Using the medial axis to locate mesh singularities

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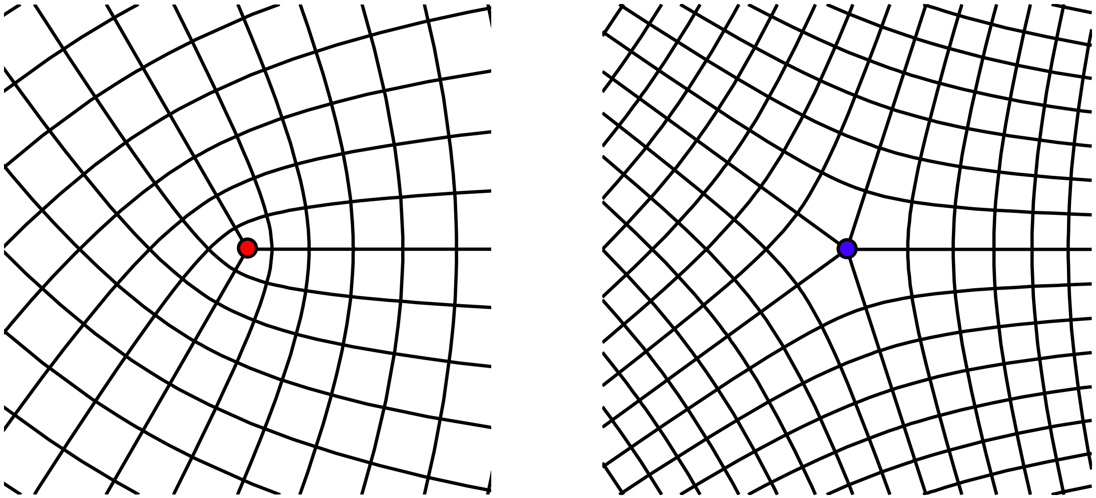
Key Messages

- Mesh singularities are crucial features
- Existing mesh generation methods often don't create the ideal singularity configurations
- The medial axis can be used to locate effective positions of mesh singularities

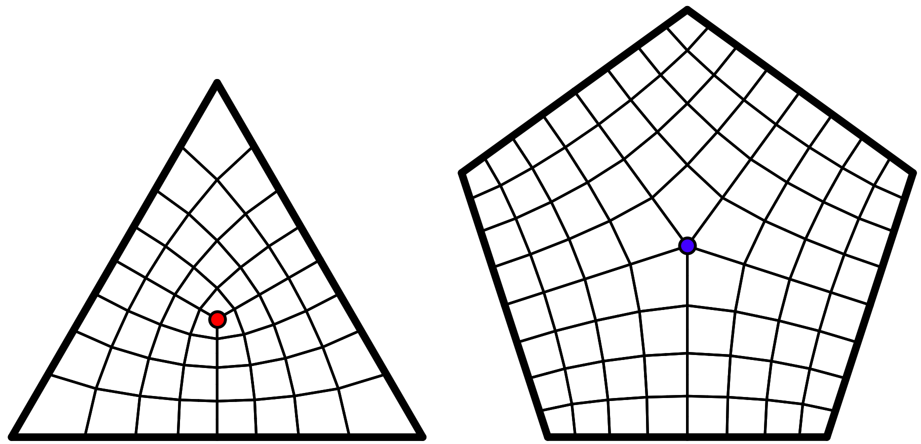
Mesh singularities

- Nodes where regular grid structure is disrupted

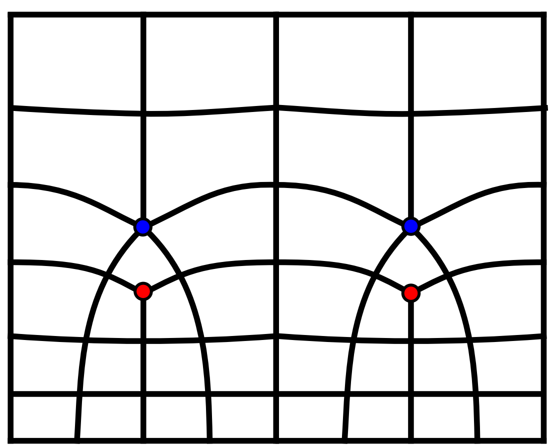


- Undesirable because
  - they cause element distortion
  - can't take advantage of grid properties

- However, singularities are necessary:
  - to satisfy boundary alignment constraints



  - to control/facilitate mesh resolution change



Review of existing methods
...paying attention to occurrence of singularities

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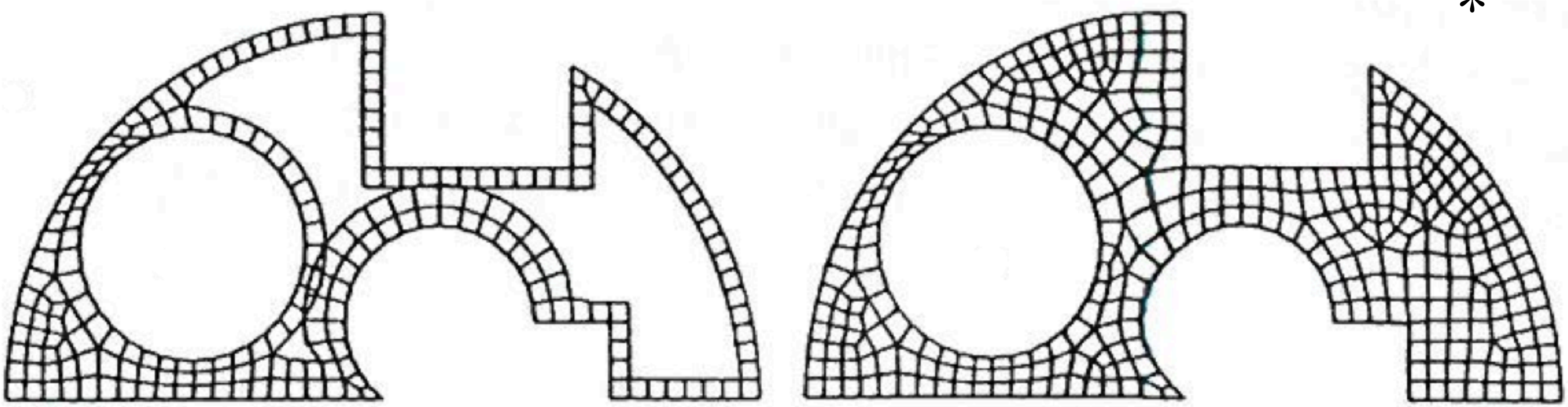
Paving

- Disordered array of sings. in interior



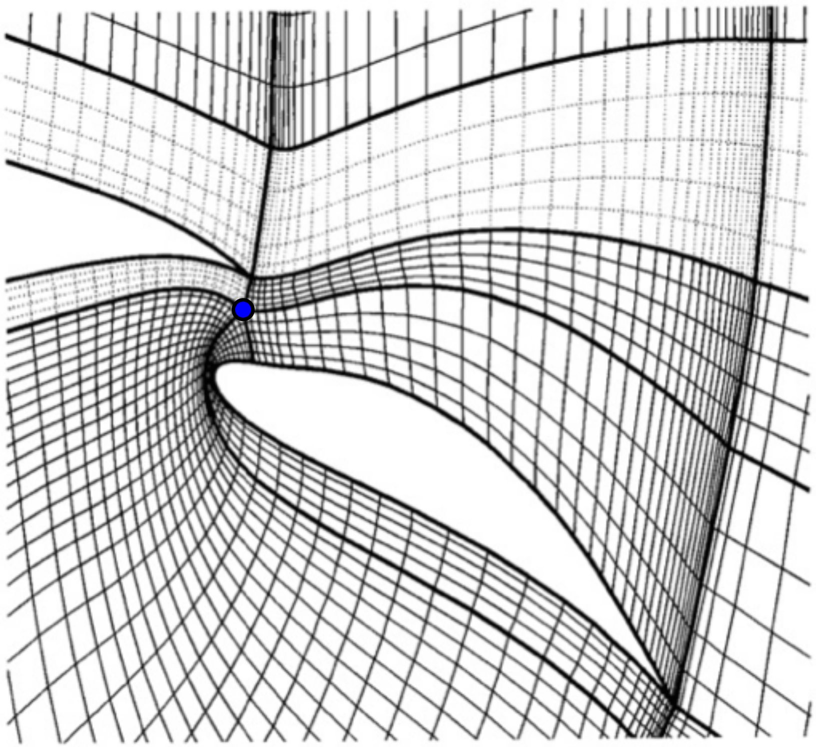
*

*T. Blacker and M. Stephenson.** Paving: A new approach to automated quadrilateral mesh generation. International Journal for Numerical Methods in Engineering, 32(4):811–847, 1991.

