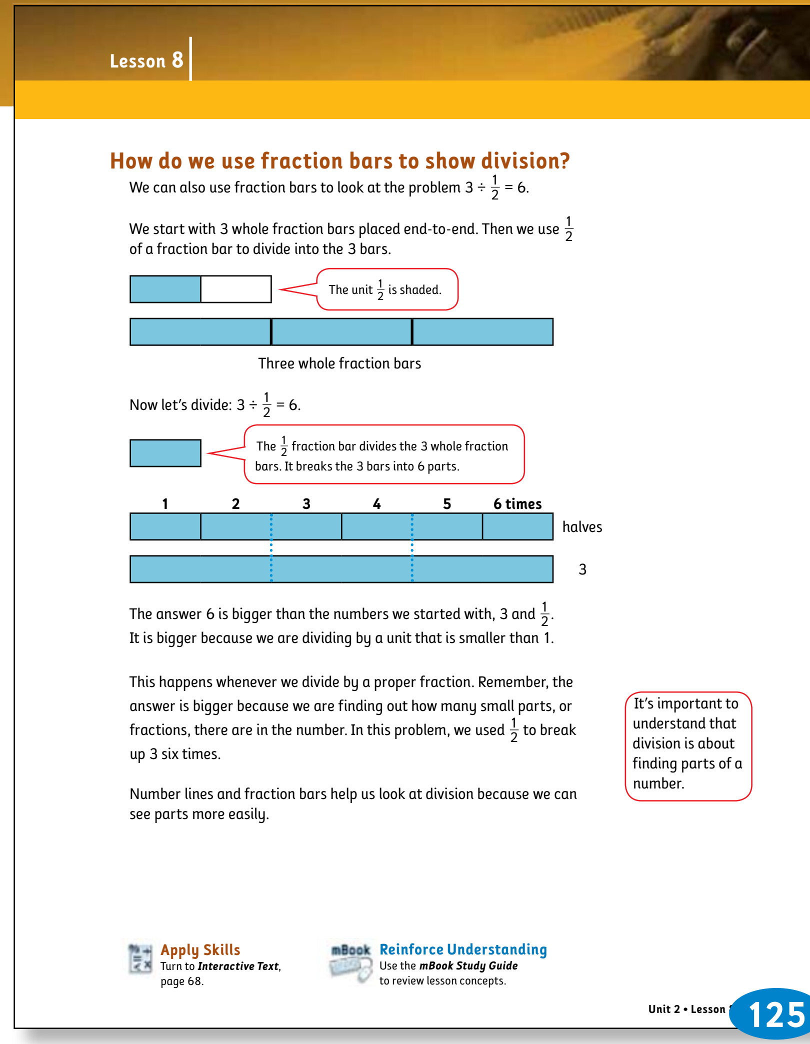
(*Student Text*, page 125)

Demonstrate

- Have students read page 125 of the *Student Text*, which shows division of fractions using fraction bars. Walk through the process as outlined in the *Student Text*. The problem again is $\mathbf{3 \div \frac{1}{2}}$.
- Demonstrate division with fraction bars by first pointing out our unit is $\frac{1}{2}$ of a fraction bar. Then look at how the whole number is three whole fraction bars.
- Compare the three whole fraction bars with the $\frac{1}{2}$ fraction bar. There are 6 fractional parts called $\frac{1}{2}$ in 3 whole fraction bars. The answer is **6**.
- Point out that both number lines and fraction bars help us see division more clearly because they are pictures that can be partitioned. Explain to students that it is important for them to remember that division is about partitioning.
- Ask students to summarize what they learned about division with fractions.

Listen for:

- *You can divide fractions the traditional way using the invert-and-multiply rule.*
- *When you divide whole numbers, the quotient is smaller than the number you start with. The quotient is bigger when you divide fractions.*
- *It's hard to picture what invert and multiply means. You can use fraction bars and number lines to see it more clearly.*



Lesson 8

How do we use fraction bars to show division?

We can also use fraction bars to look at the problem $3 \div \frac{1}{2} = 6$.

We start with 3 whole fraction bars placed end-to-end. Then we use $\frac{1}{2}$ of a fraction bar to divide into the 3 bars.

The unit $\frac{1}{2}$ is shaded.

Three whole fraction bars

Now let's divide: $3 \div \frac{1}{2} = 6$.

The $\frac{1}{2}$ fraction bar divides the 3 whole fraction bars. It breaks the 3 bars into 6 parts.

1 2 3 4 5 6 times

halves

3

The answer 6 is bigger than the numbers we started with, 3 and $\frac{1}{2}$. It is bigger because we are dividing by a unit that is smaller than 1.

This happens whenever we divide by a proper fraction. Remember, the answer is bigger because we are finding out how many small parts, or fractions, there are in the number. In this problem, we used $\frac{1}{2}$ to break up 3 six times.

Number lines and fraction bars help us look at division because we can see parts more easily.

It's important to understand that division is about finding parts of a number.

Apply Skills Turn to ***Interactive Text***, page 68.

mBook Reinforce Understanding Use the ***mBook Study Guide*** to review lesson concepts.

Unit 2 • Lesson 8 125

- *Division is about partitioning things. When you partition something by a fractional part, you need a lot more of it than when you partition something by a whole number.*

Apply Skills
(*Interactive Text*, page 68)

Have students turn to page 68 in the *Interactive Text* and complete the activities.

Activity 1

Students solve the division problems using fraction bars. Explain the instructions carefully to students. The whole number is shown on the bottom of the diagram. The fraction is shown on the top of the diagram. The task is to determine how many of the units are in the whole number. Be sure students understand what the unit is in each of the problems. You might want to read all the problems together before having students begin so that they fully understand the concept.

Activity 2

Students use the traditional invert-and-multiply rule to solve division problems. Then they simplify their answers.

Monitor students' work as they complete the activities.

Watch for:

- Can students identify the portion of the diagram that is the whole number?
- Can students identify the portion of the diagram that is the fraction?
- Can students determine the quotient from the fraction bar?
- Can students adapt when the unit has a numerator other than 1?
- Do students invert the correct fraction accurately?
- Do students multiply across correctly?

Lesson 8 | Apply Skills

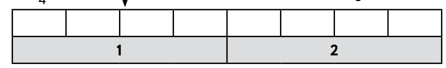
Name ______________________ Date ______________

Apply Skills
Dividing Fractions

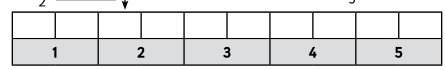
Activity 1

Use the fraction bars to help you solve these division problems.

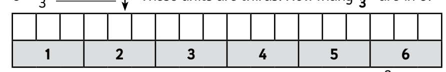
1. $2 \div \frac{1}{4}$ __8__ These units are fourths. How many are in 2?



2. $5 \div \frac{1}{2}$ __10__ These units are halves. How many are in 5?



3. $6 \div \frac{2}{3}$ __9__ These units are thirds. How many $\frac{2}{3}$* are in 6?



Note: The units are thirds, but you want to look at units of $\frac{2}{3}$. Be sure to count sections of $\frac{2}{3}$ and not $\frac{1}{3}$.

Activity 2

Use the traditional method of invert and multiply to solve the division problems. Simplify the quotients.

1. $\frac{2}{1} \div \frac{1}{3}$ __6__
2. $\frac{3}{5} \div \frac{5}{8}$ __$\frac{24}{25}$__
3. $\frac{4}{4} \div \frac{3}{4}$ __$1\frac{1}{3}$__
4. $\frac{1}{2} \div \frac{3}{8}$ __$1\frac{1}{3}$__
5. $\frac{6}{1} \div \frac{1}{2}$ __12__
6. $\frac{3}{8} \div \frac{5}{9}$ __$\frac{27}{40}$__
7. $\frac{4}{12} \div \frac{3}{16} =$ __$1\frac{7}{9}$__
8. $\frac{11}{12} \div \frac{7}{8} =$ __$1\frac{1}{21}$__

68 Unit 2 • Lesson 8

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Problem Solving:
▶ Using Measurement Tools With a Map

How do we use measurement tools with a map?

(*Student Text*, page 126)

Connect to Prior Knowledge

Remind students of the map they used that showed landmarks on a snowmobile trail.

Link to Today's Concept

In today's lesson, students are given a blank map, and they put the landmarks in the specified locations using measurement information (e.g., angles and line segments).

Demonstrate

- Read page 126 of the *Student Text*. In today's lesson, students use their metric rulers and their protractors with a map.
- Explain to students the job of city planners. In simple terms, they are people who make decisions about what is built in the city and where it is located.
- Have students use their measurement tools to measure the angles and line segments. Have students look at the map and the first angle in the table. Help students locate the angle on the map. Then have them measure the angle with their protractor.
- Have students check to see that the first angle, ∠***BFC***, measures **50 degrees**. Have students continue measuring the rest of the angles in the table.

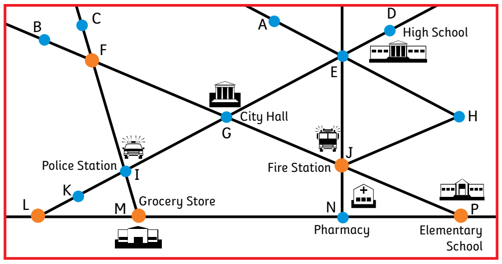
Lesson 8

▶Problem Solving: Using Measurement Tools With a Map

How do we use measurement tools with a map?

In Lesson 4, we discussed how surveyors use angles to measure the steepness of a road. Another type of worker who uses this kind of measurement is a city planner. City planners make recommendations about where buildings and other structures should be located. They determine this by looking at where all the buildings, parks, stadiums, housing developments, and so on, are in a particular city. City planners then record all of the information about the city.

We can use a protractor to measure the angles on the map. The angles and their measurements are listed in the table below.



Angle Name	Estimated Measurements
∠BFC	50°
∠KIF	100°
∠IGJ	130°
∠DEH	55°
∠JNP	90°

Problem-Solving Activity Turn to *Interactive Text*, page 69.

Reinforce Understanding Use the *mBook Study Guide* to review lesson concepts.

126 Unit 2 • Lesson 8

Problem-Solving Activity

(*Interactive Text*, page 69)

Have students turn to *Interactive Text*, page 69. Read the instructions carefully. Students place landmarks on the city map based on the instructions given.

Be sure students know they can either write the name or draw a picture of the landmark on the map. It does not have to be perfectly drawn, just something to indicate the landmark. Instruct students to make the measurement requested and record it in the table. Place students in pairs or groups, or have them work individually.

Monitor students' work as they complete this activity.

Watch for:

- Can students find the location of the landmark?
- Can students measure the angles correctly with the protractor?
- Can students measure the line segment correctly with the ruler?
- Can students transfer the information from the map to the table and from the table to the map?

Go over the answers once students finish the activity. Remind them that some answers vary slightly because of the imprecision of measurement, but the numbers should be close.

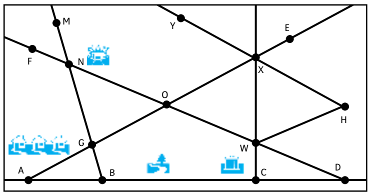
mBook Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Name ______________________ Date ______________

Problem-Solving Activity
Using Measurement Tools With a Map

We will design a city on a map using a protractor and ruler. Place certain landmarks on the map. Draw the picture of the landmark or write the words that describe the landmark, or both. Make the measurement and record it in the table.



1. Place the courthouse at the vertex of ∠BCW. Measure the angle and record its measurement in the table.
2. Place the park on line segment BC. Measure the line segment in cm. and record the measurement in the table.
3. Place Quiet Lane Townhouses on line segment AG. Measure the line segment in centimeters and record the measurement in the table. Answers will vary.
4. Place the stadium at the vertex of ∠MNO. Measure the angle and record its measurement in the table.

Landmark Name	Picture	Location	Measurement
Courthouse		∠BCW	90°
Park		Line segment BC	4cm
Quiet Lane Townhouses		Line Segment AG	2cm
Stadium		∠MNO	130°

mBook Reinforce Understanding
Use the **mBook** ***Study Guide*** to review lesson concepts.

Homework

Go over the instructions on page 127 of the *Student Text* for each part of the homework.

Activity 1

Students multiply fractions using the traditional algorithm. Student answers should be in simplest form.

Activity 2

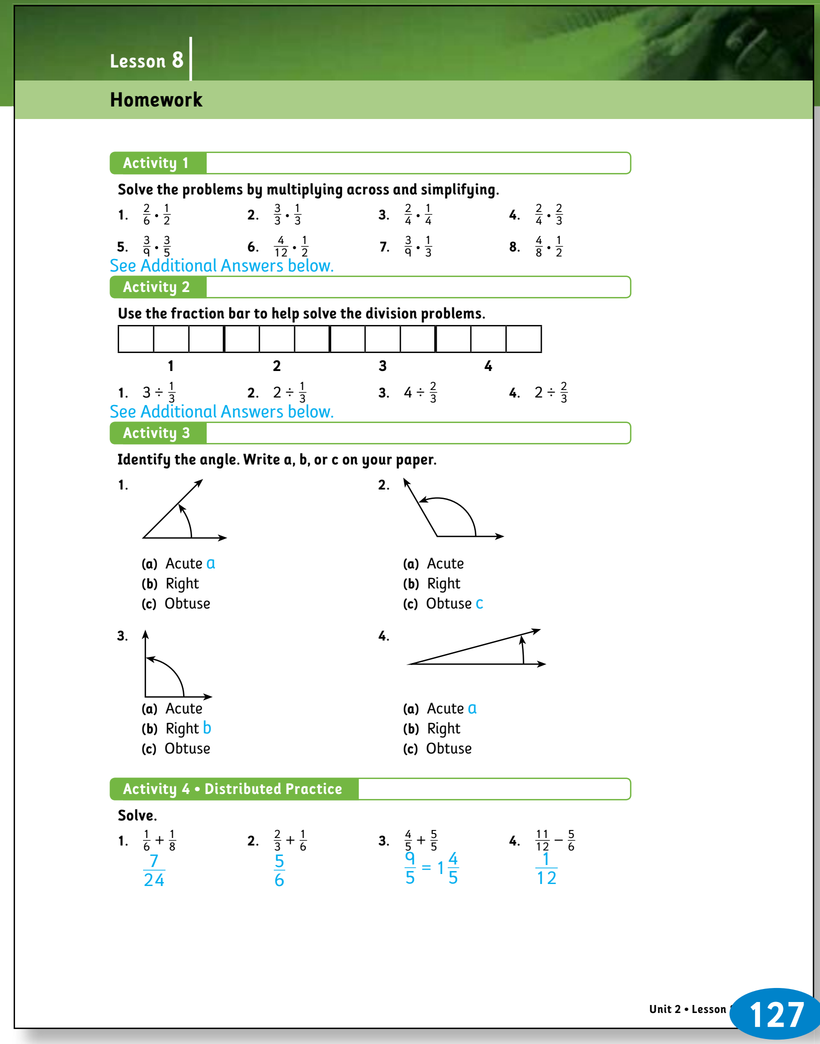
Students use the fraction bar to divide. Tell students that the problems are all represented by the fraction bar shown.

Activity 3

Students identify the type of angle.

Activity 4 • Distributed Practice

Students continue practicing addition and subtraction of fractions.



Lesson 8

Homework

Activity 1

Solve the problems by multiplying across and simplifying.

1. $\frac{2}{6} \cdot \frac{1}{2}$ 2. $\frac{3}{3} \cdot \frac{1}{3}$ 3. $\frac{2}{4} \cdot \frac{1}{4}$ 4. $\frac{2}{4} \cdot \frac{2}{3}$

5. $\frac{3}{9} \cdot \frac{3}{5}$ 6. $\frac{4}{12} \cdot \frac{1}{2}$ 7. $\frac{3}{9} \cdot \frac{1}{3}$ 8. $\frac{4}{8} \cdot \frac{1}{2}$

See Additional Answers below.

Activity 2

Use the fraction bar to help solve the division problems.

1 2 3 4

1. $3 \div \frac{1}{3}$ 2. $2 \div \frac{1}{3}$ 3. $4 \div \frac{2}{3}$ 4. $2 \div \frac{2}{3}$

See Additional Answers below.

Activity 3

Identify the angle. Write a, b, or c on your paper.

1.
(a) Acute a
(b) Right
(c) Obtuse

2.
(a) Acute
(b) Right
(c) Obtuse c

3.
(a) Acute
(b) Right b
(c) Obtuse

4.
(a) Acute a
(b) Right
(c) Obtuse

Activity 4 • Distributed Practice

Solve.

1. $\frac{1}{6} + \frac{1}{8}$ $\frac{7}{24}$ 2. $\frac{2}{3} + \frac{1}{6}$ $\frac{5}{6}$ 3. $\frac{4}{5} + \frac{5}{5}$ $\frac{9}{5} = 1\frac{4}{5}$ 4. $\frac{11}{12} - \frac{5}{6}$ $\frac{1}{12}$

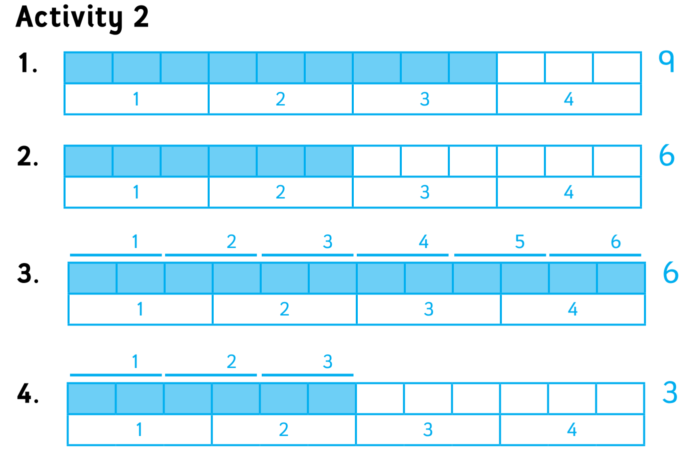
Unit 2 • Lesson 127

Additional Answers

Activity 1

1. $\frac{2}{12} = \frac{1}{6}$
2. $\frac{3}{9} = \frac{1}{3}$
3. $\frac{2}{16} = \frac{1}{8}$
4. $\frac{4}{12} = \frac{1}{3}$
5. $\frac{9}{45} = \frac{1}{5}$
6. $\frac{4}{24} = \frac{1}{6}$
7. $\frac{3}{27} = \frac{1}{9}$
8. $\frac{4}{16} = \frac{1}{4}$

Activity 2



Lesson 9 | Dividing Fractions by Fractions

Problem Solving: Measuring Angles of Ramps

Lesson Planner

Skills Maintenance

Multiplying and Simplifying

Building Number Concepts:

Dividing Fractions by Fractions

In this lesson, we use fraction bars to divide one fraction by another fraction. Fraction bars provide a good visual representation of this process. Fraction-by-fraction division is a little more difficult to represent than whole number by fraction division. It is important to first define the unit and then carefully line up the fraction bars.

Objective

Students will use fraction bars to divide one fraction by another fraction.

Problem Solving:

Measuring Angles of Ramps

In this lesson, we look at an example of how an engineer designs motorbike tracks or courses. We focus mainly on the ramps that are used for jumps and how to measure the angle of the ramp.

Objective

Students will measure angles in the real-world context of motorbike track design.

Homework

Students multiply and simplify fractions; divide fractions using the traditional approach, then simplify and draw points, lines, angles and rays. In Distributed Practice, students continue to practice addition and subtraction of fractions.

Name ______________________ Date ______________

Skills Maintenance
Multiplying and Simplifying

Activity 1

Multiply the fractions, and simplify the answers.

1. $\frac{1}{3} \cdot \frac{4}{2}$ $\frac{4}{6}, \frac{2}{3}$
2. $\frac{4}{6} \cdot \frac{5}{4}$ $\frac{20}{24}, \frac{5}{6}$
3. $\frac{4}{3} \cdot \frac{3}{6}$ $\frac{12}{18}, \frac{2}{3}$
4. $\frac{2}{5} \cdot \frac{5}{6}$ $\frac{10}{30}, \frac{1}{3}$
5. $\frac{1}{3} \cdot \frac{2}{5}$ $\frac{2}{15}, \frac{2}{15}$

Skills Maintenance

Multiplying and Simplifying

(*Interactive Text*, page 70)

Activity 1

Students multiply across and then simplify their answers using the GCF.

Building Number Concepts:
▶ Dividing Fractions by Fractions

How do we use a fraction bar to divide a whole number by a fraction?
(*Student Text*, pages 128–129)

Connect to Prior Knowledge
Remind students that we have used fraction bars to represent the division of a whole number and a fraction.

Link to Today's Concept
Tell students that in today's lesson, we use fraction bars to divide fractions by other fractions.

Demonstrate
Engagement Strategy: Teacher Modeling

Demonstrate how we represent a whole number divided by a fraction with fraction bars.

mBook: Use the *mBook Teacher Edition* for page 128 of the *Student Text*. m

Overhead Projector: Reproduce the fraction bars on a transparency, and modify as discussed.

Board: Draw the fraction bars on the board, and modify as discussed.

- Show the problem $\mathbf{2 \div \frac{1}{4}}$, and elicit how we solve the problem using fraction bars. Adjust the fraction bars as students respond. m

Listen for:

- *First you have to show a fraction bar for the whole number. You put two whole fraction bars together.* m
- *Then you have to show the fraction bar for $\frac{1}{4}$. There are 4 fourths in 1 whole.* m

Lesson 9 | ▶Dividing Fractions by Fractions
Problem Solving:
▶Measuring Angles of Ramps

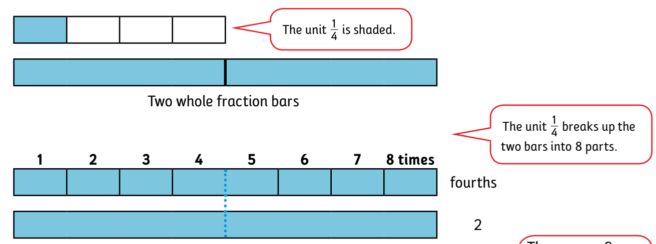
▶Dividing Fractions by Fractions

How do we use a fraction bar to divide a whole number by a fraction?

Let's review how to use a fraction bar to divide a whole number by a fraction.

$2 \div \frac{1}{4}$

For this problem, we take a $\frac{1}{4}$ fraction bar and see how many times it breaks up two whole fraction bars.



The answer is $2 \div \frac{1}{4} = 8$.

There are 8 units of $\frac{1}{4}$ in the number 2.

The answer 8 is bigger than the numbers we started with, 2 and $\frac{1}{4}$, because we divided by a unit smaller than 1.

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- *That means there are 8 fourths in 2 wholes. Show the fraction bar divided into 8 parts right above the first fraction bar.* m
- *Now see how many fourths line up with 2. There are eight of them.* m
- *The answer is 8:* $\mathbf{2 \div \frac{1}{4} = 8}$. *There are 8 units of $\frac{1}{4}$ in the number 2.*

- Point out that the answer 8 is bigger than the numbers we started with because we divided by a unit smaller than 1.
- Make sure students understand these steps before you move on to today's material. The new material builds on this information.

How do we use a fraction bar to divide a whole number by a fraction? *(continued)*

Demonstrate

- Continue the discussion of how to use fraction bars to divide.
- Have students look at **Example 1** on page 129 of the *Student Text*. Have students look at the problem $\mathbf{3 \div \frac{3}{4}}$. Walk through each of the steps with them.

STEP 1

- Note there are three whole fraction bars put together on the bottom. There are fourths shown in the top fraction bar.

STEP 2

- Compare the fourths with the 3 wholes. There are 4 fourths in 1, 8 fourths in 2, and 12 fourths in 3. To look at $\mathbf{\frac{3}{4}}$ we have to look at fourths, but we need to look at them three at a time. Define the unit of $\frac{3}{4}$.

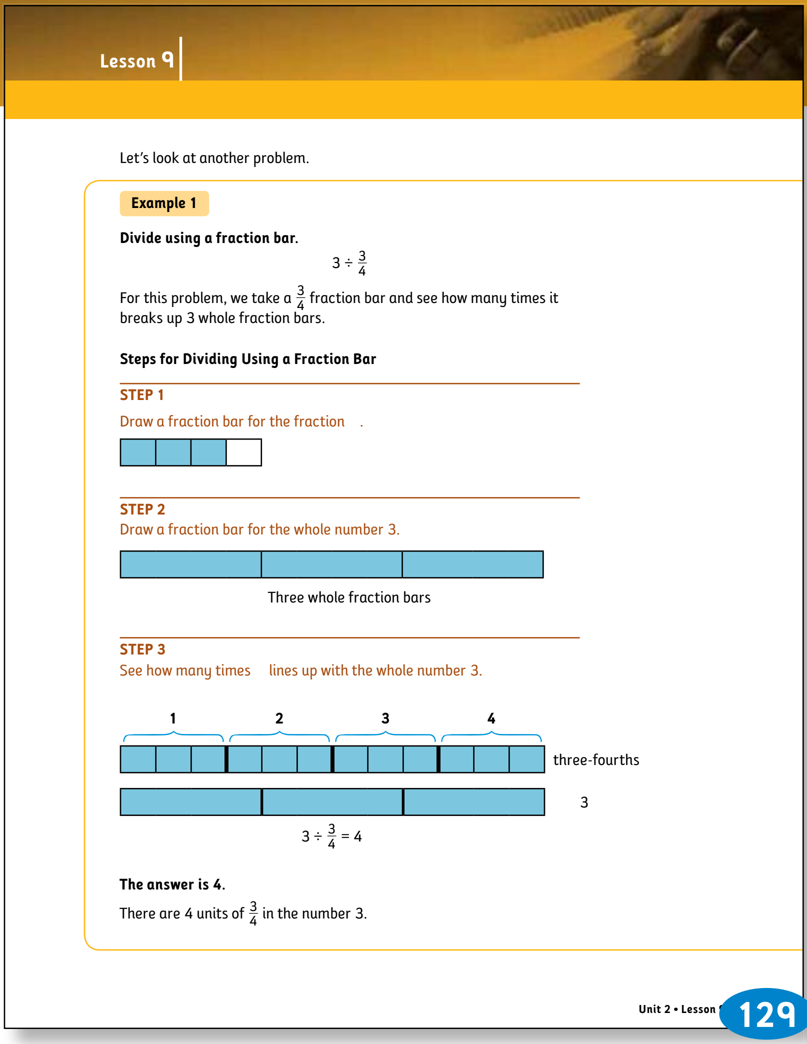
STEP 3

- Put the units on the fraction bar for 3 until there are enough. We see that there are **4 units of** $\mathbf{\frac{3}{4}}$ in 4. This process can be confusing. Go through more examples of this method if students need the extra practice.

Check for Understanding

Engagement Strategy: Look About

Write the problem $\mathbf{3 \div \frac{3}{5}}$ on the board. Tell students that they solve the problem together as a class (*$3 \div \frac{3}{5} = 5$*). Students should draw their fraction bars and write their solutions in large writing on a piece of paper or a dry erase board.


Lesson 9

Let's look at another problem.

Example 1

Divide using a fraction bar.

$3 \div \frac{3}{4}$

For this problem, we take a $\frac{3}{4}$ fraction bar and see how many times it breaks up 3 whole fraction bars.

Steps for Dividing Using a Fraction Bar

STEP 1
Draw a fraction bar for the fraction .

STEP 2
Draw a fraction bar for the whole number 3.

Three whole fraction bars

STEP 3
See how many times lines up with the whole number 3.

$3 \div \frac{3}{4} = 4$

The answer is 4.

There are 4 units of $\frac{3}{4}$ in the number 3.

Unit 2 • Lesson 129

When the students finish their work, they should hold up their answers for everyone to see.

If students are not sure about the answer, prompt them to look about at other students' solutions to help with their thinking. Review the answers after all students have held up their solutions.

Reinforce Understanding

Here are some practice problems to use if your students need more work on this concept:

$\mathbf{6 \div \frac{4}{6}}$ (9)

$\mathbf{2 \div \frac{2}{3}}$ (*3*)

$\mathbf{3 \div \frac{3}{2}}$ (*2*)

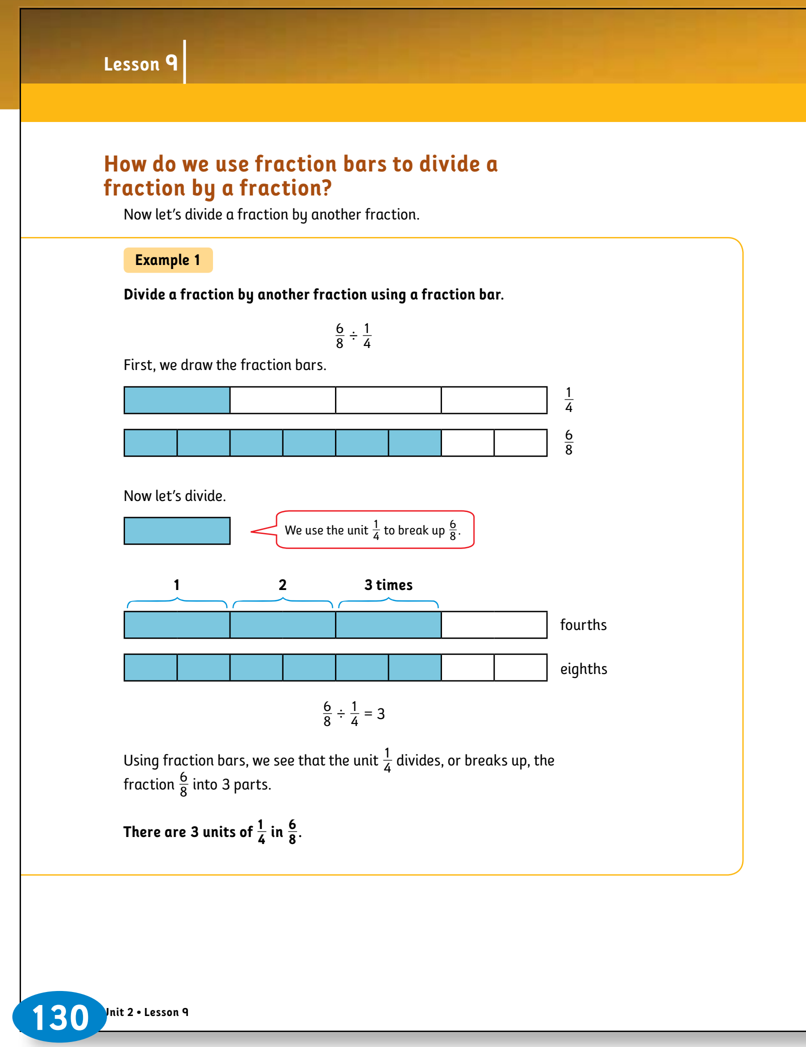
$\mathbf{1 \div \frac{3}{6}}$ (*2*)

How do we use fraction bars to divide a fraction by a fraction?

