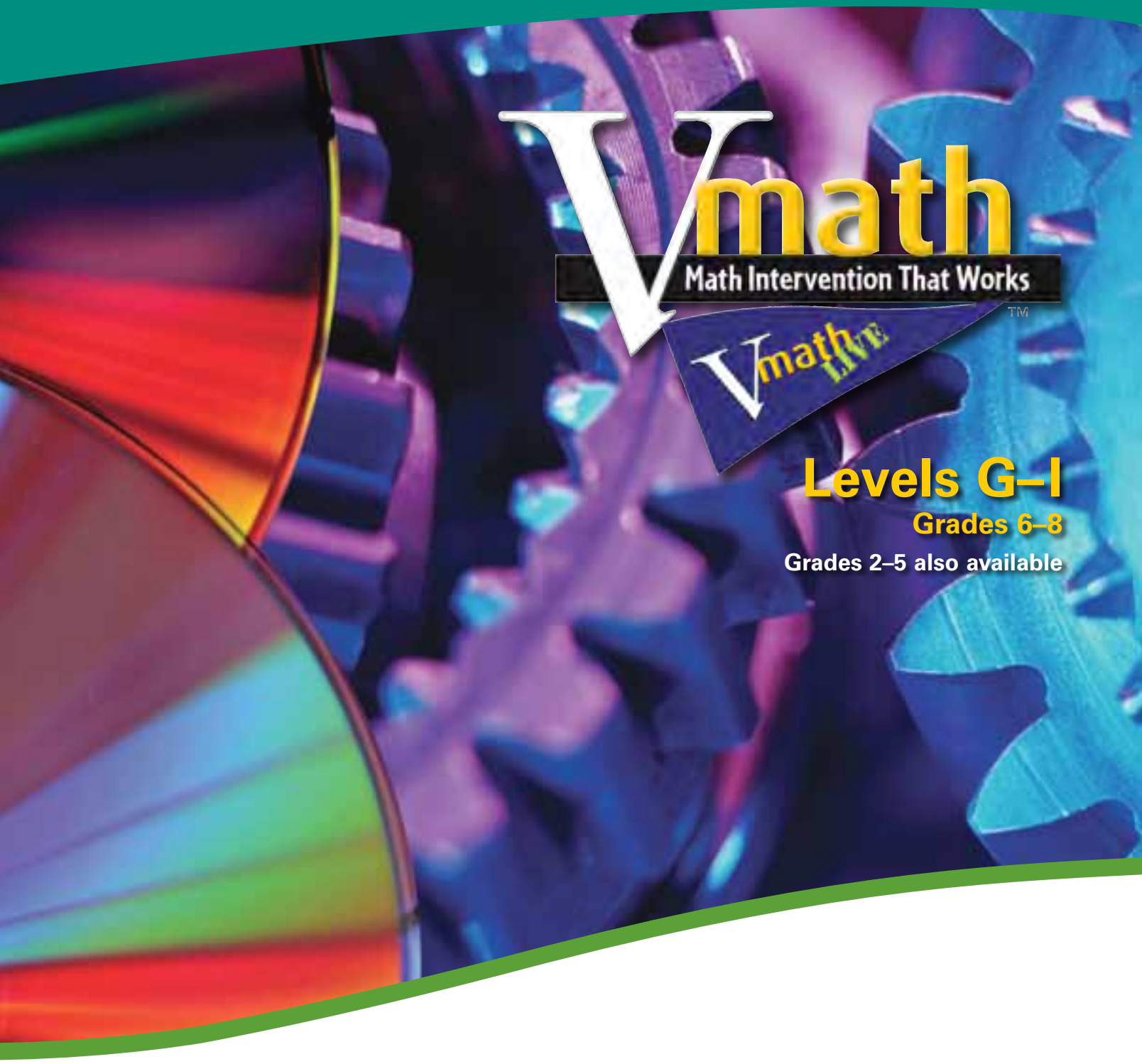
Vmath® Overview

Strategic Intervention



Levels G–I

Grades 6–8

Grades 2–5 also available

Cambium LEARNING® Group | Voyager

Voyager—Your RtI Partner

Dedicated to the success of every student, Voyager provides strategic and intensive interventions designed to accelerate skill acquisition for all struggling students.

Voyager is a member of Cambium Learning® Group, the leading educational company focused primarily on serving the needs of at-risk and special student populations. The company operates three core divisions: Voyager, which provides comprehensive interventions; Sopris, which is known for supplemental and behavioral interventions; and Cambium Learning Technologies (CLT) which includes IntelliTools®, Kurzweil Educational Systems®, Learning A–Z, and ExploreLearning.

Partnering for RtI Solutions

As your intervention partner, we will work with you to develop a customized and integrated solution to meet your Response to Intervention (RtI) needs.

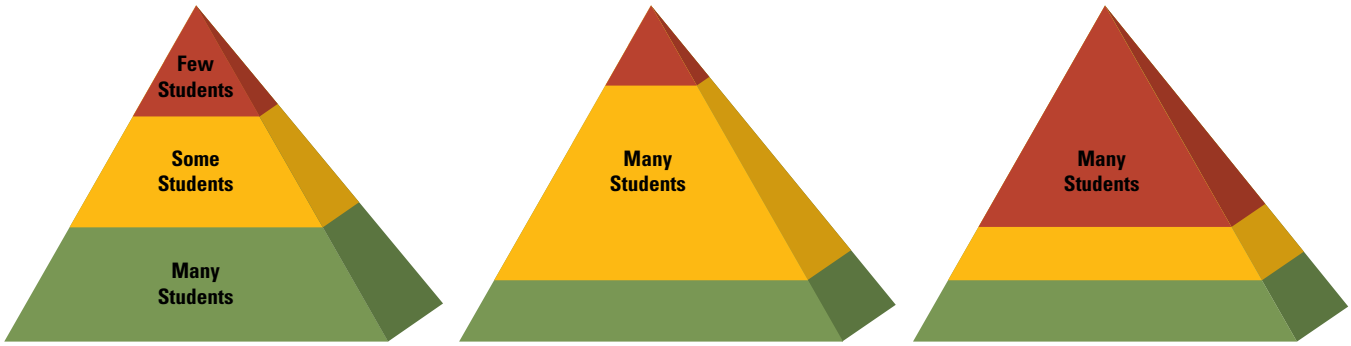
- Our math and literacy interventions are research based and research validated
- Experienced consultants and practitioners will work with you to develop a customized intervention plan to meet your unique system wide needs and goals
- Our unparalleled implementation support team will provide onsite and online staff development to ensure fidelity of implementation

Voyager's powerful and effective interventions, support services and educational technology help accelerate all struggling students to grade-level proficiency. English language learners (ELLs) and students with disabilities derive particular benefits from the interventions and make dramatic gains.

Effective RtI Key Features and Benefits

There is no one-size-fits-all solution for struggling learners. Each system and student has specific needs—some only require occasional additional instruction, while others require more comprehensive, long-term support.

What Does Your School Look Like?



Our interventions and support services are designed to meet the needs of struggling students by providing multitiered instructional interventions aligned to content standards and benchmarks, including the Common Standards.

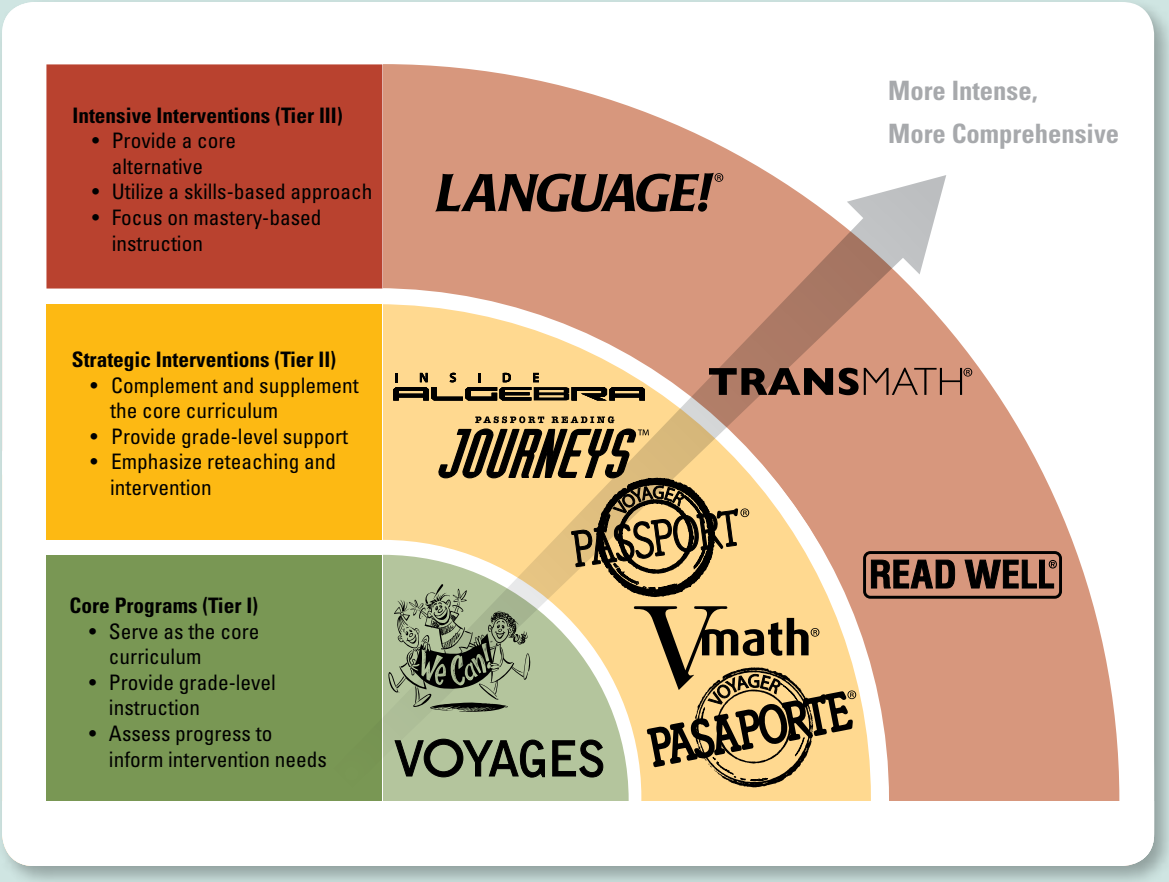
Key Features of Voyager Interventions	Benefit to Your School/District
Multitiered, systematic, scalable approach with supports and tools for differentiated instruction	**Implement an effective and comprehensive intervention plan**
Universal screening and embedded progress monitoring along with a comprehensive Web-based data-management system	**Easily access RtI documentation online and regularly assess and monitor every student's progress**
Research-based and -validated	**Deploy interventions that are proven to work and turn around low-performing schools**
Onsite and online professional development to increase fidelity of implementation	**Build teacher capacity to ensure interventions are implemented as intended and increase struggling students' academic achievement**

Voyager—Your RtI Partner

A Continuum of Interventions for Your Struggling Students

Voyager's interventions are intended to identify struggling students early—before they fall behind—and provide the support they need to be successful.

We provide a continuum of academic interventions in reading and math designed to address the needs of your struggling learners—from strategic interventions that support core curricula to intensive interventions for students who need a completely different approach.



	Literacy	Grades	Description
Intensive	READ WELL	K–2	Mastery-based, intensive reading and language arts intervention with focus on decoding, comprehension, spelling, and writing • 60–90 minutes daily
Strategic	VOYAGER PASSPORT	K–5	Small group reading intervention designed to wrap around core curriculum and accelerate students to grade level using a blended approach of teacher-led instruction and online interactive practice • 30–45 minutes daily
Strategic	VOYAGER PASAPORTE	K–3	Small group Spanish reading intervention designed to build a strong foundation in students' native language to help strengthen their transition to English • 30 or 40 minutes daily
Intensive	LANGUAGE!	3–12	Mastery-based, intensive reading and language arts intervention that targets the needs of nonreaders, struggling readers, and English learners • 90 or 120 minutes daily
Strategic	JOURNEYS	6–9	High-interest reading intervention for middle and high school students that builds academic vocabulary, comprehension, and fluency through motivating topics, teacher-led instruction, and student-centered technology • 50 minutes daily

	Mathematics	Grades	Description
Intensive	TRANSMATH	5–10	Mastery-based, intensive intervention that focuses on the foundational concepts and problem-solving strategies needed for successful entry into algebra • 50–60 minutes daily
Strategic	Vmath	2–8	Strategic intervention with a modular approach for targeted skill intervention to reach grade-level expectations • 40–45 minutes daily
Strategic	INSIDE ALGEBRA	8–12	Mastery-based, strategic intervention that provides additional strategies for algebra success • 50–60 minutes daily
Core	VOYAGES	K–5	Flexible elementary curriculum, organized by grade-level content and broken into two components: *Anchors* and *Excursions* • 50–60 minutes daily

What Is *Vmath?*

Vmath® is a results-driven, research-based math intervention program for struggling students in grades 2–8. It provides a blended solution of teacher-led instruction and student-centered technology to address the different needs of students. With *Vmath*, students experience improved results and develop the foundational knowledge they need for future success in algebra.

Increase Math Proficiencies With *Vmath*

The program:

- Delivers an **additional focus** on essential concepts and skills
- Provides the **error-analysis** tools teachers need to identify students' misconceptions and provide targeted, **corrective feedback**
- **Differentiates instruction** with hands-on activities, computer simulations, and multiple formats for practice
- **Increases students' time on task** with *VmathLive*®

Endorsed by:

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case
COUNCIL OF ADMINISTRATORS OF SPECIAL EDUCATION

Blended Learning Solution of Print and Technology

Teacher-Led Instruction

- Research-based intervention
- Aligns to key findings of the National Math Panel
- Balanced instructional approach
- Data-driven

Student–Centered, Integrated Technology

- Web-based with 24/7 access
- Reinforces instruction with more time on task
- *VmathLive* online technology
- ExploreLearning *Gizmos*™ activities

Implementation Partnership

- On-site and online training
- Consultative resources and support
- Ongoing professional development

The National Mathematics Advisory Panel says ...

> A balanced approach is recommended for concept development, computational fluency, and problem solving.

To best prepare students for algebra, a curriculum should employ a balanced approach focused on conceptual understanding, developing fluency in procedures and number combinations, and building strong problem-solving skills.

Vmath

Math Intervention That Works

Vmath LIVE™

At-A-Glance

Who Is *Vmath* for?

