

*Exceptional service in the national interest*



# LayTracks3D: Hex Meshing using MAT

William Roshan Quadros



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. SAND NO. 2014-18464 C

# Goal of LayTracks3D

- Handle General Solids
- Orientation Insensitive
- Boundary Sensitive
- Preserve Sharp Features
- Preserve Imprints in Assemblies
- Geometry Adaptive
- Potential All-Hex
- Fast Remeshing
- Morphable Meshes
- Parallel Friendly

# Medial Axis Transform

## Two-Way Mapping

$$f: p \rightarrow Mp$$

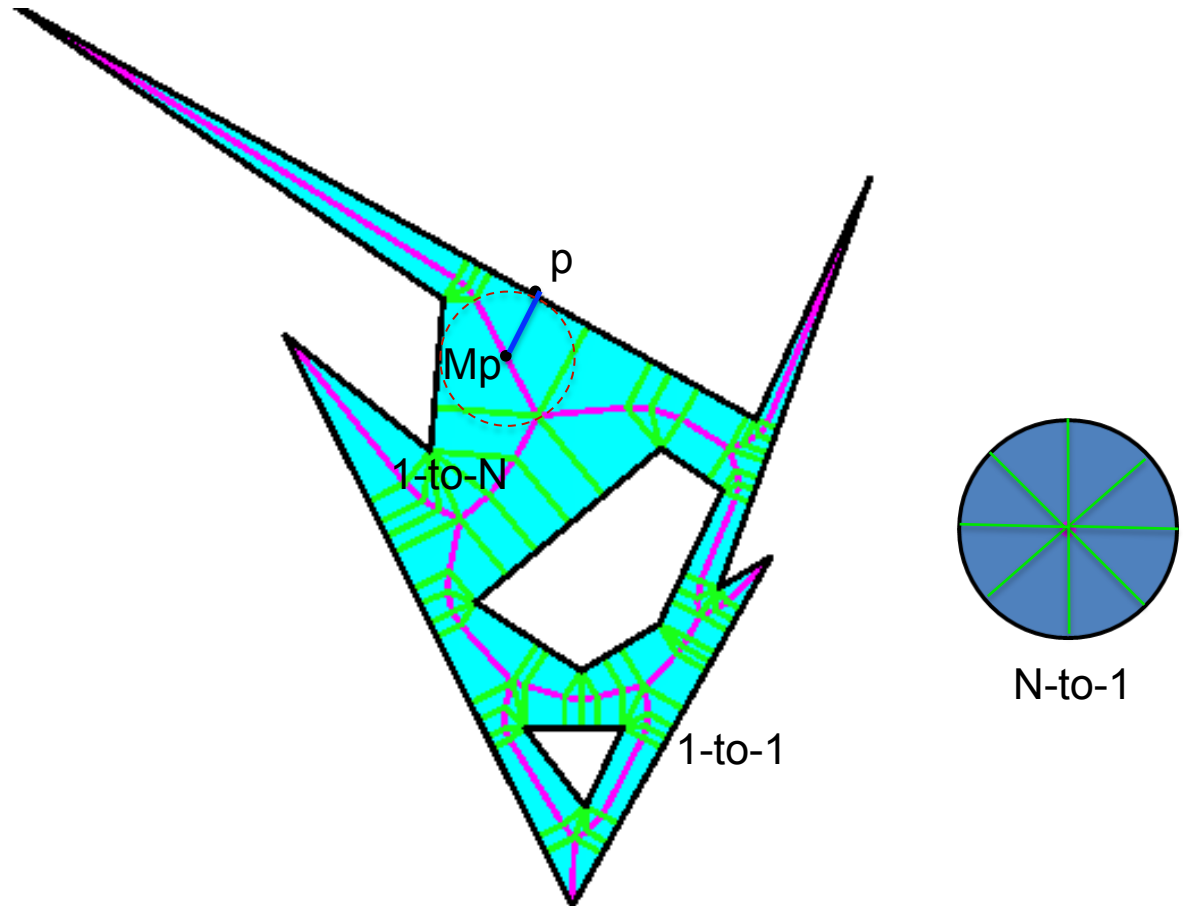
$$g: Mp \rightarrow p$$

## Types of Mapping

*1-to-1*

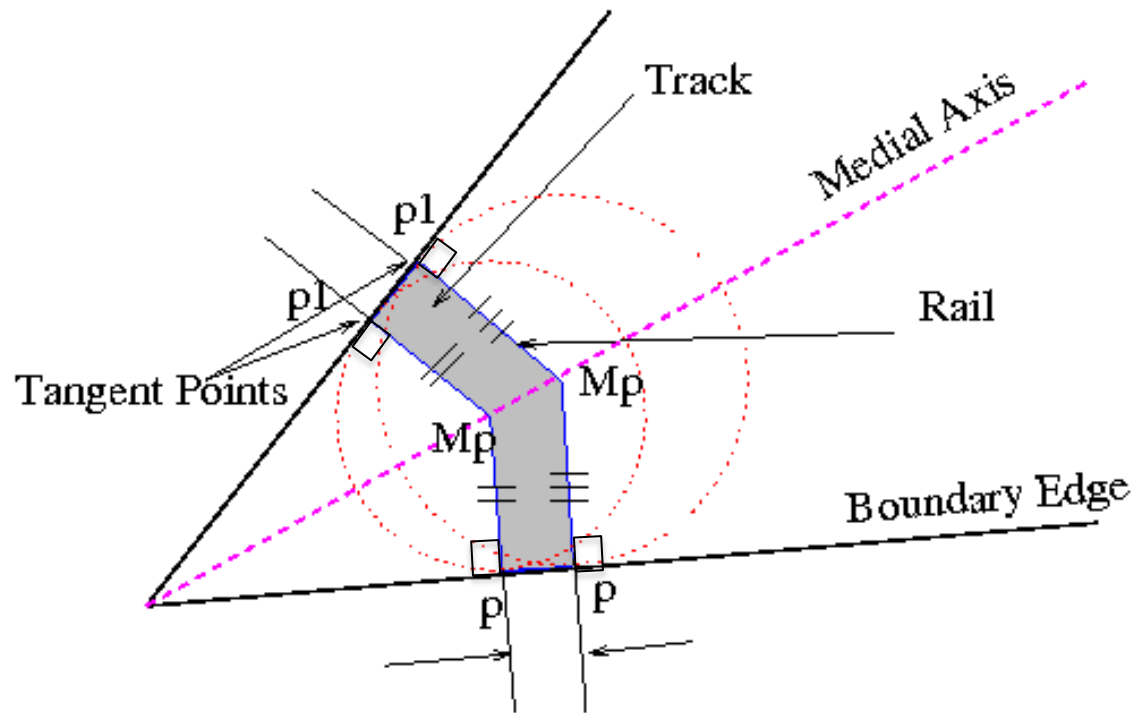
*1-to-N*

*N-to-1*



# Projection Operator

- Uses map  $f$  to connect  $p \rightarrow M_p$
- Uses map  $g$  to connect  $M_p \rightarrow p$

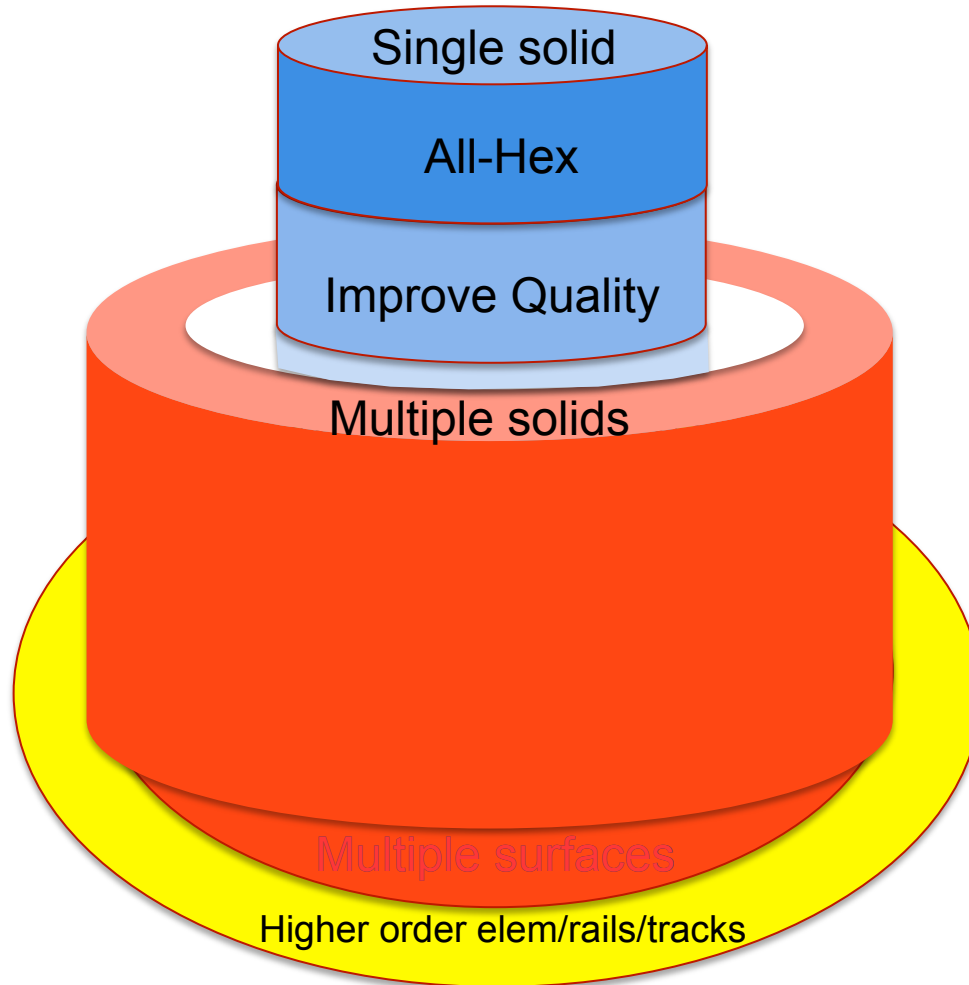




# Advantages of MAT

- Orientation Insensitive:
  - MA of a solid does not change with orientation of the solid
- Boundary Sensitive:
  - Advancing fronts meet at MA by generating structured mesh at the boundary and restricting irregular nodes at MA
- Robust Geometry Decomposition:
  - Medial branch points (critical singularities) are used for decomposition
- Dimension Reduction:
  - Hex meshing can be reduced to surface meshing on the medial surface
- Geometry Adaptive:
  - Medial radius function provides local feature size to control mesh size
- Symmetric Skeleton:
  - Simplifies interval assignment and provides theoretical foundation for all-hex
- Homotopy Equivalence:
  - Handles general solids with sharp boundary feature preservation

# Layout of Presentation

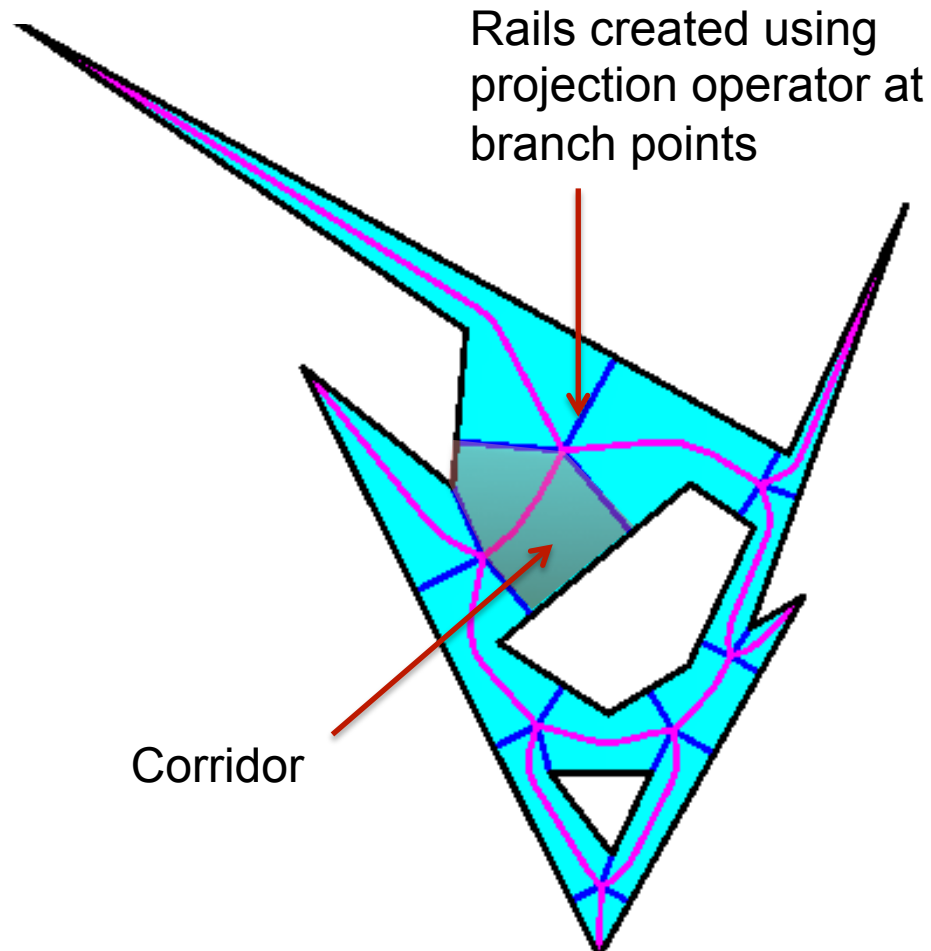


# LayTracks: All-Quad Meshing, 9<sup>th</sup> IMR, 2000

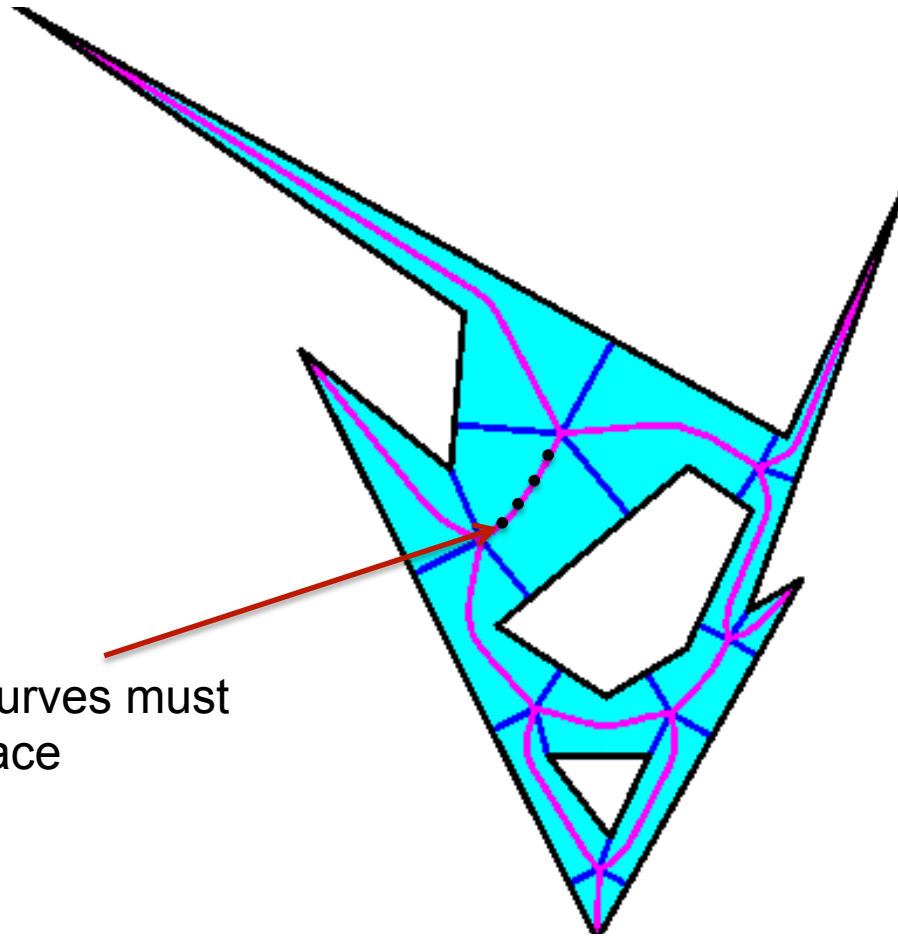
Combines merits of decomposition and advancing front methods

- **Generate MAT:** establish two-way map between domain boundary and medial
- **Generate Corridors:** decompose domain using medial branch points via projection operator
- **Mesh Medial:** surface meshing is reduced to curve meshing. Medial radius function can be used to control mesh size.
- **Generate Tracks:** further decompose the domain/Corridors into simpler/meshable Tracks using projection operator
- **Mesh Tracks:** generate all-quad mesh in each track using symmetric property of the medial

# Generate Corridors: Domain Decomposition

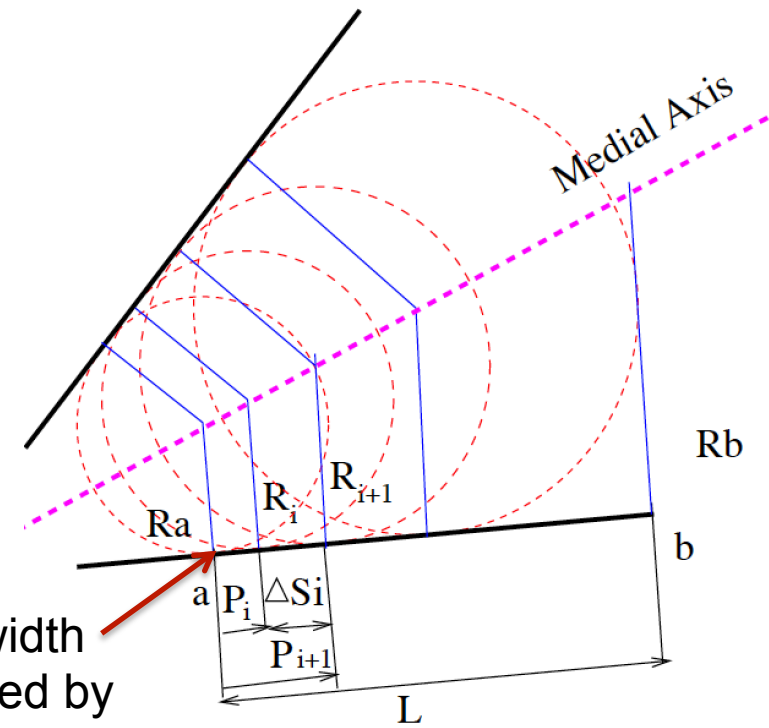
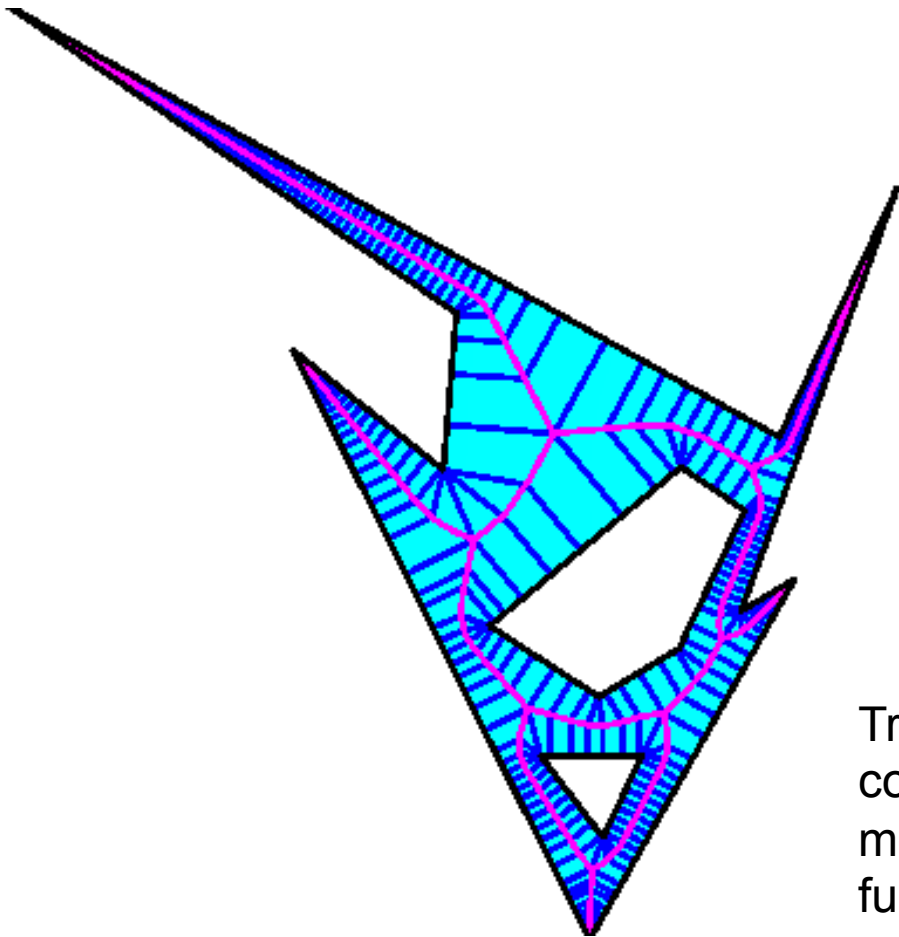


