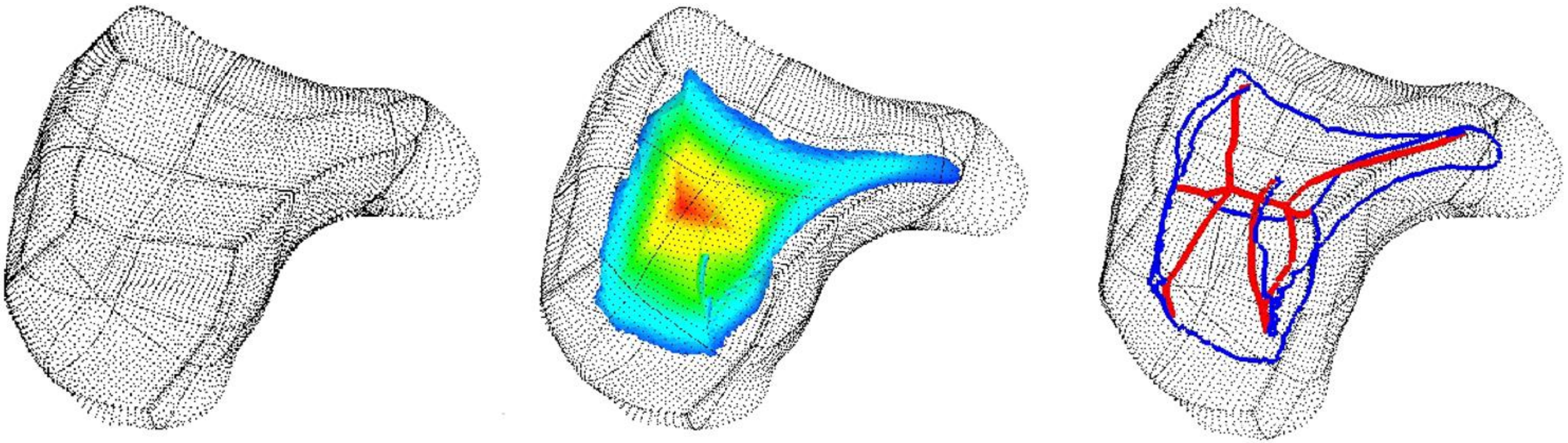
# Medial Scaffolds for 3D data

Medial Object Workshop, organised by TranscentData Europe Ltd.
Cambridge, UK, 9-10 Oct. 2014




**Frederic Fol Leymarie**

Goldsmiths
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# Outline

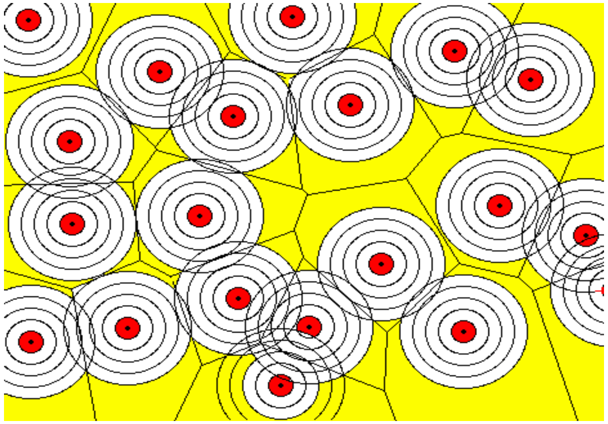
**Background**

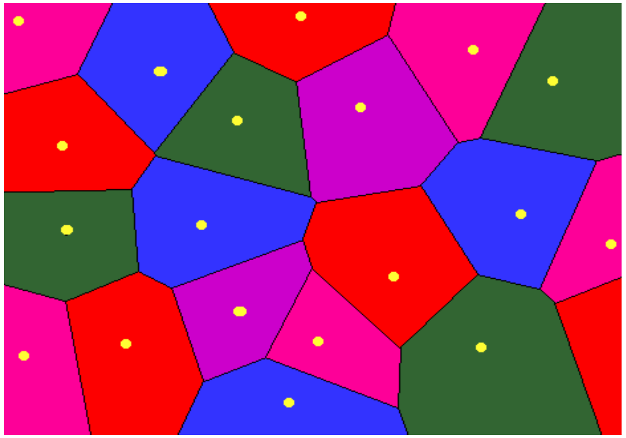
Method and some algorithmic details

Applications

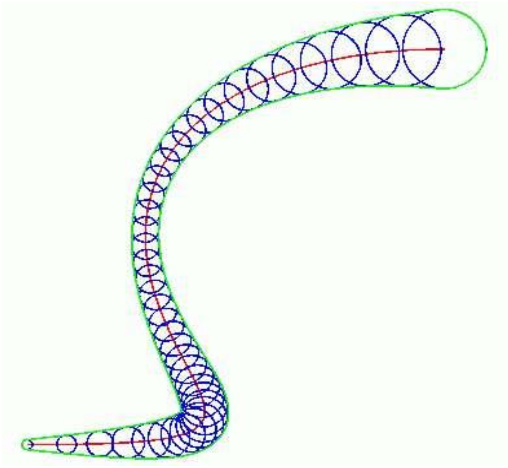
# Shape representation: the Medial Axis

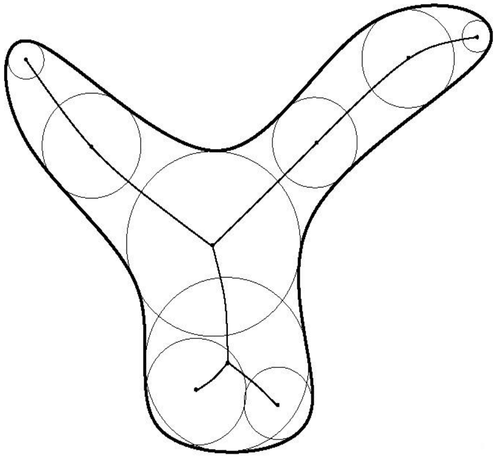
**Wave propagation**
Blum,
Voronoi,
Turing, *et al.*



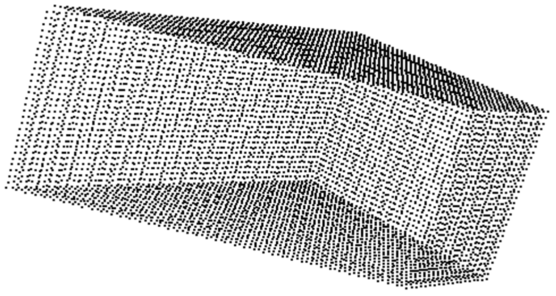


**Maximal disks**
Blum, Wolter,
Leyton, Kimia,
Giblin, *et al.*





# Study 3D shape with minimal assumptions

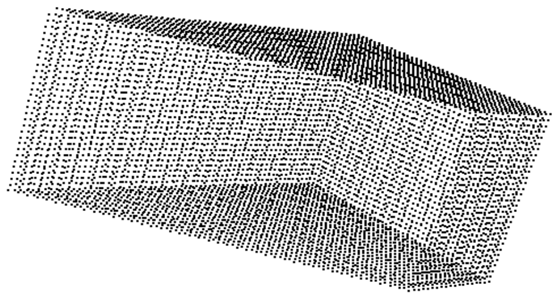


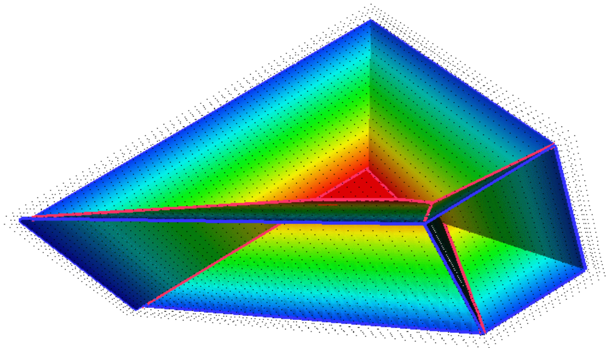
Unorganised point set (sampling)

Context: 1$^{st}$ reconstruct a surface mesh from ***unorganized*** points, with a “minimal” set of assumptions: the samples are nearby a “possible” surface *(thick volumetric traces not considered here).*

Benefit: reconstruction across **many types of surfaces**.

# Study 3D shape with minimal assumptions



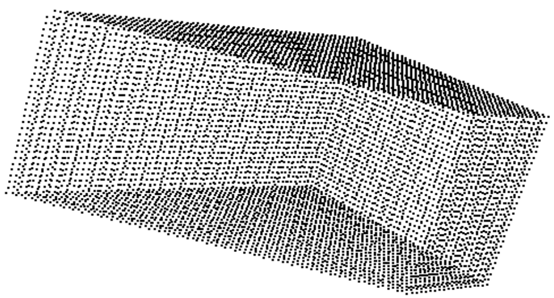


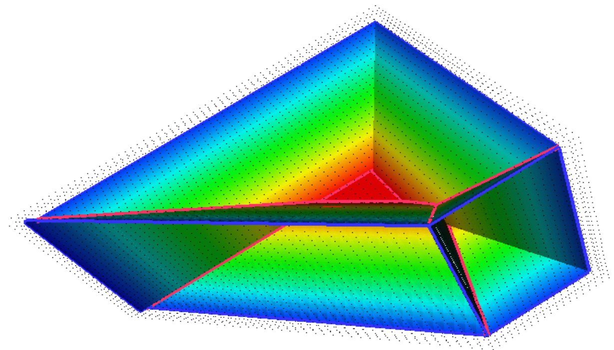
“classical” 3D Medial Axis
(color indicates radius flow)

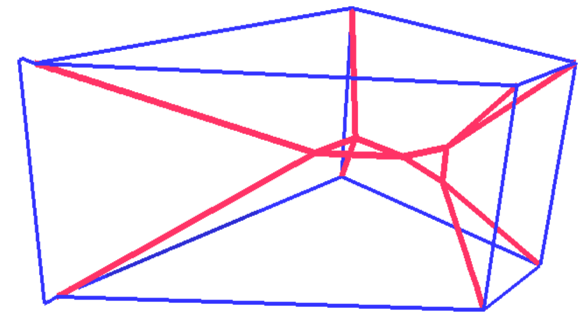
Context: 1st reconstruct a surface mesh from *unorganized* points, with a “minimal” set of assumptions:
the samples are nearby a “possible” surface
*(thick volumetric traces not considered here).*

Benefit: reconstruction across many types of surfaces.

# Study 3D shape with **minimal** assumptions







Medial scaffold as two
3D curve sets (oriented graph)

Context: 1st reconstruct a surface mesh from *unorganized* points, with a “minimal” set of assumptions:
the samples are nearby a “possible” surface
*(thick volumetric traces not considered here).*

Benefit: reconstruction across many types of surfaces.

# Study shape with **minimal** assumptions

To find a *general* approach, applicable to various topologies, without assuming strong *input constraints*, *e.g.:*

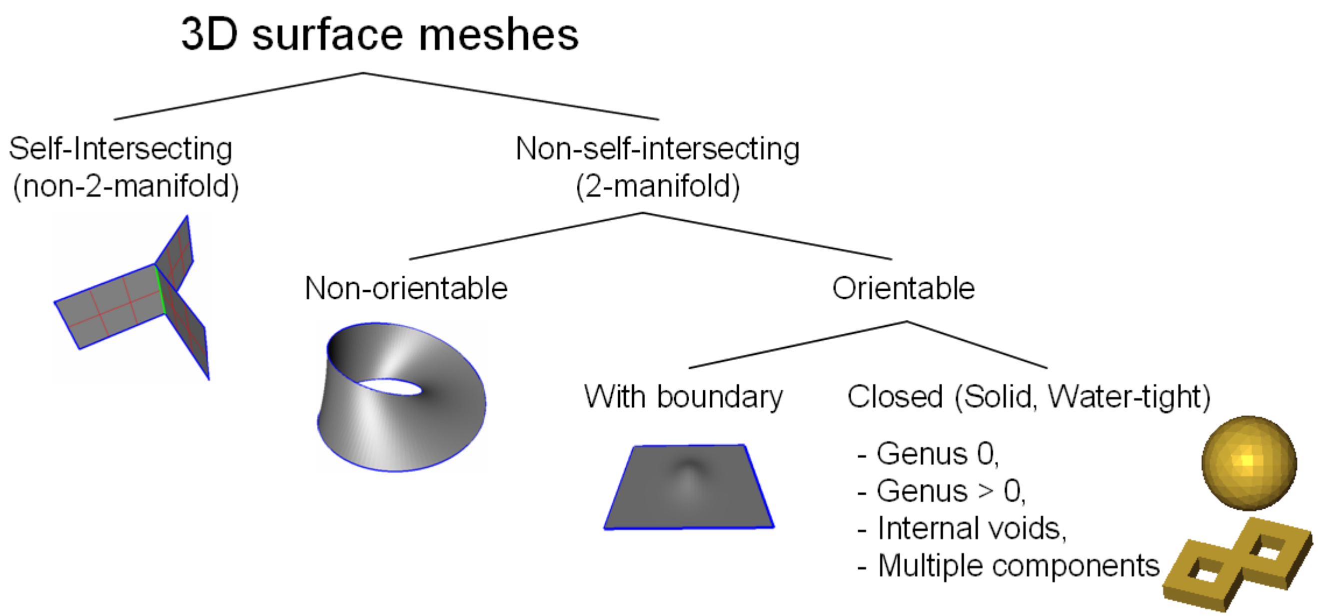
–No surface normal information.