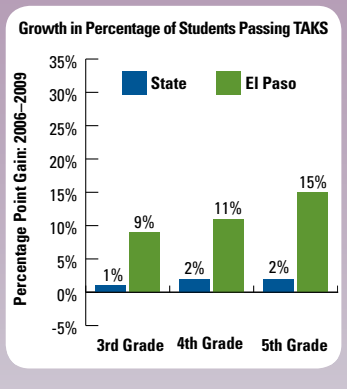
Vmath is for struggling students who:

- Lack foundational understanding of grade-level math concepts, skills, and problem-solving strategies
- Score below grade level on state assessments and would benefit from additional instruction

Page 10

Research and Effectiveness

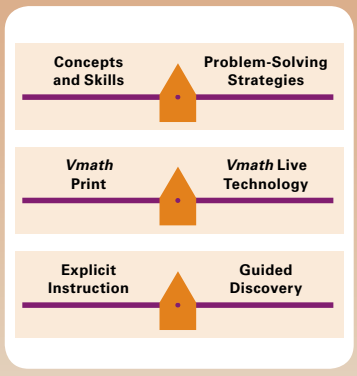
Vmath was founded on the scientific evidence of best practices when working with struggling students. Research shows that struggling students consistently demonstrate improvement in math when an explicit instructional approach is followed. In schools and districts across the country, students in *Vmath* demonstrated substantial math gains after instruction.



Pages 11–13

How It Works

The balanced, systematic approach of *Vmath* ensures that students learn the essential skills and strategies to build a strong foundation in mathematics so they are better positioned for higher-level matehmatics and algebra in the future.



Pages 14–19

Cambium Learning Group is the leading educational company focused primarily on at-risk and special student populations.

Modular Intervention

Vmath provides the flexibility teachers need to effectively reach struggling students. Each level of the program includes eight modules that focus on specific math topics.

Pages 20–23

Vmath in the Classroom

Vmath uses explicit instruction that includes teacher modeling, opportunities for students to think about and talk through the steps, purposeful practice, and extensive feedback opportunities from teachers. Each *Vmath* lesson follows a clear and consistent four-step process for students and teachers.

Pages 24–29

Professional Development

At Cambium Learning Group, we understand that intervention solutions don't come from programs alone. Voyager's professional development partnership provides ongoing training and implementation support to maximize the effectiveness of instruction.

Pages 30–31

Materials

Vmath materials include everything needed to implement the program in the classroom.

Pages 32–33

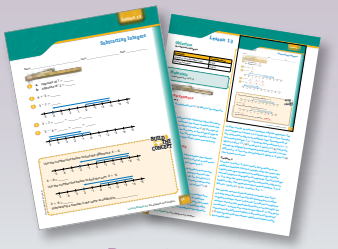
Program Scope and Sequence

The Program Scope and Sequence pinpoints where math concepts and skills are taught, introduced, or reinforced in each of the *Vmath* modules.

Pages 34–36

Sample Pages

Review a sample lesson from both the teacher and student books.



Page 37–47

Who Is *Vmath* for?

Vmath is for struggling students who:

- Lack foundational understanding of grade-level math concepts, skills, and problem-solving strategies
- Score below grade level on state assessments and would benefit from additional instruction

Unlike traditional programs that merely teach students what they should have already learned, *Vmath* divides instruction into smaller, more manageable objectives, allowing students the time they need to **build a solid math foundation**, **gain greater confidence** in their abilities, and **improve their results**.

Vmath accelerates struggling students to grade level using a results-driven, balanced approach of concept development, computational fluency, and problem solving delivered in a modular format.

> *Vmath* is a great instructional program that provides students with basic learning tools in a building, sequential order to be successful in math, especially when they have to meet state standardized test expectations. I truly believe in the program. In fact, I have all my students doing it, not just as an intervention program.

—Sergio Baca, Bilingual Teacher
El Paso ISD, Texas

Research and Effectiveness

Vmath was founded on the scientific evidence of best practices when working with struggling students. Research shows that struggling students consistently demonstrate improvement in math when an explicit instructional approach is followed. Through explicit instruction designed to meet the needs of struggling students, *Vmath* accelerates student performance in math.

Voyager acknowledges the research contributions of the *Vmath* Advisory Board:

Michael Hynes, University of Central Florida
Vmath Senior Advisor

David Rock, Columbus State University
Vmath Advisor

Terri Belcher, University of California–Berkeley
Vmath Advisor

Russell Gersten, University of Oregon
Vmath Advisor, Research and Evaluation

Mark Shinn, National Louis University
Vmath Advisor, Assessment

> Explicit instruction involving both teacher modeling and kids going through the steps in the models—in small groups or with the whole class or individually—seems to lead consistently to higher gains for kids that are struggling.

—Russell Gersten
Professor Emeritus
College of Education, University of Oregon;
Chairman of the Instructional Practices Committee
National Math Panel

Research and Effectiveness

To view more results, go to
www.voyagerlearning.com/results

National Results: Impact of *Vmath* on Student Math Proficiency

In a nationwide study of more than 15,000 students, each grade level increased its overall mathematics content knowledge and rapidly accelerated mastery of essential math skills and strategies. The program effect size* ranged from 0.96 to 1.49, which is considered educationally meaningful and generally quite large.

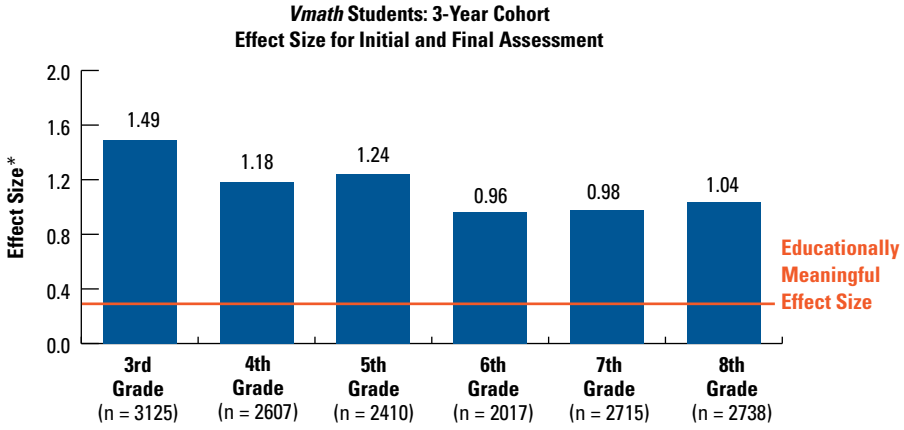
Key Details

Total Participants: 15,612

Grade Levels: 3–8

Instructional Period: 3-year cohort—2007–2009

Measures: Initial and Final Assessment



Texas Statewide: Impact of *Vmath* on State Assessment Performance

Vmath had a powerful impact on the mathematics performance of Texas students. The program effect size ranged from 1.21 to 1.51, which is considered educationally meaningful, and generally quite large.

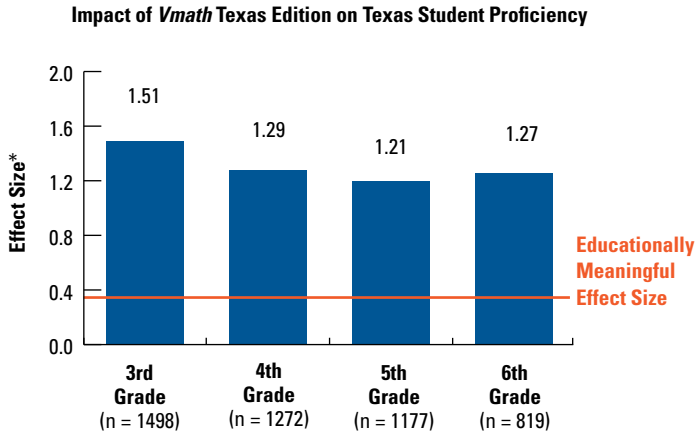
Key Details

Total Participants: 4,760

Grade Levels: 3–6

Instructional Period: 2007–2008

Measures: Initial and Final Assessment



*Effect sizes were calculated using the Initial and Final Assessment scores. Effect sizes (for differences expressed as means) of 0.2 are considered small, 0.5 are moderate, and 0.8 are large (Cohen, 1988). An effect size of 0.3 is considered to be educationally meaningful.

Bethel, OR: Impact of *Vmath* on State Assessment Performance

Results indicate that Cascade Middle School has made steady, sustained progress with students receiving math intervention instruction using *Vmath*. School officials attribute the performance gains to the *Vmath* curriculum and teachers' increasing familiarity with the program.

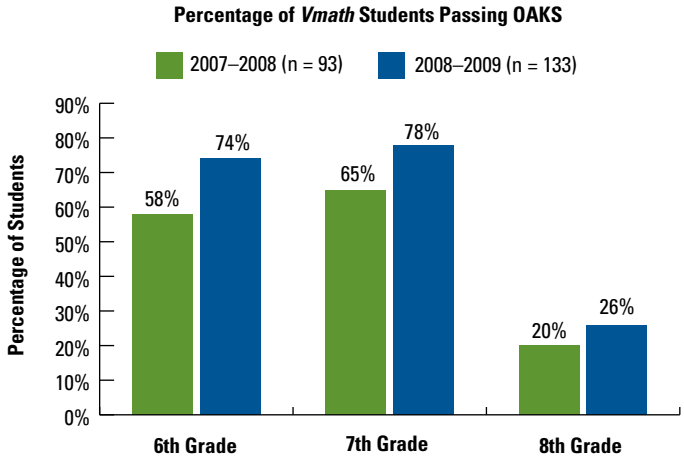
Key Details

Total Participants: 206

Grade Levels: 6–8

Instructional Period: 2008–2009

Measure: Oregon Assessment of Knowledge and Skills (OAKS)



El Paso ISD, TX: Performance Gain on TAKS Math Section

El Paso ISD began implementing *Vmath* in the 2005–2006 school year to boost student achievement in math. After four years with the Voyager program, each grade made substantial gains and has outperformed the state as measured by the percentage of students passing the state assessment (TAKS) between 2006 and 2009.

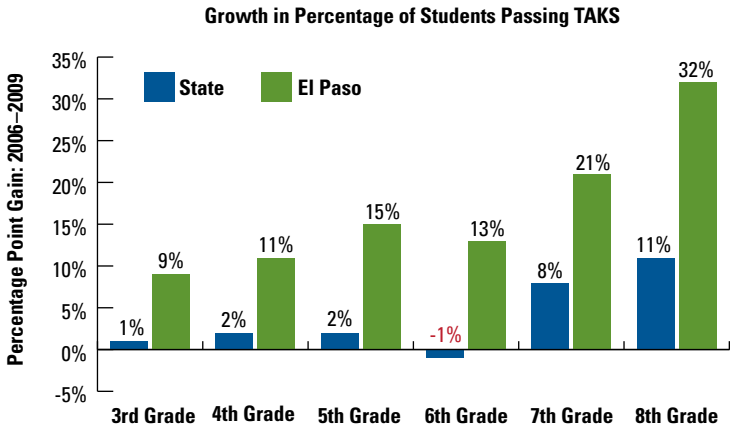
Key Details

Total Participants: Varied by grade level

Grade Levels: 3–8

Instructional Period: 2005–2009

Measure: Texas Assessment of Knowledge and Skills (TAKS)



How It Works

The balanced, systematic approach of *Vmath* ensures that students learn the essential skills and strategies to build a strong foundation in mathematics.

Teacher-Led Instruction

Research shows that **explicit instruction** positively affects math achievement for students who have difficulty learning math. Based on scientific research, *Vmath* uses an explicit instructional approach that includes:

- Systematic, scaffolded instruction with teacher modeling
- Lessons that target essential concepts, skills, and problem-solving strategies
- Time to think and talk through problems
- Purposeful practice

Powerful Assessments

The assessments embedded in *Vmath* utilize Quantiles and have been carefully designed to **evaluate all aspects of student learning** and **inform instruction**. The data gathered from these assessments fit together like pieces of a puzzle to help teachers:

- Tailor intervention to meet student needs
- Deliver appropriate preteaching and reteaching strategies
- Match instruction to a student's optimal learning range
- Continually monitor student progress

Student–Centered, Integrated Technology

Students can access *VmathLive*, the online component of *Vmath*, anytime, anywhere. **Effort** increases as students spend more time on task **practicing** the math they are learning, **preparing** for high-stakes assessments, and **playing** in real-time math competitions with students around the world.

See page 17 to learn more about *VmathLive*.

Effective Data Management System

The key to effective instruction is **real-time data** that **track student progress** throughout the year. VPORT,® Voyager's online data management system, collects and reports student assessment data, enabling district leaders and teachers to make effective instructional decisions.

Administrators can:

- Track progress toward reaching district goals
- Identify district and school professional development needs
- Evaluate the effectiveness of the intervention

Teachers can:

- Identify student needs
- Differentiate instruction
- Monitor student progress
- Evaluate student learning

See page 18 to learn more about VPORT.

How It Works

A Blended Solution of Teacher-Led Instruction and Student-Centered Technology

Teacher-Led Instruction

Vmath instructional materials are explicit and easy to implement. Each lesson follows a clear four-step instructional sequence that includes teacher modeling, students and teachers talking through the steps they take and decisions they make, purposeful practice, and targeted feedback.

Vmath instruction focuses first on foundational skills and concepts and then quickly moves students forward to grade-level learning. Using a balanced, systematic approach to instruction, *Vmath* blends print and technology to differentiate instruction for every type of learner.

Concepts and Skills ◆ **Problem-Solving Strategies**	Instruction balances conceptual development, computational fluency, and problem solving.
***Vmath* Print** ◆ ***Vmath* Live Technology**	Blended instructional model combines teacher-led instruction with print materials, and individualized instruction and practice with technology.
Explicit Instruction ◆ **Guided Discovery**	Explicit instruction, effective for struggling students, is balanced with procedural fluency and conceptual development.

The National Math Panel says ...

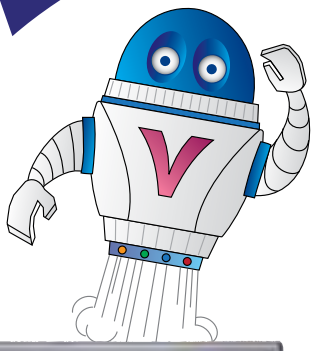
"Effort matters."

A student's success in math is not solely dependent on his or her ability. In fact, recent studies show that when students shift their focus from ability to effort, engagement increases, ultimately improving achievement in math.

Student-Centered Technology

VmathLive is the fun and motivating student-centered technology component that addresses the different learning modalities of struggling students through interactive practice that engages and motivates them to learn more. Through 24/7 access *VmathLive* extends the school day and increases math time on task. The Web-based program:

- Engages students in **real-time math competitions** with other students from around the world
- Builds **conceptual understanding** through animated visualizations
- Develops **academic math vocabulary** presented in both English and Spanish
- Offers immediate, **targeted error analysis** as students prepare for high-stakes assessments



For more information about *VmathLive* visit www.vmathlive.com

BEST IN TECH >>2008 SCHOOL ADMINISTRATOR

2009 SIIA CODiE FINALIST

The topics and content in *VmathLive* directly align to *Vmath*.