# Mesh Medial: Surface meshing is Reduced to Curve Meshing



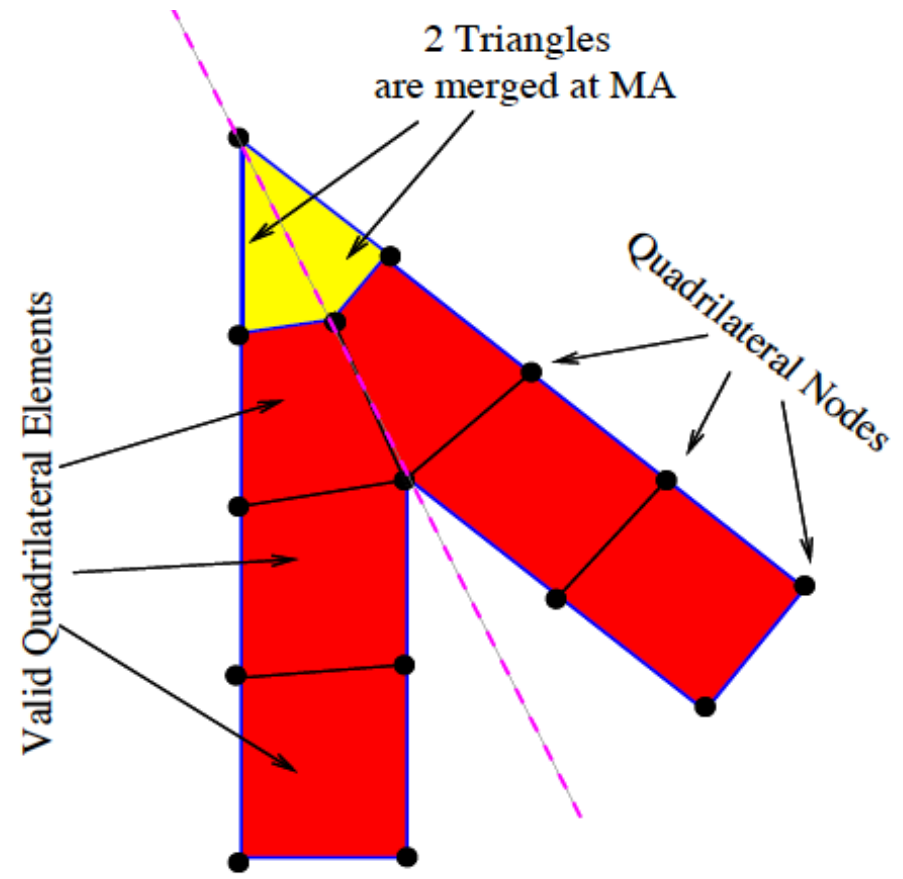
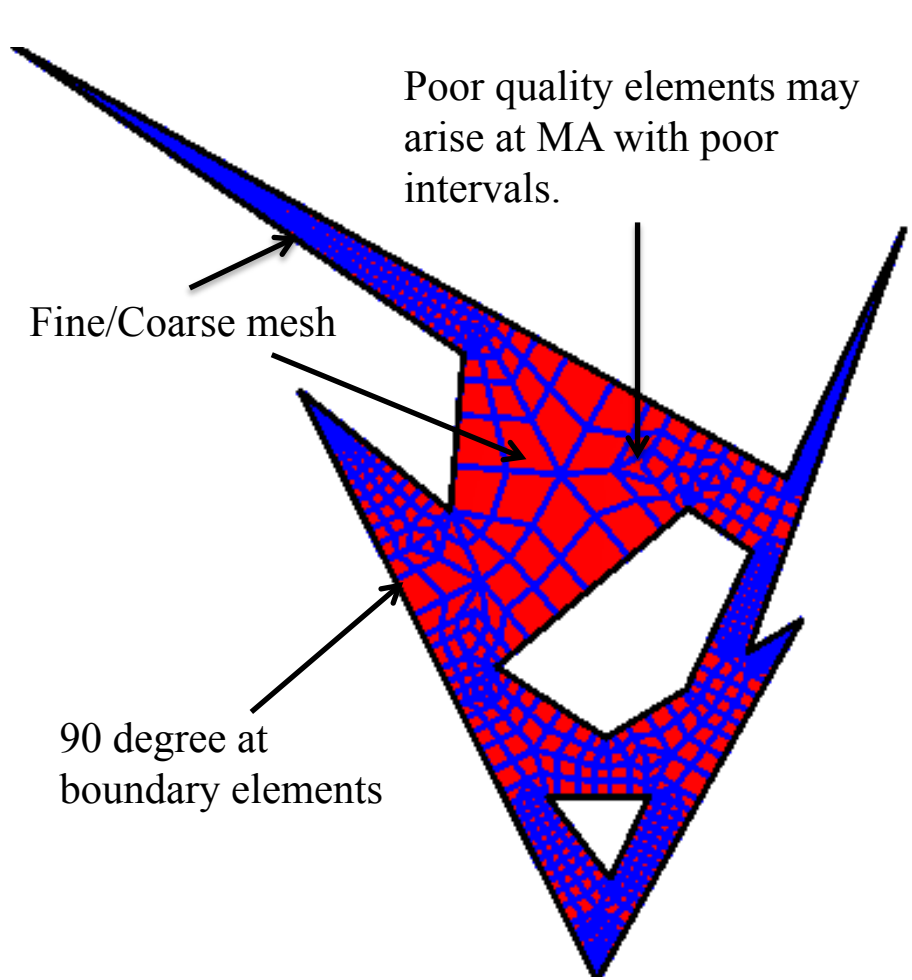
Meshing all medial curves must cover the entire surface

# Generate Tracks: Projection Operator Connects MA Nodes to Boundary



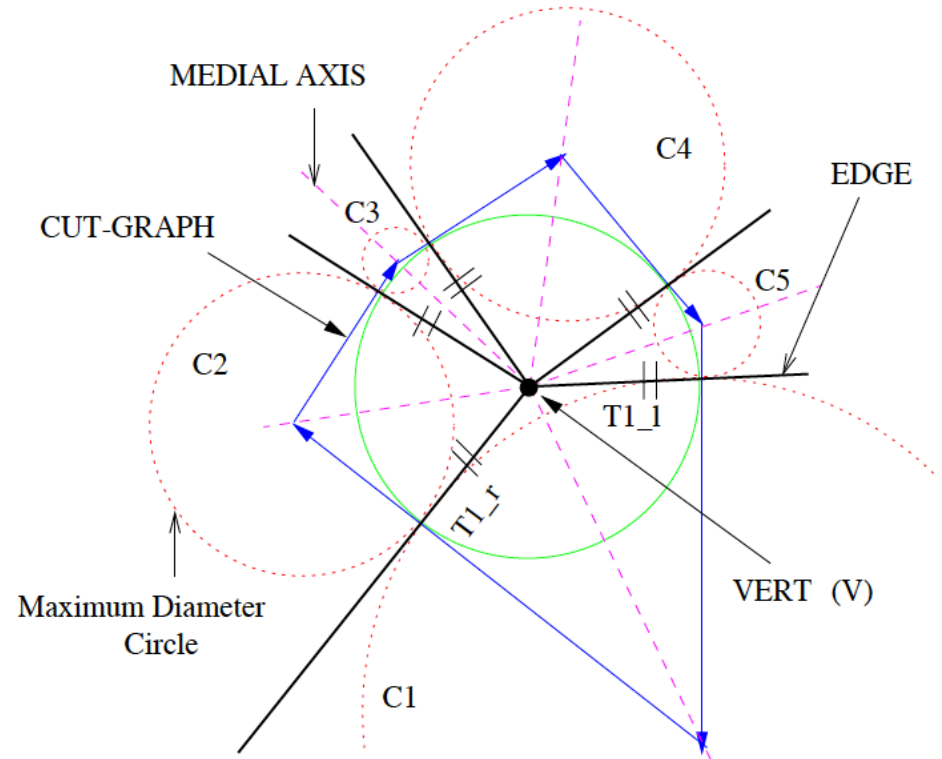
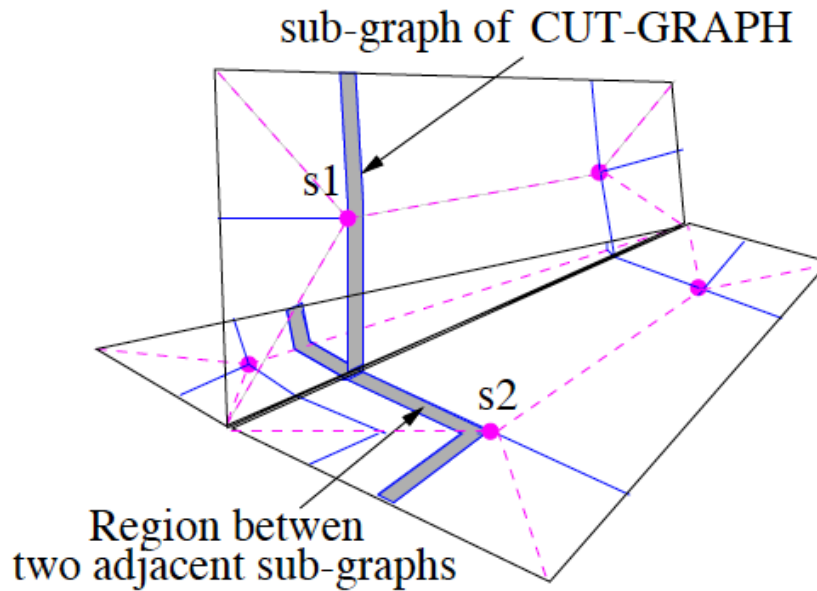
Track width  
controlled by  
medial radius  
function

# Mesh Tracks: All-Quad Mesh can be Generated in 2D



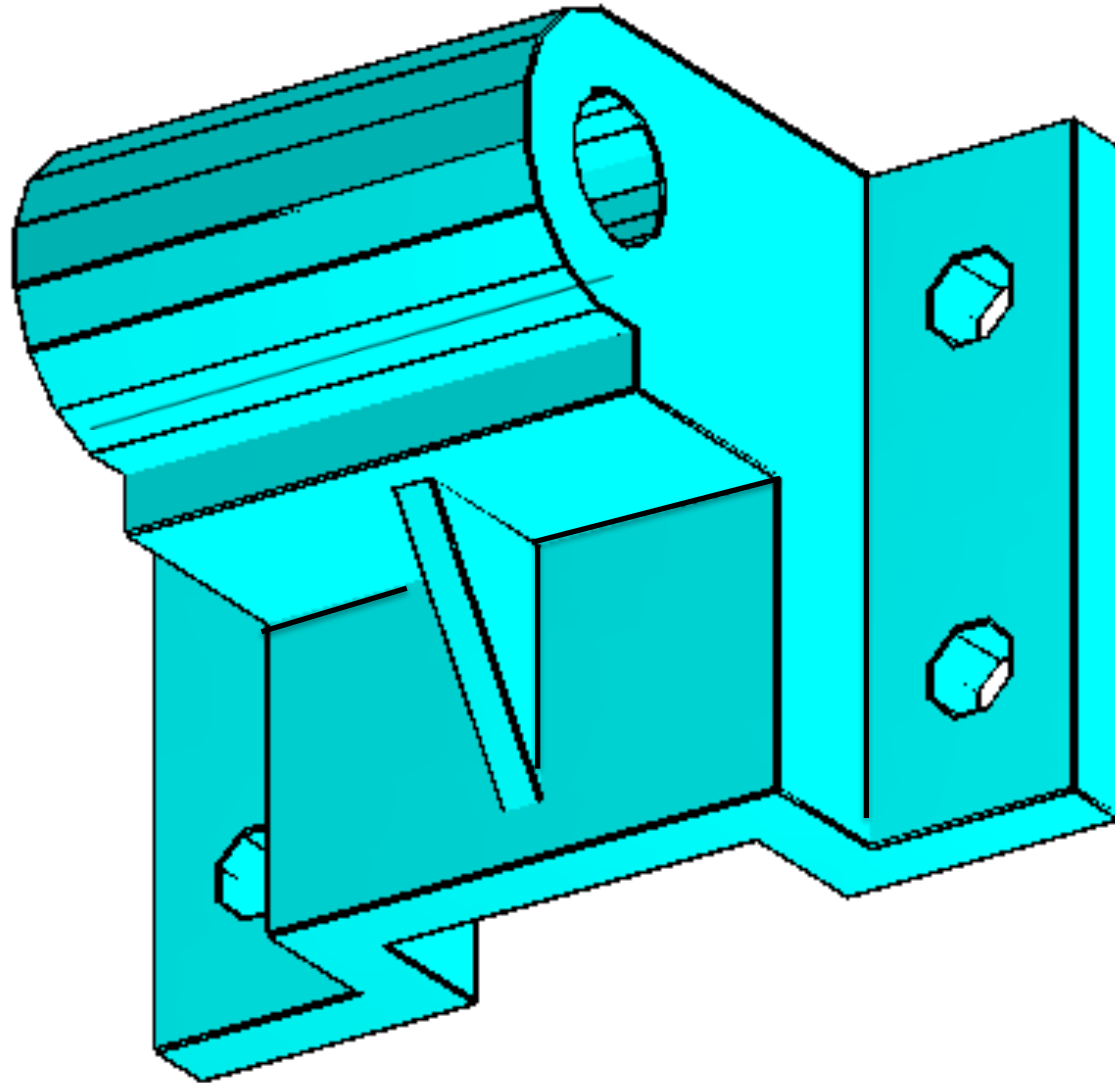
All-Quad: A track is bounded by even number of edges due to symmetric property