(*Student Text*, pages 130–131)

Demonstrate

- Have students turn to page 130 of the *Student Text*. Discuss how the answers to fraction division problems are often bigger than the numbers we start with. This method is different from whole-number division. When we divide by a fraction, we find the number of sets of the fraction in the dividend.
- Look at **Example 1**, where we divide $\mathbf{\frac{6}{8}}$ by $\mathbf{\frac{1}{4}}$ using fraction bars.
- Draw the two fraction bars. Define the unit $\frac{1}{4}$.
- Draw the unit $\frac{1}{4}$ on the fraction bar until you have enough for $\frac{6}{8}$. Note that it takes 3 units of $\frac{1}{4}$: $\mathbf{\frac{6}{8} \div \frac{1}{4} = 3}$.



Lesson 9

How do we use fraction bars to divide a fraction by a fraction?

Now let's divide a fraction by another fraction.

Example 1

Divide a fraction by another fraction using a fraction bar.

$\frac{6}{8} \div \frac{1}{4}$

First, we draw the fraction bars.

Now let's divide.

$\frac{6}{8} \div \frac{1}{4} = 3$

Using fraction bars, we see that the unit $\frac{1}{4}$ divides, or breaks up, the fraction $\frac{6}{8}$ into 3 parts.

There are 3 units of $\frac{1}{4}$ in $\frac{6}{8}$.

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How do we use fraction bars to divide a fraction by a fraction? *(continued)*

Demonstrate

- Have students look at **Example 2** on page 131 of the *Student Text*: $\frac{3}{4} \div \frac{1}{8}$. Draw the fraction bars for the two fractions. Define the unit $\frac{1}{8}$.
- Draw it on the fraction bar for $\frac{3}{4}$ until you have enough units. Point out it takes 6 units: $\frac{3}{4} \div \frac{1}{8} = 6$.
- Ask students to tell you in their own words the steps for fraction-by-fraction multiplication with fraction bars.

Listen for:

- *You start by looking at the fraction bars for both fractions.*
- *You define the unit that you are dividing by.*
- *You draw units on the other fraction bar until you have reached enough units (filled up the other fraction bar).*
- *You count how many you have. That's the answer.*

Check for Understanding

Engagement Strategy: Pair/Share

Have students solve the problem $\frac{3}{6} \div \frac{1}{6}$ (*3*) on their own. Write the problem on the board. When they finish, have them compare their solutions with a partner and talk through the steps they used to solve the problem. When pairs finish their discussion, have volunteers share their process and answer with the class.

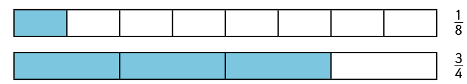
Let's look at one more problem.

Example 2

Divide $\frac{3}{4}$ by $\frac{1}{8}$ using fraction bars.

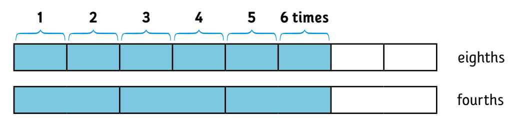
$$\frac{3}{4} \div \frac{1}{8}$$

First, we draw the fraction bars.



Then we divide.

We use the unit $\frac{1}{8}$ to break up $\frac{3}{4}$.



$$\frac{3}{4} \div \frac{1}{8} = 6$$

Using fraction bars, we see that the unit $\frac{1}{8}$ divides, or breaks up, the fraction $\frac{3}{4}$ into 6 parts.

There are 6 units of $\frac{1}{8}$ in $\frac{3}{4}$.

Apply Skills
Turn to ***Interactive Text***, page 71.

mBook Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

Reinforce Understanding

If students need more practice, try these problems:

$\frac{4}{5} \div \frac{1}{5}$ (*4*)

$\frac{1}{2} \div \frac{1}{4}$ (*2*)

$\frac{1}{2} \div \frac{1}{8}$ (*4*)

Apply Skills

(*Interactive Text*, page 71)

Have students turn to page 71 in the *Interactive Text*, which provides students an opportunity to practice dividing fractions by fractions on their own.

Activity 1

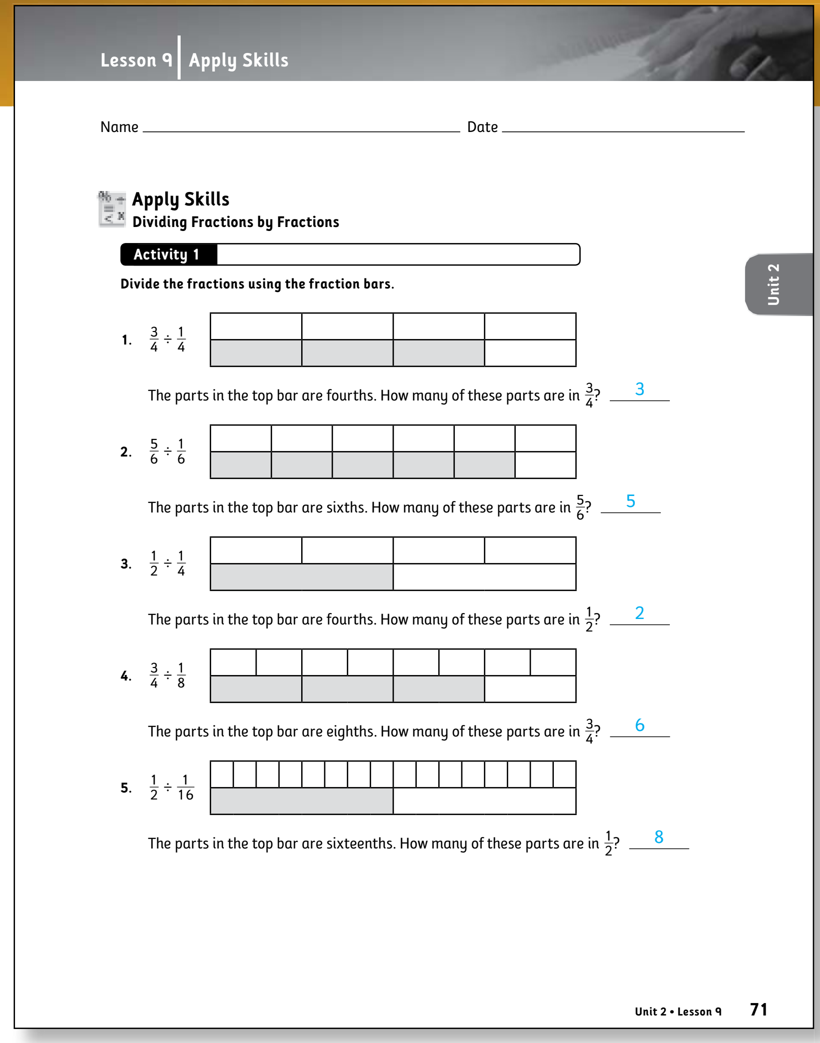
Students divide fractions using the given fraction bars. Monitor students' work as they complete the activity.

Watch for:

- Can students identify the unit?
- Can students shade the unit appropriately on the top fraction bar to tell how many units there are?

mBook **Reinforce Understanding**

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.



Lesson 9 | Apply Skills

Name ______________ Date ______________

Apply Skills
Dividing Fractions by Fractions

Activity 1

Divide the fractions using the fraction bars.

1. $\frac{3}{4} \div \frac{1}{4}$

The parts in the top bar are fourths. How many of these parts are in $\frac{3}{4}$? 3

2. $\frac{5}{6} \div \frac{1}{6}$

The parts in the top bar are sixths. How many of these parts are in $\frac{5}{6}$? 5

3. $\frac{1}{2} \div \frac{1}{4}$

The parts in the top bar are fourths. How many of these parts are in $\frac{1}{2}$? 2

4. $\frac{3}{4} \div \frac{1}{8}$

The parts in the top bar are eighths. How many of these parts are in $\frac{3}{4}$? 6

5. $\frac{1}{2} \div \frac{1}{16}$

The parts in the top bar are sixteenths. How many of these parts are in $\frac{1}{2}$? 8

Unit 2

Unit 2 • Lesson 9 71

Problem Solving:
▶ Measuring Angles of Ramps

How do we draw a specific angle?
(*Student Text*, page 132)

Connect to Prior Knowledge
Begin by asking students to recall how we estimated the measure of angles using a formal protractor.

Link to Today's Concept
In today's lesson we find the specific measure of angles.

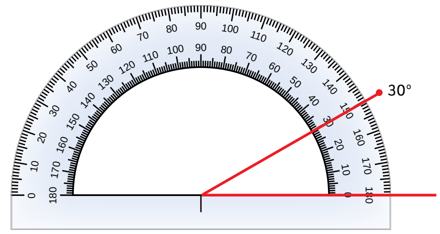
Demonstrate
- Have students turn to page 132 in the *Student Text*. Guide them in discussing how an engineer's job might entail measuring angles on a motorbike race course to see if the ramps are too steep.
- Explain that there are regulations for safety regarding how steep jump ramps are.
- Demonstrate how to measure a 30-degree ramp and a 12-degree ramp. Be sure students see from the diagrams in the book how they should line up their protractors to measure these ramps.

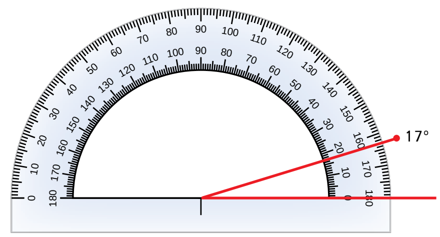
Lesson 9
▶Problem Solving: Measuring Angles of Ramps

How do we draw a specific angle?

Suppose an engineer working for a construction company needs to design a motocross race course. The engineer is in charge of mapping out the course and designing the ramps.

Let's use a protractor to draw a 30-degree ramp and a 17-degree ramp.





It is important for us to be accurate in our measurements. This will help us avoid errors.

Problem-Solving Activity
Turn to *Interactive Text*, page 72.

Reinforce Understanding
Use the *mBook Study Guide* to review lesson concepts.

Problem-Solving Activity

(*Interactive Text*, page 72)

Have students turn to page 72 in the *Interactive Text*, which provides students an opportunity to measure angles.

Students measure the angle of each of six ramps designed for the course layout and determine if any of the angles are over 35°.

Monitor students' work as they complete this activity.

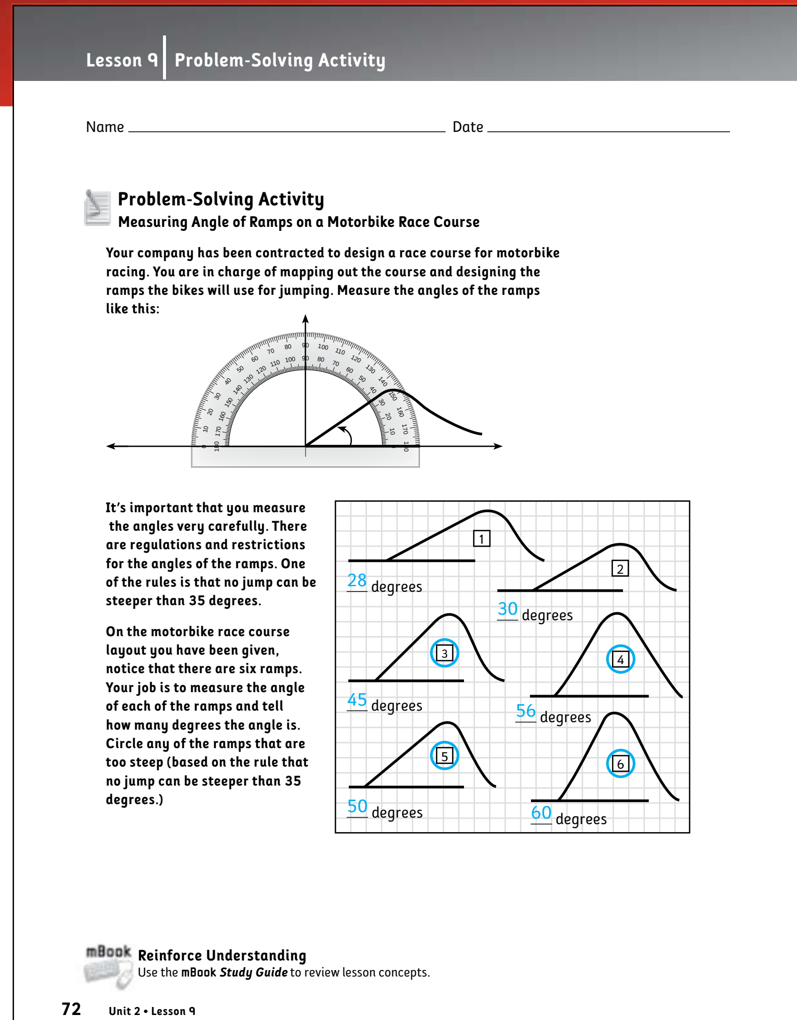
Watch for:

- Are students lining up the protractor correctly?
- Can students read the correct number from the protractor?
- Can students identify if the angle is larger than 35°?

Once students complete the activity, have them compare answers with other students. Remind students that some of their numbers might vary, but they should all be in the same range.

mBook Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.



Lesson 9 | Problem-Solving Activity

Name ______________________ Date ______________

Problem-Solving Activity
Measuring Angle of Ramps on a Motorbike Race Course

Your company has been contracted to design a race course for motorbike racing. You are in charge of mapping out the course and designing the ramps the bikes will use for jumping. Measure the angles of the ramps like this:

It's important that you measure the angles very carefully. There are regulations and restrictions for the angles of the ramps. One of the rules is that no jump can be steeper than 35 degrees.

On the motorbike race course layout you have been given, notice that there are six ramps. Your job is to measure the angle of each of the ramps and tell how many degrees the angle is. Circle any of the ramps that are too steep (based on the rule that no jump can be steeper than 35 degrees.)

mBook Reinforce Understanding
Use the mBook *Study Guide* to review lesson concepts.

72 Unit 2 • Lesson 9

Homework

Go over the instructions on page 133 of the *Student Text* for each part of the homework.

Activity 1

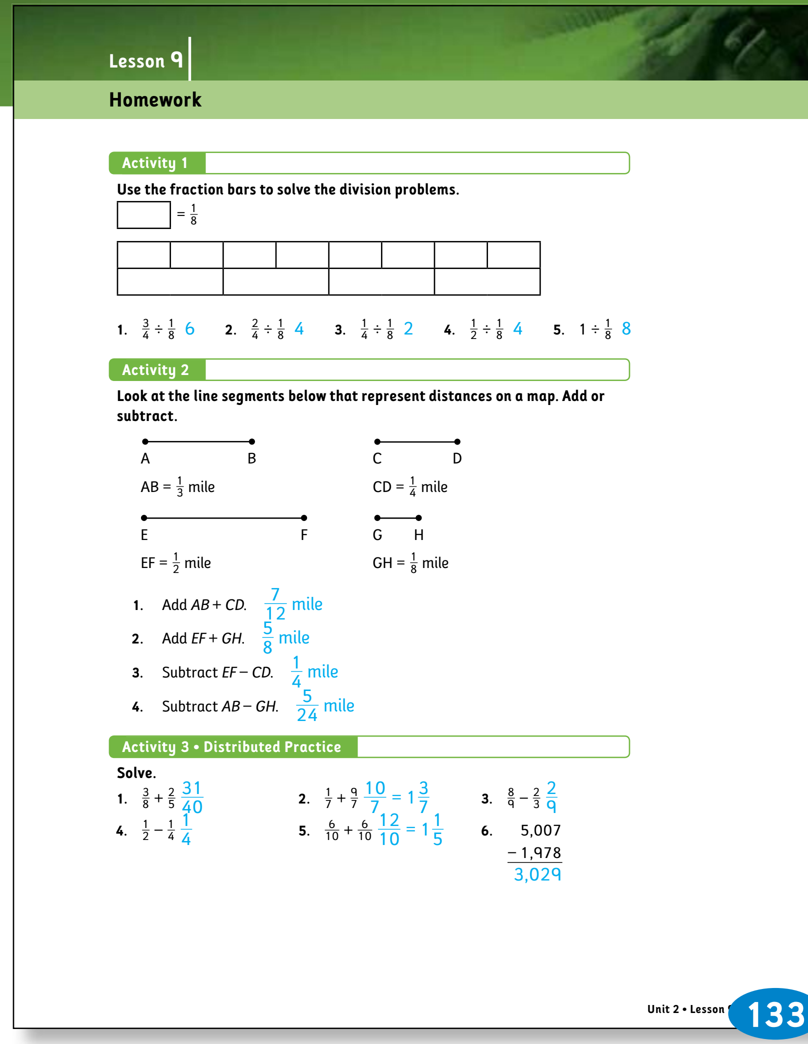
Students look at the picture of the fraction bars and solve the problems. The pictures should provide the assistance students need to avoid drawing more fraction bars.

Activity 2

Students look at the line segments that represent fractions of a mile. Then they add or subtract the line segments.

Activity 3 • Distributed Practice

Students continue to practice addition and subtraction of fractions and subtraction of whole numbers.


Lesson 9

Homework

Activity 1

Use the fraction bars to solve the division problems.

$\square = \frac{1}{8}$

1. $\frac{3}{4} \div \frac{1}{8}$ 6
2. $\frac{2}{4} \div \frac{1}{8}$ 4
3. $\frac{1}{4} \div \frac{1}{8}$ 2
4. $\frac{1}{2} \div \frac{1}{8}$ 4
5. $1 \div \frac{1}{8}$ 8

Activity 2

Look at the line segments below that represent distances on a map. Add or subtract.

A B

AB = $\frac{1}{3}$ mile

C D

CD = $\frac{1}{4}$ mile

E F

EF = $\frac{1}{2}$ mile

G H

GH = $\frac{1}{8}$ mile

1. Add *AB* + *CD*. $\frac{7}{12}$ mile
2. Add *EF* + *GH*. $\frac{5}{8}$ mile
3. Subtract *EF* − *CD*. $\frac{1}{4}$ mile
4. Subtract *AB* − *GH*. $\frac{5}{24}$ mile

Activity 3 • Distributed Practice

Solve.

1. $\frac{3}{8} + \frac{2}{5}$ $\frac{31}{40}$
2. $\frac{1}{7} + \frac{9}{7}$ $\frac{10}{7} = 1\frac{3}{7}$
3. $\frac{8}{9} - \frac{2}{3}$ $\frac{2}{9}$
4. $\frac{1}{2} - \frac{1}{4}$ $\frac{1}{4}$
5. $\frac{6}{10} + \frac{6}{10}$ $\frac{12}{10} = 1\frac{1}{5}$
6. $\begin{array}{r} 5{,}007 \\ -\ 1{,}978 \\ \hline 3{,}029 \end{array}$

Unit 2 • Lesson 9 133

Lesson 10 ▶The Traditional Method for Dividing Fractions

Monitoring Progress: ▶Quiz 2

Lesson Planner

Vocabulary Development

traditional method

Skills Maintenance

Multiplying and Simplifying, Angles

Building Number Concepts: ▶ The Traditional Method for Dividing Fractions

Students use the traditional method for dividing fractions: invert and multiply. This method works for any fractions. Fraction bars and number lines work for fractions that neatly line up but not for all fractions. The traditional method is more efficient but does not visually show the partitions. The traditional method usually results in quotients that are not in lowest terms. The answers have to be simplified, just as in multiplication.

Objective

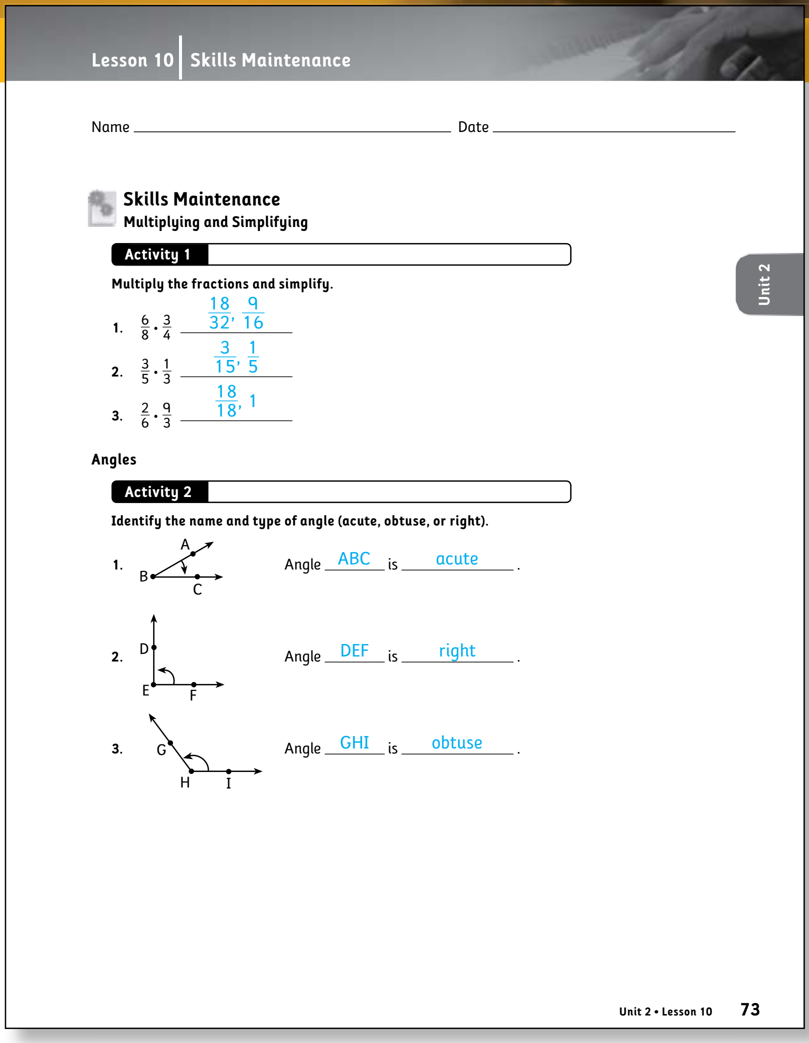
Students will divide fractions using the traditional method.

Monitoring Progress: ▶ Quiz 2

Distribute the quiz, and remind students that the questions involve material covered over the previous lessons in the unit.

Homework

Students multiply and simplify fractions, divide fractions using the traditional method and simplify the answers, and draw objects such as points, lines, angles, and rays. In Distributed Practice, students add and subtract fractions.



Lesson 10 | Skills Maintenance

Name ______ Date ______

Skills Maintenance

Multiplying and Simplifying

Activity 1

Multiply the fractions and simplify.

1. $\frac{6}{8} \cdot \frac{3}{4}$ $\frac{18}{32}, \frac{9}{16}$
2. $\frac{3}{5} \cdot \frac{1}{3}$ $\frac{3}{15}, \frac{1}{5}$
3. $\frac{2}{6} \cdot \frac{9}{3}$ $\frac{18}{18}, 1$

Angles

Activity 2

Identify the name and type of angle (acute, obtuse, or right).

1. Angle ABC is acute.
2. Angle DEF is right.
3. Angle GHI is obtuse.

Unit 2 • Lesson 10 73

Skills Maintenance

Multiplying and Simplifying, Angles

(*Interactive Text*, page 73)

Activity 1

Students multiply the fractions, then simplify the answer.

Activity 2

Students identify the name and type of the angle: acute, obtuse, or right.

Building Number Concepts:
▶ The Traditional Method for Dividing Fractions

How do we divide fractions using the traditional method?

(*Student Text*, pages 134–135)

Connect to Prior Knowledge

Remind students about division of fractions using fraction bars. Elicit observations about drawing fraction bars for the problem $\frac{6}{23} \div \frac{10}{20}$.

Listen for:

- *It would be really difficult. You would have to draw fraction bars in 23rds and in 20ths.*
- *Your unit would be $\frac{10}{20}$. That would be hard to try to repeat on the $\frac{6}{23}$ fraction bar.*

Link to Today's Concept

Tell students that in today's lesson, we use the **traditional method** to divide fractions.

Build Vocabulary

Explain to students that we need the traditional method for dividing fractions because some fractions do not work with fraction bars or number lines.

Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate how to use the traditional method in one of the following ways:

mBook: Use the *mBook Teacher Edition* for page 134 of the *Student Text*. m

Overhead Projector: Write the problem $\frac{5}{4} \div \frac{1}{2}$ on a transparency, and modify as discussed.

Lesson 10 ▶The Traditional Method for Dividing Fractions

Monitoring Progress:

▶Quiz 2

▶The Traditional Method for Dividing Fractions

Vocabulary
traditional method

How do we divide fractions using the traditional method?

Sometimes it is hard to use fraction bars to divide fractions. For example, it would be difficult and time-consuming to draw a fraction bar for a fraction with a large denominator, such as $\frac{3}{100}$. In those cases, we use the **traditional method** for dividing fractions.

When we use the traditional method, we invert and multiply. We saw an example of this in Lesson 8. Invert means to flip over the fraction. In this example, we flip over the second fraction and multiply across. The traditional method works for any division problem with fractions.

Example 1

Divide these fractions using invert and multiply.

$$\frac{5}{4} \div \frac{1}{2} = \frac{5}{4} \cdot \frac{2}{1} = \frac{10}{4}$$

Flip over the second fraction.

The answer is $\frac{10}{4}$.

$$\frac{2}{3} \div \frac{3}{100} = \frac{2}{3} \cdot \frac{100}{3} = \frac{200}{9}$$

Flip over the second fraction.

The answer is $\frac{200}{9}$.

This method works every time. It's a nice shortcut.

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Board: Write the problem $\frac{5}{4} \div \frac{1}{2}$ on the board, and modify as discussed.

- Display the problem $\frac{5}{4} \div \frac{1}{2}$. m
- Remind students that we invert and multiply. Show $\frac{5}{4} \cdot$ to the right of the problem. Explain that we need to invert the second fraction before we multiply. Show an arrow from $\frac{1}{2}$ to the space where the inverted fraction goes. Then show the inverted fraction $\frac{2}{1}$. m
- Multiply across to come up with the answer $\frac{10}{4}$. m
- Go through the problem $\frac{2}{3} \div \frac{3}{100}$ in the same way. m

Explain

Help students notice that the answers $\frac{10}{4}$ and $\frac{200}{9}$ in the example are not simplified. State that this method is efficient, but the answers are usually not in simplest form. We have to simplify the answers just like in multiplication. It is the same process.

Demonstrate

- Have students look at **Example 2** on page 135 of the *Student Text*, where we divide two fractions and simplify the answer.
  The quotient after the division is $\frac{6}{16}$. This fraction can be simplified. The GCF of 6 and 16 is 2. We pull out the 2 from the numerator and the denominator. The result is $\frac{3}{8}$, the simplest form.

Check for Understanding

Engagement Strategy: Think Tank

Ask students to use the traditional method to divide $\frac{2}{3} \div \frac{3}{6}$ $\left(\frac{4}{3}, \textit{or } 1\frac{1}{3}\right)$. Write the problem on the board. Distribute strips of paper for students to write their name and answer on. When students finish solving the problem, collect the strips, and draw out an answer. Read the answer out loud. If it is correct, congratulate that student. If it is incorrect, invite a volunteer to share the correct answer. Go over the steps with the class.

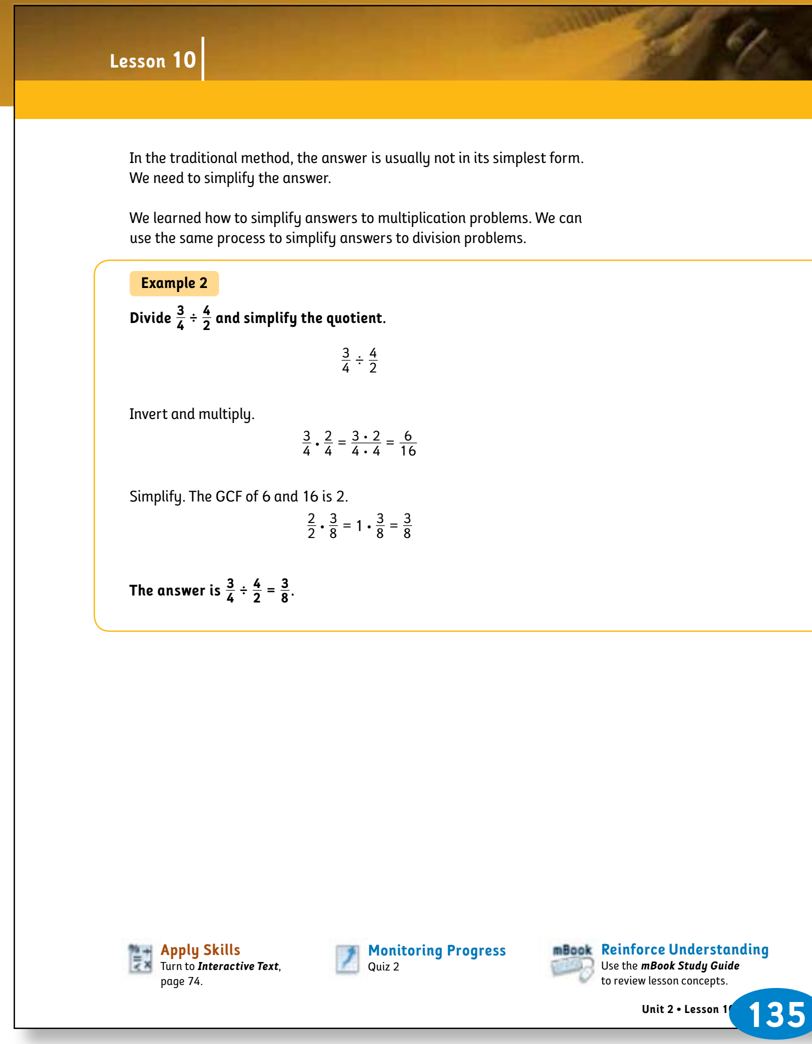
Reinforce Understanding

If students need additional practice, have them solve the following problems:

$\frac{4}{8} \div \frac{2}{3}$ $\left(\frac{3}{4}\right)$

$\frac{3}{4} \div \frac{2}{6}$ $\left(\frac{9}{4}, \textit{or } 2\frac{1}{4}\right)$

$\frac{1}{2} \div \frac{4}{12}$ $\left(\frac{3}{2}, \textit{or } 1\frac{1}{2}\right)$



Lesson 10

In the traditional method, the answer is usually not in its simplest form. We need to simplify the answer.

We learned how to simplify answers to multiplication problems. We can use the same process to simplify answers to division problems.

Example 2

Divide $\frac{3}{4} \div \frac{4}{2}$ and simplify the quotient.

$$\frac{3}{4} \div \frac{4}{2}$$

Invert and multiply.

$$\frac{3}{4} \cdot \frac{2}{4} = \frac{3 \cdot 2}{4 \cdot 4} = \frac{6}{16}$$

Simplify. The GCF of 6 and 16 is 2.

$$\frac{2}{2} \cdot \frac{3}{8} = 1 \cdot \frac{3}{8} = \frac{3}{8}$$

The answer is $\frac{3}{4} \div \frac{4}{2} = \frac{3}{8}$.

Apply Skills
Turn to ***Interactive Text***, page 74.

Monitoring Progress
Quiz 2

Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

Unit 2 • Lesson 10 135

Apply Skills

(*Interactive Text*, page 74)

Have students turn to page 74 in the *Interactive Text*, which provides students an opportunity to practice dividing fractions.

Activity 1

Students divide the fractions using the traditional method, then simplify the answer. Monitor students' work as they complete this activity.

Watch for:

- Do students know which fraction to invert?
- Can students multiply across?
- Can students simplify the answer?