VmathLive Level E Modules	*Vmath* Level E Modules
Decimals	1. Decimals
Number Theory and Fractions	2. Number Theory and Fractions
Adding and Subtracting Fractions	3. Adding and Subtracting Fractions
Multiplying and Dividing Fractions	4. Multiplying and Dividing Fractions
Ratio, Proportion, and Percent	5. Ratio, Proportion, and Percent
Geometry and Measurement	6. Geometry and Measurement
Data, Probability, and Statistics	7. Data, Probability, and Statistics
Pre-Algebra and Integers	8. Pre-Algebra and Integers

How It Works

Track and Manage Student Performance

Teachers and administrators can instantly access reports and tools to measure and track student progress with the VPORT data management system. The secure, Web-based system houses data from *Vmath*'s embedded assessments. Teachers can input and view their students' data; coaches and principals can view all building-level data; and district administrators can view district, school, and even student data.

Educators use VPORT to:

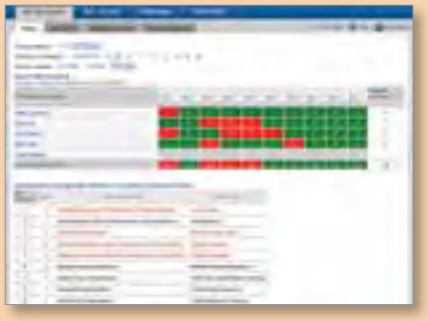
- Identify individual instructional needs and goals
- Adjust instruction based on skill need
- Monitor progress against goals
- Communicate progress to the instructional team
- Generate parent reports in English and Spanish

VPORT Reports

Vmath assessments are processed online, and the real-time data allow educators instant access to multiple reports showing student performance at various levels.

Individual Student Reports show student progress and rate of improvement, and how those results compare with the upcoming benchmark goal.

School Reports provide a comprehensive snapshot of progress across all classrooms and grade levels.



Class Reports show students' individual results and provide reteaching recommendations.

Powerful Assessment Informs Instruction

The *Vmath* assessments fit together like pieces of a puzzle as they help teachers identify student needs, differentiate instruction to accelerate learning, and monitor progress to ensure mastery. The diagnostic and prescriptive assessments evaluate all aspects of student learning in math.

Initial/Final Assessments
Identify student weaknesses and determine (along with district criteria) appropriate entry points into instruction; measure student growth and mastery throughout the course.

Progress Assessments
Identify a student's optimal learning range and monitor progress toward grade-level goals. The difficulty of each *Vmath* lesson is measured against the developmental scale of the Quantile Framework.

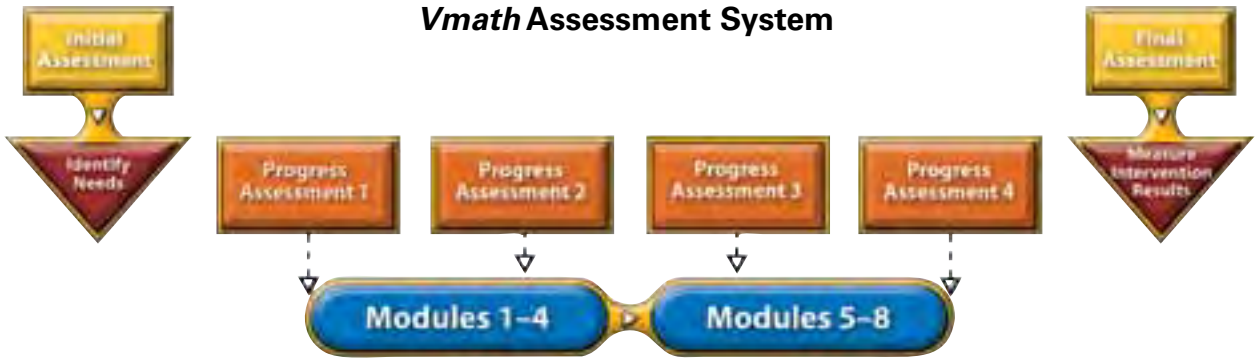
Module Pre- and Posttests
Taken online or by paper and pencil. When students take the module pre- and posttests online, teachers can easily create reports that identify student needs and recommend preteaching and reteaching activities.

Computational Fluency Benchmarks
Given three times a year and require written student responses. Results indicate student progress over the entire grade level, and the measures are predictors of future success.

Assessment Design

The *Vmath* assessment system evaluates student learning and monitors progress throughout the intervention.

***Vmath* Assessment System**



Modular Intervention

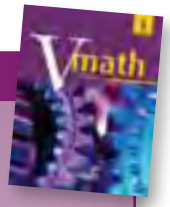
Vmath is a modular intervention system that provides the flexibility teachers need to effectively reach struggling students. Each level of the program includes eight modules that focus on specific math topics.

Level G

1. Decimals
2. Number Theory and Fractions
3. Adding and Subtracting Fractions
4. Multiplying and Dividing Fractions
5. Ratio, Proportion, and Percent
6. Geometry and Measurement
7. Data, Probability, and Statistics
8. Pre-Algebra and Integers

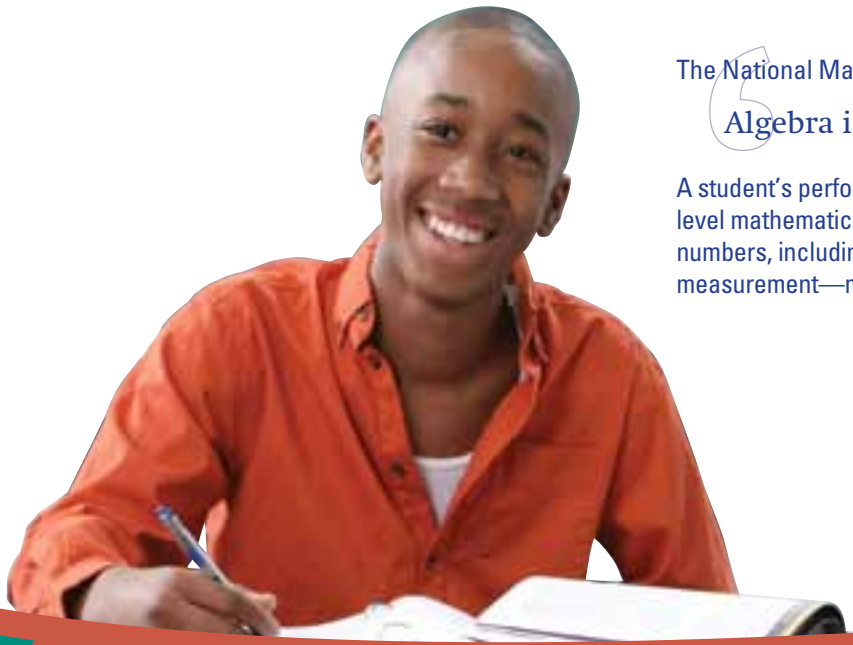
Level H

1. Decimals
2. Integers and Number Theory
3. Operations with Fractions
4. Pre-Algebra
5. Geometry and Measurement
6. Measurement
7. Ratio, Proportion, and Percent
8. Data, Probability, and Statistics



Level I

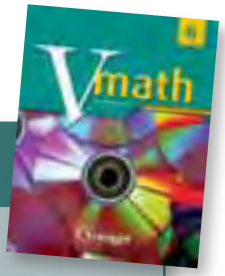
1. Integers and Exponents
2. Rational Numbers
3. Ratio, Proportion, and Percent
4. Expressions and Equations
5. Geometry
6. Measurement
7. Data, Probability, and Statistics
8. Coordinate Geometry and Inequalities



The National Math Panel says ...

> Algebra is the gateway to success in math.

A student's performance in algebra strongly correlates to success in upper-level mathematics. The critical foundations of algebra—understanding whole numbers, including fractions and decimals, and aspects of geometry and measurement—must be taught early to maximize student success.



Vmath Level G–Modules and Lessons

Module 1: Decimals

Using a Problem-Solving Plan • Reading and Writing Decimals • Hands-On Lesson: Comparing Decimals Using Decimal Grid Paper • Comparing and Ordering Decimals • Rounding Decimals • Gizmos Lesson: Sums and Differences with Decimals • Adding Decimals • Hands-On Lesson: Subtracting Decimals Using Decimal Grid Paper • Subtracting Decimals • Estimating Decimal Sums and Differences • Multiplying Decimals • Hands-On Lesson: Modeling Decimal Division Using Decimal Grid Paper • Dividing Decimals by Whole Numbers • Dividing Decimals by Decimals • Estimating Decimal Products and Quotients • Reasonableness and Estimation

Module 2: Number Theory and Fractions

Factors • Greatest Common Factor • Exponents • Prime and Composite Numbers • Prime Factorization Using Exponents • Multiples • Hands-On Lesson: Finding the Least Common Multiple Using Number Lines • Least Common Multiple • Problem-Solving: Finding Patterns • Understanding Fractions • Equivalent Fractions • Simplest Form of a Fraction • Comparing and Ordering Fractions

Module 3: Adding and Subtracting Fractions

Changing Mixed Numbers to Improper Fractions • Gizmos Lesson: Improper Fractions and Mixed Numbers • Changing Improper Fractions to Mixed Numbers • Adding Fractions With Like Denominators • Hands-On Lesson: Adding Fractions With Unlike Denominators Using Fraction Strips • Adding Fractions With Unlike Denominators • Adding Mixed Numbers • Adding Mixed Numbers and Estimating • Hands-On Lesson: Subtracting Fractions With Unlike Denominators Using Fraction Strips • Subtracting Fractions • Subtracting Mixed Numbers • Subtracting Mixed Numbers and Estimating • Problem-Solving: Working Backward • Choosing an Operation • Subtracting Mixed Numbers With Renaming

Module 4: Multiplying and Dividing Fractions

Multiplying Fractions by Whole Numbers • Hands-On Lesson: Modeling Fraction Multiplication Using Paper-Folding • Multiplying Fractions by Fractions • Multiplying Fractions and Mixed Numbers • Multiplying Mixed Numbers • Estimating Fraction and Mixed Number Products • Problem-Solving: Solving a Simpler Problem • Using Reciprocals • Dividing Fractions • Operations With Fractions

Module 5: Ratio, Proportion, and Percent

Ratios • Rates • Hands-On Lesson: Understanding Proportions Using Counters • Solving Proportions • Solving Proportion Word Problems • Modeling Percents, Decimals, and Fractions • Gizmos Lesson: Percents, Fractions, and Decimals • Writing Percents as Decimals and Fractions • Writing Decimals as Percents and Fractions • Writing Fractions as Decimals and Percents • Problem Solving: Using a Table • Finding the Percent of a Whole Number

Module 6: Geometry and Measurement

Points and Lines • Angles • Triangles • Quadrilaterals • Properties of Triangles and Quadrilaterals • Hands-On Lesson: Understanding Congruent and Similar Figures Using Grid Paper • Congruent and Similar Figures • Transformations • Problem-Solving: Finding Patterns • Perimeter • Area • Gizmos Lesson: Perimeter, Circumference, and Area–Activity A • Circumference and Area of Circles • Three-Dimensional Figures • Surface Area • Hands-On Lesson: Finding Volume Using Three-Dimensional Shapes and Unit Cubes • Volume • Conversion of Customary Units • Conversion of Metric Units • Estimating Length, Weight, and Capacity

Module 7: Data, Probability, and Statistics

Bar Graphs • Line Graphs • Circle Graphs • Stem-and-Leaf Plots • Comparing Graphical Representations • Gizmos Lesson: Mean, Median, and Mode • Measures of Central Tendency • Counting Possible Outcomes • Hands-On Lesson: Understanding Probability Using Cereal • Simple Probability and Sample Spaces • Probabilities of Complements • Problem Solving: Using a Table

Module 8: Pre-Algebra and Integers

Order of Operations • Gizmos Lesson: Using Algebraic Expressions • Variables and Expressions • Evaluating Expressions • Hands-On Lesson: Representing Equations Using a Scale • Solving Addition and Subtraction Equations • Solving Multiplication and Division Equations • Formulate an Equation From a Word Problem • Input-Output Tables • Understanding Integers • Gizmos Lesson: Real Number Line–Activity A • Integers on a Number Line • Coordinate Planes • Comparing and Ordering Integers • Hands-On Lesson: Adding Integers Using Counters • Adding Integers • Subtracting Integers • Problem Solving: Using Patterns

Modular Intervention

Vmath Level H–Modules and Lessons

Module 1: Decimals
Using a Problem-Solving Plan • Modeling Decimals • Comparing and Ordering Decimals • Adding Decimals • Subtracting Decimals • Hands-On Lesson: Multiplying Decimals Using Decimal Grid Paper • Multiplying Decimals • Dividing Decimals by Whole Numbers • Dividing Decimals by Decimals • Rounding and Estimating Decimals • Gizmos Lesson: Order of Operations • Order of Operations • Reasonableness and Estimation

Module 2: Integers and Number Theory
Understanding Integers • Comparing and Ordering Integers • Absolute Value • Adding Integers • Hands-On Lesson: Subtracting Integers Using Counters • Gizmos Lesson: Adding and Subtracting Integers • Subtracting Integers • Multiplying Integers • Dividing Integers • Problem-Solving: Using a Table • Factors and Multiples • Greatest Common Factor • Least Common Multiple • Square Roots • Hands-On Lesson: Estimating Square Roots Using Grid Paper • Estimating Square Roots

Module 3: Operations with Fractions
Equivalent Fractions and Simplest Form of a Fraction • Comparing and Ordering Fractions • Changing Improper Fractions and Mixed Numbers • Adding Fractions • Subtracting Fractions • Estimating Fraction Sums and Differences • Hands-On Lesson: Adding Mixed Numbers Using Fraction Strips • Adding Mixed Numbers • Subtracting Mixed Numbers • Problem-Solving: Solving a Simpler Problem • Gizmos Lesson: Multiplying Fractions • Multiplying Fractions by Fractions and Whole Numbers • Hands-On Lesson: Multiplying Mixed Numbers and Whole Numbers Using Fraction Strips • Multiplying Fractions and Mixed Numbers • Estimating Mixed Number and Whole Number Products • Reciprocals • Dividing Fractions and Whole Numbers by Fractions • Dividing Fractions and Mixed Numbers

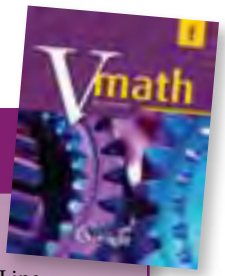
Module 4: Pre-Algebra
Exponents • Variables and Expressions • Evaluating Expressions Using Order of Operations • Hands-On Lesson: Solving Equations Using Algebra Tiles • Solving One-Step Equations Using Addition and Subtraction • Solving One-Step Equations Using Multiplication and Division • Solving Two-Step Equations • Input-Output Tables • Function Rules • Problem-Solving: Using a Table • Gizmos Lesson: Points in the Coordinate Plane–Activity A • Coordinate Plane • Graphing Linear Equations • Finding Slope • Gizmos Lesson: Finding Factors with Area Models • Prime Factorization • Powers of Ten and Scientific Notation • Inequalities