# Extension to Multiple Surfaces

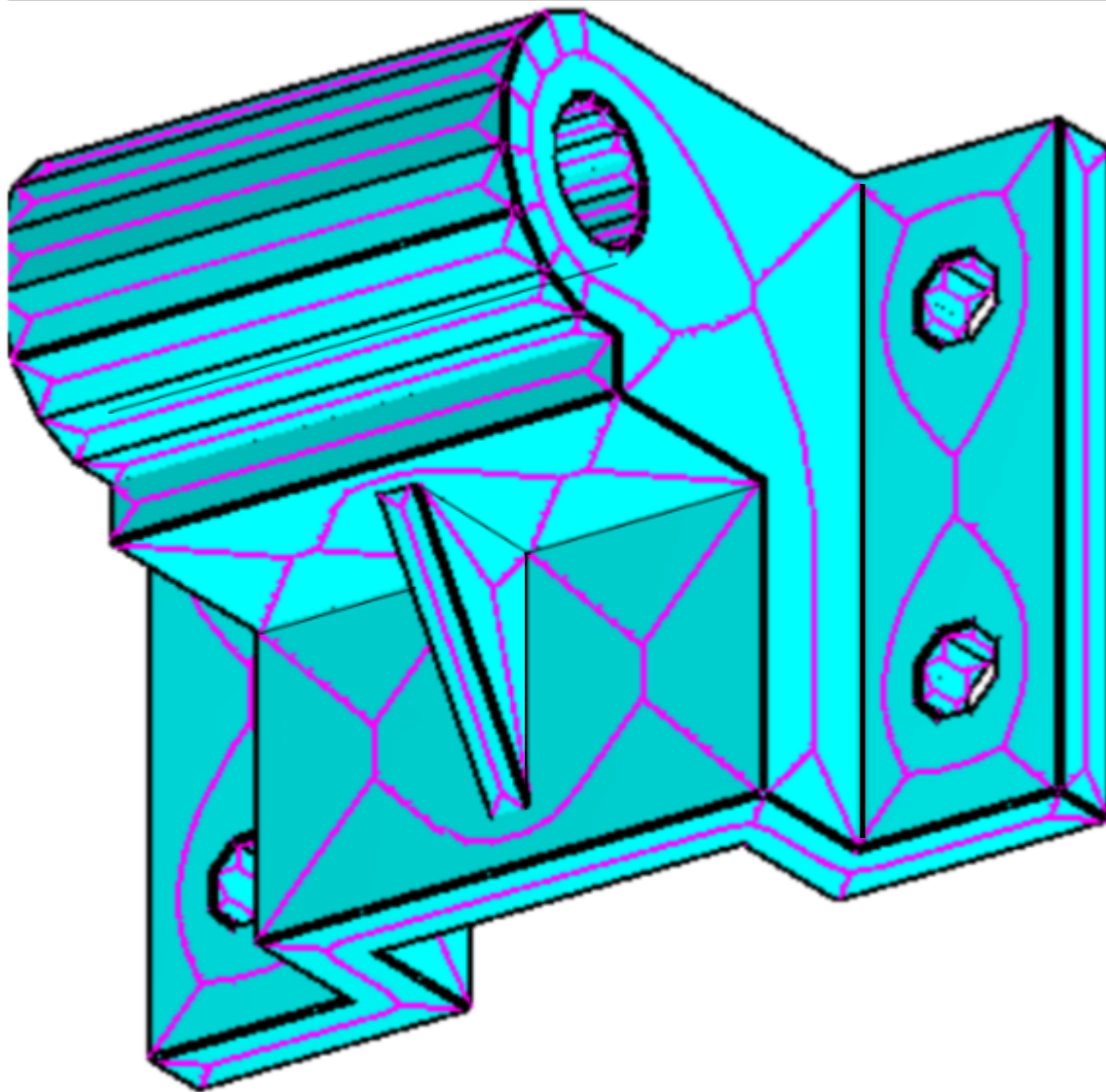




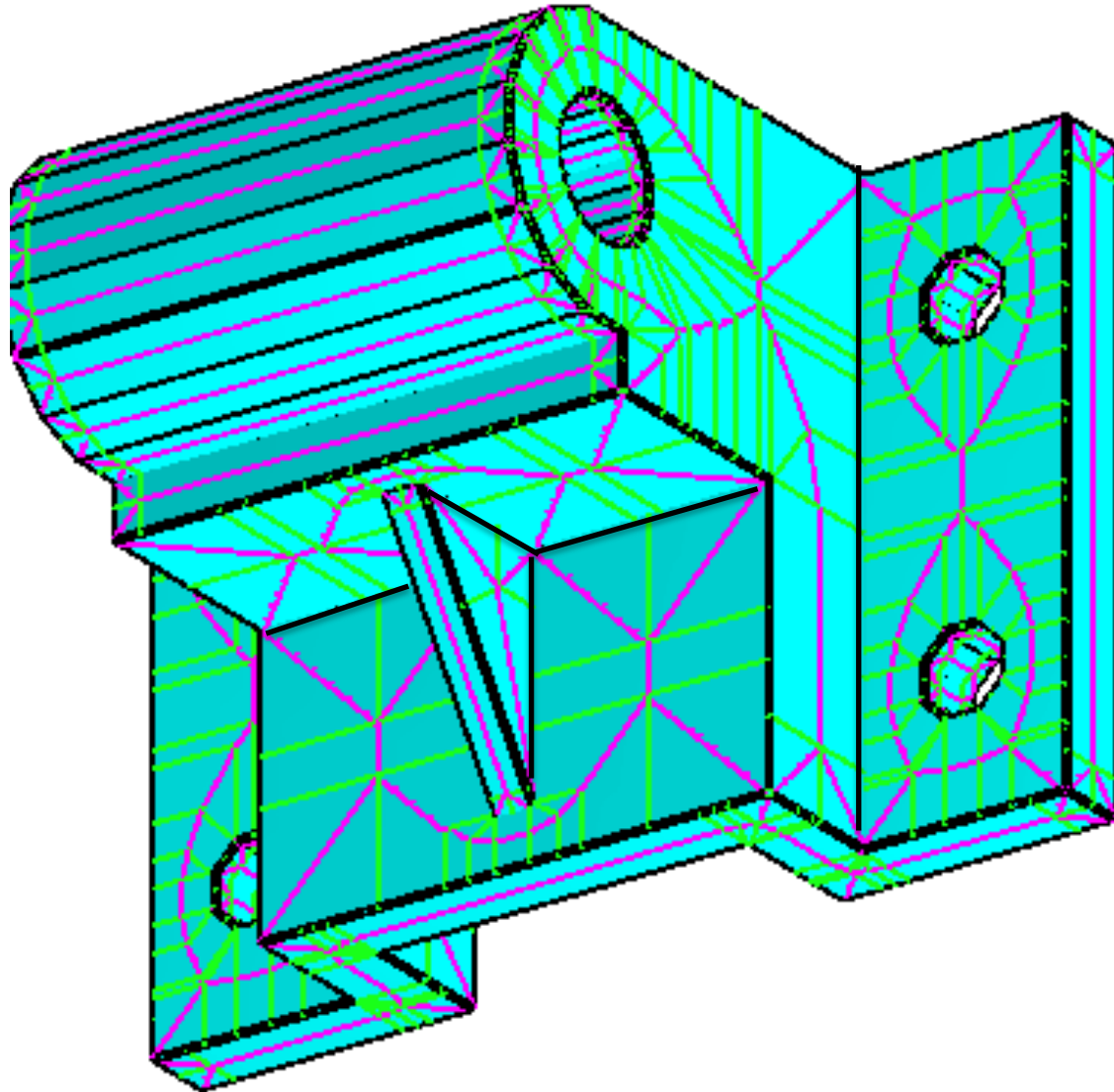
# Multiple Surfaces



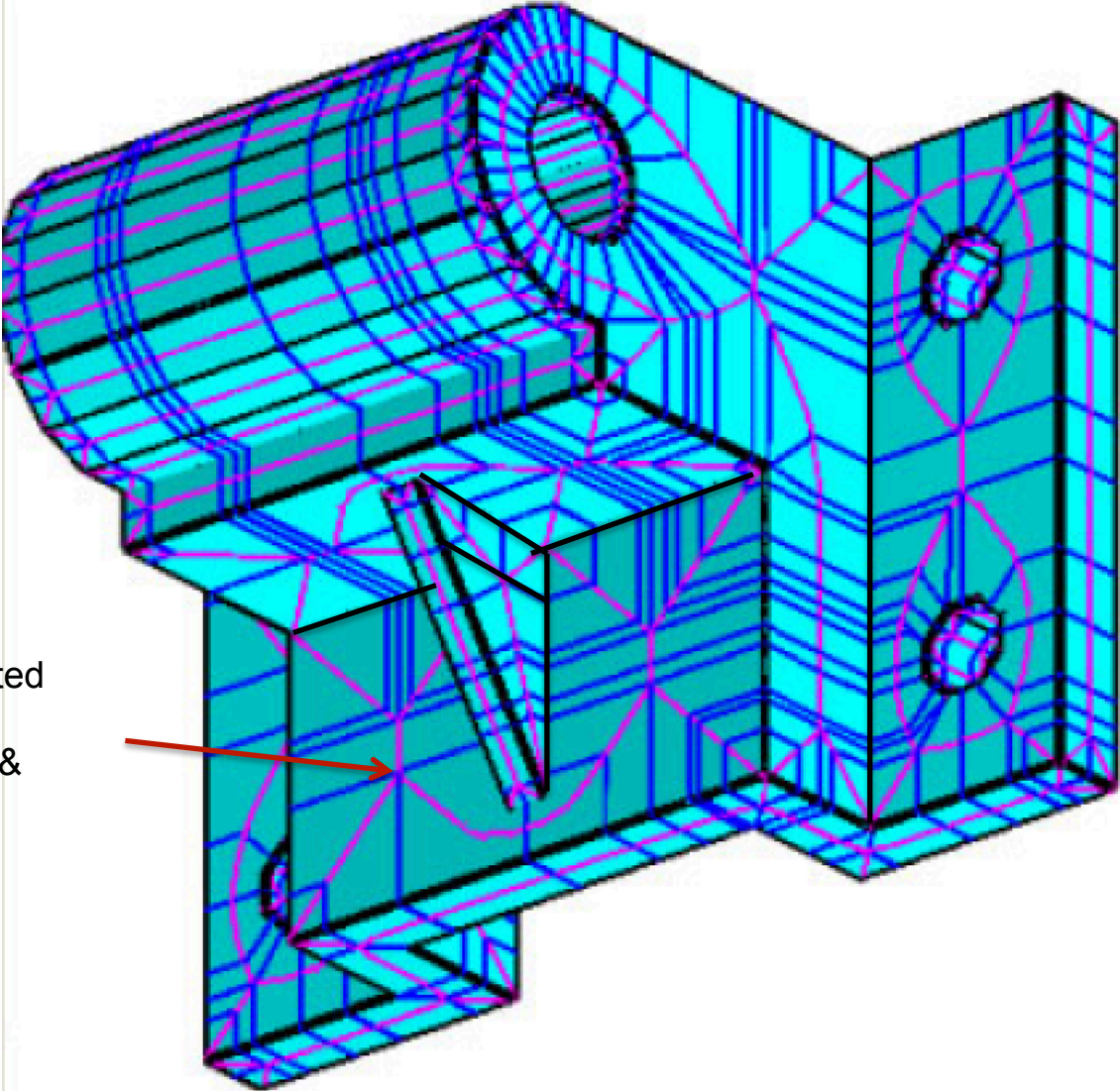
# Medial of Multiple Surfaces



# Two-Way Map on Multiple Surfaces

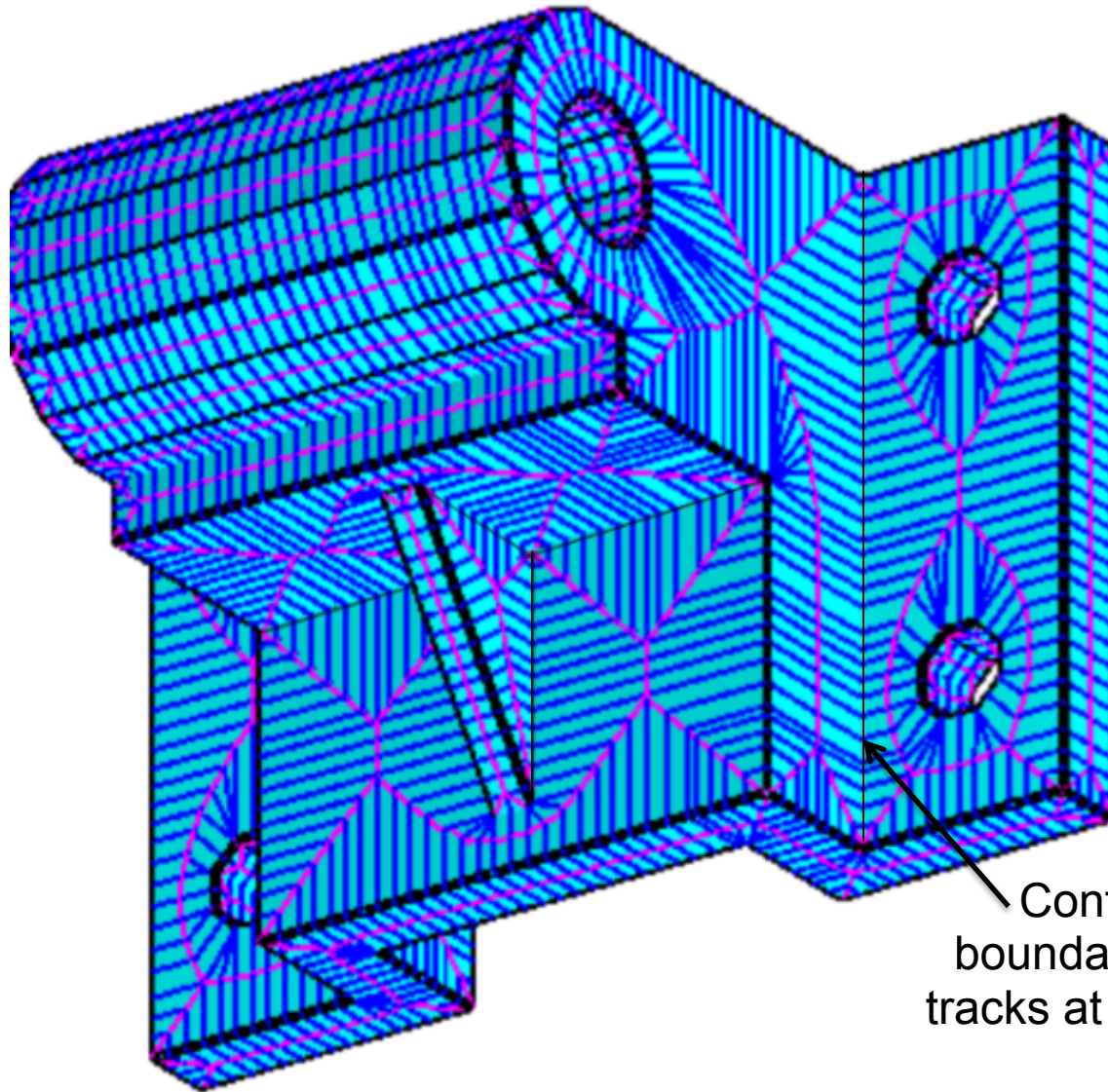


# Automatic Domain Decomposition



Rails propagated  
using medial  
branch points &  
imprints

# Tracks on Multiple Surfaces

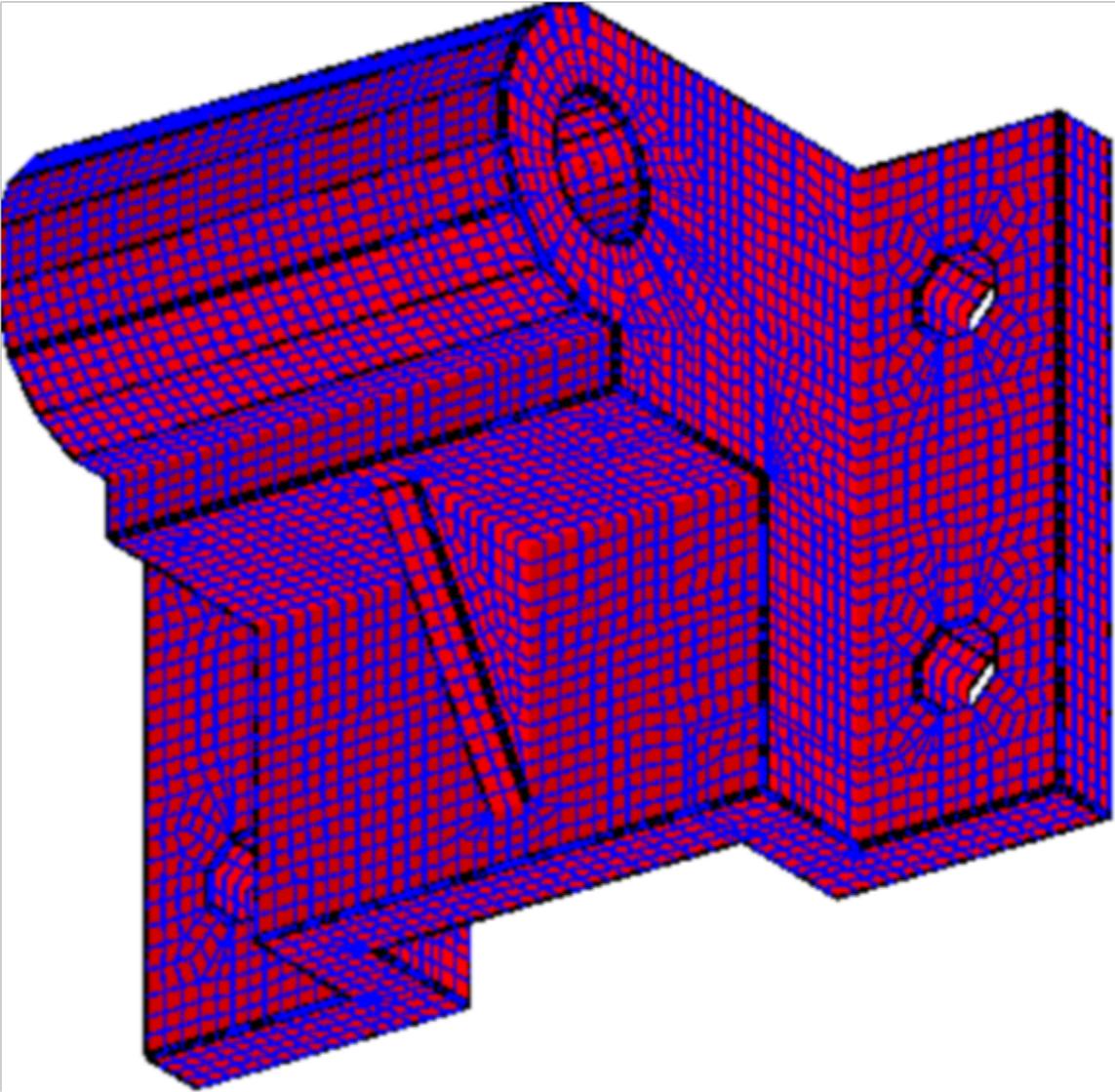


Conformal &  
boundary sensitive  
tracks at the interface

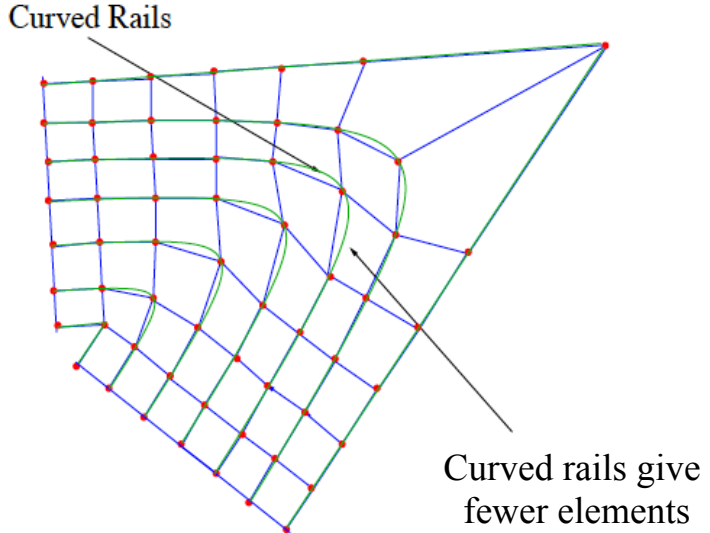
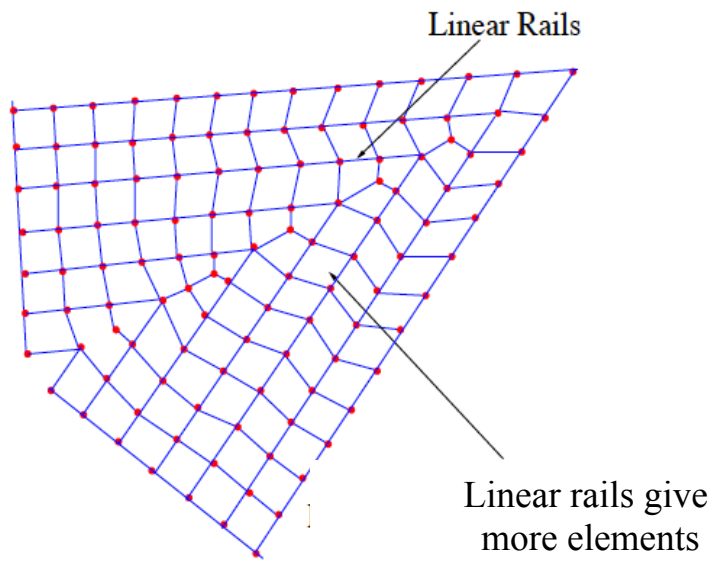
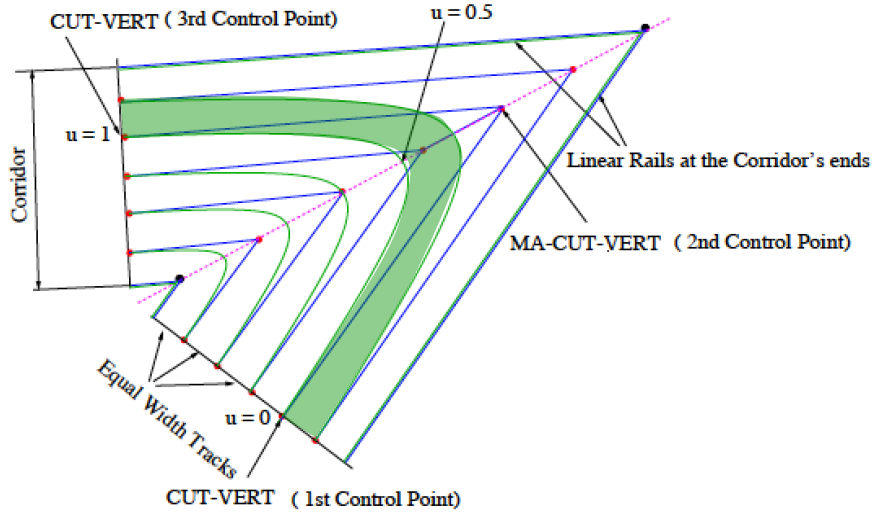