Manual multiblock decomposition

- well-positioned singularities

*



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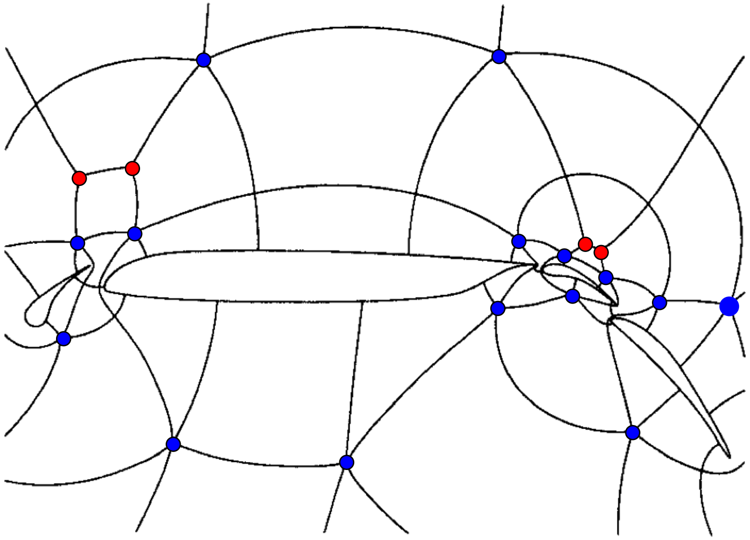
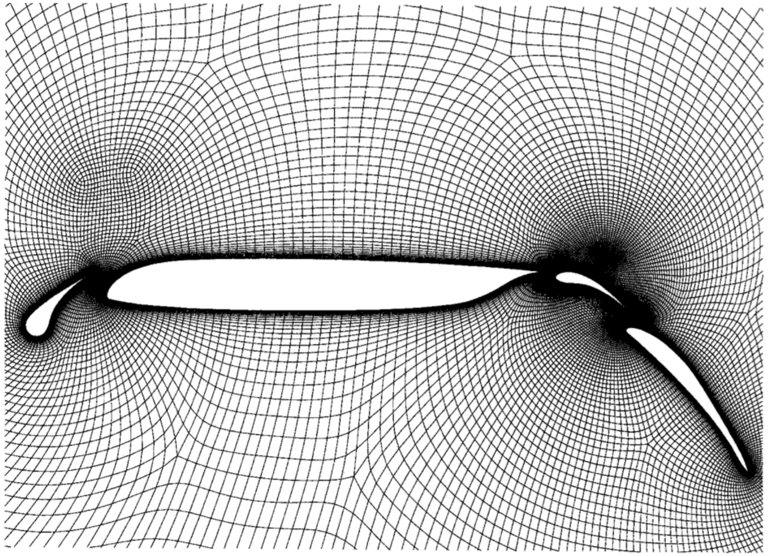
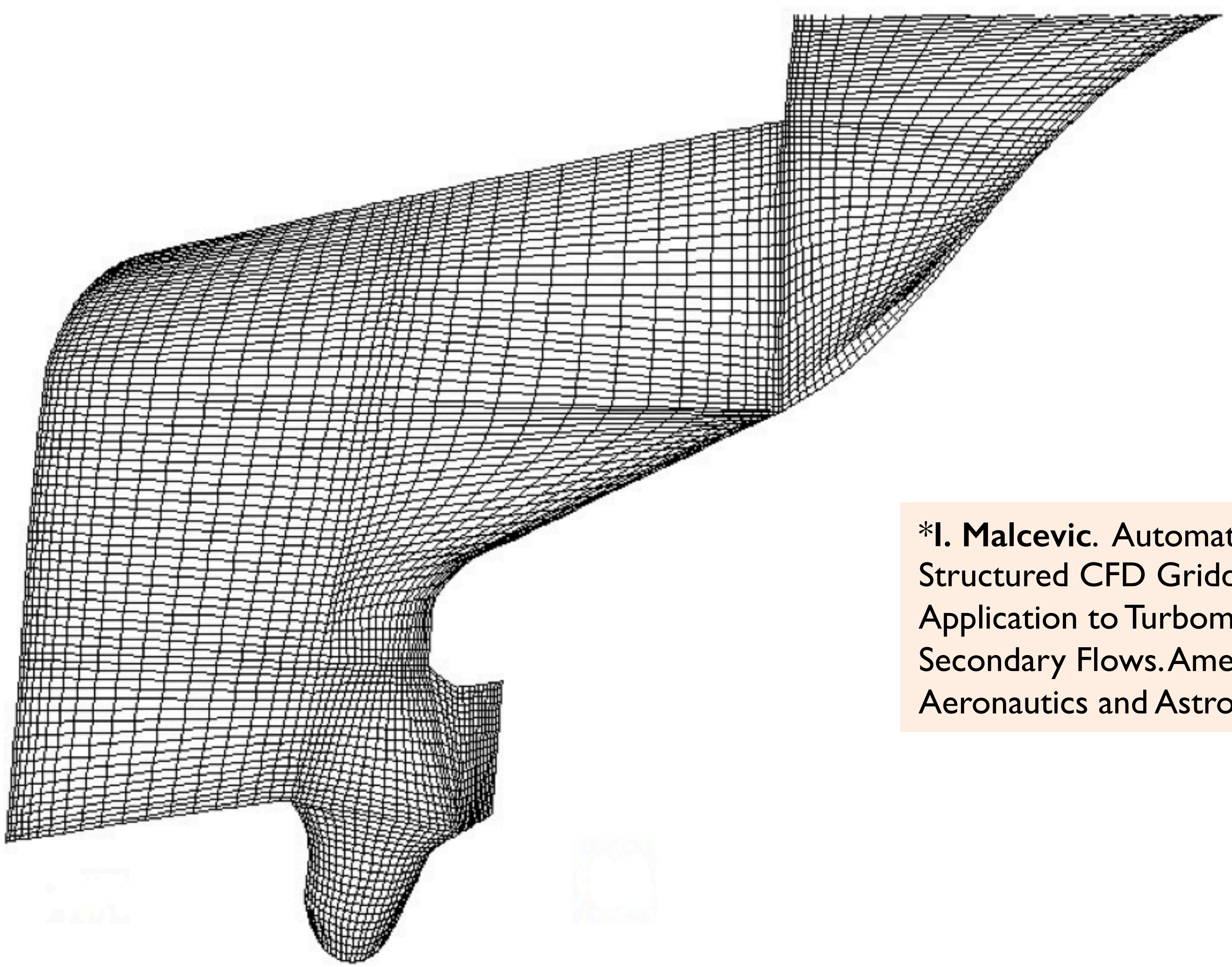




* **L. Dubuc et al.** A grid deformation technique for unsteady flow computations. Int. J. Numer. Meth. Fluids 2000
† **J. Hauser et al.** Parallel multi-block structured grids. In Handbook of grid generation, 1998

Cartesian fitting method* (submapping)

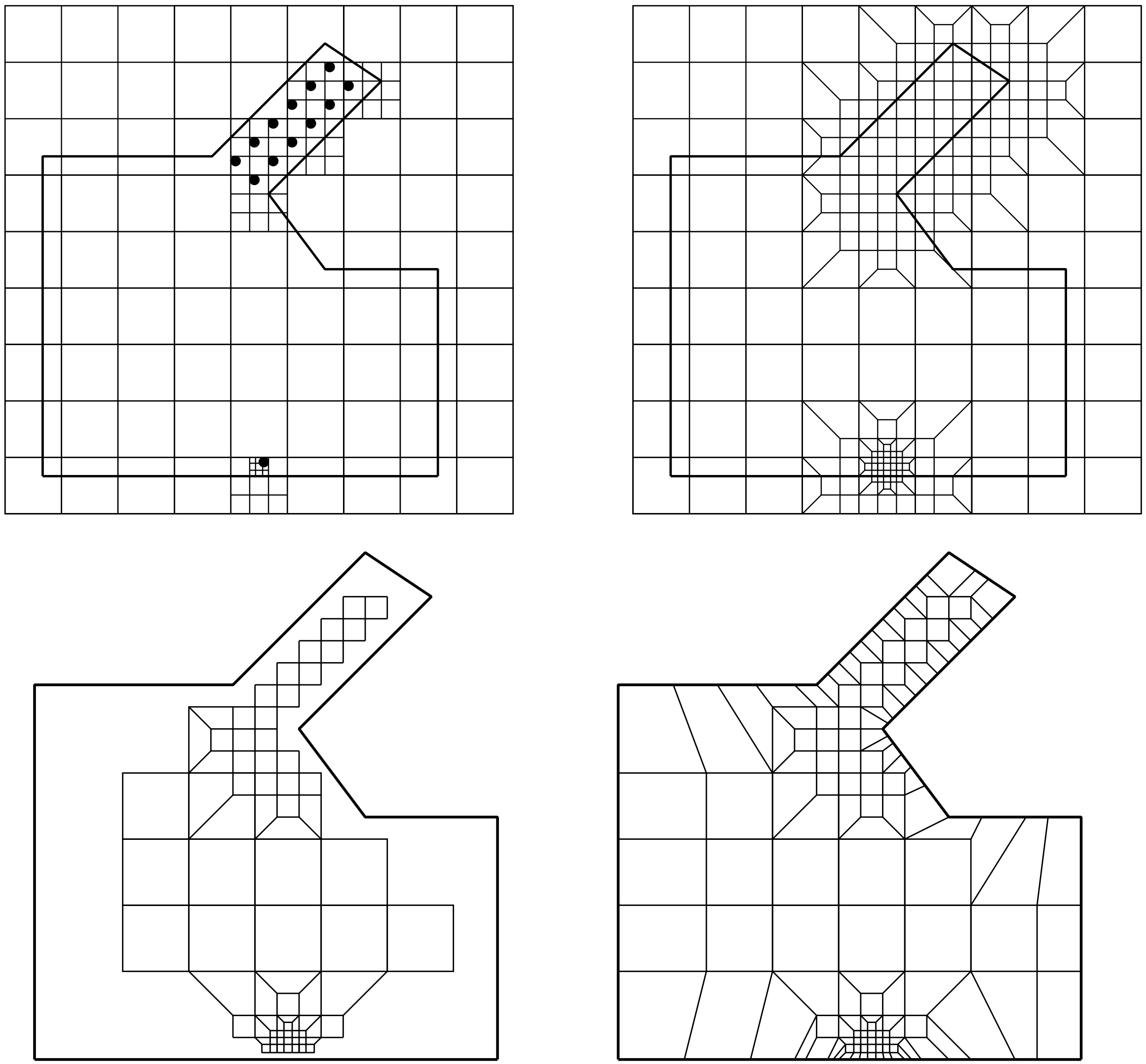
- Singularities pushed to boundaries



*I. Malcevic. Automated Blocking for Structured CFD Gridding with an Application to Turbomachinery Secondary Flows. American Institute of Aeronautics and Astronautics, 2011.

Octree meshing

- Many pos-neg singularity pairs along boundaries



*

*R. Schneiders et al. Octree-based Generation of Hexahedral Element Meshes. 5th IMR, 1996.

Medial axis (Tam & Armstrong) method*

- Geometry partitioned into sub-regions with at most one central singularity

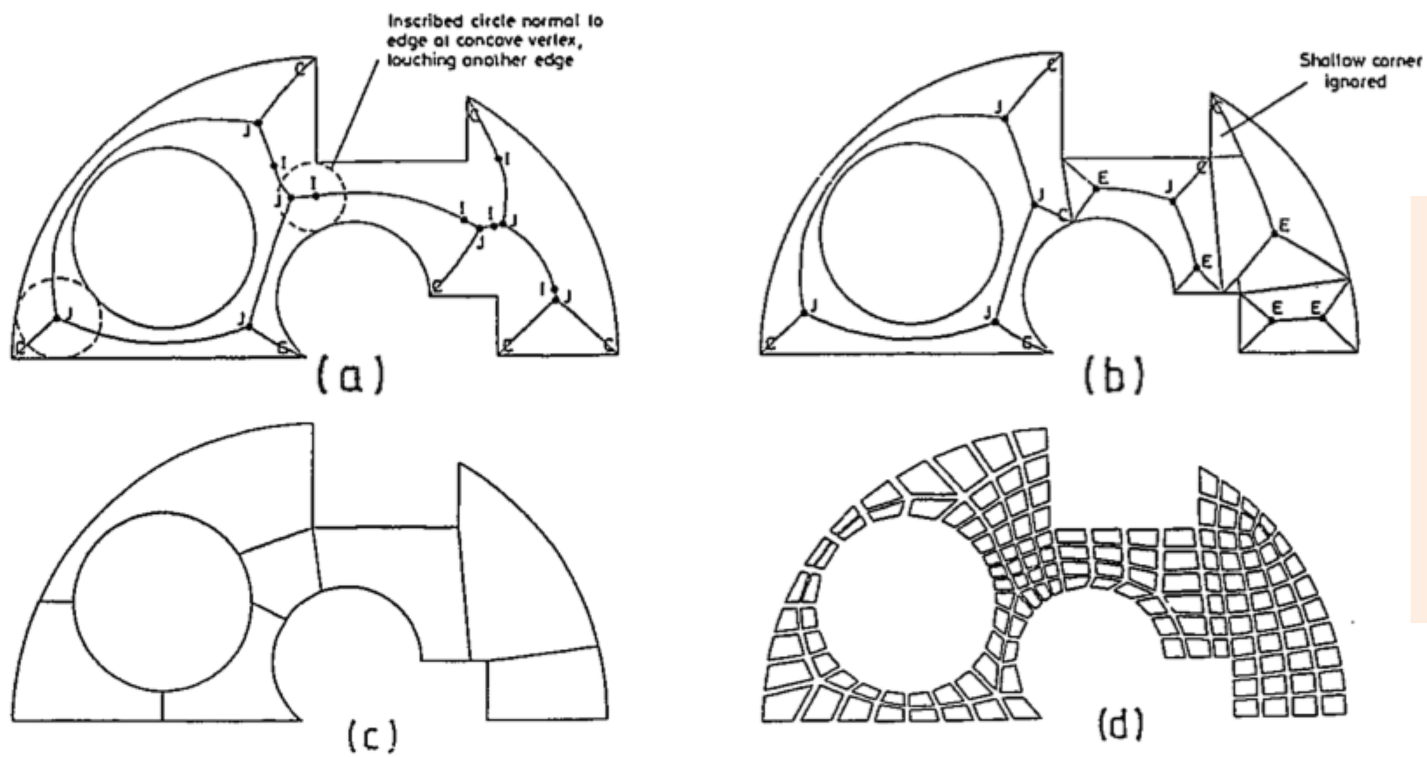
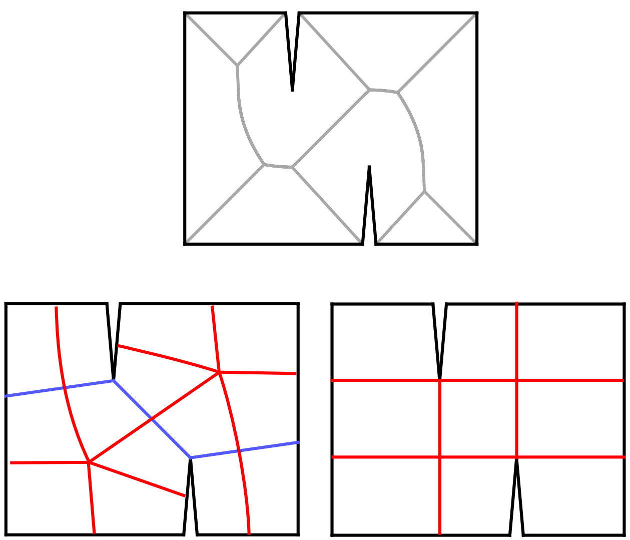


Fig. 14. Stages in mesh generation by medial axis subdivision. (a) the medial axis. (b) concavity removal. (c) chain splitting. (d) subregion meshing.

