Hedemöjligheter för clearinghus under upplösningen av en medlemsdefault

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Clearing House Hedging Opportunities in the Aftermath of a Member Default

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Abstract

Most research on clearing house default management is focused on preventive initiatives to minimize the impact of member defaults. This thesis will examine the effects of reactively applying hedging strategies on an inherited portfolio in the aftermath of a clearing member default. We have compared the impact of two different hedging strategies, a 3-factor PCA- and a parallel shift immunization, on three different portfolio characteristics. Value-at-Risk, Expected Shortfall, worst P&L outcome, volatility, and worst cumulative mark-to-market loss were evaluated through backtesting on historical data. Our results imply that by applying the hedging strategies, all of the evaluated risk parameters are subject to improvement. However, the 3-factor PCA strategy systematically outperforms the parallel shift approach and the latter may even increase risk during certain market conditions.

Key-words: Clearing house, CCP, Default management, Clearing member default, Principal component analysis, PCA, Parallel shift, Immunization, Hedge, Interest rate swap
Sammanfattning


Nyckelord: Clearinghus, CCP, Default processer, Clearingmedlemsdefault, Principalkomponentanalys, PCA, Parallelskifte, Immunisering, Hedging, Ränteswappar
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CHAPTER 1

1. Introduction

In their 2011 annual report, Ian Axe, CEO of the clearing house group LCH.Clearnet\(^1\) describes the development of today’s financial market from his perspective:

“\textit{In 2008 Lehman collapsed and the world changed. The markets we all operate in are now more risky and more turbulent. Clearing houses are no longer seen as just back office operations. Regulators and most of the market recognize us as key in managing risk whether company-specific, or market-wide.}” (LCH.Clearnet, 2011, p. 11)

For the last decades, the OTC derivatives market has been subject to tremendous growth. Notional principal outstanding of interest rate swaps, currency swaps and interest rate options has grown from around US$ 63 trillion in 2000 to more than US$434 trillion in 2010, illustrated in Figure 1. In relation, according to International Swaps and Derivatives Association (2005), credit exposure via mark-to-market P&L is typically in the range of 1% to 4% of notional principal amounts.

Historically, OTC derivatives have not been subject to government regulation but Title VII of the Dodd-Frank Wall Street Reform and Consumer Protection Act (2010) now requires clearing of such instruments. Consequently, as the derivatives market has grown, together with new legislation; more emphasis on central counterparty clearing house (CCP\(^2\)) crisis management has been requested.

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\(^1\) A European based clearing house which roots can be traced back to the London Clearing House and the Paris based clearing house, Clearnet. Through SwapClear it is currently one of the leading clearer of swaps, bonds, and repos in the world (LCH.Clearnet, 2013).

\(^2\) In this report we chose to consider clearing counterparty (CCP) as a synonym for clearing house.

\(^3\) Bengt Jansson holds a M.Sc. in Engineering Physics from the Royal Institute of Technology and has been working with risk management in the financial industry (including Nasdaq OMX and Folksam Asset Management) since the late 1990s. He is the co-founder of Tail Financial Consulting and is currently employed...
CCPs are entities in the infrastructures of current financial markets. They are structured to mitigate counterparty risk by interposing itself in every trade; hence becoming the buyer to every seller and vice versa. Clearing houses have a limited number of clients, called clearing members, who can directly engage in contracts with them, all other market participants must go through members when trading.

To comply with the absorbed counterparty risks, CCPs practice default risk management where commonly used tools are; default funds consisting of capital contributed by the clearing members, variation margins based on actual market changes, and initial margins based on possible future losses incurred by each contract. As an example, SwapClear sets the initial margin to the portfolio’s observed worst-case five-day loss over the last five years (SwapClear, 2013a). Because of how the default management processes (concerning member defaults) are designed and implemented, there are potential scenarios in where extreme market changes may result in a defaulting CCP. An event like this would have shocking effects on the whole market as well as on the entire financial system. As the issue recently has been subject to increased attention, clearing members now request improved default management processes. This provides competitive opportunities for the CCPs. Also, we argue that there is an unbalanced focus on pre-default procedures as opposed to post-default procedures. Consequently, the purpose of this study is to examine the effects of introducing post-default strategies in the default management processes.

In this paper, we provide an alternative to address this challenge by applying reactive hedging strategies on OTC derivatives portfolios and more specifically, interest rate swap portfolios. From this, the following research question has been determined:
- Can CCPs use the application of hedging techniques on a defaulted member’s interest rate swap portfolio to cover more extreme market events?

From the research question, two hypotheses have been derived and evaluated:

- Hypothesis 1: The risk of a defaulted member’s interest rate swap portfolio can be lowered by applying hedging techniques.

- Hypothesis 2: Even the application of a more simplistic hedge will, from a risk perspective, have a positive effect on the inherited portfolio.

Our results imply that when applying the examined hedging strategies, the outcomes of the evaluated risk parameters may be subject to improvements. However, the sophisticated hedge systematically outperforms the simplistic approach and the latter may even worsen the outcomes during certain market conditions.

The methodology used to obtain our results can be segmented into three modules; (1) Analyzing the data, (2) Evaluating the outcomes, and (3) Computing the hedge. The historical data used consists of USD LIBOR spot rates and USD LIBOR par swap rates starting 2001-01-02 and ending 2013-04-04. From these data sets, the zero curves were constructed and analyzed using bootstrapping techniques and principal component analysis (PCA). The results were used to compute optimal positions for two different hedging strategies, parallel shift- and 3-factor PCA immunization. We evaluated the performance of each strategy applied on three portfolios (with different cash flow characteristics) through a rolling historical simulation, yielding profit and loss (P&L) outcomes given a default on any day throughout the set. Value-at-Risk (VaR), Expected Shortfall (ES), Worst P&L Outcome (WPO), and Volatility ($\sigma$) were computed based on these results and then compared between the portfolios. Also, we examine the liquidity impact on the CCP by calculating the maximum cumulative mark-to-market loss (later denoted Worst Cumulative Mark-to-Market (WCM)).

The report is structured as follows; the remaining part of Chapter 1 describes the background of a clearing house and the implications followed by a clearing member default. Chapter 2 presents the theory used in this study. Chapter 3 presents a critical review of previous research as well as our contribution to this scientific field. Chapter 4 describes the methodology as well as limitations and delimitations of the study. Chapter 5 presents the results. Chapter 6 presents a discussion of our findings. Chapter 7 concludes the report as well propose potential avenues for future research.

1.1 Background

As described above, a clearing house reduces the risk of counterparty failure on contracts by interposing itself in the transactions. The original bilateral contracts between the market participants are extinguished and replaced by new contracts where the clearing house becomes the buyer to every seller and the seller to every buyer. The bilateral, variable quality counterparty risks are now replaced by a single, high quality counterparty risk against the CCP (see Figure 2 and Figure 3). There are also
other benefits to collect from this arrangement such as post-trade anonymity, netting (see Figure 4), and reduction of operational risk (Knott & Mills, 2002; Gibson & Murawski, 2013; Norman, 2011). Furthermore, since the market participants no longer need to be concerned about the creditworthiness of their different counterparties (as they are only exposed to the CCP), less capital is required to be held against the risk of counterparty defaults (Knott & Polenghi, 2006).

Clearing houses are typically specialized in the clearing of transactions in different markets. Some offer clearing of commodities whilst others offer derivatives and cash instrument clearing (EACH, 2012). The range of instruments cleared differs between the clearing houses. An example of CCPs of particular interest in this paper is SwapClear which is a part of LCH.Clearnet and a global clearing service for OTC interest rate swaps. It currently clears more than 50% of the global interest rate swap market, and is the second largest clearer of bonds and repos in the world (LCH.Clearnet, 2013).

**Figure 2, Pre-CCP trading**

**Figure 3, CCP trading today**

In early trading, traders were in direct contact with each other. Figures are illustrative and denote bilateral cash flows between the counterparties.

A CCP interposes itself as the buyer to every seller and the seller to every buyer but only interact with clearing members. Numbers are based on the example in Figure 2.

**Figure 4, CCP netting**

The effect of netting cash flows; less capital is transferred, hence the counterparty risk is decreased. Numbers are the net positions from the example given in Figure 3.
Only a limited number of so called clearing members are approved to directly engage in contracts with the CCPs. The other market participants enjoy the same benefits as clients of these members (Knott & Mills, 2002). There are typically two categories of clearing members; general clearing members (GCM) and direct clearing members (DCM). A GCM may clear its own trades, the trades of its clients, as well as the trades of a non-clearing member (NCM). There is no straightforward definition of a DCM as all CCPs have their own definitions where in some, it only clears its own trades whilst in others, it may also clear its clients, as well as trades of NCMs that are affiliated to the DCM (Norman, 2011).

As described above, another advantage using CCPs is the possibility of netting exposures. When becoming the counterparty to every trade, it can allow some of the original bilateral exposures to be netted. This means that the total exposures on the market are being reduced (Hills, et al., 1999). The decreased exposures results in that lesser collateral is required from the clearing members, hence allowing for increased liquidity in the market (Gibson & Murawski, 2013).

By definition, when assuming the responsibility for contract performance, the CCP itself is exposed to the risk of a clearing member default. In a default scenario, the contracts between the clearing house and the member are terminated and all cleared trades are inherited and transferred to what we choose to denote as the default portfolio. Hence, the clearing house now has to meet the financial obligations of the defaulter (LCH.Clearnet, 2012; Knott & Mills, 2002; Gemmill, 1994).