# Quad Mesh on Multiple Surfaces

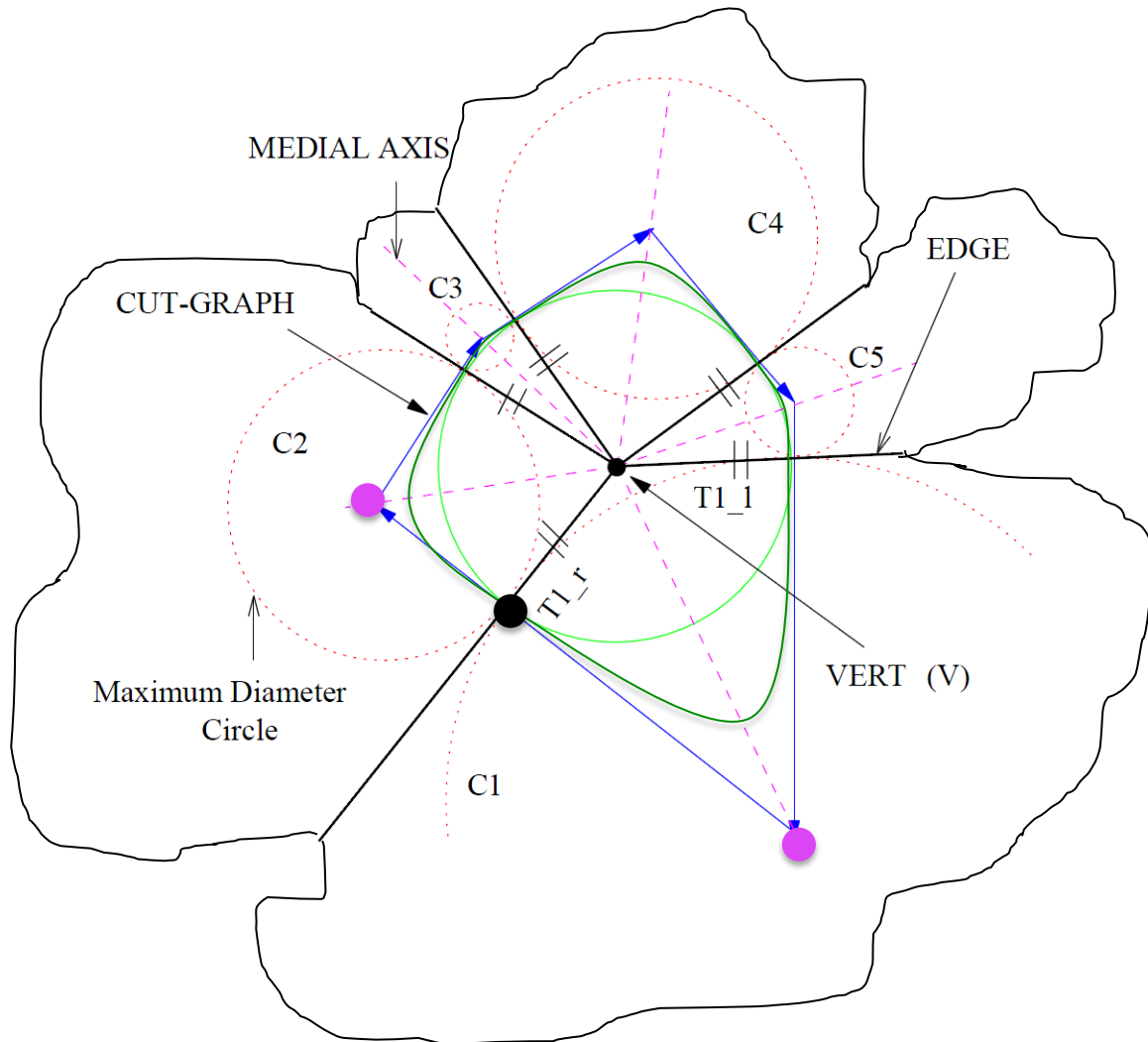
(with no post-meshing operations)



# Extension to Nonlinear Rails & Tracks

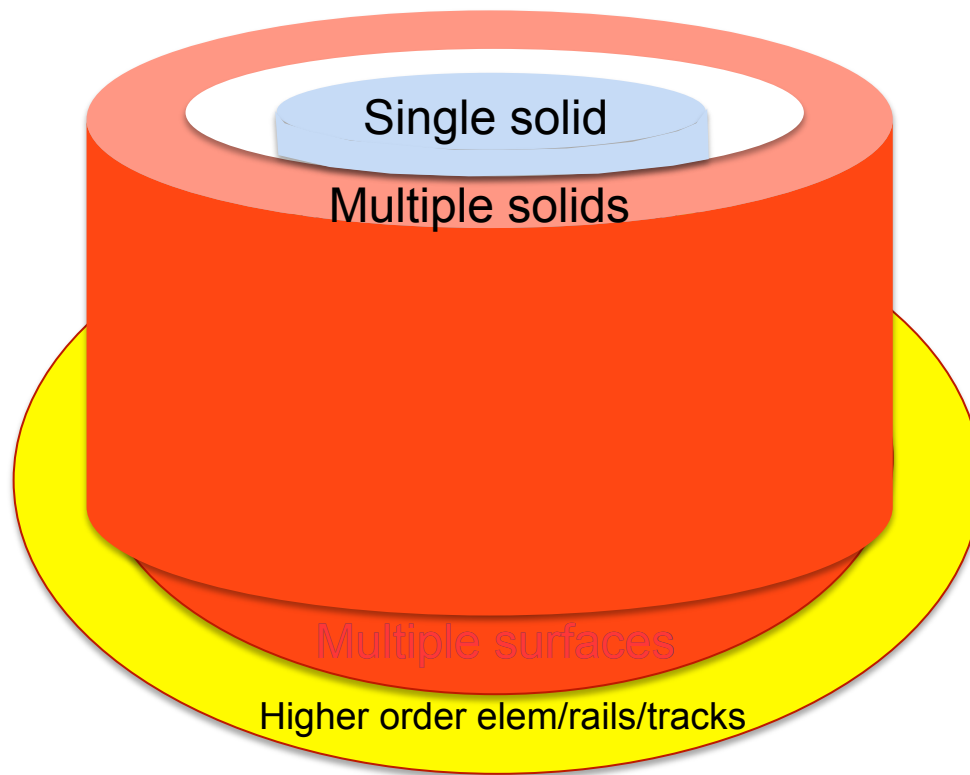


# Rail as Control Polygon Satisfies Orthogonality



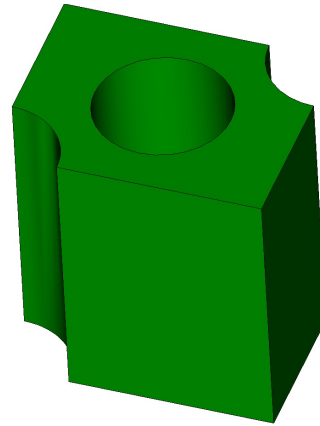
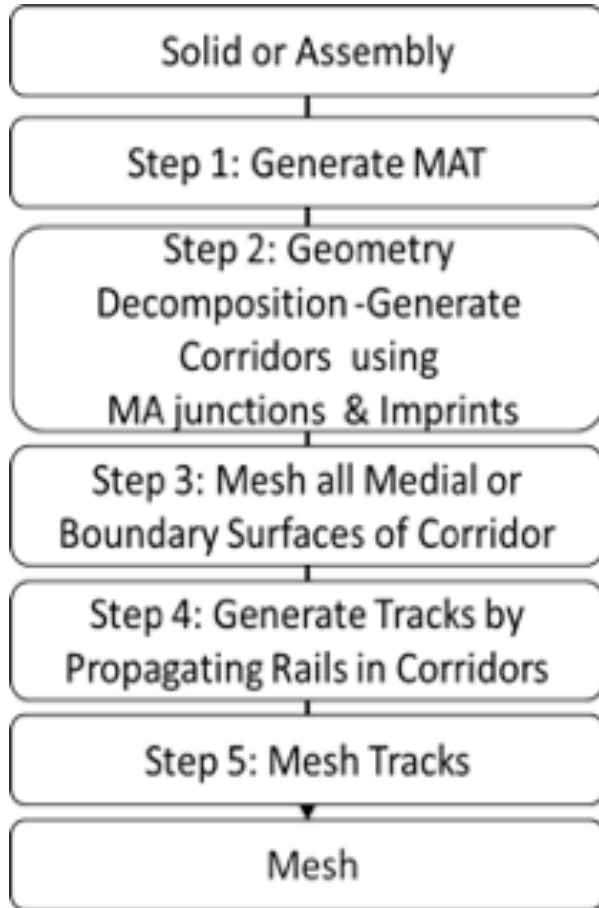