*T.K.H. Tam and C.G. Armstrong, 2-D finite element mesh generation by medial axis subdivision. Advances in Engineering Software, 13(5/6): 313–324, 1991.

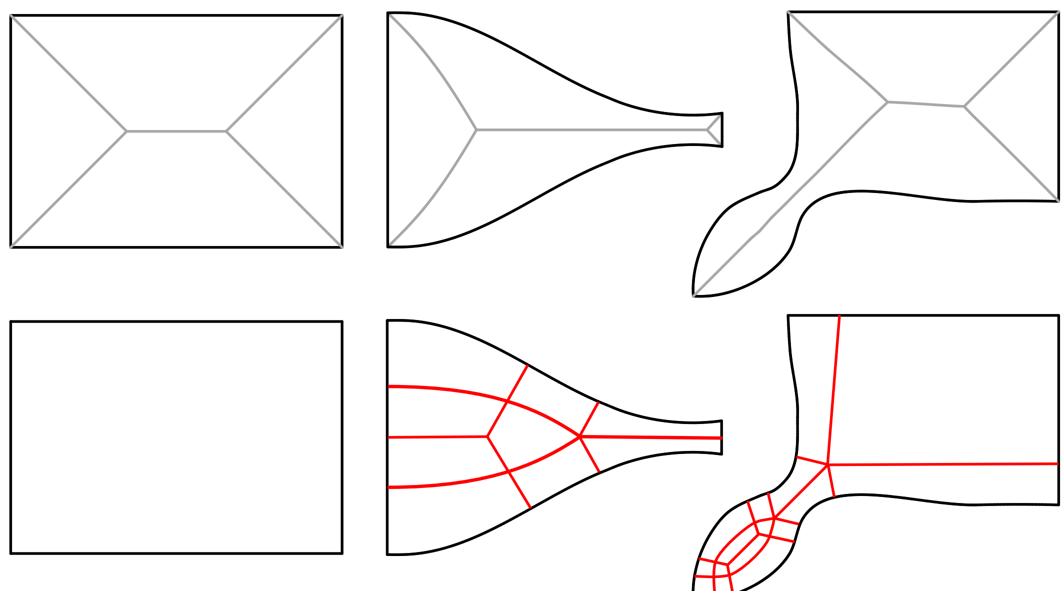
- Tam & Armstrong MA based method generates high quality meshes…
- But it has shortcomings:
  - crude treatment of concavities
  - only takes into account the topological info in MA and not geometric info

Indents



T&A decomp.   preferred decomp.

Topological rectangles:



preferred decomps.

Theory

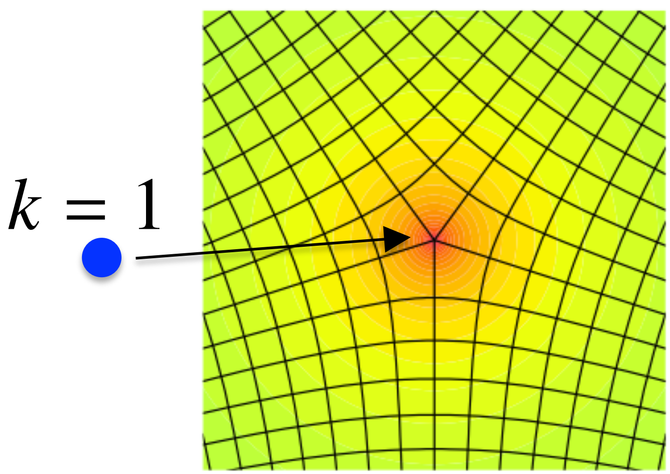
- Governing eqn. of orthogonal mesh*:

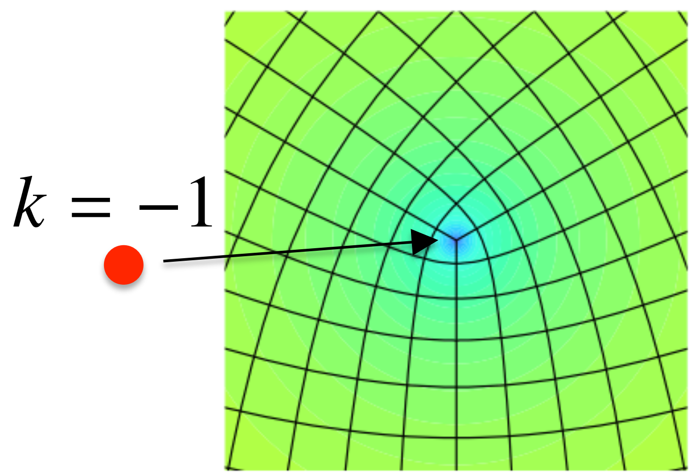
  Poisson Eqn.

$$\Delta_s \phi = K + \sum k_i \frac{\pi}{2} \delta_{\mathbf{p}_i}$$

scalar field relating to element size — *Gaussian curvature surface* — *point sources at mesh singularities*

* **Guy Bunin**. A continuum theory for unstructured mesh generation in two dimensions. Comput. Aided Geom. Des., 25(1):14–40, January 2008.





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- ϕ can be solved numerically (FEM)
  - isotropic element size given by

$$h = e^{-\phi}$$

  - derivatives of ϕ describe change in angle of cross-field
- But singularities must be positioned first

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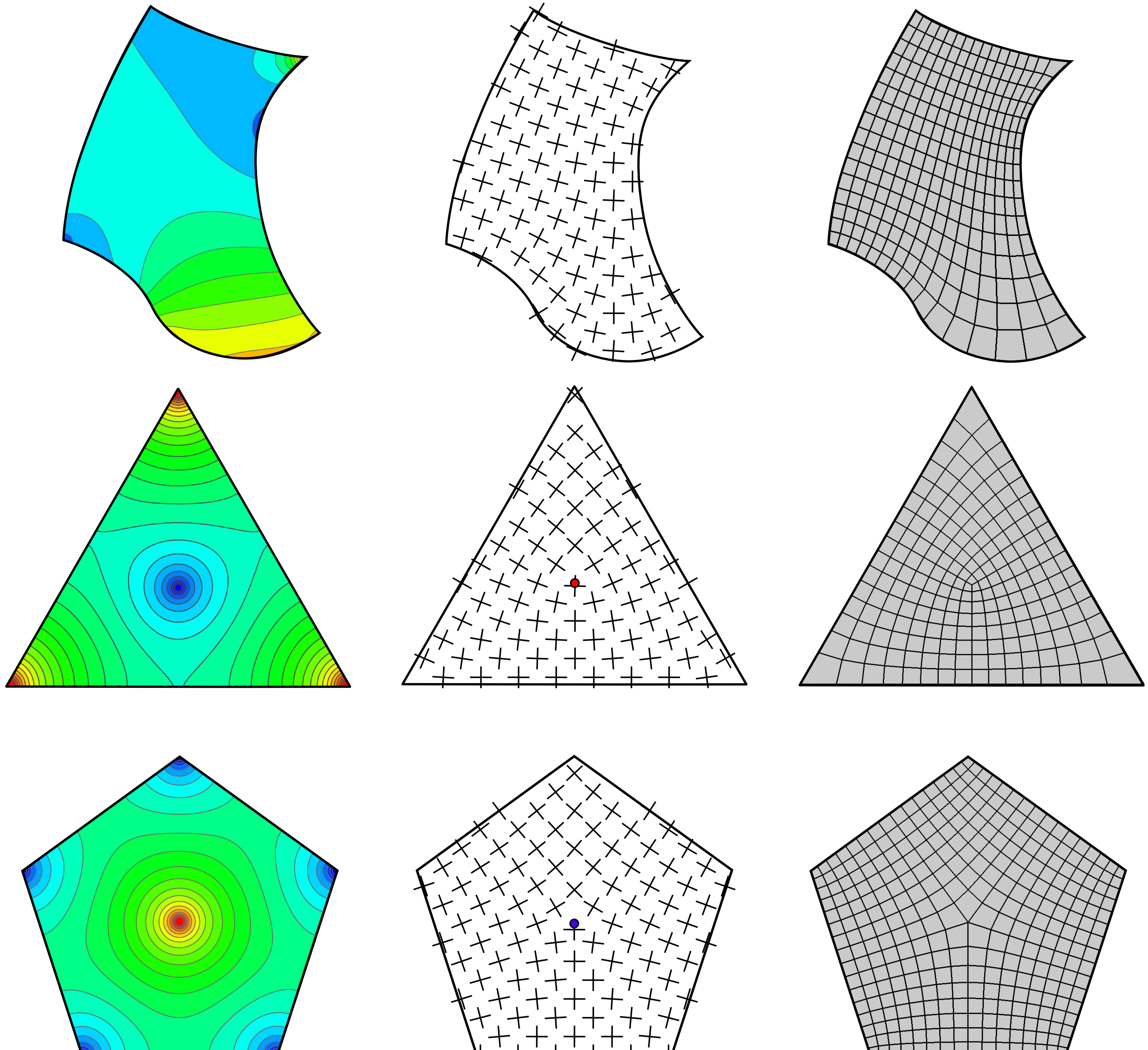
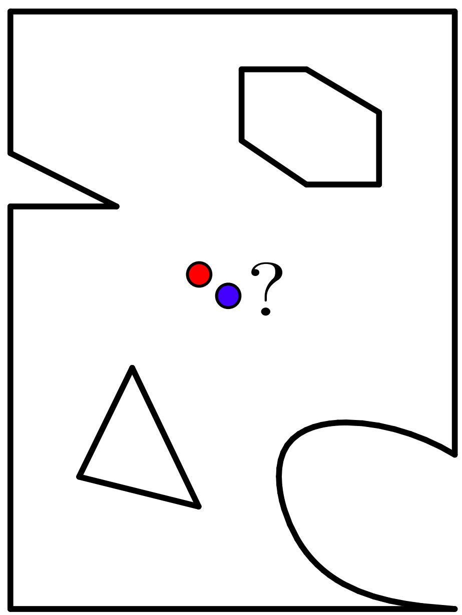
(−)ϕ-field | cross-field | mesh



Placing singularities

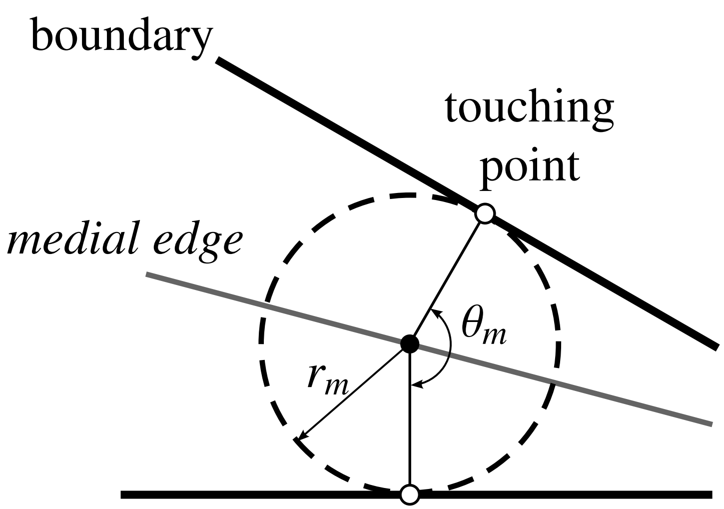
- The crucial step
- Inverse Poission problem . . . no general efficient solution method