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Name ______________________ Date ______________________

Apply Skills

The Traditional Method for Dividing Fractions

Activity 1

Divide the fractions using the traditional method. Write the quotient in its simplest form.

1. $\frac{2}{3} \div \frac{1}{8}$ $\frac{16}{3}$, $5\frac{1}{3}$
2. $\frac{1}{4} \div \frac{2}{8}$ $\frac{8}{8}$, 1
3. $\frac{5}{6} \div \frac{3}{4}$ $\frac{20}{18}$, $1\frac{1}{9}$
4. $\frac{3}{100} \div \frac{1}{4}$ $\frac{12}{100}$, $\frac{3}{25}$
5. $\frac{3}{25} \div \frac{6}{7}$ $\frac{21}{150}$, $\frac{7}{50}$
6. $\frac{6}{16} \div \frac{5}{6}$ $\frac{36}{80}$, $\frac{9}{20}$

Reinforce Understanding

Use the **mBook** ***Study Guide*** to review lesson concepts.

Monitoring Progress: ▶ Quiz 2

Assess Quiz 2

- Administer Quiz 2 Form A in the *Assessment Book*, pages 25–26. (If necessary, retest students with Quiz 2 Form B from the *mBook Teacher Edition* following differentiation.)

Students	Assess	Differentiate
	Day 1	Day 2
All	Quiz 2 *Form A*	
Scored 80% or above		Extension
Scored Below 80%		Reinforcement

Differentiate

- Review Quiz 2 Form A with class.
- Identify students for Extension or Reinforcement.

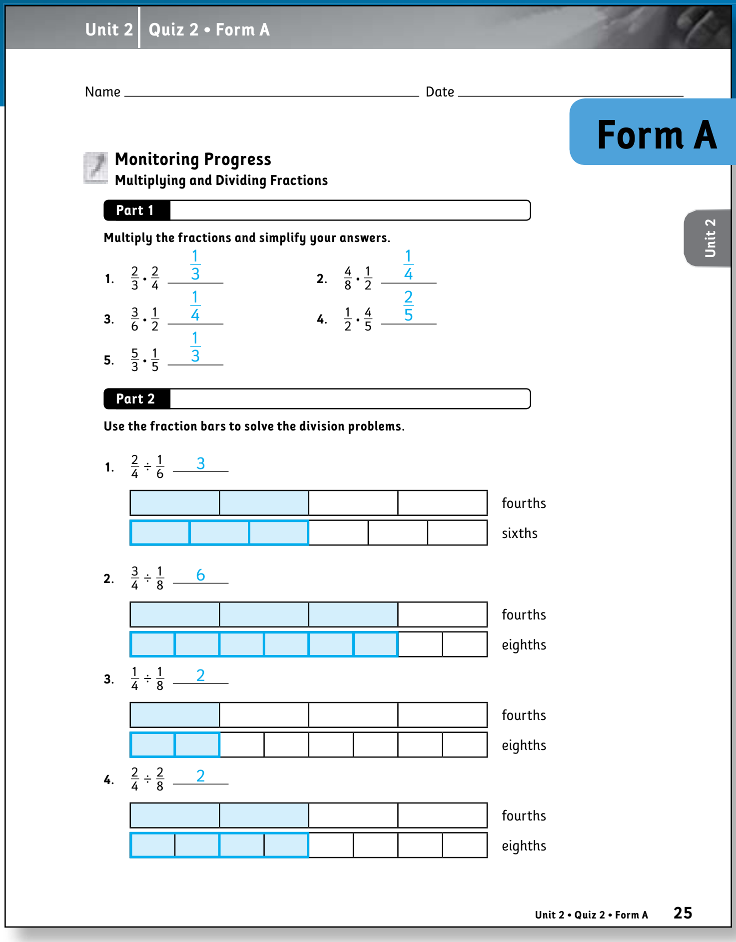
Extension

For those students who score 80 percent or better, provide the On Track! Activities from Unit 2, Lessons 6–10, from the *mBook Teacher Edition.*

Reinforcement

For those students who score below 80 percent, provide additional support in one of the following ways:

- Have students access the online tutorial provided in the *mBook Study Guide.*
- Have students complete the Interactive Reinforcement Exercises for Unit 2, Lessons 5–9, in the *mBook Study Guide.*
- Provide teacher-directed reteaching of unit concepts.



Unit 2 | Quiz 2 • Form A

Name ______________ Date ______________

Form A

Unit 2

Monitoring Progress
Multiplying and Dividing Fractions

Part 1

Multiply the fractions and simplify your answers.

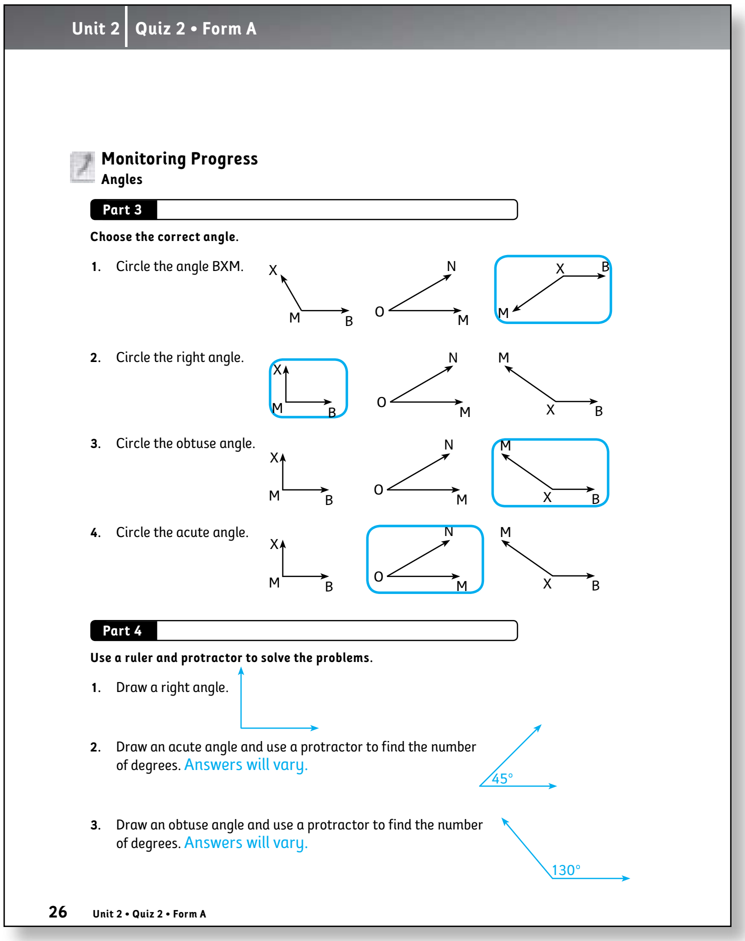
1. $\frac{2}{3} \cdot \frac{2}{4}$ $\frac{1}{3}$
2. $\frac{4}{8} \cdot \frac{1}{2}$ $\frac{1}{4}$
3. $\frac{3}{6} \cdot \frac{1}{2}$ $\frac{1}{4}$
4. $\frac{1}{2} \cdot \frac{4}{5}$ $\frac{2}{5}$
5. $\frac{5}{3} \cdot \frac{1}{5}$ $\frac{1}{3}$

Part 2

Use the fraction bars to solve the division problems.

1. $\frac{2}{4} \div \frac{1}{6}$ 3
fourths
sixths
2. $\frac{3}{4} \div \frac{1}{8}$ 6
fourths
eighths
3. $\frac{1}{4} \div \frac{1}{8}$ 2
fourths
eighths
4. $\frac{2}{4} \div \frac{2}{8}$ 2
fourths
eighths

Unit 2 • Quiz 2 • Form A 25



Unit 2 | Quiz 2 • Form A

Monitoring Progress
Angles

Part 3

Choose the correct angle.

1. Circle the angle BXM.
2. Circle the right angle.
3. Circle the obtuse angle.
4. Circle the acute angle.

Part 4

Use a ruler and protractor to solve the problems.

1. Draw a right angle.
2. Draw an acute angle and use a protractor to find the number of degrees. Answers will vary.
45°
3. Draw an obtuse angle and use a protractor to find the number of degrees. Answers will vary.
130°

26 Unit 2 • Quiz 2 • Form A

Name ______________________ Date ______________

Form B

mBook

Monitoring Progress
Multiplying and Dividing Fractions

Part 1

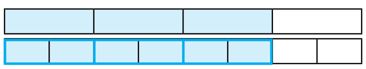
Multiply the fractions and simplify your answers.

1. $\frac{3}{2} \cdot \frac{2}{4}$ $\frac{3}{4}$
2. $\frac{6}{8} \cdot \frac{1}{2}$ $\frac{3}{8}$
3. $\frac{3}{6} \cdot \frac{1}{2}$ $\frac{1}{4}$
4. $\frac{1}{2} \cdot \frac{3}{5}$ $\frac{3}{10}$
5. $\frac{5}{3} \cdot \frac{2}{5}$ $\frac{2}{3}$

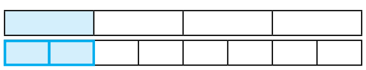
Part 2

Use the fraction bars to solve the division problems.

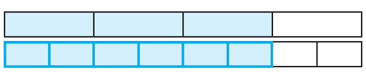
1. $\frac{3}{4} \div \frac{2}{8}$ 3



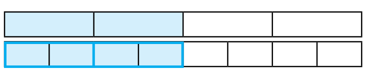
2. $\frac{1}{4} \div \frac{1}{8}$ 2



3. $\frac{3}{4} \div \frac{1}{8}$ 6



4. $\frac{2}{4} \div \frac{2}{8}$ 2



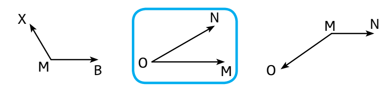
Name ______________________ Date ______________

Monitoring Progress
Angles

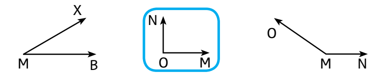
Part 3

Choose the correct angle.

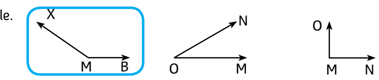
1. Circle the angle NOM.



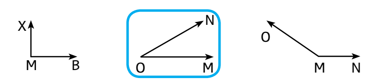
2. Circle the right angle.



3. Circle the obtuse angle.



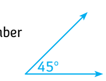
4. Circle the acute angle.



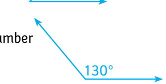
Part 4

Use a ruler and protractor to solve the problems.

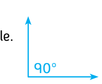
1. Draw an acute angle and use a protractor to find the number of degrees. Answers will vary.



2. Draw an obtuse angle and use a protractor to find the number of degrees. Answers will vary.



3. Draw a right angle.



Homework

Go over the instructions on page 136 of the *Student Text* for each part of the homework.

Activity 1

Students multiply and simplify fractions.

Activity 2

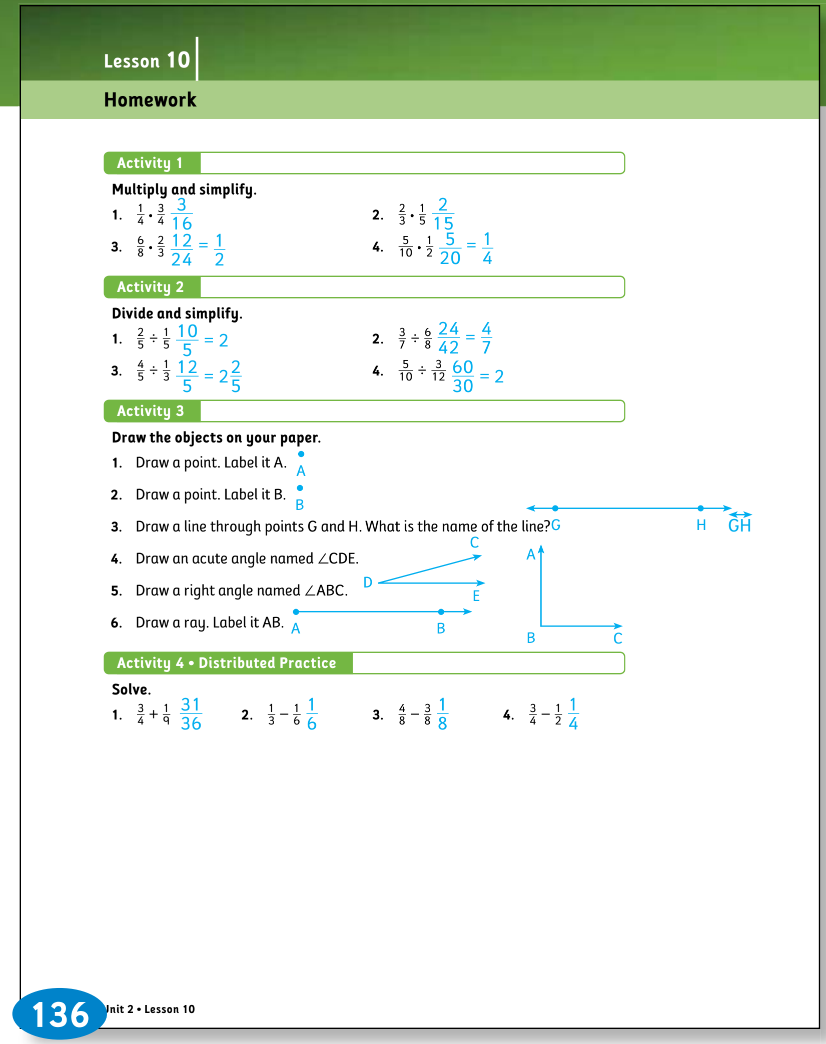
Students divide fractions using the traditional method and then simplify the answers.

Activity 3

Students draw different objects, such as points, lines, angles, and rays on their papers.

Activity 4 • Distributed Practice

Students add and subtract fractions.



Homework

Activity 1

Multiply and simplify.

1. $\frac{1}{4} \cdot \frac{3}{4}$ $\frac{3}{16}$
2. $\frac{2}{3} \cdot \frac{1}{5}$ $\frac{2}{15}$
3. $\frac{6}{8} \cdot \frac{2}{3}$ $\frac{12}{24} = \frac{1}{2}$
4. $\frac{5}{10} \cdot \frac{1}{2}$ $\frac{5}{20} = \frac{1}{4}$

Activity 2

Divide and simplify.

1. $\frac{2}{5} \div \frac{1}{5}$ $\frac{10}{5} = 2$
2. $\frac{3}{7} \div \frac{6}{8}$ $\frac{24}{42} = \frac{4}{7}$
3. $\frac{4}{5} \div \frac{1}{3}$ $\frac{12}{5} = 2\frac{2}{5}$
4. $\frac{5}{10} \div \frac{3}{12}$ $\frac{60}{30} = 2$

Activity 3

Draw the objects on your paper.

1. Draw a point. Label it A.
2. Draw a point. Label it B.
3. Draw a line through points G and H. What is the name of the line? $\overleftrightarrow{GH}$
4. Draw an acute angle named ∠CDE.
5. Draw a right angle named ∠ABC.
6. Draw a ray. Label it AB.

Activity 4 • Distributed Practice

Solve.

1. $\frac{3}{4} + \frac{1}{9}$ $\frac{31}{36}$
2. $\frac{1}{3} - \frac{1}{6}$ $\frac{1}{6}$
3. $\frac{4}{8} - \frac{3}{8}$ $\frac{1}{8}$
4. $\frac{3}{4} - \frac{1}{2}$ $\frac{1}{4}$

Lesson 11 | Problem Solving: Measuring With a U.S. Customary Ruler

Lesson Planner

Skills Maintenance

Simplifying Fractions

Problem Solving: Measuring With a U.S. Customary Ruler

Students learn to measure with a U.S. customary ruler and round to the nearest $\frac{1}{4}$ inch. Students need to be familiar with the U.S. customary system of measurement as well as the metric system. Inches are more difficult to measure with because fractions are needed to get an accurate measurement. Inches are bigger than centimeters and millimeters, and it is hard to get an accurate measurement of smaller objects (e.g., line segments) using a larger unit of measure.

Objective

Students will measure using a U.S. customary ruler.

Homework

Students measure line segments and round to the nearest $\frac{1}{4}$ inch. In Distributed Practice, students solve addition and subtraction problems involving fractions and they solve a subtraction problem with whole numbers.

Name ____________________ Date ____________________

Skills Maintenance
Simplifying Fractions

Activity 1

Multiply the fractions. Then simplify the answer.

1. $\frac{3}{9} \cdot \frac{2}{3}$ $\frac{6}{27}, \frac{2}{9}$
2. $\frac{4}{8} \cdot \frac{4}{6}$ $\frac{16}{48}, \frac{1}{3}$
3. $\frac{6}{9} \cdot \frac{3}{6}$ $\frac{18}{54}, \frac{1}{3}$

Activity 2

Divide the fractions using invert and multiply. Then simplify the answer using the GCF. Write the answer in simplest form.

1. $\frac{1}{2} \div \frac{1}{8}$ $\frac{8}{2}, 4$
2. $\frac{3}{4} \div \frac{5}{6}$ $\frac{18}{20}, \frac{9}{10}$
3. $\frac{4}{6} \div \frac{5}{4}$ $\frac{16}{30}, \frac{8}{15}$

Unit 2

Skills Maintenance

Simplifying Fractions

(*Interactive Text*, page 75)

Activity 1

Students solve multiplication problems, then simplify the answers.

Activity 2

Students solve division problems, then simplify the answers.

Problem Solving: ▶ Measuring With a U.S. Customary Ruler

What is the difference between a U.S. customary ruler and a metric ruler?

(*Student Text*, pages 137–138)

Connect to Prior Knowledge

Begin by reminding students about the metric ruler. Ask students to make observations about the units on a metric ruler.

Listen for:

- *The small lines are millimeters.*
- *There's a bigger line after 5 millimeters.*
- *There's an even bigger line after 10 millimeters. That's a centimeter. There's a number for the centimeters.*

Be sure students are familiar with the units on a metric ruler.

Link to Today's Concept

In today's lesson, we compare metric and U.S. customary units.

Demonstrate

Engagement Strategy: Teacher Modeling

Compare the metric ruler to the U.S. customary ruler in one of the following ways:

mBook: Use the *mBook Teacher Edition* for *Student Text*, page 137. m

Overhead Projector: Use Transparencies 7 and 8, and modify as discussed.

- Display the U.S. customary ruler. Tell students that this ruler is used to measure inches. m

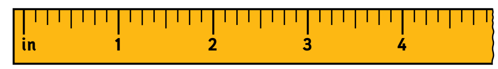
Lesson 11 | Problem Solving: ▶ Measuring With a U.S. Customary Ruler

▶ Problem Solving: Measuring With a U.S. Customary Ruler

What is the difference between a U.S. customary ruler and a metric ruler?

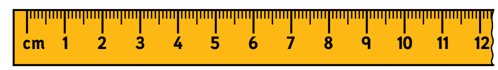
We learned to measure line segments using a metric ruler. We measured in units of millimeters or centimeters. Now let's look at the U.S. customary ruler. We use it to measure in units of inches. Let's compare these two measuring tools.

The *U.S. customary ruler* looks like this:



The units on this ruler are fractional parts of an inch. The marks represent eighths, fourths, and halves of inches, as well as whole inches.

The *metric ruler* looks like this:



The units on this ruler are:

- millimeters (shortest lines)
- centimeters (longest lines)

The mid-length lines are $\frac{1}{2}$ centimeters, which are equal to 5 millimeters.

Millimeters and centimeters are easier to work with than inches because they are smaller units. Because a millimeter is a very small unit, it is easy to use it to measure in whole numbers. The small unit also allows us to get a very accurate measurement.

Unit 2 • Lesson 11 137

- Point out that the marks on this ruler represent fractional parts of an inch. Point out the $\frac{1}{4}$ inch, $\frac{1}{2}$ inch, and whole inch marks. m
- Display the metric ruler. Remind students that this ruler measures centimeters. m
- Review the markings on this ruler with students. Point out both the centimeter and millimeter markings. m
- Display both rulers. Ask students to make observations about similarities and differences between the two rulers.

Listen for:

- *The smallest units on a metric ruler are millimeters.*
- *The smallest units on a customary ruler are fractions of an inch.*

What is the difference between a U.S. customary ruler and a metric ruler? *(continued)*

Demonstrate

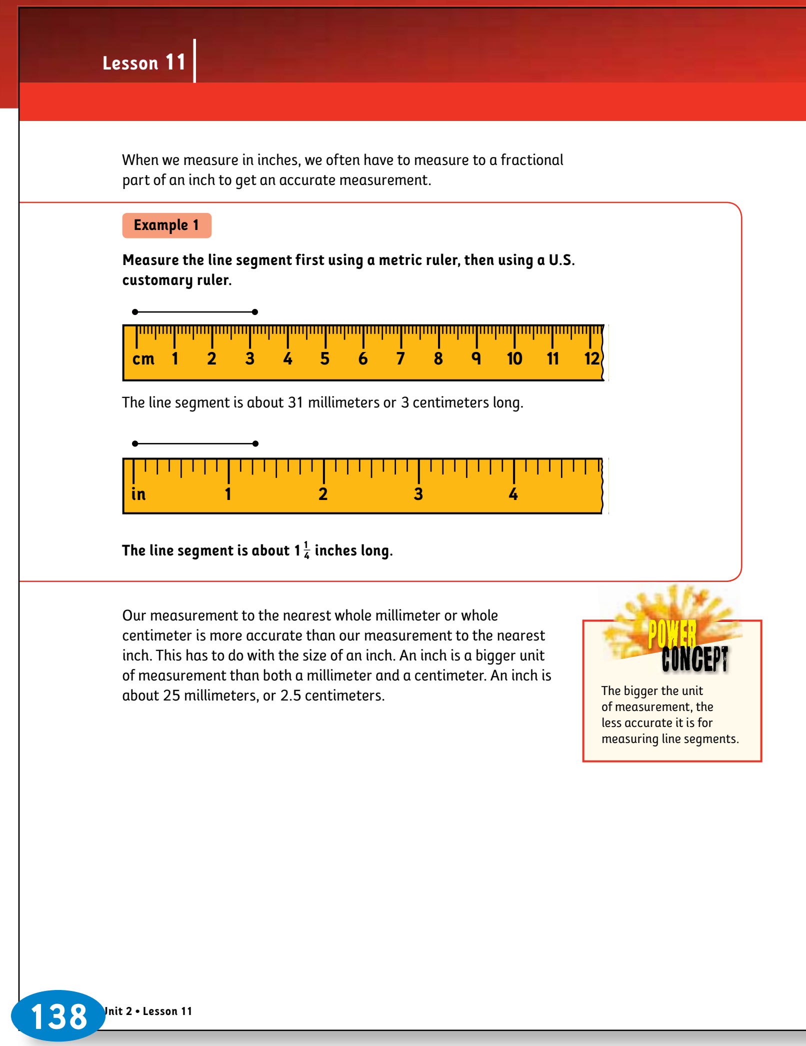
- Look at **Example 1** on page 138 of the *Student Text*. In this example, we measure a line segment both in metric and U.S. customary units. This example shows that it is much easier to get a whole-number measurement in metric units than in inches.
- Explain to students that this is because of the size of the units. Millimeters are very small. It is easier to get close to the next millimeter. Inches are much larger. Explain to students that there are about 25 millimeters in one inch. We have to use fractions to measure as accurately in inches.

Discuss

Call students' attention to the Power Concept, and point out that it will be helpful as they complete the activities.

POWER CONCEPT

The bigger the unit of measurement, the less accurate it is for measuring line segments.


Lesson 11

When we measure in inches, we often have to measure to a fractional part of an inch to get an accurate measurement.

Example 1

Measure the line segment first using a metric ruler, then using a U.S. customary ruler.

cm 1 2 3 4 5 6 7 8 9 10 11 12

The line segment is about 31 millimeters or 3 centimeters long.

in 1 2 3 4

The line segment is about $1\frac{1}{4}$ inches long.

Our measurement to the nearest whole millimeter or whole centimeter is more accurate than our measurement to the nearest inch. This has to do with the size of an inch. An inch is a bigger unit of measurement than both a millimeter and a centimeter. An inch is about 25 millimeters, or 2.5 centimeters.

POWER CONCEPT

The bigger the unit of measurement, the less accurate it is for measuring line segments.

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How do we measure to the nearest $\frac{1}{4}$ inch?

(*Student Text,* page 139–140)

Demonstrate

- Turn to page 139 of the *Student Text*, and read through the text at the top of the page.
- Make sure students understand that not all measurements are represented in fourths; they can be any number equal to a multiple of $\frac{1}{4}$, such as $\frac{1}{2}$ or $\frac{3}{4}$.
- Have students look at **Example 1**. In this example, we demonstrate how to measure to the nearest $\frac{1}{4}$ inch. The first line segment shows that the nearest $\frac{1}{4}$ inch is a whole number. The second line segment shows that the nearest $\frac{1}{4}$ inch is $\mathbf{\frac{3}{4}}$.

How do we measure to the nearest $\frac{1}{4}$ inch?

Measuring line segments in inches is not very accurate unless we measure to a fractional part of an inch, such as fourths. This might be confusing because not all measurements can be expressed in fourths.

Here are some fractional parts that are fourths:

$\frac{1}{4}$ $\quad$ $\frac{2}{4}$ (or $\frac{1}{2}$) $\quad$ $\frac{3}{4}$ $\quad$ $\frac{8}{4}$ (or 2) $\quad$ $\frac{12}{4}$ (or 3)

When we measure to the nearest $\frac{1}{4}$ inch, our measurements could be any number that is a multiple of $\frac{1}{4}$. Here are some examples:

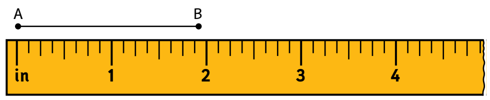
$1\frac{1}{4}$ $\quad$ $15\frac{1}{2}$ $\quad$ $27\frac{3}{4}$ $\quad$ 50

Any of these numbers could represent the nearest $\frac{1}{4}$ inch.

Example 1

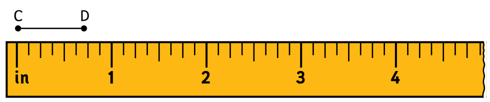
Measure the line segments using a U.S. customary ruler. Round the answers to the nearest $\frac{1}{4}$ inch.

Measure $\overline{AB}$ to the nearest $\frac{1}{4}$ inch.



The measure of $\overline{AB}$, rounded to the nearest $\frac{1}{4}$ inch, is 2 inches.

Measure $\overline{CD}$ to the nearest $\frac{1}{4}$ inch.

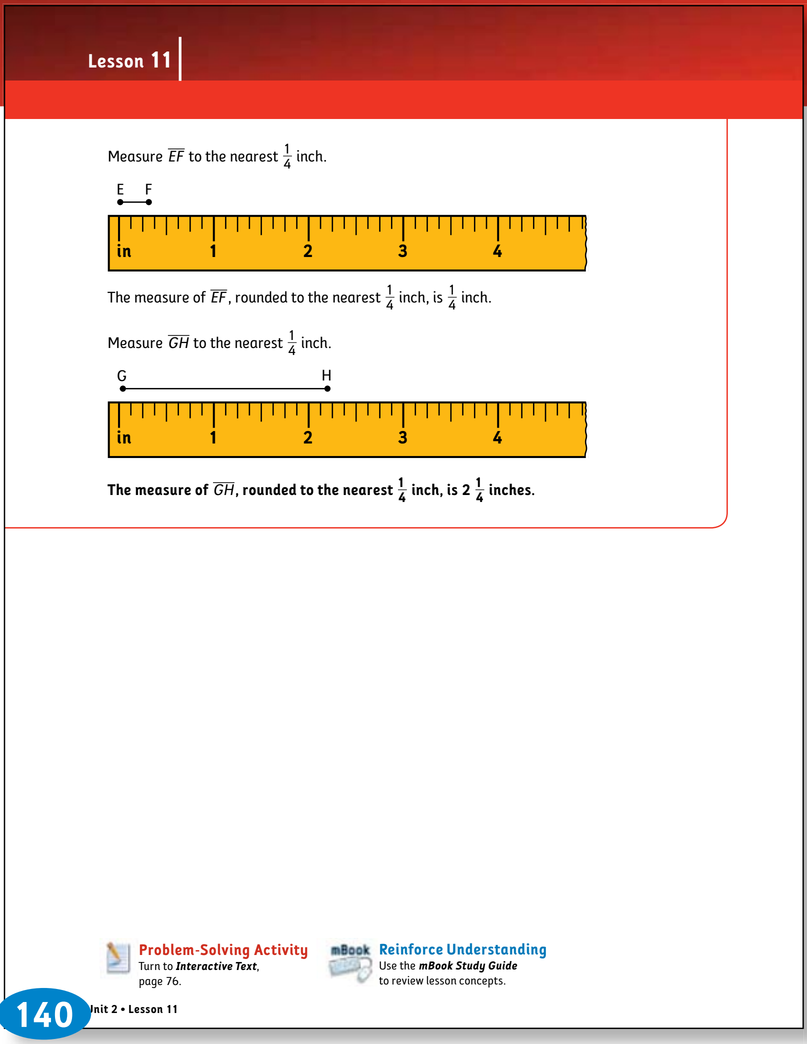


The measure of $\overline{CD}$, rounded to the nearest $\frac{1}{4}$ inch, is $\frac{3}{4}$ inch.

How do we measure to the nearest $\frac{1}{4}$ inch? *(continued)*

Demonstrate

- Continue going over the example on page 140 of the *Student Text*.
- Note to students that the first line segment on the page rounds to the nearest $\frac{1}{4}$ inch, in this case, $\mathbf{\frac{1}{4}}$. The second line segment rounds to the nearest $\frac{1}{4}$ inch, $\mathbf{2\frac{1}{4}}$. It is important that students see these estimates are all to the nearest $\frac{1}{4}$ inch.



Lesson 11

Measure $\overline{EF}$ to the nearest $\frac{1}{4}$ inch.

The measure of $\overline{EF}$, rounded to the nearest $\frac{1}{4}$ inch, is $\frac{1}{4}$ inch.

Measure $\overline{GH}$ to the nearest $\frac{1}{4}$ inch.

The measure of $\overline{GH}$, rounded to the nearest $\frac{1}{4}$ inch, is $2\frac{1}{4}$ inches.

Problem-Solving Activity
Turn to ***Interactive Text***, page 76.

Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

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Check for Understanding

Engagement Strategy: Think, Think

Ask students the following questions. Tell them that you will call on one of them to answer a question after you ask it. Tell them to listen for their names. After each question, allow time for students to think of the answer. Then call on a student.