–Unknown topology (with boundary, for a solid, with holes, non-orientable).

–No a priori surface smoothness assumptions.

–Practical sampling condition: non-uniformity, with varying degrees of noise.

–Practical large input size (> millions, billions of points).



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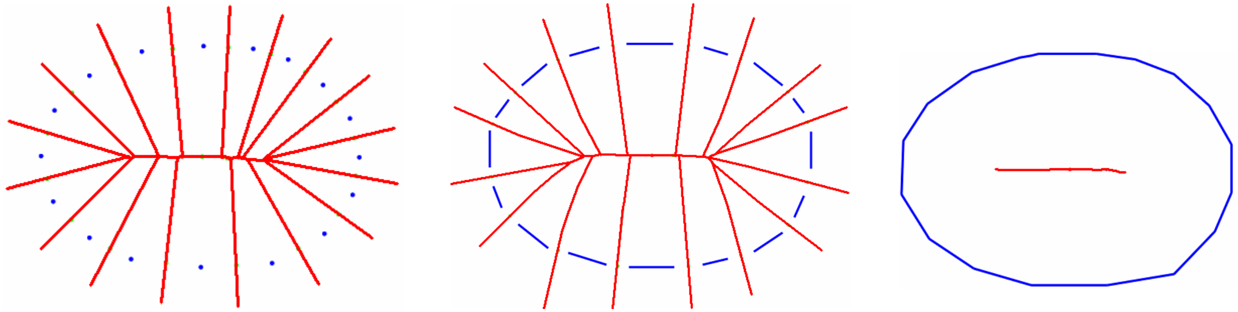
# How: Overview of Our Approach (2D)

Not many clues from the assumed loose input constraints.

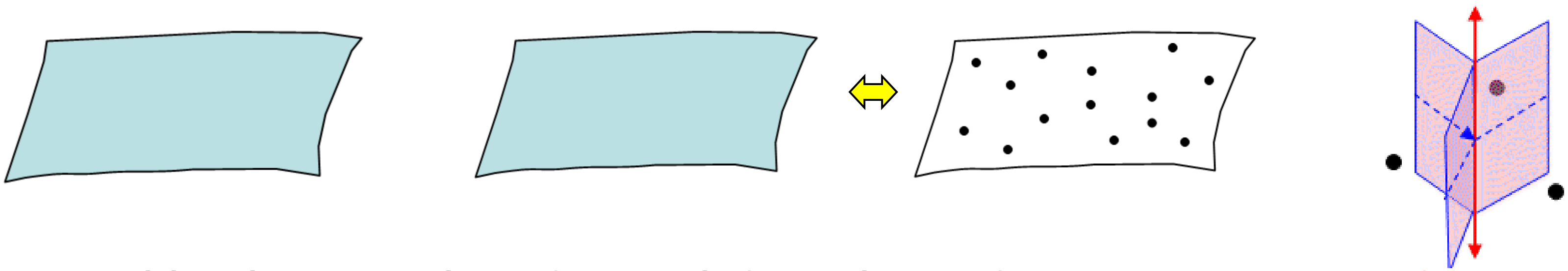
Work on the shape itself to recover the sampling process.



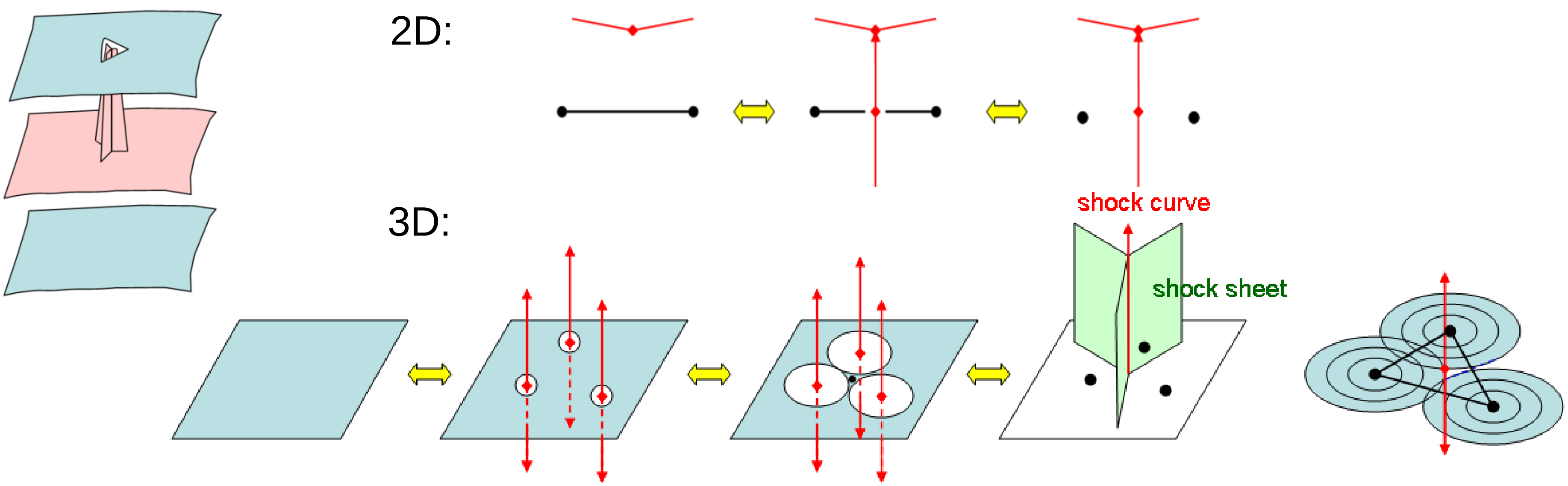
In 2D: work with shock graphs (after Kimia *et al*.).

# How: Sampling / Meshing as Deformations

Schematic view of sampling: infinitesimal holes grows, remaining are the samples.



We consider the removing of a patch from the surface as a Gap Transform.



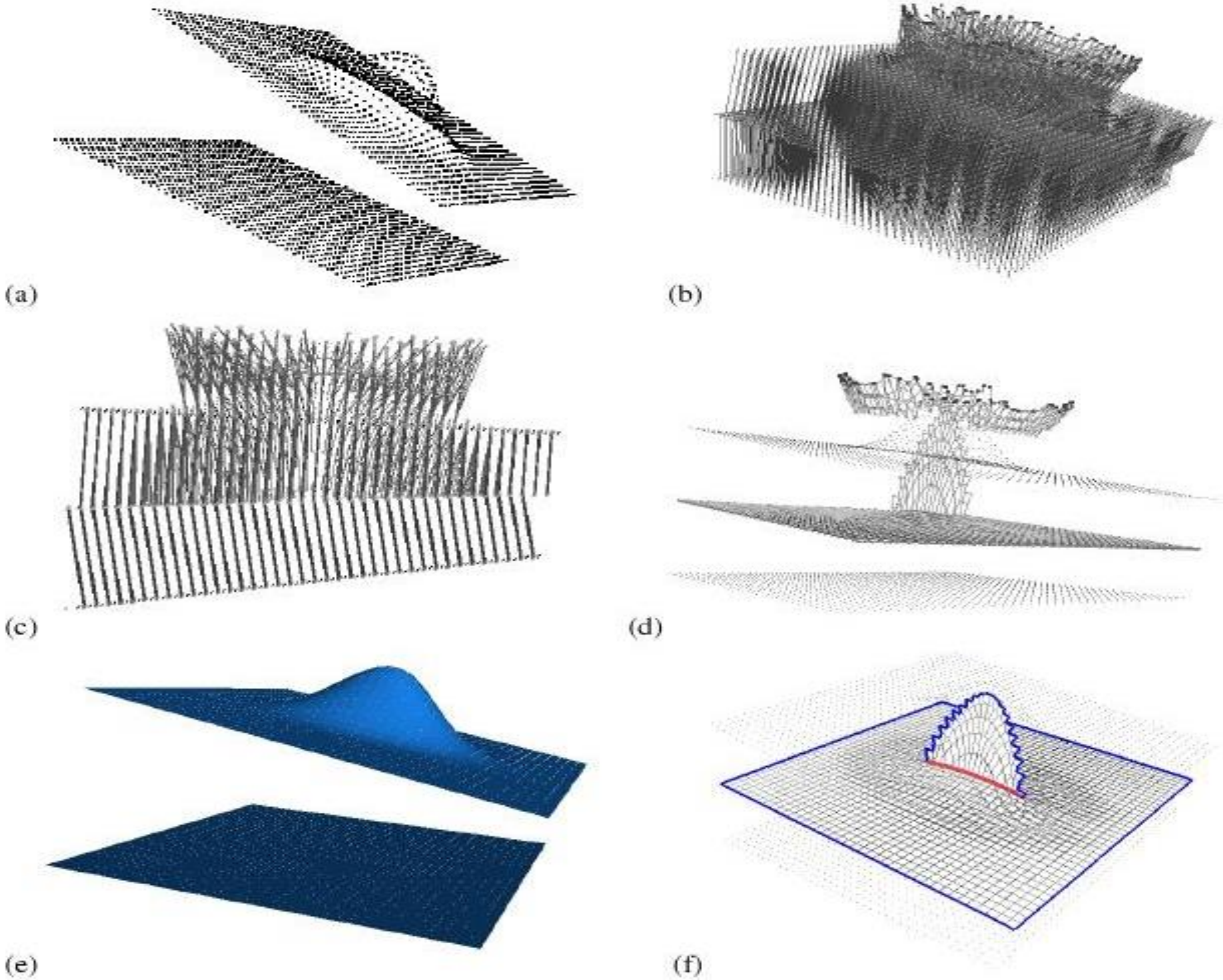

Surface

Infinitesimal holes

Holes merging, surface between holes degenerating into points

The shock curve with 3 shock sheets are used to recover gaps.

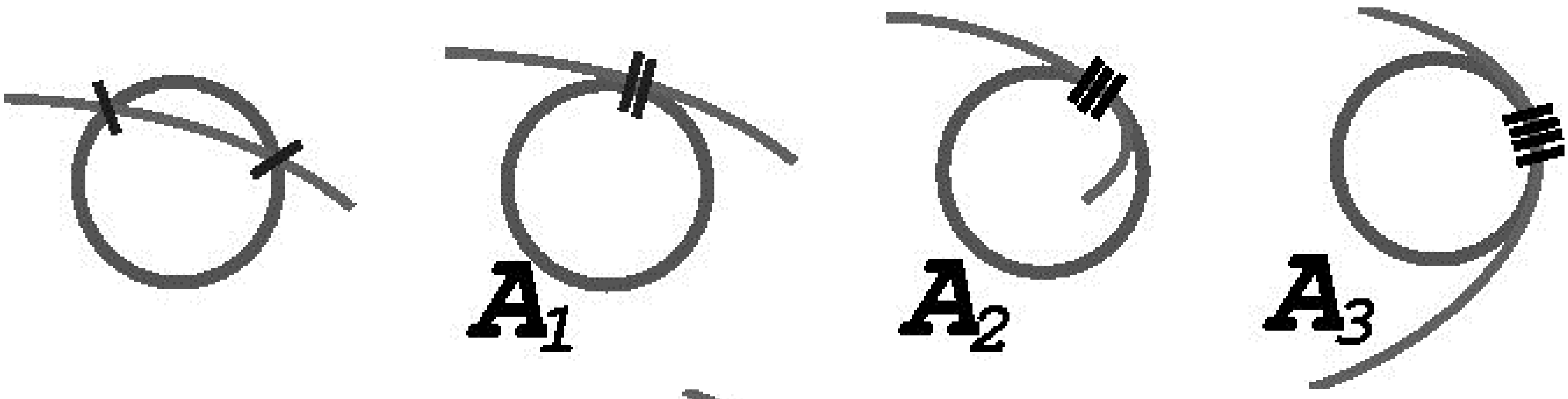
# How: Overview of Our Approach (3D)

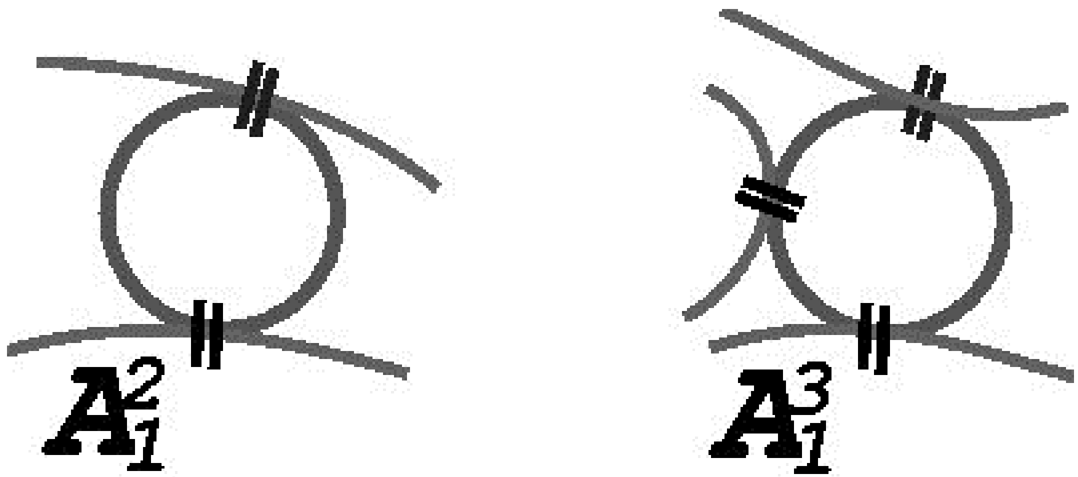



PhD 2002-3 of Fol Leymarie.

