

# Advanced FEA Compaction Model Using CEL Method

L. Chi<sup>1</sup> and M. Zhang<sup>2</sup>

1 Machine & Machine System Research, PD & GT, Caterpillar Inc. [Chi\\_Liqun@CAT.com](mailto:Chi_Liqun@CAT.com)

2 Engineering, Industrial & Waste Products, Caterpillar Inc. [Zhang\\_Ming\\_Greg@CAT.com](mailto:Zhang_Ming_Greg@CAT.com)

*Abstract: Compaction is one of the most important processes in landfill operations. The process of compaction crushes out the air void in the waste mass, increasing waste density, increasing landfill life, and increasing site profitability while reducing required air space. FEA-based, numerical landfill compaction models were developed at Caterpillar to predict compactor performance and to guide optimum compactor designs. Early models representing compactors with smooth drums used the traditional Lagrangian method, which was adequate to predict average compaction. However, excessive mesh distortions limited its use for the compactor model with detailed tip designs, especially when the simulation of multiple machine passes is required. This paper presents an advanced FEA compaction model using coupled Eulerian-Lagrangian (CEL) method. The developed model realistically simulates the interaction between wheel and ground with large distortions caused by the complex tip shapes. The application of the CEL method significantly enhances the capability of the compaction model for simulating multiple machine passes on level ground and slopes, traction behavior under large wheel slip, as well as blade dozing operations. An elasto-plastic model is used for highly compressible waste materials. The material parameters were determined from the laboratory and in-situ shear and compaction tests. The developed CEL compaction models have been validated against field test data with two different sizes of landfill compactors in the US and in China. The paper also includes some application examples for optimizing the design of a new generation of landfill compactors.*

*Keywords: Coupled Eulerian Lagrangian Method, Inelastic material model, Compaction, Explicit method, Work-Hardening, user defined constitutive material model, Landfill, Element Distortion*

## 1. Introduction

Disposing of municipal solid waste in a landfill has been the most economically feasible solution throughout the world. As a result, in most countries, the primary method of municipal solid waste (MSW) disposal is a sanitary landfill. Modern landfills are a great asset to enhance our standard of living by providing a safe, healthy way to dispose of residential and industrial waste in an environmentally protective manner. Increased land costs and government regulations for landfills have made the optimized placement of waste a crucial process. Compaction is one of the most important aspects of operating a landfill. Good compaction reduces the use of air space and increases the landfill life. High performance compactors and proper compaction procedures are key components for efficient landfill operations.

Cat® waste handling equipment has been sold worldwide. Performance of the equipment is heavily affected by the type and behavior of the waste stream in different regions of the world.

Characteristics of municipal waste, such as composition and water content, are affected by socio-economic development, degree of industrialization, life-styles, and climate. To fully understand the implications of different regional waste streams, Caterpillar collaborated with the University of Florida in US and Tsinghua University in China to investigate regional waste characteristics and their impact on its mechanical behavior. The study found that Asian waste, particularly in mainland China, contains significantly higher food and organic waste and higher water content than waste in industrialized regions like North America and Western Europe (Chi et-al, 2008). The laboratory studies at U. of Florida (Cho et-all, 2011) studied the relationship of food waste contents on the compressibility and shear strength of the waste material. The mechanical behavior of waste obtained from laboratory tests was also used to build robust numerical compaction models.

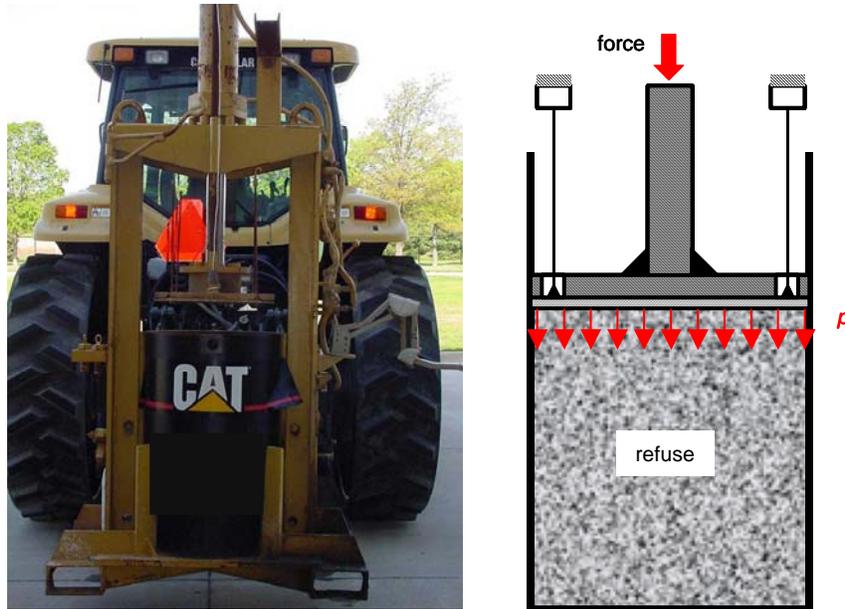
Traditional methods used to predict the settlement of landfill consisted of analytical and empirical methods (Ling, et al., 1998; Welsh, 1983; Steinberg and Lukas, 1984). These models focused on the secondary compaction during long-term settlement after initial compaction during the waste placement. The finite element method (FEM) has been generally considered a powerful computer simulation tool in solid mechanics. It can be used to predict compacted waste density, machine power, and operating efficiency. In order to accurately predict the compaction, Caterpillar developed special material models and finite element models to simulate landfill compaction processes. Smooth drums were used in the early versions of the FEM compaction model (Chi, 2000). The model successfully predicted the effect of the drum size, machine weight and number of passes on the average compaction density. Next generation of compaction models included details of compactor tips. The excessive mesh distortion problem was the major obstacle when detailed tip shapes were included in the model. A number of modeling techniques were explored including Arbitrary Lagrangian-Eulerian (ALE) and other distortion control methods. Limited success was obtained by using ABAQUS/Explicit with enhanced hourglass control with element distortion controls (2008) in Lagrangian domain. The model accurately predicted wheel torque and compaction density after the first pass of the compactor on a level ground. Excessive mesh distortion occurred during the simulation of the following passes and on slopes with high wheel slips.