Ask:

Why are millimeters and centimeters easier to work with than inches? (*They are smaller units than inches, so it is easy to measure in whole numbers. Also, the measurements allow for more accuracy.*)

How can we get a more accurate measurement using a U.S. customary ruler? (*We can measure to a fractional part of an inch, such as the nearest $\frac{1}{4}$ inch.*)

When we measure to the nearest $\frac{1}{4}$ inch, will the measurements always be expressed in fourths? (*No, they can be any number that is equal to a multiple of $\frac{1}{4}$.*)

Problem-Solving Activity
(*Interactive Text*, page 76)

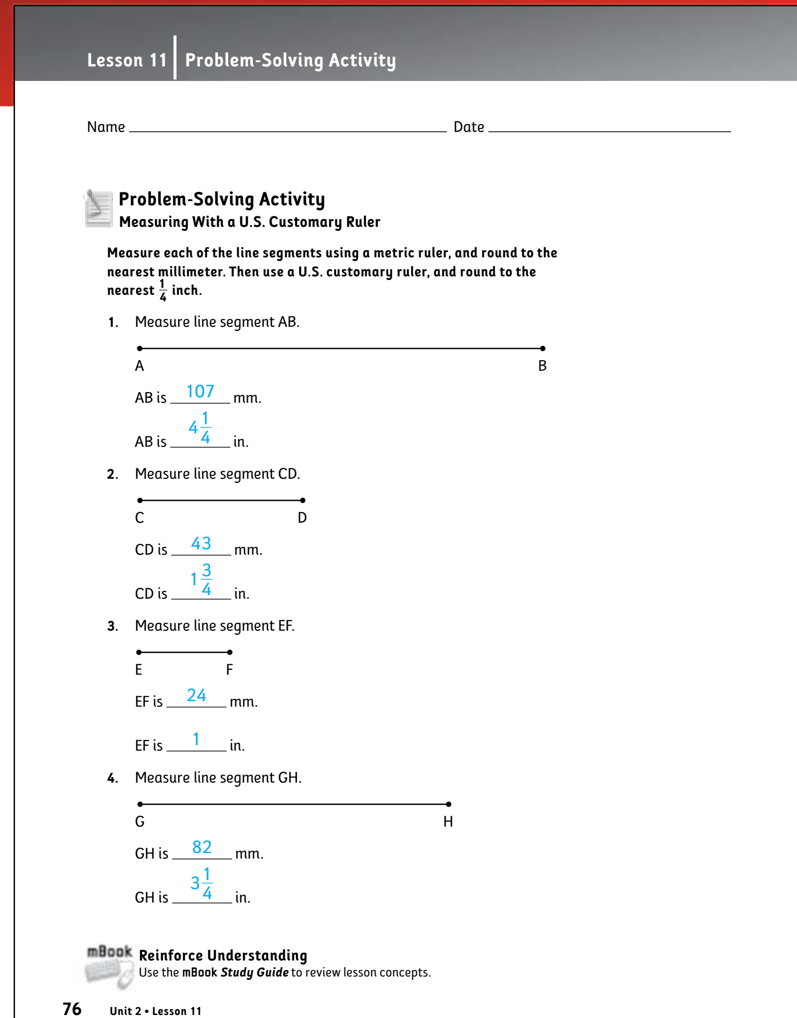
Have students turn to page 76 in the *Interactive Text*, which provides students an opportunity to practice measuring with U.S. customary and metric rulers. In this activity, students measure line segments first to the nearest millimeter, then to the nearest $\frac{1}{4}$ inch.

Monitor students' work as they complete the activity.

Watch for:

- Can students measure with a metric ruler to the nearest millimeter?
- Can students measure with a U.S. customary ruler to the nearest $\frac{1}{4}$ inch?
- Do students understand that the nearest $\frac{1}{4}$ means $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or a whole number?
- Are students lining up the ruler correctly to get the most accurate measurement?

Reinforce Understanding
Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.


Lesson 11 | Problem-Solving Activity

Name ______________________ Date ______________________

Problem-Solving Activity
Measuring With a U.S. Customary Ruler

Measure each of the line segments using a metric ruler, and round to the nearest millimeter. Then use a U.S. customary ruler, and round to the nearest $\frac{1}{4}$ inch.

1. Measure line segment AB.

   A B

   AB is 107 mm.

   AB is $4\frac{1}{4}$ in.

2. Measure line segment CD.

   C D

   CD is 43 mm.

   CD is $1\frac{3}{4}$ in.

3. Measure line segment EF.

   E F

   EF is 24 mm.

   EF is 1 in.

4. Measure line segment GH.

   G H

   GH is 82 mm.

   GH is $3\frac{1}{4}$ in.

mBook **Reinforce Understanding**
Use the mBook *Study Guide* to review lesson concepts.

76 Unit 2 • Lesson 11

Homework

Go over the instructions on page 141 of the *Student Text* for each part of the homework.

Activity 1

Students measure line segments with a U.S. customary ruler and round to the nearest $\frac{1}{4}$ inch.

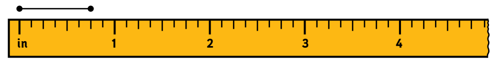
Activity 2 • Distributed Practice

Students solve addition and subtraction problems involving fractions and a subtraction problem with whole numbers.

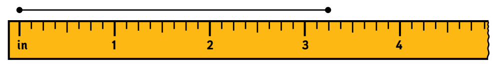
Homework

Activity 1

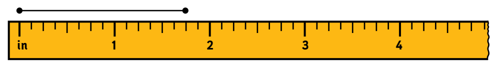
Measure the line segments using a U.S. customary ruler. Round to the nearest $\frac{1}{4}$ inch.

1.


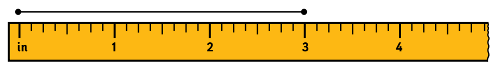
$\frac{3}{4}$ inch

2.


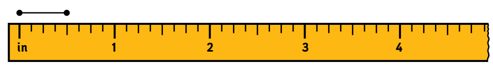
$3\frac{1}{4}$ inches

3.


$1\frac{3}{4}$ inches

4.


3 inches

5.


$\frac{1}{2}$ inch

Activity 2 • Distributed Practice

Solve.

1. $\frac{1}{3} + \frac{1}{4}$ $\frac{7}{12}$
2. $\frac{1}{2} + \frac{1}{6}$ $\frac{4}{6} = \frac{2}{3}$
3. $\frac{1}{9} + \frac{3}{6}$ $\frac{11}{18}$
4. $\frac{7}{8} - \frac{3}{4}$ $\frac{1}{8}$
5. $\frac{9}{10} - \frac{8}{10}$ $\frac{1}{10}$
6. $5{,}009 - 1{,}239$ $3{,}770$

Lesson 12 ▶ Multiplying Three Fractions and Simplifying Using the GCF

Problem Solving: ▶ Introduction to the Compass

Lesson Planner

Build Vocabulary

multiplicand

Skills Maintenance

Multiply, Divide, and Simplify Fractions

Building Number Concepts: ▶ Multiplying Three Fractions and Simplifying Using the GCF

Students solve multiplication problems involving three fractions. Students learn that when multiplying three fractions together, the resulting numerator and denominator might be quite large.

Simplifying becomes more challenging in this case. More rules of numbers, such as divisibility, factors, GCF, and powers of 10, need to be applied. Students need to look at factoring in more complex situations to prepare for factoring in algebra.

Objective

Students will multiply three fractions and use the GCF to simplify.

Problem Solving: ▶ Introduction to the Compass

We introduce a new measuring tool: the compass. Students learn that an angle, such as a right angle (and a perpendicular line), can be constructed using a compass and a ruler.

Objective

Students will use a compass to draw right angles.

Homework

Students multiply, divide, and simplify fractions, multiply three fractions and simplify, and measure line segments to the nearest $\frac{1}{4}$ inch. In Distributed Practice, students add and subtract fractions.

Lesson 12 | Skills Maintenance

Name ______________________ Date ______________

Skills Maintenance

Multiply, Divide, and Simplify Fractions

Activity 1

Use traditional algorithms to multiply and divide the fractions. Simplify the answers.

1. $\frac{3}{4} \cdot \frac{2}{9}$ $\frac{6}{36}, \frac{1}{6}$
2. $\frac{4}{5} \div \frac{3}{4}$ $\frac{16}{15}, 1\frac{1}{15}$
3. $\frac{7}{8} \cdot \frac{4}{6}$ $\frac{28}{48}, \frac{7}{12}$
4. $\frac{3}{8} \div \frac{6}{2}$ $\frac{6}{48}, \frac{1}{8}$

Unit 2

Unit 2 • Lesson 12 77

Skills Maintenance

Multiply, Divide, and Simplify Fractions

(*Interactive Text*, page 77)

Activity 1

Students multiply and divide fractions, then simplify the answers.

Building Number Concepts:
▶ Multiplying Three Fractions and Simplifying Using the GCF

How do we simplify fractions using the GCF?

(*Student Text*, page 142)

Connect to Prior Knowledge

Begin by asking students to recall the process for multiplying fractions.

Link to Today's Concept

In today's lesson we multiply three fractions and simplify the answer.

Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate how to simplify fractions in one of the following ways:

- **mBook**: Use the *mBook Teacher Edition* for page 142 of the *Student Text*. m
- **Overhead Projector**: Copy the fractions onto a transparency, and modify as discussed.
- **Board**: Copy the fractions onto the board, and modify as discussed.

- Show the fraction $\frac{8}{10}$ on the board, and have students list the steps for simplifying it. m

Listen for:

- *The numbers are both even so you know you can factor out a 2.*
- *Think about all the factors for 8 and 10 and find the GCF.*
- *Pull out a 2 from both the numerator and the denominator.*

Lesson 12 ▶Multiplying Three Fractions and Simplifying Using the GCF

Problem Solving:
▶Introduction to the Compass

▶Multiplying Three Fractions and Simplifying Using the GCF

How do we simplify fractions using the GCF?

Vocabulary
multiplicand

Let's review how to simplify a fraction using the GCF. We'll simplify the fraction $\frac{8}{10}$.

We know that both the numerator and denominator are even, so we can "pull out," or factor out, a 2. We pull out this factor because 2 is the GCF of 8 and 10.

$$\frac{8}{10} = \frac{2}{2} \cdot \frac{4}{5}$$
$$= 1 \cdot \frac{4}{5} = \frac{4}{5}$$

The simplified fraction is $\frac{4}{5}$.

Let's try with $\frac{70}{100}$.

We know that both the numerator and denominator are powers of 10, so we can factor out a 10. We pull out this factor because 10 is the GCF of 70 and 100.

$$\frac{70}{100} = \frac{10}{10} \cdot \frac{7}{10}$$
$$= 1 \cdot \frac{7}{10} = \frac{7}{10}$$

The simplified fraction is $\frac{7}{10}$.

142 Unit 2 • Lesson 12

- Walk through the steps with students. Explain that we can factor out a 2 from both the numerator and the denominator. m
- Show $\frac{8}{10} = \frac{2}{2}$ on the board. Then show the fraction $\frac{4}{5}$ to complete the statement, reminding students that **2 • 4 = 8** and **2 • 5 = 10**. Remind students that $\frac{2}{2}$ = 1, so $\frac{4}{5}$ is the simplified fraction. m
- Remind students that we can also rewrite this as $\mathbf{\frac{8}{10} = 1 \cdot \frac{4}{5} = \frac{4}{5}}$. m
- Next show $\frac{70}{100}$ on the board. Walk through the steps to simplify the fraction. Explain to students that because both numbers are powers of 10, we can pull out a 10. The fraction becomes $\frac{7}{10}$, and these numbers are much easier to work with. m

How do we multiply three fractions together?

(*Student Text*, page 143)

Build Vocabulary

Introduce the term **multiplicands** to students and explain that they are the numbers multiplied together in a problem.

Demonstrate

- Tell students that we know how to multiply two fractions together and simplify the answer. Now we look at how to multiply three fractions together.
- Have students turn to page 143 in the *Student Text* and discuss **Example 1**. In this example, we multiply three fractions together. We multiply across the first two from left to right. Then we multiply the third.
- Point out that the numbers get large fast when we multiply three numbers. The result needs to be simplified in most cases. Make sure students understand how to simplify the answer by finding the GCF.

Check for Understanding

Engagement Strategy: Look About

Write the problem $\frac{1}{3} \cdot \frac{2}{5} \cdot \frac{3}{4}$ on the board. Tell students that they will multiply the three fractions with the help of the whole class $\left(\frac{1}{3} \cdot \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{60} = \frac{1}{10}\right)$. Students should write their solutions in large writing on a piece of paper or a dry erase board. When students finish their work, they should hold up their answers for everyone to see.

If students are not sure about the answer, prompt them to look about at other students' solutions to help with their thinking. Review the answers after all students have held up their solutions.

Lesson 12

How do we multiply three fractions together?

Multiplicands are the numbers being multiplied together in a problem.

We have learned to multiply two fractions together and simplify the answer. Now we will learn to multiply three fractions together. The product of three fractions will most likely have large numbers in the numerator and denominator.

Example 1

Multiply and simplify.

$$\frac{1}{4} \cdot \frac{2}{5} \cdot \frac{1}{3}$$

Start with the first two fractions.

$$\frac{1}{4} \cdot \frac{2}{5} = \frac{2}{20}$$

Multiply the product by the remaining fraction.

$$\frac{2}{20} \cdot \frac{1}{3} = \frac{2}{60}$$

The answer is not in the simplest form.
Think about the factors of 2 and 60.
The GCF is 2.

$$\frac{2}{60} = \frac{2}{2} \cdot \frac{1}{30}$$

$$= 1 \cdot \frac{1}{30} = \frac{1}{30}$$

The simplified answer is $\frac{1}{30}$.

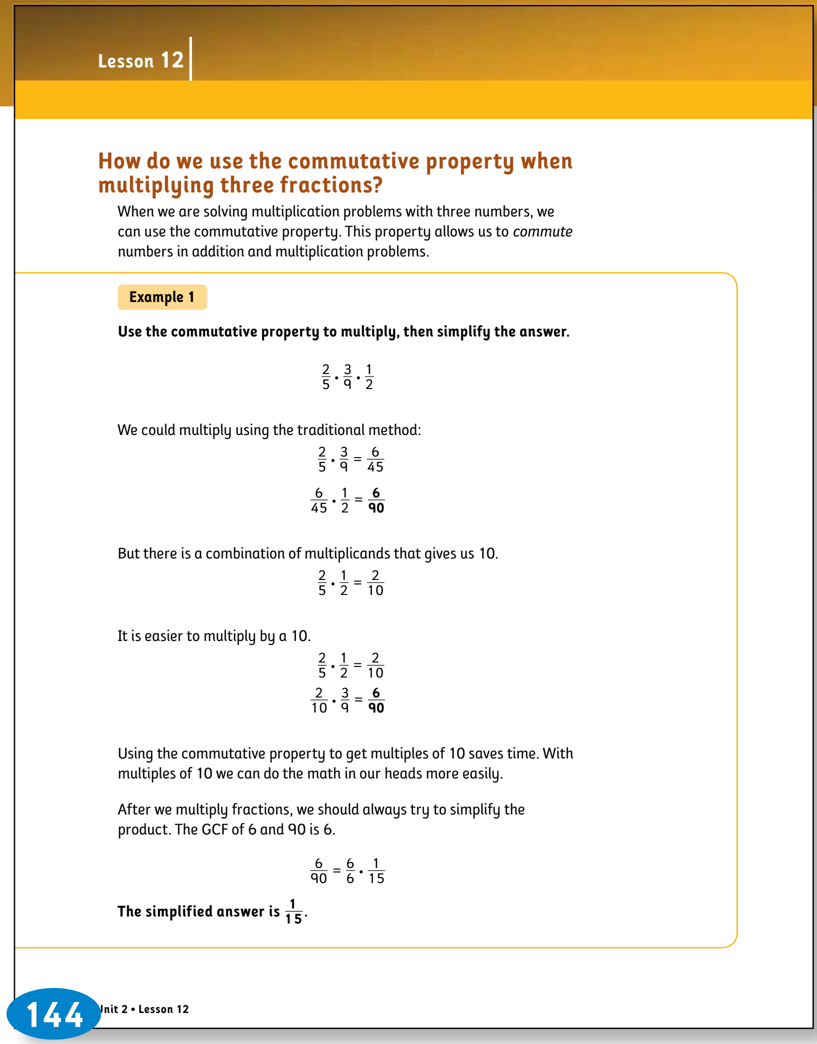
Unit 2 • Lesson 12 143

How do we use the commutative property when multiplying three fractions?

(*Student Text*, page 144)

Demonstrate

- Have students turn to **Example 1** on page 144 of the *Student Text*. In this example, we use commutation to move the numbers around so there are easier combinations to work with. For instance, $2 \cdot 5 = 10$. Tens are easier to multiply using mental math.
- Walk through the steps as outlined to show students first how we solve the problem using the traditional method and then using the commutative property.
- Point out that the answers are the same, but using multiples of 10 in this case saves us time.



Lesson 12

How do we use the commutative property when multiplying three fractions?

When we are solving multiplication problems with three numbers, we can use the commutative property. This property allows us to *commute* numbers in addition and multiplication problems.

Example 1

Use the commutative property to multiply, then simplify the answer.

$$\frac{2}{5} \cdot \frac{3}{9} \cdot \frac{1}{2}$$

We could multiply using the traditional method:

$$\frac{2}{5} \cdot \frac{3}{9} = \frac{6}{45}$$

$$\frac{6}{45} \cdot \frac{1}{2} = \mathbf{\frac{6}{90}}$$

But there is a combination of multiplicands that gives us 10.

$$\frac{2}{5} \cdot \frac{1}{2} = \frac{2}{10}$$

It is easier to multiply by a 10.

$$\frac{2}{5} \cdot \frac{1}{2} = \frac{2}{10}$$

$$\frac{2}{10} \cdot \frac{3}{9} = \mathbf{\frac{6}{90}}$$

Using the commutative property to get multiples of 10 saves time. With multiples of 10 we can do the math in our heads more easily.

After we multiply fractions, we should always try to simplify the product. The GCF of 6 and 90 is 6.

$$\frac{6}{90} = \frac{6}{6} \cdot \frac{1}{15}$$

The simplified answer is $\frac{1}{15}$.

144 Unit 2 • Lesson 12

What other properties do we use to simplify fractions?

(*Student Text*, page 145)

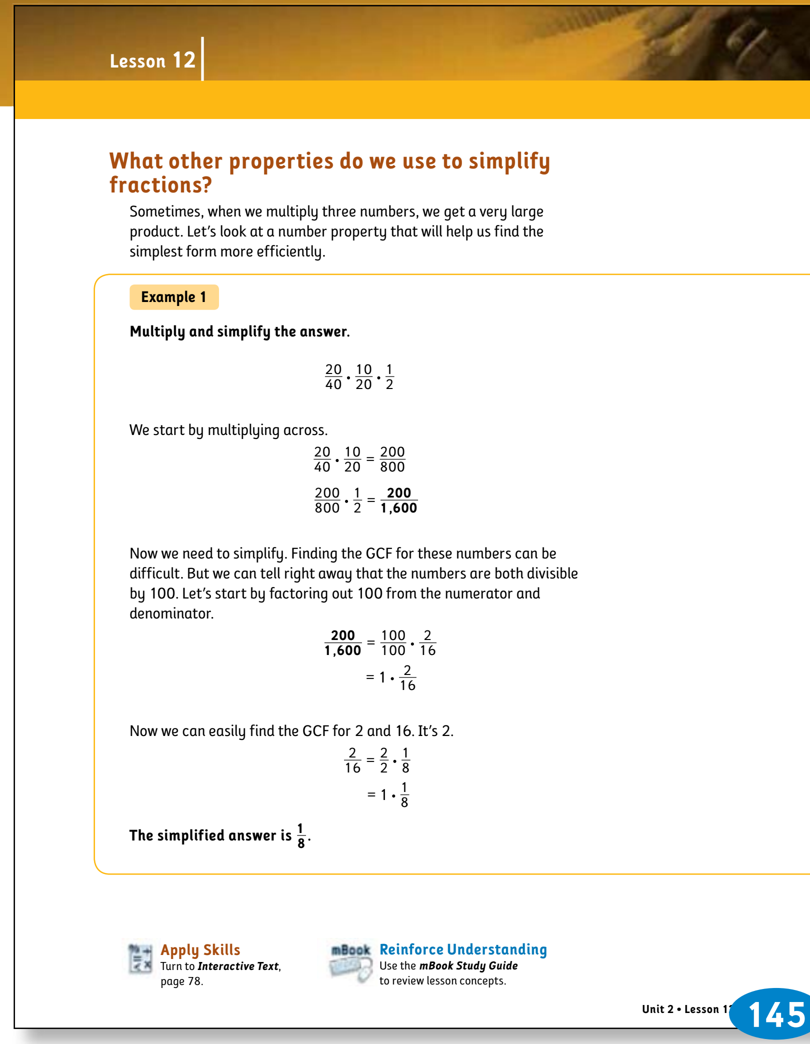
Demonstrate

- Turn to page 145 of the *Student Text*, and explain that we can use another number property to help us find the simplest form more efficiently.
- Direct students' attention to **Example 1**. In this example, we multiply three fractions together and get a very large answer, $\mathbf{\frac{200}{1600}}$.
- Explain that it might be hard to find the GCF for these numbers. Instead we can use the divisibility rule to arrive at numbers that are easier to work with.
- With $\frac{200}{1600}$, the numerator and denominator are both divisible by 100. We factor that out of each first. Then we are left with $\mathbf{\frac{2}{16}}$. Now it is easy to find the GCF and simplify, $\mathbf{\frac{2}{16} = \frac{1}{8}}$.

Check for Understanding

Engagement Strategy: Think Tank

Distribute strips of paper to students, and have them write their names on them. Then have students solve the following multiplication problem and simplify the answer using good number sense: $\mathbf{\frac{2}{3} \cdot \frac{3}{5} \cdot \frac{1}{2}}$. Collect the strips of paper, and put them in a container. Draw a strip of paper, and read the answer out loud. If correct, congratulate that student. If incorrect, invite a student volunteer to explain the solution $\left(\frac{1}{5}\right)$.



Lesson 12

What other properties do we use to simplify fractions?

Sometimes, when we multiply three numbers, we get a very large product. Let's look at a number property that will help us find the simplest form more efficiently.

Example 1

Multiply and simplify the answer.

$$\frac{20}{40} \cdot \frac{10}{20} \cdot \frac{1}{2}$$

We start by multiplying across.

$$\frac{20}{40} \cdot \frac{10}{20} = \frac{200}{800}$$

$$\frac{200}{800} \cdot \frac{1}{2} = \mathbf{\frac{200}{1{,}600}}$$

Now we need to simplify. Finding the GCF for these numbers can be difficult. But we can tell right away that the numbers are both divisible by 100. Let's start by factoring out 100 from the numerator and denominator.

$$\mathbf{\frac{200}{1{,}600}} = \frac{100}{100} \cdot \frac{2}{16}$$

$$= 1 \cdot \frac{2}{16}$$

Now we can easily find the GCF for 2 and 16. It's 2.

$$\frac{2}{16} = \frac{2}{2} \cdot \frac{1}{8}$$

$$= 1 \cdot \frac{1}{8}$$

The simplified answer is $\frac{1}{8}$.

Apply Skills
Turn to *Interactive Text*, page 78.

Reinforce Understanding
Use the *mBook Study Guide* to review lesson concepts.

Unit 2 • Lesson 12 145

Reinforce Understanding

If students need additional practice, use these problems:

$$\mathbf{\frac{1}{2} \cdot \frac{2}{25} \cdot \frac{3}{4}} \left(\frac{3}{100}\right)$$

$$\mathbf{\frac{7}{10} \cdot \frac{3}{4} \cdot \frac{2}{5}} \left(\frac{21}{100}\right)$$

Apply Skills

(*Interactive Text*, page 78)

Have students turn to page 78 in the *Interactive Text*, which provides students an opportunity to practice multiplying and simplifying fractions on their own.

Activity 1

Students multiply three fractions and simplify the answers. Monitor students' work as they complete the activity.

Watch for:

- Can students multiply the three fractions?
- Do students use commutation to move "friendly" numbers next to each other for easier computations?
- Can students use more complex number concepts to make the numbers more reasonable for finding the GCF?

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Name ______________________ Date ______________

Apply Skills
Multiplying Three Fractions and Simplifying Using the GCF

Activity 1

Solve the multiplication problems with three fractions. Simplify the answer.

1. $\frac{4}{5} \cdot \frac{1}{3} \cdot \frac{1}{2}$ $\frac{4}{30}, \frac{2}{15}$
2. $\frac{1}{8} \cdot \frac{1}{2} \cdot \frac{1}{5}$ $\frac{1}{80}, \frac{1}{80}$
3. $\frac{3}{5} \cdot \frac{3}{4} \cdot \frac{1}{5}$ $\frac{9}{100}, \frac{9}{100}$
4. $\frac{1}{3} \cdot \frac{24}{25} \cdot \frac{1}{4}$ $\frac{24}{300}, \frac{2}{25}$

Problem Solving:
▶ Introduction to the Compass

How does a compass help us draw right angles?
(*Student Text*, pages 146–147)

Connect to Prior Knowledge
Remind students of the concept of a right angle. Draw a right angle on the board and then extend both lines so that you show horizontal and vertical lines that intersect. Describe how perpendicular lines are defined by the fact that they form right angles. Ask students how they could tell if they had drawn a perpendicular line.

Listen for:

- *The lines would just look like they were perpendicular.*
- *I could check to see if it was a right angle by putting something square in the corner (i.e., in the right angle).*
- *I could use a protractor to check.*

Link to Today's Concept
In today's lesson, we present the compass as a tool for constructing right angles. Students also use a ruler and check their work with a protractor.

Demonstrate
- Direct students' attention to page 146 of the *Student Text*.
- Distribute the compasses and show students how to construct circles and arcs using the compass. One of the most common problems that students have with compasses is that they try to rotate the compass by holding the sides of the compass rather than the nub on top. If they hold the sides, the width of the compass often changes resulting in a misshapen circle.
- Have students draw circles and arcs, and remind them of what these terms mean.
- Have them turn over their papers and follow and complete the steps, which show how to construct a perpendicular line using two circles.

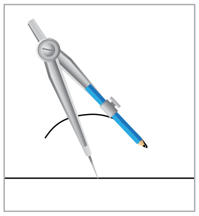
Lesson 12 ▶Problem Solving: Introduction to the Compass

How does a compass help us draw right angles?

We used a ruler and protractor to draw angles. The compass is another tool we use to make shapes and draw angles.

The most common use of a compass is to make arcs and circles.

We start by putting the sharp end of the compass where we want to make the center of a circle. Then we hold the top of the compass as we draw the shape. We keep the sharp end of the compass in place as we move the pencil end.

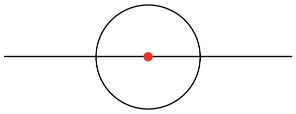


We can also use the compass to make a right angle. When perpendicular lines intersect, they create right angles. We need both a compass and a ruler to draw right angles.

Steps for Finding a Right Angle Using a Compass and Ruler

STEP 1
Draw a line segment with the ruler and put a point near the middle of the line segment.

STEP 2
Next, use the compass to draw a circle. Use the point on the line segment as the center of the circle.



146 Unit 2 • Lesson 12

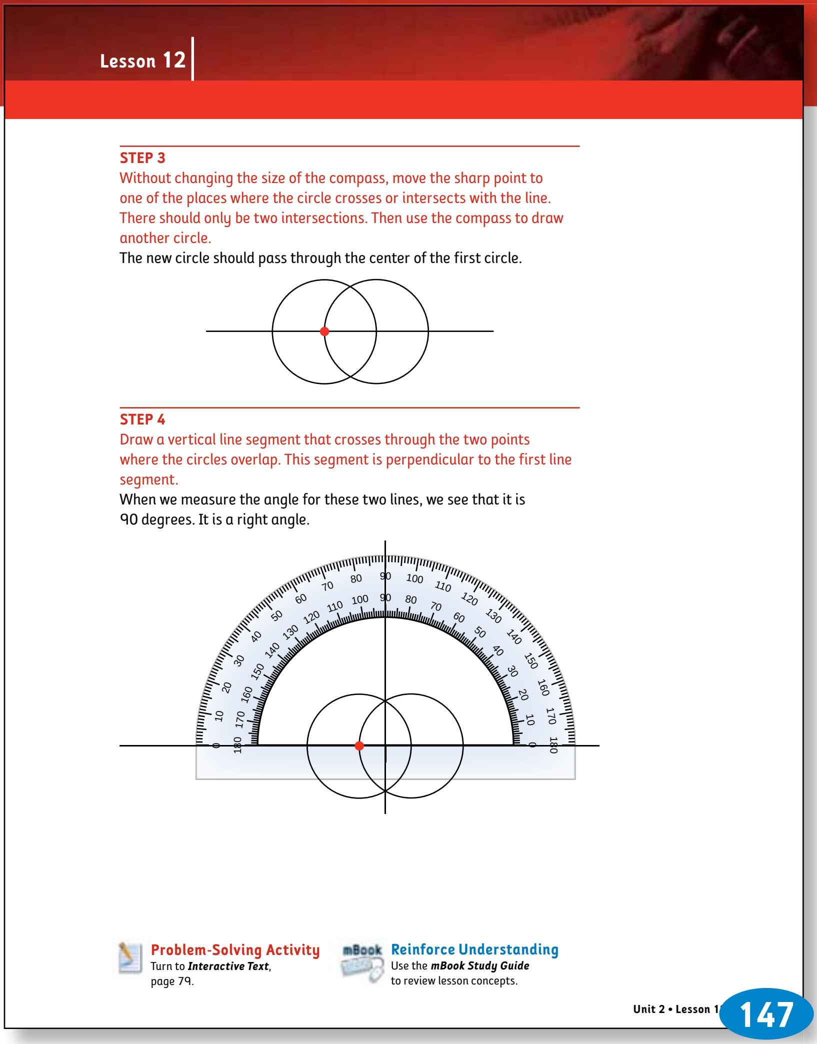
STEP 1
- Direct students to draw a line segment using a ruler. Then place a point in the middle of the segment.

STEP 2
- Demonstrate how to use the compass to draw a circle, using the point on the line segment as the circle's center.

How does a compass help us draw right angles? *(continued)*

Demonstrate

- Turn to page 147 of the *Student Text* to continue walking through the steps to draw a perpendicular line.



STEP 3

Without changing the size of the compass, move the sharp point to one of the places where the circle crosses or intersects with the line. There should only be two intersections. Then use the compass to draw another circle.

The new circle should pass through the center of the first circle.

STEP 4

Draw a vertical line segment that crosses through the two points where the circles overlap. This segment is perpendicular to the first line segment.

When we measure the angle for these two lines, we see that it is 90 degrees. It is a right angle.

Problem-Solving Activity Turn to *Interactive Text*, page 79.

Reinforce Understanding Use the *mBook Study Guide* to review lesson concepts.

STEP 3

- Demonstrate how to move the sharp point of the compass to a place where the circle intersects with the line. Tell students not to change the size of the compass. The circles should be the same size.
- Draw another circle using the compass.

STEP 4

- Draw a vertical line segment that crosses through the two points where the circles overlap.
- Show students how to use the protractor to check the line. Remind students that the angles must be 90 degrees.
- If time permits, have students create a second perpendicular line using the compass on their own. Have students check to see if the angle is 90 degrees by using their protractors.

Watch for:

- Are students holding and using the compass correctly?
- Are students following the steps correctly, especially locating the sharp point of the compass?
- Are their vertical lines perpendicular?

- Discuss the value of the compass as a tool for constructing perpendicular lines. Point out how the compass was able to help students construct a perpendicular line that was accurate without the need for a protractor.

Problem-Solving Activity
(*Interactive Text*, page 79)

Have students turn to page 79 in the *Interactive Text*, which provides students an opportunity to use a compass and protractor on their own.