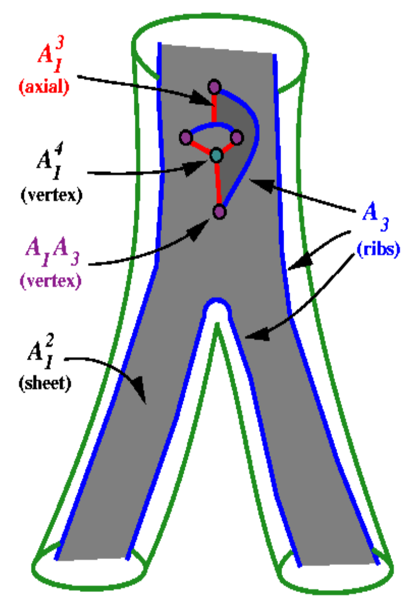
# How: Medial Scaffolds for **3D** Shapes

## A graph structure for the 3D Medial Axis




**A**$_k^n$: contact (max. ball) at *n* distinct points, each with *k+1* degree of contact.



# How: Medial Scaffolds for **3D** Shapes

## A graph structure for the 3D Medial Axis

Classify shock points into **5 general types**,

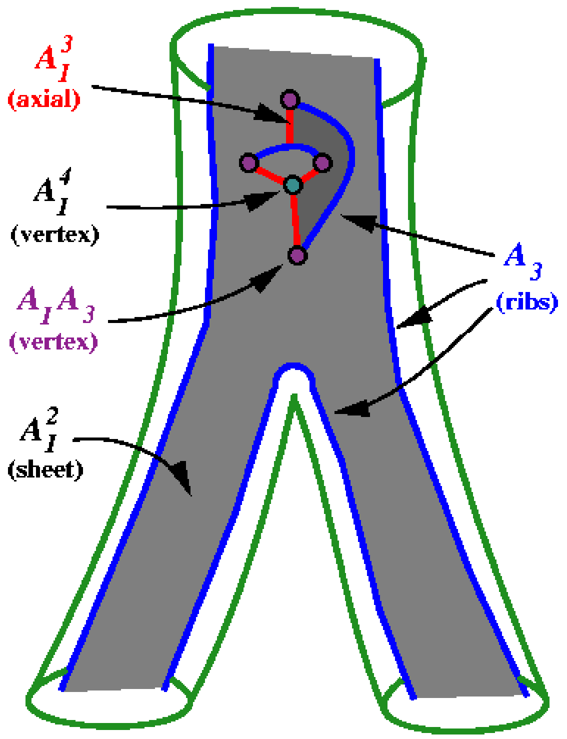
and organized into a hyper-graph form

[Leymarie PhD 2002-3, Giblin&Kimia PAMI’04, Leymarie&Kimia PAMI'07]:

–Shock Sheet: $A_1^2$

–Shock Curves: $A_1^3$ (***Axial***), $A_3$ (***Rib***)

–Shock Vertices: $A_1^4$, $A_1A_3$

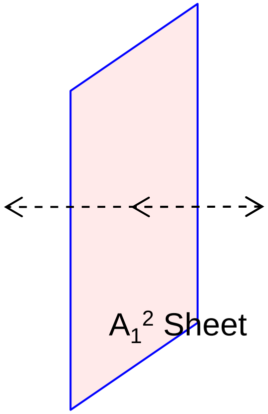


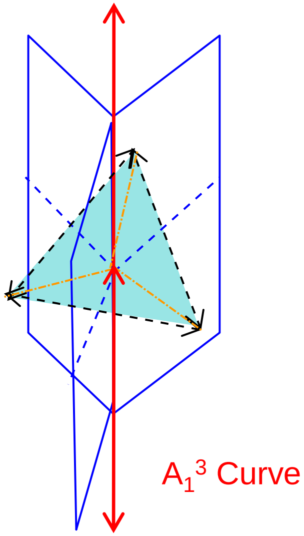

$\mathbf{A}_{\boldsymbol{k}}^{\boldsymbol{n}}$: contact (max. ball) at *n* distinct points, each with *k+1* degree of contact.

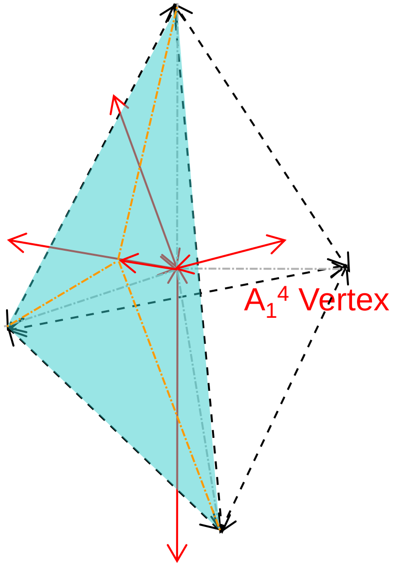
# How: Sampling / Meshing as Deformations

Special case where input consists only of points (in 3D), then the Medial Scaffold consists of only:

$A_1^2$ Sheets, $A_1^3$ Curves, $A_1^4$ Vertices.



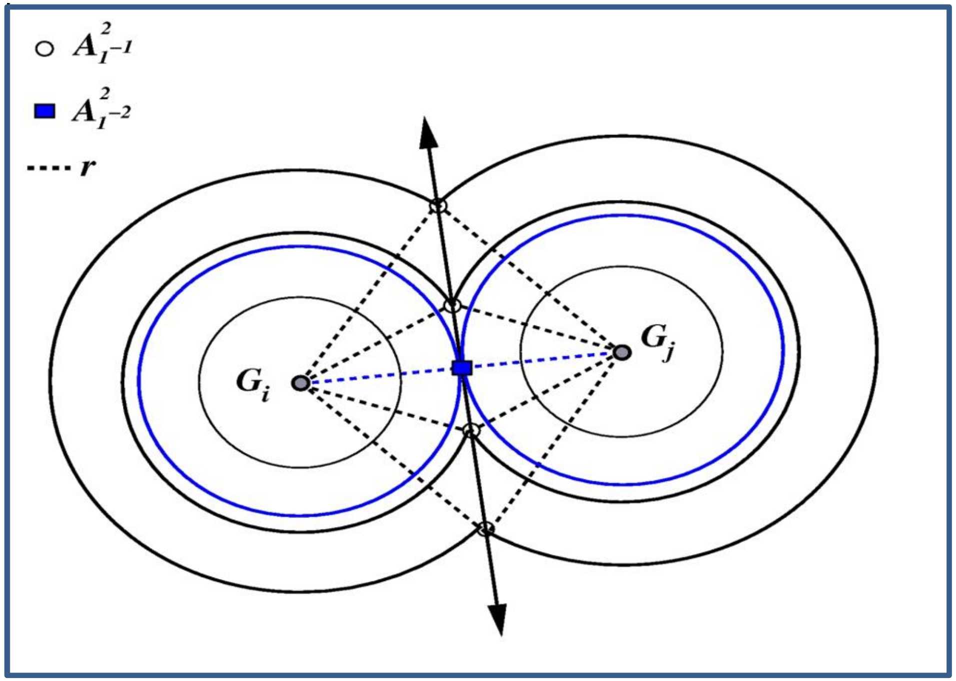



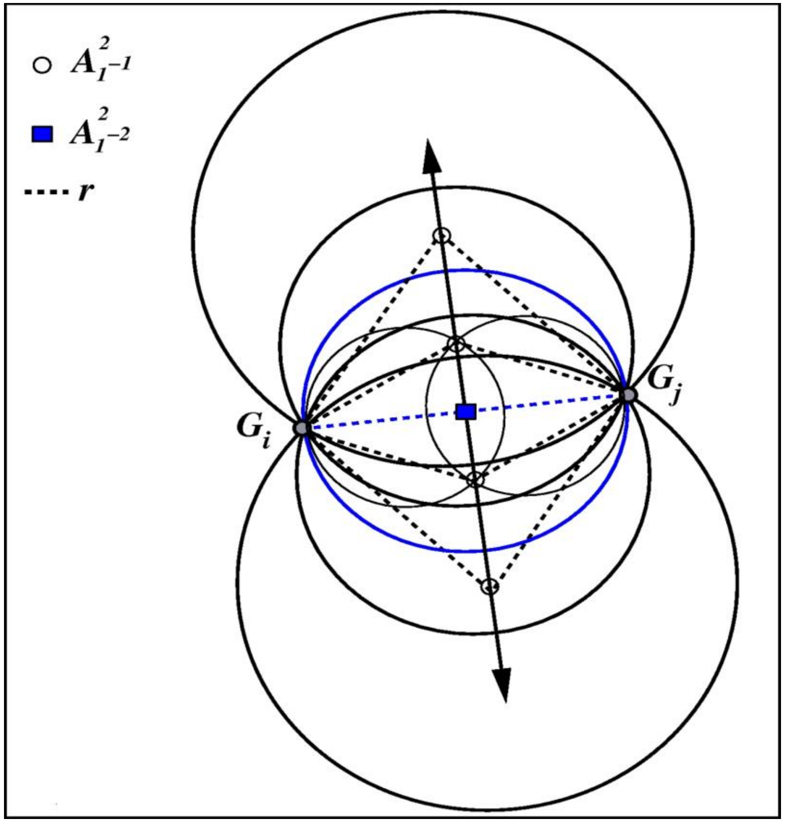



# How: Medial Scaffolds for **3D** Shapes

## A graph structure for the 3D Medial Axis

Each point sample is a generator (G_i).
Idea: (1) Pair generators to find initial shock sources of shock sheets & curves. Then, (2) pair shock sources to find higher degree shocks (sheets to curves, curves to vertices).

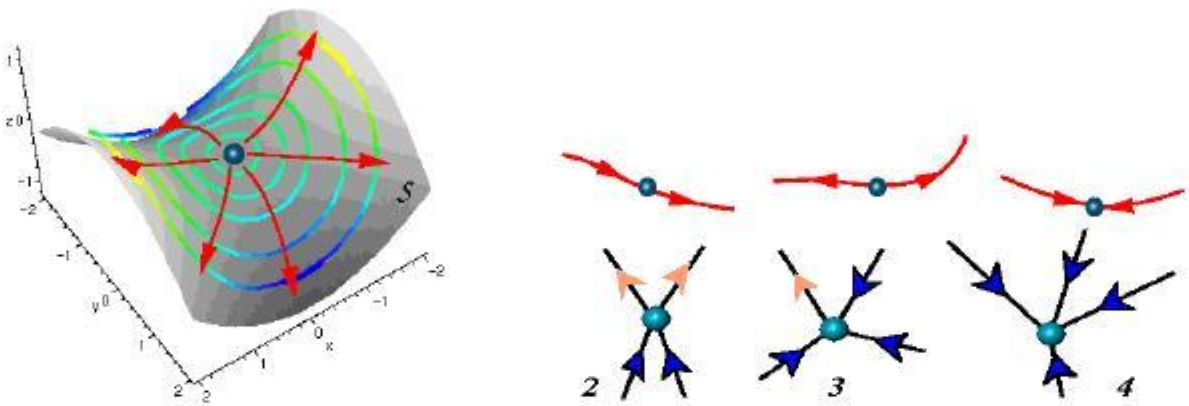



Details in PAMI 2007 paper (Leymarie+Kimia).

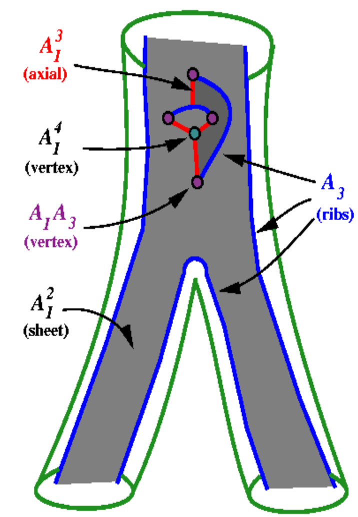
# How: Medial Scaffolds for **3D** Shapes

## A graph structure for the 3D Medial Axis



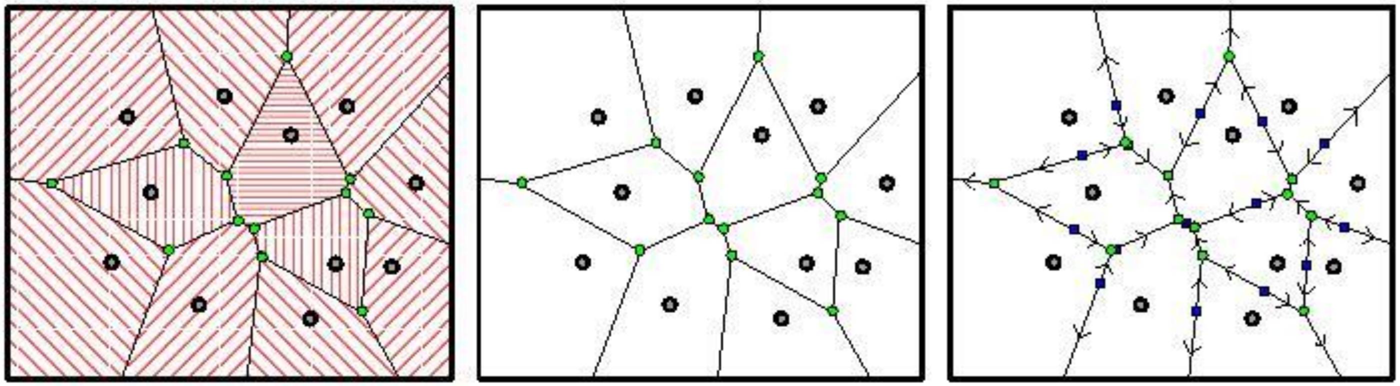

(a) (b)

Figure 1.3: Types of flows along $\mathcal{MA}$ structures. (a) For $\mathcal{MA}$ sheets, the flow is generally initiated at a single point, and the sheet is grown outward and radially from that point. (b) At the top are shown the typical flows along $\mathcal{MA}$ curves, *i.e.*, regular, initial and final. At the bottom are shown the typical sets of inward and outward flows (along $\mathcal{MA}$ curves) at $\mathcal{MA}$ vertices where the number of inward flows is indicated.