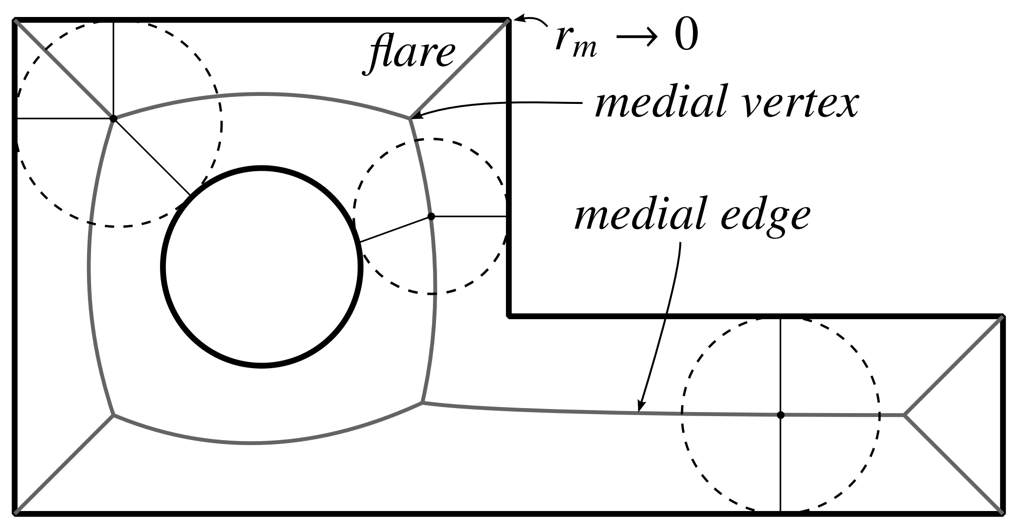


- Once singularities are fixed the rest of the mesh solution follows

The medial axis (MA)

- It provides a means to assess the geometry locally – links proximate boundaries





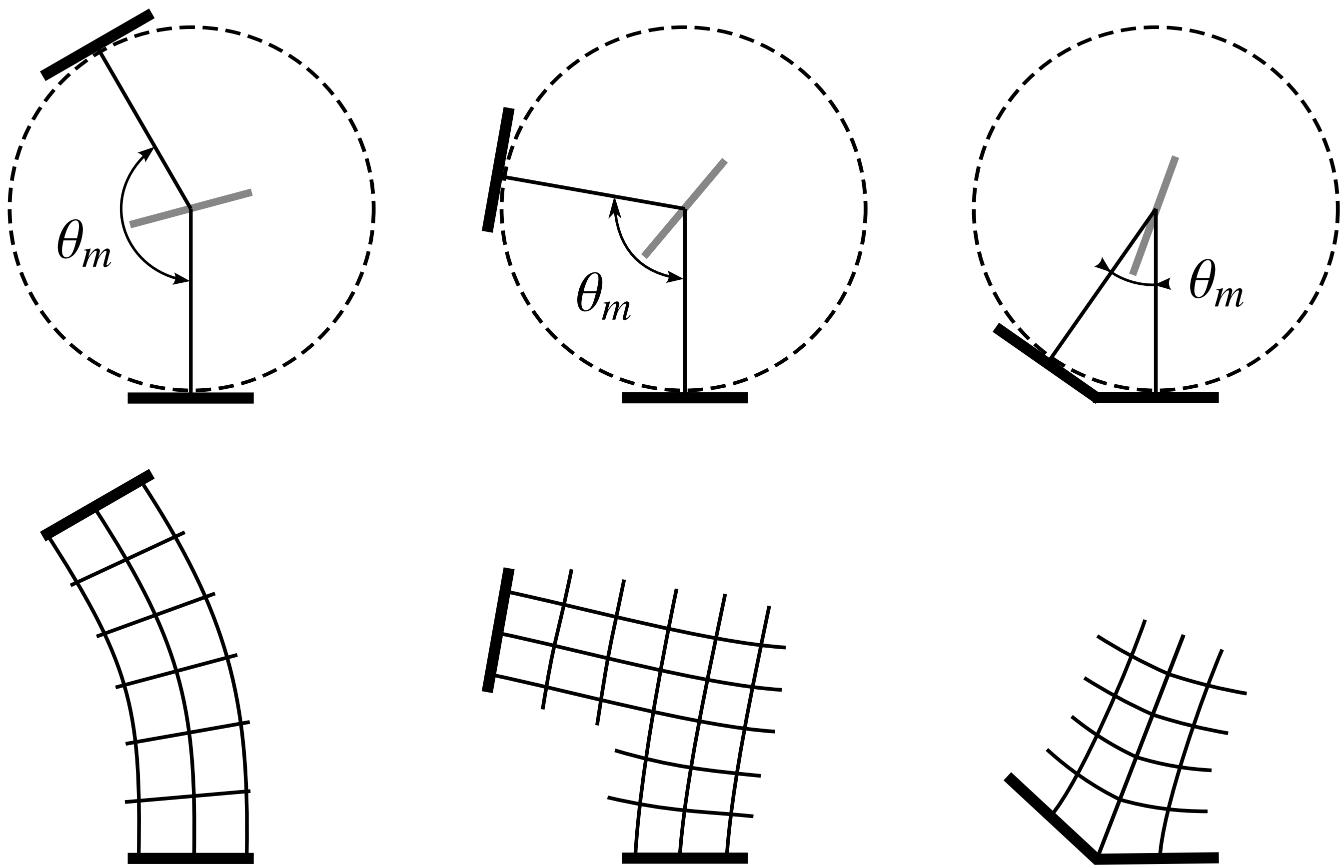
- It can be used to identify critical locations for mesh singularities*

* **H. Fogg, C. Armstrong, and T. Robinson**. New techniques for enhanced medial axis based decompositions in 2-D. In Proc. of 23rd IMR, 2014

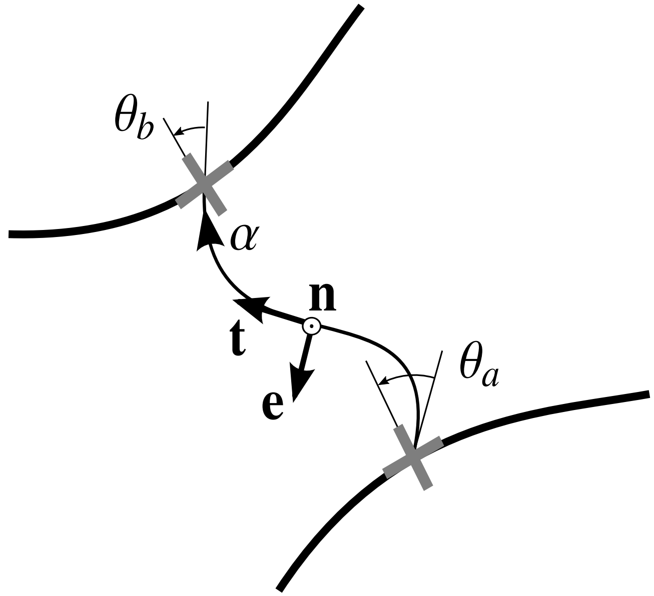
A simple idea…

- θ_m – relative orientation of proximate boundaries

  $\Rightarrow$ indicates preferred mesh pattern



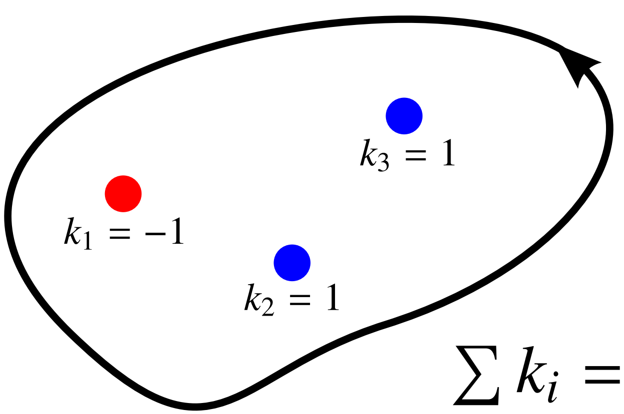
Theory



Between boundaries:

$$\Phi_\alpha \equiv \int_\alpha \frac{\partial \phi}{\partial e}\, ds = \Delta\theta$$

change in angle of cross-field



Closed loop:

$$\Phi_{\text{in}} \equiv \oint_\alpha \frac{\partial \phi}{\partial e}\, ds = -\sum_{\text{enc.}} k_i \frac{\pi}{2}$$

$$\sum k_i = 1 \Leftrightarrow \Phi_{\text{in}} = -\pi/2$$

Optimum mesh flow

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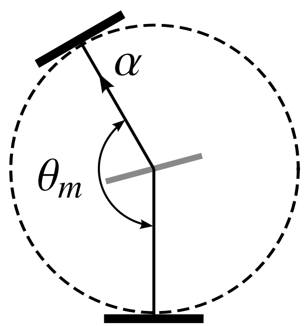
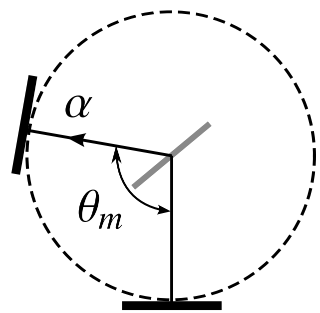
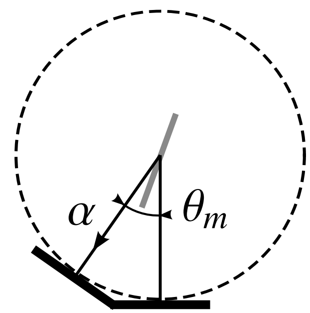
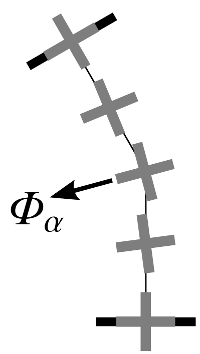
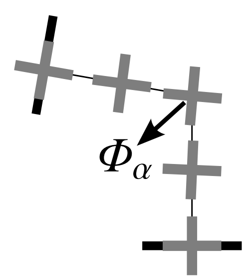
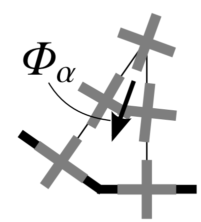
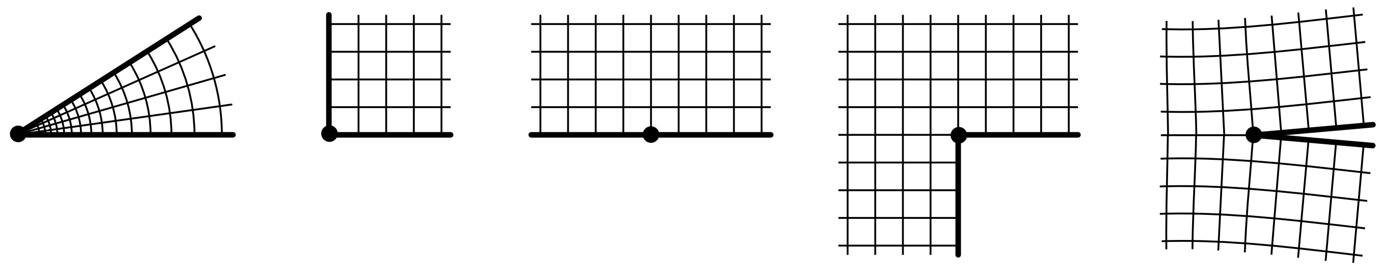
θ_m range	$[\pi, 3\pi/4)$	$[3\pi/4, \pi/4)$	$[\pi/4, 0]$
$\Phi_\alpha\vert_{\text{optimum}}$	$\pi - \theta_m$	$\pi/2 - \theta_m$	$-\theta_m$
Medial axis appearance			
Cross-field behaviour			
Colour			