To manage these scenarios, the clearing house requires all members to submit collateral in terms of margin and in some cases contribute to a so called clearing fund (LCH.Clearnet, 2011; Eurex Exchange, 2013; Knott & Mills, 2002; Arnsdorf, 2011). The collateral is intended to cover for the losses originated from the default situation. There are two methods for deciding the proportion of margin submitted to the clearing house; net margining and gross margining. Net margining means that the clearing member is allowed to net long and short positions held by their clients and post their margin on aggregate net positions. Gross margining is when the member submits margin based on their gross positions (Knott & Mills, 2002).

Depending on the product type and terms of the deal, the tenor of a trade can vary between days to decades and during the life of a trade considerable market movement can take place. This puts pressure on the clearing house regarding risk and liquidity management (LCH.Clearnet, 2011; Knott & Mills, 2002).

As discussed above, CCPs are structured to mitigate counterparty risk through multilateral netting, high levels of collateralization together with loss mutualization. Hence, a CCP is a risk sharing arrangement where every member is liable for the performance of all the other members (Hills, et al., 1999). Arnsdorf (2011) argues that moving from bilateral to central clearing transforms, although not entirely, the risk of counterparty default into the risk of losses on the mutualized collateral pool. Also, in the event of a clearing member default, all uncollateralized losses arising from the liquidation of the defaulted portfolio will be shared pro-rate amongst the surviving members.
Not frequently but indubitably, clearing members will be declared in default by the CCP. This can be triggered through different scenarios, e.g. when a clearing member’s trading capacity is suspended or when it cannot make an intraday margin call (EACH, 2010). Should a member become financially distressed and unable to comply with margin calls, the CCP has legal right to use the margins from the member’s account to settle the outstanding obligations (Knott & Polenghi, 2006). As described above, the clearing house will now take over the cleared financial obligations of the member and consequently be subject to the same risk exposure as if the CCP itself had made the trades.

To comply with defaulting members, CCPs have default management processes. The exact procedures may differ between the clearing houses but the European Association of CCP Clearing Houses (EACH) have developed guidelines on how to outline the default procedures. The guidelines involve a so called “waterfall of resources” to describe how to distribute the losses associated with a default (EACH, 2010). To close out the positions inherited from the defaulted member, the CCP may host an auction process in which tranches of the default portfolio is subject to bids from the non-defaulted members (Eurex Clearing, 2013; SwapClear, 2013b). As such, the winning bidder will take over the financial obligations from the CCP.

Even though the default management processes are relatively comprehensive and include dedicated resources to cover for potential losses, CCPs would benefit from more developed methods to manage the risk of price movements (i.e. changes in present value) and/or liquidity shortage in the defaulted portfolio. This could involve a clear post member default hedging scheme with the objective to lower fluctuations in the portfolio between the actual member default and until the positions have been closed (i.e. until the CCP no longer is exposed). Figure 5 describes the scenario of a clearing member default \( t_0 \) followed by the unwinding period, i.e. the critical phase where changes in present value and short term cash flow could stress the CCP.

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**Figure 5, Post member default cash flow illustration**

![Diagram showing cash flow illustration](image)

*Illustrative description of the cash flow after a clearing member default where the CCP is exposed to price and liquidity risk.*

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Theoretically, in the ideal environment, reducing the risk during the unwinding period could be achieved by simply taking the exact opposite cash flow obligations of the swap, thus creating a perfect
hedge of the portfolio. Practically, however, this is not possible due to the nature of how interest rate swaps are traded (Koch, 2009). Consequently, in order to hedge the cash flows of an existing swap one usually has to match it with swaps of different settlement dates.

Typically, in a member default scenario, the margins submitted will be sufficient to cover the losses incurred with the unwinding of the defaulted portfolio, at least in theory. As an example, Lehman Brothers subsidiaries were clearing members of CCPs around the world and in the case of the default, the margins proved sufficient to protect nearly all the clearing houses from the effect of Lehman’s default. Only one clearing house in Hong Kong was obliged to dip into the default fund (Norman, 2011).

Were a CCP to be strained by a member default, not only the market in which it operates might be disrupted but it can also act as a channel of contagion into other markets. As an example; if a market changes abruptly it may cause the CCP to make intraday margin calls on other members to protect against defaults. To meet the margin call, some members could now have to sell assets in another market. The price changes in that market might in turn cause another margin call (Knott & Mills, 2002).

Fortunately, clearing member defaults can be considered to be rare events and in most situations, the CCPs can cover for the associated losses. However, there are examples where even clearing houses have defaulted (Hills, et al., 1999; Knott & Mills, 2002). Therefore, it is not unreasonable to consider that a CCP default will ever happen again. Collateralization together with mutualization via a CCP is an efficient and acknowledged technique of reducing risk, but it should be pointed out that it is not entirely risk eliminating.

1.2 Delimitations

This thesis focused on portfolios solely consisting of plain vanilla swaps. In reality, this is a rather unrealistic portfolio composition. Still, one could argue that the results from this research indicate a recommendation for the CCPs and could be transmittable to larger and more complex compositions. Also, due to the relatively low liquidity in the 30-year USD swap, we have chosen to discard it from the data set.

As described earlier, the clearing house holds a margin from all members which are used to cover for potential losses associated with the defaulted portfolio. Because the report only examined the P&L performance during the unwinding period between a hedged portfolio and an unhedged portfolio, collateral implications were left out from the analysis. Also, the application of interest rate immunization is commonly associated with transaction costs. Calculating this cost is not easy, not least as prices are likely to be increasingly volatile during the default. There are plenty of sophisticated algorithms for this kind of computations but in relation to the research question for this study, transaction costs were considered to be out of scope and thus neglected. Because of this, no re-hedging strategy was employed during the unwinding period, i.e. the hedge positions remained constant during
the complete evaluation period. As a consequence, our results were expected to underestimate the impact of the hedges.

In this study, we assumed daily mark-to-market settlements for the portfolios P&L performances during the unwinding period. Hence, payments corresponding to the daily changes in present value were made on each trading day. Another assumption was that, as we were looking at trading days, we decided to normalize the number of days per year from 365 days to 250 trading days.
2. Theory

2.1 Bond and Swap Contracts

To model the portfolio value, the contracts of which it consist of has to be modeled. Following are the definitions of the formulas used for this.

We denote the known interest rate from time $t$ to time $T > t$ as $r_{t,T}$. The simplest form of bond is the zero coupon bond where the buyer is entitled to a single cash flow in the future when the bond matures. The zero coupon bond is the implied discount factor of the market and it is commonly used for valuation of contract. The interest rate of this bond is called the zero rate. We choose to denote the value at time $t_0$ of a zero coupon bond with maturity $t_m > t_0$, as $Z_{0,m}$. A bond which pays one unit at time $t_m$ can be expressed in continuous compounding form as:

$$Z_{0;m} \cdot e^{r_{0,m}(t_m-t_0)} = 1$$  \hspace{1cm} (1)

A coupon bond pays a future stream of cash flows (coupons) and can be described as a linear combination of zero coupon bonds. If we describe the coupons $c_1, ..., c_m$ which are received at times $t_1, ..., t_m$, the bond price at time $t$, named $p_{Bond,0:m}$, is given by:

$$p_{Bond,0;m} = \sum_{i=1}^{m} c_i Z_{0;i}$$  \hspace{1cm} (2)

Plain vanilla interest rate swaps are highly liquid OTC instruments between two counterparties, where the holders of the contract periodically exchange interest rate payments on a hypothetical loan. One counterparty pays a periodic fixed coupon, while the other counterparty pays a variable amount that periodically resets on a benchmark interest rate index, for example 3-month LIBOR (Schmidt, 2011; Sadr, 2009). Opposed to a bond, no principal payment is made neither at the beginning nor end of the swap. Hence, only the net interest payments on the hypothetical loan are paid by the counterparties. Since receiving coupons and paying periodic financing the receiver in a swap (the floating-rate payer)
is akin to being long in a bond. Similarly, paying in a swap (the fixed-rate payer) is akin to being short in a bond (Sadr, 2009).

The cash flow streams made by both counterparties are referred to as legs. Hence, the swap consists of a floating leg and a fixed leg. The first/last date that interest begins/ends accruing is called the effective date and maturity date, respectively. Typically, both legs of the swap have the same effective and maturity dates and thus referred to as the swap’s effective and maturity dates. Furthermore, the interest rate payments are based on a hypothetical loan with the same principal for both legs and are referred to as the notional of the swap (Sadr, 2009).

USD plain vanilla swaps are quoted as mid-market par swap rates for a fixed rate payer in return for receiving 3-month USD LIBOR. Table 1 illustrates a selection of par swap rates quoted over three consecutive trading days.

**Table 1, Quoted USD plain vanilla swap**

<table>
<thead>
<tr>
<th>Date</th>
<th>Maturity</th>
<th>1-year</th>
<th>2-year</th>
<th>3-year</th>
<th>4-year</th>
<th>5-year</th>
<th>7-year</th>
<th>10-year</th>
<th>30-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010-05-05</td>
<td>0.71%</td>
<td>1.21%</td>
<td>1.72%</td>
<td>2.19%</td>
<td>2.57%</td>
<td>3.12%</td>
<td>3.58%</td>
<td>4.18%</td>
<td></td>
</tr>
<tr>
<td>2010-05-06</td>
<td>0.72%</td>
<td>1.20%</td>
<td>1.69%</td>
<td>2.14%</td>
<td>2.53%</td>
<td>3.07%</td>
<td>3.54%</td>
<td>4.16%</td>
<td></td>
</tr>
<tr>
<td>2010-05-07</td>
<td>0.81%</td>
<td>1.18%</td>
<td>1.63%</td>
<td>2.06%</td>
<td>2.42%</td>
<td>2.97%</td>
<td>3.41%</td>
<td>3.98%</td>
<td></td>
</tr>
</tbody>
</table>

*A selection of USD swap par rates quoted at three consecutive dates for eight different maturities*

Both legs of a swap are comprised of contiguous calculation periods. A 1-year USD swap has two semiannual calculation periods on the fixed leg, and four quarterly calculation periods on the floating leg, demonstrated in Figure 6.