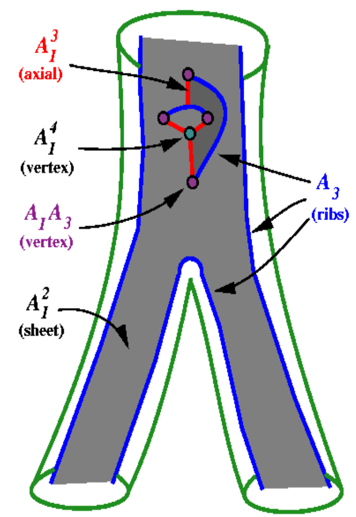
# How: Medial Scaffolds for **3D** Shapes

## A graph structure for the 3D Medial Axis

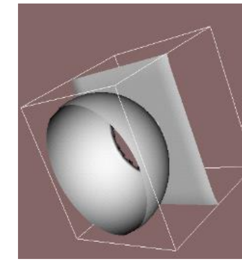


(a) $\mathcal{VD}$ (b) $\mathcal{MA}$ (c) $\mathcal{SC}$

Figure 3.14: Example of a 2D Voronoi diagram (a), medial axis (b), and shock scaffold (c) for a set of eleven point generators (large grey disks) in the plane. Voronoi or shock vertices are indicated as smaller **green** disks, Voronoi edges or shock curves are drawn as straight lines, Voronoi regions are hashed in **red**. In (c), $A_1^2{-}2$ shock sources are indicated as **blue** squares. In (b) we see that the $\mathcal{VD}$ minus the interior of its Voronoi regions coincides with the $\mathcal{MA}$.



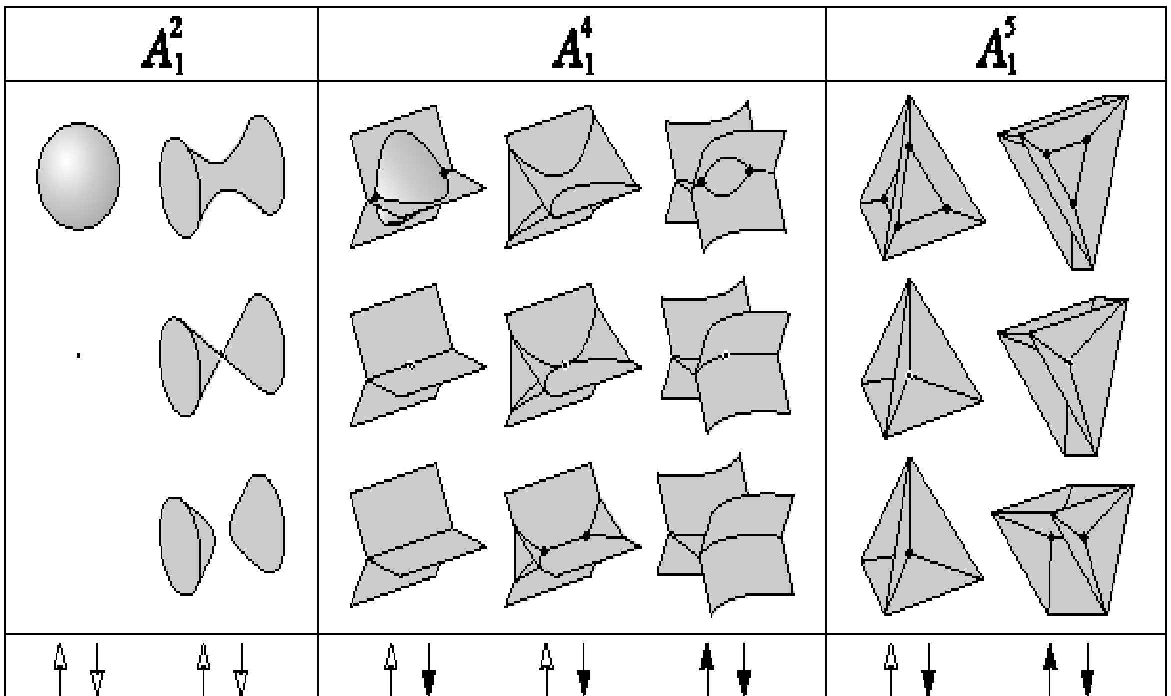
# **Transitions** of the 3D graph structure



Study the topological events of the graph structure under perturbations and shape deformations.

Singularity theory (Arnold *et al.*, since the 1990's):

In 3D, 26 topologically different perestroikas of linear shock waves.




“Perestroikas of shocks and singularities of minimum functions”
I. Bogaevsky, 2002.

# **Transitions** of the 3D graph structure

Study the topological events of the graph structure under **perturbations** and **shape deformations**.

Transitions of the MA (Giblin, Kimia, Pollit, PAMI 2009):

Under a 1-parameter family of deformations, only **seven transitions** are relevant.

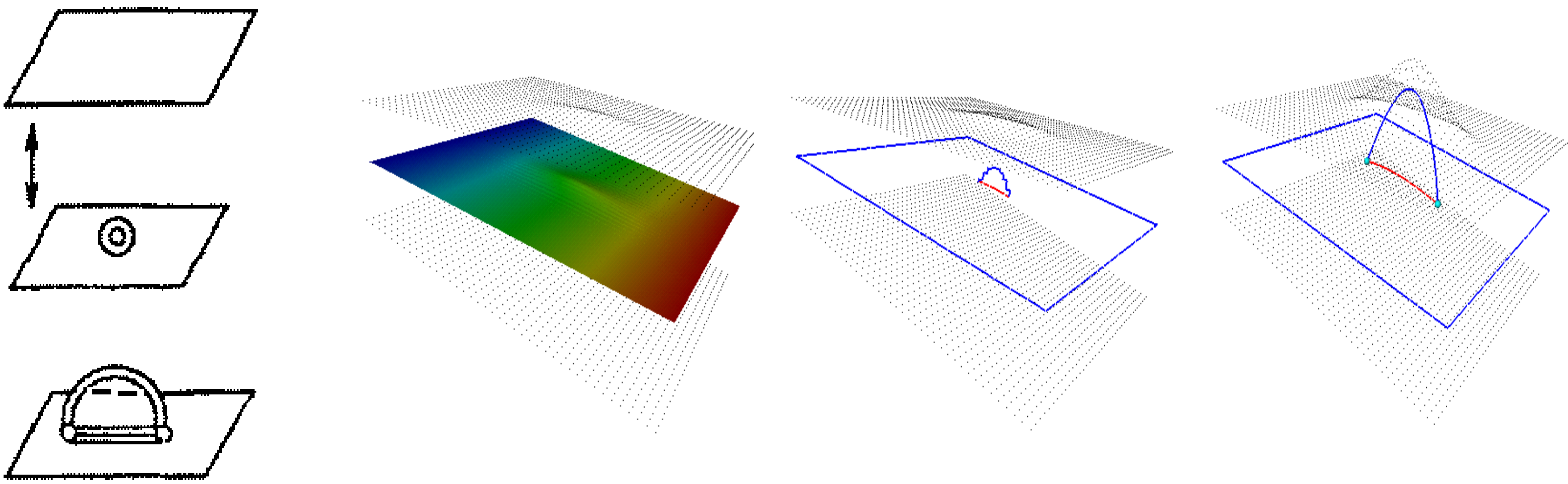
| Transition | Collision of Types |
|---|---|
| $A_1^4$ | $A_1^3$ - $A_1^3$ |
| $A_1^5$ | $A_1^4$ - $A_1^4$, $A_1^4$ - $A_1^3$ |
| $A_5$ | $A_1A_3$ - $A_1A_3$, $A_3$ - $A_3$ |
| $A_1A_3 - I$ | $A_1A_3$ - $A_1A_3$ |
| $A_1A_3 - II$ | $A_1A_3$ - $A_1A_3$, $A_1^3$ - $A_3$ |
| $A_1^2A_3 - I$ | $A_1^4$ - $A_1A_3$ |
| $A_1^2A_3 - II$ | $A_1^3$ - $A_1A_3$ |

# **Transitions** of the 3D graph structure

Study the topological events of the graph structure
under **perturbations** and **shape deformations**.

Transitions of the MA:

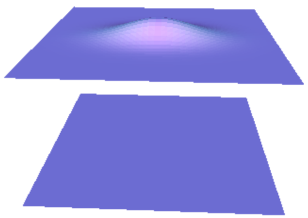
Under a 1-parameter family of deformations, only **seven transitions** are relevant.



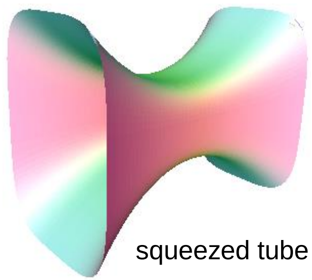
$A_1A_3$-I (protrusion-like, Leymarie, PhD, 2003)

# **Transitions** of the 3D graph structure

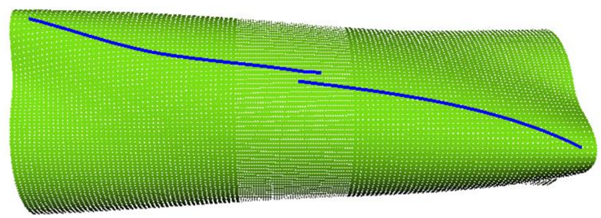
Study the topological events of the graph structure
under **perturbations** and **shape deformations**.