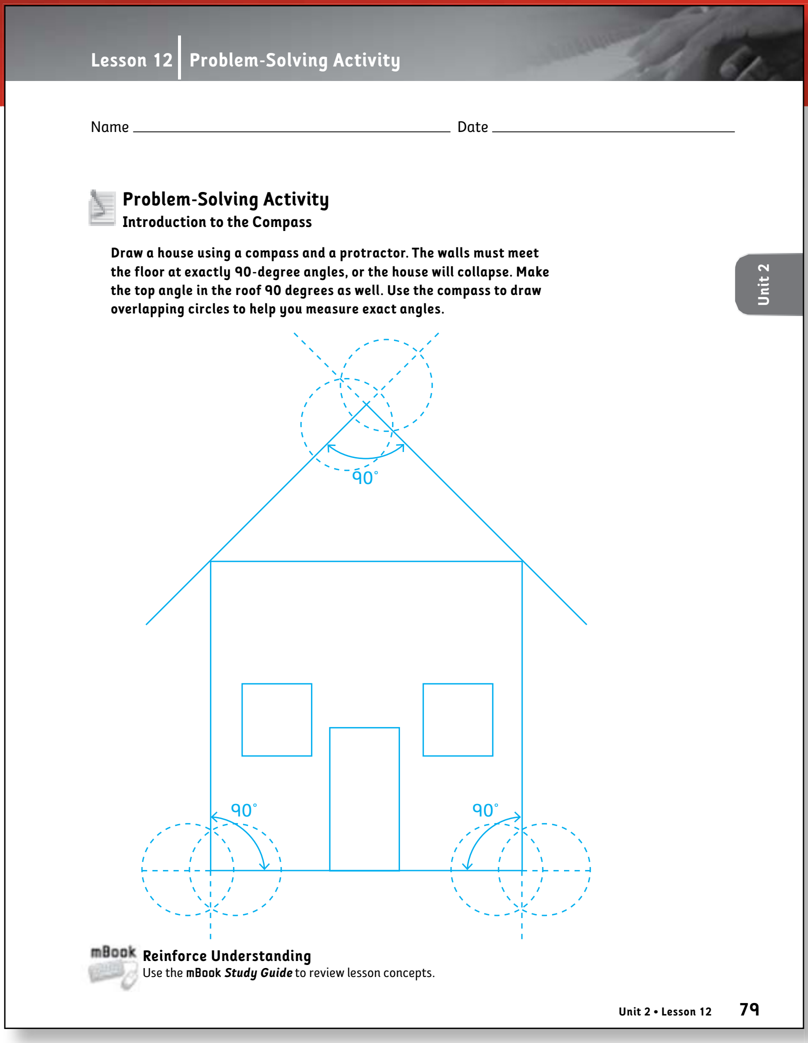
Students draw a house using a compass and protractor. Make sure students understand that the walls must meet the floors at exactly 90 degrees and that the angle of the roof is also 90 degrees.

Monitor students' work as they complete this activity.

Watch for:

- Are students holding and using the compass correctly?
- Are they following the steps correctly, especially locating the sharp point of the compass?
- Are their vertical lines perpendicular?

mBook Reinforce Understanding
Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.


Lesson 12 | Problem-Solving Activity

Name ______________________ Date ______________

Problem-Solving Activity
Introduction to the Compass

Draw a house using a compass and a protractor. The walls must meet the floor at exactly 90-degree angles, or the house will collapse. Make the top angle in the roof 90 degrees as well. Use the compass to draw overlapping circles to help you measure exact angles.

Unit 2

mBook **Reinforce Understanding**
Use the mBook *Study Guide* to review lesson concepts.

Unit 2 • Lesson 12 79

Homework

Go over the instructions on page 148 of the *Student Text* for each part of the homework.

Activity 1

Students multiply and divide, then simplify fractions.

Activity 2

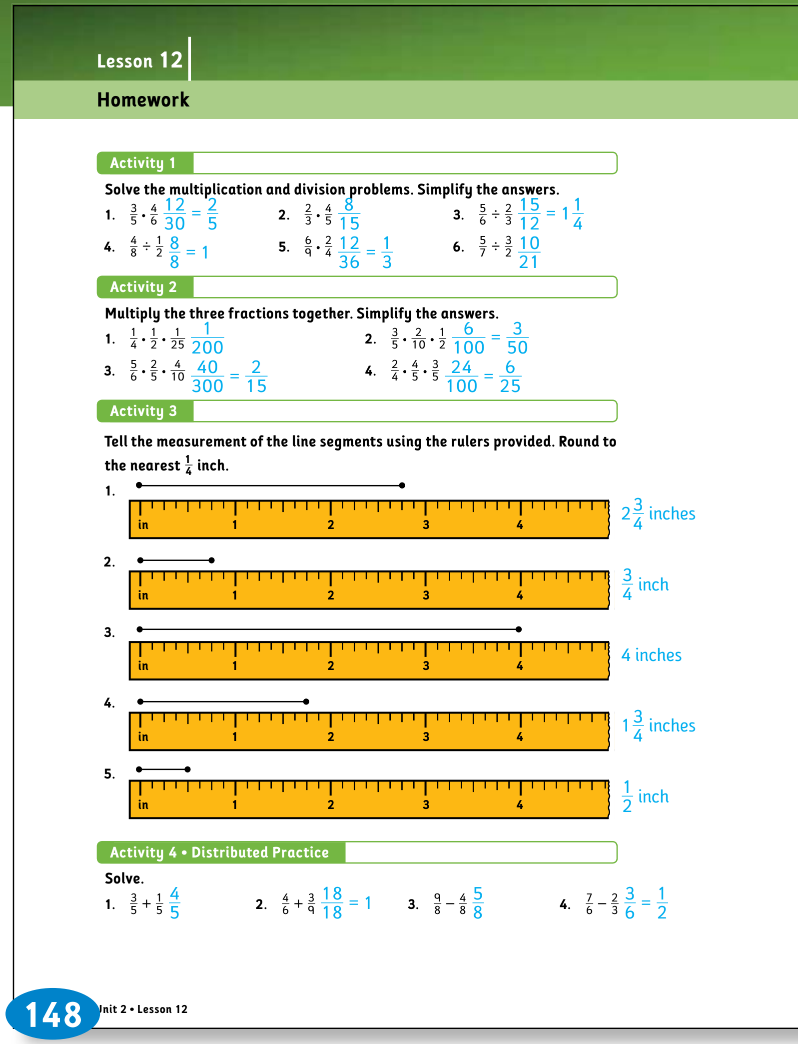
Students multiply three fractions and simplify.

Activity 3

Students measure line segments to the nearest $\frac{1}{4}$ inch.

Activity 4 • Distributed Practice

Students solve problems that involve addition and subtraction of fractions.



Lesson 12

Homework

Activity 1

Solve the multiplication and division problems. Simplify the answers.

1. $\frac{3}{5} \cdot \frac{4}{6}$ $\frac{12}{30} = \frac{2}{5}$
2. $\frac{2}{3} \cdot \frac{4}{5}$ $\frac{8}{15}$
3. $\frac{5}{6} \div \frac{2}{3}$ $\frac{15}{12} = 1\frac{1}{4}$
4. $\frac{4}{8} \div \frac{1}{2}$ $\frac{8}{8} = 1$
5. $\frac{6}{9} \cdot \frac{2}{4}$ $\frac{12}{36} = \frac{1}{3}$
6. $\frac{5}{7} \div \frac{3}{2}$ $\frac{10}{21}$

Activity 2

Multiply the three fractions together. Simplify the answers.

1. $\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{25}$ $\frac{1}{200}$
2. $\frac{3}{5} \cdot \frac{2}{10} \cdot \frac{1}{2}$ $\frac{6}{100} = \frac{3}{50}$
3. $\frac{5}{6} \cdot \frac{2}{5} \cdot \frac{4}{10}$ $\frac{40}{300} = \frac{2}{15}$
4. $\frac{2}{4} \cdot \frac{4}{5} \cdot \frac{3}{5}$ $\frac{24}{100} = \frac{6}{25}$

Activity 3

Tell the measurement of the line segments using the rulers provided. Round to the nearest $\frac{1}{4}$ inch.

1. $2\frac{3}{4}$ inches
2. $\frac{3}{4}$ inch
3. 4 inches
4. $1\frac{3}{4}$ inches
5. $\frac{1}{2}$ inch

Activity 4 • Distributed Practice

Solve.

1. $\frac{3}{5} + \frac{1}{5}$ $\frac{4}{5}$
2. $\frac{4}{6} + \frac{3}{9}$ $\frac{18}{18} = 1$
3. $\frac{9}{8} - \frac{4}{8}$ $\frac{5}{8}$
4. $\frac{7}{6} - \frac{2}{3}$ $\frac{3}{6} = \frac{1}{2}$

148 Unit 2 • Lesson 12

Lesson 13 | Comparing Multiplication and Division of Fractions

Problem Solving: Working With a Compass

Lesson Planner

Vocabulary Development

bisect

Skills Maintenance

Multiplying Three Fractions and Simplifying the Answer

Building Number Concepts: Comparing Multiplication and Division of Fractions

Students compare the algorithms for multiplying and dividing fractions. Students learn tips for avoiding the common error of confusing the two algorithms. Discrimination of the algorithms is challenging when they are introduced together.

Objective

Students will compare the multiplication and division of fractions.

Problem Solving: Working With a Compass

Students use a different way to create perpendicular lines with the arcs of a compass. They also bisect an angle. In both cases, they use a protractor to check their measurements.

Objective

Students will create perpendicular lines with arcs of a compass and bisections of angles.

Homework

Students solve multiplication and division problems and simplify answers, identify perpendicular lines, and identify triangles with perpendicular lines. In Distributed Practice, students solve four problems that involve addition and subtraction of fractions and two whole-number operations.

Lesson 13 | Skills Maintenance

Name ________________ Date ________________

Skills Maintenance
Multiplying Three Fractions and Simplifying the Answer

Activity 1

Multiply the fractions. Simplify your answer.

1. $\frac{1}{3} \cdot \frac{2}{5} \cdot \frac{4}{2}$ $\frac{8}{30}$, $\frac{4}{15}$
2. $\frac{2}{4} \cdot \frac{3}{6} \cdot \frac{5}{8}$ $\frac{30}{192}$, $\frac{5}{32}$
3. $\frac{4}{6} \cdot \frac{3}{2} \cdot \frac{2}{3}$ $\frac{24}{36}$, $\frac{2}{3}$

80 Unit 2 • Lesson 13

Skills Maintenance

Multiplying Three Fractions and Simplifying the Answer

(*Interactive Text*, page 80)

Activity 1

Students solve multiplication problems involving three fractions and use more advanced number concepts to find easier numbers to simplify using the GCF.

Building Number Concepts:
▶ Comparing Multiplication and Division of Fractions

How do we compare the algorithms for multiplication and division of fractions?

(*Student Text,* pages 149–152)

Connect to Prior Knowledge

Begin by putting this problem on the board: $\frac{3}{10} \cdot \frac{4}{8}$. Tell students that we are going to begin solving it by inverting the second fraction and multiplying across. Ask if this is correct. If students correctly identify that this procedure is for division, reinforce their good observation. However if students incorrectly identify this as the correct procedure, prompt them to tell the procedures for multiplying and dividing fractions until they recognize the procedure for division. Explain to students that we have to be very careful not to confuse the operations for fractions once we know them all.

Link to Today's Concept

In this lesson, students compare the algorithms for multiplication and division of fractions.

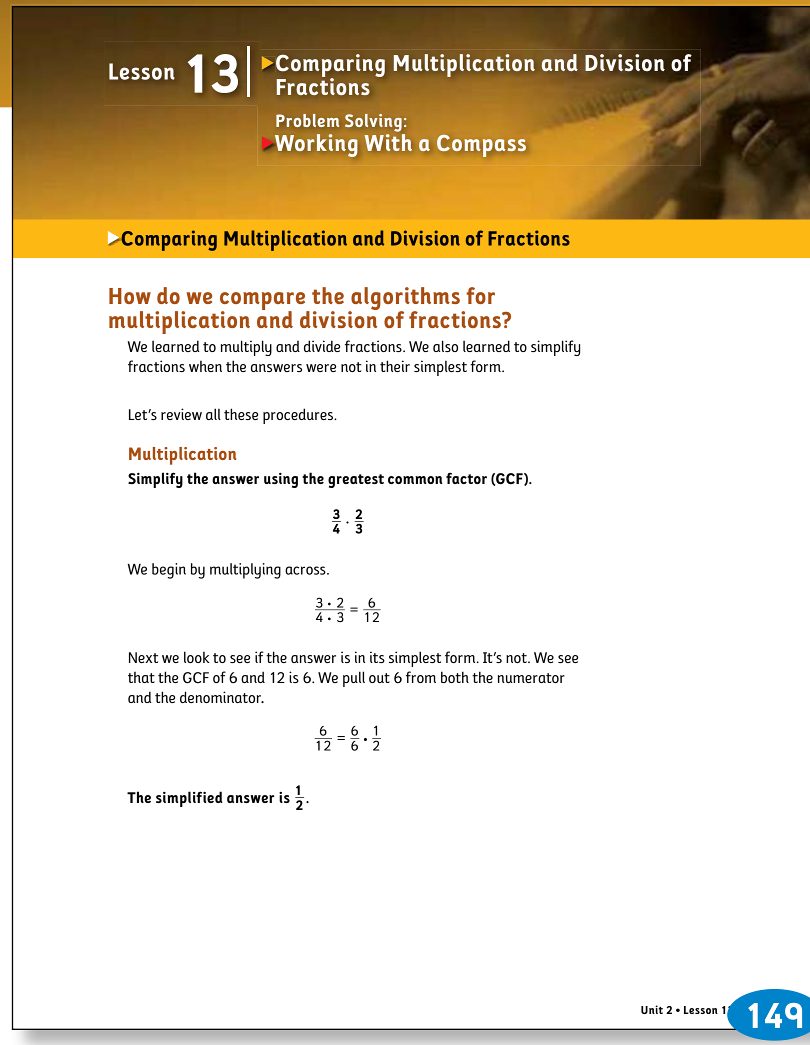
Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate how to multiply fractions and simplify the answer using the GCF. Do this in one of the following ways:

mBook: Use the *mBook Teacher Edition* for pages 149–150 of the *Student Text.* m

Overhead Projector: Copy the fractions onto a transparency, and modify as discussed.


Lesson 13 ▶Comparing Multiplication and Division of Fractions
Problem Solving: ▶Working With a Compass

▶Comparing Multiplication and Division of Fractions

How do we compare the algorithms for multiplication and division of fractions?

We learned to multiply and divide fractions. We also learned to simplify fractions when the answers were not in their simplest form.

Let's review all these procedures.

Multiplication

Simplify the answer using the greatest common factor (GCF).

$$\frac{3}{4} \cdot \frac{2}{3}$$

We begin by multiplying across.

$$\frac{3 \cdot 2}{4 \cdot 3} = \frac{6}{12}$$

Next we look to see if the answer is in its simplest form. It's not. We see that the GCF of 6 and 12 is 6. We pull out 6 from both the numerator and the denominator.

$$\frac{6}{12} = \frac{6}{6} \cdot \frac{1}{2}$$

The simplified answer is $\frac{1}{2}$.

Unit 2 • Lesson 1 149

Board: Copy the fractions onto the board, and modify as discussed.

Multiplication

- Show the problem $\frac{3}{4} \cdot \frac{2}{3}$. Remind students that we multiply across to get the answer $\frac{6}{12}$. m
- Remind students we must simplify the answer. We find the GCF of 6 and 12, which is 6. We can pull out 6 from the numerator and denominator.
- Erase the answer and show $\frac{6}{6}$, reminding students that this equals 1. Then remind students that we use the other factor for the numerator and denominator for the other fraction, in this case $\frac{1}{2}$. Recall with students that anything times 1 is itself, so **the simplified answer to $\frac{3}{4} \cdot \frac{2}{3}$ is $\frac{1}{2}$.** m

Demonstrate

- Remind students that we can use the commutative property and then simplify when we multiply.
- Display the problem $\frac{3}{4} \cdot \frac{2}{3}$ using the commutative property. Point out that there is a 3 in the numerator of one fraction and in the denominator of the other.
- Point out that the answer is the same as the last problem; we just used the commutative property to make the fractions easier to work with.

Division

- Go over the division of fractions with students by displaying the problem $\frac{3}{4} \div \frac{5}{6}$. Remind students how to invert the second fraction and multiply across.
- Then walk through how to simplify the answer.

Check for Understanding

Ask students to compare the multiplication and division algorithms.

Listen for:

- *In both algorithms, you multiply across. Division has the extra step of inversion first.*
- *In both algorithms, you usually end up with an answer that needs to be simplified.*
- *In multiplication, the answer is usually smaller. You are taking a fraction of a fraction.*
- *In division, the answer is usually bigger. You are telling how many very small units are found in the whole. You need a lot of very small units to make up the whole.*

Use the commutative property, then simplify.
We have a 3 in the numerator and a 3 in the denominator. This means we can use the commutative property.

$$\frac{3}{4} \cdot \frac{2}{3} = \frac{3 \cdot 2}{3 \cdot 4}$$
$$= \frac{3}{3} \cdot \frac{2}{4}$$
$$= 1 \cdot \frac{2}{4}$$

In this case, the fraction still needs to be simplified. We pull out the GCF of 2.

$$\frac{2}{4} = \frac{2}{2} \cdot \frac{1}{2}$$
$$= 1 \cdot \frac{1}{2}$$

The simplified answer is $\frac{1}{2}$.

Division

Divide, then simplify the answer.

$$\frac{3}{4} \div \frac{5}{6}$$

We begin by inverting the second fraction.

$$\frac{3}{4} \cdot \frac{6}{5}$$

Then we multiply across.

$$\frac{3 \cdot 6}{4 \cdot 5} = \frac{18}{20}$$

Now we simplify the answer by pulling out the GCF.

$$\frac{18}{20} = \frac{2}{2} \cdot \frac{9}{10}$$
$$= 1 \cdot \frac{9}{10}$$

The simplified answer is $\frac{9}{10}$.

How do we compare the algorithms for multiplication and division of fractions? *(continued)*

Improve Your Skills

- Direct students' attention to page 151 of the *Student Text*. In this illustration, we see a common error. Students often confuse the algorithms and try to invert the second fraction in a multiplication problem. This results in a larger fraction, which is the number sense cue that something is wrong.
- Explain to students why the result of taking a fraction of a fraction should be quite small. The answer $\frac{18}{15}$ is greater than 1, which is quite large in terms of fractions.
- Walk through the correct process with students to get the simplified answer of $\frac{3}{10}$. Explain that $\frac{3}{10}$ is much smaller than $\frac{18}{15}$, which we can see on the number line.
- Remind students that these are important concepts that need to be practiced and applied to move students toward using good number sense in their computations.

Check for Understanding

Engagement Strategy: Think, Think

Ask students the following question. Tell them that you will call on one of them to answer the question after you ask it. Tell them to listen for their names. After asking the question, allow time for students to think of the answer. Then call on a student.

When we learn both multiplication and division at the same time, we can sometimes confuse the procedures. However, we can learn certain things that teach us good number sense to avoid making mistakes.

Improve Your Skills

It is easy to make a mistake if you forget what operation you are working with.

$$\frac{3}{5} \cdot \frac{3}{6}$$

To solve a multiplication problem, you begin by multiplying across. If you forget what operation you are working with, you might invert and multiply.

$$\frac{3}{5} \cdot \frac{3}{6} = \frac{3 \cdot 6}{5 \cdot 3} = \frac{18}{15} \qquad \textbf{ERROR}$$

To check that your answer is correct, look at the size of the answer. Remember, when we take a fraction of a fraction, we usually end up with a smaller number. In terms of fractions, $\frac{18}{15}$ is not a small number. It is greater than 1 because $\frac{15}{15}$ is 1, and there are $\frac{3}{15}$ left over. This should clue us in that our answer is not correct.

When we solve the problem the correct way, the answer looks like this:

$$\frac{3}{5} \cdot \frac{3}{6} = \frac{9}{30} \qquad \textbf{CORRECT}$$

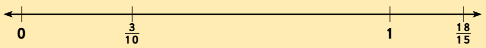
It's important to think about the size of the fractions and the size of the answer.

To simplify the answer, we pull out the GCF of 3:

$$\frac{9}{30} = \frac{3}{3} \cdot \frac{3}{10} = 1 \cdot \frac{3}{10}$$

The simplified answer is $\frac{3}{10}$.

This is much smaller than $\frac{18}{15}$. It is closer to 0 on the number line than 1. Here is where the numbers are located on a number line.



Ask:

How might a larger number give us a clue that we made an error in multiplying fractions?
(*When multiplying fractions, we usually end up with a smaller number. If the answer is greater than one, it can signal there was an error.*)

Improve Your Skills

- Tell students to turn to page 152 of the *Student Text*. In this illustration, we can see another common error. Here the first step of division was not completed. Instead the problem was solved multiplying across. The answer, $\frac{4}{25}$, is very small, which is the number sense cue that something is wrong in how the problem was completed. The answer to a division problem should be bigger because we are looking for many small parts.
- Walk students through the correct way to complete this problem to get the answer, **4**. **There are 4 units of $\frac{1}{5}$ in the number $\frac{4}{5}$.** The answer is much greater than $\frac{4}{25}$. We can view these results on the number line.
- Remind students that they must be sure of the type of computation they are doing before beginning their work. Division of fractions involves the first initial step of inverting the second fraction within the problem. Remembering the difference in these two types of algorithms leads to better number sense.

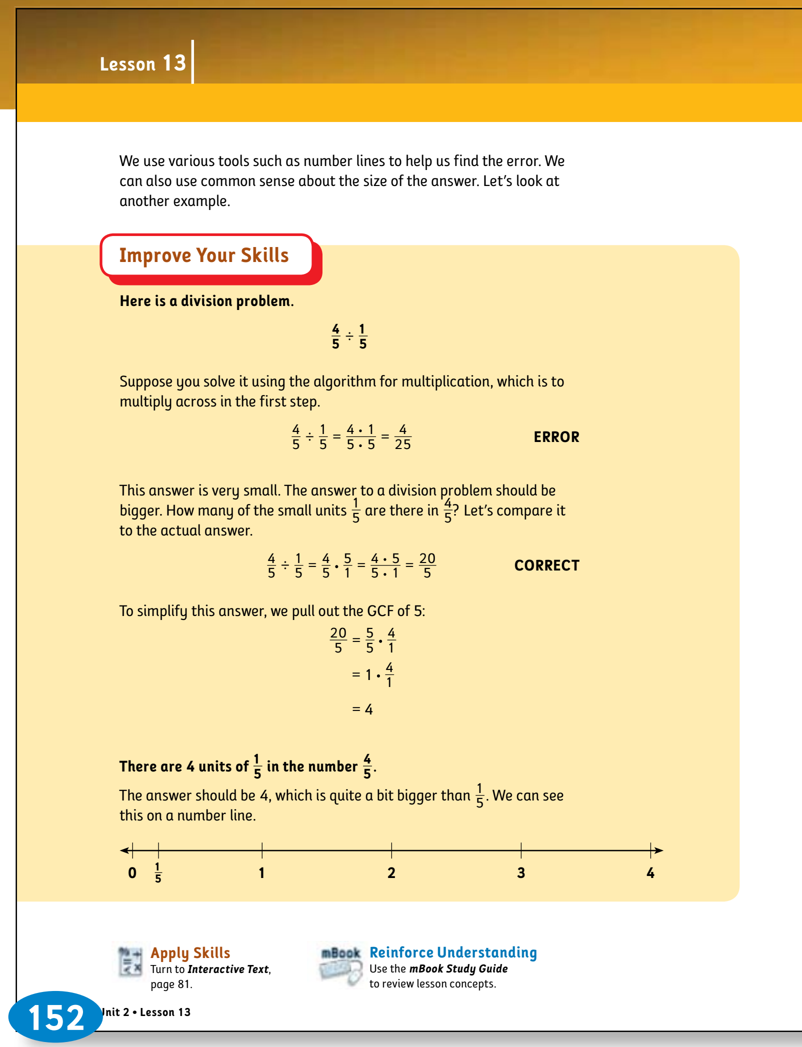
Check for Understanding

Engagement Strategy: Pair/Share

Have students pick a partner. Write the following two problems on the board:

$\frac{2}{3} \div \frac{1}{3}$ *(2)* and $\frac{3}{4} \div \frac{1}{4}$ *(3)*

Have each student in the pair work one of the problems. When students finish, have them switch papers and examine each other's work. If one of the problems is incorrect, have the partners work together to get the correct answer. When all work is complete, have volunteers raise their hands to tell how they solved their problems. If additional help is needed, work with students to correctly solve the problems.



We use various tools such as number lines to help us find the error. We can also use common sense about the size of the answer. Let's look at another example.

Improve Your Skills

Here is a division problem.

$$\frac{4}{5} \div \frac{1}{5}$$

Suppose you solve it using the algorithm for multiplication, which is to multiply across in the first step.

$$\frac{4}{5} \div \frac{1}{5} = \frac{4 \cdot 1}{5 \cdot 5} = \frac{4}{25} \qquad \textbf{ERROR}$$

This answer is very small. The answer to a division problem should be bigger. How many of the small units $\frac{1}{5}$ are there in $\frac{4}{5}$? Let's compare it to the actual answer.

$$\frac{4}{5} \div \frac{1}{5} = \frac{4}{5} \cdot \frac{5}{1} = \frac{4 \cdot 5}{5 \cdot 1} = \frac{20}{5} \qquad \textbf{CORRECT}$$

To simplify this answer, we pull out the GCF of 5:

$$\frac{20}{5} = \frac{5}{5} \cdot \frac{4}{1}$$
$$= 1 \cdot \frac{4}{1}$$
$$= 4$$

There are 4 units of $\frac{1}{5}$ in the number $\frac{4}{5}$.

The answer should be 4, which is quite a bit bigger than $\frac{1}{5}$. We can see this on a number line.

Apply Skills Turn to *Interactive Text*, page 81.

Reinforce Understanding Use the *mBook Study Guide* to review lesson concepts.

Apply Skills

(*Interactive Text*, page 81)

Have students turn to page 81 in the *Interactive Text*, which provides students an opportunity to practice finding errors in problems regarding fractions on their own.

Activity 1

In this activity, an error is made in each problem. Students are to find the error and tell what it is. Next they are to tell how they know it is an error and fix it. Monitor students' work as they complete this activity.

Watch for:

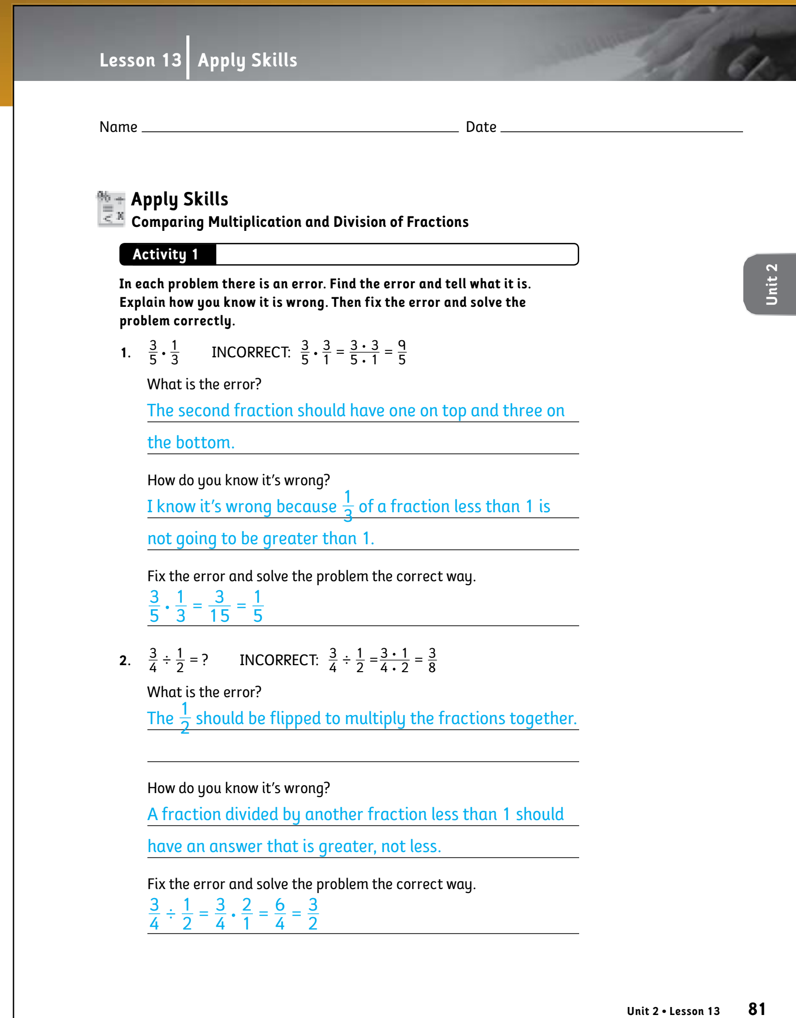
- Can students identify the error?
- Can students explain why they know it is an error?
- Can students use good number sense about the size of the fractions?
- Can students fix the error and solve the problem correctly?

Be sure to discuss the answers after students are done. Look for the discussion about size of the resulting fractions as an indicator that students are starting to use good number sense.

Reinforce Understanding

mBook

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide.*



Lesson 13 | Apply Skills

Name ______________________ Date ______________________

Unit 2

Apply Skills

Comparing Multiplication and Division of Fractions

Activity 1

In each problem there is an error. Find the error and tell what it is. Explain how you know it is wrong. Then fix the error and solve the problem correctly.

1. $\frac{3}{5} \cdot \frac{1}{3}$ INCORRECT: $\frac{3}{5} \cdot \frac{3}{1} = \frac{3 \cdot 3}{5 \cdot 1} = \frac{9}{5}$

What is the error?

The second fraction should have one on top and three on the bottom.

How do you know it's wrong?

I know it's wrong because $\frac{1}{3}$ of a fraction less than 1 is not going to be greater than 1.

Fix the error and solve the problem the correct way.

$\frac{3}{5} \cdot \frac{1}{3} = \frac{3}{15} = \frac{1}{5}$

2. $\frac{3}{4} \div \frac{1}{2} = ?$ INCORRECT: $\frac{3}{4} \div \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$

What is the error?

The $\frac{1}{2}$ should be flipped to multiply the fractions together.

How do you know it's wrong?

A fraction divided by another fraction less than 1 should have an answer that is greater, not less.

Fix the error and solve the problem the correct way.

$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \cdot \frac{2}{1} = \frac{6}{4} = \frac{3}{2}$

Unit 2 • Lesson 13 81

Problem Solving:
▶ Working With a Compass

How do we make a right angle using a compass?

(*Student Text*, pages 153–154)

Connect to Prior Knowledge

Review the method used in the previous lesson to make a perpendicular line. Draw a line and circles on the board as a reminder. Also draw an angle on the board and bisect it. Point out that now we have two angles. Then ask students how they could tell if the two angles that were split in half, or bisected, were equal.