**Figure 6, 1-year USD plain vanilla swap**

Illustrative cash flows of a 1-year USD swap. The deterministic cash flows are determined by the par swap rate (C in figure) and the stochastic cash flows are determined by the 3-month LIBOR (L_{3m}) between each settlement.

Pricing of swaps boils down to discounting the series of future cash flows associated with each leg. Again opposed to a bond, where the present value of all cash flows is related to a single yield via the flat yield assumption, the term structure of interest rates is acknowledged in a swap perspective. Hence
it is recognized that deposits for different maturities earn different interest rates and there is no single yield-to-maturity that is applicable to all cash flows (Sadr, 2009).

### 2.2 Zero Curve

Zero rates for different times to maturity are not typically equal. The relationship between these rates could depict both investors’ preferences for different maturities as well as market expectation about future inflation and financial climate. The zero curve is a graphical representation showing several zero rates across different contract lengths for debt contracts with similar credit risk (see Figure 7), hence displaying the cost of borrowing in relation to the time to maturity (Asgharian & Nordén, 2011).

![Illustrative bootstrapped zero rate curve based on the swap par rates displayed in Table 1.](image)

As explained above, pricing of swaps relies on discounting a series of future cash flows associated with each leg. Unfortunately, the discount curve is not quoted directly on the market; instead there are a variety of methods to obtain it indirectly. Note that we would like to have the daily discount factors for a rather long time period (in this study; ten years), yielding 3650 daily prices. At each point in time, there are active and liquid markets for at most 40 to 50 instruments that provide sufficient information. Consequently, the solution is not unique (Sadr, 2009).

A common method for constructing the curve is called bootstrapping. Through arranging the traded instruments into increasing maturity and starting with the shortest, one could derive the discount curve up to its maturity, and move on to the next instrument (Sadr, 2009).

The LIBOR rates depict the short term components of the curve whilst the swap contracts are used to compute the long term zero rates. The formulas used for calculating the discount factors differ depending on what maturities are available on the market. The swaps used in this study are only
quoted in tenors of: 1, 2, 3, 4, 5, 7, and 10 years. In this case, one must interpolate between greater gaps in time for the longer maturities as can be seen in equation (7) and (8). Finally, as we need the daily discount factors to construct the entire zero curve (as in Figure 7), a new set of linear interpolations are exercised.

This report assumes that USD LIBOR for 3-months and 6-months equals the zero rates for the corresponding maturity. Furthermore, discount factors for time frames between one and ten years have been calculated through the quoted USD LIBOR par swap rates with corresponding formulas displayed below.

\[ Z_{0.1} = \frac{1 - 0.5r_{par\,1\,year}Z_{0.05}}{1 + 0.5r_{par\,1\,year}} \]  
\[ Z_{0.2} = \frac{1 - 0.5r_{par\,2\,year}(Z_{0.05} + 1.5Z_{0.1})}{1 + 0.75r_{par\,2\,year}} \]  
\[ Z_{0.3} = \frac{1 - 0.5r_{par\,3\,year}(Z_{0.05} + Z_{0.1} + Z_{0.15} + 1.5Z_{0.2})}{1 + 0.75r_{par\,3\,year}} \]  
\[ Z_{0.4} = \frac{1 - 0.5r_{par\,4\,year}(Z_{0.05} + \cdots + Z_{0.2.5} + 1.5Z_{0.3})}{1 + 0.75r_{par\,4\,year}} \]  
\[ Z_{0.5} = \frac{1 - 0.5r_{par\,5\,year}(Z_{0.05} + \cdots + Z_{0.3.5} + 1.5Z_{0.4})}{1 + 0.75r_{par\,5\,year}} \]  
\[ Z_{0.7} = \frac{1 - 0.5r_{par\,7\,year}(Z_{0.05} + \cdots + Z_{0.4.5} + 2.5Z_{0.5})}{1 + 1.25r_{par\,7\,year}} \]  
\[ Z_{0.10} = \frac{1 - 0.5r_{par\,10\,year}(Z_{0.05} + \cdots + Z_{0.6.5} + \frac{21}{12}Z_{0.7})}{1 + \frac{21}{12}r_{par\,10\,year}} \]

2.3 Principal Component Analysis

Principal component analysis relies on linear algebra and assists in explaining a high dimensional data set with correlated factors, in terms of a few uncorrelated factors – the principal components. Furthermore, PCA could be considered rather as a rotation tool than an actual model. The mathematics behind the PCA is described below.
Let $f$ be a column vector of different factors and $\Sigma_f$ the covariance matrix of $f$.

$$f = (f_1, \ldots, f_m)^T$$

$$\Sigma_f = \text{Cov}(f)$$

Furthermore, a symmetric and positive semidefinite matrix with non-negative eigenvalues $\Sigma_f$ can be written as a product of a diagonal matrix $D$ with non-negative eigenvalues $(\lambda_1, \ldots, \lambda_m)$ and an orthogonal matrix $O$ whose columns are eigenvectors of $\Sigma_f$, orthogonal and of length one, hence:

$$\Sigma_f = \text{Cov}(f) = OD^T$$

Without loss of generality, we may assume that the columns of $O$ and $D$ are structured in a way that the diagonal elements are in a descending order. Next, $f$ has to be transformed so that the main axes are parallel to the coordinate axes.

By choosing

$$f^* = O^T(f - E[f]),$$

we can show that

$$\text{Cov}(f^*) = E[O^T(f - E[f])(f - E[f])^T O] = O^T \text{Cov}(f) O = D.$$  

Which means that the components of $f^*$ are uncorrelated and have variances $\lambda_1 \geq \cdots \geq \lambda_m$, in that order. One could interpret this transformation geometrically as $f$ now has rotated until the main axes are parallel to the coordinate axes. Also, $f^*$ is the principal component transform of $f$ and can be considered as a rotation and a re-centering of $f$ (Hult, et al., 2012).

To demonstrate the transformation from correlated components to uncorrelated components of $f$, we put $e_k$ as the $k$th standard unit vector in $\mathbb{R}^m$, using formula (13) such that

$$\sum_{k=1}^m f_k e_k = f = O O^T f = E[f] + O f^* = E[f] + O \sum_{k=1}^m f_k^* e_k = E[f] + \sum_{k=1}^m f_k^* o_k.$$
This indicates that the components of \( f \) are correlated when expressed in terms of the standard basis \( \{ e_1, ..., e_m \} \), however uncorrelated when expressed in the alternative orthonormal basis \( \{ o_1, ..., o_m \} \) (Hult, et al., 2012).

In order to measure how much of the total variance of \( f \) is explained by the first principal components, it is necessary to define the total variance:

\[
\sum_{k=1}^{m} \text{var}(f_k^*) = \sum_{k=1}^{m} \lambda_k = \sum_{k=1}^{m} \text{var}(f_k)
\]  

(16)

Hence, the following ratio represents the percentage of total movements that can be explained by the first \( j \) principal components.

\[
\frac{\sum_{k=1}^{j} \lambda_k}{\sum_{k=1}^{m} \lambda_k}
\]  

(17)

Typically, the ratio is close to one for \( j = 1 \) which indicates that movements of \( f \) is mainly in the direction of the first component \( o_1 \) (corresponding to a parallel shift in the yield curve), and that

\[
f \approx E[f] + f_k^* o_1.
\]  

(18)

is a reasonably accurate approximation.

Next, by arranging historical changes in interest rates with different maturities, specific principal components describing the movements can be identified. The idea is then to express the observed interest rates as a linear sum of the factors by solving a set of simultaneous equations. The quantity of a particular factor in the interest rate changes on a particular day is the factor score for that day. The importance of a principal component is then measured by the standard deviation. Furthermore, the variance of the factor scores has the property that they add up to the total variance of the data (shown in equation (16)). Hence it is possible to calculate how much each principal component accounts for the total variance of the data (shown in equation (17)). When the PCA is completed, it is now possible to relate the risk in an interest rate dependent portfolio to a total of, let’s say, three factors instead of considering all the factors (Hult, et al., 2012).