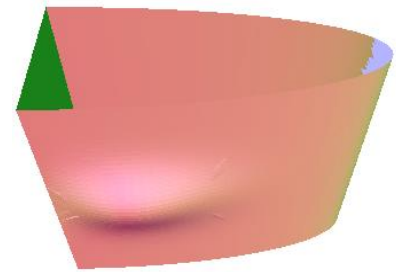
parallel plane with a bump



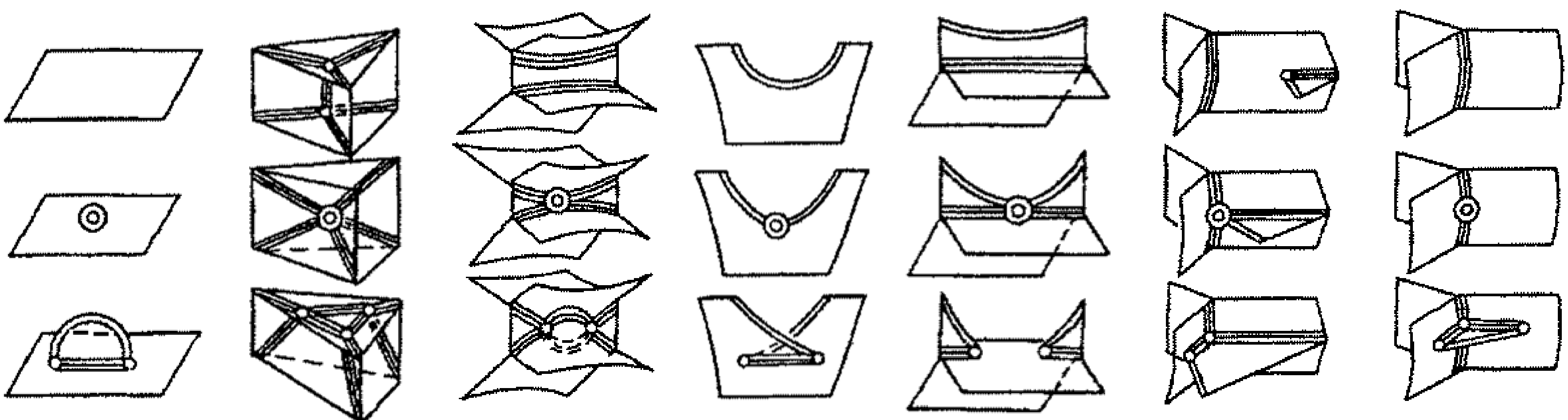
squeezed tube



twisted parabolic gutter



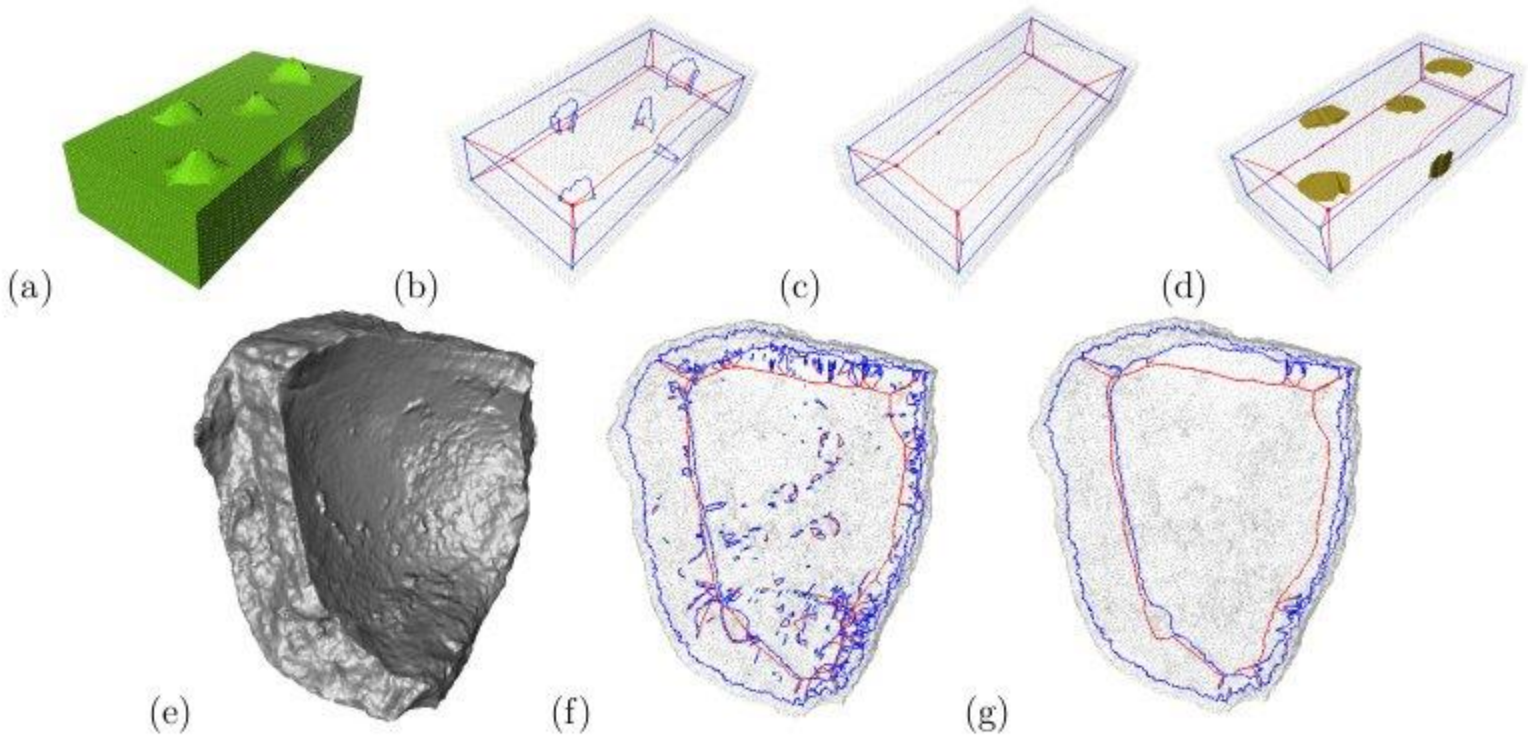
parabolic gutter with a bump



$A_1A_3$-I  $A_1^5$  $A_1^4$  $A_5$  $A_1A_3$-II  $A_1^2A_3$-I  $A_1^2A_3$-II

# **Transitions** of the 3D graph structure

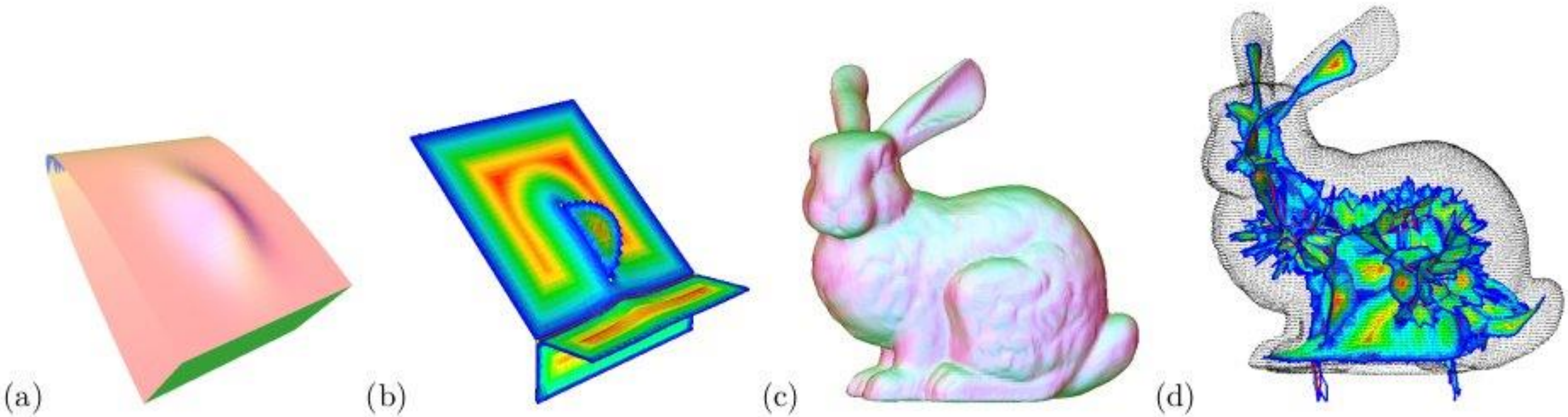
Study the topological events of the graph structure under perturbations and shape deformations.




Towards surface regularisations via transitions (Leymarie, Giblin, Kimia, 2004)

# **Transitions** of the 3D graph structure

Study the topological events of the graph structure under perturbations and shape deformations.




Capture transitions via geodesy on MA (Chang, Kimia, Leymarie, on-going)

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Method and **some algorithmic details**

Applications

# Algorithmic Method

- Consider Gap Transforms on *all* $A_1^3$ shock curves in a ranked-order fashion:

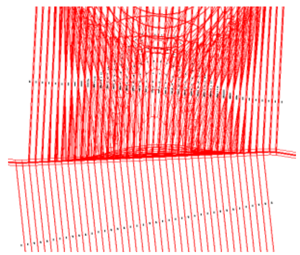
–best-first (greedy) with error recovery.

- Cost reflects:

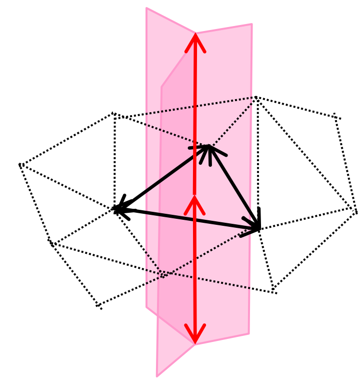
–Likelihood that a shock curve (triangle) represents a surface patch.

–Consistency in the local context (neighboring triangles).

–Allowable (local surface patch) topology.

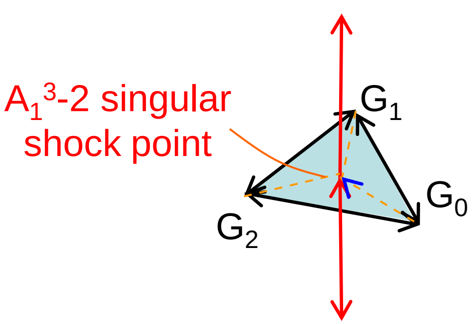


$A_1^3$ shock curve

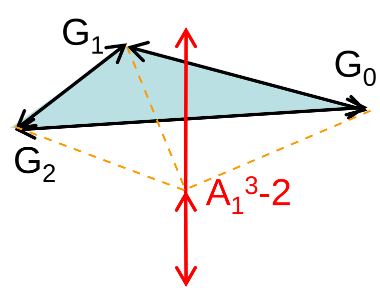


Three $A_1^2$ shock sheets

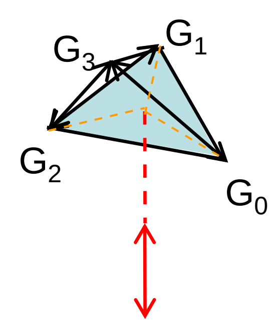
3 Types of $A_1^3$ shock curves (dual Delaunay triangles):
Represented in the MS by “singular shock points” ($A_1^3$-2)




Type I




Type II




Type III

(unlikely to be correct candidate)

# Algorithmic Method

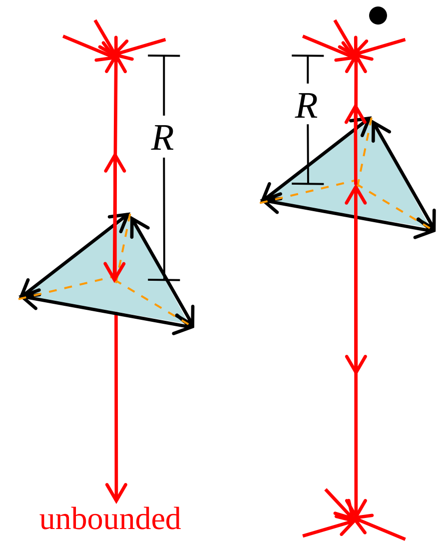
How we **order gap transforms**:

- Favor small “compact” triangles.
- Favor recovery in “nice” (simple) areas, *e.g.*, away from ridges, corners, necks.
- Favor simple local continuity (similar orientation).
- Favor simple local topologies (2D manifold).
- BUT: allow for error recovery!

# Ranking Isolated Shock Curves (Triangles)

Triangle geometry:

$$
\begin{aligned}
D &= \max(d_1, d_2, d_3) \\
P &= d_1 + d_2 + d_3 \\
m &= (d_1 + d_2 - d_3)(d_3 + d_1 - d_2)(d_2 + d_3 - d_1) \\
A &= \sqrt{(P \cdot m)/16} \qquad \text{(Heron's formula)} \\
C &= 4\sqrt{3} \cdot A/(d_1^2 + d_2^2 + d_3^2), \quad \text{(Compactness, Gueziec's formula, } 0<C<1\text{)}
\end{aligned}
$$



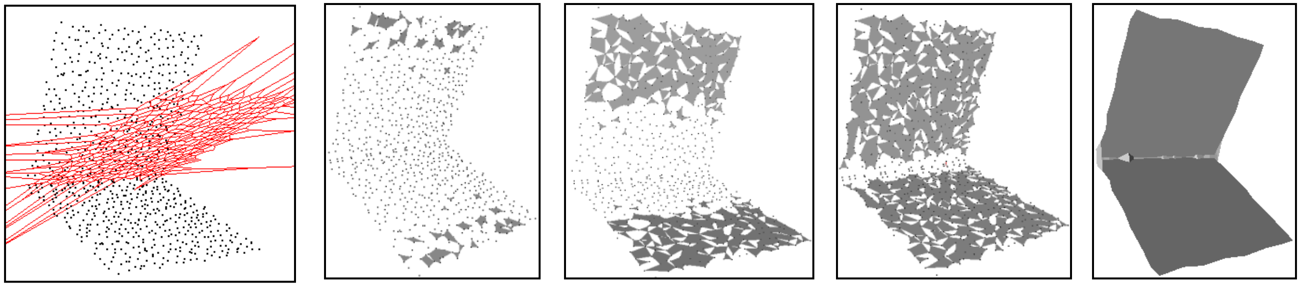

The side of smaller shock radius is more salient.

Cost: favors *small compact* triangles with large shock radius *R*.

$$
\rho_1 = \begin{cases} \frac{P}{R} \cdot \frac{1}{C^2}, & \text{if } D < d_{\max} \\ \infty, & \text{if } D \geq d_{\max} \end{cases}
$$

*R*: minimum shock radius

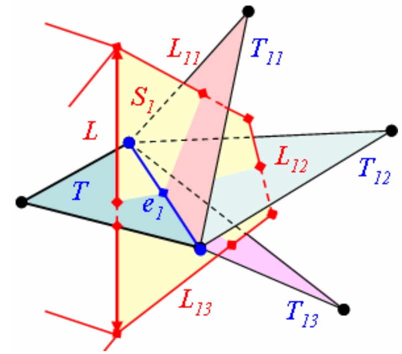
$d_{max}$: maximum expected triangle, estimated from $d_{med}$



Surface meshed from confident regions toward the sharp ridge region.