# Layout of Presentation

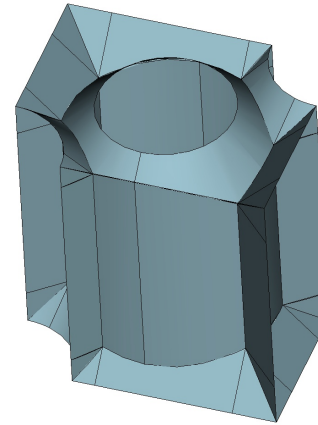


# LayTracks3D: Extension of LayTracks to 3D

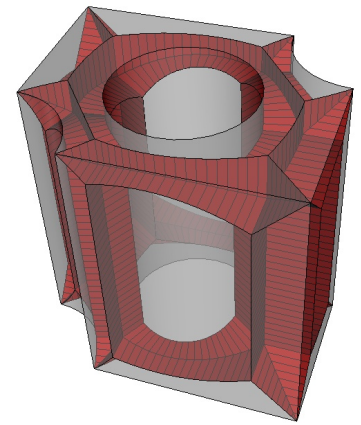
# Overview of LayTracks3D



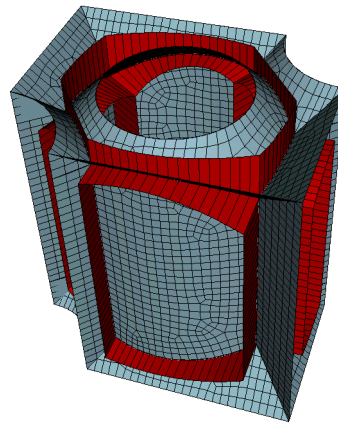
Input solid



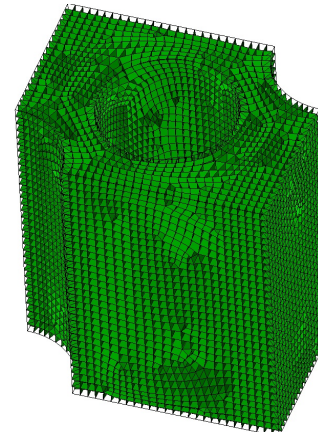
(1) Generate MAT



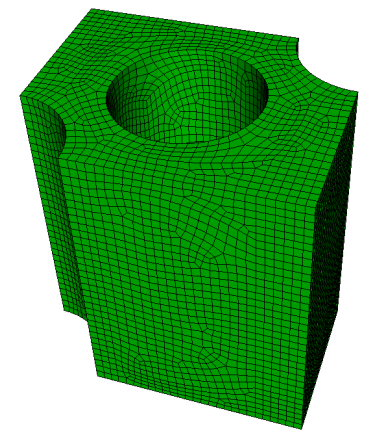
(2) Generate Corridors



(3) Mesh Medial

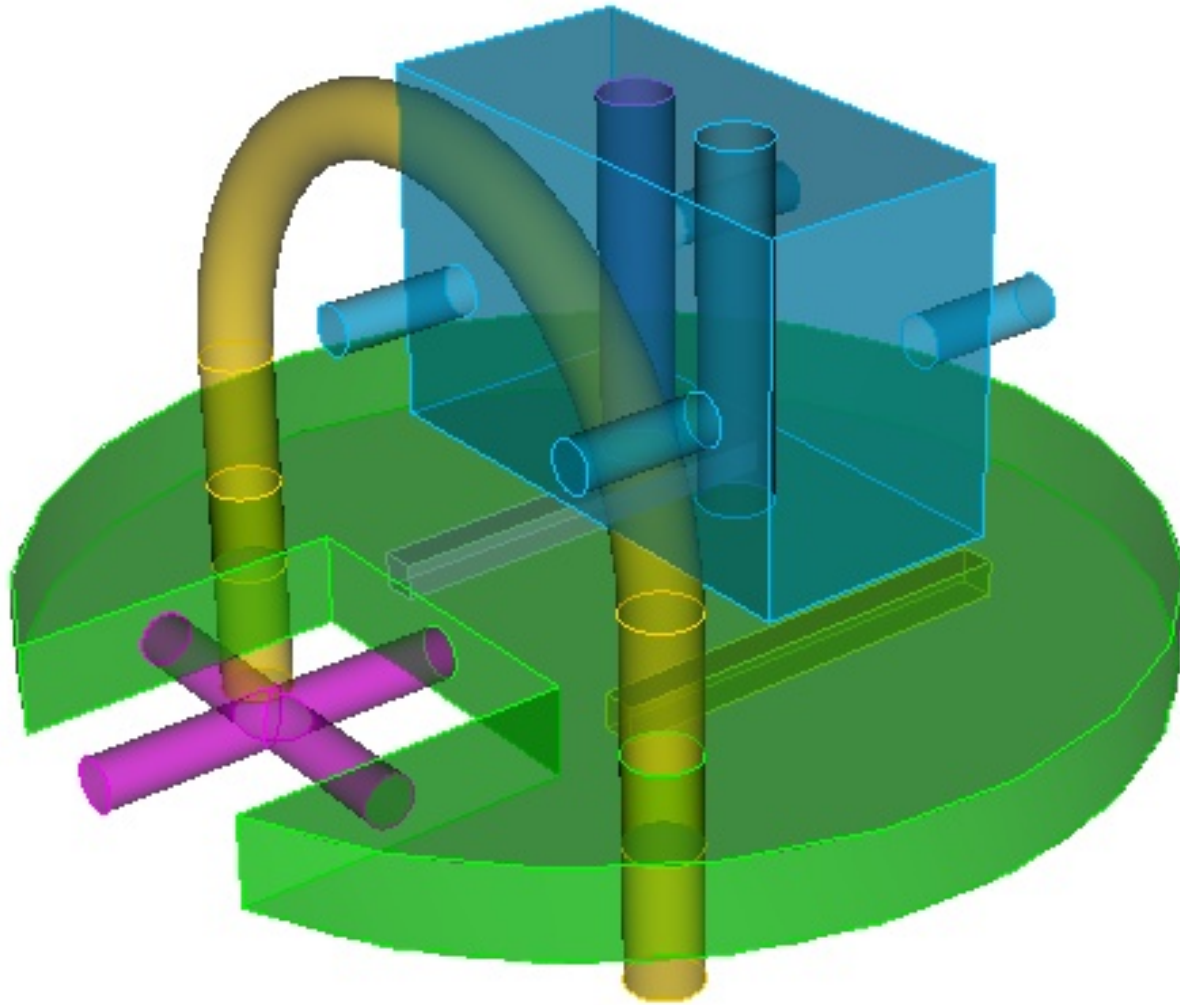


(4) Generate Tracks

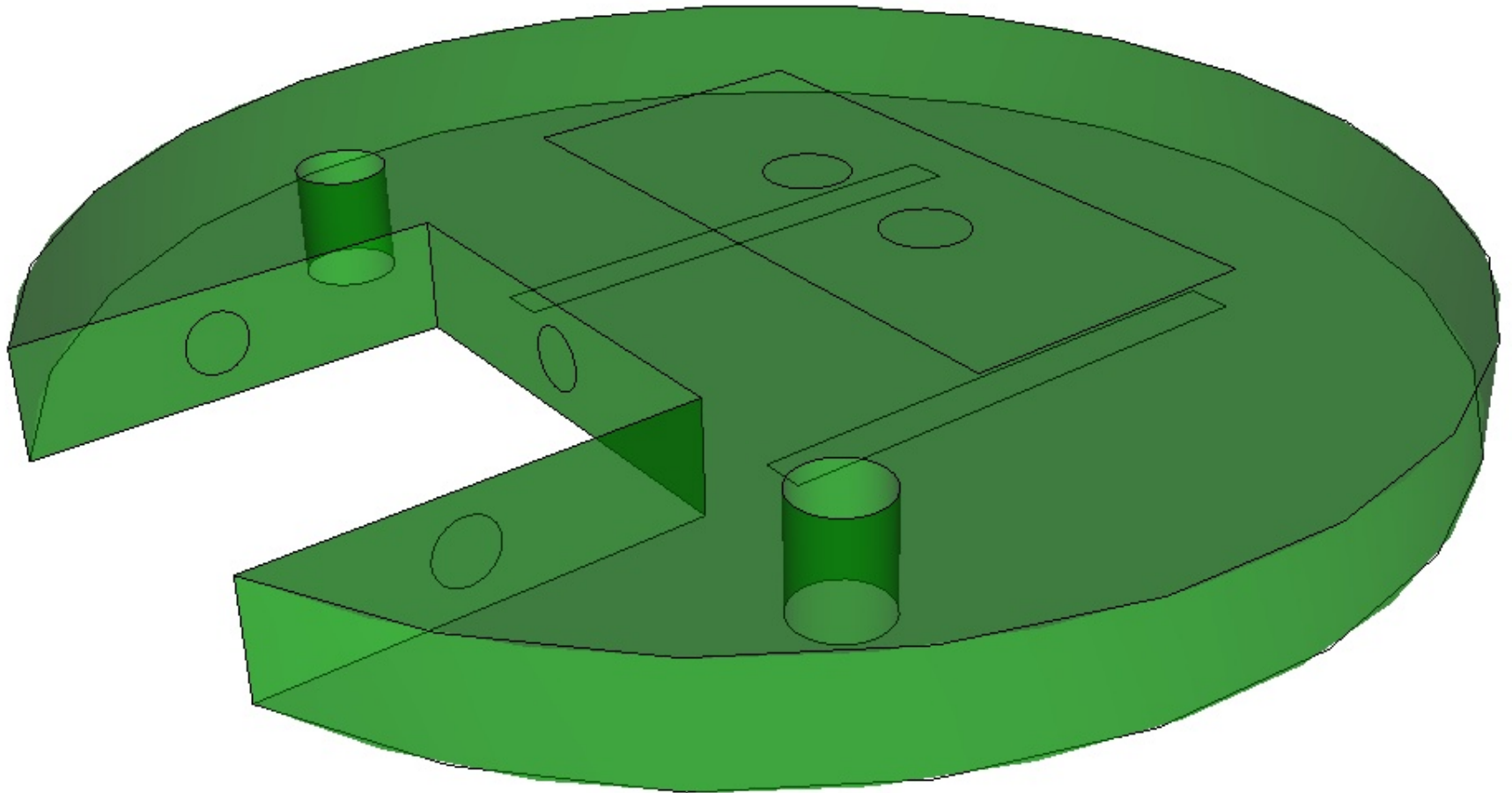


(5) Mesh Tracks

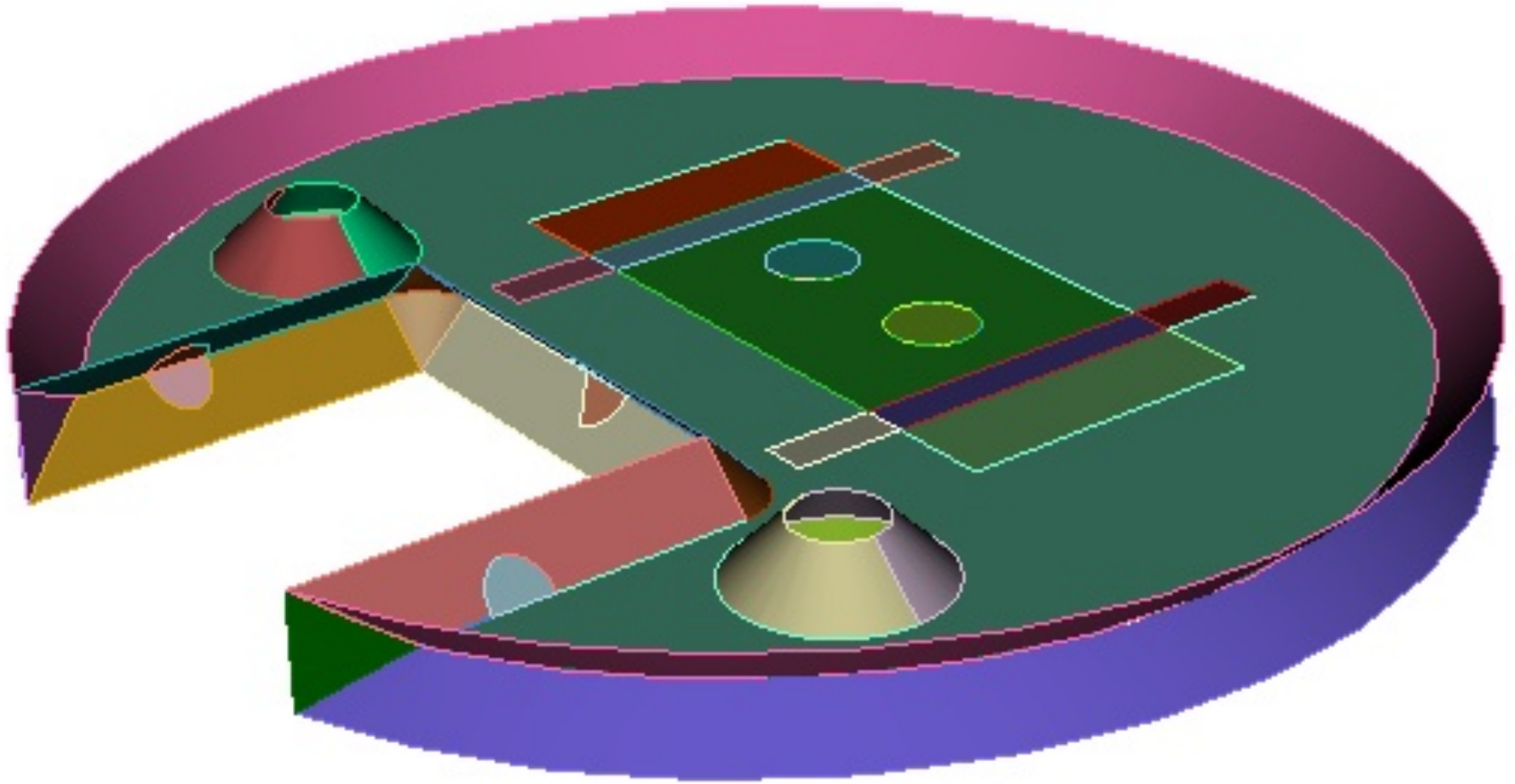
# Extension to Assembly Model



# Imprints on Top, Bottom, and Sides

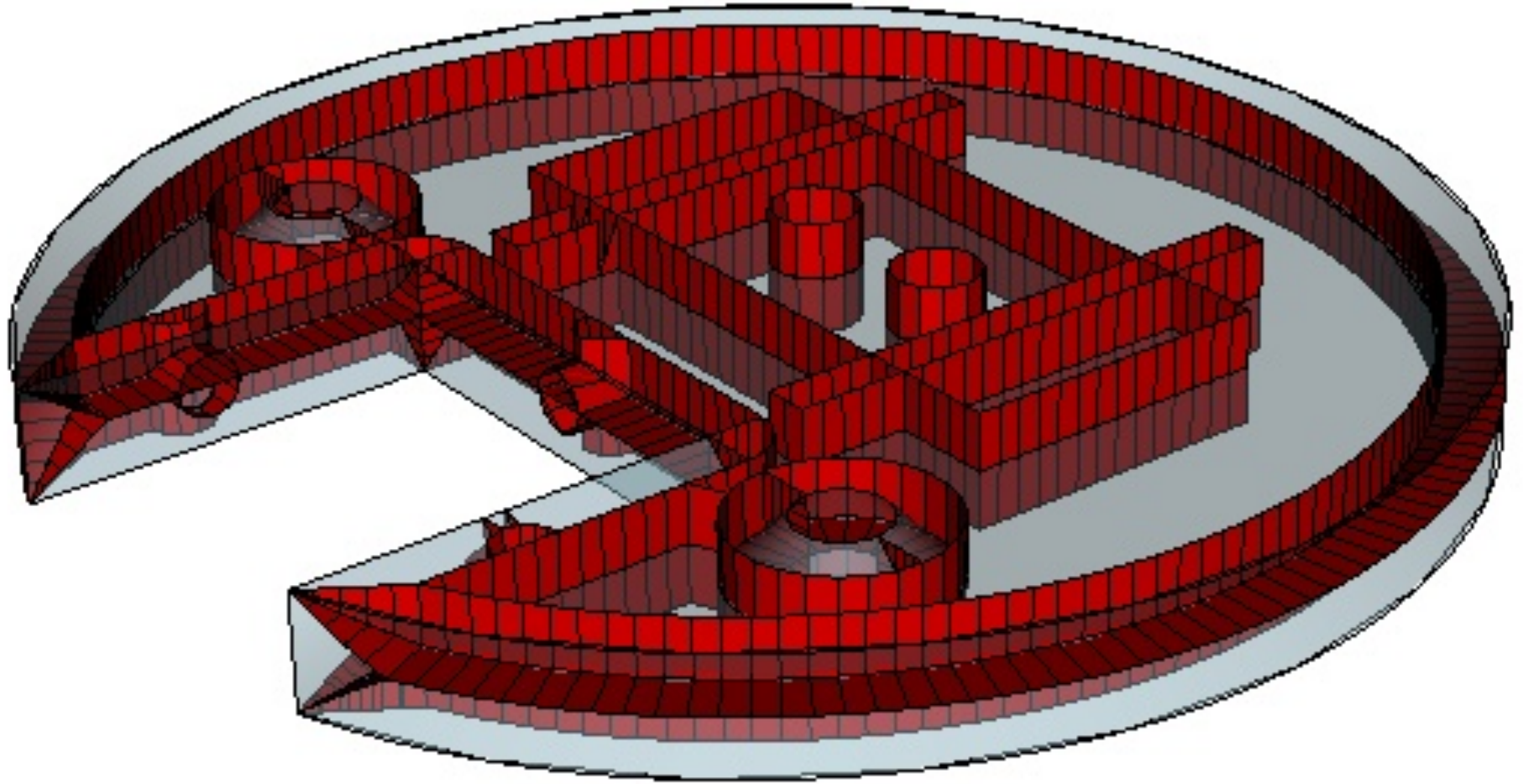


# Medial Resolves Imprints from All Directions



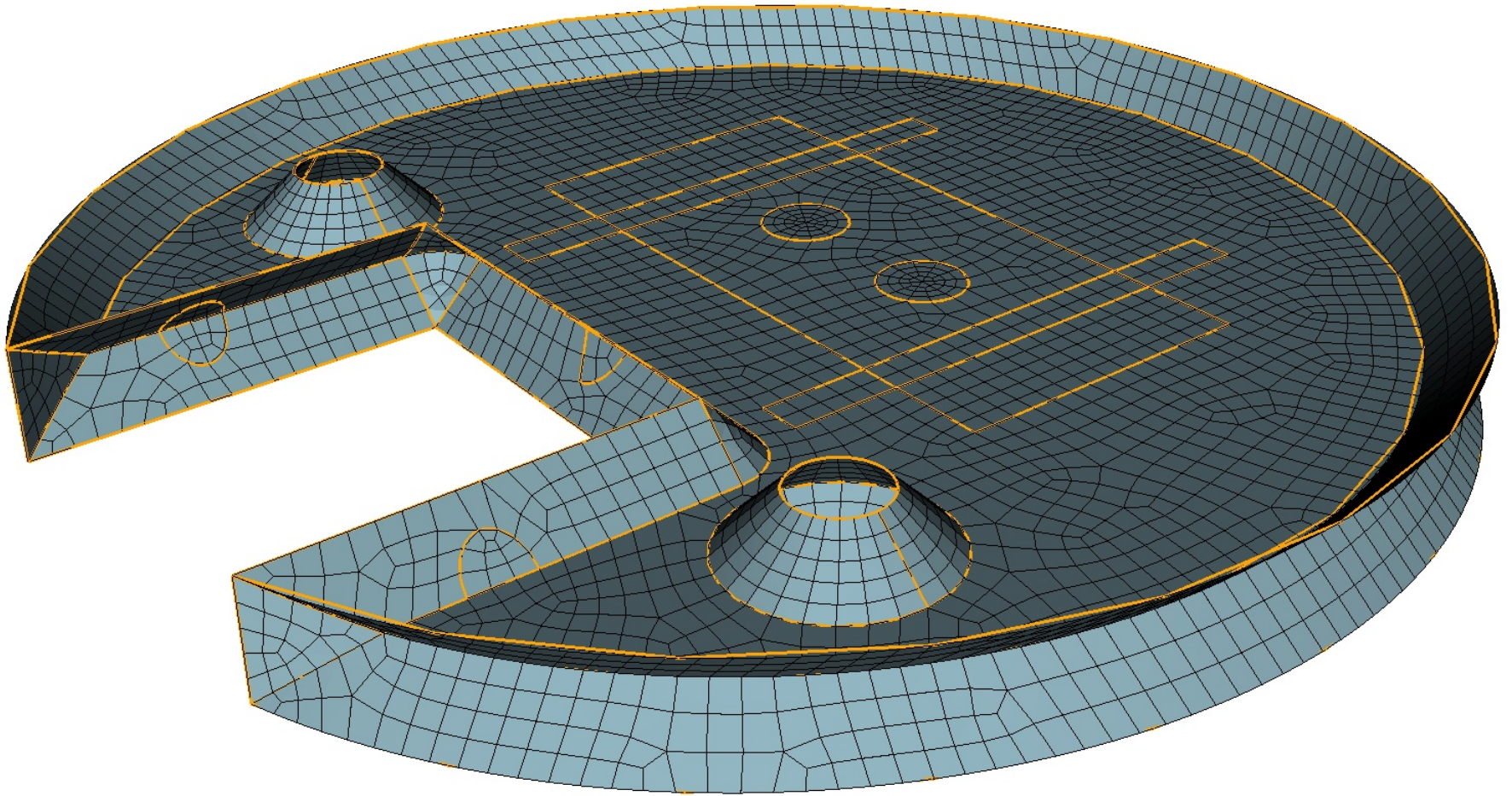


# Automatic Geometry Decomposition using Imprints and Medial Junctions



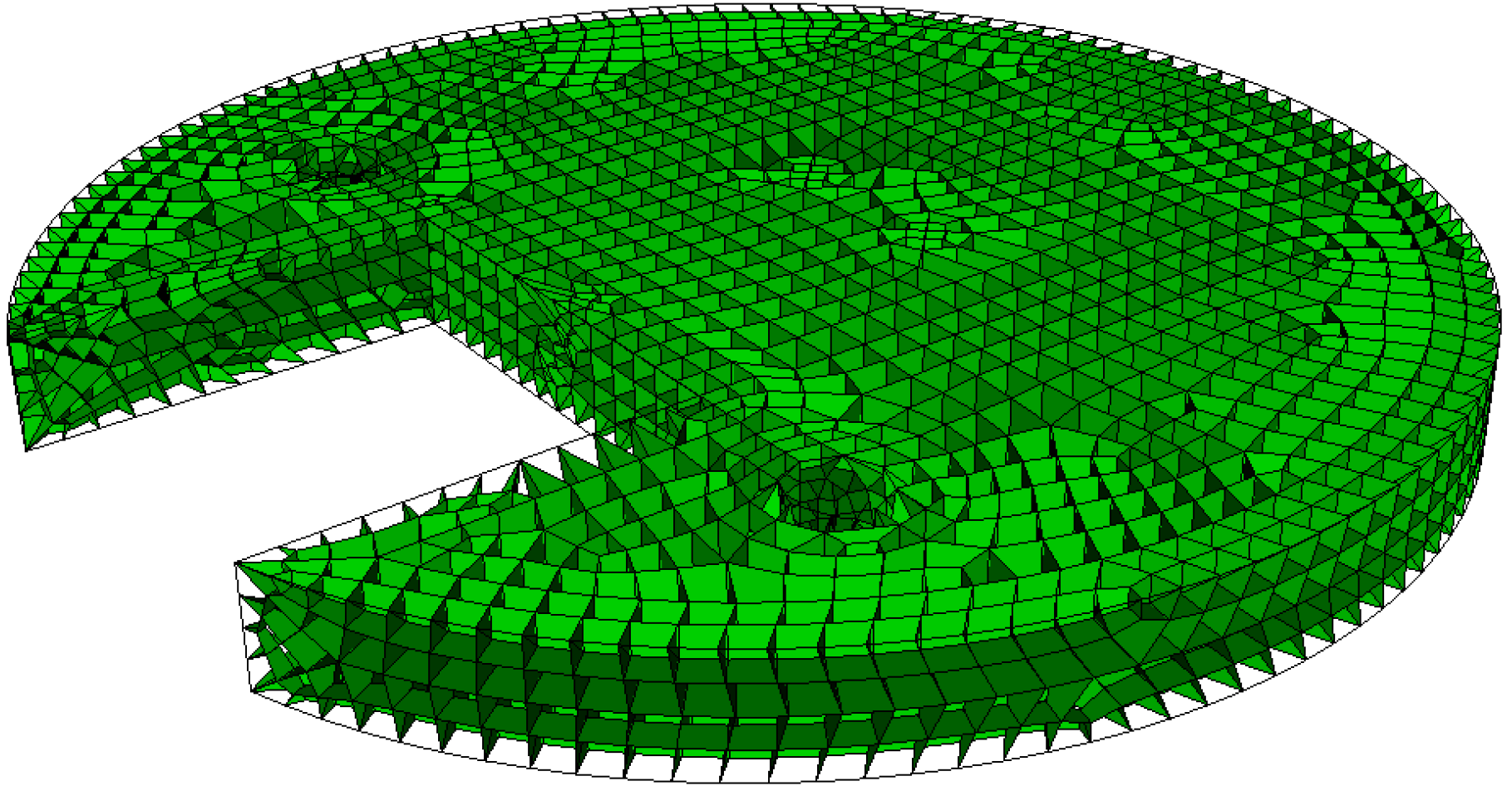