### 2.4 Hedging Techniques

Let \( r = (r_1, ..., r_n)^T \) be a vector composed of current zero rates for the maturity times \( 0 < t_1 < \cdots < t_n \), and let \( \Delta r \) be a vector whose components are instantaneous changes in the zero rates. If we consider a deterministic liability of cash flows \( \{(c_k, t_k) : k = 1, ..., n\} \) whose present value is given by \( P(r) \), then
The holder of this portfolio is exposed to the risk that the value of the liability increases due to an unfavorable outcome of $\Delta r$. Therefore, in order to manage this risk, one could purchase a hedging portfolio that costs $P(r)$ with the property that the current net value of both portfolios is zero and that the net value is insensitive to zero rate changes, $\Delta r$. This so called immunization technique is achieved by taking positions $h_1, \ldots, h_m$ in $m$ hedging derivatives (in this study; plain vanilla interest rate swaps), whose present values are given by $P_k(r)$ for $k = 1, \ldots, m$. Naturally, a perfect hedge against an instantaneous change in the zero rate curve is obtained by taking positions, such that for all $\Delta r$

$$
\sum_{k=1}^{m} h_k P_k(r + \Delta r) = P(r + \Delta r)
$$

However, due to lack of hedging derivatives, one would most likely be unable to perfectly match the liability cash flow, in both timing as well as in size. Also, during a systemic risk event, e.g. in the default of Lehman Brothers in 2008, increasing volatility were observed in the zero rates – especially with maturities below five years. However, given the rather short time period we are interested in (20 trading days, argumentation follows in Chapter 4) we will assume that zero rate changes are likely to be small (we will discuss this assumption later in Chapter 6). Now, let’s consider the first order Taylor approximation

$$
P(r + \Delta r) \approx P(r) + \nabla P(r)^T \Delta r,
$$

where

$$
\nabla P(r)^T = \left( \frac{\partial P}{\partial r_1}, \ldots, \frac{\partial P}{\partial r_n} \right)(r),
$$

and similarly for the hedging instruments. Hence, the resulting system of equations is then given by

$$
\sum_{k=1}^{m} h_k P_k(r) = P(r) \text{ and } \sum_{k=1}^{m} h_k \nabla P_k(r)^T \Delta r = \nabla P(r)^T \Delta r,
$$

and equivalently, in vector notation

$$
h^T P(r) = P(r) \text{ and } h^T \nabla P(r) \Delta r = \nabla P(r)^T \Delta r,
$$
where

\[ P(r) = (P_1(r), \ldots, P_m(r))^T \text{ and } \nabla P(r) = \begin{pmatrix} \frac{\partial P_1}{\partial r_1}(r) & \cdots & \frac{\partial P_1}{\partial r_n}(r) \\ \vdots & \ddots & \vdots \\ \frac{\partial P_m}{\partial r_1}(r) & \cdots & \frac{\partial P_m}{\partial r_n}(r) \end{pmatrix}. \tag{25} \]

Next, let’s consider changes in zero rates as a number of deterministic scenarios denoted \( \Delta r_1, \ldots, \Delta r_q \) and look for the hedging positions \( h_1, \ldots, h_m \) that makes the liability portfolio immune (or at least insensitive) to these \( q \) number of scenarios through solving the system of equations:

\[ \sum_{k=1}^{m} h_k P_k(r) = P(r), \tag{26} \]

\[ \sum_{k=1}^{m} h_k \nabla P_k(r)^T \Delta r_j = \nabla P(r)^T \Delta r_j \text{ for } j = 1, \ldots, q. \tag{27} \]

As mentioned before, changes in the zero rates for different maturities have a strong positive dependence. Typically they all move up or they all move down, i.e. a parallel shift in the zero rate curve. Therefore, let’s first consider a normalized scenario and set \( \Delta r_1 = (1, \ldots, 1) = 1 \), which corresponds to an upwards parallel shift of the zero rate curve. Immunization against this scenario is achieved by taking the positions \( h_1, \ldots, h_m \) solving

\[ h^T P = P \text{ and } h^T \nabla P 1 = \nabla P^T 1, \tag{28} \]

where \( P = P(r) \) and \( P = P(r) \). The two equations for immunizing against parallel shifts may be written in matrix notation as (note that only two hedging derivates are needed)

\[ \left( \sum_{i=1}^{n} \frac{\partial P_j}{\partial r_i} \right) \left( \sum_{i=1}^{n} \frac{\partial P_k}{\partial r_i} \right) \frac{h_j}{h_k} = \left( \sum_{i=1}^{n} \frac{\partial P}{\partial r_i} \right) \left( \sum_{i=1}^{n} \frac{\partial P}{\partial r_i} \right) \tag{29} \]

Immunization against parallel shifts in the zero rate curve is an effective and rather straight forward approach. However, other scenarios are not uncommon and might be reasonable to take into account (typically changes in slope and curvature). One efficient way to determine additional scenarios is by applying the PCA method and analyze how much of the variation in \( \Delta r \) each component explains. In general, the first component corresponds to the most likely event and could roughly be compared to a
parallel shift in all maturities. The second component is typically a vector whose first components are positive and the remaining components are negative (or vice versa). This could be interpreted as an increase in zero rates for short maturities and a decrease in zero rates for long maturities (or vice versa). As argued above, a good choice of a hedging portfolio is obtained by solving

\[ \sum_{k=1}^{m} h_k P_k(r) = P(r), \]

\[ \sum_{k=1}^{m} h_k \nabla P_k(r)^T a_j = \nabla P(r)^T a_j \text{ for } j = 1, ..., q, \]

where \( a_j \) is the principal components and \( q \) is the number of considered principal components (scenarios). Typically, it is sufficient to consider two or three components.
CHAPTER 3

3. Literature Review

3.1 Default Risk Management

According to Arnsdorf (2011), an important difference between bilateral counterparty risk and the risk associated with facing a CCP is that the latter is not primarily driven by the exposure to a member’s own portfolio. Hence, the member’s CCP risk can increase even if the portfolio does not change. This can be explained by the insurance on the tail losses of all other clearing members provided by each member via the default fund. As a consequence, this puts pressure on CCPs risk control and its member default management processes.

Arnsdorf (2011) describes the typical multi-layer capital structure a CCP has to protect itself and its members from losses derived from member defaults; (1) Variation margin is charged or credited daily to clearing member accounts to cover any portfolio mark-to-market changes. (2) Initial margin is posted by clearing members to the CCP in order to cover any potential losses during the unwinding of a defaulted portfolio. The initial margin is typically set to cover all losses up to a pre-defined confidence level in normal market conditions (e.g. five-day VaR$_{99\%}$). (3) A CCP will in general have an equity buffer provided by shareholders. However, the position of the equity buffer in the multi-layer capital structure can vary between CCPs. (4) Every member contributes to the clearing house default fund. This could be viewed as a type of mutualized insurance for uncollateralized losses. (5) Lastly, in addition to the default fund, each clearing member is often committed to providing further funds if necessary. This is known as an unfunded default fund. The maximum amount of additional funds varies between CCPs and is in some cases uncapped.

Generally, in the event of a member default, losses incurred during the unwinding period will first be covered by the defaulting member’s initial margin and default fund contribution. Uncollateralized losses will subsequently be charged against the CCP’s equity buffer and ultimately the mutualized default fund. In the event of additional outstanding losses after the depletion of all funded and unfunded default fund contributions the CCP could find itself in default (Arnsdorf, 2011).
As described by International Swaps and Derivatives Association (2005) collateral does not reduce the likeliness of a counterparty default and does not change the value of a defaulted transaction. Also, it does not turn a bad counterparty into a good counterparty – it does not eliminate credit risk, rather it mitigates that risk. However, in many financial disasters, including: Metallgesellschaft, Long-Term Capital Management, and more recently Bear Sterns, Lehman Brothers and American International Group, collateral through margining has played an ambivalent role. Gibson & Murawski (2013) argues that the use of margining might have exacerbated the recent financial crisis via identifying situations in which margining of derivatives decreases trading volume, increases default rates and default severity, and reduces welfare in the banking sector. Their analysis shows that margining presents derivatives counterparties and regulators with a delicate trade-off. Margins reduce default severity by reducing banks’ exposure to the default of their counterparty but do also generate several types of costs, particularly when banks use derivatives for hedging purposes. Firstly, margin requirements can limit the number of derivatives contracts traded by a bank and thus prevent it from implementing its optimal hedging position. Secondly, increased margin requirements can indirectly constrain a bank’s hedging strategy by reducing the number of contracts outstanding of other banks. Moreover, Gibson & Murawski (2013) suggests that increased margin requirements can reduce the credit quality of a bank’s counterparties by constraining the counterparties’ hedging strategy.

3.2 Interest Rate Risk

By definition, the payoffs of interest rate derivatives are in some way dependent on the level of interest rates. According to Hull (2002), difficulties to value interest rate derivatives are primarily driven by four reasons; (1) the behavior of an individual interest rate is more complicated than that of a stock price or an exchange rate, (2) for the valuation of many products, it is necessary to develop a model describing the behavior of the entire zero curve, (3) the volatilities of different points on the zero curve are different, and (4) interest rates are used for discounting as well as for defining the payoff from the derivative.

Hull (2002) describes three different theories on what determines the shape of the zero curve. Firstly, the expectations theory, which conjectures that long-term interest rates should reflect expected future short-term interest rates. Hence, it argues that a forward interest rate corresponding to a certain future period is equal to the expected future zero interest rate for that period. Secondly, the segmentation theory, suggests that there need to be no relationship between short-, medium, and long-term interest rates. This is explained through a major investor, such as a large pension fund, that invests in bonds of a certain maturity and does not readily switch from one maturity to another. Hence, all different interest rates segments (e.g. short-, medium-, and long-term) are determined by supply and demand in that particular segment. Thirdly, the liquidity preference theory, argues that forward rates should always be higher than expected future zero rates. The fundamental assumption underlying this theory is that investors prefer to preserve their liquidity and invest funds for short periods of time. On the other hand, borrowers tend to prefer to borrow at fixed rates for long periods of time. In order to match
depositors with borrowers and avoid interest rate risk, financial intermediaries raise long-term interest rates relative to expected future short-term interest rates. This reduces the demand for long-term fixed-rate borrowing and encourages investors to deposit their funds for longer terms. Consistent with empirical results that the yield curve tend to be upward sloping more often than downward sloping, the liquidity preference theory leads to a situation where forwards rates are greater than expected future zero rates.

Soto (2004) discusses how duration models attempt to measure the sensitivity of bond prices to changes in interest rates with a wide variety of different approaches and how they differ in the assumed behavior of the term structure of interest rates. The author was especially interested in verifying whether the differences that may exist can be attributed to the particular model chosen or to the number of risk factors considered. The empirical results suggest that a principal component analysis model provides the best result and thus demonstrates the attractiveness of this model for interest rate risk management.