Module 5: Geometry and Measurement
Terms and Figures in Geometry • Angles • Gizmos Lesson: Classifying Quadrilaterals–Activity A • Polygons and Quadrilaterals • Hands-On Lesson: Finding Sums of Angles in Triangles • Triangles • Problem-Solving: Finding Patterns • Congruent Figures • Hands-On Lesson: Determining Rotational Symmetry of a Rectangle • Symmetry • Transformations • Three-Dimensional Figures • Front, Side, and Top Views of Three-Dimensional Figures • Conversion of Customary Units • Conversion of Metric Units

Module 6: Measurement
Perimeter • Circumference • Area of Rectangles • Hands-On Lesson: Modeling Area of Rectangles and Triangles Using Grid Paper • Area of Triangles • Gizmos Lesson: Area of Parallelograms–Activity A • Area of Parallelograms • Area of Trapezoids • Area of Circles • Nets of Three-Dimensional Figures • Surface Area of Rectangular Prisms • Volume of Rectangular Prisms • Volume of Triangular Prisms • Volume of Cylinders • Problem-Solving: Using a Table • Pythagorean Theorem • Problem-Solving: Working Backward

Module 7: Ratio, Proportion, and Percent
Ratios • Rates • Solving Proportions • Solving Proportion Word Problems • Proportions and Similar Figures • Scale Drawings and Maps • Hands-On Lesson: Modeling Fraction, Decimals, and Percents Using Decimal Grid Paper • Understanding Percents • Decimals and Percents • Fractions and Percents • Finding the Percent of a Number • Solving Percent Equations • Percent of Change • Gizmos Lesson: Percent of Change • Applications of Percent • Problem-Solving: Solving a Simpler Problem

Module 8: Data, Probability, and Statistics
Bar Graphs and Histograms • Line Graphs • Circle Graphs • Stem-and-Leaf Plots • Gizmos Lesson: Scatter Plots–Activity A • Scatter Plots • Venn Diagrams • Measures of Central Tendency • Comparing Measures of Central Tendency • Counting Possible Outcomes • Hands-On Lesson: Conducting an Experiment Using Number Cubes • Simple Probability and Sample Spaces • Problem-Solving: Using a Table • Box-and-Whisker Plots



Vmath Level I–Modules and Lessons

Module 1: Integers and Exponents

Using a Problem-Solving Plan • Understanding Integers • Hands-On Lesson: Understanding Absolute Value Using a Number Line • Absolute Value • Gizmos Lesson: Real Number Line–Activity B • Comparing and Ordering Integers • Adding Integers • Subtracting Integers • Multiplying Integers • Dividing Integers • Reasonableness and Estimation • Exponents • Order of Operations • Integer Exponents • Properties of Exponents • Scientific Notation

Module 2: Rational Numbers

Understanding Rational Numbers • Comparing and Ordering Rational Numbers • Adding Positive and Negative Decimals • Adding Positive and Negative Fractions • Subtracting Positive and Negative Decimals and Fractions • Multiplying Positive and Negative Decimals • Multiplying and Dividing Positive and Negative Fractions • Dividing Positive and Negative Decimals • Adding and Subtracting Positive and Negative Mixed Numbers • Hands-On Lesson: Multiplying Mixed Numbers Using Fraction Strips • Multiplying and Dividing Positive and Negative Mixed Numbers • Problem-Solving: Finding a Pattern

Module 3: Ratio, Proportion, and Percent

Rates • Solving One-Step Equations Using Addition and Subtraction • Solving One-Step Equations Using Multiplication and Division • Solving Proportions • Hands-On Lesson: Identifying Properties of Similar Figures Using Grid Paper • Proportions and Similar Figures • Understanding Percents • Fractions, Decimals, and Percents • Repeating Decimals • Finding the Percent of a Number • Gizmos Lesson: Percents and Proportions • Solving Percent Equations • Percent of Change • Problem-Solving: Solving a Simpler Problem

Module 4: Expressions and Equations

Evaluating Algebraic Expressions • Writing Algebraic Expressions • Simplifying Algebraic Expressions • Simplifying Algebraic Expressions Using the Distributive Property • Hands-On Lesson: Solving Two-Step Equations Using Algebra Tiles • Solving Two-Step Equations • Solving Multistep Equations • Solving Equations With Variables on Both Sides • Gizmos Lesson: Using Algebra Equations • Writing Equations • Using an Equation • Problem-Solving: Using a Table

Module 5: Geometry

Angle Relationships • Polygons and Quadrilaterals • Properties of Triangles and Quadrilaterals • Congruent Figures • Gizmos Lesson: Proving Triangles Congruent • Congruent Triangles • Three-Dimensional Figures • Front, Side, and Top Views of Geometric Figures • Problem-Solving: Using Patterns • Gizmos Lesson: Square Roots • Square Roots • Hands-On Lesson: Estimating Square Roots Using Grid Paper • Estimating Square Roots • Hands-On Lesson: Representing the Pythagorean Theorem Using Grid Paper • Pythagorean Theorem • Coordinate Plane • Transformations • Rotations • Dilations on a Coordinate Plane

Module 6: Measurement

Conversion of Customary Units • Conversion of Metric Units • Perimeter • Circumference • Area of Rectangles and Compound Figures • Area of Triangles and Compound Figures • Gizmos Lesson: Area of Parallelograms–Activity B • Area of Parallelograms • Area of Trapezoids • Area of Circles • Perimeter, Area, and Proportional Dimension Changes • Hands-On Lesson: Creating Three-Dimensional Figures to Find Surface Area Using Grid Paper • Surface Area • Volume of Prisms and Pyramids • Volume of Cylinders, Spheres, and Cones • Volume and Proportional Dimension Changes • Problem-Solving: Using a Table

Module 7: Data, Probability, and Statistics

Line Plots and Histograms • Double Line Graphs • Circle Graphs • Comparing Graphical Representations • Stem-and-Leaf Plots • Scatter Plots • Gizmos Lesson: Graphing Skills • Misleading Graphs • Measures of Central Tendency • Box-and-Whisker Plots • Venn Diagrams • Counting Possible Outcomes • Probability and Sample Spaces • Problem-Solving: Using a Table • Hands-On Lesson: Conducting an Experiment: Independent and Dependent Events • Independent and Dependent Events • Making Predictions

Module 8: Coordinate Geometry and Inequalities

Functions • Function Rules • Problem-Solving: Finding Patterns • Hands-On Lesson: Graphing a Linear Equation Using Grid Paper • Graphing Linear Equations • Gizmos Lesson: Slope–Activity A • Slope of a Line • Vertical and Horizontal Lines on a Coordinate Plane • Identifying Linear and Nonlinear Functions • Inequalities • Graphing Inequalities on a Number Line • Finding Points on a Coordinate Plane • Solving Addition or Subtraction Inequalities • Gizmos Lesson: Solving Inequalities Using Multiplication and Division • Solving Multiplication Inequalities • Solving Division Inequalities • Solving Two-Step Inequalities • Solving Systems of Linear Equations by Graphing

Vmath In The Classroom

Vmath uses explicit instruction that includes teacher modeling, opportunities for students to think about and talk through the steps, purposeful practice, and extensive feedback opportunities from teachers. Each *Vmath* lesson follows a clear and consistent four-step process for students and teachers.

GET STARTED

The teacher **models the preskills and new concept, skill, or strategy** being targeted that day.

TRY IT TOGETHER

The teacher provides scaffolded instructional support as students and teachers **practice together** and talk through the steps of the concept, skill, or strategy.

WORK ON YOUR OWN

Students independently apply their understanding of the preskills and new learning through **purposeful practice**.

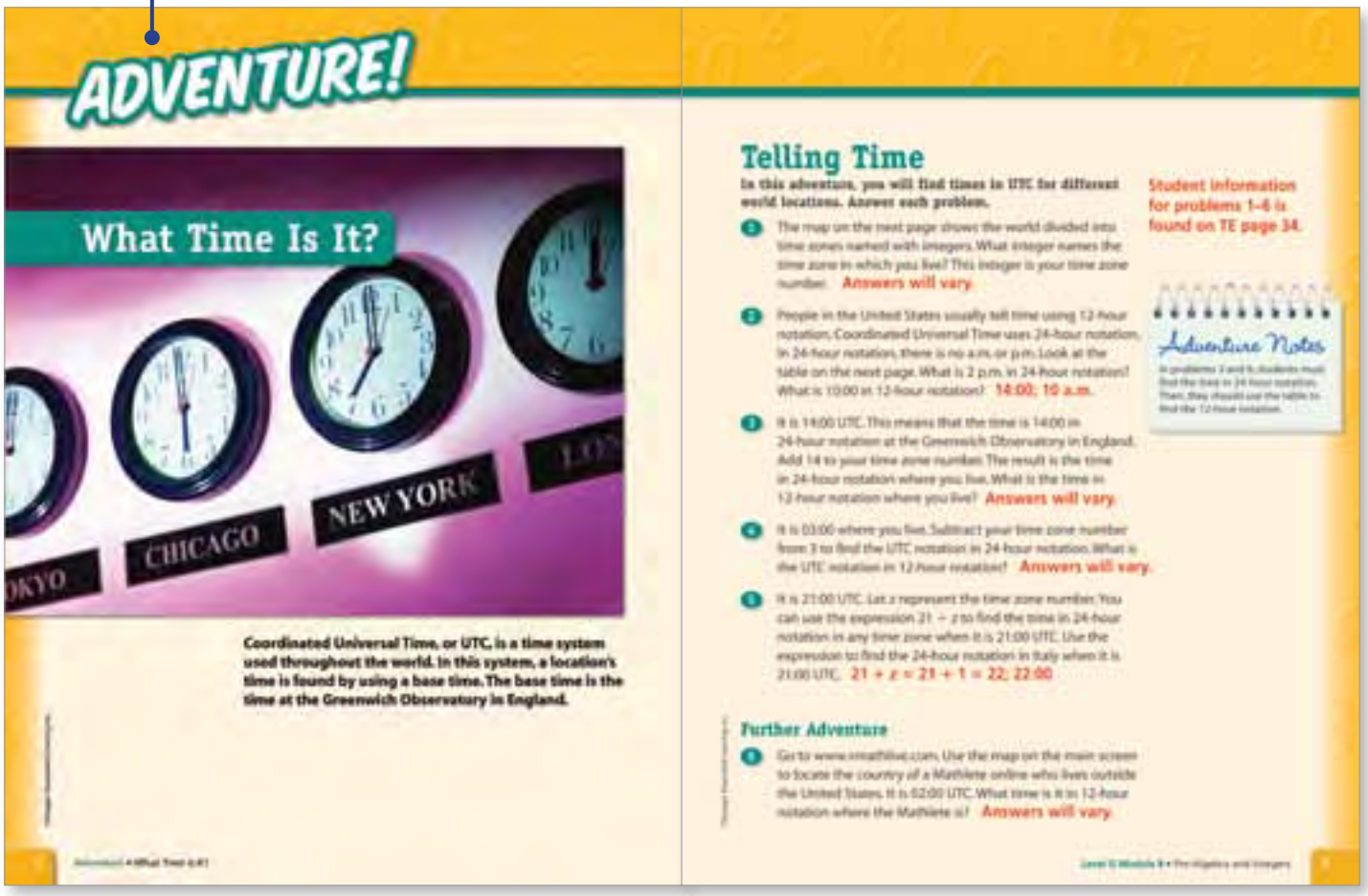
CHECK UP

Daily informal assessments allow teachers to **monitor student understanding** and provide immediate, targeted feedback.

Adventure Opener

Each module begins with an Adventure Opener featuring exciting photographs and real-life math situations that pose relevant questions.

Students think about math in different ways and make **cross-curricular connections.**



ADVENTURE!

What Time Is It?

Coordinated Universal Time, or UTC, is a time system used throughout the world. In this system, a location's time is found by using a base time. The base time is the time at the Greenwich Observatory in England.

Telling Time

In this adventure, you will find times in UTC for different world locations. Answer each problem.

Student information for problems 1–6 is found on TE page 34.

1. The map on the next page shows the world divided into time zones named with integers. What integer names the time zone in which you live? This integer is your time zone number. Answers will vary.
2. People in the United States usually tell time using 12-hour notation. Coordinated Universal Time uses 24-hour notation. In 24-hour notation, there is no a.m. or p.m. Look at the table on the next page. What is 2 p.m. in 24-hour notation? What is 10:00 in 12-hour notation? 14:00; 10 a.m.
3. It is 14:00 UTC. This means that the time is 14:00 in 24-hour notation at the Greenwich Observatory in England. Add 14 to your time zone number. The result is the time in 24-hour notation where you live. What is the time in 12-hour notation where you live? Answers will vary.
4. It is 03:00 where you live. Subtract your time zone number from 3 to find the UTC notation in 24-hour notation. What is the UTC notation in 12-hour notation? Answers will vary.
5. It is 21:00 UTC. Let z represent the time zone number. You can use the expression $21 + z$ to find the time in 24-hour notation in any time zone when it is 21:00 UTC. Use the expression to find the 24-hour notation in Italy when it is 21:00 UTC. $21 + z = 21 + 1 = 22$; 22:00

Adventure Notes

Further Adventure

6. Go to www.mathlive.com. Use the map on the main screen to locate the country of a Mathlete online who lives outside the United States. It is 02:00 UTC. What time is it in 12-hour notation where the Mathlete is? Answers will vary.