Optimum mesh patterns at corners



n_c:	0	1	2	3	4
Optimum θ_c range:	$[0, \frac{\pi}{4})$	$[\frac{\pi}{4}, \frac{3\pi}{4})$	$[\frac{3\pi}{4}, \frac{5\pi}{4})$	$[\frac{5\pi}{4}, \frac{7\pi}{4})$	$[\frac{7\pi}{4}, 2\pi]$

Find singularities by flux balancing

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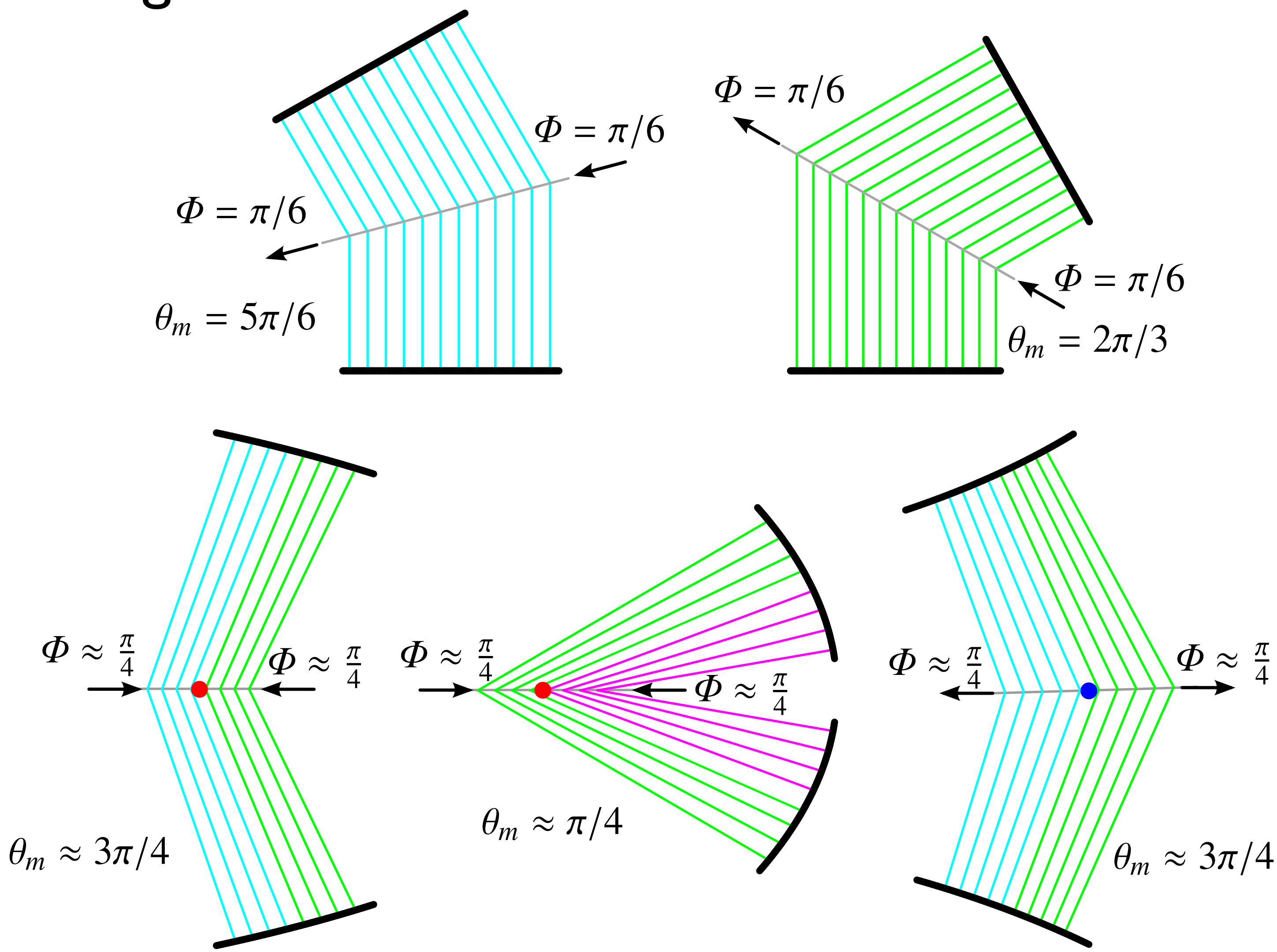
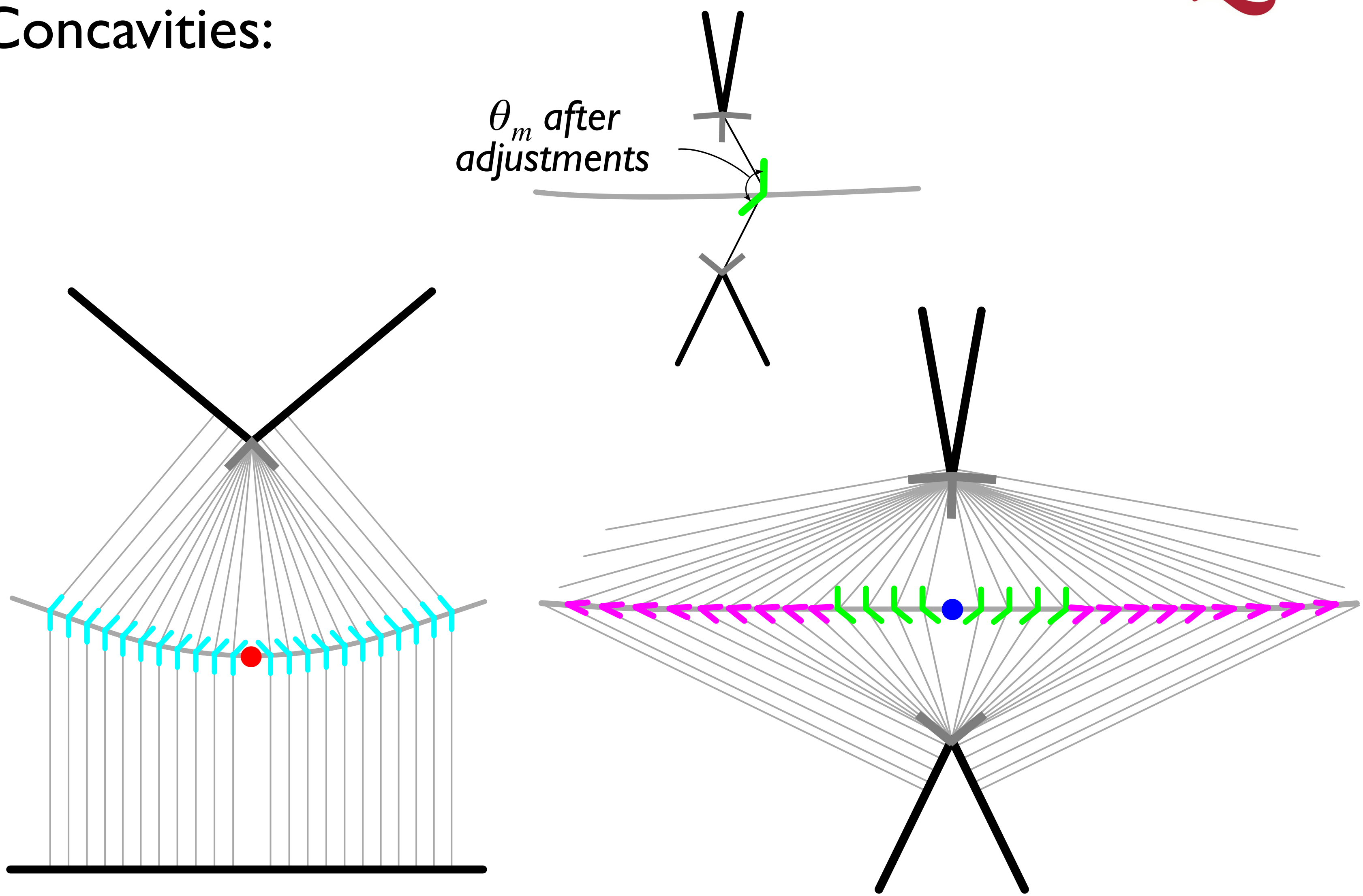
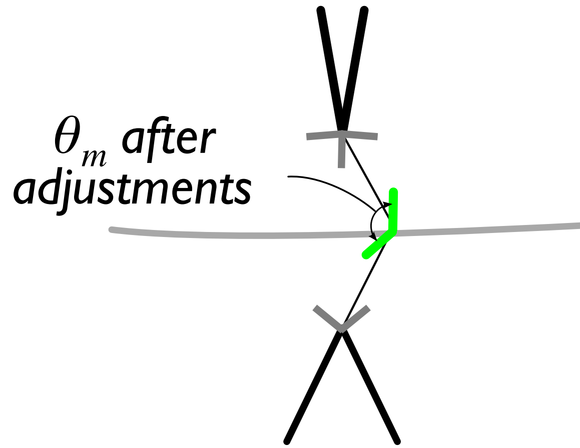
Medial edges:

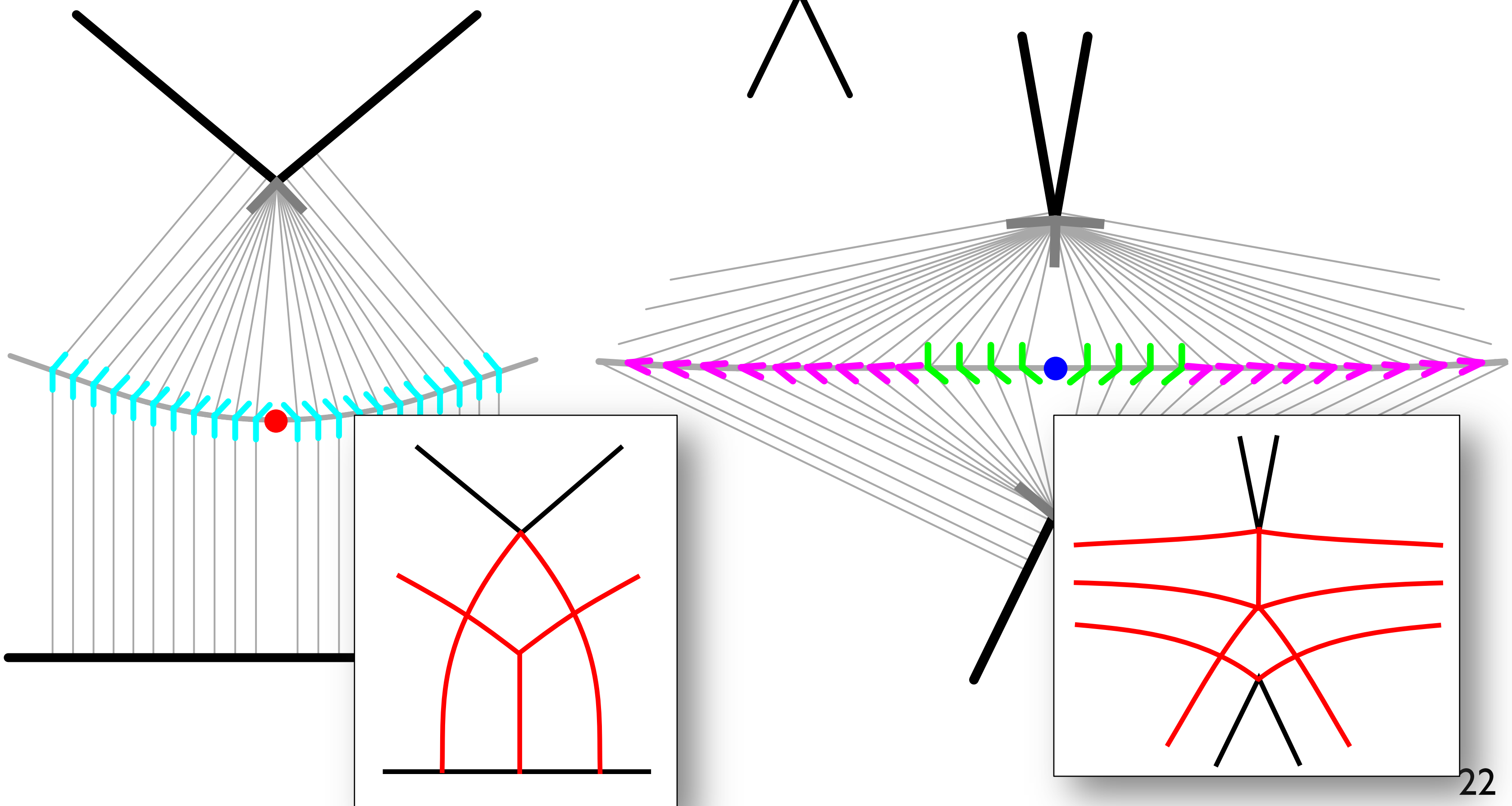


Concavities:

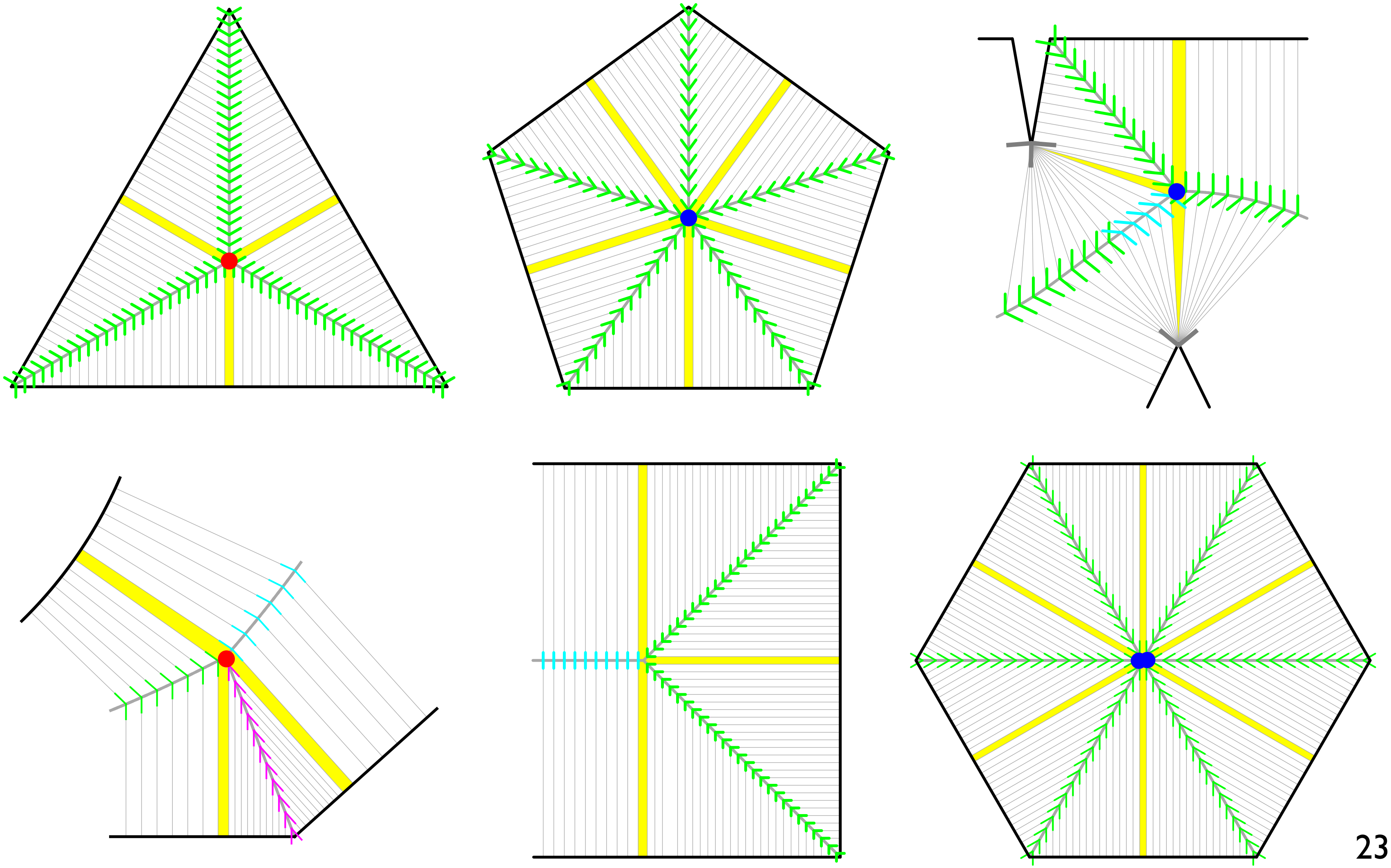


Concavities:



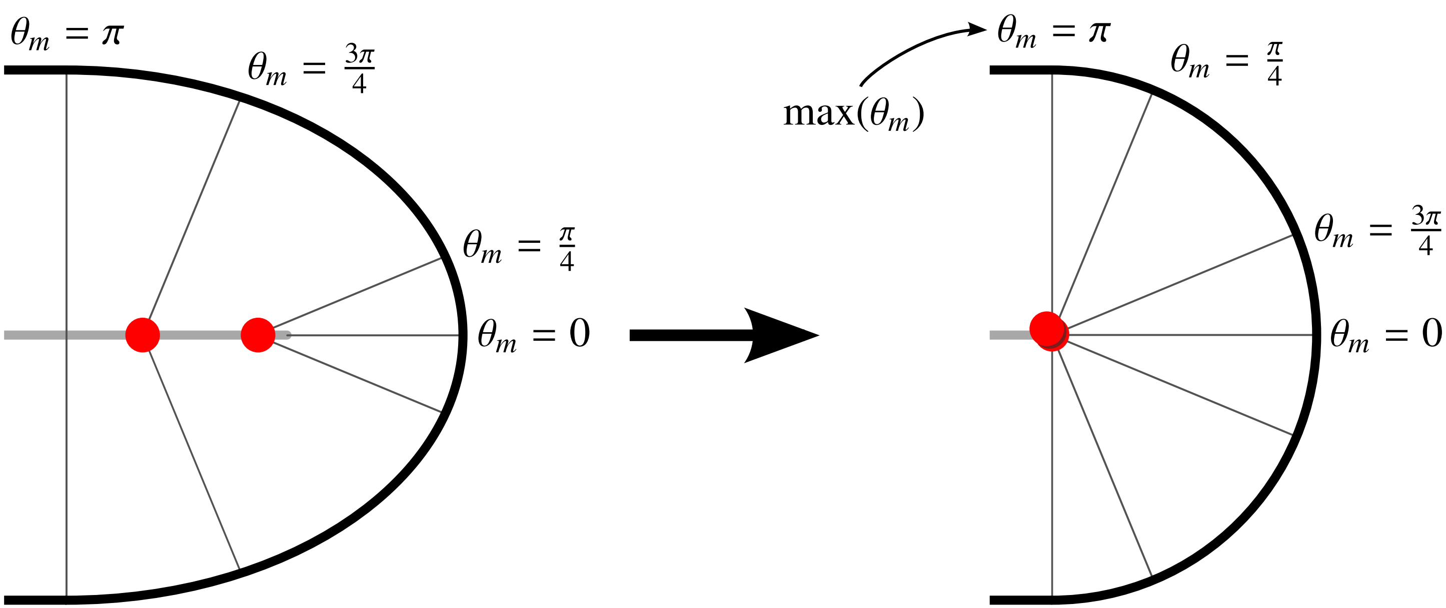


Medial vertices:



Finite contact:

$$k = -\text{floor}\left(\frac{\max(\theta_m)}{\pi/2}\right)$$

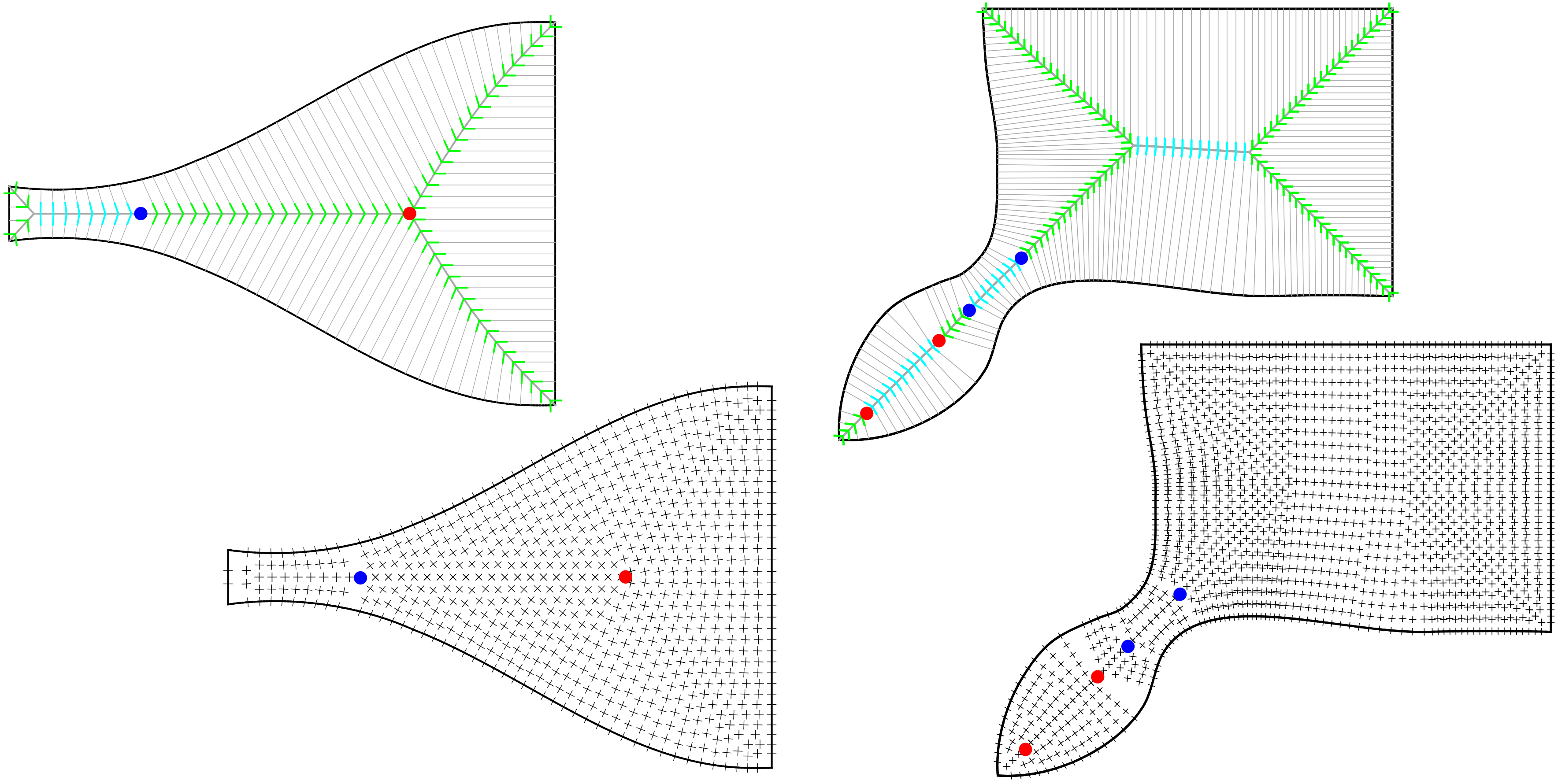


Basic Algorithm

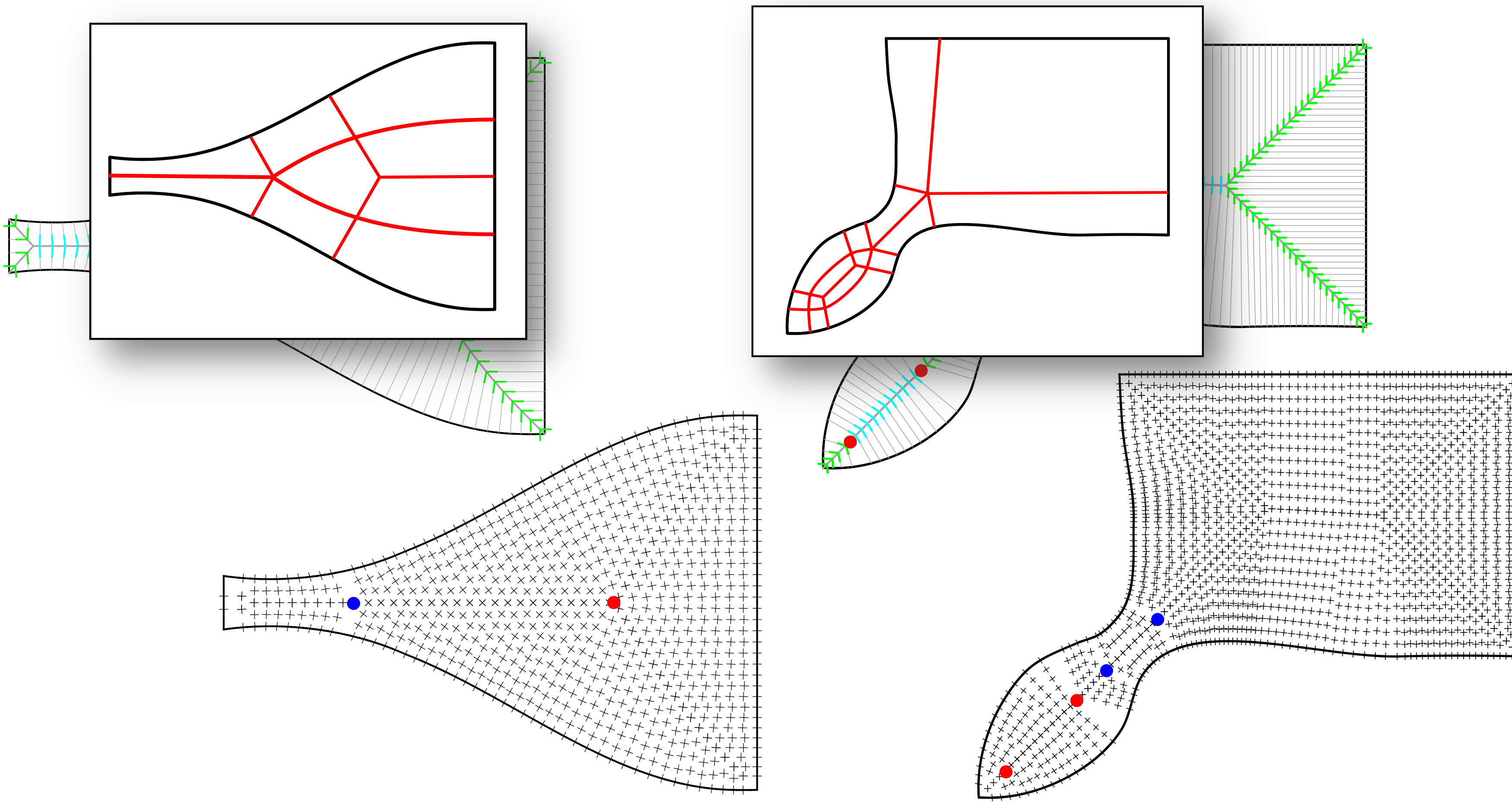
1. Generate MA (CADfix).
2. Assemble analysis positions on MA.
3. Perform flux balance calculation in slivers.

$\Rightarrow$ Place mesh singularities on MA.

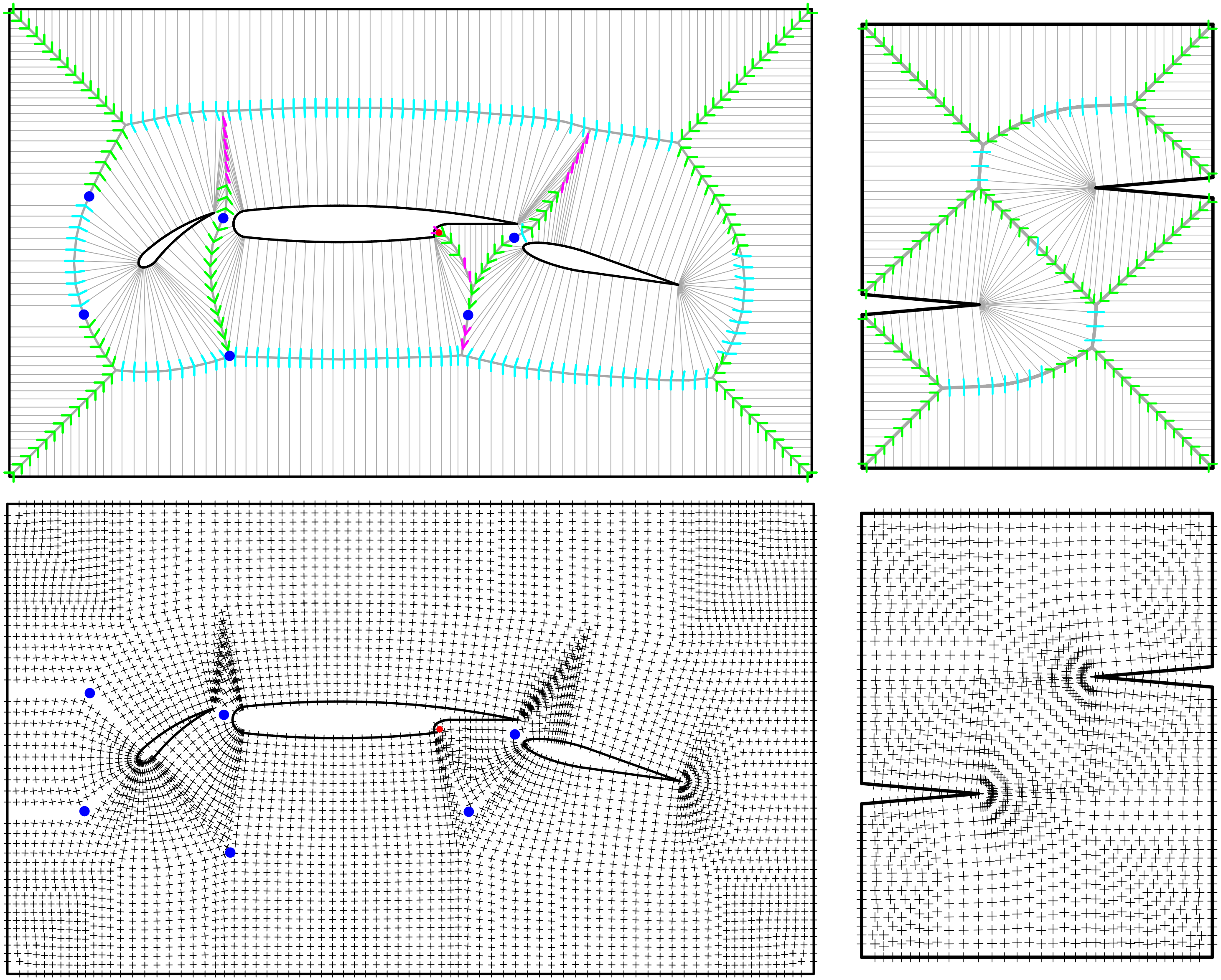
Examples



Examples



Examples



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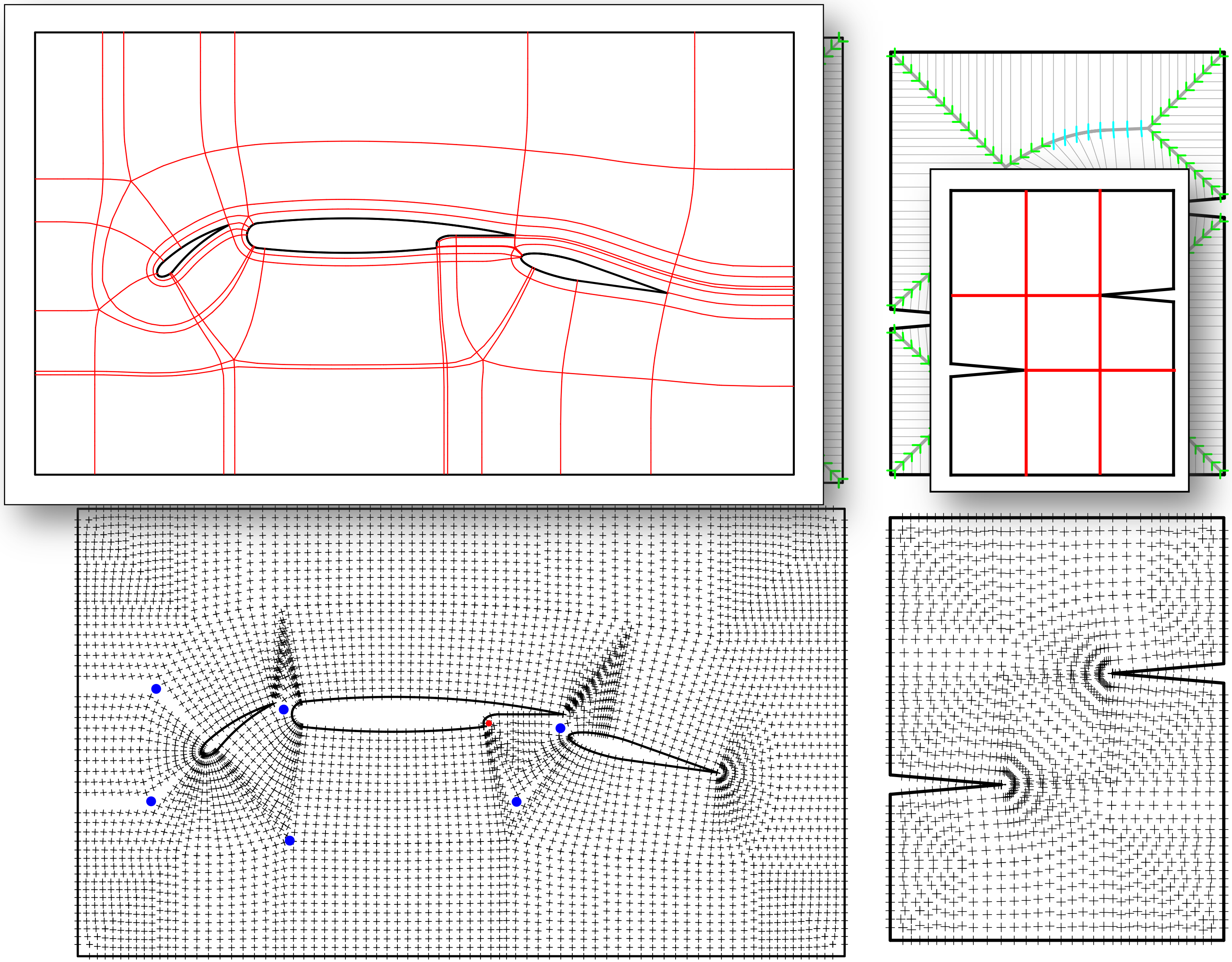