Two classical descriptions of motion in continuous mechanics are the Lagrangian formulation and the Eulerian formulation. In the Lagrangian formulations (commonly used in structural mechanics), each individual node of the computational mesh follows the associated material particles during the motion. The Lagrangian formulation allows easy tracking of free boundary and interfaces between different materials, and is capable of handling complicated, history dependent constitutive relations. The weakness of the Lagrangian formulation is its inability to follow large distortion of a computational domain. Eulerian formulations are widely used in fluid dynamics, in which the computation mesh is fixed in space and the material moves with respect to the computational mesh. The Eulerian formulation can handle large distortion of continuum motion with relative ease. The ALE method was developed in an attempt to combine the advantages of both methods (Donea et-al, 2004). ABAQUS/Explicit offers the ALE adaptive meshing capability (Simulia, 2011). The method allows the mesh smoothing by relocating the nodes without altering the element connectivity and the shapes of the free boundaries and interfaces between different materials. ABAQUS offered several individual smoothing algorithms and an option of weighted combination, which resulted in infinite numbers of different mesh smoothing methods. Searching for the most effective smoothing method is problem dependent and often cumbersome. For our compaction model, the most severe mesh distortion occurred at wheel

and ground interface. We were not able to find an effective smoothing method in ALE adaptive meshing to solve our mesh distortion problem. Coupled Eulerian-Lagrangian method was developed in recent years to address the mesh distortion for solid mechanics (Benson, 1992; Benson and Okazawa, 2004; Perry and Carrol, 2000). The method splits each time increment of the computation into two steps: 1) Lagrangian step and 2) Eulerian step. The standard Lagrangian formulation is used in the first step. The deformed mesh calculated from the first step then is remapped back to the original mesh in the second Eulerian step. Unlike elements in ALE adaptive meshing that are always filled with single material, the elements in CEL can be partially filled by single or multiple materials, or completely void. This CEL method is very effective for solving the excessive mesh distortion problem in our landfill compaction model. This paper documents recent research performed at Caterpillar to advance the available technology in landfill compaction simulation. The model used the CEL method, which was introduced in the recent ABAQUS releases.

## 2. Constitutive Model of Waste Material

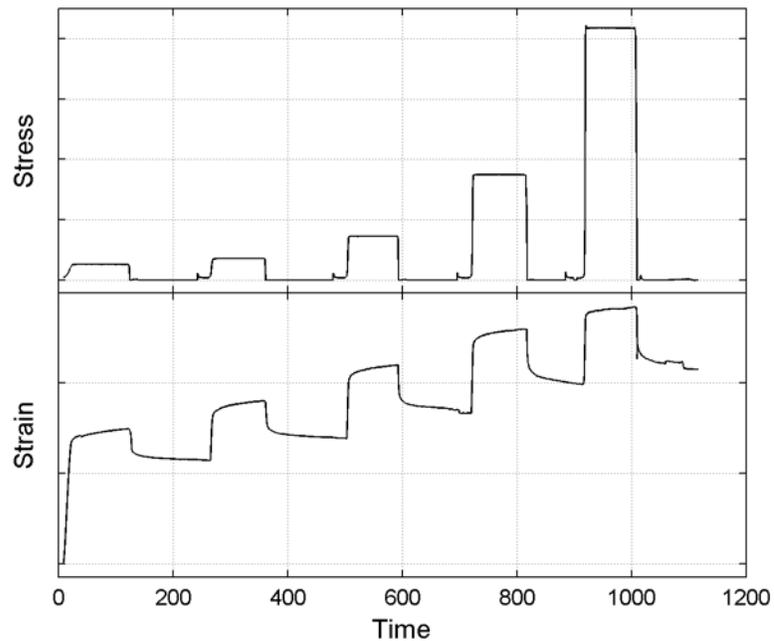
The material model of loose and un-compacted refuse is a critical component of the whole compaction model. The behavior of this material model directly affects the accuracy of model prediction of compaction behavior, as well as wheel torque and mobility. As the compactor wheels roll over the loose refuse, the ground undergoes both recoverable elastic deformation and irreversible plastic deformation. After wheel passage, the elastic deformation will rebound back and the plastic deformation remains as a permanent volume change.