Carano & Dall’O (2011) define zero curve risk as the risk that the value of a financial asset might change due to shifts in one or more points of the relevant zero curve. The zero curve risk represents one of the most widely spread financial risk: each institution having to match future streams of assets and liabilities is, up to a certain extent, exposed to it. Theoretically, an efficient way to cope with this risk is simply to match positive with negative cash-flows. Practically however, the dates and the amounts of future cash-flows are often subject to constraints, so that implementing an accurate matching might either not be possible or be very expensive. In these cases, immunization techniques can be employed in order to manage the zero curve risk. Immunization make the sensitivity of the assets and the liabilities to zero curve changes similar to each other, thus the overall balance sheet will not be largely affected by these changes (Carcano & Dall’O, 2011; Hult, et al., 2012; Hull, 2002).

Carcano & Dall’O (2011) suggests three different classes of models corresponding to zero curve hedging techniques. Firstly, they describe hedging strategies relying on duration-, convexity-matching, and further-order approximations of the price-yield relationship. Secondly, they discuss the principal component analysis which identifies orthogonal factors explaining the largest possible proportion of the variance of interest rate changes. Litterman & Scheinkman (1991) suggests that most of the important characteristics namely; level, steepness, and curvature would be captured by applying a 3-factor PCA. Consequently, strategies matching the sensitivity of assets and liabilities to these three components should lead to high-quality hedging (Carcano & Dall’O, 2011). Thirdly, they explain the key rate duration concept which argues that changes in all rates along the zero curve can be represented as linear interpolations of the changes in a limited number of rates, the key rates.

3.3 Margining

In their article, Knott & Polenghi (2006) found that the margin coverage probabilities are likely to fall under more volatile market conditions. The risk of margin exceedances is also stressed in an earlier
paper where it is concluded that there will always be a non-zero probability that circumstances arise under which margins are exhausted (Knott & Mills, 2002).

Some clearing houses, e.g. SwapClear, employ VaR-measures to compute their initial margins (SwapClear, 2013a). When VaR was first introduced it achieved great popularity among both regulators and financial institutions. In short, VaR is an estimator of the maximum loss over a predefined period of time and probability level. Hence, it is the lower quantile on the tail of the portfolio’s distribution of possible values at the target horizon. Even though the resilient popularity of VaR, it does have some shortcomings. The main criticism is that VaR estimates do not take into account the magnitude of extreme or rare losses present outside the VaR-quantile. Actually, VaR mainly considers the frequency of losses, while the severity of a loss is often more important from a CCP risk management perspective as it might exceed available default resources if it is large enough. Also, since its introduction in the early 1990’s, several estimation methodologies have been proposed to either overcome the limitations of standard methodologies (e.g. Monte Carlo), or more closely match the distributional properties of the underlying risk factors (e.g. historical simulation). Nevertheless, whatever method applied, VaR is always defined as the maximum possible loss at a given critical level and does not consider rare but possible losses that are significantly larger than the VaR itself (Goannopoulos & Tunaru, 2005). According to Hult et al. (2012), VaR is likely to be the most commonly used risk measure for financial risk control, albeit its deficiencies. One could argue that this might be due to its conceptual simplicity and rather straightforward implementation. The biggest weakness, as described above, is that it disregards any loss beyond the specified level of probability and thus fails to describe what happens in a worst case scenario. Actually, the fact that it is a quantile value implies that it ignores most of the distribution. Consequently, it allows a careless or dishonest risk manager to hide rare but catastrophic risks in the tail (Hult, et al., 2012).

3.4 Contribution

There has been recent concern regarding the risk that clearing houses may be tempted to relax financial and other standards in order to attract more business (Grant, 2011; Cameron, 2011). Also, as described above, the way initial margins are computed will by definition result in scenarios when the total margin will be insufficient to cover for the losses incurred by a member default. Finally, in EACH default procedure guidelines (EACH, 2010), there seems to be a focus on preventive measures, i.e. what to do in order to prepare for a default.

Our research will address this concern by applying reactive hedging strategies on an inherited default portfolio. By doing so, we hope to address the concern of what reactive strategies should be applied when a member default has already occurred.
CHAPTER 4

4. Methodology

4.1 Overview

We have chosen to present our method in three different modules; (1) Data set, (2) Evaluation, and (3) Hedging positions. Two different hedging strategies have been backtested on historical data for three different portfolio compositions. The general idea was to explore:

- Given a default on any date in our set, what would have been the different portfolio performances from the point of default until the portfolio is closed if the CCP had used our hedging strategies compared to if it had not?

A more detailed description of the algorithm used during the research is illustrated in Figure 11.

4.2 Data Set

Data was collected from U.S. Federal Reserve in the form of USD LIBOR spot rates and USD LIBOR par swap rates. These data sets were converted into zero rates by applying bootstrapping methods (for details, see Chapter 2). There are more sophisticated methods for computing the zero curve and some companies are even specialized in the offering of such services. However, after consultation with an industry expert\textsuperscript{3} (Jansson, 2013) it was concluded that the bootstrapping technique was good enough. Also, one might argue that the study addresses the concern of default management given a certain set of scenarios and that those scenarios could be arbitrarily chosen, thus making our zero rate curve assumption less heroic. The full data set contained 3064 data points on 3-month USD LIBOR and 6-month USD LIBOR spot rates as well as 1, 2, 3, 4, 5, 7, and 10 year USD LIBOR par swap rates starting 2001-01-02 and ending 2013-04-04.

As we were backtesting, the data was segmented into smaller sets; this was done by selecting calibration and unwinding periods which are in relation to the default date (illustrated in Figure 8).

\textsuperscript{3} Bengt Jansson holds a M.Sc. in Engineering Physics from the Royal Institute of Technology and has been working with risk management in the financial industry (including Nasdaq OMX and Folksam Asset Management) since the late 1990s. He is the co-founder of Tail Financial Consulting and is currently employed as Director at Crescore.
The length of these periods were decided to be 1001 and 20, respectively (the argumentation for these assumptions will follow below). Furthermore, we let the default date vary through the complete data set; thus creating 2044 individual sets. The entire data set for each simulation can therefore be described by the expression \([t_{i-1001}, t_{i+20}]\), where \(i \in [1001,3044]\).

In this study, we chose to alternate \(i\) by one (i.e. \(i = 1001,1002, \ldots,3044\)) between each iteration. This means that there was overlapping between the sets. In order to eliminate the risk that the evaluated measures were influenced by these overlaps, we have also run the program alternating \(i\) by 20 (i.e. \(i = 1001,1021, \ldots,3041\)). The outcomes from this assessment are stated in Chapter 5.

![Figure 8, Data set segmentation](image)

*Illustration of the data sets in the rolling historical simulation where every iteration is performed on a new unique set.*

As mentioned above, the length of the post-default unwinding period needed to be determined to conduct our research. Again, after consultation with Jansson (2013), a 20-trading day unwinding period (approximately one month) was chosen to represent a reasonable time frame until the CCP has been able to close its inherited positions. In relation, the swap portfolios inherited by LCH.Clearnet after the Lehman Brothers default in 2008 were successfully auctioned after 14 days (de Terán, 2008). Consequently, the calibration period data, which consisted of daily changes in interest rates, was compounded into 20-day-changes (i.e. the total change in interest over approximately one month), summing up to a total of 50 changes in each calibration period.

In statistical analysis, the model output is dependent on what data it uses. Hence, data sets are needed to be relevant and large enough which is why it can be viewed as subjectively chosen by the model user (Hult, et al., 2012). For example, the dynamics of turbulent financial data (e.g. during a financial crisis) is expected to differ from stable financial data, resulting in different model outcomes. Also,
using an insufficiently large set of data would most likely generate unreliable results. In this study, for each time point simulation \( t_i \), the preceding 1001 days of trading (roughly four years of historical data) have been chosen, i.e. the first trading day in the set is \( t_{i-1001} \), where \( i = 1001 \).

4.3 Evaluation

The relative size of price movements in a swap is mainly dependent on two factors; (1) changes in interest rate, and (2) the time the cash flows are allowed to accrue, i.e. the time to maturity of the swap. Clearing members possess different portfolio compositions, generating different cash flow characteristics; hence the price movements of the inherited positions may vary between defaults. Based on the rationale above we chose to analyze three portfolios based on cash flow timing; (1) the Front-Loaded Portfolio (FLP), (2) the Mid-Loaded Portfolio (MLP), and (3) the End-Loaded Portfolio (ELP). The cash flows of these are illustrated in Figure 9.

**Figure 9, Portfolio composition with corresponding cash flows**

Illustrative future cash flow distributions of the different portfolio loadings.

The analyzed portfolios were constructed by selecting arbitrary positions in various USD swap maturities. This was done by simulating trading activity during a period prior to the default where the positions taken were chosen so that all the cash flow scenarios in Figure 9 were represented. After discussions with Jansson (2013), a time frame of 40 trading days was chosen (roughly 2 months of trading). Table 2 presents the positions taken on a daily basis for these days. As an example, for the MLP on the day of default, the artificial member portfolio contained 40 long 5-year USD swaps and 40 long 7-year USD swaps – with contiguous effective dates (see Table 3).
Each scenario was evaluated in terms of its observed post default outcome. The parameters considered are based on the P&Ls from the portfolios. As each portfolio composition was assessed on all data sets, we had 2044 observations for every portfolio loading. We chose five different measures for the analysis; Value-at-Risk 99% (VaR_{99%}), Expected Shortfall 99% (ES_{99%}), Worst P&L Outcome (WPO), Volatility (\sigma), and Worst Cumulative Mark-to-Market (WCM). VaR is a well-known measure because of its relative simplicity and as it is intuitively easy to comprehend. VaR has however been subject to some critique (Goannopoulos & Tunaru, 2005; Hult, et al., 2012) as what happens in the cases outside the chosen confidence level is left unknown and can have devastating effects. As argued by Hult, et al (2012), one way of exploring the, from a VaR-perspective, hidden tail events is the application of ES which can be described as the expected loss given a tail event outside the chosen confidence level. Also, the WPO was used to depict the absolute extreme value incurred by the default, an assessment necessary for the CCP.