Listen for:

- *The angles would look about the same.*
- *You could use a protractor to check if they were the same.*

Link to Today's Concept

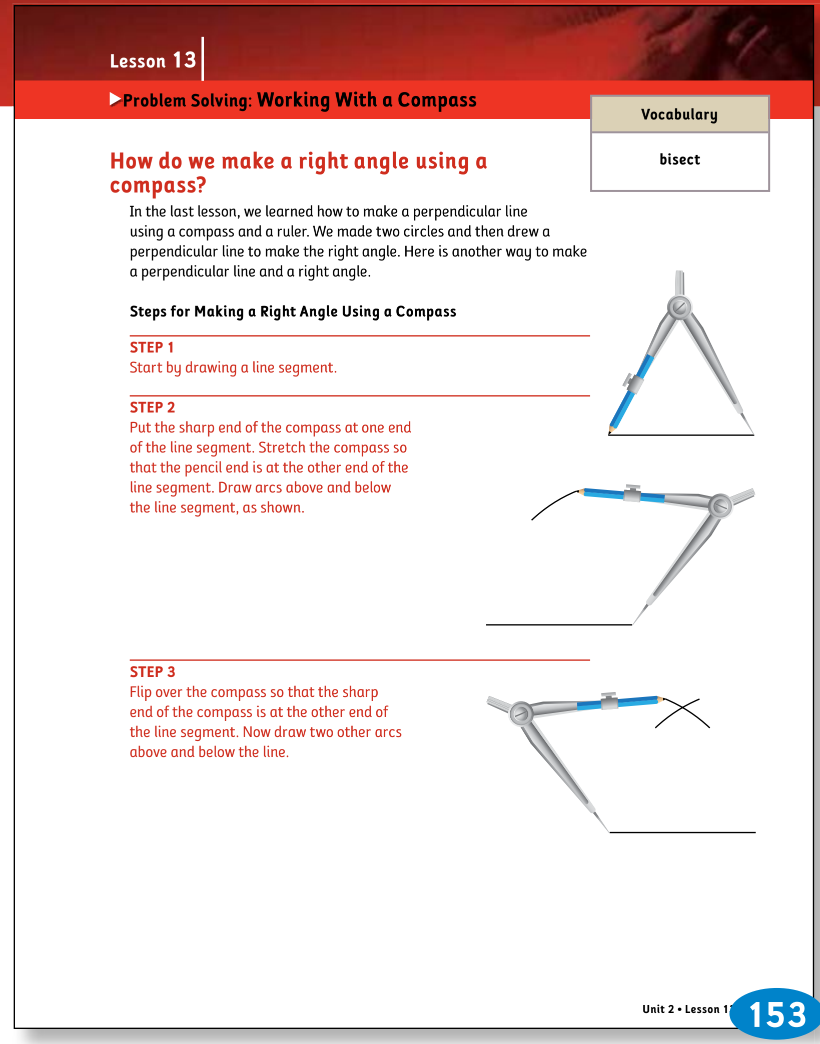
Today's lesson builds off the previous lesson by showing another way to construct a perpendicular line. We also look at how to bisect an angle.

Demonstrate

- Have students follow the steps that show how to construct a perpendicular line using arcs. Have students get out blank or lined paper and complete the steps as you walk though them. Make sure students have rulers, compasses, and protractors before they begin.

STEP 1

- Have students draw a line segment with enough room above and below to draw arcs.



Lesson 13

▶Problem Solving: Working With a Compass

Vocabulary
bisect

How do we make a right angle using a compass?

In the last lesson, we learned how to make a perpendicular line using a compass and a ruler. We made two circles and then drew a perpendicular line to make the right angle. Here is another way to make a perpendicular line and a right angle.

Steps for Making a Right Angle Using a Compass

STEP 1
Start by drawing a line segment.

STEP 2
Put the sharp end of the compass at one end of the line segment. Stretch the compass so that the pencil end is at the other end of the line segment. Draw arcs above and below the line segment, as shown.

STEP 3
Flip over the compass so that the sharp end of the compass is at the other end of the line segment. Now draw two other arcs above and below the line.

Unit 2 • Lesson 1 153

STEP 2

- Show students how to line up the point of the compass at one end of the line segment to draw an arc above and below the line.

STEP 3

- Then have them move the compass point to the other end to draw arcs.

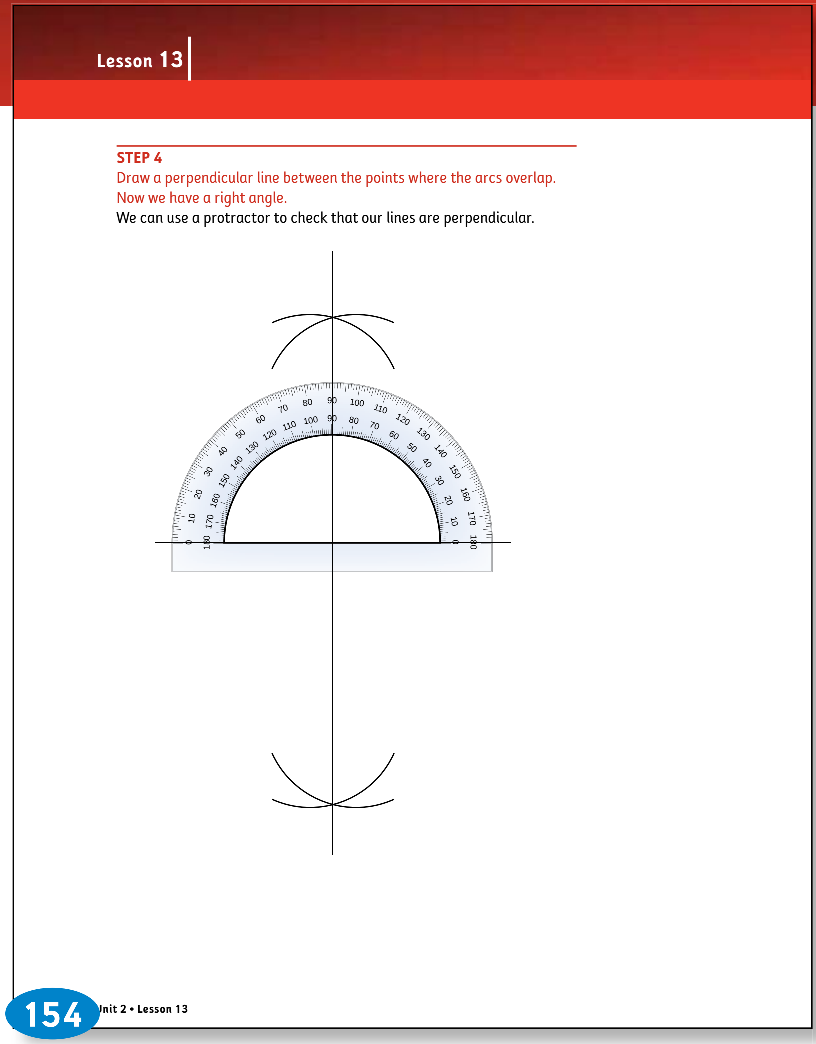
How do we make a right angle using a compass? *(continued)*

Demonstrate

- Continue to the last step to draw the perpendicular line.

STEP 4

- Point out how to draw the line between the points where the arcs intersect. Then show students that we can check the angle with a protractor.
- If time permits, have students create a second perpendicular line using the compass on their own. Then explain that we also see how to use a compass and ruler to bisect an angle into equal parts.


Lesson 13

STEP 4
Draw a perpendicular line between the points where the arcs overlap.
Now we have a right angle.
We can use a protractor to check that our lines are perpendicular.

154 Unit 2 • Lesson 13

How do we bisect an angle using a compass?

(*Student Text*, pages 155–156)

Demonstrate

- Have students turn to page 155 of the *Student Text* to follow the steps for drawing and bisecting angles. Have students complete each step on a sheet of paper as you discuss it.

STEP 1

- Have students use their rulers to draw an angle like the one shown in the *Student Text*.

STEP 2

- Show students how to put the compass point at the vertex of the angle to draw an arc. Make sure students draw the arc across both sides.

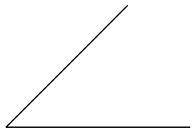
We can do more than make perpendicular lines with a compass. We can also use a compass and ruler to split an angle into two equal parts. This is called bisecting an angle. It doesn't matter how big the angle is, we just need a compass and a ruler.

How do we bisect an angle using a compass?

Steps for Bisecting an Angle Using a Compass

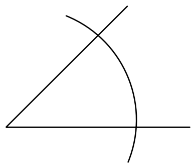
STEP 1

Begin by drawing an angle with only a ruler.



STEP 2

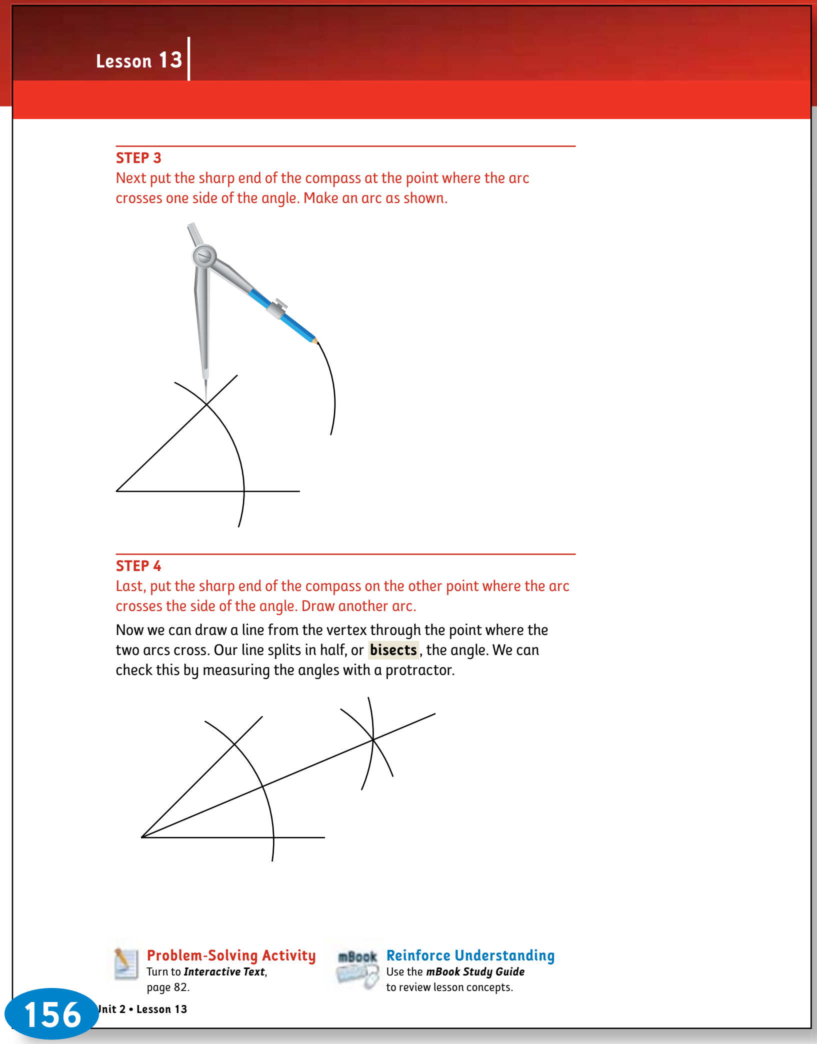
Put the sharp end of the compass at the vertex of the angle and draw an arc that crosses both sides of the angle.



How do we bisect an angle using a compass? *(continued)*

Demonstrate

- Walk through the steps on page 156 of the *Student Text* to **bisect** the angle.



STEP 3
Next put the sharp end of the compass at the point where the arc crosses one side of the angle. Make an arc as shown.

STEP 4
Last, put the sharp end of the compass on the other point where the arc crosses the side of the angle. Draw another arc.

Now we can draw a line from the vertex through the point where the two arcs cross. Our line splits in half, or **bisects**, the angle. We can check this by measuring the angles with a protractor.

Problem-Solving Activity Turn to ***Interactive Text***, page 82.

mBook **Reinforce Understanding** Use the ***mBook Study Guide*** to review lesson concepts.

STEP 3

- Show students how to put the compass point at the point where the arc crosses one side of the angle to draw an arc outside the angle.

STEP 4

- Then have the students move the compass point to the other side where the arc crosses and draw another arc. Point out that the two arcs intersect.
- Show students how to draw a line from the vertex to where the arcs intersect. Explain that this line splits the angle in half, or bisects, the angle.
- Have students check that the two angles are the same by measuring them with a protractor.
- Once students complete this activity, discuss the value of the compass as a tool for constructing lines.

Check for Understanding

Engagement Strategy: Pair/Share

Divide the class into pairs. Have students draw an angle on a sheet of paper. Then have them exchange their papers. Have each partner bisect the angle that their partner drew, using a compass and a ruler. When students finish, have pairs again exchange papers. Ask them to use their protractors to check that they correctly bisected the angles.

Watch for:

- Are students holding and using the compass correctly, especially in drawing arcs that are long enough?
- Are they putting the compass points in the correct places to draw the arcs (the vertex and the two points where the arc crosses each side)?
- Can they use the protractor to measure the bisected lines?

Problem-Solving Activity

(*Interactive Text*, page 82)

Have students turn to page 82 in the *Interactive Text*, which provides students an opportunity to practice bisecting angles.

Tell students to use their compasses to bisect the angle formed between the moon and the earth and then the angle formed between the moon and the sun. Then have them find the angle measurements.

Monitor students' work as they complete this activity.

Watch for:

- Are students holding and using the compass correctly, especially in drawing arcs that are long enough?
- Are their vertical lines perpendicular?
- Can students use the protractor to measure the bisected lines?

mBook Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Lesson 13 | Problem-Solving Activity

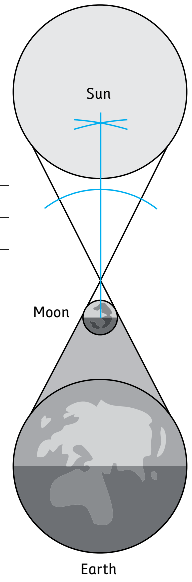
Name ______________________ Date ______________________

Problem-Solving Activity
Working With a Compass

A solar eclipse occurs when the moon passes between the Earth and the Sun. This causes the sun to seem to disappear in the middle of the day. This phenomenon used to frighten people, but astronomers know now that there is a mathematical explanation.

Bisect the angle that is formed between the Moon and the Sun. Find the measurement of the two new angles.

The new angles are each 25 degrees.



mBook **Reinforce Understanding**
Use the mBook *Study Guide* to review lesson concepts.

82 Unit 2 • Lesson 13

Homework

Go over the instructions on page 157 of the *Student Text* for each part of the homework.

Activity 1

Students solve multiplication and division problems, then simplify the answers.

Activity 2

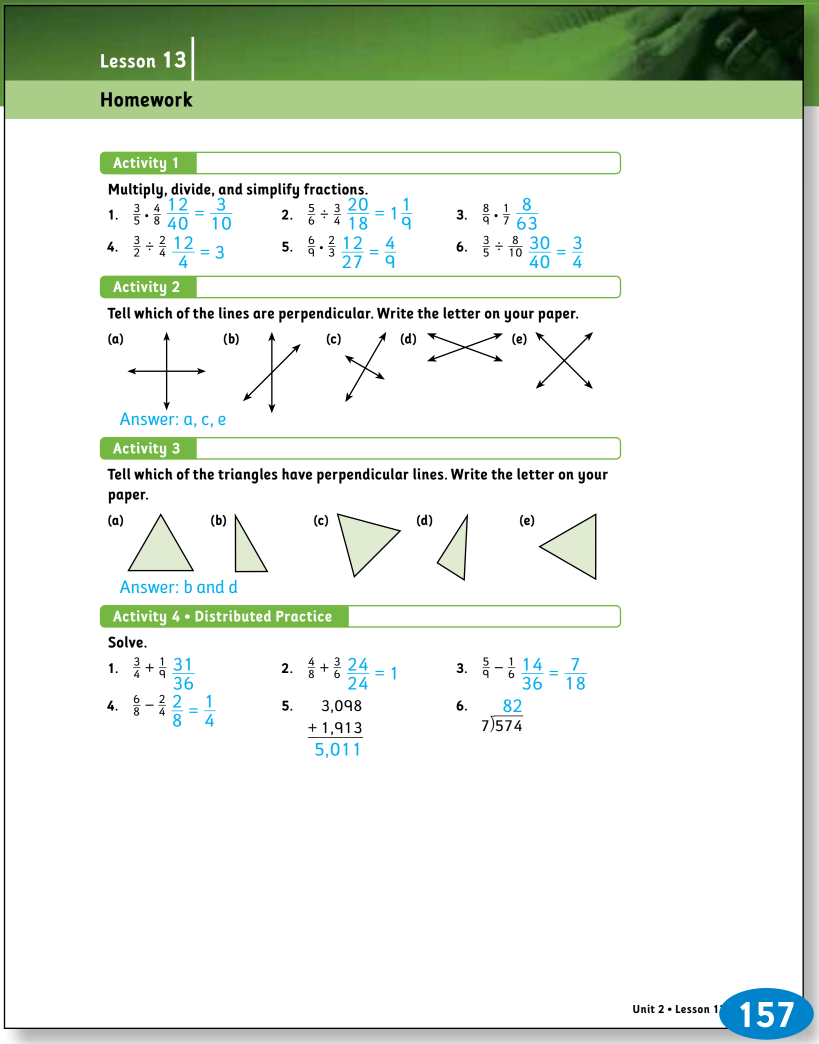
Students identify the perpendicular lines.

Activity 3

Students identify the triangles with perpendicular lines.

Activity 4 • Distributed Practice

Students solve four problems that involve addition and subtraction of fractions and two whole-number operations.



Lesson 13

Homework

Activity 1

Multiply, divide, and simplify fractions.

1. $\frac{3}{5} \cdot \frac{4}{8}$ $\frac{12}{40} = \frac{3}{10}$
2. $\frac{5}{6} \div \frac{3}{4}$ $\frac{20}{18} = 1\frac{1}{9}$
3. $\frac{8}{9} \cdot \frac{1}{7}$ $\frac{8}{63}$
4. $\frac{3}{2} \div \frac{2}{4}$ $\frac{12}{4} = 3$
5. $\frac{6}{9} \cdot \frac{2}{3}$ $\frac{12}{27} = \frac{4}{9}$
6. $\frac{3}{5} \div \frac{8}{10}$ $\frac{30}{40} = \frac{3}{4}$

Activity 2

Tell which of the lines are perpendicular. Write the letter on your paper.

(a) (b) (c) (d) (e)

Answer: a, c, e

Activity 3

Tell which of the triangles have perpendicular lines. Write the letter on your paper.

(a) (b) (c) (d) (e)

Answer: b and d

Activity 4 • Distributed Practice

Solve.

1. $\frac{3}{4} + \frac{1}{9}$ $\frac{31}{36}$
2. $\frac{4}{8} + \frac{3}{6}$ $\frac{24}{24} = 1$
3. $\frac{5}{9} - \frac{1}{6}$ $\frac{14}{36} = \frac{7}{18}$
4. $\frac{6}{8} - \frac{2}{4}$ $\frac{2}{8} = \frac{1}{4}$
5. $\begin{array}{r} 3{,}098 \\ +\ 1{,}913 \\ \hline 5{,}011 \end{array}$
6. $7\overline{)574}$ 82

Unit 2 • Lesson 13 157

Lesson 14 ▶ Common Fraction Errors: Keeping It All Straight

Problem Solving: ▶ Drawing Triangles With a Compass

Lesson Planner

Vocabulary

equilateral triangle

Skills Maintenance

Fractions

Building Number Concepts: ▶ Common Fraction Errors: Keeping It All Straight

Students discuss possible confusion with the traditional algorithms for fraction operations. It is one skill to learn an algorithm for one operation in isolation. It is a completely different skill to discriminate between the four operations and the four algorithms. Alerting students to common errors helps them gain number sense.

Objective

Students will analyze common errors in dividing and multiplying fractions.

Problem Solving: ▶ Drawing Triangles With a Compass

Students learn methods for making an equilateral triangle. We define an equilateral triangle as having three sides of equal measure.

Objective

Students will draw equilateral triangles with a compass.

Homework

Students solve a mix of fraction operations, identify names and types of angles, and measure line segments to the nearest millimeter and the nearest $\frac{1}{4}$ inch. In Distributed Practice, students solve four problems involving fractions and whole-numbers.

Lesson 14 | Skills Maintenance

Name ____________ Date ____________

Skills Maintenance
Fractions

Activity 1

Multiply, divide, and simplify the fractions.

1. $\frac{2}{3} \cdot \frac{3}{5} \cdot \frac{1}{2}$ $\frac{6}{30}, \frac{1}{5}$
2. $\frac{3}{9} \cdot \frac{4}{8}$ $\frac{12}{72}, \frac{1}{6}$
3. $\frac{8}{12} \div \frac{1}{4}$ $\frac{32}{12}, 2\frac{2}{3}$
4. $\frac{3}{9} \div \frac{1}{3}$ $\frac{9}{9}, 1$

Unit 2

Unit 2 • Lesson 14 83

Skills Maintenance

Fractions

(*Interactive Text*, page 83)

Activity 1

Students multiply, divide, and simplify fractions.

Building Number Concepts: ▶ Common Fraction Errors: Keeping It All Straight

What are some common errors we make when multiplying or dividing fractions?

(*Student Text*, pages 158–159)

Connect to Prior Knowledge

Remind students that they looked at the algorithms for operations with fractions. Explain that it is important to remember the correct steps for each operation to avoid making errors.

Link to Today's Concept

Tell students that in today's lesson, we look at common errors in multiplying and dividing fractions.

Improve Your Skills

Engagement Strategy: Teacher Modeling

Demonstrate common fraction errors in one of the following ways:

mBook: Use the *mBook Teacher Edition* for *Student Text*, pages 158–159. m

Overhead Projector: Copy the fractions onto a transparency, and modify as discussed.

Board: Copy the fractions onto the board, and modify as discussed.

- Begin by showing a common error that occurs with multiplication. Display the problem $\frac{3}{5} \cdot \frac{1}{2}$. m
- Work through the problem by first finding the common denominator. Check to see if students notice that this is an incorrect method for multiplication. We do not need to find a common denominator when we multiply fractions. m
- Continue by showing the division problem $\frac{4}{6} \div \frac{2}{3}$. m
- Walk students through this problem, and help them notice the error. Like in multiplication, students do not need to find the common denominator when dividing fractions. m

Lesson 14 ▶Common Fraction Errors: Keeping It All Straight

Problem Solving: ▶Drawing Triangles With a Compass

▶Common Fraction Errors: Keeping It All Straight

What are some common errors we make when multiplying or dividing fractions?

We learned the algorithms for operating on fractions in the past several lessons. Now we mix all the operations together in one lesson. The challenge becomes remembering the correct steps for the problem we are working on. It is easy to confuse the different procedures.

Improve Your Skills

Common Denominators

Sometimes we forget that we do not need common denominators for multiplication and division. Here are some common errors:

Multiplication

$\frac{3}{5} \cdot \frac{1}{2}$

The common denominator is 10. **ERROR**

$\frac{3}{5} \cdot \frac{2}{2} = \frac{6}{10}$ and $\frac{1}{2} \cdot \frac{5}{5} = \frac{5}{10}$

$\frac{6}{10} \cdot \frac{5}{10} = \frac{30}{10}$

This answer is incorrect.

Remember: We do not need a common denominator for multiplication and division.

Division

$\frac{4}{6} \div \frac{2}{3}$

The common denominator is 6. **ERROR**

$\frac{2}{3} \cdot \frac{2}{2} = \frac{4}{6}$

$\frac{4}{6} \cdot \frac{6}{4} = \frac{24}{24}$

This answer is incorrect.

Remember: We do not need a common denominator for multiplication and division.

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Improve Your Skills

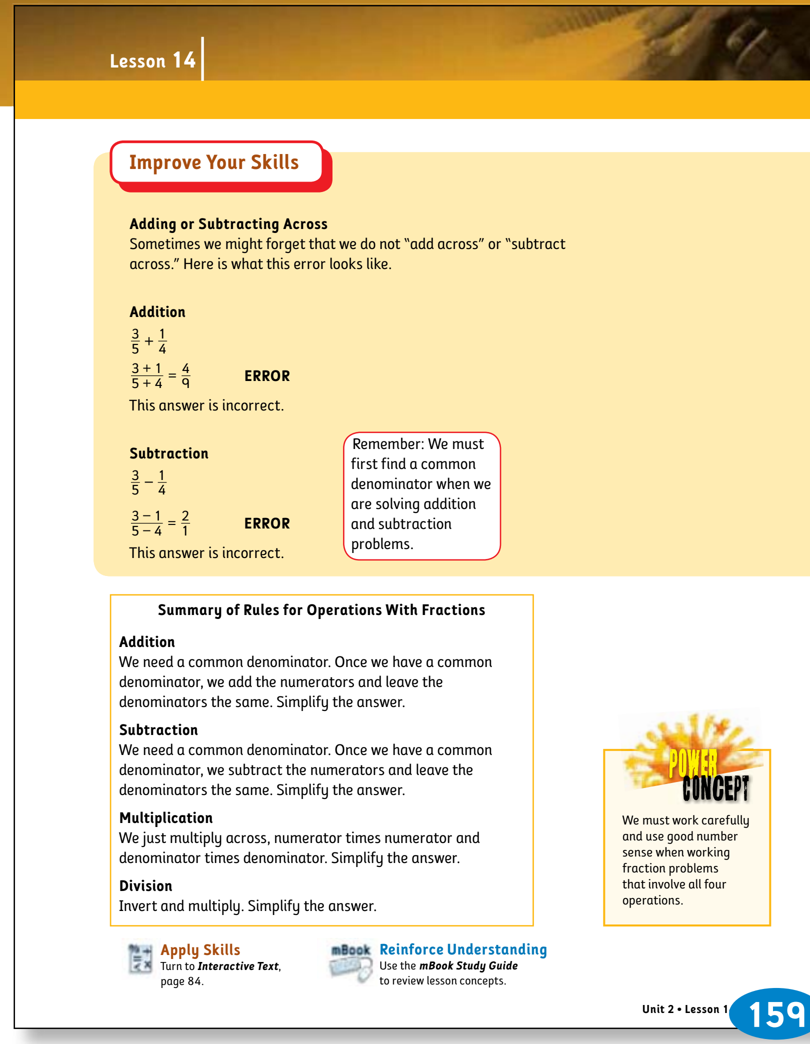
- Display the problems $\frac{3}{5} + \frac{1}{4}$ and $\frac{3}{5} - \frac{1}{4}$, which show another very common error. Students forget to find a common denominator and just subtract or add across.
- Go over both problems and help students notice this error. Remind students that they need to find a common denominator before adding or subtracting fractions.

Check for Understanding

Engagement Strategy: Pair/Share

Have students work in pairs to summarize the rules for the four operations with fractions. Have them write the rules on a sheet of paper. When students finish, invite pairs to share their summaries.

Then go through the Summary of Rules for Operations With Fractions with students, and have them check that their summaries match.



Improve Your Skills

Adding or Subtracting Across

Sometimes we might forget that we do not "add across" or "subtract across." Here is what this error looks like.

Addition

$\frac{3}{5} + \frac{1}{4}$

$\frac{3+1}{5+4} = \frac{4}{9}$ **ERROR**

This answer is incorrect.

Subtraction

$\frac{3}{5} - \frac{1}{4}$

$\frac{3-1}{5-4} = \frac{2}{1}$ **ERROR**

This answer is incorrect.

Remember: We must first find a common denominator when we are solving addition and subtraction problems.

Summary of Rules for Operations With Fractions

Addition
We need a common denominator. Once we have a common denominator, we add the numerators and leave the denominators the same. Simplify the answer.

Subtraction
We need a common denominator. Once we have a common denominator, we subtract the numerators and leave the denominators the same. Simplify the answer.

Multiplication
We just multiply across, numerator times numerator and denominator times denominator. Simplify the answer.

Division
Invert and multiply. Simplify the answer.

POWER CONCEPT

We must work carefully and use good number sense when working fraction problems that involve all four operations.

Apply Skills Turn to *Interactive Text*, page 84.

Reinforce Understanding Use the *mBook Study Guide* to review lesson concepts.

Apply Skills

(*Interactive Text*, page 84)

Have students turn to page 84 in the *Interactive Text*, which provides students an opportunity to practice finding fraction errors.

Activity 1

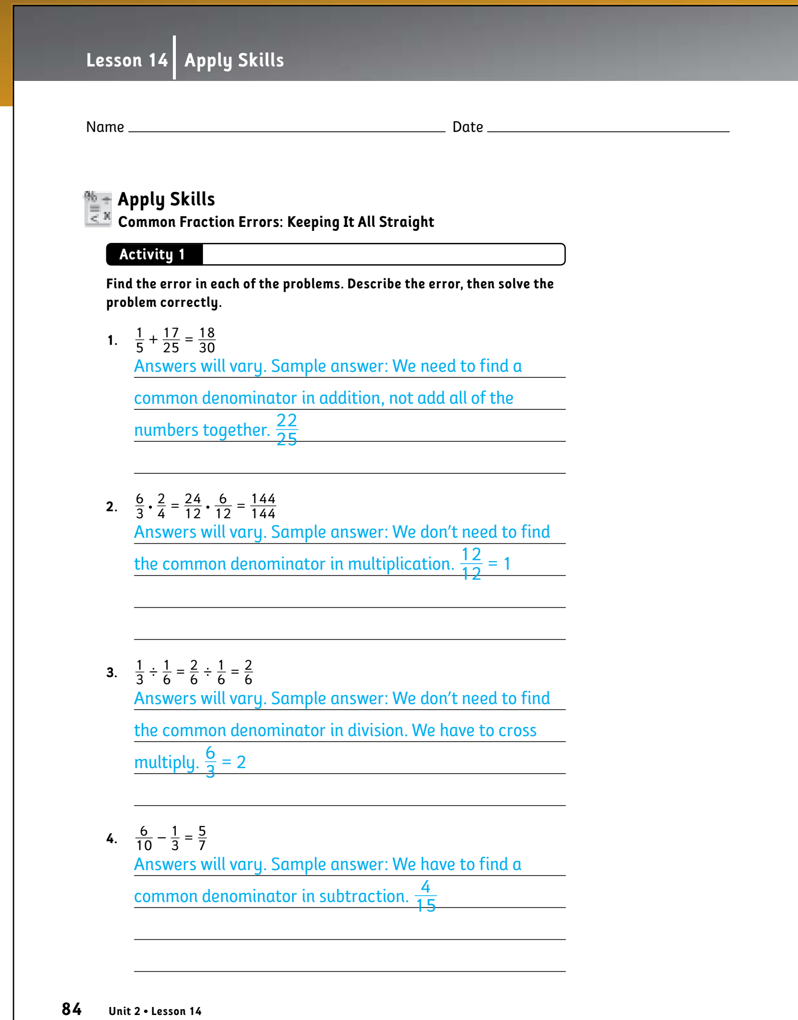
Students find and describe the error in each problem. Then they solve the problem correctly. Monitor students' work as they complete the activity.

Watch for:

- Can students identify the error in each problem?
- Can students describe what the error is?
- Are students able to determine the best method for solving the problem correctly?