**Figure 1. A portable test device for compaction behavior of waste**

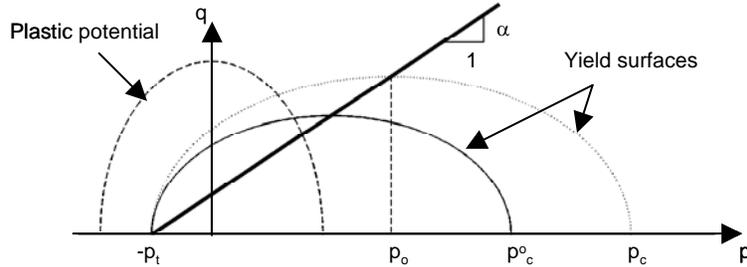
A portable device was developed at Caterpillar for testing the compressibility of waste on site (Figure 1). The device can be mounted on a general-purpose tractor and uses the hydraulic power

output of the tractor to apply the compaction forces. A similar device was designed and manufactured for the China market. During the test, the waste material was loaded into the cylinder with little or no previous compaction effort. A set of vertical compression pressure cycles were applied and released to the waste material in the confined cylinder to obtain both elastic rebound and permanent volume changes. The device was capable of testing the compaction behavior of waste from a very loose state to its maximum density achieved during the primary densification by the landfill compactors. Typically, the final volume after test is only a fraction of that before the test. The large deformation and finite strains are used in the FEM analysis. The test results (Figure 2) show a nonlinear relationship between applied stress and permanent plastic deformation. As the waste material densified, higher stress was required to produce further permanent compaction. The results also indicate obvious nonlinear, stress-dependent elastic behavior. Both stress dependent elastic and nonlinear plastic behavior need to be considered in the material model.



**Figure 2. Typical test results of compaction behavior of waste in the confined cylinder.**

In this study, a modified crushable foam model was selected to represent the mechanical behavior of the loose waste. This elasto-plastic constitutive model contains volumetric hardening, which is suitable for modeling the compaction process of lower-density waste materials. The crushable foam model with volumetric hardening is shown in Figure 3.



**Figure 3. Yield surfaces of crushable foam model.**

The yield surface,  $f$ , and plastic flow potential,  $g$ , for the crushable foam models are defined as follows:

$$f = \sqrt{q^2 + \alpha^2(p - p_o)^2} - \alpha(p_o + p_t) \quad (1)$$

$$g = \sqrt{q^2 + \beta^2 p^2} \quad (2)$$

Where  $p$  is defined as the pressure stress and  $q$  is defined as the Mises equivalent stress. Both yield function and flow potential are represented as ellipses in the meridian ( $p$ - $q$ ) stress plane. The center of the yield ellipse is defined by  $p_o$ . All the yield surfaces intersect the  $p$ -axis at  $p_c$ , the yield stress under hydrostatic compression. This stress term is further related to the plastic volumetric strain through a nonlinear volumetric hardening curve.

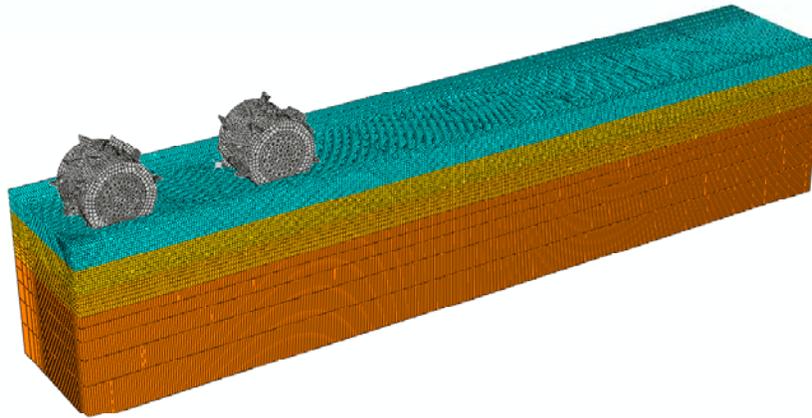
The elastic part of the response of the crushable foam model in ABAQUS/explicit only allows using linear elasticity. However, Figure 2 showed strong stress dependency in elastic rebound of waste. Therefore, a user defined material subroutine (VUMAT) is developed with combined stress dependent elasticity with crushable foam plasticity. Backward Euler method is used for both stress-dependent elastic and nonlinear plastic responses.

### 3. Landfill Compaction Model with CEL

To better understand the behavior of waste while being compacted and to continuously improve machine design, Caterpillar has invested significant effort and resources over the years in the development of a numerical model to predict compaction performance of landfill compactors. The model is based on the Finite Element Method (FEM) with dynamic explicit integration algorithms. The model is designed to understand the interaction at the wheel and ground interface. ABAQUS/Explicit simulation software was used to build these models.

Figure 4 shows the CEL finite element model of Cat® compactor wheels for China market. The detailed tip and drum shape were modeled with rigid elements. The whole drum and tip assembly was further represented by a rigid body with its reference points at the centers of the wheels. The total mass and rotation inertia are applied at the reference points. The machine chassis was modeled as a rigid body with reference points at its center of gravity (CG) of the frame and two additional nodal points at the centers of the front and rear wheels. The point mass and rotation

inertia are applied at the CG of the machine chassis. The hinge-type connector elements were used to link the machines chassis with two compactor wheels. The static load of machine weight is applied as gravity force to the point masses of machine chassis and wheels. Different weight distribution between front and rear axles is modeled by the CG location of the frame. The effect of dynamic weight transfer due to ground slope or drawbar force is automatically encountered in this model setup.



**Figure 4 CEL finite element model of landfill compaction with tip-wheels**

The landfill ground, shown in Figure 4, consisted of three physical layers of different refuse materials: a) a top layer representing the loose refuse material to be compacted, b) a pre-compacted waste layer, and c) a firm sub-base. Different material models were applied to properly represent their behavior under compaction. For CEL method, several layers of Eulerian element mesh were also defined above the top refuse layer. These elements were initially void. The refuse material is allowed to flow in and out of this void layer. Eight node solid elements were used for the ground models. To improve the simulation accuracy, the top surface of the potential contact area was discretized with a fine hexahedral mesh. Coarser element meshes were used in other places to minimize the total size of the model required. In general, CEL required a finer mesh compared to the traditional Lagrangian mesh. The model shown in Figure 4 consisted of 420,305 elements and 445,521 nodes, and had a total of degrees of freedom of 1,336,551.