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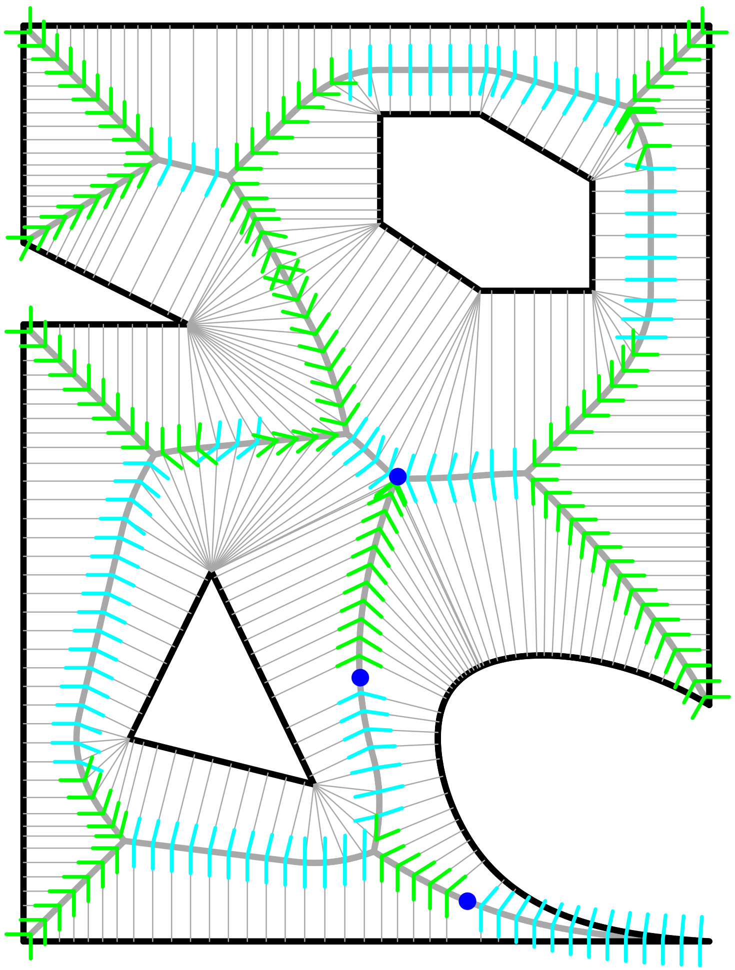

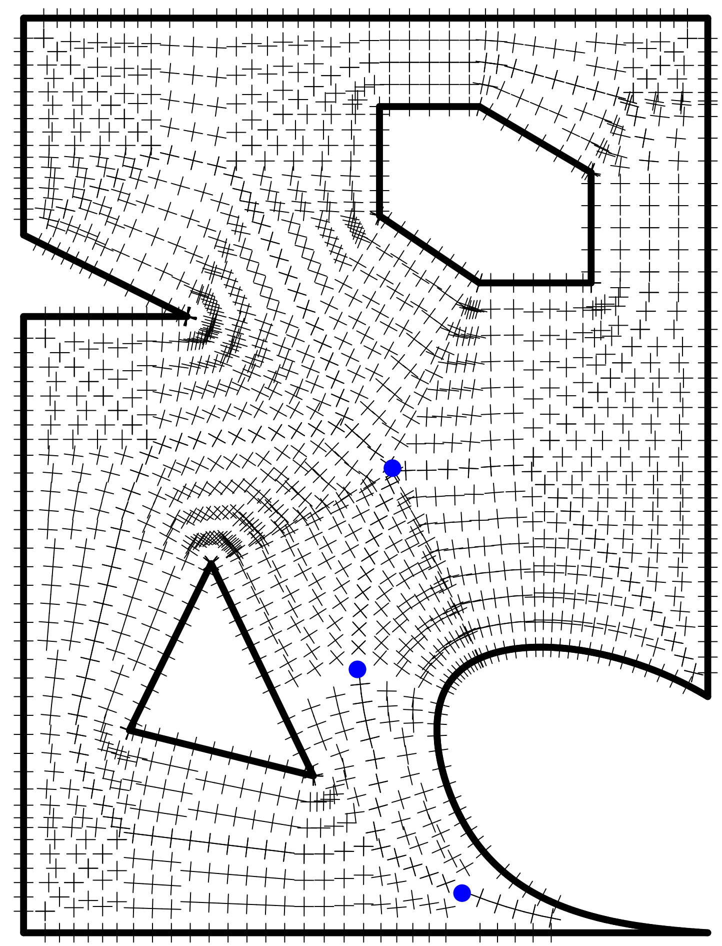

Examples

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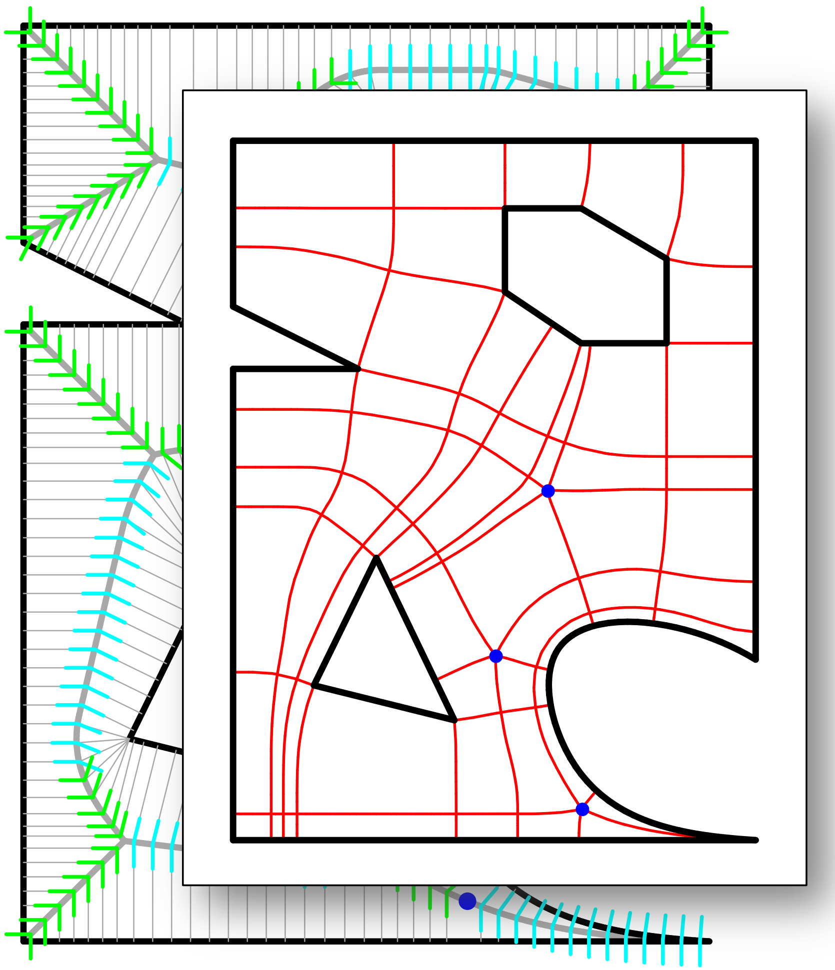
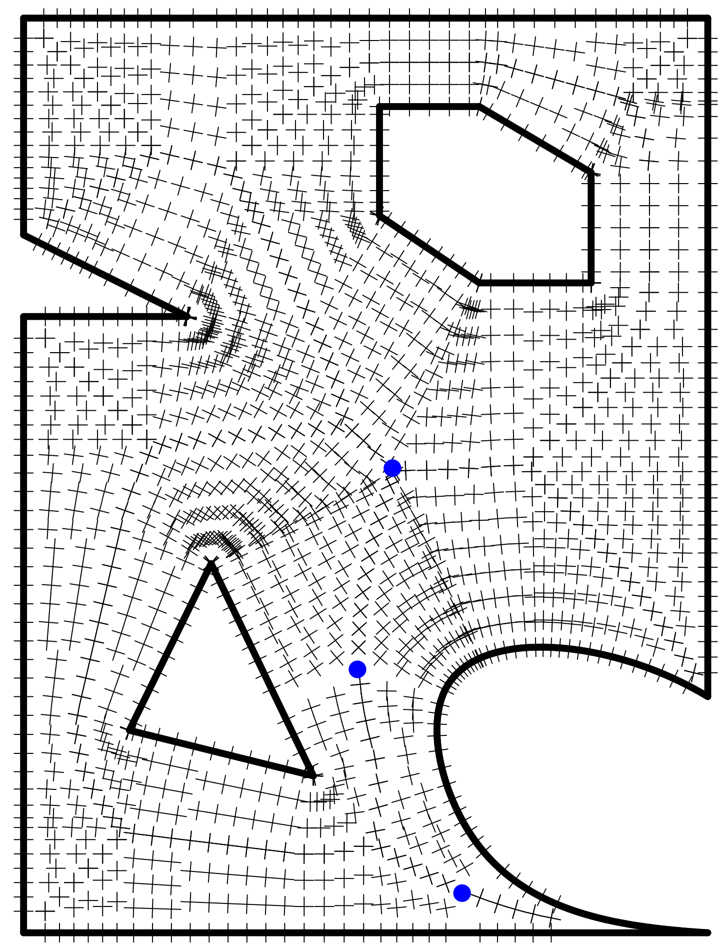
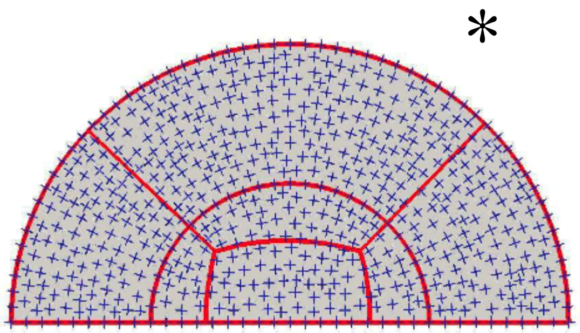
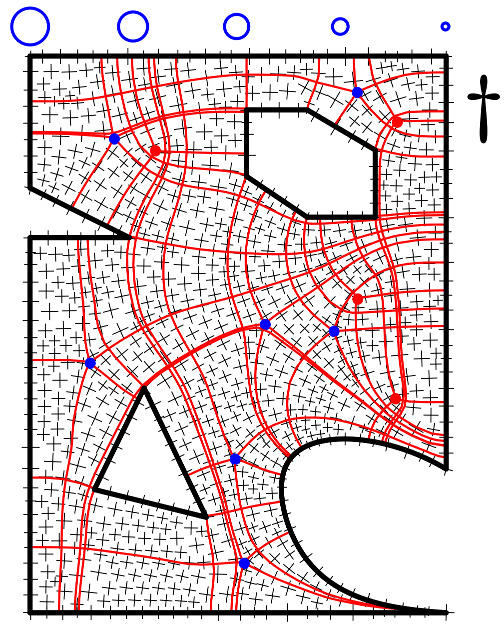




Generating the decomposition

- Trace separatrices of cross-field…been done.





* **N. Kowalski et al.** A pde based approach to multidomain partitioning and quadrilateral meshing. In Proc. of the 21st IMR, San Jose, Oct. 2012.
† **H. Fogg et al.** Multi-block decomposition using cross-fields. In Proc. of ADMOS, Lisbon, June 2013.
H. Fogg et al. New techniques for enhanced medial axis based decompositions in 2-D. In Proc. of 23rd IMR, London, Oct, 2014.

- Niche of MA-based mesh generation:
  - ‘lightweight’, cheap, fast, robust.
- Require simple operations to split surface satisfying implicit solution…see [Fogg, 2014]

Summary

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- Mesh singularities are the key features in all-quad meshes
- The MA can be used to effectively locate positions of singularities.
- Natural treatment of concavities and responsiveness to geometric shape