We have also looked at the Volatility of the P&L outcomes to see how much the losses differ between the data sets. This also provides a good measurement of the hedge performance between different scenarios.

Furthermore, during the unwinding period, the CCP pays for daily mark-to-market changes (driven by changes in interest rates) and are consequently exposed to the risk of liquidity shortage. To address this concern, we have also evaluated the Worst Cumulative Mark-to-Market (WCM) for all portfolios in every data set. An illustrative presentation of the mark-to-market cash flow can be seen in Figure 10. In this example, the portfolio’s total P&L seen over 20 trading days is equal to zero. However, in order for the CCP to manage the daily mark-to-market payments during these days, it requires available cash of USD 8 millions.
Also, as a complement to the five measures; VaR, ES, σ, WPO, and WCM, we analyzed the P&L time series using plots, histograms, and empirical quantile plots. This was to make sure that no hidden results were overlooked by biases in the selection of samples.

4.4 Hedging Positions

For each portfolio composition, two corresponding hedge portfolios were constructed with the intention of immunizing against changes in the zero rate curve. To minimize the impact of transaction costs, we chose to hedge over the entire expected unwinding period, i.e. no re-hedging was executed. Hence, the hedge ratios were computed to immunize against movements incurred over 20-day-changes in interest rates.

Two types of hedging techniques have been employed in this study; parallel shift- and 3-factor PCA immunization. PCA is a mathematical tool used for finding patterns in data of high dimensions but can also be applied for constructing feasible scenarios of changes within data sets (in our case zero rates). We have chosen the first three components based on that, generally speaking, these factors will be expected to account for roughly 97% of the variation in the zero curve. Also, parsimony in number of components is important because each additional factor imposes an additional constraint on the immunization problem. Not only is complexity increased by additional constraints, but it also reduces the number of feasible set of solutions (Barber & Copper, 1996). Immunization against parallel shifts in the zero rate curve was applied in order to evaluate differences between simplistic and more sophisticated hedging strategies (3-factor PCA being the latter). Mathematical explanations of both methods are found in Chapter 2.
As described in section 2.4, the first step when calculating the hedge ratios was to compute the gradient for both the defaulted portfolio as well as for the available hedging derivatives. Engaging in a swap does not change the present value of a portfolio (the par swap rate is set as such that the present value of the swap equals zero and we assumed that trades existed solely on the primary market). Consequently, this study discarded equation (30) and only used equation (31) to calculate the hedging positions, i.e. the hedges only considered changes in value.

Finally, by solving equation (28) with $j = 1$ and $\Delta r_1 = (1, ..., 1) = 1$ as well as equation (31) with $l = 3$, we obtained the positions for the parallel shift- and the 3-factor PCA immunization, respectively.

As the hedge positions were expected to be large, suggesting that a price impact could be at risk, another aspect emphasized was liquidity. Therefore, we were constrained to using only highly liquid assets for hedging. This entails, not only lower transaction costs (smaller spreads), but also less supply of hedging instruments. Consequently, we only used USD LIBOR plain vanilla swaps with maturities equal to, or less than, seven years. Hence, a total of six different contracts were used as hedging instruments. Please note that in our case, the number of principal components chosen equals the number of hedging positions (and also the minimum number of hedging instruments needed) taken in each default scenario. Here, the 3-factor PCA immunization implies that a minimum of three hedging instruments are needed.

The number of hedging instruments required for the parallel shift immunization is decided by the equation system displayed in equation (28). Hence, only one contract was needed in the hedge portfolio to match the inherited portfolio gradient. As we were using six different hedging instruments, equation (28) was an overdetermined system. However, if one were to hedge a portfolio consisting of swaps with short maturities (e.g. one and two year tenors); it would seem unreasonable to take one position in a contract with long maturity (e.g. seven and ten year tenors). This remark would be disregarded by the mathematics of the hedge ratio calculations. We chose to address this issue by adding a complementary constraint for the FLP parallel shift immunization (which consists of short maturities); namely that only one, two, and three year maturities may be used for hedging. For the 3-factor PCA immunization we chose not to apply this constraint as three contracts with different tenors will be chosen by default. The positions taken in each hedging portfolio are displayed in the Appendix.
Figure 11. Iterative flowchart of the research

Flow chart illustrating our iterative algorithm for a rolling historical simulation. Every iteration represents a unique default date which yields P&L measures for the corresponding data set.
CHAPTER 5

5. Results

5.1 Overview

As described in Chapter 4, two hedging techniques have been evaluated on three different portfolios (with different cash flow characteristics) using plots of the results as well as five measures of performance. We have benchmarked the strategies against the unhedged portfolios through backtesting on our data. The results are displayed in figures and tables where the figures show histograms, time series of the P&Ls, as well as worst cash flow caused by mark-to-market losses. The P&L outcomes have been computed based on the 20 post-default trading days using rolling historical simulation with time step one trading day. Hence, each figure represents 2044 unwinding period P&L performances, i.e. the outcome seen over 20 trading days if a clearing member were to be declared in default on any given historical date in the set. Our method have also been stress tested with regards to potential discrepancies in the measures caused by overlapping data sets by using the time step 20 trading days. As there were no significant differences in this assessment, the one day time step has been concluded to generate, from this aspect, valid empirical evidence.

The tables below present our evaluated measures; Value-at-Risk 99% (VaR$_{99\%}$), Expected Shortfall 99% (ES$_{99\%}$), Worst P&L Outcome (WPO), Volatility ($\sigma$), and Worst Cumulative Mark-to-Market (WCM). The values are presented so that they can be compared to the corresponding unhedged portfolio. Also, the relative improvements are given on each measure.

The results are first presented for the PCA immunization, comparing hedge performance between the three portfolio compositions. Subsequently, the parallel shift strategy is evaluated in the same manner. This is followed by a comparison between the overall performances of both strategies. An extended portrayal of the PCA results can be found in Appendix.

The notional values, as well as the portfolio positions have been arbitrarily chosen. Therefore, numerical comparisons of some measures between the portfolio compositions would prove irrelevant. As such, we have chosen to make these comparisons in terms of relative improvements and by analytically reviewing the graphs.
5.2 PCA Immunization

Table 4 shows the computed measurements obtained from applying the PCA immunization strategy. VaR\textsubscript{99\%}, ES\textsubscript{99\%}, WPO, \(\sigma\), and WCM all indicate signs of improvement. Generally, the strategy seems most efficient on the MLP, and least performing on the ELP. However, results display relative reduction levels of no less than 63.4%.

Figure 12 - Figure 17 demonstrate the distributional characteristics of the three portfolio loadings. The histograms suggest reduction of tail properties for all portfolios, which is also depicted in Table 4, where \(\sigma\) is decreased by 71.8% - 97.2%. As a consequence, not only extreme losses, but also extreme profits, are reduced. Also, closer inspection of Figure 12 reveals a skewed underlying distribution for the unhedged FLP.

Figure 18 - Figure 23 advocate that the MLP is where the immunization strategy has the largest impact. The ELP seems to have the worst performing hedge, especially for the iterations where the unhedged portfolio displays more volatile P&L outcomes but does however reduce the worst losses.

Figure 24 - Figure 29 present a plot over each default date’s worst cumulative cash flow, incurred by daily mark-to-market losses. Again, the figures indicate a difference in performance between portfolio loadings, where the immunization strategy on the MLP has largest impact. As before, the ELP shows improvements, however, not in the same magnitude as with the other portfolios.

Another remark is that volatility differs between the unhedged portfolio loadings. Figure 18 - Figure 23 suggest that the P&L volatilities also vary between the portfolios within corresponding subsets (see iterations 1600-2000 for Figure 18, Figure 20, and Figure 22), this has also been confirmed mathematically.

Table 4. Evaluated measures from 3-factor PCA immunization

<table>
<thead>
<tr>
<th>Measure</th>
<th>Front-loaded portfolio (FLP)</th>
<th>Mid-loaded portfolio (MLP)</th>
<th>End-loaded portfolio (ELP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unhedged</td>
<td>Hedged</td>
<td>Reduction</td>
</tr>
<tr>
<td>VaR\textsubscript{99%}</td>
<td>0,63</td>
<td>0,07</td>
<td>89,3%</td>
</tr>
<tr>
<td>ES\textsubscript{99%}</td>
<td>0,87</td>
<td>0,08</td>
<td>90,6%</td>
</tr>
<tr>
<td>WPO</td>
<td>1,16</td>
<td>0,11</td>
<td>90,4%</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0,46</td>
<td>0,03</td>
<td>93,4%</td>
</tr>
<tr>
<td>WCM</td>
<td>1,43</td>
<td>0,12</td>
<td>91,3%</td>
</tr>
</tbody>
</table>

The table presents values on the evaluated measures as well as relative improvements before and after the application of a 3-factor PCA immunization strategy.
Figures illustrate histograms on P&L outcomes for all the 2044 post-default unwinding periods from the backtested data. The three portfolios evaluated, FLP, MLP, and ELP, are presented in a descending order. The left column represents the unhedged portfolios and the right represents the hedged equivalent using 3-factor PCA immunization.
Figures illustrate time series of the P&L outcomes for all the 2044 post-default unwinding periods from the backtested data. Iteration corresponds to each evaluated default date. The three portfolios evaluated, FLP, MLP, and ELP, are presented in a descending order. The left column represents the unhedged portfolios and the right represents the hedged equivalent using 3-factor PCA immunization.
Figures illustrate WCM for all the 2044 post-default unwinding periods from the backtested data. Iteration corresponds to each evaluated default date. The three portfolios evaluated, FLP, MLP, and ELP, are presented in a descending order. The left column represents the unhedged portfolios and the right represents the hedged equivalent using 3-factor PCA immunization.
5.3 Parallel Shift Immunization

The parallel shift immunization represents the simplistic technique. Table 5 presents our results on the five evaluated measures. As with the 3-factor PCA approach, hedging performance seems to differ between the three portfolio characteristics. The greatest impact seems to be when the hedge is applied on the MLP. The hedge on the ELP and FLP has less impact, especially for certain subsets.