Velocity boundary conditions were applied on the sides of the ground mesh. The materials were not allowed to flow in or out of the defined Eulerian mesh. General contact is used to define interaction between two compactor wheels and refuse materials. A simple friction model with constant friction coefficient is used for interface behavior between wheels and refuse. In the CEL method, the interface reconstruction algorithm within each element automatically handles the interface between different materials in the Eulerian mesh. No additional contact definition is required for material interfaces within in the Eulerian mesh. Both front and rear wheels are driven by pre-defined rotational velocity during the model simulation. At two ends of the ground mesh, the loose refuse is compacted by only one wheel. Only in the middle of the ground mesh is the refuse compacted by both wheels. An element set at the mesh center was defined for calculating the density of refuse during post-processing. The average density through the layer,  $\rho_{avg}$ , was calculated by using Equation 3.

$$\rho_{avg} = \frac{\sum DENSITY_{i} \cdot EVOL_{i}}{\sum EVF_{i} \cdot EVOL_{i}} \quad (3)$$

#### 4. Field Compaction Tests

Several field compaction tests were conducted for validating the compaction model predictions at actual landfill sites located in US and China. The tests were carefully prepared and controlled to obtain relatively consistent results required by the model validation. The test measurements included machine operation parameters, such as drive shaft torques and velocity, and ground survey for determining the refuse density using GPS or optical survey tool such as Trimble's Universal Total Station. The on-site barrel compression tests were also conducted for characterizing the behavior of waste used in the tests. These measurements provided a complete data set required for model set-up and validation.



a) spreading



b) field compaction test



c) survey



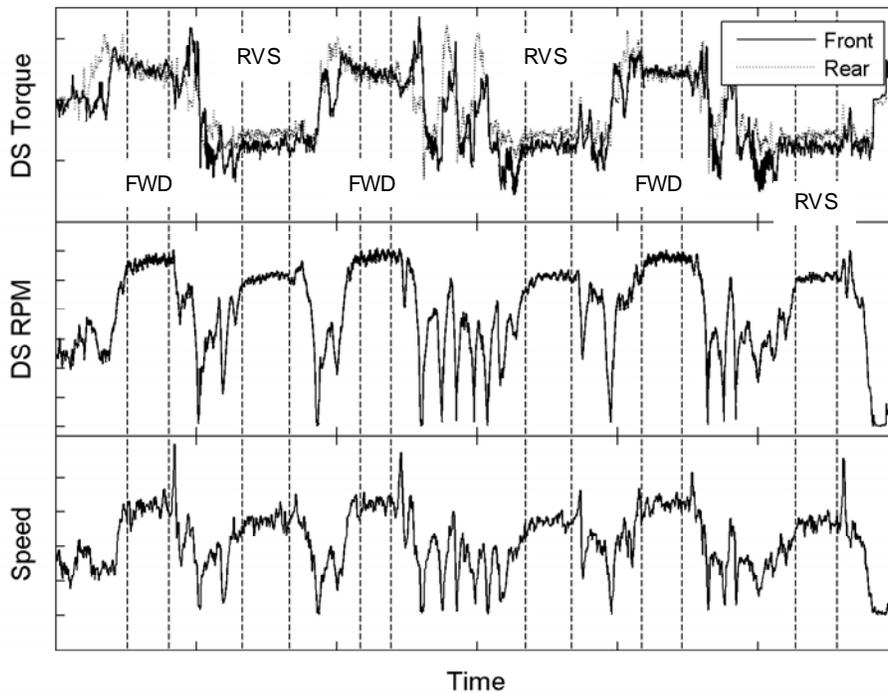
d) in-site compression test

**Figure 5 Field compaction tests for model validations**

Figure 5 shows the pictures of the field tests conducted at a landfill in China in 2010. Only residential waste was directed to the test cells to minimize the variation in the waste stream. The base was first firmly compacted by a number of passes of a heavy landfill compactor. The waste used in the test was first pushed into the testing area by track dozers and then spread evenly by

hydraulic excavators (Figure 5a). The test cell was prepared in such way that no vehicle or equipment traffic was allowed on the waste before the actual compaction by the landfill compactor took place. A compactor that was specially designed for China market was used in the field test (Figure 5b).

The number of waste collection trucks directed to the test cell and the weight of waste in each of these trucks determined the total weight of waste in the test layer. The survey (Figure 5c) was conducted on the pre-compacted base and the test layer after spreading to get total volume of the test layer. The initial density was then determined from the total weight and the volume of the waste in the test layer. To determine the density change, the survey was conducted after each pass of the compactor at the surface of the test cell. The density after each machine pass can be determined from elevation changes in the survey data. Four to five machine passes were used for this test. The survey data of the base was also used to determine the slopes of the test cell, which is important for model validation analysis.



**Figure 6. Machine torque and speed measured during compaction tests**

The original front and rear drive shaft on the compactor were replaced with instrumented drive shafts to measure propulsion torques and rotation speed of the shaft. The wheel torques and rotary speed can be determined from measured drive shaft data and the characteristics of the machine differentials and final drives. The machine travel speed was measured by the GPS mounted on the compactor. Figure 6 shows typical results from machine instrumentation during the first pass of compactions tests. Six trips (three forward and three reverse) were required to cover the entire test

cell. The compactor was maneuvered outside of the test cell to avoid any overlapping or gaps between each trip.