Corridors

# Quad Mesh on Imprinted Media

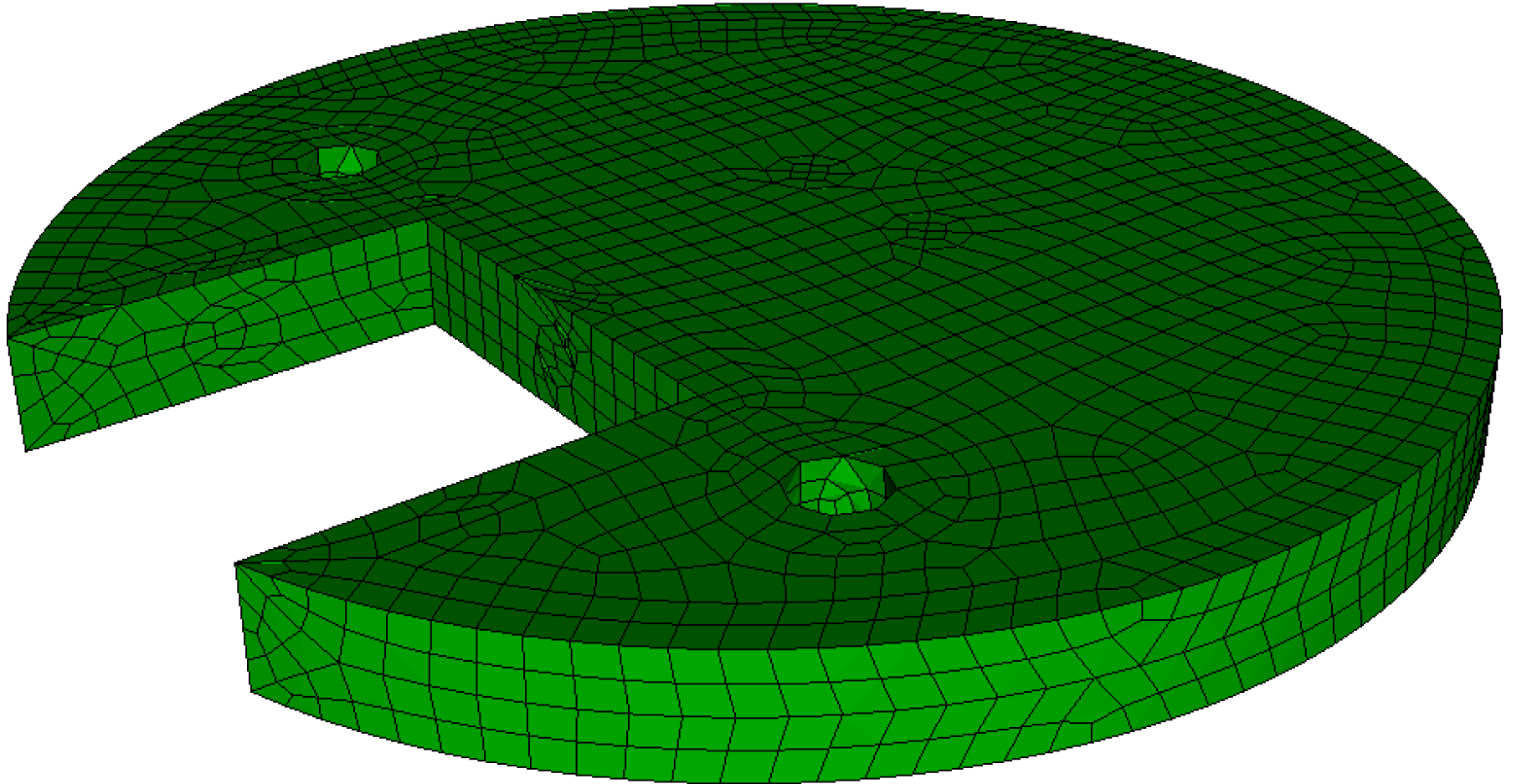




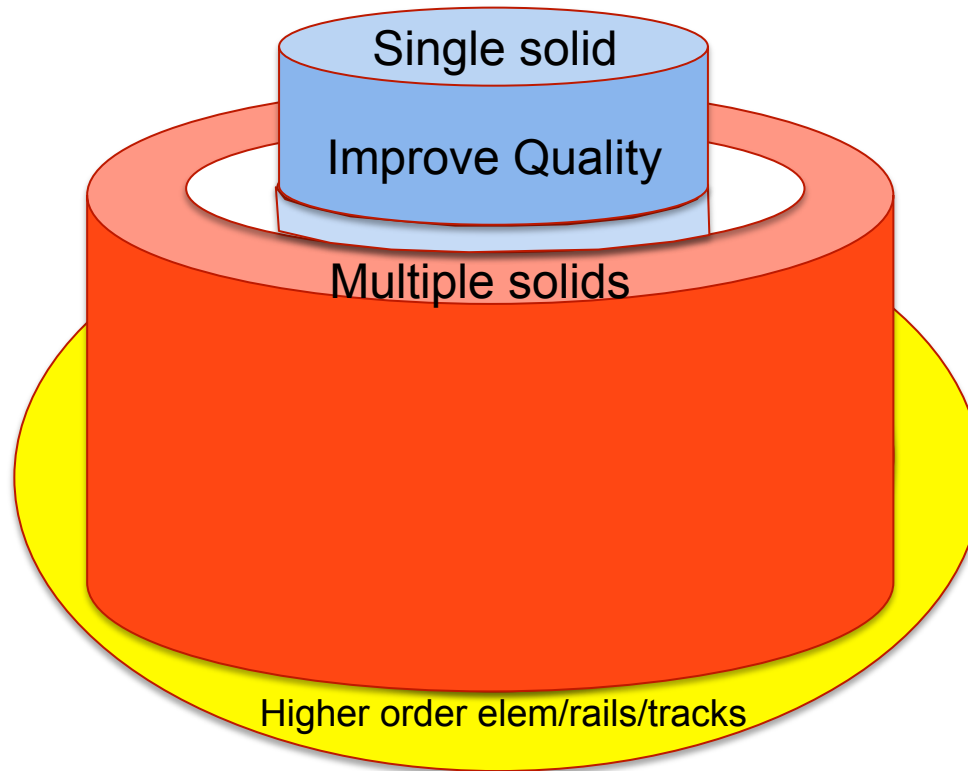
# Tracks in 3D



# Hex-Dominant Mesh

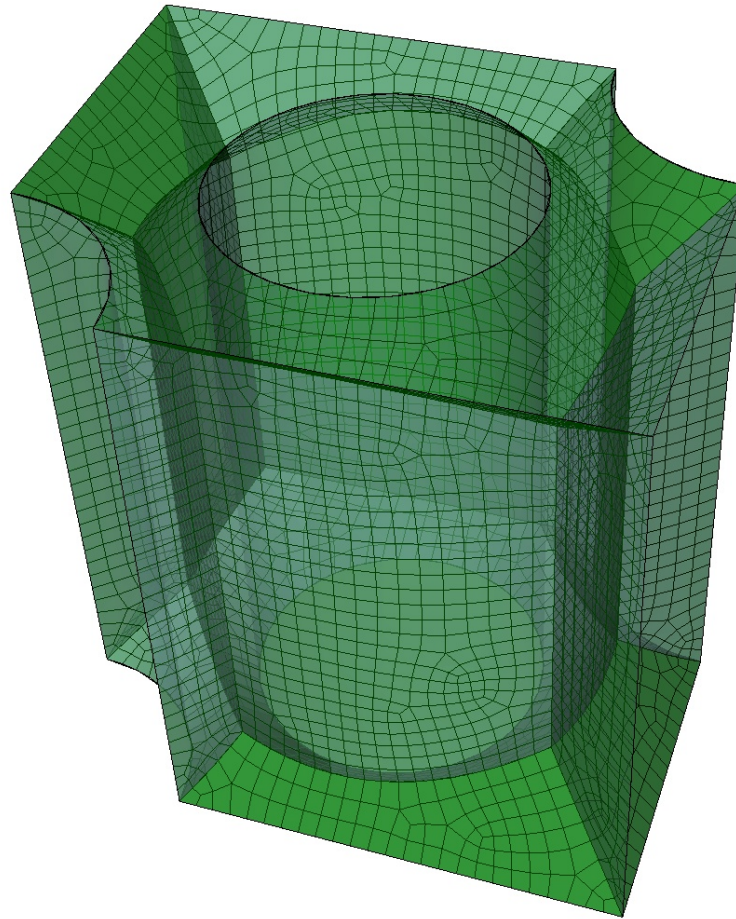


# Layout of Presentation



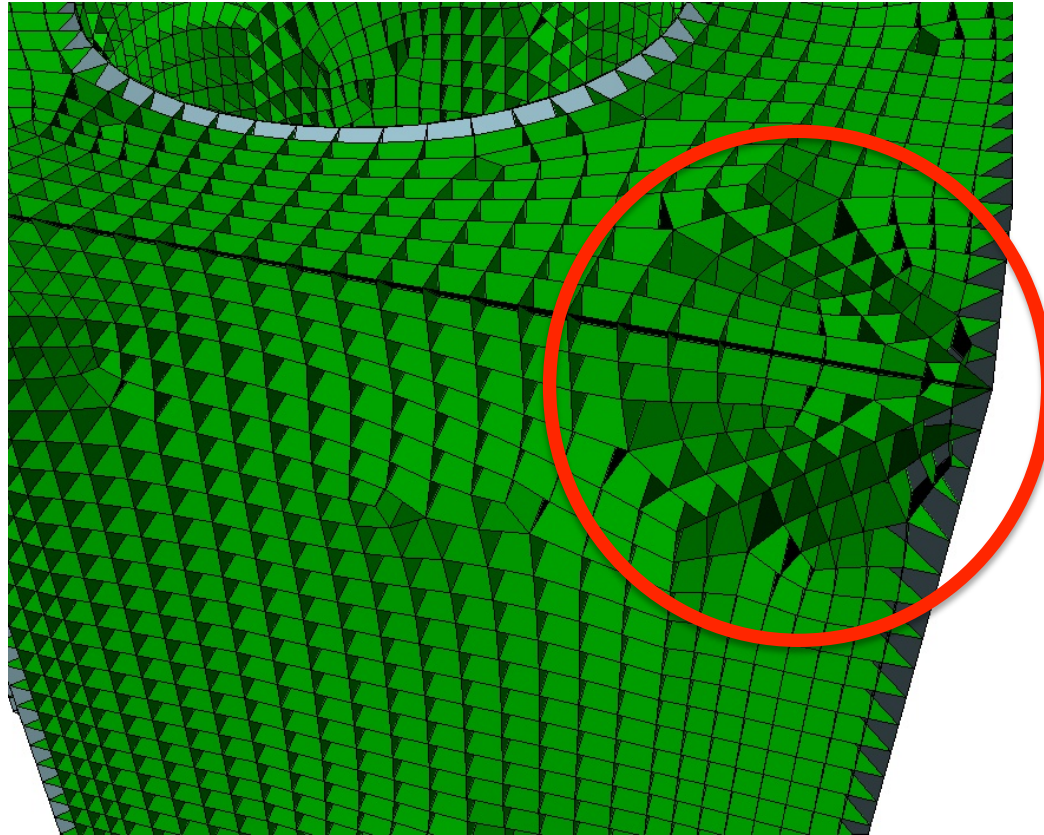
# Improve Quality at Common 3-manifold MA Curve

# All-Quad Mesh on Medial is not Ideal!



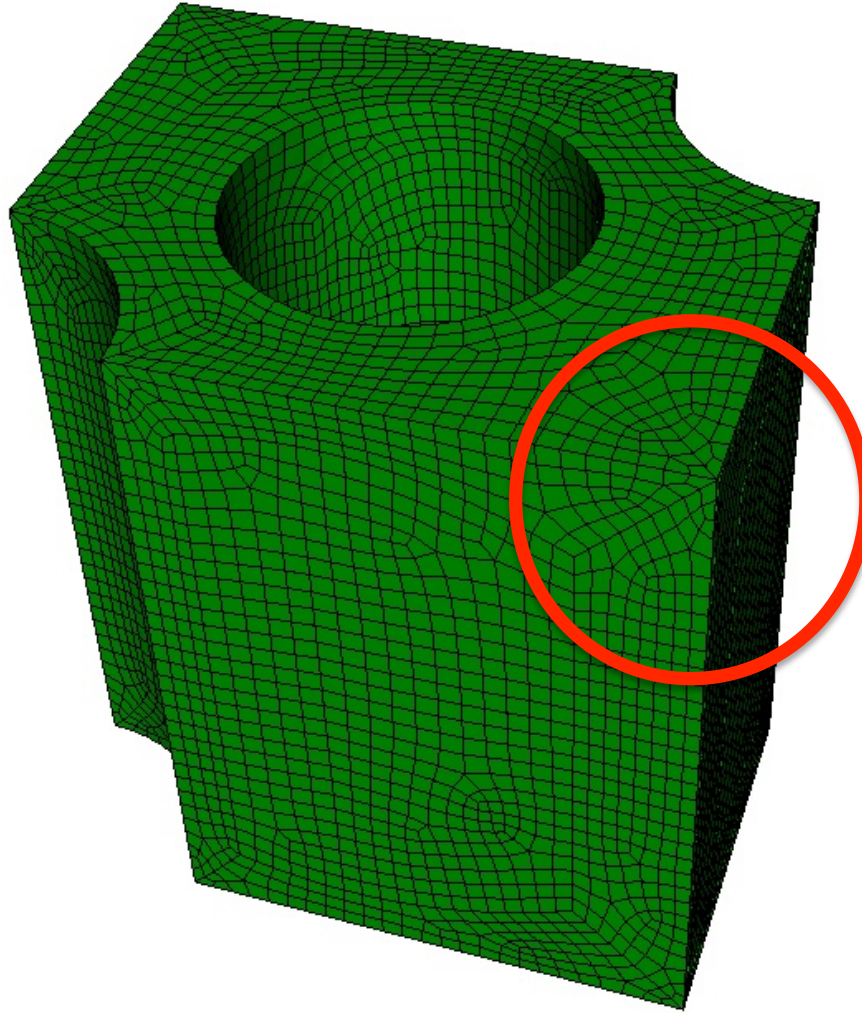
All-Quad Mesh on  
Medial Surface

# Poor Tracks at a Convex Vertex





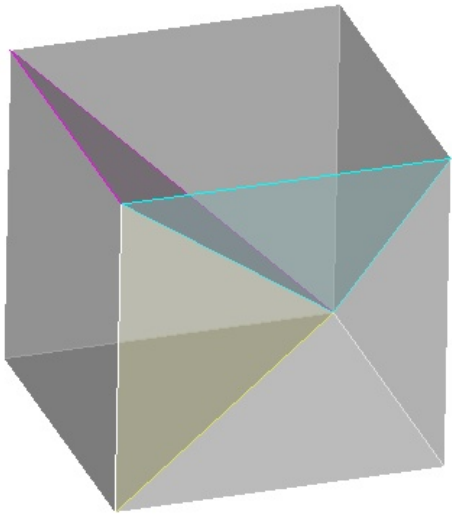
# Hex-dominant Mesh at Convex Vertex



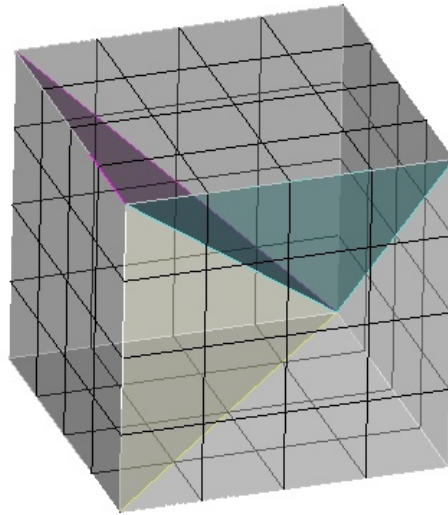
# How to Improve Hex Quality at a Convex Vertex?



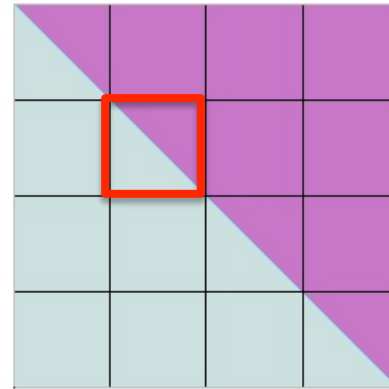
# Ideal Mesh Should Contain Tri Along 3-Manifold Medial Curve!!!