Figure 30 - Figure 35 demonstrate signs of improvements in terms of tail reduction for all portfolios. Also, the skewness of the FLP portrayed in Figure 30 seems to have decreased when comparing to the hedged portfolio in Figure 31. When comparing Figure 33 and Figure 35 against Figure 31, there seems to be differences regarding smoothness of the distributional characteristics, e.g. the hedged FLP histogram displays a diminishing frequency just around the mean. Overall, the histograms indicate that the MLP hedge outperforms the FLP and ELP, which display more or less equal performance. Looking at Table 5, the evaluated measures suggest that the ELP produce slightly higher reductions than the FLP (with the exception of $\sigma$).

Figure 36 - Figure 41 present the P&Ls timing through the time series plots. As expected, the figures substantiate the interpretation above; that the parallel shift indeed outperforms the unhedged portfolio for all compositions. Noteworthy, at first glance, Figure 37 and Table 5 indicate that the parallel shift immunization on the FLP provides an adequate hedge when evaluated over the entire set. However, there seems to be subsets in the data where the hedge not only underperforms, but in fact increases the incurred losses and general P&L volatility (e.g. the last 200 iterations).

In accordance with above, Figure 42 - Figure 47 demonstrate that the worst cumulative cash flow seems to be reduced on both the MLP and the ELP, whilst the FLP plots suggest reductions within the entire data set but increase for some subsets (especially around iteration 1350 and 1650).

**Table 5, Evaluated measures from parallel shift immunization**

<table>
<thead>
<tr>
<th>Portfolio:</th>
<th>Front-loaded portfolio (FLP)</th>
<th>Mid-loaded portfolio (MLP)</th>
<th>End-loaded portfolio (ELP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unhedged</td>
<td>Hedged</td>
<td>Reduction</td>
</tr>
<tr>
<td>VaR99%</td>
<td>0,63</td>
<td>0,53</td>
<td>15,6%</td>
</tr>
<tr>
<td>ES99%</td>
<td>0,87</td>
<td>0,61</td>
<td>28,9%</td>
</tr>
<tr>
<td>WPO</td>
<td>1,16</td>
<td>0,73</td>
<td>37,7%</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0,46</td>
<td>0,20</td>
<td>57,0%</td>
</tr>
<tr>
<td>WCM</td>
<td>1,43</td>
<td>0,88</td>
<td>38,4%</td>
</tr>
</tbody>
</table>

The table presents values on the evaluated measures as well as relative improvements before and after the application of a parallel shift immunization strategy.
Figures illustrate histograms on P&L outcomes for all the 2044 post-default unwinding periods from the backtested data. The three portfolios evaluated, FLP, MLP, and ELP, are presented in a descending order. The left column represents the unhedged portfolios and the right represents the hedged equivalent using parallel shift immunization.
Figures illustrate time series of the P&L outcomes for all the 2044 post-default unwinding periods from the backtested data. Iteration corresponds to each evaluated default date. The three portfolios evaluated, FLP, MLP, and ELP, are presented in a descending order. The left column represents the unhedged portfolios and the right represents the hedged equivalent using parallel shift immunization.
Figures illustrate WCM for all the 2044 post-default unwinding periods from the backtested data. Iteration corresponds to each evaluated default date. The three portfolios evaluated, FLP, MLP, and ELP, are presented in a descending order. The left column represents the unhedged portfolios and the right represents the hedged equivalent using parallel shift immunization.
5.4 Comparison

As our results from the two different hedging strategies have been presented against the three portfolio compositions, we will now provide a comparison between the two techniques. Figure 48 - Figure 53 represent each technique’s P&L quantile plot together with the unhedged equivalent. The 3-factor PCA approach seems to outperform the more simplistic parallel shift immunization on all three loadings. However, the differences in impact appear to be dependent on the characteristics of the defaulted portfolio.

Firstly, the FLP displayed in Figure 48 and Figure 49, suggest advantages towards the 3-factor PCA technique. As mentioned above, looking at the last 200 iterations in Figure 36 and Figure 37, the parallel shift approach will in some cases increase the P&L volatility. This is not depicted by the quantile plot since the observed P&Ls are sorted in descending order instead of chronologically.

Secondly, the MLP (see Figure 50 - Figure 51) show signs of improvements when applying both strategies. By only analyzing the graphs, it would be difficult to identify any clear dissimilarity between the two. However, when examining Table 6, the 3-factor PCA demonstrates a slightly stronger performance, looking at the evaluated measures.

Lastly, Figure 52 - Figure 53 represent the ELP and both strategies seem to reduce the tail-losses compared to the unhedged portfolio. Again, the 3-factor PCA outperforms the parallel shift approach, yielding a less volatile P&L for the post-default unwinding period.

Table 6 presents reductions of each measure for the two different strategies and thus compares the effectiveness of the hedges through relative improvements against the unhedged portfolio. The comparison suggests that even if the simplistic parallel shift immunization does show signs of improvements, the 3-factor PCA strategy outperforms on all portfolio loadings. The largest discrepancies between the strategies seem to be for the FLP whilst the MLP display rather similar reductions in the evaluated measures.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Front-loaded portfolio (FLP)</th>
<th>Mid-loaded portfolio (MLP)</th>
<th>End-loaded portfolio (ELP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3-Factor PCA</td>
<td>Parallel shift</td>
<td>3-Factor PCA</td>
</tr>
<tr>
<td>VaR99%</td>
<td>89,3%</td>
<td>15,6%</td>
<td>96,6%</td>
</tr>
<tr>
<td>ES99%</td>
<td>90,6%</td>
<td>28,9%</td>
<td>96,2%</td>
</tr>
<tr>
<td>WPO</td>
<td>90,4%</td>
<td>37,7%</td>
<td>95,9%</td>
</tr>
<tr>
<td>σ</td>
<td>93,4%</td>
<td>57,0%</td>
<td>97,2%</td>
</tr>
<tr>
<td>WCM</td>
<td>91,3%</td>
<td>38,4%</td>
<td>95,9%</td>
</tr>
</tbody>
</table>

The table presents a comparison of measure reductions between the 3-Factor PCA and the parallel shift immunization hedging strategy.
Figures illustrate P&L quantiles for the 2044 post-default unwinding periods on the backtested data. Iteration corresponds to each evaluated default date. The three portfolios evaluated, FLP, MLP, and ELP, are presented in a descending order. The left column represents the 3-factor PCA hedge and the right represents the parallel shift immunization. The hedges are shown as dashed lines and the unhedged equivalent is also presented in each figure by a solid line.
CHAPTER 6

6. Discussion

Based on the results of our research it may be tempting to assume that both of the applied hedging strategies are beneficial in terms of a reduction in the statistical measures assessed in the study. However, there were subsets in the data where the volatility of the P&Ls concerning the unwinding period actually increased by applying the simplistic hedge (e.g. Figure 37).

When conducting our research, there were indications of lack in robustness when for the PCA immunization using certain sets of data. An example of this was when the 7-day LIBOR was used as a 7-day zero rate, resulting in some unexpected behavior of the computed hedge portfolio positions. Results were obtained where very large positive and negative positions were suggested for the hedge; yielding even more volatile P&L outcomes than before the hedge was applied. This could be due to the assumption that short LIBOR rates estimates market zero rates; an assumption which, if made erroneously, would carry the implication that these function under different dynamics than the other zero rates. In this study, the problem was addressed by simply removing the 7-day LIBOR data and using the 3-month LIBOR as our first data point. Another explanation for the PCA not being robust could be that the linear bootstrapping technique applied in the study was not accurate enough, resulting in unexpected dynamics for some zero rates. The solution to such a problem would simply be to apply more advanced techniques. Alternatively, there are highly sophisticated algorithms for computing the zero rate curve which can be delivered by companies specialized in these services.

An assumption we would like to address is linked to the Taylor approximation used to calculate the hedge portfolio. The approximation assumes that there will be a “small change” in the portfolio value, why we chose only to use the first order Taylor. One might argue that, were a clearing member to default, the assumption is challenged as empirical evidence suggests that volatility increases during these events. Albeit the statement above, the expression “small change” is highly relative and the findings in this study suggests that a first order Taylor will in many outcomes produce an adequate hedge. An alternative would be to expand the approximation to a second order Taylor which would potentially generate a more robust hedge with the disadvantage of increased complexity.
As we have seen that the performance of the PCA immunization strategy is highly dependent on what data is used to calibrate the positions to engage in, the topic should be subject to discussion. As is argued in Chapter 4, one should consider the data input as a subjective decision made by the model user. In this study, we have used four years to calibrate with no emphasis on chronology, meaning that the first data point (from four years ago) will have the same model impact as the data point from yesterday. One might argue that this is an unreasonable assumption and that the dynamics of a pre-financial crisis data would largely differ from that of a post-financial crisis. The matter could be addressed by subjectively choosing “relevant” time series for each calibration. The rationale above is an argument for using the parallel shift approach, as this it does not require historical data to compute the hedge ratios. However, as a test, the PCA immunization was run under only two years of input data and still greatly outperformed the parallel shift.