# Cost Reflecting Local Context & Topology

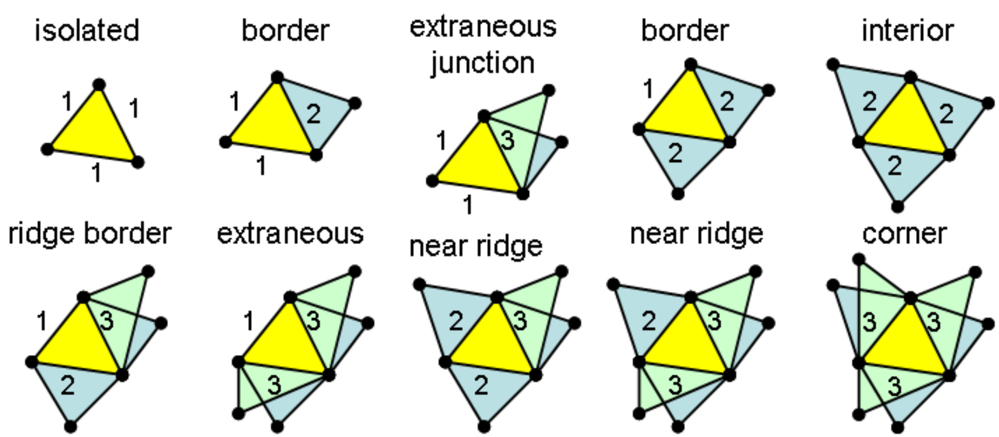
Cost to reflect smooth continuity of edge-adjacent triangles:



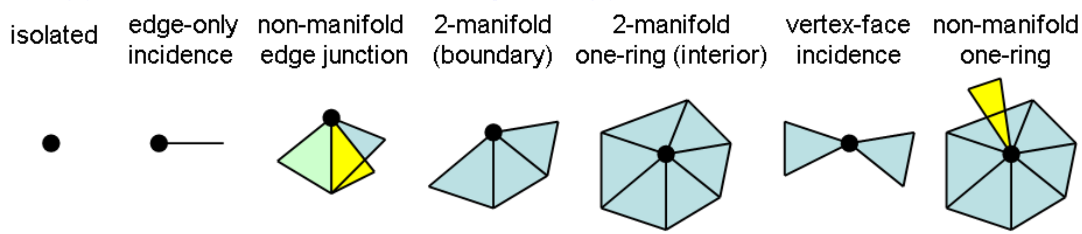

$$\rho_2 = \frac{d}{R} \cdot \frac{1}{C^2} \cdot f(\theta)\,,$$

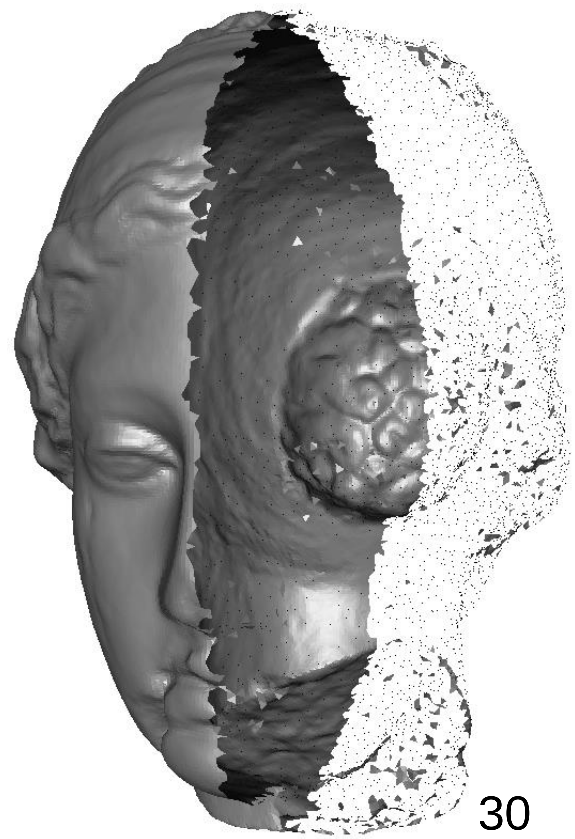
$$f(\theta) = [\exp^{\theta} - 1]^2 - 1 \begin{cases} \theta = 0,\ f(\theta) = -1 \\ \theta = 40°,\ f(\theta) \simeq 0 \\ \theta = 80°,\ f(\theta) \simeq 8.24 \end{cases}$$

Typology of triangles sharing an edge:




Typology of mesh vertex topology



Point data courtesy of Ohtake *et al*.

# Strategy in the Greedy Meshing Process

Problem: Local ambiguous decisions □ errors.

Solutions:

- Multi-pass greedy iterations

  First construct confident surface triangles without ambiguities.

- Postpone ambiguous decisions

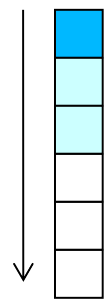
  – Delay related candidate Gap Transforms close in rank, until additional supportive triangles (built in vicinity) are available.

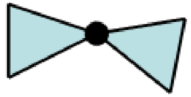
  – Delay potential topology violations.

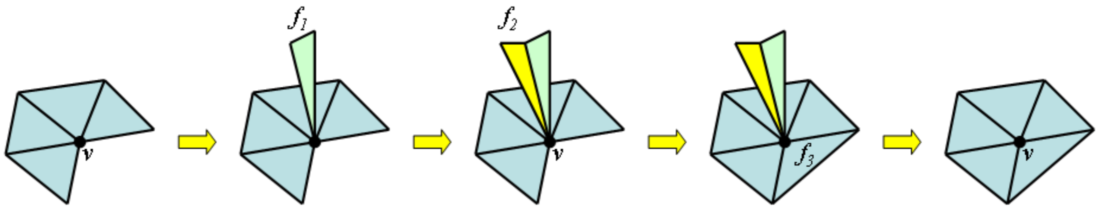
- Error recovery

  – For each Gap Transform, re-evaluate cost of both related *neighboring (already built)* & *candidate* triangles.

  – If cost of any existing triangle exceeds top candidate, undo its Gap Transform.

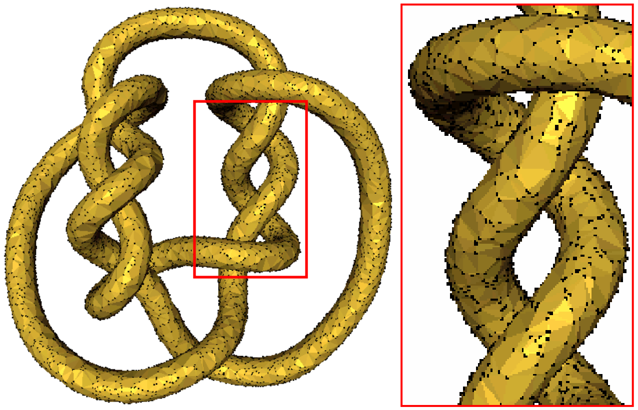


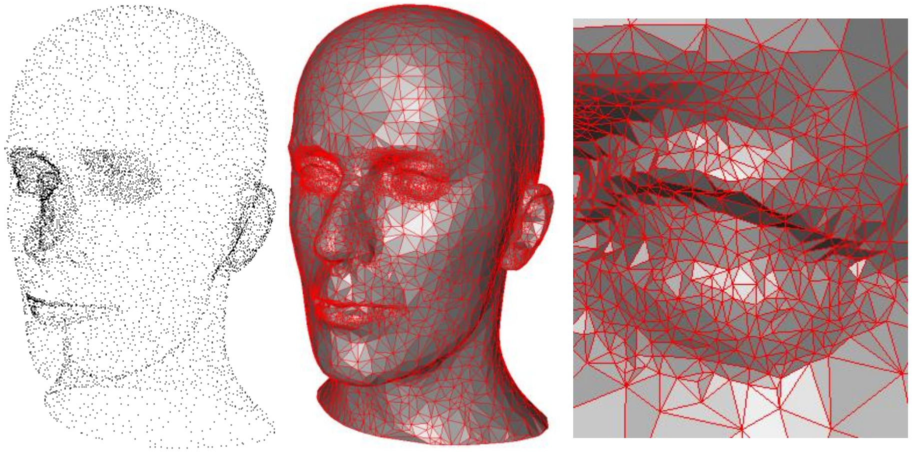



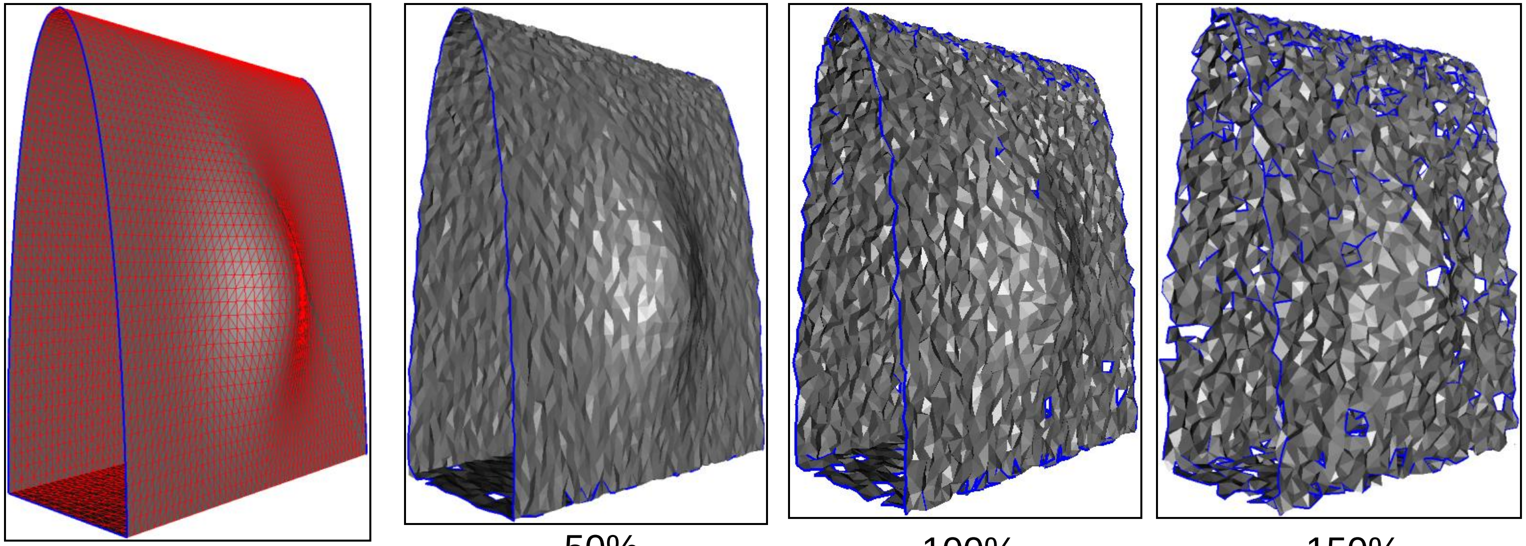
# Dealing with sampling quality

Input of non-uniform and low-density sampling:





Response to additive noise:



50% 100% 150%

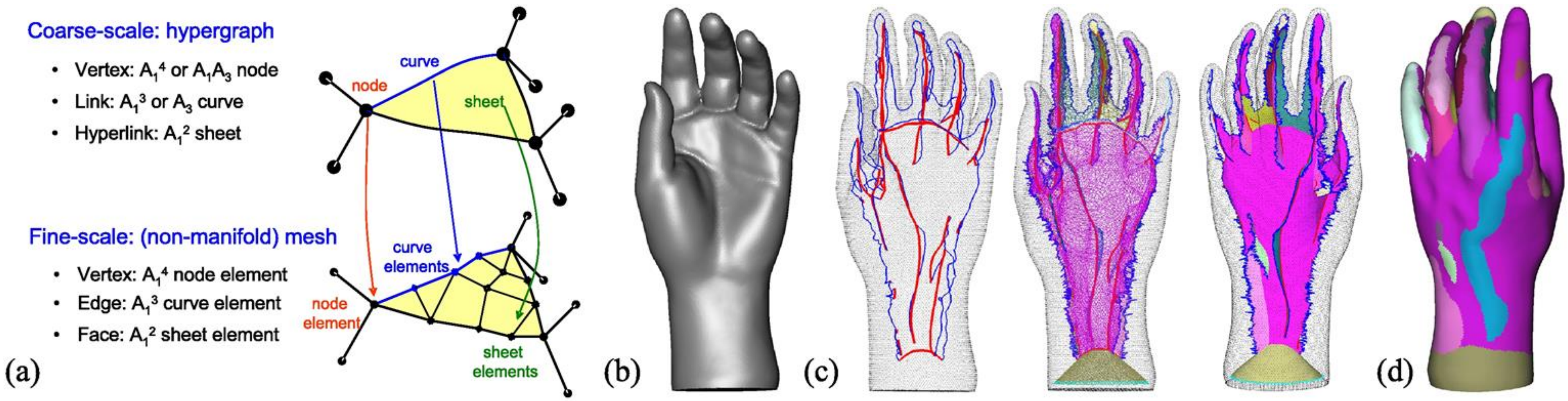
# Outline

Background

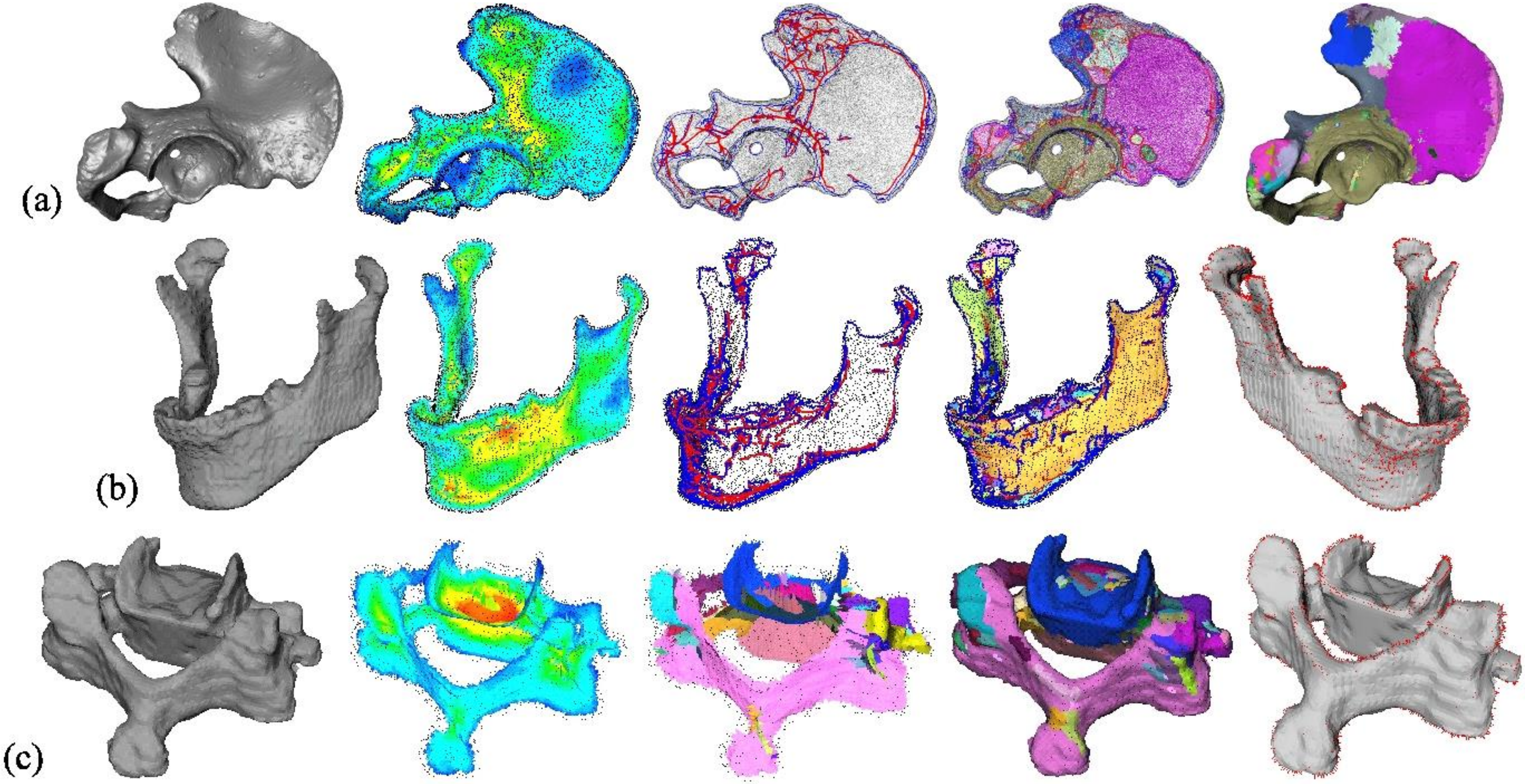
Method and some algorithmic details

**Applications**

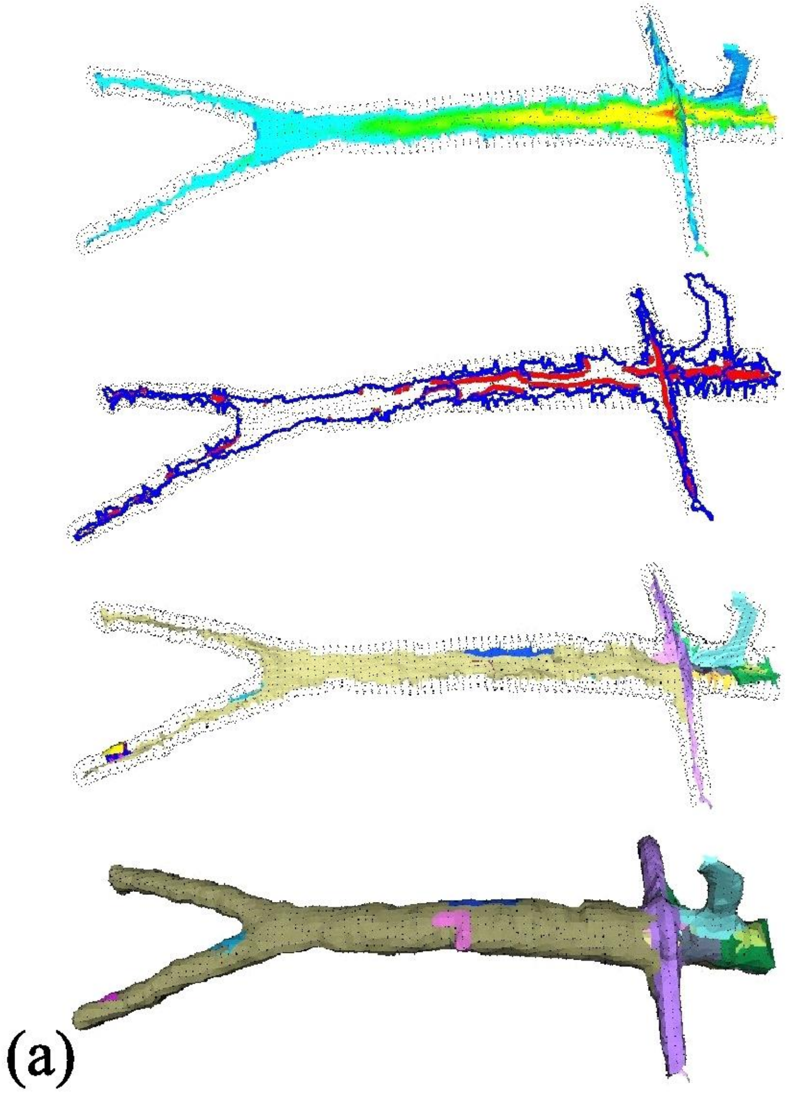
# From Fine to Coarse Scales



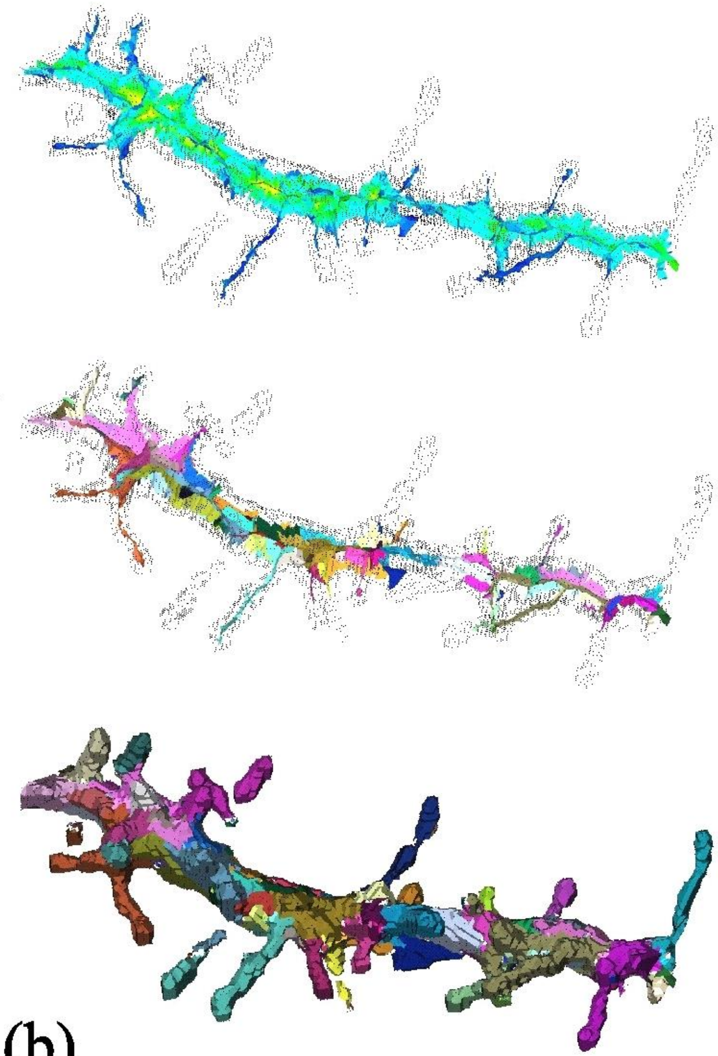
# Bone shape study



# 3D Tubular & Branching Shapes

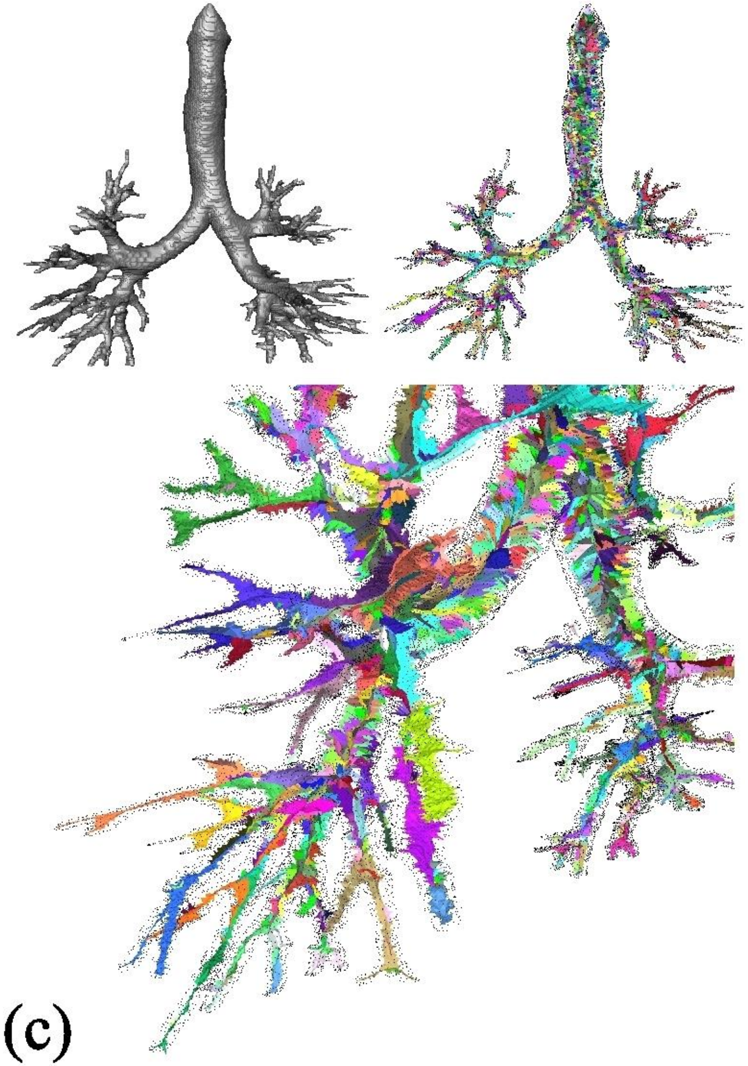


# 3D Tubular & Branching Shapes



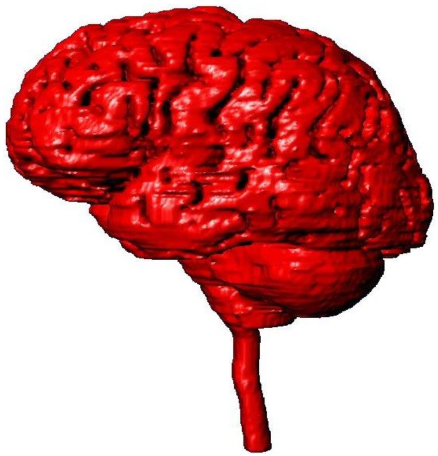
(b)

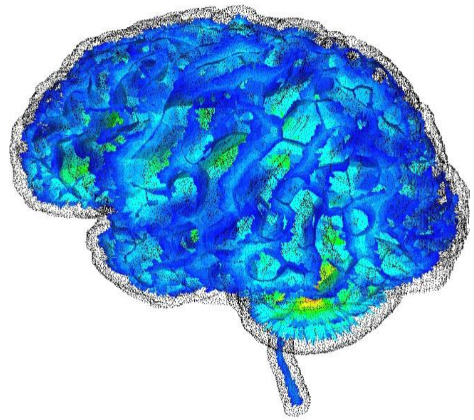
# 3D Tubular & Branching Shapes

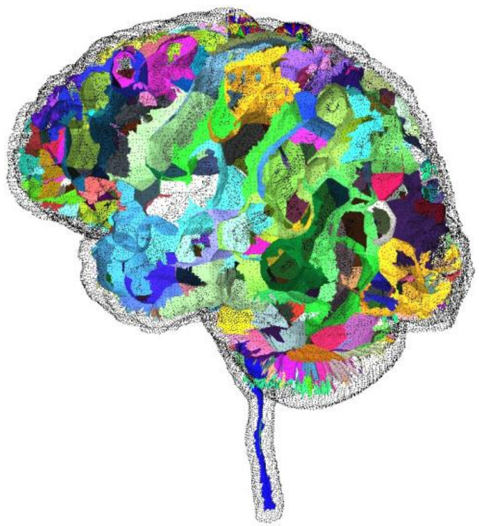


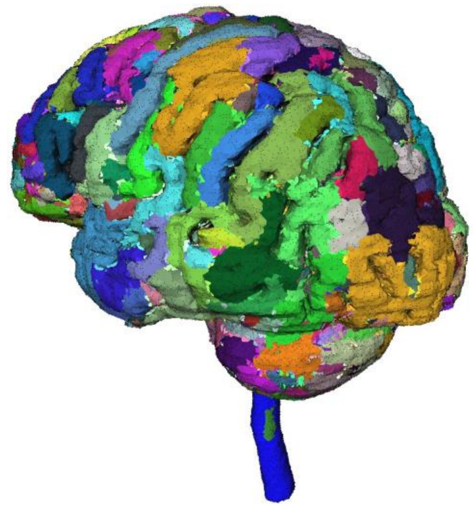
(c)