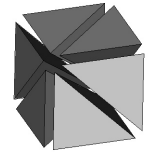
Typical 3-manifold MA



Ideal Hex Mesh

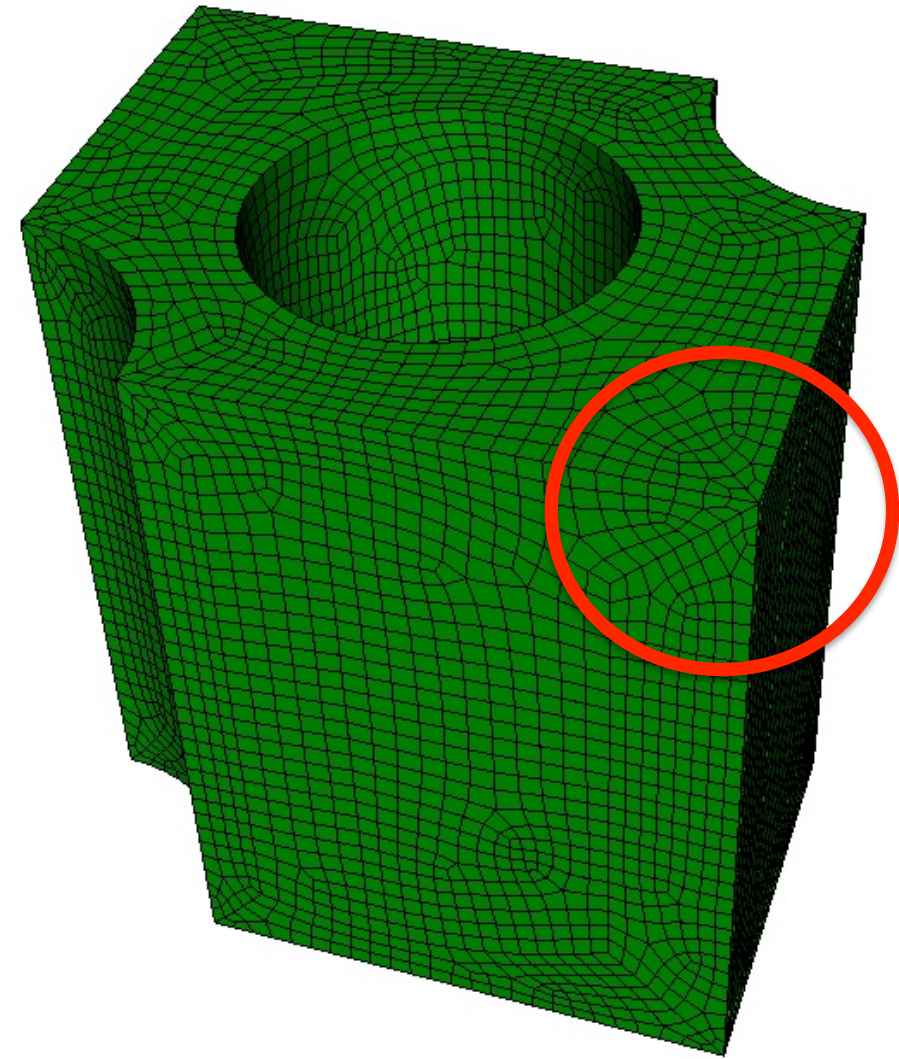


Surface Mesh on MA

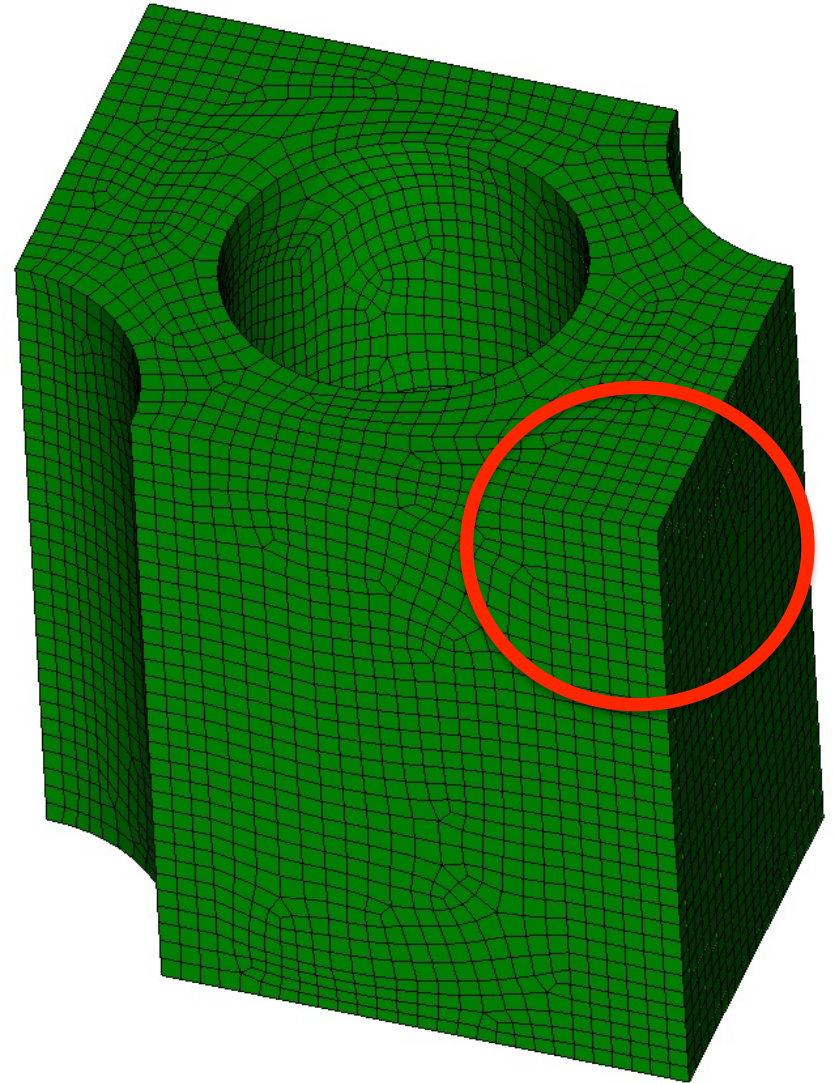


6 Tets form a Hex

# Improved Hex-dominant Mesh



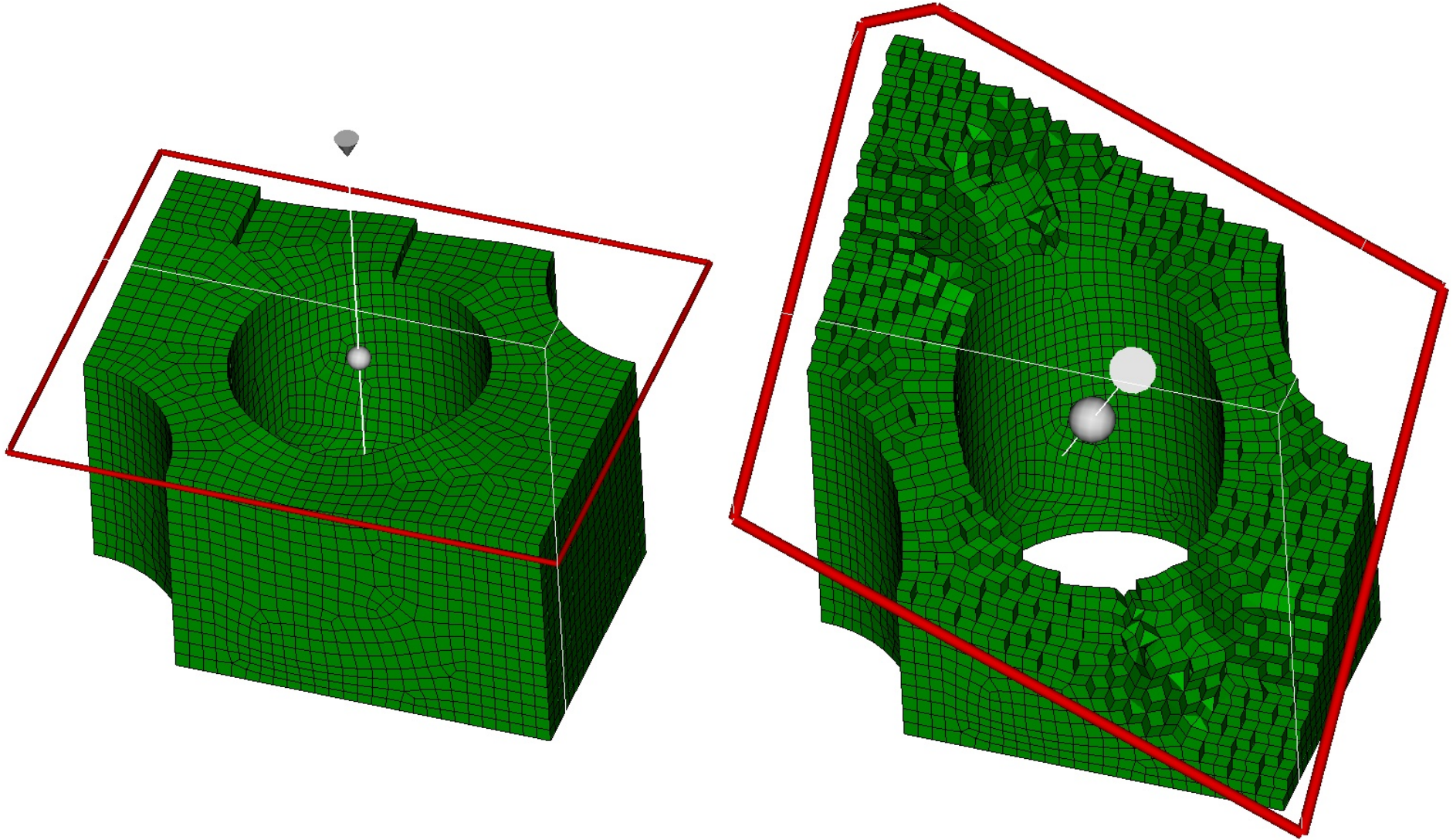
All-Quad Mesh on Medial



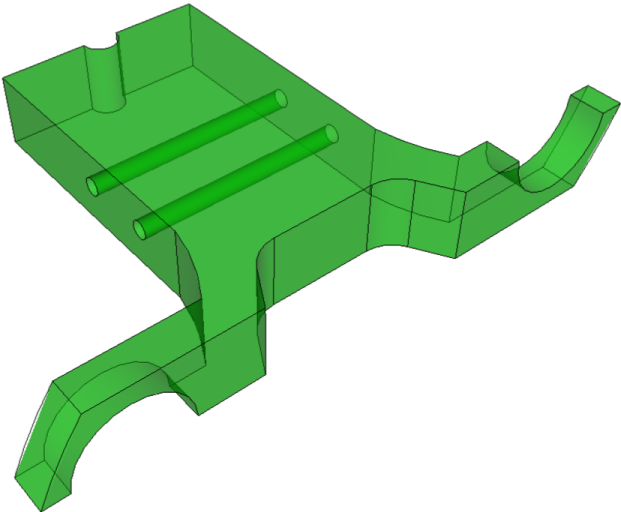
Tri at 3-Manifold Medial Curve



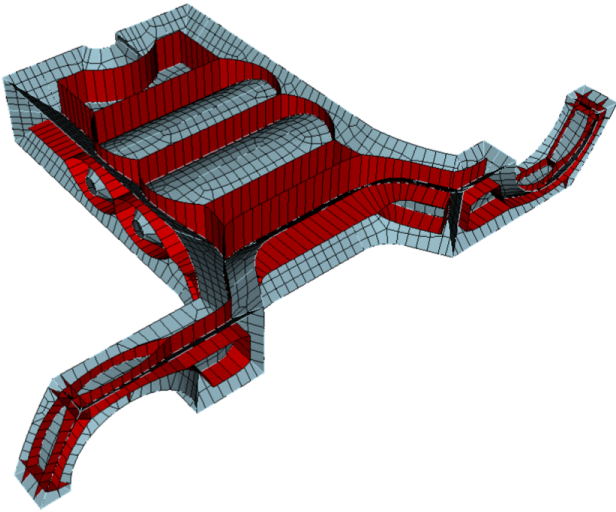
# Sectional View



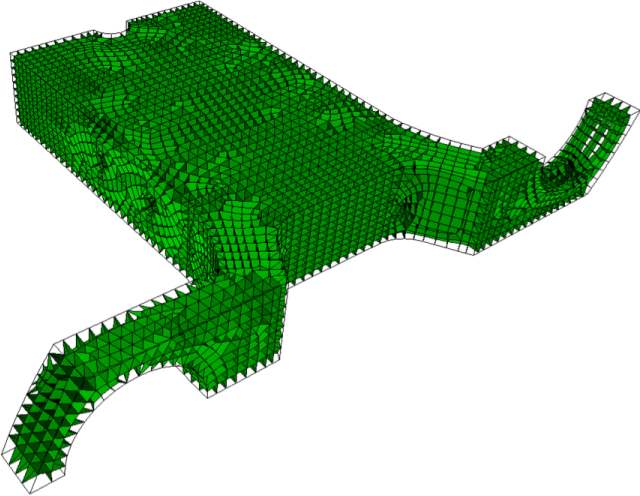
# Demo



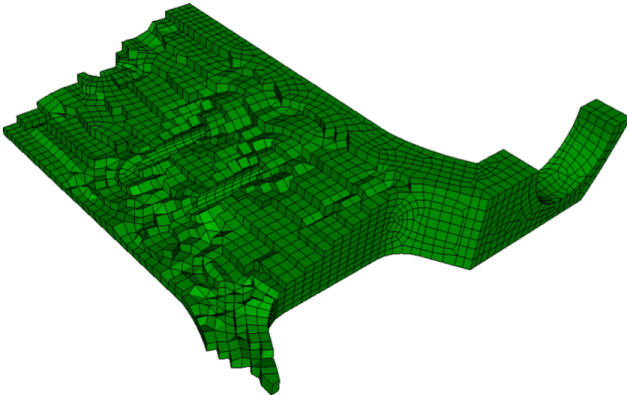
(a) Solid with holes



(b) Mesh on MA inside corridors

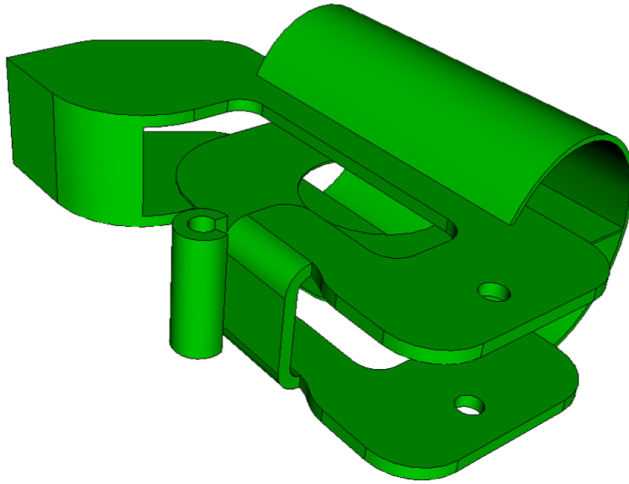


(c) Tracks in 3D

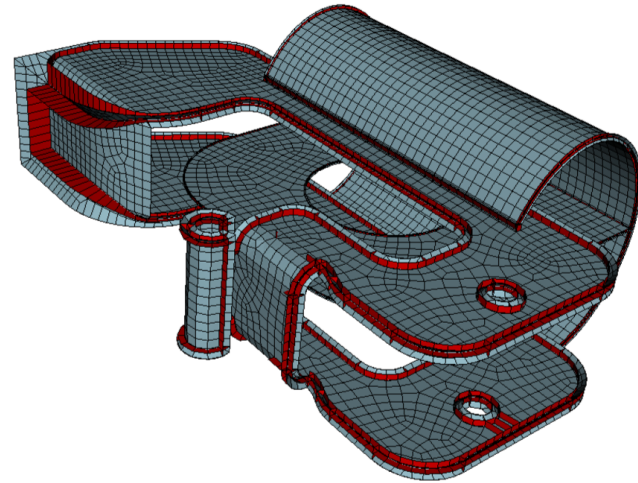


(d) Mesh cross section

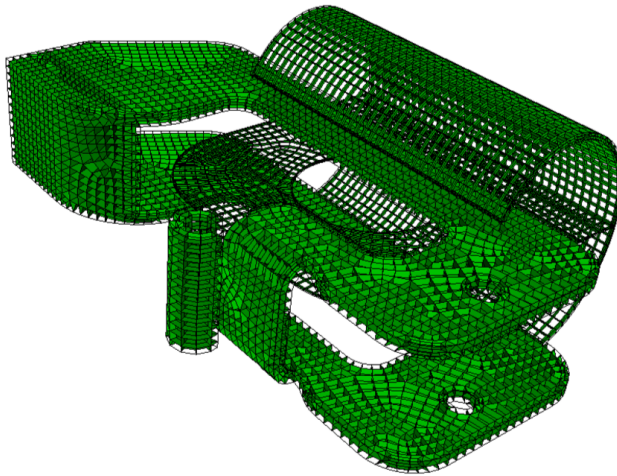
# Demo



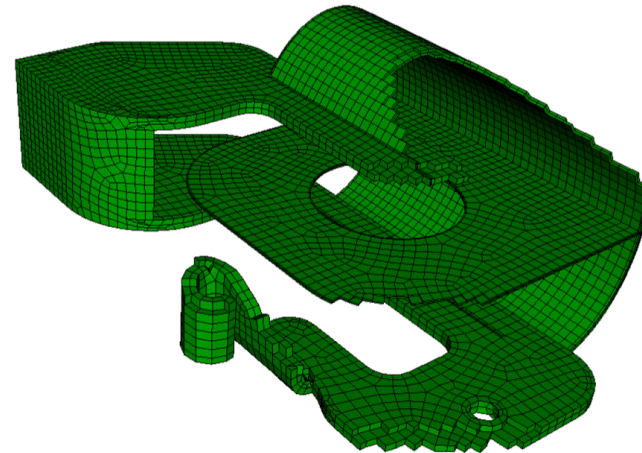
(a) Thin-wall solid



(b) Mesh on MA inside corridors

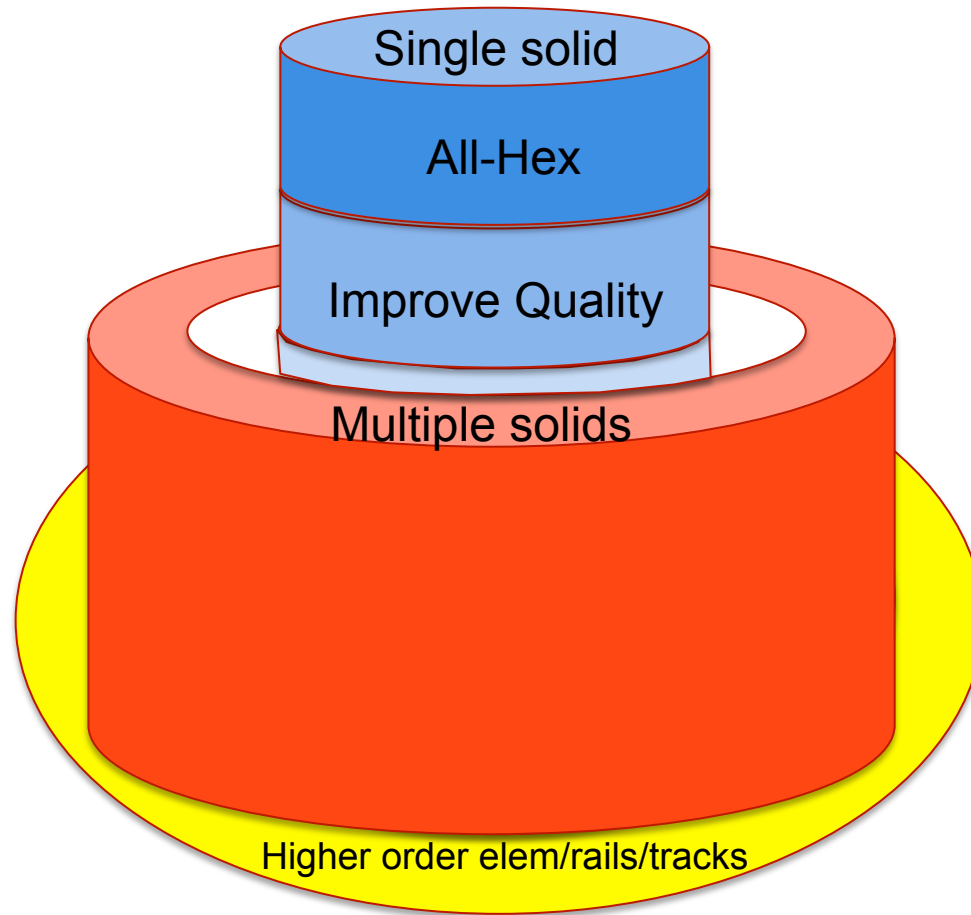


(c) Tracks in 3D

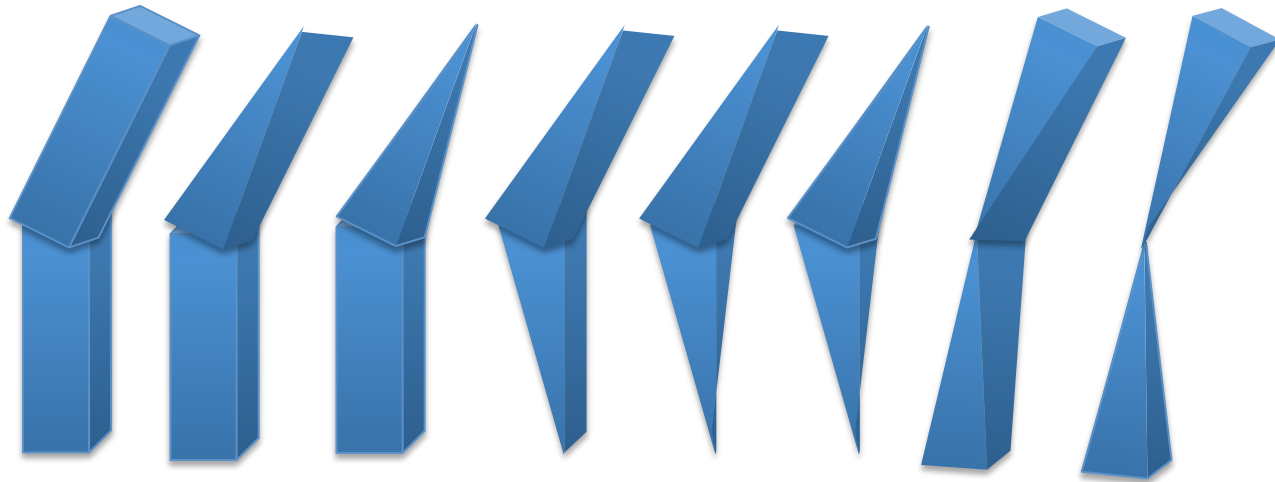
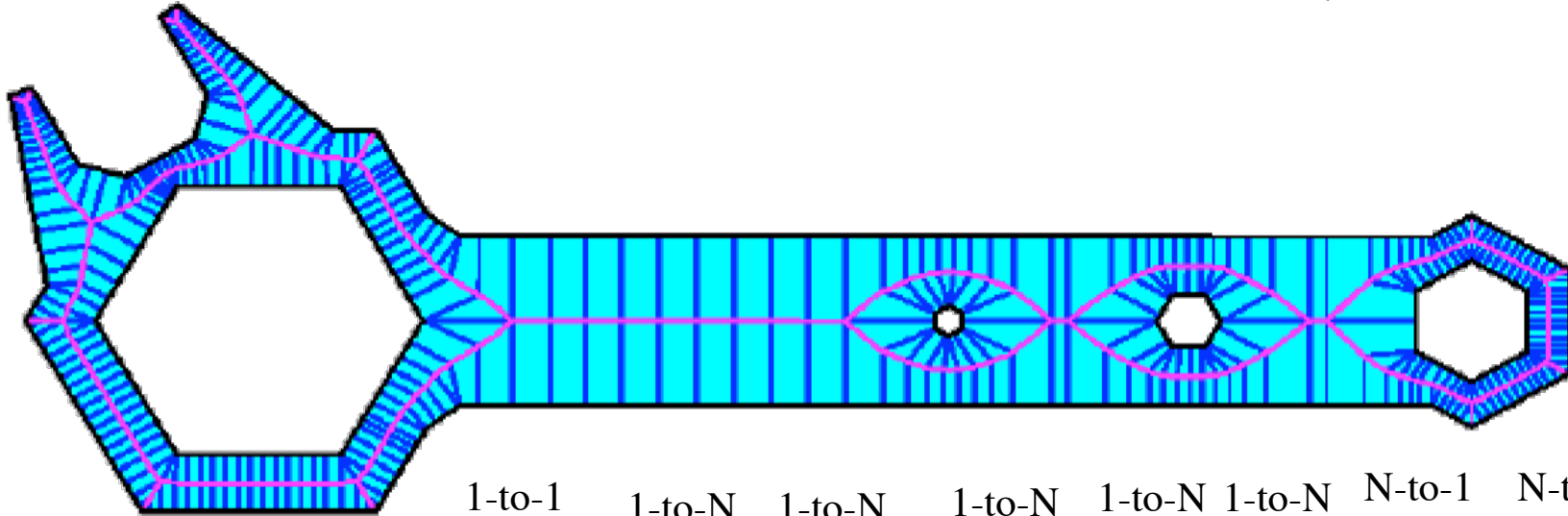