Several barrel compaction tests were conducted on-site (Figure 5d). The samples were randomly collected from the test cell to make sure the behavior of waste in the barrel test resembles that in the test cell. The results from the barrel test were used to determine the material model parameters.

## 5. Results

This section shows the model validation results and some application examples. Model validations include both material model verification against the barrel test data and compaction model against the field test data.

### 5.1 Material Model Verification

A data analysis procedure was developed to determine the parameters in elastic and plastic responses of material model. First, in the procedure, the elastic strain is extracted from rebound response of barrel test results. The stress and elastic rebound strain are used to determine the parameters in the stress dependent bulk modulus equation. The plastic strain is then calculated by subtracting the elastic strain from the total strain. In our user defined material subroutine, the work hardening curve is defined by the volumetric strain and pressure stress at hydrostatic loading path. The stress state obtained in the barrel test needs to be properly converted based on the shape of the crushable foam yield functions. The parameters of yield function,  $\alpha$  and  $p_t$ , were determined from cohesion and internal friction angle of the waste obtained from the direct shear tests at University of Florida (Cho et al, 2011).

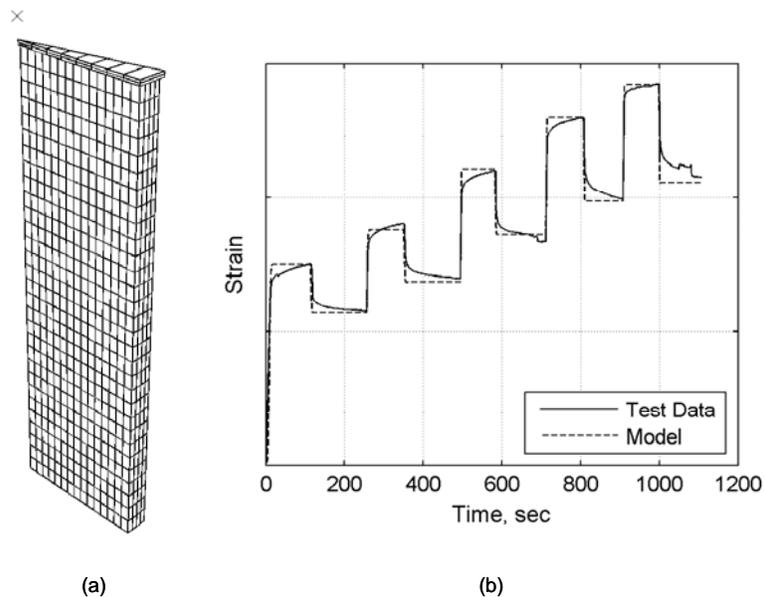
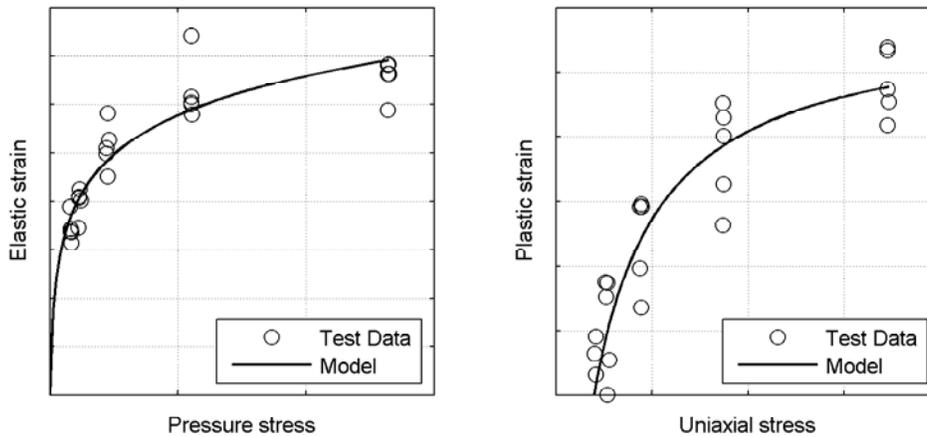


Figure 7. Material model verification

A Eulerian mesh was built to model a small portion of the cylinder to mimic axisymmetric stress case in 3D (Figure 7a). The material was confined in the Eulerian mesh. The VUMAT was verified against the test data from a single test trial. The elastic and plastic hardening material parameters were obtained by the single tests. The compaction pressure used in the test was directly used as model input. The model prediction of the maximum total strain and plastic permanent strain agreed very well with test data with  $R^2$  values greater than 0.99. Due to lack of material damping for the Eulerian element, the model was not able to predict the time dependent behavior in the test data.

## 5.2 SEM6020 Compactor Model Validation for 2010 Field Test Data in China

The SEM6020 compactor model with CEL ground mesh is shown in Figure 4. Although the VUMAT user subroutine was successful for the material model verification, the use of the user subroutine failed for real compaction models due to the element distortion that was encountered at the very start of the wheel rotation. As a result, the model validation had to use the internal crushable foam model of ABAQUS for CEL mesh. The average elastic and plastic response were obtained from all barrel test data on the testing day. The initial density of the test cell calculated from survey data was significantly higher than that of the barrel test. Therefore, the plastic strains from the barrel test were corrected based on the difference in initial densities in the field and the barrel tests. The crushable foam model in ABAQUS uses the uniaxial stress for the work-hardening curve. Figure 8 shows the elastic and plastic response of the material model developed and the data from the barrel tests. Only linear elasticity can be used by the ABAQUS's internal crushable foam material model. As a result, a single, average Young's modulus was selected based on the range of predicted compression stresses by the model.