# 3D Convoluted Shapes: Brains

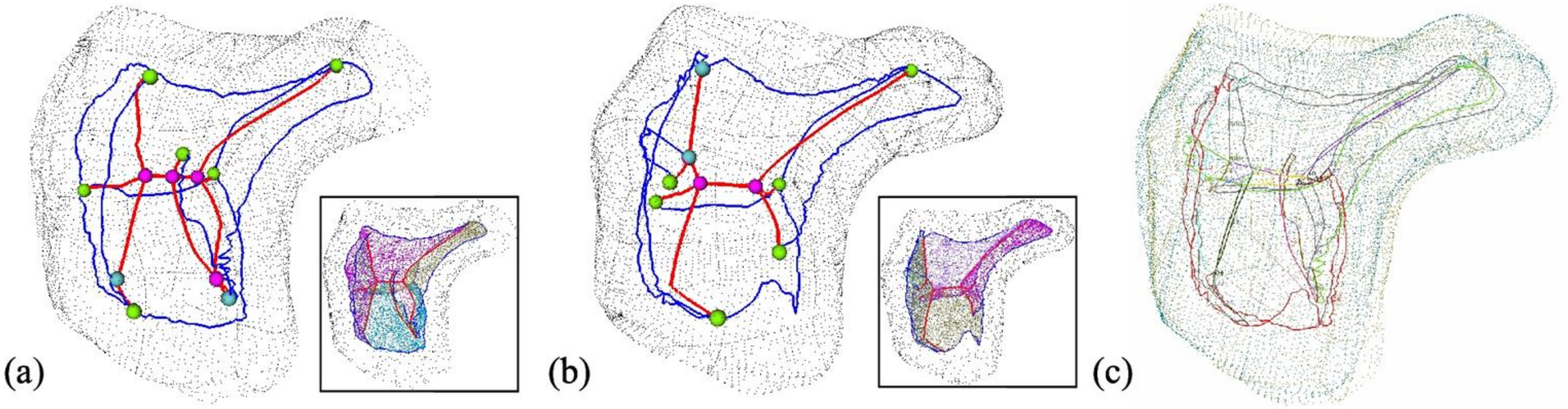






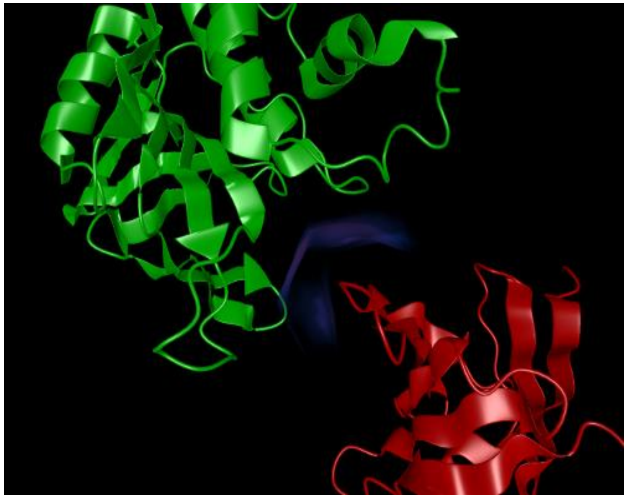


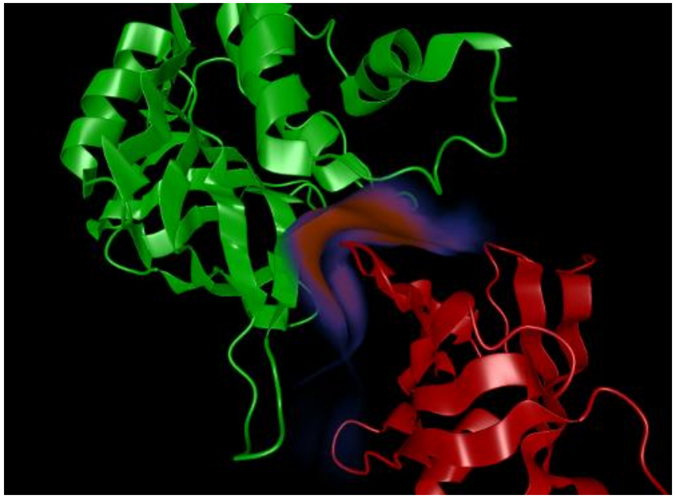
# 3D Shape Matching/Registration

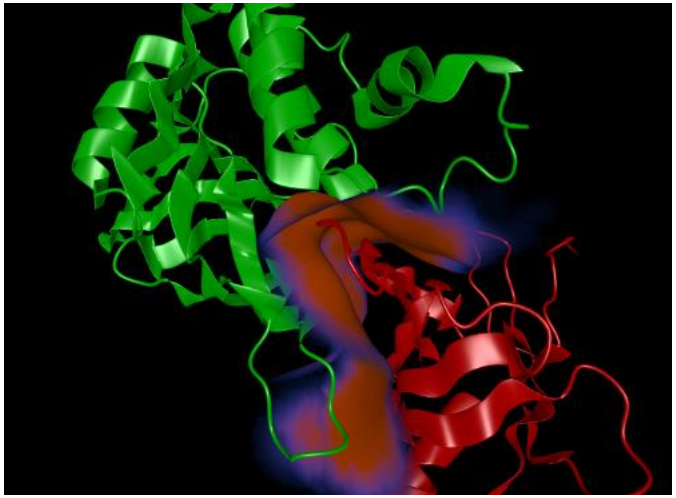



Metacarpal bones being matched/registered. Using the simplified Medial Scaffolds makes the problem tracktable.

# 3D Shape in Molecular biochemistry







Protein Docking

Goldsmiths College and Imperial College

BBSRC funded project: 2013-2017.

# Outline

Background

Method and some algorithmic details

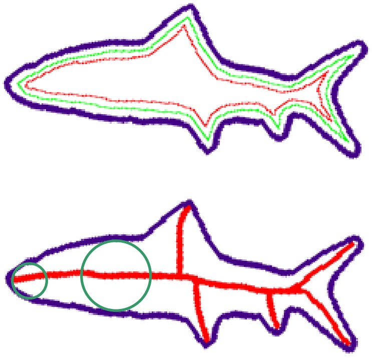
Applications

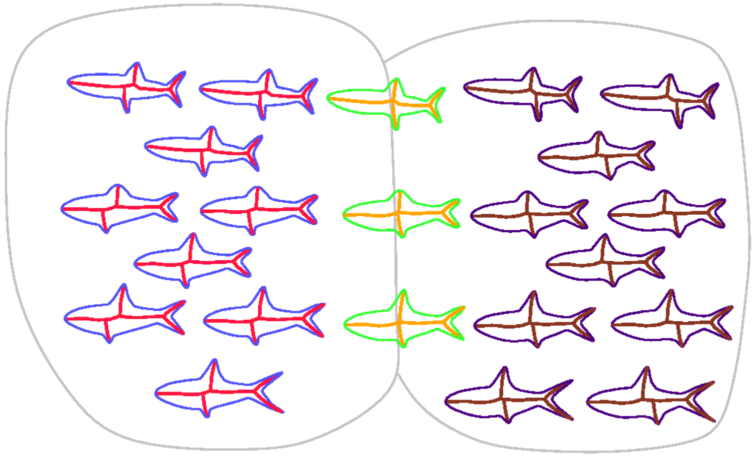
…

**Conclusions**

# Next: 3D Shape Deformation

- [Kimia, Siddiqi *et al.*] represent shape as a member of an equivalent class ('shape cell'), each defined as the set of shapes sharing a common shock graph (in 3D, we would say Medial Scaffold) topology.





# Next: 3D Shape Deformation

- Link this to **Information Models**: incorporation of human expert knowledge; *e.g.* in building taxonomies.
- Statistical analysis; definition of classes; distribution of features.
- Combine **exterior** with interior scaffolds.

# Other open issues:

- Combine or study relations with other existing main shape representations based on propagations: Voronoi, Morse/Reeb, flow complex, 3D Curve skeletons, flux-based, ..

- Interactions between 2D and 3D inputs : visual inputs/snapshots (2D) versus 3D percepts : no trivial correspondence between 2D and 3D medial representations (including Voronoi)

- Complexity, proofs of convergences for realistic data (not too smooth).

# Reference pointers:

- My PhD (available online):
  - [http://doc.gold.ac.uk/~ffl/phd/](http://doc.gold.ac.uk/~ffl/phd/)

- PAMI 2007 paper:
  - F. F. Leymarie and B. B. Kimia, "*The Medial Scaffold of 3D Unorganised Point Clouds*," IEEE Transactions on Pattern Analysis and Machine Intelligence (IEEE-PAMI), vol. 29, no. 2, pp. 313-330, February 2007.

- CVIU 2009 paper:
  - M.-C. Chang, F.F. Leymarie and B.B. Kimia, "*Surface Reconstruction from Point Clouds by Transforming the Medial Scaffold*," Computer Vision and Image Understanding (CVIU), vol. 113, no. 11, pp. 1130-46, *Special issue on new advances in 3-D imaging and modeling*, November 2009.

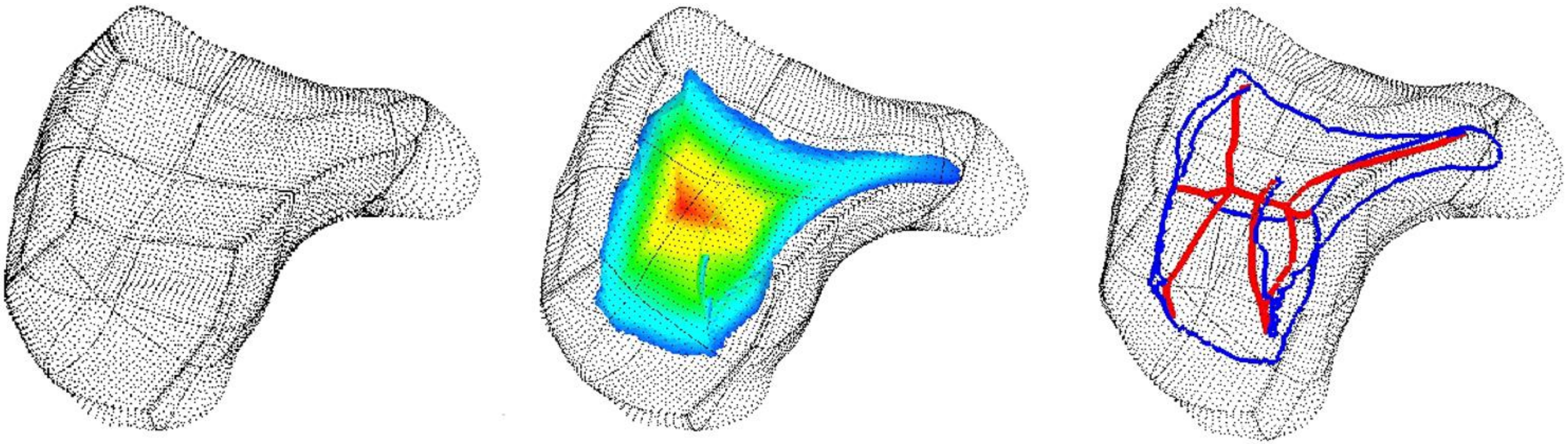
# Reference pointers:

- Survey of MA applications:
  - F. F. Leymarie and B. B. Kimia, "*From the Infinitely Large to the Infinitely Small*," Ch. 11 in "Medial Representations --- Mathematics, Algorithms and Applications," pp. 369-406, K. Siddiqi and S. M. Pizer, eds., Springer, volume 37 of *Computational Imaging and Vision series*, 2008.

- Applications in Biomedical Science:
  - F. F. Leymarie, M.-C. Chang, C. Imielinska and B.B. Kimia, *A General Approach to Model Biomedical Data from 3D Unorganised Point Clouds with Medial Scaffolds*, Proc. of Eurographics Wokshop on Visual Computing for Biology and Medicine (VCBM), D. Bartz *et al.*, eds., pp. 65-74, Leipzig, Germany, July 2010.

# Medial Scaffolds for 3D data

Medial Object Workshop, organised by TranscentData Europe Ltd.
Cambridge, UK, 9-10 Oct. 2014

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