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.



Lesson 14 | Apply Skills

Name ____________________ Date ____________________

Apply Skills

Common Fraction Errors: Keeping It All Straight

Activity 1

Find the error in each of the problems. Describe the error, then solve the problem correctly.

1. $\frac{1}{5} + \frac{17}{25} = \frac{18}{30}$
Answers will vary. Sample answer: We need to find a common denominator in addition, not add all of the numbers together. $\frac{22}{25}$

2. $\frac{6}{3} \cdot \frac{2}{4} = \frac{24}{12} \cdot \frac{6}{12} = \frac{144}{144}$
Answers will vary. Sample answer: We don't need to find the common denominator in multiplication. $\frac{12}{12} = 1$

3. $\frac{1}{3} \div \frac{1}{6} = \frac{2}{6} \div \frac{1}{6} = \frac{2}{6}$
Answers will vary. Sample answer: We don't need to find the common denominator in division. We have to cross multiply. $\frac{6}{3} = 2$

4. $\frac{6}{10} - \frac{1}{3} = \frac{5}{7}$
Answers will vary. Sample answer: We have to find a common denominator in subtraction. $\frac{4}{15}$

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Problem Solving:
▶ Drawing Triangles With a Compass

How do we make equilateral triangles with a compass?

(*Student Text,* pages 160–161)

Connect to Prior Knowledge

Ask students to describe an **equilateral triangle**.

Listen for:

- *The angles on the triangle are the same.*
- *The sides on the triangle are the same.*

Use the board or an overhead transparency to show that drawing an equilateral triangle just using a ruler is difficult. It is difficult to get the sides to line up so that they are the same length and they meet at the top.

Link to Today's Concept

In today's lesson, we show students two ways to make an equilaterial triangle.

Demonstrate

- Have students get out a sheet of blank or lined paper and ask them to follow and complete the steps to construct an equilateral triangle using arcs. Walk students through each step, allowing them time to complete their drawings.

STEP 1

- Have students draw a line segment. Remind them to leave enough room above to construct the triangle.

Lesson 14

▶Problem Solving: Drawing Triangles With a Compass

Vocabulary
equilateral triangle

How do we make equilateral triangles with a compass?

This lesson shows how to make a triangle that has equal sides using just a ruler and a compass. These triangles are called **equilateral triangles**. The first method is simple.

Steps for Making an Equilateral Triangle Using a Compass

STEP 1

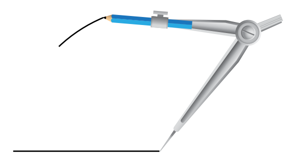
Draw a line segment.

STEP 2

Put the sharp end of the compass at one end of the line segment and stretch the compass so that the pencil tip touches the other end of the segment.

STEP 3

Draw an arc above the line.



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STEP 2

- Show students how to line up the point of the compass at one end of the line segment and stretch the compass so that the pencil point touches the other end of the segment.

STEP 3

- Draw an arc above the line.

How do we make equilateral triangles with a compass? *(continued)*

Demonstrate

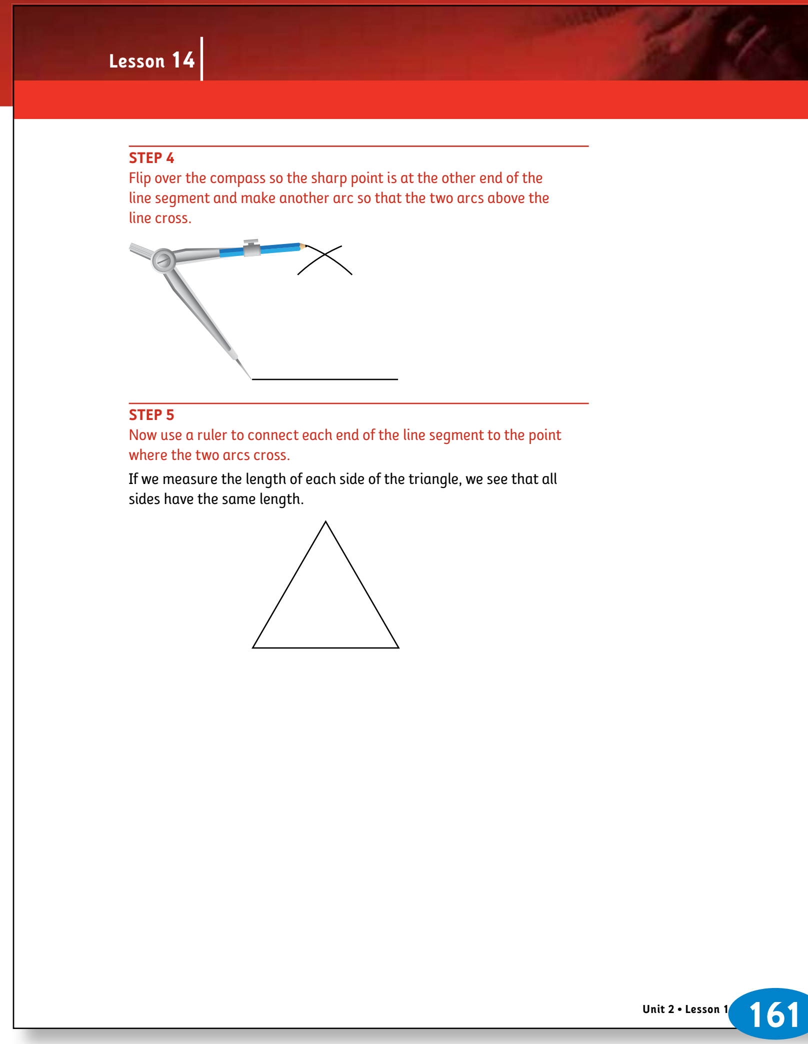
- Continue walking through the steps on page 161 of the *Student Text*.

STEP 4

- Show students how to line the point of the compass at the other end of the line segment to draw another arc. Remind students to be careful not to change the size of the opening of the compass.

STEP 5

- Explain that we must now use a ruler to connect the ends of the line segment to the point where the arcs cross. Then have students measure each side of the triangle to check that they are all equal.


Lesson 14

STEP 4
Flip over the compass so the sharp point is at the other end of the line segment and make another arc so that the two arcs above the line cross.

STEP 5
Now use a ruler to connect each end of the line segment to the point where the two arcs cross.

If we measure the length of each side of the triangle, we see that all sides have the same length.

Unit 2 • Lesson 14 161

What is another way to make an equilateral triangle with a compass?
(*Student Text*, pages 162–163)

Demonstrate

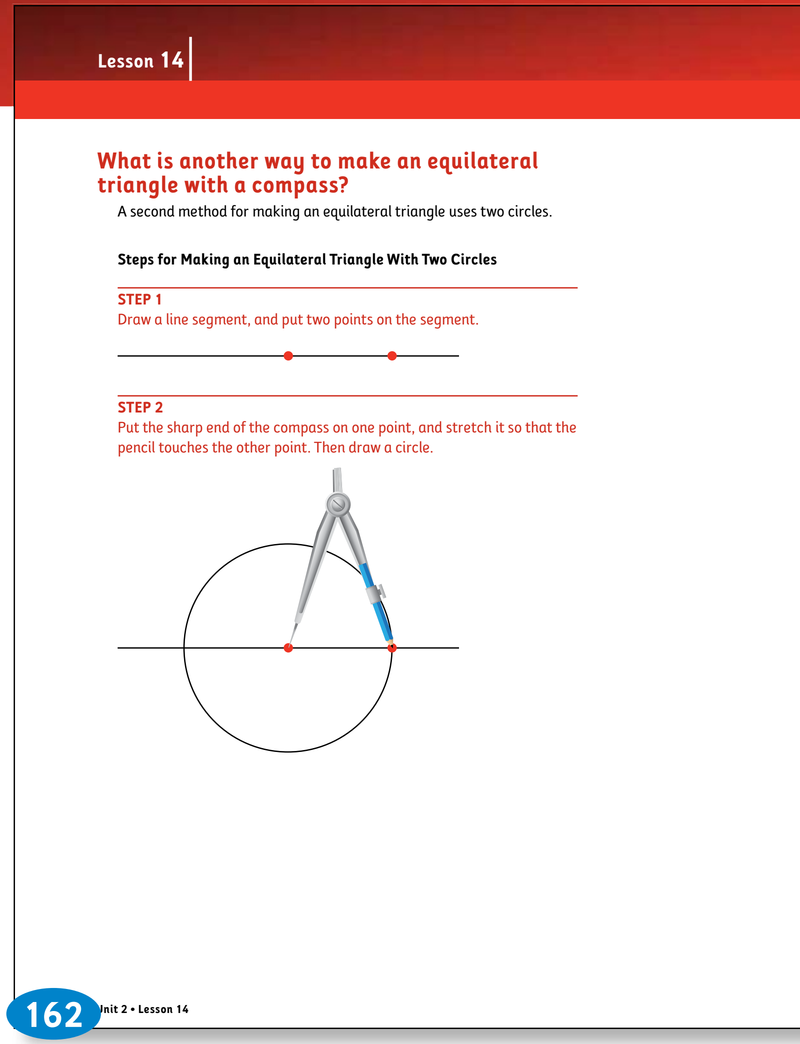
- Tell students that we can also make an equilateral triangle using the compass in a different way. Instead of drawing arcs, we can draw circles.
- Direct students' attention to page 162 of the *Student Text*. Have students turn their papers over to complete the steps to draw an equilateral triangle using circles. Again have them follow along, and complete each step as you discuss.

STEP 1

- Have students draw a line segment with two points. Again remind them to leave ample space above and below to draw.

STEP 2

- Show students how to line up the point of the compass to one point on the line and adjust the compass so that the pencil touches the other point.
- Have students draw a circle. Make sure students understand that the rim of the circle should touch the second point.


Lesson 14

What is another way to make an equilateral triangle with a compass?

A second method for making an equilateral triangle uses two circles.

Steps for Making an Equilateral Triangle With Two Circles

STEP 1
Draw a line segment, and put two points on the segment.

STEP 2
Put the sharp end of the compass on one point, and stretch it so that the pencil touches the other point. Then draw a circle.

162 Unit 2 • Lesson 14

What is another way to make an equilateral triangle with a compass? (*continued*)

Demonstrate

Continue the steps on page 163 of the *Student Text* to complete the drawing.

STEP 3

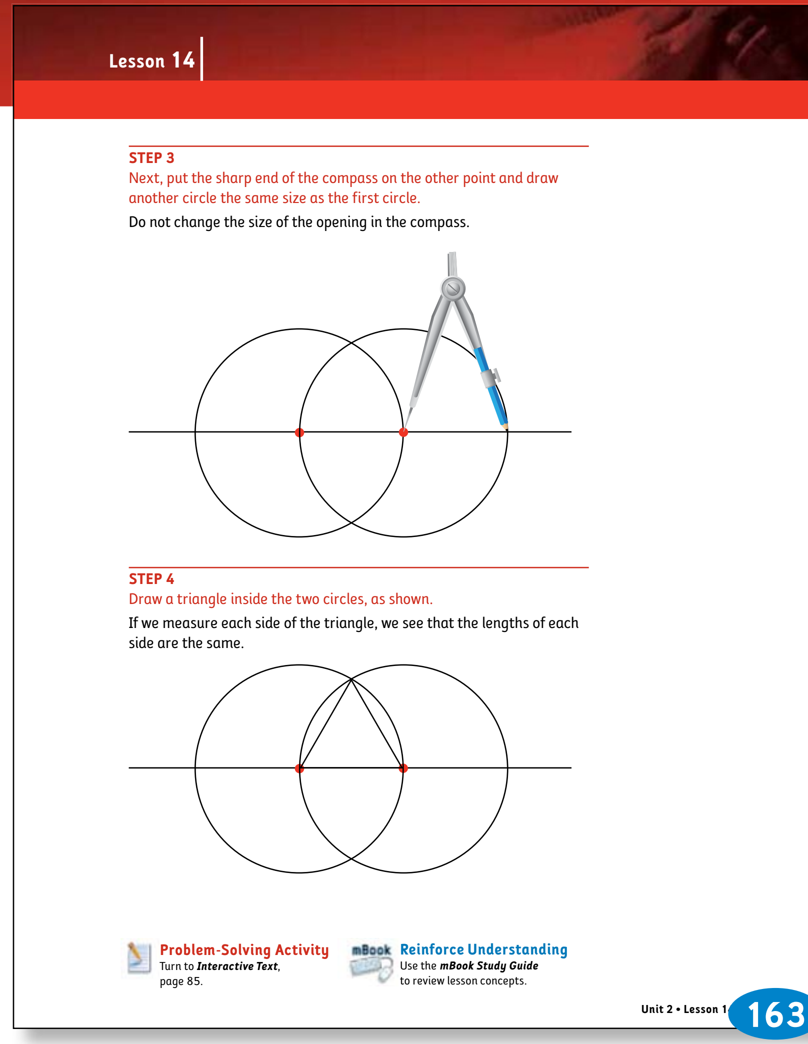
- Have students draw the second circle, this time aligning the compass point on the second point on the line and the pencil on the first point.
- Remind students again that the rim of the circle should touch the first point on the line.

STEP 4

- Show students how to draw lines that connect each point on the line to the point where the circles cross above the line. The lines should form an equilateral triangle inside the circles.
- Have students measure the sides of the triangles to see that they are all equal in length.
- If time permits, have students create a second triangle using the compass on their own, trying each method.
- Have students check to see if the sides are equal using their rulers. Suggest that students use metric measurements in this case because the lengths are easier to read.

Watch for:

- Are students holding and using the compass correctly, especially in drawing arcs that are long enough?



Lesson 14

STEP 3
Next, put the sharp end of the compass on the other point and draw another circle the same size as the first circle.

Do not change the size of the opening in the compass.

STEP 4
Draw a triangle inside the two circles, as shown.

If we measure each side of the triangle, we see that the lengths of each side are the same.

Problem-Solving Activity Turn to *Interactive Text*, page 85.

Reinforce Understanding Use the *mBook Study Guide* to review lesson concepts.

Unit 2 • Lesson 14 163

- Do the sides on the triangle using the second method meet at the intersection points on the circle?
- Are students measuring their lines accurately?
- Once students complete these activities, remind them that these methods are much more precise than trying to construct equilateral triangles with just a ruler.

Problem-Solving Activity

(*Interactive Text*, page 85)

Have students turn to page 85 in the *Interactive Text*, which provides students an opportunity to draw equilateral triangles using a compass.

Have students use their compasses and rulers to make an equilateral triangle that has sides that are 4 inches each. They should draw arcs to make this triangle. Then have students draw a second triangle, which is 33 millimeters on each side, using two circles.

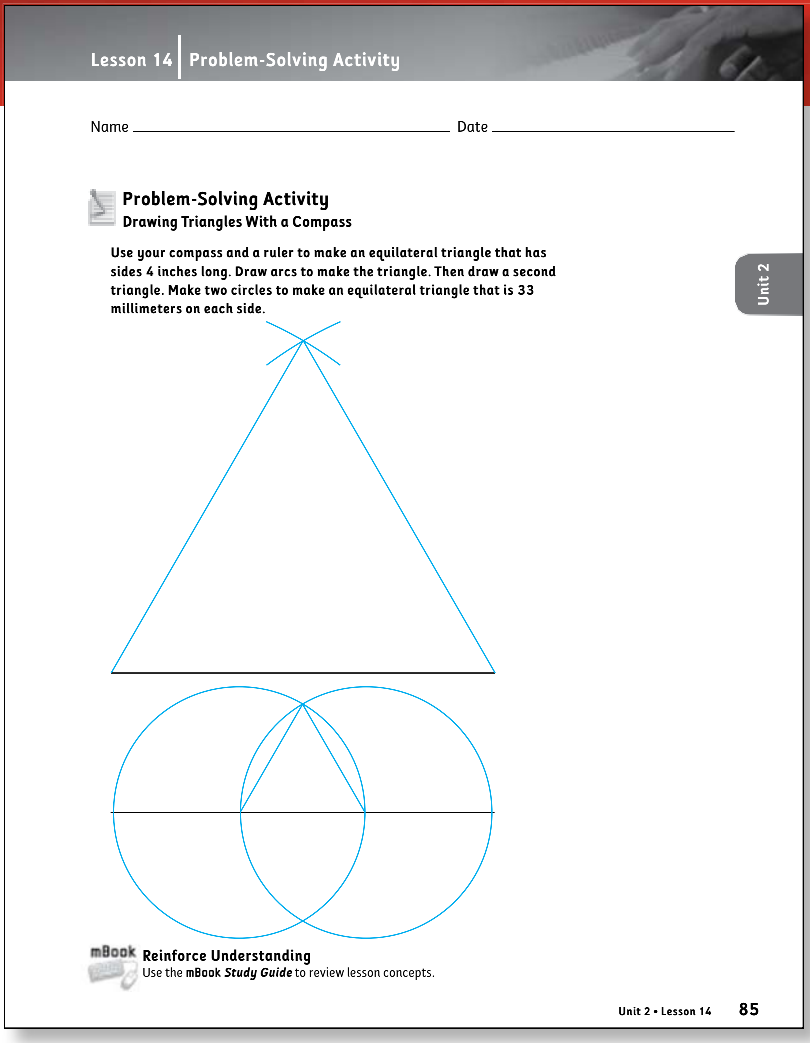
Monitor students' work as they complete the activity.

Watch for:

- Are students holding and using the compass correctly, especially when drawing arcs that are long enough?
- Do the sides on triangle using the second method meet at the intersection points on the circle?
- Are students measuring their lines accurately?

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.


Lesson 14 | Problem-Solving Activity

Name ______________________ Date ______________

Problem-Solving Activity
Drawing Triangles With a Compass

Use your compass and a ruler to make an equilateral triangle that has sides 4 inches long. Draw arcs to make the triangle. Then draw a second triangle. Make two circles to make an equilateral triangle that is 33 millimeters on each side.

Unit 2

Reinforce Understanding
Use the mBook *Study Guide* to review lesson concepts.

Unit 2 • Lesson 14 85

Homework

Go over the instructions for each activity on pages 164–165 of the *Student Text*.

Activity 1

Students solve a mix of fraction operations.

Activity 2

Students identify names and types of angles.

Homework

Activity 1

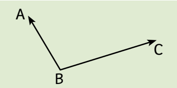
Solve the fraction problems. Simplify the answers.

1. $\frac{3}{5} + \frac{1}{2}$ $\frac{11}{10} = 1\frac{1}{10}$
2. $\frac{4}{9} - \frac{1}{3}$ $\frac{1}{9}$
3. $\frac{5}{6} \cdot \frac{2}{3}$ $\frac{10}{18} = \frac{5}{9}$
4. $\frac{4}{5} \div \frac{1}{5}$ $\frac{20}{5} = 4$
5. $\frac{2}{3} + \frac{1}{2}$ $\frac{7}{6} = 1\frac{1}{6}$
6. $\frac{3}{4} - \frac{1}{6}$ $\frac{7}{12}$
7. $\frac{3}{5} \cdot \frac{3}{4}$ $\frac{9}{20}$
8. $\frac{3}{6} \div \frac{2}{4}$ $\frac{12}{12} = 1$

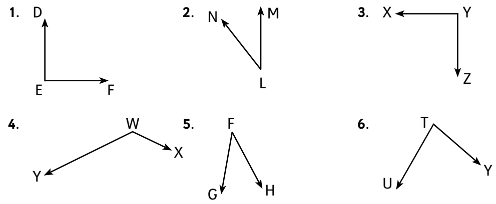
Activity 2

Tell the name and type of each of the angles (acute, obtuse, or right).

Model



Answer: ∠ABC is obtuse.



1. ∠DEF is right.
2. ∠NLM is acute.
3. ∠XYZ is right.
4. ∠YWX is obtuse.
5. ∠GFH is acute.
6. ∠UTY is acute.

Homework

Go over the instructions for each activity on page 165 of the *Student Text*.

Activity 3

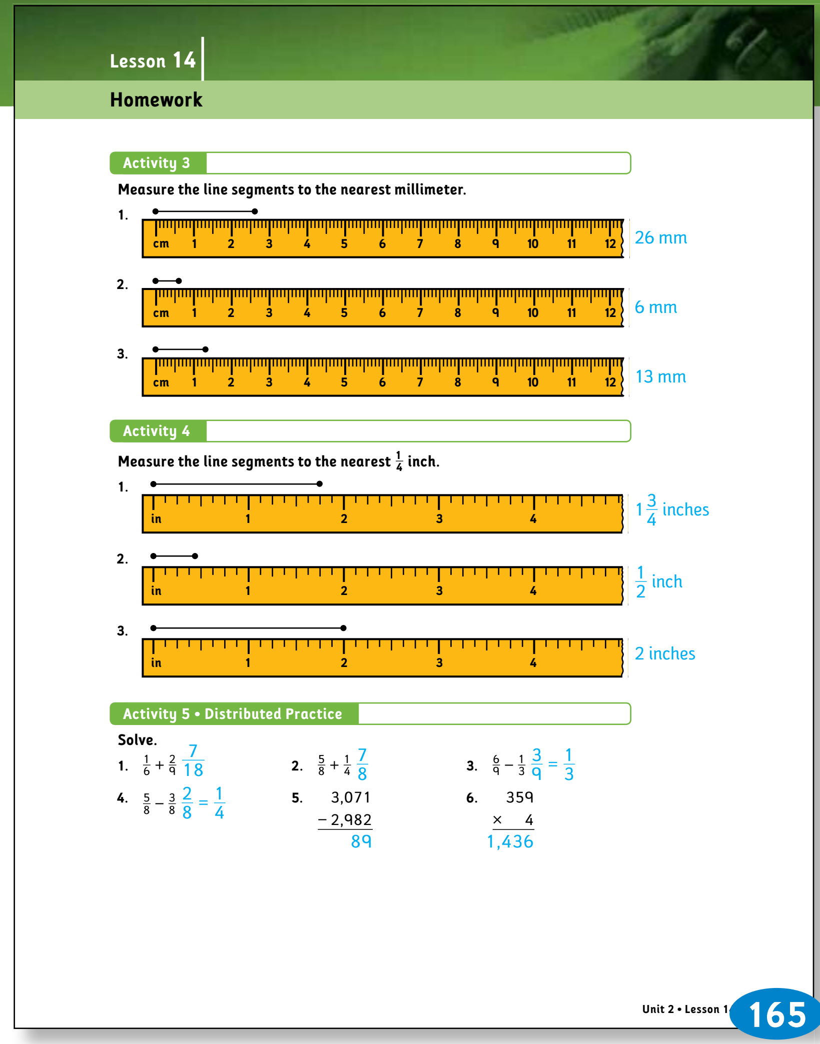
Students measure line segments to the nearest millimeter. The rulers are provided on the page.

Activity 4

Students measure line segments to the nearest $\frac{1}{4}$ inch. Again the rulers are provided on the page.

Activity 5 • Distributed Practice

Students solve four problems involving addition and subtraction of fractions and two whole-number operations.



Lesson 14

Homework

Activity 3

Measure the line segments to the nearest millimeter.

1. 26 mm
2. 6 mm
3. 13 mm

Activity 4

Measure the line segments to the nearest $\frac{1}{4}$ inch.

1. $1\frac{3}{4}$ inches
2. $\frac{1}{2}$ inch
3. 2 inches

Activity 5 • Distributed Practice

Solve.

1. $\frac{1}{6} + \frac{2}{9}$ $\frac{7}{18}$
2. $\frac{5}{8} + \frac{1}{4}$ $\frac{7}{8}$
3. $\frac{6}{9} - \frac{1}{3}$ $\frac{3}{9} = \frac{1}{3}$
4. $\frac{5}{8} - \frac{3}{8}$ $\frac{2}{8} = \frac{1}{4}$
5. $3{,}071 - 2{,}982 = 89$
6. $359 \times 4 = 1{,}436$

Unit 2 • Lesson 14 165

Lesson 15 | Unit Review

▶ Multiplication and Division of Fractions

Problem Solving:

▶ Tools for Measurement and Construction

Lesson Planner

Vocabulary Development

area model
area
factors
common factors
greatest common factor
commute
traditional method
multiplicand
point
line segment
line
ray
metric ruler
millimeters
centimeters
angle
vertex
right angle
acute angle
obtuse angle
protractor
bisect
equilateral triangle

Skills Maintenance

Identifying Angles

Building Number Concepts:

▶ Multiplication and Division of Fractions

Students review multiplication and division of fractions and traditional algorithms.

Problem Solving:

▶ Tools for Measurement and Construction

We use rulers, protractors, and compasses. Students review terms and metric and U.S. customary rulers.

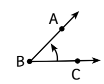
Lesson 15 | Skills Maintenance

Name ________________ Date ________________

Skills Maintenance
Identifying Angles

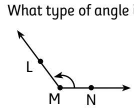
Activity 1

Give the name of the angles using the letters given and the angle symbol (remember, it looks like this: ∠). Tell what type of angle it is (right, obtuse, acute).

1. 

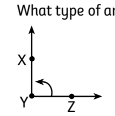
This angle is called: ∠ABC

What type of angle is it? (circle one) right obtuse acute

2. 

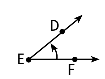
This angle is called: ∠LMN

What type of angle is it? (circle one) right obtuse acute

3. 

This angle is called: ∠XYZ

What type of angle is it? (circle one) right obtuse acute

4. 

This angle is called: ∠DEF

What type of angle is it? (circle one) right obtuse acute

Skills Maintenance

Identifying Angles

(*Interactive Text*, page 86)

Activity 1

Students name the angles and tell what type it is (right, obtuse, acute).

Building Number Concepts:
▶ Multiplication and Division of Fractions

What do we need to remember when we multiply or divide fractions?

(*Student Text*, pages 166–168)

Discuss

Begin by asking students to tell you what they learned about multiplying fractions that is different from multiplying whole numbers.

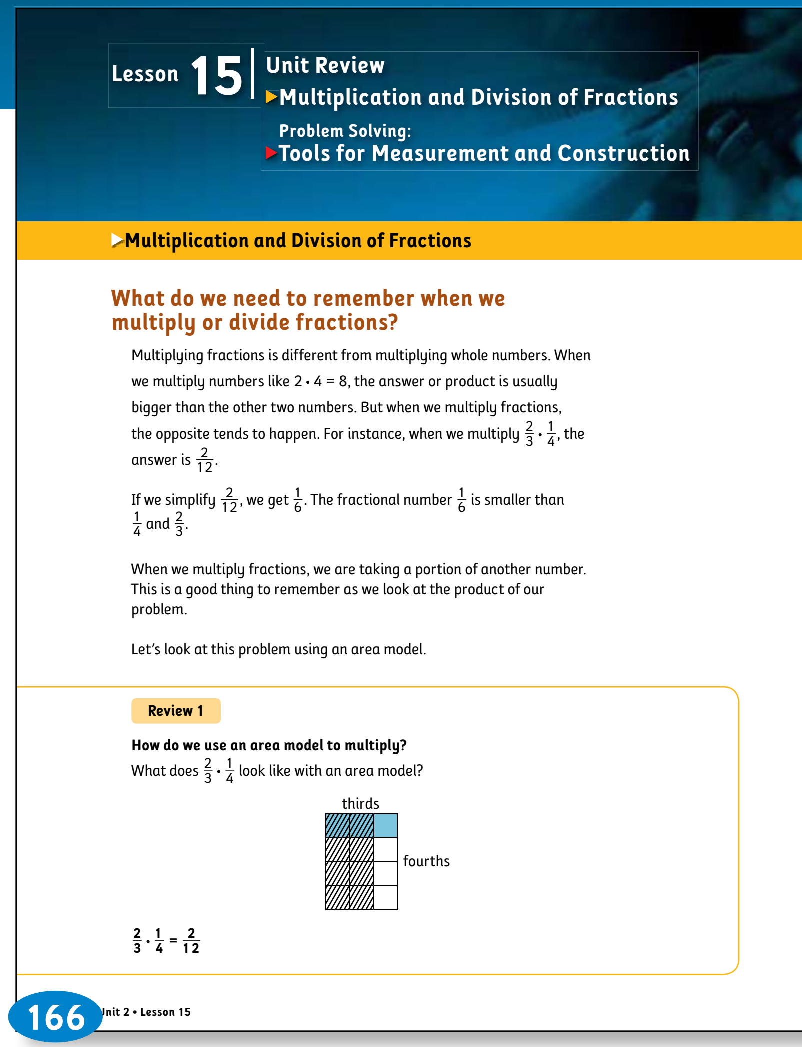
Listen for:

- *The number you end up with (the product) is usually smaller than the two other numbers.*
- *When you multiply whole numbers, the number you end up with (the product) is usually bigger than the other two numbers.*

Have students look at page 166 of the *Student Text*. Review the material at the top of the page to show why the product is usually a smaller number when multiplying fractions.

Demonstrate

- Go over **Review 1** with students to look at using an area model to multiply the fractions $\mathbf{\frac{2}{3} \cdot \frac{1}{4}}$.
- Point out the rectangle. Note that we divide the width of the rectangle into thirds by making three rows. Tell students the diagonal black shading represents $\frac{2}{3}$.
- Then we represent the multiplicand $\frac{1}{4}$ by dividing the rectangle into fourths and making four columns. Tell students the teal shading represents $\frac{1}{4}$.



Lesson 15 | Unit Review
▶ Multiplication and Division of Fractions
Problem Solving:
▶ Tools for Measurement and Construction

▶ Multiplication and Division of Fractions

What do we need to remember when we multiply or divide fractions?

Multiplying fractions is different from multiplying whole numbers. When we multiply numbers like $2 \cdot 4 = 8$, the answer or product is usually bigger than the other two numbers. But when we multiply fractions, the opposite tends to happen. For instance, when we multiply $\frac{2}{3} \cdot \frac{1}{4}$, the answer is $\frac{2}{12}$.

If we simplify $\frac{2}{12}$, we get $\frac{1}{6}$. The fractional number $\frac{1}{6}$ is smaller than $\frac{1}{4}$ and $\frac{2}{3}$.

When we multiply fractions, we are taking a portion of another number. This is a good thing to remember as we look at the product of our problem.

Let's look at this problem using an area model.

Review 1

How do we use an area model to multiply?

What does $\frac{2}{3} \cdot \frac{1}{4}$ look like with an area model?

$\mathbf{\frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12}}$

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- We can easily see the overlapping shading shows how the problem $\frac{2}{3} \cdot \frac{1}{4}$ results in the answer $\mathbf{\frac{2}{12}}$. Be sure to stress that the overlapping shaded cells are in relation to the total number of cells (i.e., 2 in relation to 12).

What do we need to remember when we multiply or divide fractions? *(continued)*

Demonstrate

- Next move to **Review 2** on page 167 of the *Student Text*. This example shows the traditional method for multiplying fractions. Remind students that answers should always be simplified.
- Remind students of using good number sense to multiply and simplify fractions, such as looking at factors and common factors to find the GCF. We can also use information we have about multiplication to commute numbers to make fractions easier to work with.