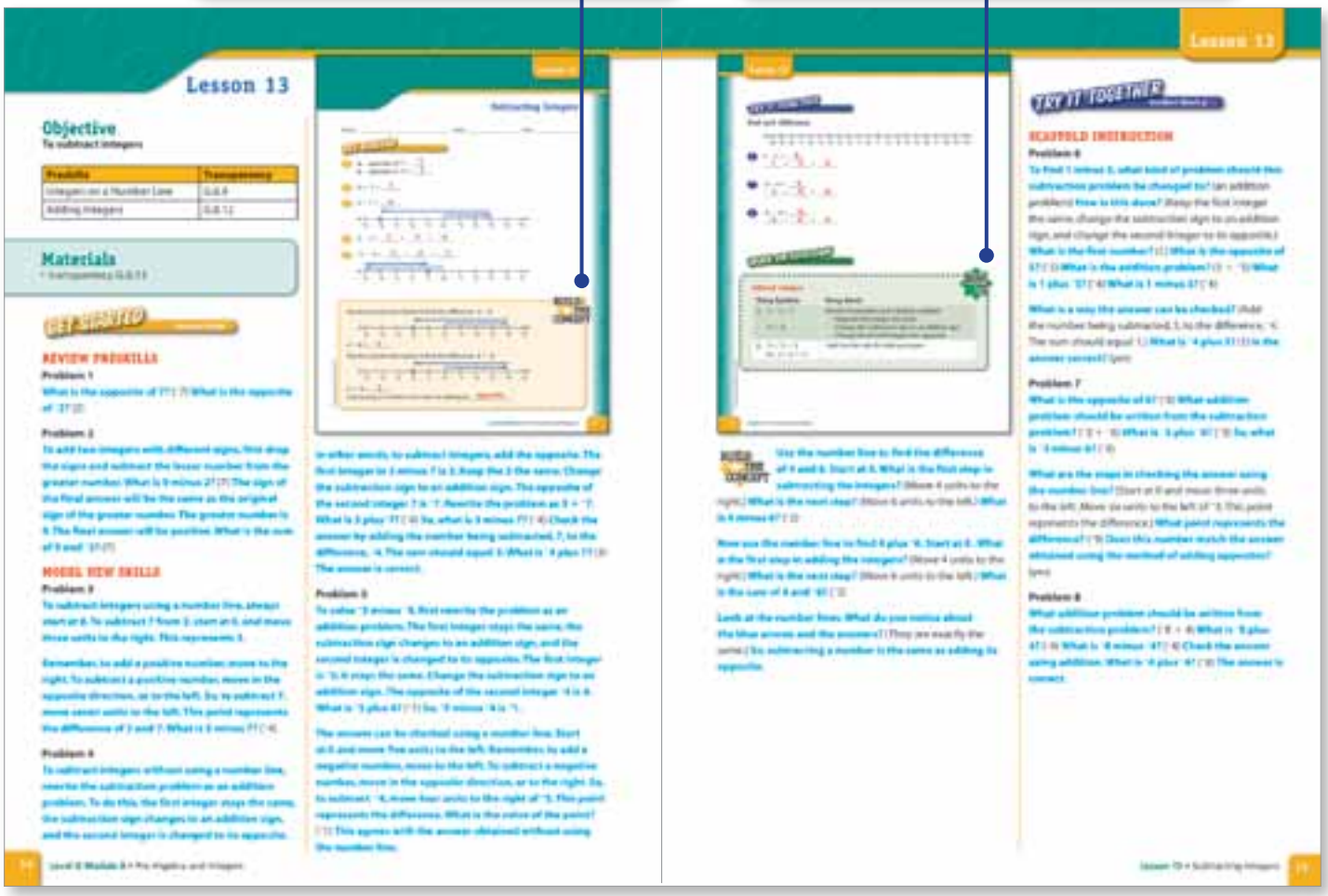
Adventure • What Time Is It?
Level G, Module 8 • Pre-Algebra and Integers

Lesson Overview

Each *Vmath* lesson is written to meet the needs of struggling students and their teachers. Each lesson provides detailed support for teachers of all specialties and experience levels.

Build the Concept uses visual models to help students **develop a deeper understanding** of the targeted mathematical concept.

The **How To box** in each lesson helps students make connections between mathematical symbols and the language of math.



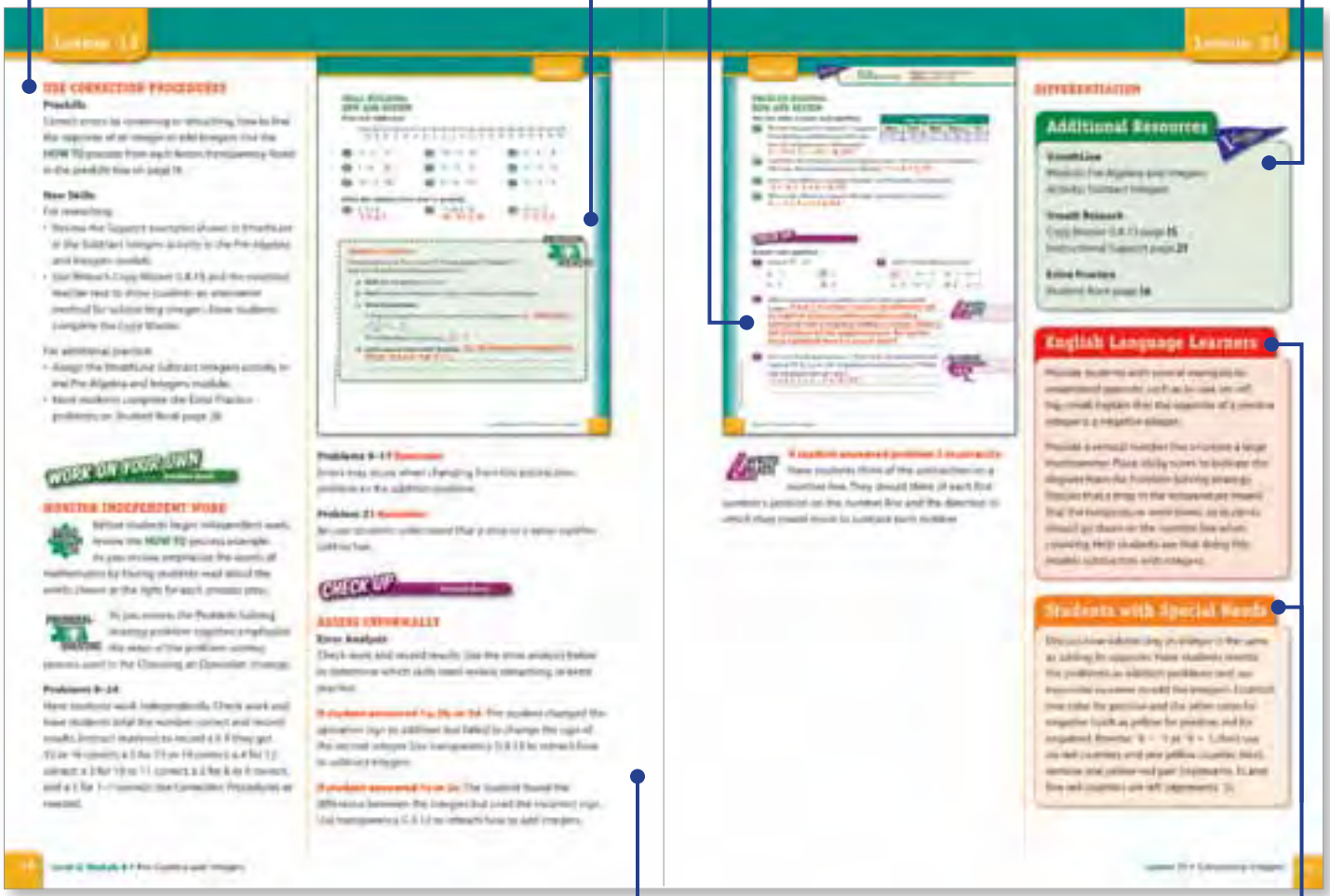
Level G, Module 8 • Pre-Algebra and Integers
Lesson 13 • Subtracting Integers

Lesson-specific **Correction Procedures** provide alternative teaching methods and additional approaches through technology to ensure mastery.

Special emphasis is placed on building **problem-solving skills** as students learn and apply strategies such as working backward, solving a simpler problem, and finding a pattern.

Students **demonstrate their understanding**, **apply mathematical reasoning**, and **justify their thinking** as they explain and write about math.

Additional resources **extend learning** and provide extra practice.



Level G, Module 8 • Pre-Algebra and Integers
Lesson 13 • Subtracting Integers

Targeted, corrective feedback recommendations help teachers **analyze student errors** and correct misconceptions.

Targeted teaching **recommendations for English language learners and students with special needs** are provided in each lesson.

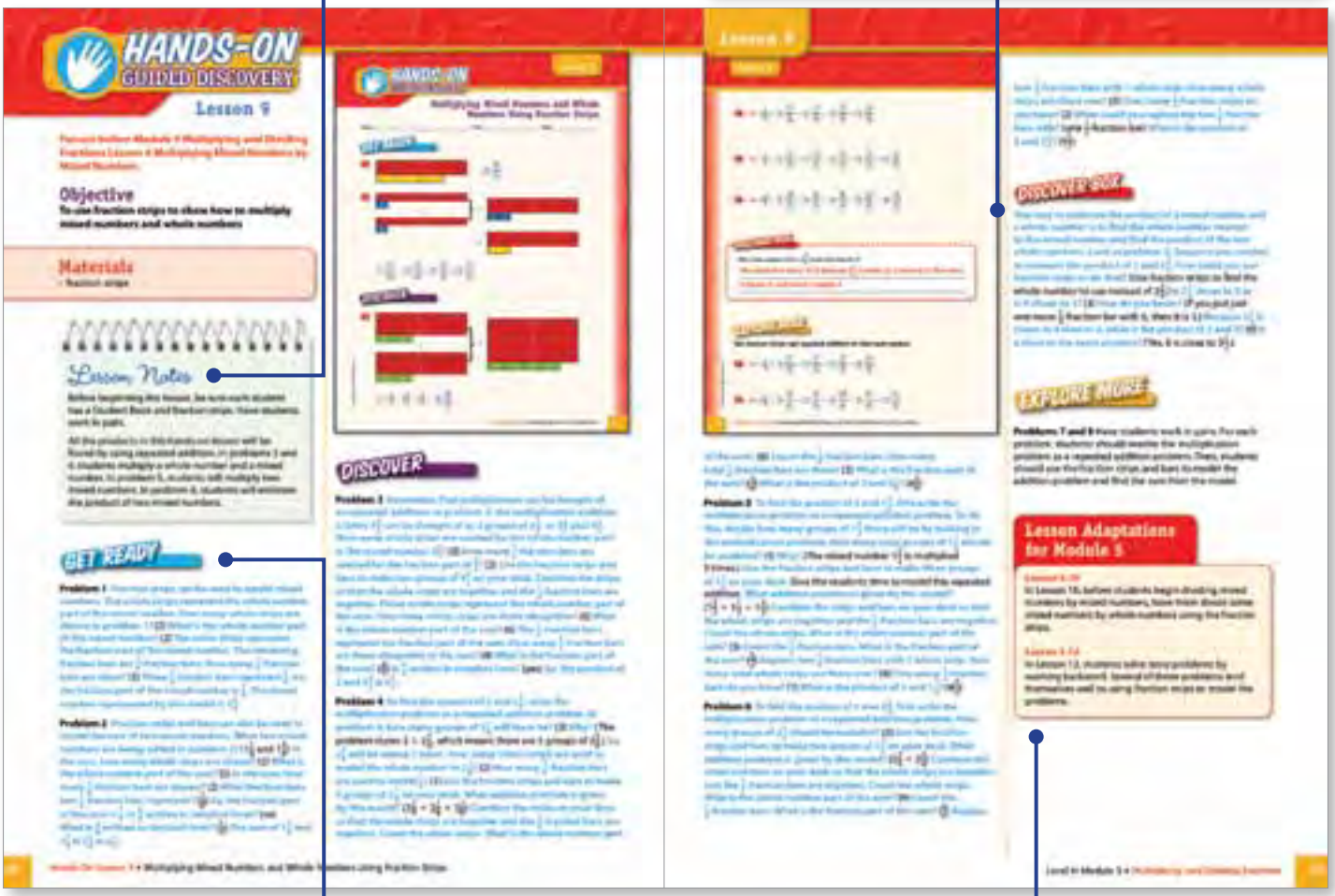
Vmath In The Classroom

Vmath Hands-On Lessons

Manipulatives or representational models are proven to help teach students essential mathematical concepts. The Hands-On Lessons in *Vmath* enable active, **hands-on explorations of abstract concepts** using manipulatives commonly found in the classroom.

Lesson Notes help teachers effectively teach the lesson.

Students **explain** what they have observed, **apply** critical thinking skills, and **use deductive reasoning** as they **communicate their conclusions** with peers and teachers.



Level H, Module 5
- Multiplying and Dividing Fractions

Hands-On Lesson 9
- Multiplying Mixed Numbers and Whole Numbers Using Fraction Strips

In the **Get Ready** section of the lesson, teachers review prerequisite skills before modeling new concepts as students respond orally and follow along in their booklets.

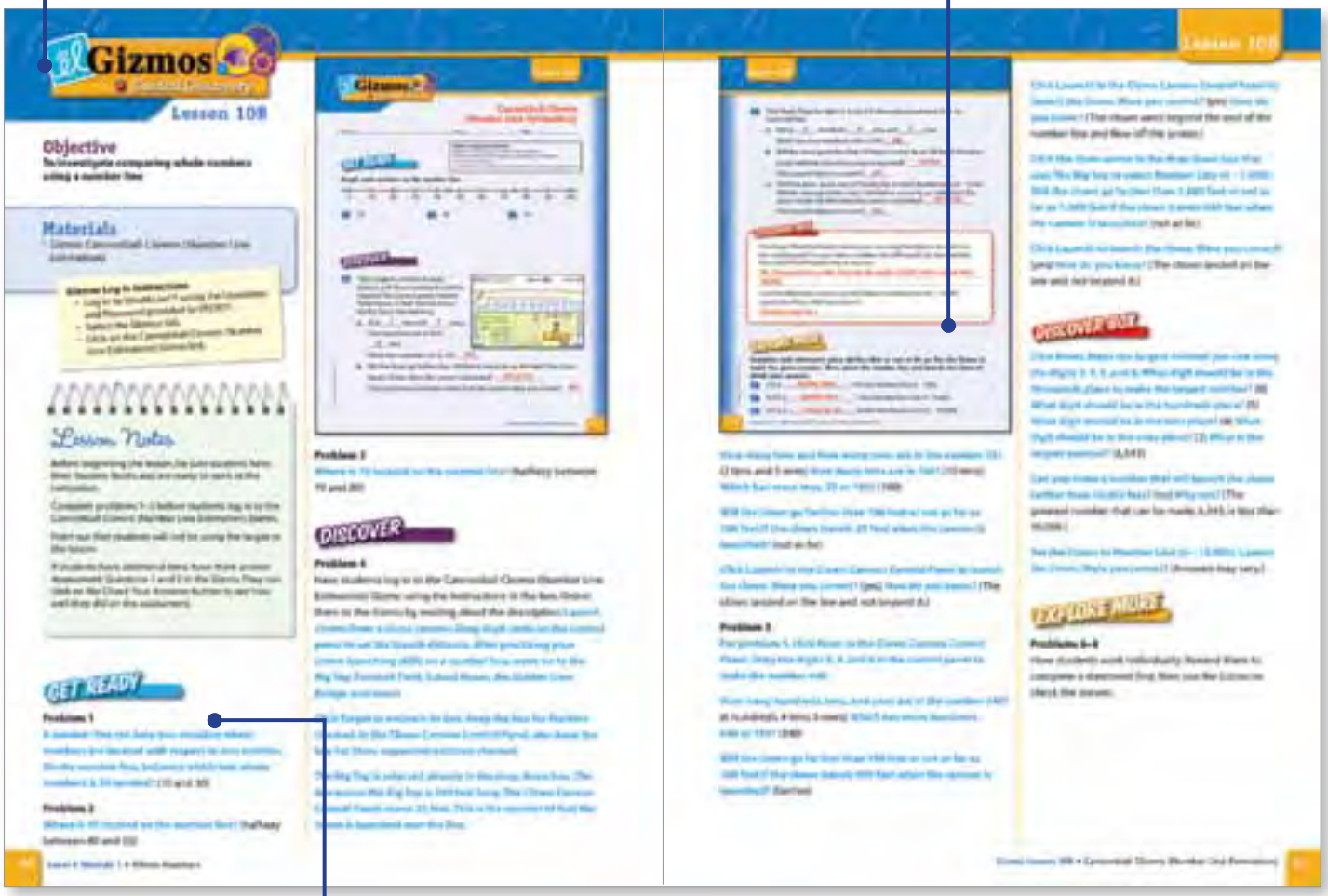
Teachers continue to use the **manipulatives** throughout the *Vmath* modules to teach related concepts.