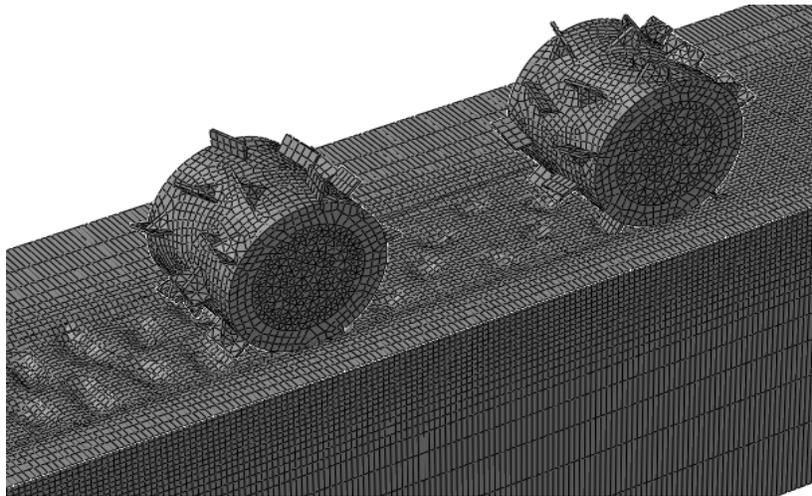


**Figure 8. Refuse material behavior used in model validation**

A slight slope (6%) existed at the base of the test cell, which affected the wheel torque and machine speed. Each compactor pass during the tests consisted of three forward trips moving down the slope and three reverse trips moving up on the slope. A total of four compactor passes were conducted in this test. Two sets of model validations were conducted to cover two sets of

different traffic patterns: 1) forward/downhill – reverse/uphill - forward/downhill – reverse/uphill; and 2) reverse/uphill - forward/downhill - reverse/uphill-forward/downhill. The measured average wheel rotation speeds for each pass were used to drive the compactor model. The effect of ground slope was modeled by changing the direction of gravity force on the machine mass and ground. The model predicts the average density change, wheel torque and machine speed.

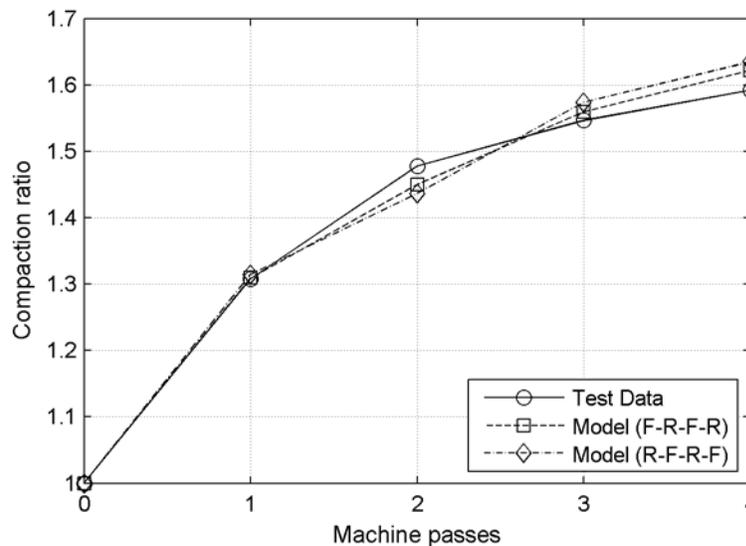
ABAQUS V6.11-1 was used for the model simulation. The models were executed on a Linux cluster with Intel Xeon processors. The size of the time increments ranged from  $3.5 \times 10^{-5}$  to  $1.5 \times 10^{-4}$  sec. The model simulation required about 520,000 time increments for all four machine passes. Double precision option was used to minimize accumulated errors for the explicit method. The average computation time was about 16 hours with 24 CPUs. Figure 9 shows the deformation of ground mesh during the first machine pass. The CEL models were able to complete all four machine passes for both traffic patterns. The validation results are shown in Figures 10-12.



**Figure 9. Mesh deformation during the first machine pass**

Average density changes from the field survey data did not distinguish the effect of different traffic patterns. The model predictions from two traffic patterns are very close, which indicates the traffic pattern did not significantly affect the average density change. Density predictions from both models are in very good agreement with the field test data. Several critical steps contributed to the success of the model validation throughout the model development and validation process. First the on-site barrel test with random sampling resulted in a good representation of waste material behavior in the test cell. Secondly, the barrel test data was analyzed carefully by considering the stress paths in the controlled test. The stress path in a confined chamber is different than that required for defining material model parameters. Different material models may require different stress terms and/or strain terms for defining their plastic behavior. The stress and strain terms from the particular controlled stress paths need to be carefully converted into proper stress and strains terms required by a particular inelastic material model. Finally, the difference in the initial state of material between barrel tests and field test cell has to be considered for defining the work hardening behavior. The plastic strain needs to be corrected to properly reflect this difference. With all these considerations, we were able to accurately predict the compaction behavior of the landfill compactor using the material model built from one-dimensional

compression data obtained from the barrel tests. The impact of this finding is significant. As mentioned in previous sections, the waste characteristics and its mechanical behavior vary significantly in different regions of the world. Even for the same landfill, the seasonal variations of waste composition are significant. It is impractical to conduct field tests to evaluate the machine performance for all landfills in a particular region and for all specific conditions due to cost and limited resources. With the success of this study, we can use the virtual simulation models to predict the compactor performance for a particular situation based on simple on-site barrel test data or pre-established relationship between waste composition and its mechanical behaviors (Cho et-al, 2011). The developed models are also very effective tools for selecting the best suitable wheel configuration and guiding the specific compactor design for a particular market.