Discuss

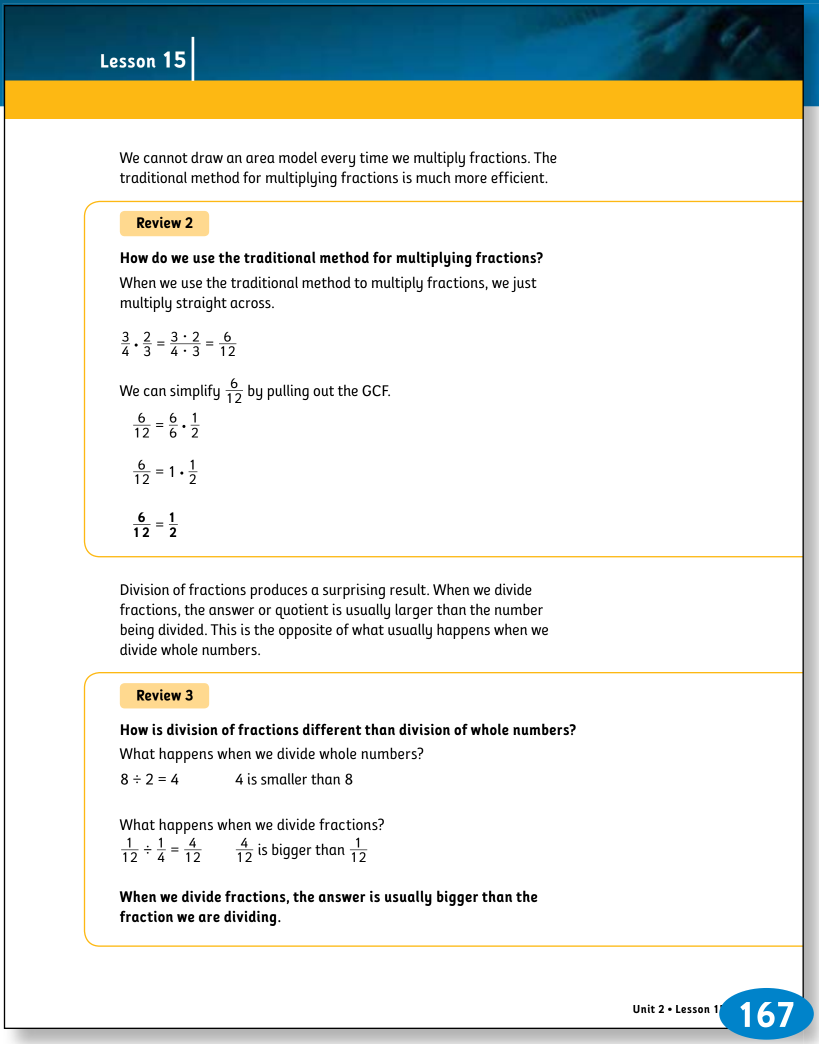
Have students tell you what they learned about division of fractions that makes them different from dividing whole numbers.

Listen for:

- *The number you end up with (the quotient) is usually bigger than the number you are dividing.*
- *When you divide whole numbers, the number you end up with (the quotient) is usually smaller than the number you are dividing.*

Demonstrate

- Then go over **Review 3**, which presents a comparison of division of whole numbers and division of fractions. When you discuss the fact $\frac{1}{12} \div \frac{1}{4} = \frac{4}{12}$, point out that it is hard to tell that $\frac{4}{12}$ is actually larger than $\frac{1}{4}$.



Lesson 15

We cannot draw an area model every time we multiply fractions. The traditional method for multiplying fractions is much more efficient.

Review 2

How do we use the traditional method for multiplying fractions?

When we use the traditional method to multiply fractions, we just multiply straight across.

$\frac{3}{4} \cdot \frac{2}{3} = \frac{3 \cdot 2}{4 \cdot 3} = \frac{6}{12}$

We can simplify $\frac{6}{12}$ by pulling out the GCF.

$\frac{6}{12} = \frac{6}{6} \cdot \frac{1}{2}$

$\frac{6}{12} = 1 \cdot \frac{1}{2}$

$\frac{6}{12} = \frac{1}{2}$

Division of fractions produces a surprising result. When we divide fractions, the answer or quotient is usually larger than the number being divided. This is the opposite of what usually happens when we divide whole numbers.

Review 3

How is division of fractions different than division of whole numbers?

What happens when we divide whole numbers?

$8 \div 2 = 4$ 4 is smaller than 8

What happens when we divide fractions?

$\frac{1}{12} \div \frac{1}{4} = \frac{4}{12}$ $\frac{4}{12}$ is bigger than $\frac{1}{12}$

When we divide fractions, the answer is usually bigger than the fraction we are dividing.

Unit 2 • Lesson 15 167

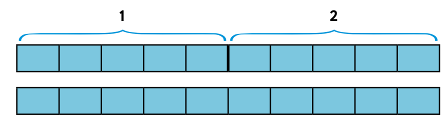
Demonstrate

- Have students turn to page 168 of the *Student Text*. To demonstrate that we usually get larger numbers when we divide fractions, work through **Review 4**, which shows how to divide using fraction bars.
- Stress that the fraction example shows that when we break up the fraction, we are dividing by a unit. In this case, it is $\mathbf{\frac{2}{8}}$. If students ask why the answer is **3**, you might want to work through the traditional method to get $\frac{48}{16}$ and state that this is equivalent to 3.
- Finally finish this discussion of division of fractions with **Review 5**, which shows the traditional method. Remind students that in the traditional method, we invert and multiply.

It is important to remember that division is about breaking up or dividing a quantity into equal groups. We see this with whole numbers and with fractions.

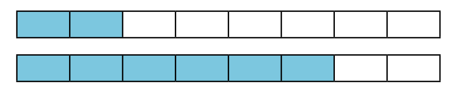
Review 4

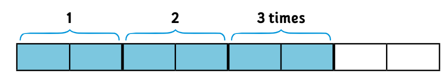
How do we show division using fraction bars? $10 \div 5 = 2$



The unit 5 breaks up 10 two times. There are two groups of the unit 5 in the quantity 10.

$\frac{6}{8} \div \frac{2}{8} = 3$





The unit $\frac{2}{8}$ breaks up $\frac{6}{8}$ three times. There are 3 groups of the unit $\frac{2}{8}$ in the quantity $\frac{6}{8}$.

A faster way to do all of this is to invert the second fraction and multiply.

Review 5

How do we use the traditional method for dividing fractions?

$\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3}$

Apply Skills
Turn to ***Interactive Text***, page 87.

Reinforce Understanding
Use the ***mBook Study Guide*** to review lesson concepts.

Unit Review: Multiplication and Division of Fractions

(*Interactive Text*, page 87)

Have students turn to page 87 in the *Interactive Text*, which provides students an opportunity to review multiplication and division of fractions.

Activity 1

Students solve division problems and simpify the answers.

Activity 2

Students solve multiplication problems and simplify the answers.

Activity 3

Students solve a mix of operations with fractions and simplify the answers.

Monitor students as they complete the activities.

Watch for:

- Can students multiply and divide fractions using the traditional algorithms, or do they get confused and use an incorrect method (e.g., converting denominators as they would in addition or subtraction)?
- Are there any multiplication fact errors?
- Do students simplify their answers?

Once students finish, discuss any difficulties that you noticed.

mBook

Reinforce Understanding

Remind students that they can review unit concepts by accessing the online *mBook Study Guide.*

Lesson 15 | Unit Review

Unit 2

Name ____________ Date ____________

Unit Review

Multiplication and Division of Fractions

Activity 1

Solve the division problems. Simplify your answers.

1. $\frac{2}{6} \div \frac{4}{3}$ $\frac{6}{24}, \frac{1}{4}$
2. $\frac{3}{8} \div \frac{1}{8}$ $\frac{24}{8}, 3$
3. $\frac{4}{5} \div \frac{1}{5}$ $\frac{20}{5}, 4$
4. $\frac{3}{9} \div \frac{2}{3}$ $\frac{9}{18}, \frac{1}{2}$
5. $\frac{4}{6} \div \frac{1}{3}$ $\frac{12}{6}, 2$
6. $\frac{5}{6} \div \frac{2}{12}$ $\frac{60}{12}, 5$

Activity 2

Multiply the fractions. Simplify your answers.

1. $\frac{2}{6} \cdot \frac{3}{7}$ $\frac{6}{42}, \frac{1}{7}$
2. $\frac{3}{8} \cdot \frac{2}{3}$ $\frac{6}{24}, \frac{1}{4}$
3. $\frac{5}{9} \cdot \frac{1}{5}$ $\frac{5}{45}, \frac{1}{9}$
4. $\frac{3}{9} \cdot \frac{2}{4}$ $\frac{6}{36}, \frac{1}{6}$
5. $\frac{4}{6} \cdot \frac{3}{4}$ $\frac{12}{24}, \frac{1}{2}$
6. $\frac{1}{6} \cdot \frac{3}{5}$ $\frac{3}{30}, \frac{1}{10}$

Activity 3

Solve a mix of operations with fractions. Remember the rules for each algorithm. Simplify the answers.

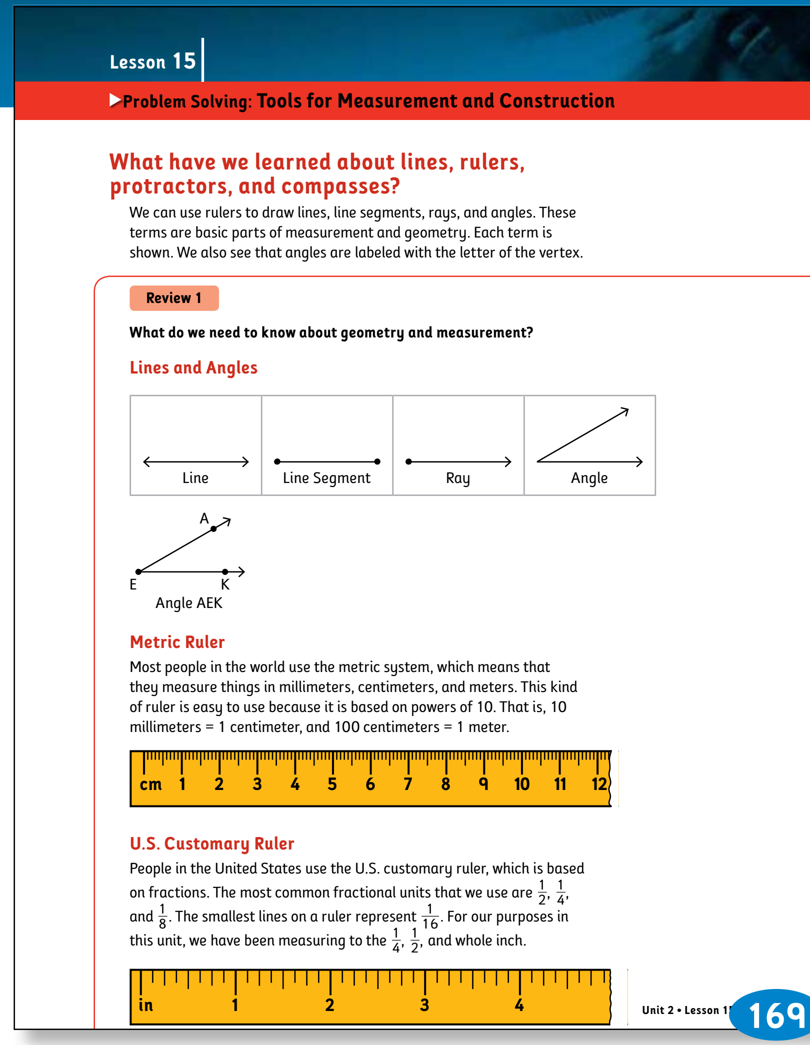
1. $\frac{3}{5} + \frac{2}{4}$ $1\frac{1}{10}$
2. $\frac{8}{9} \div \frac{2}{3}$ $\frac{24}{18}, 1\frac{1}{3}$
3. $\frac{3}{9} \cdot \frac{2}{4}$ $\frac{6}{36}, \frac{1}{6}$
4. $\frac{1}{3} - \frac{1}{9}$ $\frac{2}{9}$
5. $\frac{7}{9} - \frac{1}{3}$ $\frac{4}{9}$
6. $\frac{3}{5} \div \frac{4}{2}$ $\frac{6}{20}, \frac{3}{10}$
7. $\frac{3}{4} + \frac{4}{5}$ $1\frac{11}{20}$
8. $\frac{8}{9} \cdot \frac{2}{3}$ $\frac{16}{27}$

Unit 2 • Lesson 15 87

Problem Solving:
▶ Tools for Measurement and Construction

What have we learned about lines, rulers, protractors, and compasses?

(*Student Text*, pages 169–170)


Lesson 15

▶Problem Solving: Tools for Measurement and Construction

What have we learned about lines, rulers, protractors, and compasses?

We can use rulers to draw lines, line segments, rays, and angles. These terms are basic parts of measurement and geometry. Each term is shown. We also see that angles are labeled with the letter of the vertex.

Review 1

What do we need to know about geometry and measurement?

Lines and Angles

Line	Line Segment	Ray	Angle

A
E
K
Angle AEK

Metric Ruler

Most people in the world use the metric system, which means that they measure things in millimeters, centimeters, and meters. This kind of ruler is easy to use because it is based on powers of 10. That is, 10 millimeters = 1 centimeter, and 100 centimeters = 1 meter.

cm 1 2 3 4 5 6 7 8 9 10 11 12

U.S. Customary Ruler

People in the United States use the U.S. customary ruler, which is based on fractions. The most common fractional units that we use are $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$. The smallest lines on a ruler represent $\frac{1}{16}$. For our purposes in this unit, we have been measuring to the $\frac{1}{4}$, $\frac{1}{2}$, and whole inch.

in 1 2 3 4

Unit 2 • Lesson 15 169

Demonstrate

- Have students look at **Review 1** on page 169 of the *Student Text*. Direct their attention to the line, line segment, ray, and angle. Ask students to define the terms.

Listen for:

- *A line goes on forever.*
- *Line segments start and stop.*
- *Rays have a starting point but no end point.*
- *An angle is made up of two rays that come together at the vertex.*

- Have students look at the metric ruler and U.S. customary rulers. Ask students to tell you some differences between the two.

Listen for:

- *The metric ruler is based on 10s.*
- *We measure in fractions with the regular ruler.*

- Go over the material to discuss each ruler. Make sure students can identify the units on each ruler, specifically the millimeters and centimeters on the metric ruler and the inch, $\frac{1}{2}$-, $\frac{1}{4}$-, and $\frac{1}{8}$-inch units on the U.S. customary ruler.

What have we learned about lines, rulers, protractors, and compasses?
(*continued*)

Discuss

Review the information on page 170 of the *Student Text,* which discusses protractors as a tool for exactly measuring angles. Although we can use a protractor to measure the degrees on an entire circle, we limit our work to angles between 0 and 180 degrees. Review the different kinds of angles shown.

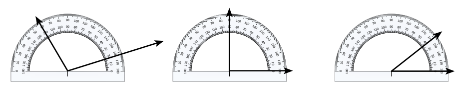
Finally discuss the uses of a compass as a tool to draw circles and precisely construct objects, such as perpendicular lines and equilateral triangles. Review the steps for bisecting an angle using a compass. Refer to pages 155–156 of the *Student Text* if necessary.

Protractors

A protractor is a kind of measuring tool that is used to measure angles. While an entire circle measures 360 degrees, a typical protractor only measures from 0 to 180 degrees. This is only half of a circle.

Most of the time we use the protractor to measure angles less than 90 degrees or more than 90 degrees. We call a 90-degree angle a right angle.

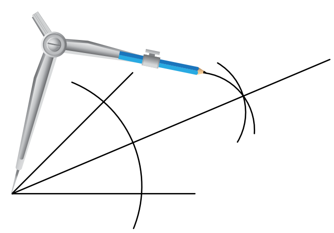
Common Types of Angles



Obtuse: more than 90° Right: exactly 90° Acute: less than 90°

Compass

Finally, a compass is a tool for drawing circles and making angles and shapes. We learned how to use a ruler and a compass to make perpendicular lines as well as triangles. Look below to see how we bisect an angle with a compass.



Problem-Solving Activity Turn to *Interactive Text*, page 88.

Reinforce Understanding Use the ***mBook Study Guide*** to review lesson concepts.

Unit Review: Tools for Measurement and Construction

(*Interactive Text*, pages 88–89)

Have students turn to pages 88–89 in the *Interactive Text*, which provides students an opportunity to review measuring line segments and angles.

Activity 1

Students measure line segments in millimeters and inches, then round to the nearest whole unit.

Activity 2

Students measure angles with a protractor, then name the type of angle.

Activity 3

Students use a protractor to make angles.

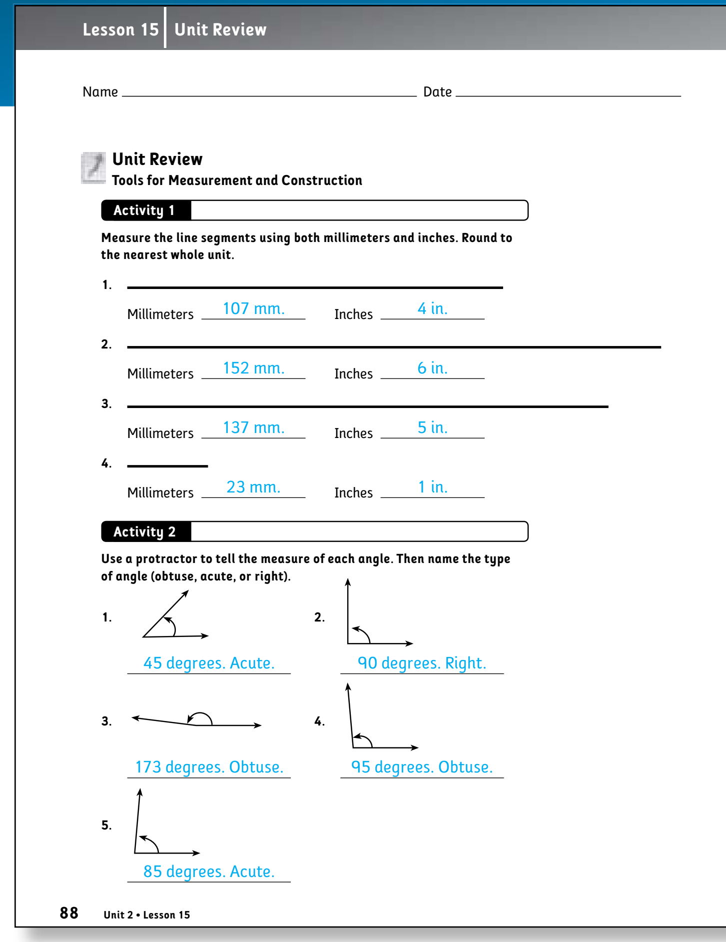
Monitor students as they complete the activities.

Watch for:

- Can students measure in both millimeters and inches?
- Can students round to the nearest whole unit?
- Can students identify and label angles?
- Can students construct angles accurately?

Reinforce Understanding

Remind students that they can review unit concepts by accessing the online *mBook Study Guide*.



Lesson 15 | Unit Review

Name ______ Date ______

Unit Review
Tools for Measurement and Construction

Activity 1

Measure the line segments using both millimeters and inches. Round to the nearest whole unit.

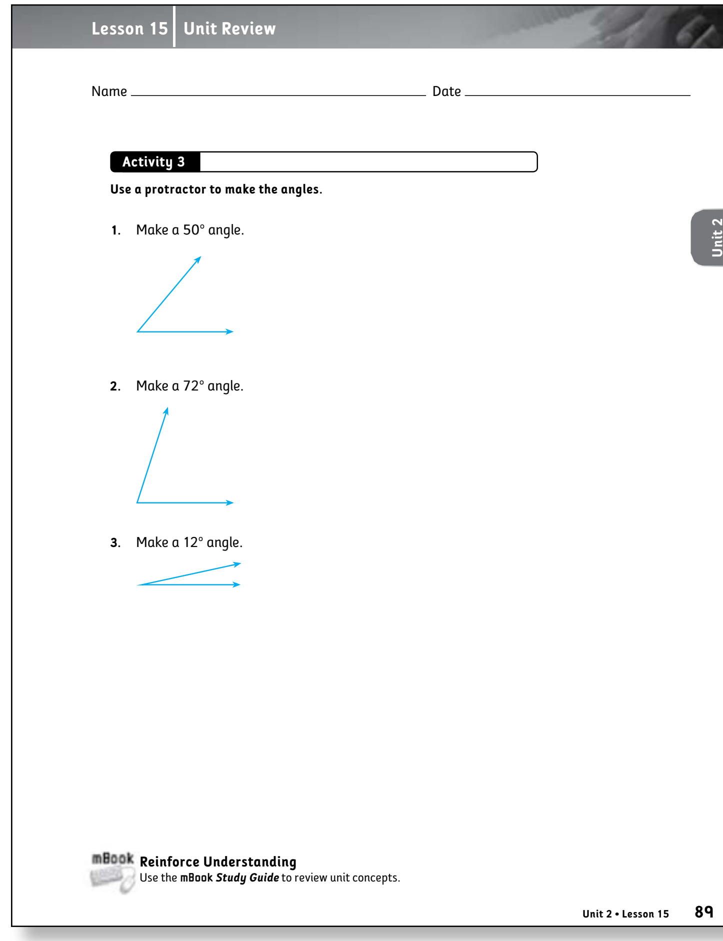
1. Millimeters 107 mm. Inches 4 in.
2. Millimeters 152 mm. Inches 6 in.
3. Millimeters 137 mm. Inches 5 in.
4. Millimeters 23 mm. Inches 1 in.

Activity 2

Use a protractor to tell the measure of each angle. Then name the type of angle (obtuse, acute, or right).

1. 45 degrees. Acute.
2. 90 degrees. Right.
3. 173 degrees. Obtuse.
4. 95 degrees. Obtuse.
5. 85 degrees. Acute.

88 Unit 2 • Lesson 15



Lesson 15 | Unit Review

Name ______ Date ______

Activity 3

Use a protractor to make the angles.

1. Make a 50° angle.
2. Make a 72° angle.
3. Make a 12° angle.

Unit 2

Reinforce Understanding
Use the mBook *Study Guide* to review unit concepts.

Unit 2 • Lesson 15 89

Assessment Planner

Students	Assess	Differentiate		Assess
	Day 1	Day 2	Day 3	Day 4
All	End-of-Unit Assessment *Form A*			Performance Assessments Unit 3 Opener
Scored 80% or above		Extension	Extension	
Scored Below 80%		Reinforcement	Retest	

Assessment Objectives

Building Number Concepts:
▶ Multiplication and Division of Fractions

- Use models to show multiplication and division of fractions
- Understand how multiplication and division of fractions is different from whole numbers
- Use the traditional methods to multiply and divide fractions

Problem Solving:
▶ Tools for Measurement and Construction

- Develop an understanding of basic geometric terms
- Measure lengths and angles using a variety of tools and units
- Use a compass to complete basic geometric constructions

Monitoring Progress:
▶ Unit Assessments

Assess
End-of-Unit Assessment

- Administer End-of-Unit Assessment Form A in the *Assessment Book*, pages 27–29.

Differentiate

- Review End-of-Unit Assessment Form A with class.
- Identify students for Extension or Reinforcement.

Extension

For those students who score 80 percent or better, provide the On Track! Activities from Unit 2, Lessons 11–15, from the *mBook Teacher Edition*.

Reinforcement

For those students who score below 80 percent, provide additional support in one of these ways:

- Have students access the online tutorial provided in the *mBook Study Guide*.
- Have students complete the Interactive Reinforcement Exercises for Unit 2, in the *mBook Study Guide*.
- Provide teacher-directed reteaching of unit concepts.

Retest

Administer End-of Unit Assessment Form B from the *mBook Teacher Edition* to those students who scored below 80 percent on Form A.

Assess
Performance Assessments

- Guide students through the Performance Assessment Model on *Assessment Book*, page 31. Then, administer the Performance Assessments on pages 32–33.

Name ______________________ Date ______________

Form A

Monitoring Progress
Multiplication and Division of Fractions

Part 1

Solve the multiplication problems. Simplify your answers.

1. $\frac{3}{8} \cdot \frac{1}{2}$ $\frac{3}{16}$
2. $\frac{4}{6} \cdot \frac{1}{3}$ $\frac{2}{9}$
3. $\frac{1}{5} \cdot \frac{2}{4}$ $\frac{1}{10}$
4. $\frac{4}{3} \cdot \frac{2}{4}$ $\frac{2}{3}$
5. $\frac{1}{2} \cdot \frac{4}{5}$ $\frac{2}{5}$

Part 2

Solve the division problems. Simplify your answers.

1. $\frac{3}{10} \div \frac{1}{2}$ $\frac{3}{5}$
2. $\frac{1}{6} \div \frac{1}{3}$ $\frac{1}{2}$
3. $\frac{2}{6} \div \frac{2}{4}$ $\frac{2}{3}$
4. $\frac{3}{8} \div \frac{1}{2}$ $\frac{3}{4}$
5. $\frac{2}{4} \div \frac{3}{4}$ $\frac{2}{3}$

Monitoring Progress
Tools for Measurement and Construction

Part 3

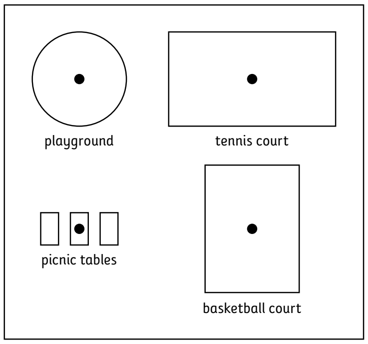
Use a ruler to draw the lines.

1. Draw a line that is $3\frac{3}{4}$ inches in length.
2. Draw a line that is $5\frac{1}{2}$ inches in length.
3. Draw a line that $4\frac{1}{2}$ inches in length.
4. Draw a line that is 13 cm in length.
5. Draw a line that is 80 mm in length.

Unit 2

Part 4

Use the distance between the dots to answer the questions.



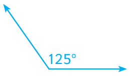
1. How many inches is it from the picnic tables to the tennis court?
   $2\frac{1}{2}$ inches
2. How many inches is it from the basketball court to the tennis court?
   $1\frac{3}{4}$ inches
3. How many inches is it from the playground to the basketball court?
   $2\frac{1}{2}$ inches
4. How many inches is it from the picnic tables to the playground?
   $1\frac{3}{4}$ inches

Name ______________________ Date ______________

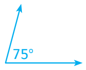
Part 5

Use a protractor to draw the angles.

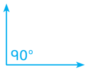
1. Draw an obtuse angle that is 125 degrees.



2. Draw an acute angle that is 75 degrees.



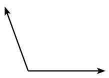
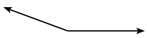
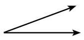
3. Draw a right angle.



4. Draw an angle that is 180 degrees.

Part 6

Use your protractor to measure the angles.

1.  Answer 110 degrees
2.  Answer 160 degrees
3.  Answer 20 degrees

Unit 2

Name ______________________ Date ______________

Form B

mBook

Monitoring Progress
Multiplication and Division of Fractions

Part 1

Solve the multiplication problems. Simplify your answers.

1. $\frac{4}{8} \cdot \frac{1}{2}$ $\frac{1}{4}$
2. $\frac{2}{5} \cdot \frac{3}{2}$ $\frac{3}{5}$
3. $\frac{6}{8} \cdot \frac{1}{2}$ $\frac{3}{8}$
4. $\frac{3}{4} \cdot \frac{1}{3}$ $\frac{1}{4}$
5. $\frac{1}{2} \cdot \frac{4}{5}$ $\frac{2}{5}$

Part 2

Solve the division problems. Simplify your answers.

1. $\frac{2}{10} \div \frac{1}{3}$ $\frac{3}{5}$
2. $\frac{2}{6} \div \frac{3}{4}$ $\frac{4}{9}$
3. $\frac{1}{8} \div \frac{2}{4}$ $\frac{1}{4}$
4. $\frac{4}{5} \div \frac{1}{2}$ $1\frac{3}{5}$
5. $\frac{3}{4} \div \frac{3}{2}$ $\frac{1}{2}$

Monitoring Progress
Tools for Measurement and Construction

Part 3

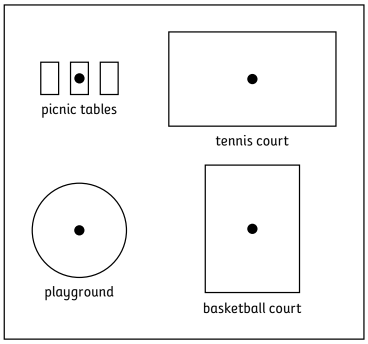
Use a ruler to draw the lines.

1. Draw a line that is $2\frac{1}{4}$ inches in length.
2. Draw a line that is 6 inches in length.
3. Draw a line that $5\frac{1}{2}$ inches in length.
4. Draw a line that is 15 cm in length.
5. Draw a line that 60 mm in length.

Name ______________________ Date ______________

Part 4

Use the distance between the dots to answer the questions.



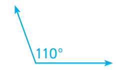
1. How many inches is it from the picnic tables to the playground?
   $1\frac{3}{4}$ inches
2. How many inches is it from the basketball court to the picnic tables?
   $2\frac{1}{2}$ inches
3. How many inches is it from the playground to the basketball court?
   2 inches
4. How many inches is it from the picnic tables to the tennis court?
   2 inches

Name ______________________ Date ______________

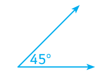
Part 5

Use a protractor to draw the angles.

1. Draw an obtuse angle that is 110 degrees.

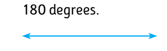


2. Draw an acute angle that is 45 degrees.



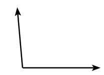
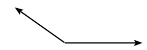
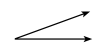
3. Draw right angle.

4. Draw an angle that is 180 degrees.



Part 6

Use your protractor to measure the angles.

1.  Answer 95 degrees
2.  Answer 145 degrees
3.  Answer 20 degrees

Name ______________________ Date ______________

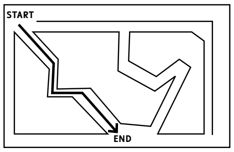
Monitoring Progress
Practice Problem 2-2

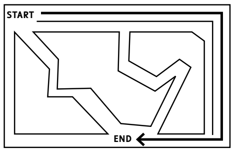
Solve the Problem

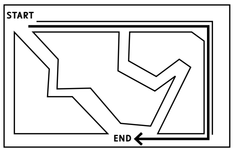
Jared is on a TV game show. In order to win, he has to run through an obstacle course by following directions exactly. Here are his instructions:

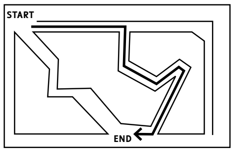
Run from Start to End by following a path that has one obtuse angle, one acute angle, and then one obtuse angle.

Which path did Jared follow so that he could win?

(a) 

(b) 

(c) 

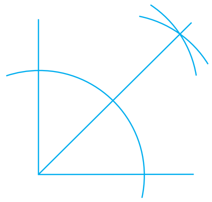
(d) 

(The answer is a.)

Monitoring Progress
Problem 2-2-A

Solve the Problem

Draw a right angle using a ruler and a compass. Then use the protractor to measure the angle to make sure it is a right angle. Use the compass to bisect the angle.



Name ______________________ Date ______________

Monitoring Progress
Problem 2-2-B

Solve the Problem

Find the path that takes you through the maze from Start to End that is made up of one acute and one obtuse angle.