Vmath Gizmos Lessons

The award-winning ExploreLearning *Gizmos* are online simulations of essential math concepts. These **interactive, virtual manipulatives** provide learning tools that are **engaging, easy to use, and flexible** enough to support many different learning styles. Students are able to visualize the targeted concepts as they explore and expand their learning.

Easy-to-Use online manipulatives provide interactive simulations that can be accessed 24/7.

Gizmos help students **make connections** between mathematical concepts and mathematical processes.



Specific **step-by-step instructions** take the guesswork out of how *Gizmos* work.

Level H, Module 5
- Multiplying and Dividing Fractions

Gizmos Lesson 8
- Multiplying Fractions

Professional Development

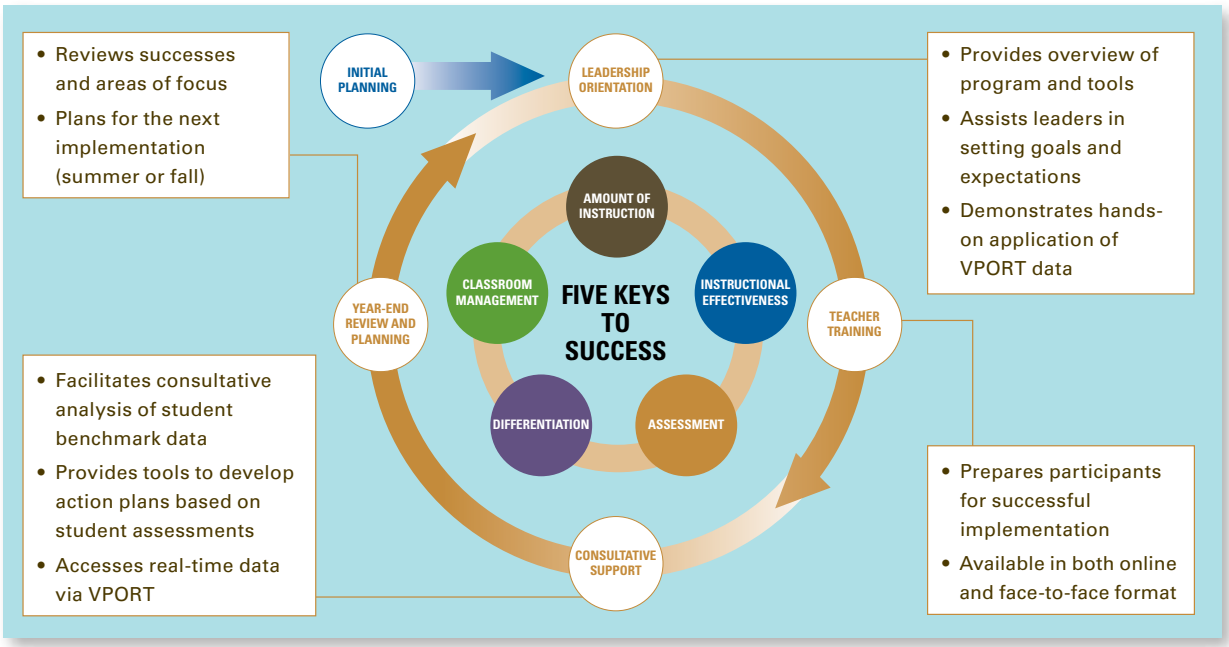
At Cambium Learning® Group, we understand that intervention solutions don't come from programs alone. Voyager's professional development partnership provides ongoing training and implementation support to maximize the effectiveness of instruction.

Focus on Fidelity

Voyager provides award-winning professional development to support effective teaching practices. The hands-on, interactive design can be used in structured environments or in self-paced individual settings to help teachers be successful from the start. Participants learn to:

- Use the VPORT data management system to assess students and differentiate instruction
- Apply new research and best practices
- Implement the program with ease and fidelity

The Voyager professional development partnership extends throughout the school year and integrates continuous training and support services with detailed reporting on student achievement for teachers and administrators. Our services embody the **five keys to success.**



Initial Planning, Leadership Orientation, and Teacher Training

The professional development partnership begins with collaborative planning between district leadership and Voyager's support staff. This **initial planning** involves customizing program training and support to align with district expectations and goals.

Voyager's **leadership orientation** provides an opportunity for school leaders to review program components and VPORT, Voyager's online data management system. Leaders establish implementation goals and expectations as well as an implementation plan and timeline for their school.

Student success depends on the strength of the teacher, and Voyager's training focuses on improving the quality of instruction by increasing teacher knowledge. Voyager's professional development is unmatched in the industry, offering **teacher training** through face-to-face sessions and an online course.

Consultative Support and Year-end Review/Planning

VPORT provides educators with immediate and transparent real-time data to track student progress throughout the year. With Voyager's **consultative support**, educators learn to:

- Identify student needs
- Monitor student progress against goals
- Evaluate student learning
- Adjust instruction based on skills and needs

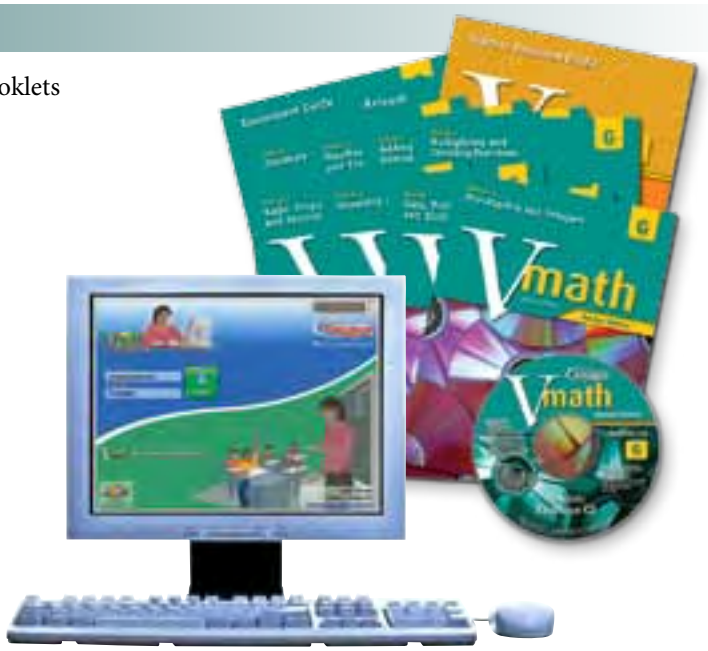
One of the most important benefits of the Voyager partnership occurs during the **year-end review and planning** stage. Administrators and Voyager support personnel review student progress made during the year and examine areas of focus for the following year. Working collaboratively, they analyze benchmark data and set goals for summer and fall implementations.

Materials

Vmath materials include everything needed to implement the program in a classroom.

Teacher Resource Kits

- Eight Teacher Edition Module Booklets
- *Vmath* Assessment Guide
- Teacher Reteach Lesson Booklet
- Teacher Resource Guide
- Instructional Transparency Set
- Teacher Resource CD-ROM
- Teacher access to VPORT
- *VmathLive*
- *Vmath* online product training



Student Math Packs

- Eight Student Edition Module Booklets
- Student Assessment books
- One *VmathLive* student school-year license

Vmath Summer Adventure

Prevent the summer math slide with this powerful instructional tool targeting the essential concepts and skills that students must master before moving to the next grade level in math.

Research shows that some students experience learning loss during the summer months across grade levels and subject areas. The greatest learning loss is experienced in math.

Vmath Summer Adventure has been proven to:

- Improve promotion rates
- Maintain academic year gains
- Accelerate struggling students toward grade-level proficiency

Key features of *Vmath Summer Adventure* include:

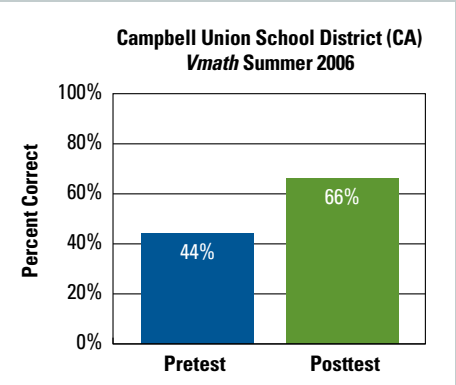
- Explicit instruction shown to be effective with struggling students
- Purposeful practice that helps students achieve mastery
- Built-in assessments and data management to monitor student progress
- Targeted, corrective feedback to guide student learning
- Optional student-centered technology to engage and motivate students

Vmath Summer Adventure Promotes Success

Vmath Summer Adventure delivers results that prevent the summer math slide and prepare students for the upcoming school year.

"Our initial data from *Vmath* was very promising. We saw growth in our students at all grade levels. We are anxious to continue our use of *Vmath* in the years to come."

—Michael O'Laughlin
Director of Curriculum
Campbell Union School District, CA



After four weeks in *Vmath Summer Adventure*, Campbell Union third grade students made a 22 percentage point gain in the number of correct answers from the pretest to the posttest in the *Vmath* program.

Program Scope and Sequence

WHOLE NUMBERS	*Vmath* Level						
	C	D	E	F	G	H	I
Number Patterns	●	▲	▲	▲			
Skip Counting	●	▲	▲				
Basic Addition and Subtraction Facts	●	●					
Odd and Even Numbers		●					
Read and Write	●	●	●	●	▲		
Round		●	●	●	▲		
Place Value	●	●	●	▲	▲		
Expanded Notation	●	●	●	▲			
Number Line	●	●	▲	▲			
Compare and Order	●	●	●	●	▲		
Addition and Subtraction Fact Families	●	●	●	▲	▲		
Addition	●	●	●	●	▲	▲	▲
Subtraction	●	●	●	●	▲	▲	▲
Estimation of Sums and Differences		●	●	▲	▲	▲	▲
Multiplication and Division Fact Families		●	●	●	▲	▲	▲
Multiplication	●	●	●	●	▲	▲	▲
Division	●	●	●	●	▲	▲	▲
Estimation of Products			●	▲	▲	▲	▲
Estimation of Quotients			●	▲	▲	▲	▲

DECIMALS	*Vmath* Level						
	C	D	E	F	G	H	I
Money	●	●	●	▲	▲	▲	▲
Read and Write		●	●	●	●		
Compare and Order		●	●	●	●	●	▲
Number Line		●	●	▲	▲	▲	
Addition		●	●	●	●	●	▲
Subtraction		●	●	●	●	●	▲
Round			●	●	●	●	▲
Estimation of Sums			●	●	●	▲	▲
Estimation of Differences			●	●	●	▲	▲
Multiplication					●	●	▲
Estimation of Products				●	●	●	▲
Division					●	●	▲
Estimation of Quotients				●	●	●	▲
Repeating Decimals							●

NUMBER THEORY	*Vmath* Level						
	C	D	E	F	G	H	I
Commutative Properties	●	●	●	●	▲	▲	▲
Associative Properties		●	●	●	▲	▲	▲
Distributive Property							●
Factors		●	●	●	●	●	▲
Multiples		●	●	●	●	●	▲
Prime and Composite Numbers				●	●	▲	
Least Common Multiple				●	●	●	▲
Greatest Common Factor				●	●	●	▲
Prime Factorization				●	●	●	
Exponents					●	●	●
Powers of Ten/Scientific Notation					●	●	●

KEY ● = Taught ▲ = Introduced or Reinforced

FRACTIONS	*Vmath* Level						
	C	D	E	F	G	H	I
Understanding Fractions	●	●	▲	▲	▲		
Fractional Parts	●	●	●	●	●	▲	▲
Compare and Order	●	●	●	●	●	●	▲
Equivalent Fractions		●	●	●	●	●	▲
Number Line		●	●	▲	▲	▲	
Greater than One			●	▲	▲	▲	
Addition of Fractions		●	●	●	●	●	●
Subtraction of Fractions		●	●	●	●	●	●
Simplest Form			●	●	●	●	▲
Addition of Mixed Numbers			●	●	●	●	●
Subtraction of Mixed Numbers			●	●	●	●	●
Relating Fractions and Decimals			●	●	▲	▲	▲
Estimation of Sums				●	●	●	▲
Estimation of Differences				●	▲	●	▲
Improper Fractions and Mixed Numbers				●	●	●	▲
Multiplication of Fractions				●	●	●	●
Multiplication of Mixed Numbers				●	●	●	●
Estimation of Products				●	●	●	▲
Reciprocals					●	●	▲
Division of Fractions					●	●	●
Division of Mixed Numbers						●	●

DATA, PROBABILITY, AND STATISTICS	*Vmath* Level						
	C	D	E	F	G	H	I
Frequency Tables	●	●	●	●			
Pictographs	●	●	●	●			
Measures of Central Tendency		●	●	●	●	●	●
Bar Graphs	●	●	●	●	●	●	▲
Line Graphs		●	●	●	●	●	●
Circle Graphs			●	●	●	●	●
Histograms					●	●	●
Misleading Graphs							●
Stem-and-Leaf Plots					●	●	●
Compare Graphical Representations				●	●	▲	●
Scatter Plots						●	●
Counting Possible Outcomes			●	●	●	●	●
Probability	●	●	●	●	●	●	●
Making Predictions			●	●	▲	▲	●
Independent and Dependent Events							●
Box-and-Whisker Plots						●	●
Line Plots							●
Graph Data to Represent Relationships						●	●
Venn Diagrams						●	●

MEASUREMENT	*Vmath* Level						
	C	D	E	F	G	H	I
Reading a Calendar	●						
Time and Temperature	●	●	●	●			
Measure to the Nearest 1 in. and 1/2 in.	●	●	●				
Measure to the Nearest 1 cm and 1/2 cm	●	●	●				
Using Benchmark Measurements	●	●	●	●	▲	▲	▲
Customary Measurements	●	●	●	●	●	●	●

Program Scope and Sequence

KEY ● = Taught ▲ = Introduced or Reinforced

MEASUREMENT (continued)	*Vmath* Level						
	C	D	E	F	G	H	I
Metric Measurements	●	●	●	●	●	●	●
Perimeter		●	●	●	●	●	●
Area of Rectangles and Squares		●	●	●	●	●	●
Area of Triangles				●	●	●	●
Area of Parallelograms				●	●	●	●
Area of Trapezoids						●	●
Area of Compound Figures				▲	▲	▲	●
Area of Circles					●	●	●
Volume			●	●	●	●	●
Estimation of Lengths	●	●	●	●	●		
Circumference of Circles				●	●	●	●
Surface Area					●	●	●
Perimeter and Area with Dimension Changes						▲	●
Volume and Dimension Changes							●