**Figure 10. Model prediction of average density change.**

The ground slope showed significant effects on the wheel torque and speed measured in the field. Higher wheel torque was measured when the compactor was moving up on the slope (Figures 11a and 12a). The machine speeds were also lower for uphill motion due to limited engine power (Figures 11b and 12b). The compaction model slightly over-predicted the machine speeds for both cases. Predicted speeds correctly reflected the effect of slopes. The predicted trend of changes in machine speeds between passes is also in good agreement with the test data.

The torque predictions are significantly lower than test data. However, the compaction model predicted the correct trend of wheel torque changes between each pass and the effect of ground slope. Previous compaction models that used Lagrangian mesh, user defined material subroutine and material damping resulted in more accurate predictions of wheel torque. The current version of ABAQUS is not capable of including material damping with elasto-plastic models for Eulerian elements. The current CEL model seems to be able to correctly simulate the wheel/ground interface and predict the additional traction force for pushing a machine upward on a slope. However, lack of material damping resulted in under-prediction of overall wheel torques. The previous model with Lagrangian mesh and VUMAT also resulted in more accurate predictions of

wheel torque splits between front and rear axles. During the first compactor pass, the first wheel engaged with very soft ground and the following second wheel interacted with much denser ground from the first wheel pass. The VUMAT that included both stress dependent elasticity and nonlinear plastic behavior better reflected changes in material response at front and rear wheels, while a linear elastic model could not accurately reflect the stress dependent elastic response.

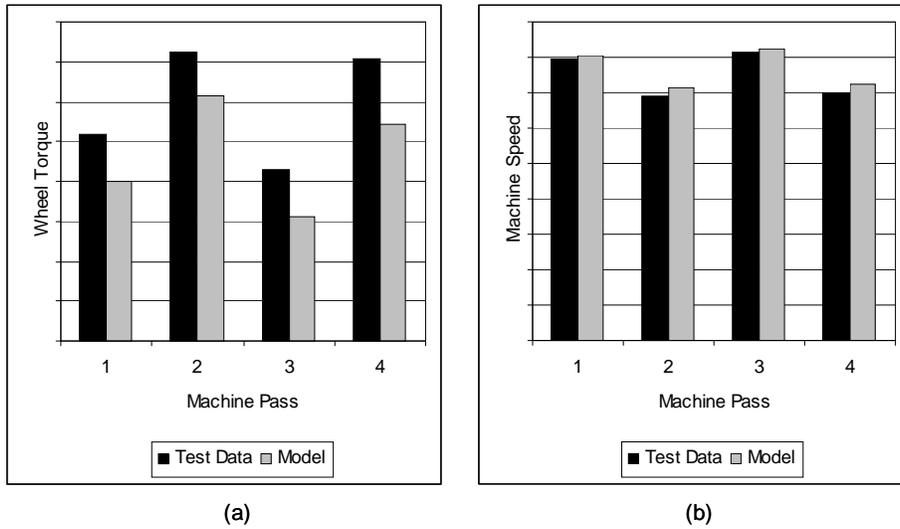


Figure 11. Model prediction of machine torque and speed (fwd-rvs-fwd-rvs)

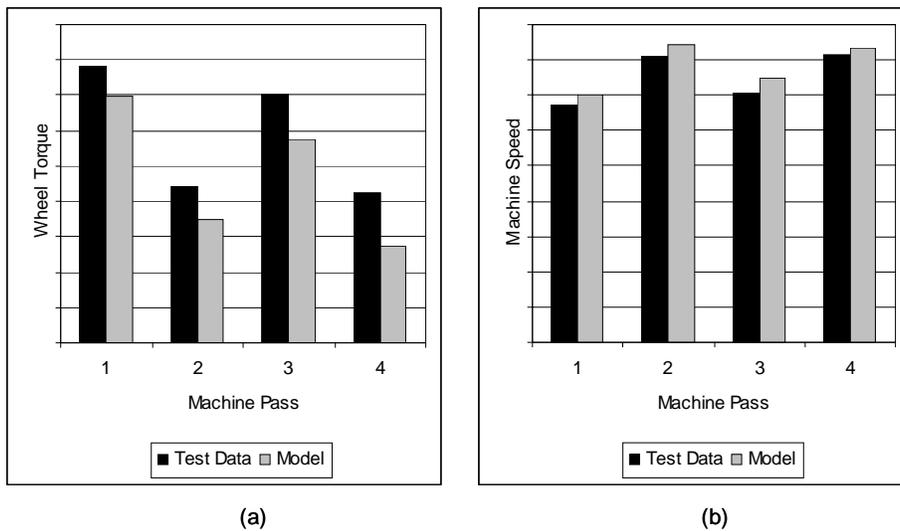


Figure 12. Model prediction of machine torque and speed (rvs-fwd-rvs-fwd)

### 5.3 Model Application Examples

Compaction simulation is frequently used for product development at Caterpillar. For landfill compactors, machine and waste in the domain of the real world are modeled using Dynasty (machine system modeling) and ABAQUS Explicit (Compaction modeling) in the virtual world. Together, we can predict machine performance and structural life, as shown in Figure 13.