(d) Mesh cross section

# Layout of Presentation



# #1 Root Cause for Hex-Dominant: N-to-1 or 1-to-N Creates Non-Quad Tracks

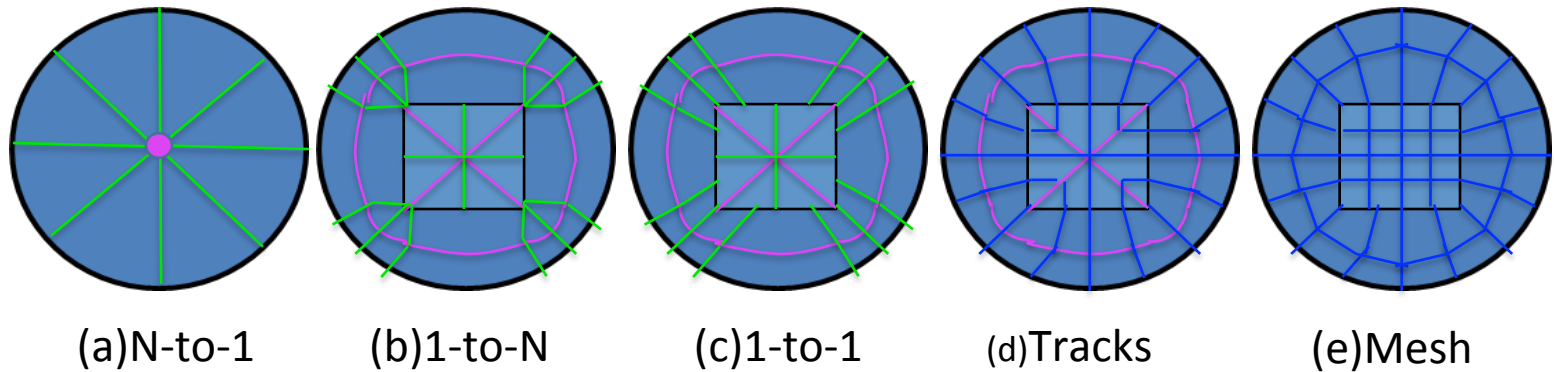
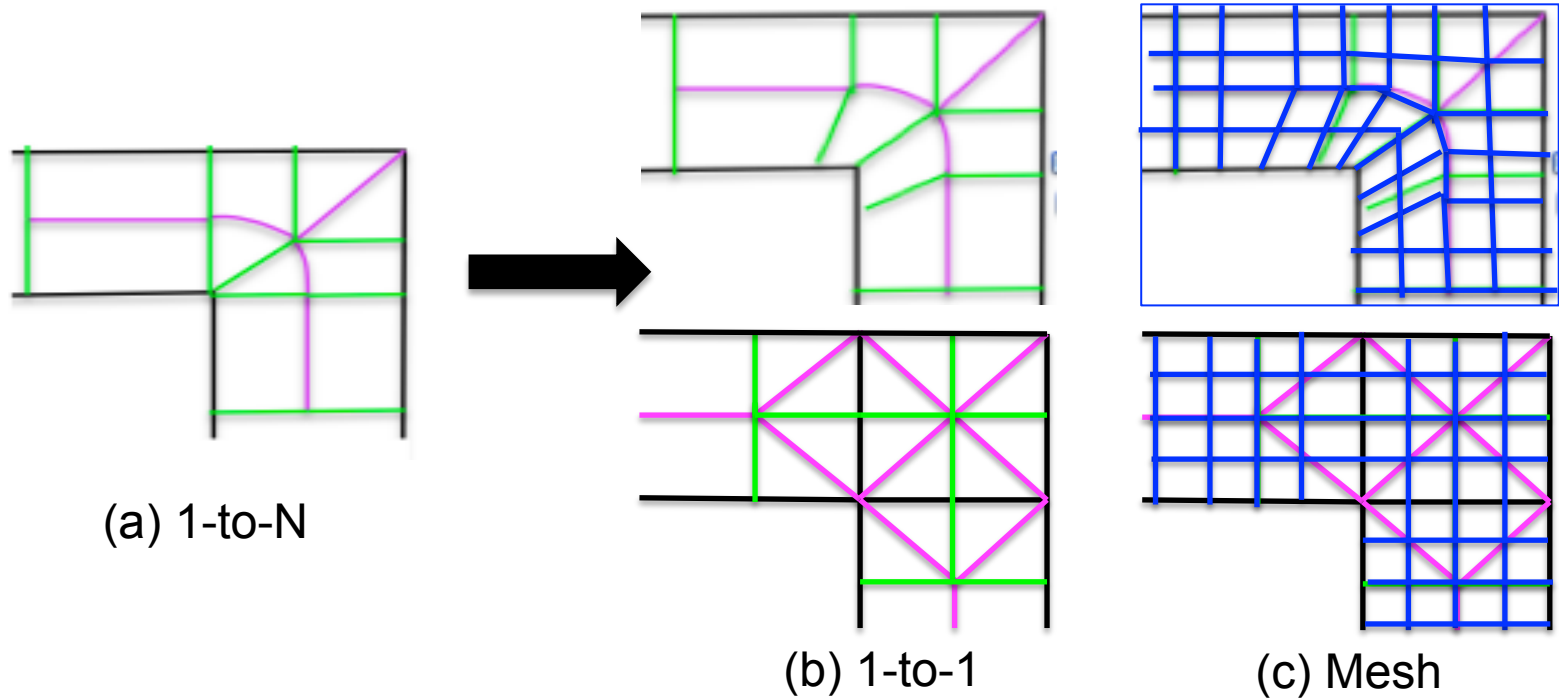


1-to-1    1-to-1    1-to-1    1-to-N    1-to-N    1-to-N    N-to-1    N-to-1



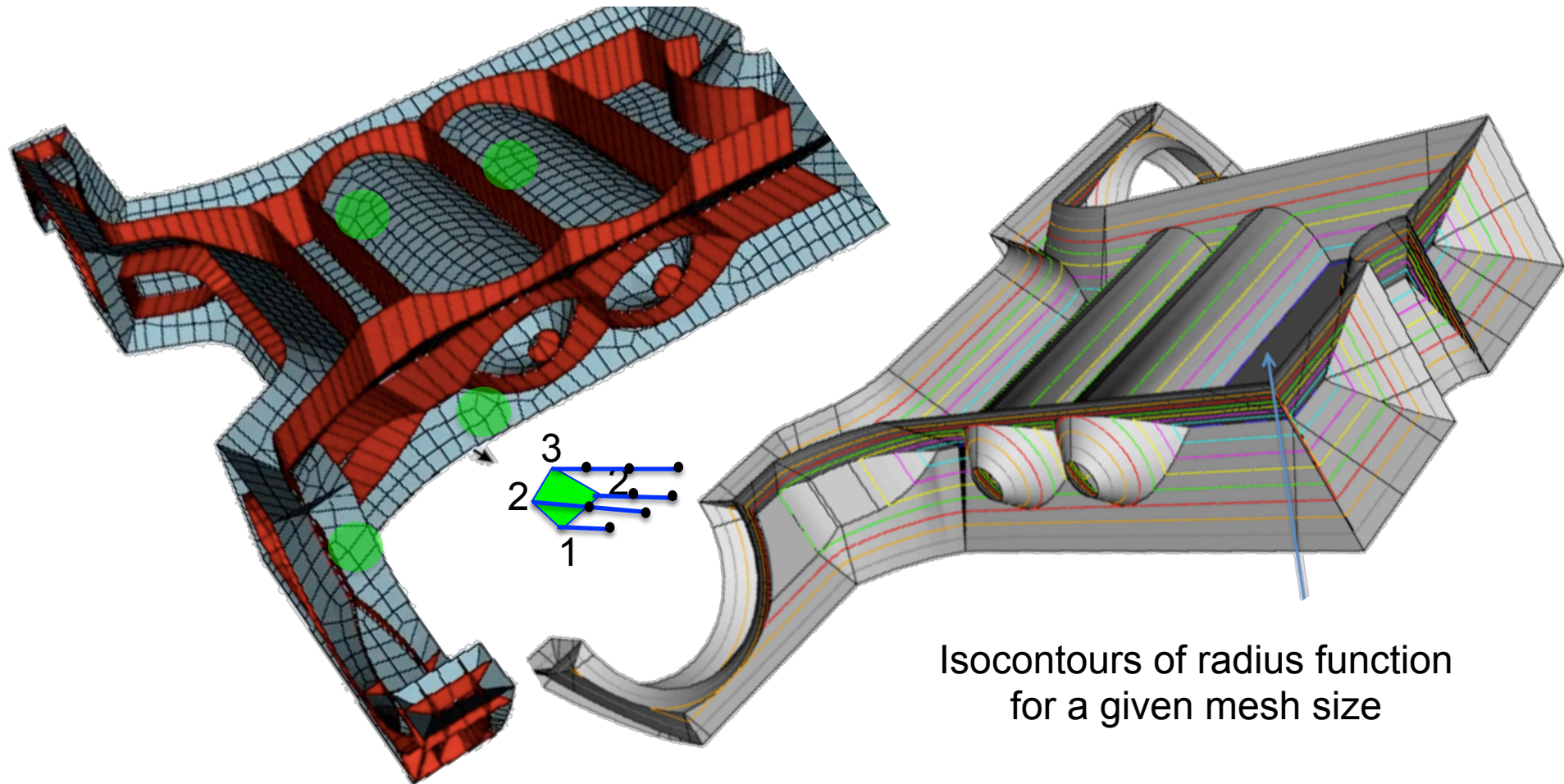
# 1-to-N & N-to-1 Track

## Transformed into 1-to-1 Track





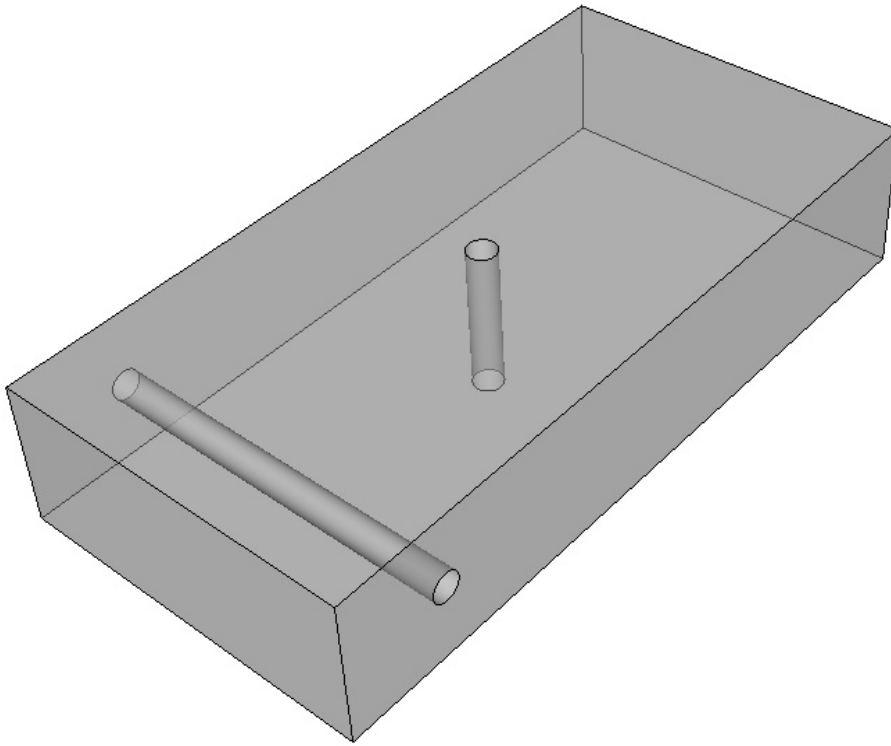
# #2 Root Cause for Hex-Dominant: Invalid intervals on 1-to-1 tracks



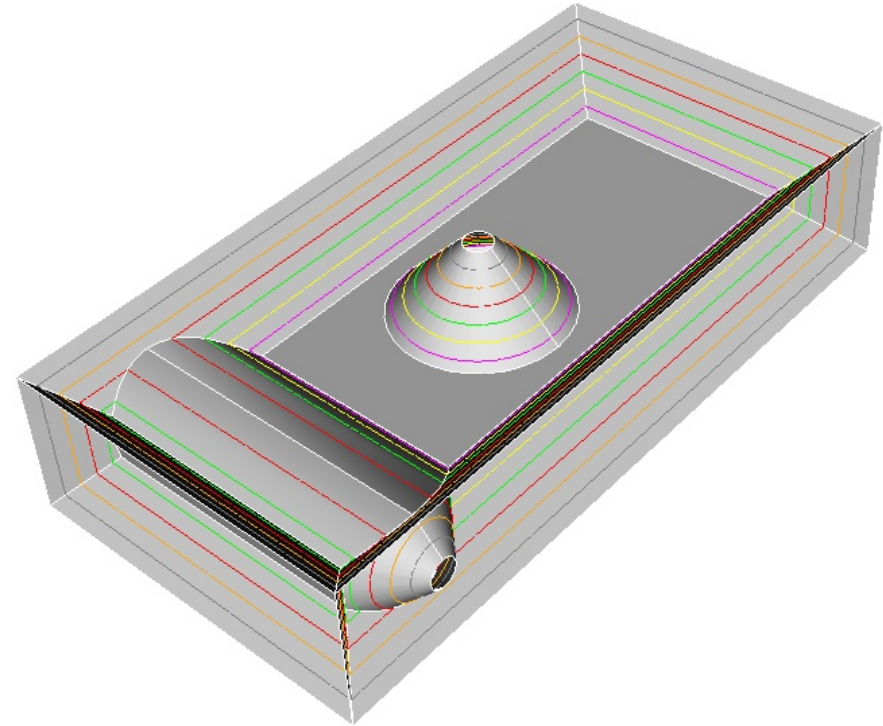
Isocontours of radius function  
for a given mesh size

Quad mesh generated without using  
radius function gives hex-dominant mesh

# Isocontours of Medial Radius Function Provide Correct Mesh Intervals on Rails



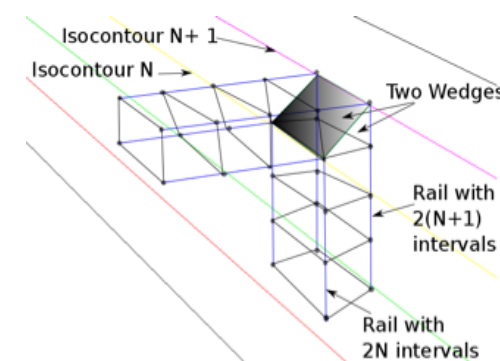
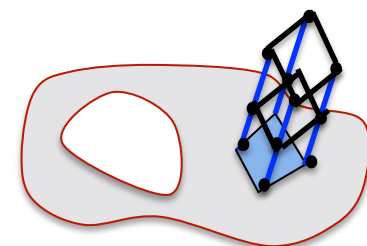
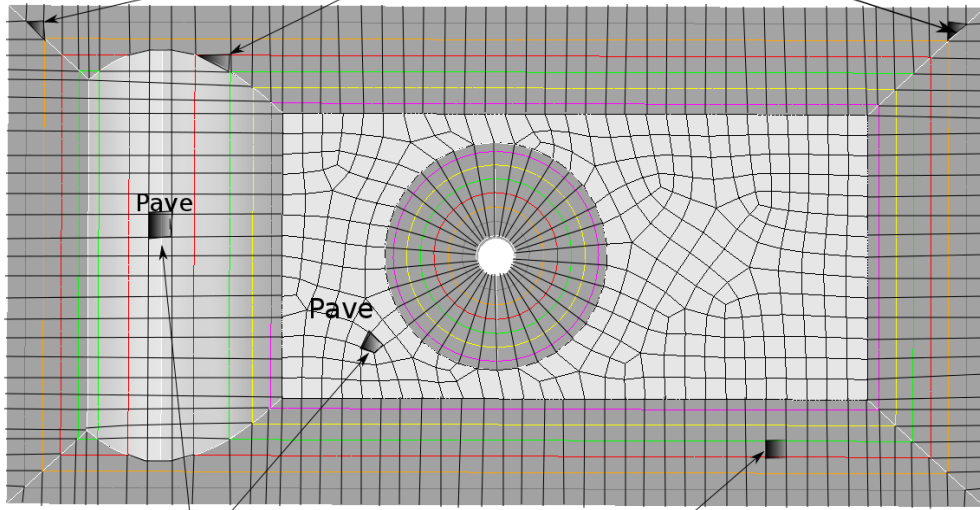
Non-Sweepable Solid



Isocontours of MA Radius Function

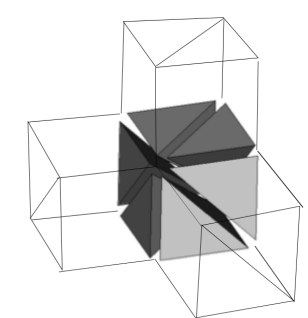
# All-Hex Mesh Topology Visualization on Isocontours of MA

Tri along 3-manifold MA edges



(a) Case1: equal no. of hexes exist on both sides of MA

(b) Case2: two wedges form a hex



(c) Six tets form a hex at 3-manifold MA

Case1: quad in region bounded by one isocontour

Case 2: quad incident on N & N+1 isocontours

Meshing MA using isocontours guarantees correct intervals on rails to satisfy Case 1 or Case 2 in each track

# Conclusions

- LayTracks has been successfully extended to 3D
  - Projection operator extends very well from 2D to 3D in performing geometry decomposition respecting imprints via rails, corridors, and tracks.
  - Automatic, Handles General Solids, Structured Boundary Sensitive Mesh, Irregular Nodes restricted at MA, Respects Sharp Features, Respects Imprints, Orientation insensitive,...
- Meshing medial surfaces without radius function gives 1% to 5% non-hex elements at medial
- Extension to All-Hex looks very promising
  - Requires transforming 1-to-N and N-to-1 maps to 1-to-1
  - Requires meshing medial surface via isocontours of radius function

# Related Publications

- W. R. Quadros, “**LayTracks3D: A New Approach to Meshing General Solids using Medial Axis Transform**”, 23rd International Meshing Roundtable, London, Oct 12-15, 2014.
- W. R. Quadros, “**LayTracks3D: Mesh Generator for General Assembly Models using Medial Axis Transform**”, 22<sup>nd</sup> International Meshing Roundtable -Research Notes, Orlando, Oct 13-16, 2013.
- W. R. Quadros, K. Ramaswami, F. B. Prinz, B. Gurumoorthy “**LayTracks: A New Approach to Automated Geometry Adaptive Quadrilateral Mesh Generation using Medial Axis Transform**”, in International Journal for Numerical Methods in Engineering, Vol. 61, Issue 2, pp. 209-237, Sept. 2004.
- W. R. Quadros, K. Ramaswami, F. B. Prinz, B. Gurumoorthy, “**Automated Adaptive Quadrilateral Mesh Generation using MAT**”, in Proc. of ASME DETC Design Automation Conference, Pittsburgh, Sept. 2001.
- W. R. Quadros, K. Ramaswami, F. B. Prinz, B. Gurumoorthy, “**Lay Tracks: A New Approach to Automated Quadrilateral Mesh Generation using MAT**”, in Proc. of 9<sup>th</sup> International Meshing Roundtable, pp. 239 - 250, New Orleans, Oct. 2000.

# Acknowledgements

*Thanks to Henry Bucklow, Robin Fairey,  
and Mark Gammon at ITI TranscenData  
Europe Ltd for providing the CADfix medial  
object library.*

Thank You

Questions?

[wrquadr@sandia.gov](mailto:wrquadr@sandia.gov)