GEOMETRY	*Vmath* Level						
	C	D	E	F	G	H	I
Points and Lines		●	●	●	●	●	▲
Angles		●	●	●	●	●	●
Polygons/Triangles/Quadrilaterals/Circles	●	●	●	●	●	●	●
Making and Separating Plane Figures	●						
Three-Dimensional Figures	●	●	●	●	●	●	●
Three-Dimensional Nets					▲	●	●
Sketch Three-Dimensional Figures from Different Views						●	●
Congruent Figures	●	●	●	●	●	●	●
Similar Figures				●	●	●	▲
Line Symmetry	●	●	●	●			
Rotational Symmetry						●	
Transformations	●		●	●	●	●	●
Congruent Triangles							●
Rotations							●
Dilations on a Coordinate Plane							●
Pythagorean Theorem						●	●

RATIO, PROPORTION, AND PERCENT	*Vmath* Level						
	C	D	E	F	G	H	I
Ratios				●	●	●	▲
Rates					●	●	●
Percents				●	●	●	●
Proportions					●	●	●
Proportion Word Problems					●	●	▲
Convert Fractions, Decimals, and Percents			●	●	●	●	●
Percent of a Number					●	●	●
Percent of Increase and Decrease					●	●	●
Solving Percent Equations						●	●
Scale Drawings and Maps						●	▲
Find Missing Lengths in Similar Figures						●	●

Sample Teacher Pages

Module 8

Pre-Algebra and Integers

G

Vmath®

Second Edition

Teacher Edition

Expanded Learning Voyager®

Lesson 13

Objective

To subtract integers

Preskills	Transparency
Integers on a Number Line	G.8.9
Adding Integers	G.8.12

Materials

- transparency G.8.13

REVIEW PRESKILLS

Problem 1

What is the opposite of 7? ($^{-}7$) What is the opposite of $^{-}2$? (2)

Problem 2

To add two integers with different signs, first drop the signs and subtract the lesser number from the greater number. What is 9 minus 2? (7) The sign of the answer will be the same as the original sign of the greater number. The greater number is 9. The answer will be positive. What is the sum of 9 and $^{-}2$? (7)

MODEL NEW SKILLS

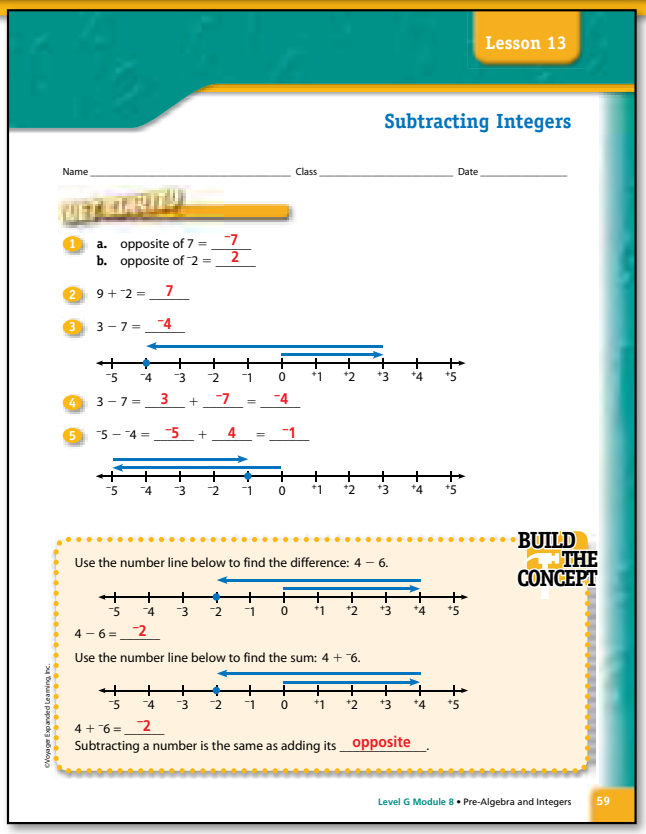
Problem 3

To subtract integers using a number line, always start at 0. To subtract 7 from 3, start at 0, and move three units to the right. This represents 3.

Remember, to add a positive number, move to the right. To subtract a positive number, move in the opposite direction, or to the left. So, to subtract 7, move seven units to the left. This point represents the difference of 3 and 7. What is 3 minus 7? ($^{-}4$)

Problem 4

To subtract integers without using a number line, rewrite the subtraction problem as an addition problem. To do this, the first integer stays the same, the subtraction sign changes to an addition sign, and the second integer is changed to its opposite.

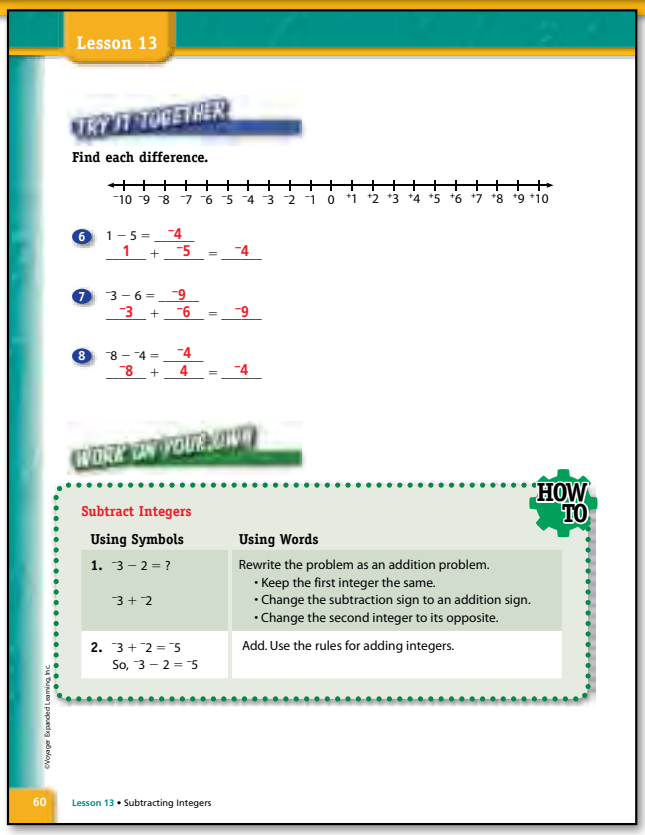


In other words, to subtract integers, add the opposite. The first integer in 3 minus 7 is 3. Keep the 3 the same. Change the subtraction sign to an addition sign. The opposite of the second integer, 7, is $^{-}7$. Rewrite the problem as $3 + {}^{-}7$. What is 3 plus $^{-}7$? ($^{-}4$) So, what is 3 minus 7? ($^{-}4$) Check the answer by adding the number being subtracted, 7, to the difference, $^{-}4$. The sum should equal 3. What is $^{-}4$ plus 7? (3) The answer is correct.

Problem 5

To solve $^{-}5$ minus $^{-}4$, first rewrite the problem as an addition problem. The first integer stays the same, the subtraction sign changes to an addition sign, and the second integer is changed to its opposite. The first integer is $^{-}5$; it stays the same. Change the subtraction sign to an addition sign. The opposite of the second integer, $^{-}4$, is 4. What is $^{-}5$ plus 4? ($^{-}1$) So, $^{-}5$ minus $^{-}4$ is $^{-}1$.

The answer can be checked using a number line. Start at 0 and move five units to the left. Remember, to add a negative number, move to the left. To subtract a negative number, move in the opposite direction, or to the right. So, to subtract $^{-}4$, move four units to the right of $^{-}5$. This point represents the difference. What is the value of the point? ($^{-}1$) This agrees with the answer obtained without using the number line.


Lesson 13

TRY IT TOGETHER

Find each difference.

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9 +10

6. 1 − 5 = −4
1 + −5 = −4

7. −3 − 6 = −9
−3 + −6 = −9

8. −8 − −4 = −4
−8 + 4 = −4

WORK ON YOUR OWN

HOW TO

Subtract Integers

Using Symbols	Using Words
1. −3 − 2 = ? −3 + −2	Rewrite the problem as an addition problem. • Keep the first integer the same. • Change the subtraction sign to an addition sign. • Change the second integer to its opposite.
2. −3 + −2 = −5 So, −3 − 2 = −5	Add. Use the rules for adding integers.

60 Lesson 13 • Subtracting Integers

BUILD THE CONCEPT

Use the number line to find the difference of 4 and 6. Start at 0. What is the first step in subtracting the integers? (Move 4 units to the right.) **What is the next step?** (Move 6 units to the left.) **What is 4 minus 6?** (−2)

Now use the number line to find 4 plus −6. Start at 0. What is the first step in adding the integers? (Move 4 units to the right.) **What is the next step?** (Move 6 units to the left.) **What is the sum of 4 and −6?** (−2)

Look at the number lines. What do you notice about the blue arrows and the answers? (They are exactly the same.) **So, subtracting a number is the same as adding its opposite.**

TRY IT TOGETHER Student Book p60

SCAFFOLD INSTRUCTION

Problem 6

To find 1 minus 5, what kind of problem should this subtraction problem be changed to? (an addition problem) **How is this done?** (Keep the first integer the same, change the subtraction sign to an addition sign, and change the second integer to its opposite.) **What is the first number?** (1) **What is the opposite of 5?** (−5) **What is the addition problem?** (1 + −5) **What is 1 plus −5?** (−4) **What is 1 minus 5?** (−4)

What is a way the answer can be checked? (Add the number being subtracted, 5, to the difference, −4. The sum should equal 1.) **What is −4 plus 5?** (1) **Is the answer correct?** (yes)

Problem 7

What is the opposite of 6? (−6) **What addition problem should be written from the subtraction problem?** (−3 + −6) **What is −3 plus −6?** (−9) **So, what is −3 minus 6?** (−9)

What are the steps in checking the answer using the number line? (Start at 0 and move three units to the left. Move six units to the left of −3. This point represents the difference.) **What point represents the difference?** (−9) **Does this number match the answer obtained using the method of adding opposites?** (yes)

Problem 8

What addition problem should be written from the subtraction problem? (−8 + 4) **What is −8 plus 4?** (−4) **What is −8 minus −4?** (−4) **Check the answer using addition. What is −4 plus −4?** (−8) **The answer is correct.**

USE CORRECTION PROCEDURES

Preskills

Correct errors by reviewing or reteaching how to find the opposite of an integer or add integers. Use the **HOW TO** process from each lesson transparency listed in the preskills box on page 60.

New Skills

For reteaching:

- Review the Math Coach examples shown in VmathLive in the Subtract Integers activity in the Pre-Algebra and Integers module.
- Use Reteach Copy Master G.8.13 and the modeled teacher text to show students an alternative method for subtracting integers. Have students complete the Copy Master.

For additional practice:

- Assign the VmathLive Subtract Integers activity in the Pre-Algebra and Integers module.
- Have students complete the Extra Practice problems on Student Book page 75.

WORK ON YOUR OWN Student Book p60

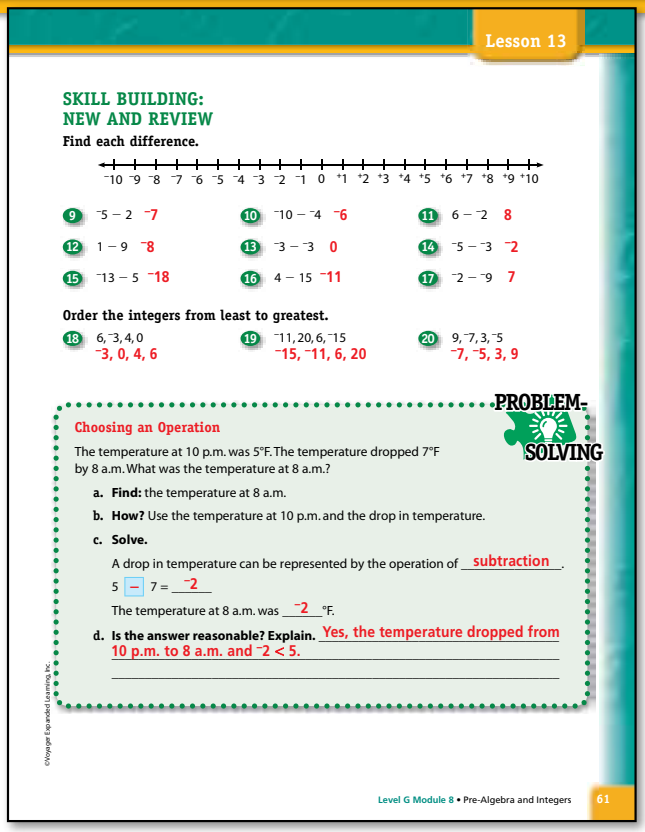
MONITOR INDEPENDENT WORK

HOW TO Before students begin independent work, review the **HOW TO** process example. As you review, emphasize the words of mathematics by having students read aloud the words shown at the right for each process step.

PROBLEM-SOLVING As you review the Problem-Solving strategy problem together, emphasize the steps of the problem-solving process used in the Choosing an Operation strategy.

Problems 9–24

Have students work independently. Check work and have students total the number correct and record results. Instruct students to record a 6 if they got 15 or 16 correct, a 5 for 13 or 14 correct, a 4 for 12 correct, a 3 for 10 or 11 correct, a 2 for 8 or 9 correct, and a 1 for 1–7 correct. Use Correction Procedures as needed.



Lesson 13

SKILL BUILDING: NEW AND REVIEW

Find each difference.

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9 +10

9. −5 − 2 −7
10. −10 − −4 −6
11. 6 − −2 8
12. 1 − 9 −8
13. −3 − −3 0
14. −5 − −3 −2
15. −13 − 5 −18
16. 4 − 15 −11
17. −2 − −9 7

Order the integers from least to greatest.

18. 6, −3, 4, 0 — −3, 0, 4, 6
19. −11, 20, 6, −15 — −15, −11, 6, 20
20. 9, −7, 3, −5 — −7, −5, 3, 9

PROBLEM-SOLVING

Choosing an Operation

The temperature at 10 p.m. was 5°F. The temperature dropped 7°F by 8 a.m. What was the temperature at 8 a.m.?

a. **Find:** the temperature at 8 a.m.

b. **How?** Use the temperature at 10 p.m. and the drop in temperature.

c. **Solve.**

A drop in temperature can be represented by the operation of subtraction.

5 − 7 = −2

The temperature at 8 a.m. was −2 °F.

d. **Is the answer reasonable? Explain.** Yes, the temperature dropped from 10 p.m. to 8 a.m. and −2 < 5.