The compaction model not only can predict compaction, mobility and power train loads, but also can predict traction, rolling resistance and radius which could be used in machine system models to further predict fuel efficiency, productivity, heat loads, structural loads and life. The compaction model has been successfully used in product development and commercial applications for 1) tip/wheel designs to achieve optimized machine performance, power train, cooling and structural integrity; 2) China specific wheel/tip designs to suit the characteristics of Chinese waste; 3) belly guard designs to reduce drag; 4) customer support to help market the products; 5) competitive studies to understand our products' strengths and weaknesses.

The CEL method could also be used in the model that simulates dozing and digging. The initial evaluation is very promising. It is in the process of being validated.

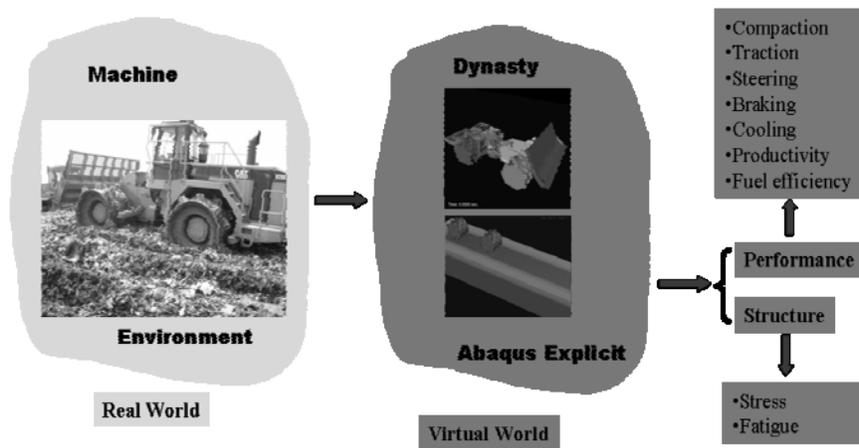


Figure 13. Cat® Landfill Compactor VPD Modeling Domain

## 6. Conclusion

The CEL method was first introduced in ABAQUS v6.9. Significant improvements in robustness were made in the following releases. The compaction model developed using the CEL method resolved the element distortion problems experienced in the previous compaction models using Lagrangian method. The new model can simulate multiple passes for the compaction wheels with detailed tip shapes. The model accurately predicts the average density changes made by landfill compactor and machine speed.

Lack of material damping for Eulerian elements results in under-prediction of wheel torque by the current landfill compaction models. Therefore, numerical material damping for soil like plastic

material models is critical. However, the model was able to predict the correct trend of changes in wheel torque between passes and capture the effect of slopes on the wheel torque.

The material properties significantly affect the accuracy of model predictions. The material behavior in controlled barrel tests needs to be carefully analyzed based on the material model chosen. The difference in initial states of barrel test and that in the field needs to be considered for defining the plastic behavior of the materials. The study showed that we can accurately predict the compaction performance of a landfill compactor by using material models determined from laboratory or on-site one-dimensional compression tests.

The stress-dependent elasticity is significant for elastic response of waste materials. The VUMAT was successfully used for simulating the barrel tests. The model accurately predicted both elastic rebound and permanent plastic strain. However, the used of VUMAT of the same material subroutine failed for the full landfill compactor model. The robustness of using user-defined material subroutine (VUMAT) with Eulerian elements in ABAQUS needs to be further improved.

## 7. Reference

1. Benson, D. J., "Computational methods in Lagrangian and Eulerian hydrocodes," *Computer Methods in Applied Mechanics and Engineering*, vol. 99, pp. 235–394, 1992.
2. Benson, D.J. and S. Okazawa, "Contact in a multi-material Eulerian finite element formulation", *Methods in Applied Mechanics and Engineering*, vol. 193, pp. 4277–4298, 2004.
3. Chi, L., "Numerical landfill compaction model". Presented at 2004 ISWA conference at Hong Kong, China, 2004
4. Chi, L., T. Townsend, K. Xia and W. Debord. "Compaction behavior of Chinese waste and its impact on machine configurations". Presented at ISWA/WMRAS World Congress 2008 at Singapore, 2008
5. Cho, Y. M., J. H. Ko, L. Chi, and T. G. Townsend, "Food waste impact on municipal solid waste angle of internal friction," *Waste Management* 31(1):26-32, 2011.
6. Donea, A. H., J.-Ph. Ponthot, and A. Rodriguez-Ferran., "Arbitrary Lagrangian-Eulerian method", *Encyclopedia of Computational Mechanics*, Edited by Erwin Steein, Rene de Borst and Thomas J.R. Hughes. 1-38, 2004.
7. Ling, H.I., Leshchinsky, D., Mohri Y., Kawabata T., "Estimation of municipal solid waste landfill settlement", *Journal of Geotechnical and Geoenvironmental Engineering ASCE* 124(1), pp21-28, 1998.
8. Peery, J. S., and D. E. Carroll, "Multi-material ALE methods in unstructured grids," *Computer Methods in Applied Mechanics and Engineering*, vol. 187, pp. 591–619, 2000.
9. Simulia, ABAQUS analysis user's manual, v6.11, 2011
10. Steinberg, S.B., and R.G Lukas, *Densifying landfill for a commercial development*, *Proceedings of International Conference on Case Histories in Geotechnical Engineering*, Vol. 3, St. Louis, 1984.
11. Welsh, J.P., *Dynamic deep compaction of sanitary landfill to support superhighway*, *Proceedings of 8th European Regional Conference on Soil Mechanics and Foundation*, 1983.