Problem 21 Reminder

Be sure students understand that a drop in a value signifies subtraction.

CHECK UP Student Book p62

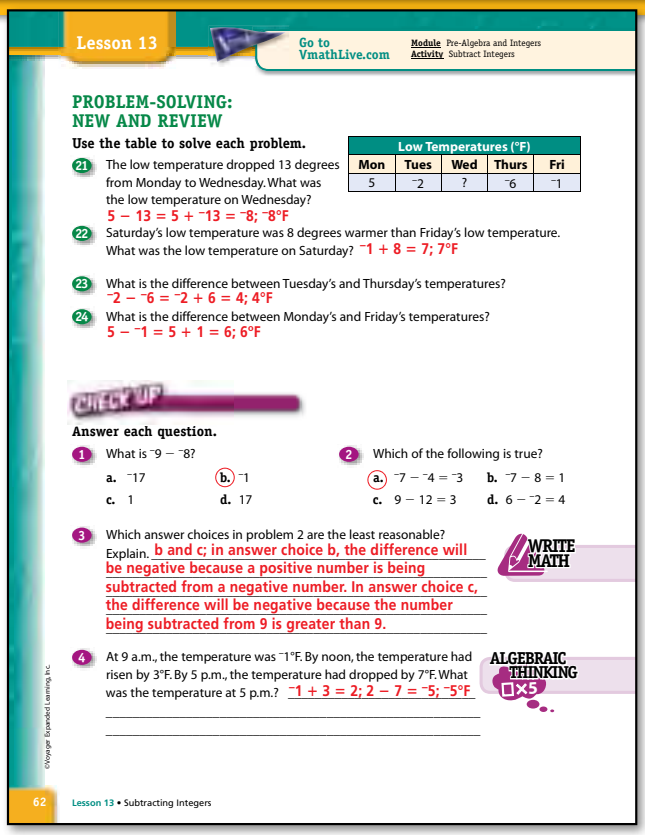
ASSESS INFORMALLY

Error Analysis

Check work and record results. Use the error analysis to determine which skills need review, reteaching, or extra practice.

If student answered 1a, 2b, or 2d: The student changed the operation sign to addition but failed to change the sign of the second integer. Use Correction Procedures in G.8.13 to reteach how to subtract integers.

If student answered 1c or 2c: The student found the difference between the integers but used the incorrect sign. Use Correction Procedures in G.8.12 to reteach how to add integers.



Lesson 13

Go to VmathLive.com

Module Pre-Algebra and Integers
Activity Subtract Integers

PROBLEM-SOLVING: NEW AND REVIEW

Use the table to solve each problem.

Low Temperatures (°F)				
Mon	Tues	Wed	Thurs	Fri
5	−2	?	−6	−1

21 The low temperature dropped 13 degrees from Monday to Wednesday. What was the low temperature on Wednesday?
5 − 13 = 5 + −13 = −8; −8°F

22 Saturday's low temperature was 8 degrees warmer than Friday's low temperature. What was the low temperature on Saturday? −1 + 8 = 7; 7°F

23 What is the difference between Tuesday's and Thursday's temperatures?
−2 − −6 = −2 + 6 = 4; 4°F

24 What is the difference between Monday's and Friday's temperatures?
5 − −1 = 5 + 1 = 6; 6°F

CHECK UP

Answer each question.

1 What is −9 − −8?
a. −17 (b.) −1
c. 1 d. 17

2 Which of the following is true?
(a.) −7 − −4 = −3 b. −7 − 8 = 1
c. 9 − 12 = 3 d. 6 − −2 = 4

3 Which answer choices in problem 2 are the least reasonable? Explain. b and c; in answer choice b, the difference will be negative because a positive number is being subtracted from a negative number. In answer choice c, the difference will be negative because the number being subtracted from 9 is greater than 9.

WRITE MATH

4 At 9 a.m., the temperature was −1°F. By noon, the temperature had risen by 3°F. By 5 p.m., the temperature had dropped by 7°F. What was the temperature at 5 p.m.? −1 + 3 = 2; 2 − 7 = −5; −5°F

ALGEBRAIC THINKING

□×5

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62 Lesson 13 • Subtracting Integers

WRITE MATH

If student answered problem 3 incorrectly: Have students think of the subtraction on a number line. They should think of each first number's position on the number line and the direction in which they would move to subtract each number.

DIFFERENTIATION

Additional Resources

VmathLive
Module: Pre-Algebra and Integers
Activity: Subtract Integers

Vmath Reteach
Copy Master G.8.13 page 147
Instructional Support page 49

Extra Practice
Student Book page 75

English Language Learners

Provide students with several examples to understand *opposite*, such as in/out, on/off, big/small. Explain that the opposite of a positive integer is a negative integer.

Provide a vertical number line or create a large thermometer. Place sticky notes to indicate the degrees from the Problem-Solving strategy problem. Discuss that a drop in the temperature means that the temperature went down, so students should go down on the number line when counting. Help students see that doing this models subtraction with integers.

Students with Special Needs

Discuss how subtracting an integer is the same as adding its opposite. Have students rewrite the problems as addition problems and use two-color counters to add the integers. Establish one color for positive and the other color for negative (such as yellow for positive, red for negative). Rewrite −6 − −1 as −6 + 1, then use six red counters and one yellow counter. Next, remove one yellow-red pair (represents 0), and five red counters are left (represents −5).

Sample Student Pages

Module 8

Pre-Algebra and Integers

G

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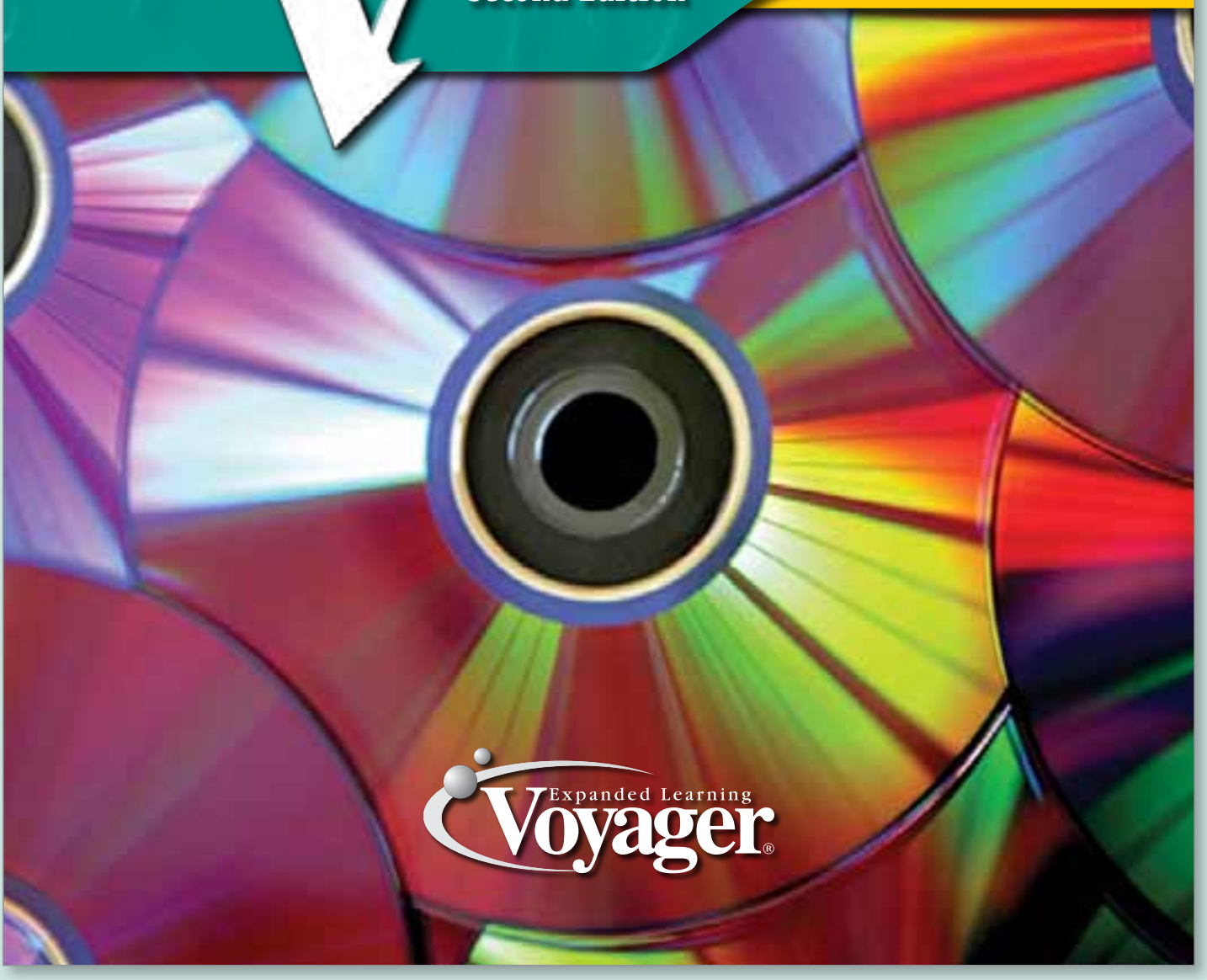


Expanded Learning Voyager®

Subtracting Integers

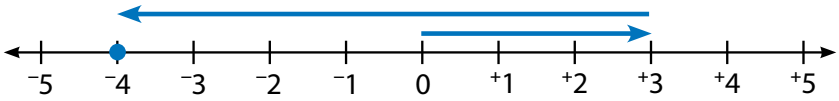
Name _____ Class _____ Date _____

GET STARTED

1 **a.** opposite of 7 = _____
b. opposite of $^{-}2$ = _____

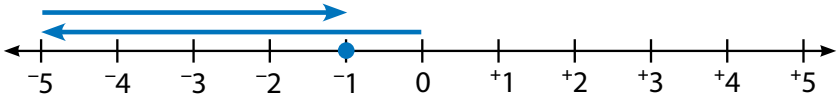
2 $9 + {}^{-}2 =$ _____

3 $3 - 7 =$ _____



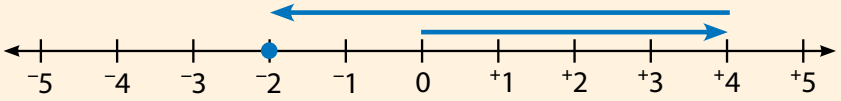
4 $3 - 7 =$ _____ + _____ = _____

5 $^{-}5 - {}^{-}4 =$ _____ + _____ = _____



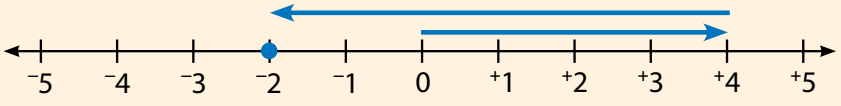
BUILD THE CONCEPT

Use the number line below to find the difference: $4 - 6$.



$4 - 6 =$ _____

Use the number line below to find the sum: $4 + {}^{-}6$.

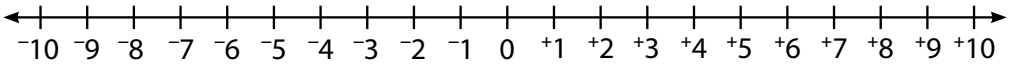


$4 + {}^{-}6 =$ _____

Subtracting a number is the same as adding its _____.

TRY IT TOGETHER

Find each difference.



6 $1 - 5 =$ _____

_____ + _____ = _____

7 $^{-}3 - 6 =$ _____

_____ + _____ = _____

8 $^{-}8 - {}^{-}4 =$ _____

_____ + _____ = _____

WORK ON YOUR OWN

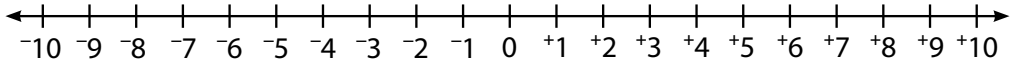
HOW TO

Subtract Integers

Using Symbols	Using Words
1. $^{-}3 - 2 = ?$ $^{-}3 + {}^{-}2$	Rewrite the problem as an addition problem. • Keep the first integer the same. • Change the subtraction sign to an addition sign. • Change the second integer to its opposite.
2. $^{-}3 + {}^{-}2 = {}^{-}5$ So, $^{-}3 - 2 = {}^{-}5$	Add. Use the rules for adding integers.

SKILL BUILDING: NEW AND REVIEW

Find each difference.



9. $^{-}5 - 2$
10. $^{-}10 - {}^{-}4$
11. $6 - {}^{-}2$
12. $1 - 9$
13. $^{-}3 - {}^{-}3$
14. $^{-}5 - {}^{-}3$
15. $^{-}13 - 5$
16. $4 - 15$
17. $^{-}2 - {}^{-}9$

Order the integers from least to greatest.

18. $6, {}^{-}3, 4, 0$
19. $^{-}11, 20, 6, {}^{-}15$
20. $9, {}^{-}7, 3, {}^{-}5$

PROBLEM-SOLVING

Choosing an Operation

The temperature at 10 p.m. was 5°F. The temperature dropped 7°F by 8 a.m. What was the temperature at 8 a.m.?

a. Find: the temperature at 8 a.m.

b. How? Use the temperature at 10 p.m. and the drop in temperature.

c. Solve.

A drop in temperature can be represented by the operation of _____.

5 ☐ 7 = _____

The temperature at 8 a.m. was _____°F.

d. Isthe answer reasonable? Explain. _____

_____

_____

PROBLEM-SOLVING: NEW AND REVIEW

Use the table to solve each problem.

Low Temperatures (°F)				
Mon	**Tues**	**Wed**	**Thurs**	**Fri**
5	$^{-}2$	?	$^{-}6$	$^{-}1$

21 The low temperature dropped 13 degrees from Monday to Wednesday. What was the low temperature on Wednesday?

22 Saturday's low temperature was 8 degrees warmer than Friday's low temperature. What was the low temperature on Saturday?

23 What is the difference between Tuesday's and Thursday's temperatures?

24 What is the difference between Monday's and Friday's temperatures?

CHECK UP

Answer each question.

1 What is $^{-}9 - {}^{-}8$?

a. $^{-}17$ **b.** $^{-}1$

c. 1 **d.** 17

2 Which of the following is true?

a. $^{-}7 - {}^{-}4 = {}^{-}3$ **b.** $^{-}7 - 8 = 1$

c. $9 - 12 = 3$ **d.** $6 - {}^{-}2 = 4$

3 Which answer choices in problem 2 are the least reasonable? Explain. _____

WRITE MATH

_____

_____

_____

_____

4 At 9 a.m., the temperature was $^{-}1$°F. By noon, the temperature had risen by 3°F. By 5 p.m., the temperature had dropped by 7°F. What was the temperature at 5 p.m.? _____

ALGEBRAIC THINKING □×5

_____

_____

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One Hickory Centre
1800 Valley View Lane, Ste. 400
Dallas, TX 75234-8923
800-547-6747
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