The 3-factor PCA immunization was used to represent a sophisticated hedging technique for the study. This means that only three components from the PCA were used to calculate the hedge. Technically, even though there is a decrease in marginal benefit received from using more components; if one had many hedge instruments, more components could be utilized resulting in greater accuracy. There are however some benefits to collect with parsimony in the selection of components used for the hedge. One advantage is that there is room for other conditions in the optimization problem and another that the general complexity of the problem is decreased. Extra conditions could be used to exclude unfeasible results from the hedge, e.g. if there are some limitations regarding liquidity in some hedging instruments, it can be addressed by using an extra condition for the problem. The amount of extra conditions obtained is decided by the ratio between hedging instruments and principal components used. Based on this, we would like to lift the notion that by using swaps, with present value equal to zero, we remove equation (30) with the implication that one less hedging instrument is needed. Hence, by using swap contracts for hedging, the possibility of an additional constraint is obtained.

We have seen that for the 3-factor PCA, the hedge performs best when applied to the FLP, second best for the MLP, and worst for the ELP. One could argue that this might be because of the rather high sensitivity of changes in zero rates for cash flows far into the future. Also, because we have constrained ourselves to not using 10-year swaps for hedging, it may be somewhat difficult to match the gradients between the hedge instruments and the inherited defaulted portfolio. We would like to extend this interpretation to our findings from the parallel shift approach, where the best outcomes were obtained for the MLP, second best for the ELP, and worst for the FLP. It seems that both techniques displayed similar limitations on the ELP (supporting the argument given above) whilst performing well on the MLP. The largest difference was for the FLP, where one possible explanation could be that parallel shifts are less likely to occur in the shorter zero rates during unstable financial times. If this were to be true, an immunization against a parallel shift would naturally prove less efficient. However, no proofs of this claim, other than the anomalies observed in the hedge performances, are provided in this study.
CHAPTER 7

7. Conclusion

There will always be a non-zero probability of a clearing member default. Consequently, CCPs need to prove to their clients, regulators, as well as the remaining stakeholders in the financial industry that their services are associated with minimum counterparty risks. Even though extensive efforts have been made to improve the practices of default management processes, there seems to be a lack of guidelines concerning how to manage the risks associated with the inherited portfolio after the member default.

Our empirical findings suggest that by applying reactive hedging strategies, CCPs may enjoy the benefits of a less risky inherited portfolio during a member default. However, the performance of the hedge is seemingly dependent on at least two factors; (1) portfolio composition as well as (2) choice of hedging strategy. Furthermore, for both the simplistic and the complex hedging strategy, the outcomes of the evaluated risk measures show improvements. However, the sophisticated hedge seems to systematically outperform the simplistic approach and the latter may even worsen the outcomes during certain market conditions.

As encouraging as our results may appear, a number of aspects should be kept in mind. Firstly, the evaluated portfolios have been artificially designed to represent clearing member positions. Hence, there is room for assessment regarding if our findings would hold for more complex compositions, perhaps in the form of case studies on real portfolios. This would possibly introduce new challenges to the problem in the form of netting computations and gradient computations.

Secondly, the zero rate curve used should be considered as primitive and the linear interpolation assumptions may have influenced our findings. As there are companies specialized in the offering of highly sophisticated algorithms to compute the zero rates, a study using data from these should prove more accurate.

Thirdly, there is additional room for constraints to complete the overdetermined system of equations which can and should be used to obtain more relevant hedging positions. These could be used for tailoring the hedge portfolio to account for various practical issues that may arise when moving from theoretical into applied areas of conduct.
Finally, there are some modifications to the hedge approaches that would further increase the quality of the immunization strategies. For one, to minimize the impact of the assumption to exclude the transaction costs from the research, we also chose to disregard the possibility of re-hedging the portfolio during the unwinding period. Naturally this poses the question of how the hedges were to perform, would one take into account both of these options. Alternatively, with origin that the founding assumption of the immunization strategies are based on the first order Taylor approximation, there could be an extended approach. To account for larger changes in the interest rate and to further distance oneself from the need of frequent re-hedging, one could expand the approximation to the second order Taylor. There are also additional avenues of improvements such as; time weighting of the input data as well as the application of an error-adjusted PCA immunization technique, which is advocated by Carcano & Dall’O (2011) in their paper.

As argued above, assumptions and simplifications have been made to obtain our results. Hence, the findings should be treated with this taken into consideration and as complexity increases in the default portfolio, losses of generality may be infused to the interpretations given in this report. However, we are cautiously optimistic that our initial findings will provide sufficient motivation to take on these challenges.
APPENDIX

Risk Properties

Consider two time points, time 0 which corresponds to today and a future time point $\Delta t > 0$. As time may be measured in units of $\Delta t$, the future time point is therefore chosen as 1. Next, let $V_1$ be the value of a portfolio at time 1. Naturally, this portfolio involves both an opportunity of a favorable outcome as well as a risk of an unfavorable outcome. As the CCP do not want to speculate on risk they would only focus on analyzing the part of the probability distribution of $V_1$ corresponding to the unfavorable outcomes (Hult, et al., 2012). Consequently, CCPs may request a clearing member with a non-acceptable position to adjust its exposure in order to be allowed to continue with its business. This is usually done via increasing margins or re-allocating the portfolio positions.

In order to understand the quality of a portfolio one could analyze the probability distribution of its future value $V_1$. However, comparison of different probability distributions is a rather complex and difficult operation. Therefore, it is desirable to apply a method to summarize the entire probability distribution from a risk measurement perspective with a single quantifiable number (Hult, et al., 2012).

According to Artzner, et al. (1999) and Hult et al. (2012), there are four different properties that have been suggested as natural requirements for good risk measures.

Consider a linear vector space $X$ of random variables where its elements are corresponding to time 1 values of every position of the portfolio. Also, the function $\rho$ assigns a real number to each $X$ in $X$. The function $\rho$ is chosen so that $\rho(X) \leq 0$ mean that $X$ is an acceptable future net worth. Consequently, $\rho(X) > 0$ implies the minimum number of units in the particular currency at time 0 that need to be added to the position and invested in a reference instrument with percentage return $R_{rf}$ until time 1 in order to make the position acceptable (Hult, et al., 2012).

Translation invariance

$$\rho(X + cR_{rf}) = \rho(X) - c, \text{ for all real numbers } c$$ (32)
This can be interpreted as by adding the amount $c$ of cash and buying zero-coupon bonds for this amount will reduce the risk by the same amount. In particular, by adding the amount $\rho(X)$ to an unacceptable future net worth $X$, makes the position acceptable:

$$\rho(X + \rho(X)R_{rf}) = \rho(X) - \rho(X) = 0$$  \hspace{1cm} (33)

**Monotonicity**

If $X_2 \leq X_1$, then $\rho(X_1) \leq \rho(X_2)$ \hspace{1cm} (34)

This states that if a position has a greater future net worth for sure, then that position must be considered less risky.

**Positive homogeneity**

$$\rho(\lambda X) = \lambda \rho(X), \text{ for all } \lambda \geq 0$$ \hspace{1cm} (35)

This property implies that the risk increases linearly with the size of the position. Hence, if we double the size of the position, then we also double the risk.

**Subadditivity**

$$\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$$  \hspace{1cm} (36)

Subadditivity states that diversification should be rewarded. An investor consisting of two different units should be required to put aside less buffer capital compared to if they were considered as two separate entities.

A risk measure $\rho$ satisfying all of the above properties is called a coherent measure of risk and does generally possess the functionality of an acknowledged good risk measure.

**Value-at-Risk**

Hult et al. (2012) describes VaR for a position with value $X$ at time 1 as the smallest amount of money that if added to the position at time 0 and invested in a risk-free asset ensures that the probability of a strictly negative net worth at time 1 is not greater than $\alpha \in (0,1)$, hence:

$$VaR_\alpha = \min \{m: P(mR_{rf} + X < 0) \leq \alpha\}$$  \hspace{1cm} (37)
Expected Shortfall

Expected Shortfall (ES), or sometimes Conditional Value-at-Risk (CVaR), can be interpreted as:

Given a rare event, below or on the predefined level $\alpha$, what is the average loss. ES at level $\alpha$ is the average VaR for levels $p \leq \alpha$ and can be described by the following expression:

$$ES_{\alpha}(X) = \frac{1}{\alpha} \int_{0}^{\alpha} VaR_{p}(X) dp$$  \hspace{1cm} (38)

Particularly, if $X$ has a continuous distribution function and loss is defined as $L = -X/\text{Ref}$, then $ES_{\alpha}(X)$ may be expressed as:

$$ES_{\alpha}(X) = E[L | L \geq VaR_{\alpha}(X)]$$  \hspace{1cm} (39)
PCA Results

Figure 54 and Table 7 contains illustrative results from the PCA for the calibration period on an arbitrary chosen default date. The same computations were performed on all the unique default dates throughout the complete data set, generating 2044 individual sets of principal components. The first component could roughly be interpreted as a parallel shift in the zero curve, given all of its units are either all positive. In this example, if the 2-year zero rate increases by 0.346 basis points, the 3-year zero rate is expected to increase by 0.389 basis points. In the same manner, the second principal component could be interpreted as a change in slope of the zero curve; if zero rates between 3-months and 3-year increases, zero rates between 4-year and 10-year will be expected to decrease. Lastly, the third component could be described as a change in curvature; if short-term and long-term zero rates increases, mid-term zero rates is expected to decrease and of course vice versa.

Figure 54, Principal Components

The first three principal components corresponding to a theoretical default on 2010-05-05.

Table 7, Principal components

<table>
<thead>
<tr>
<th>PC Number</th>
<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
<th>PC4</th>
<th>PC5</th>
<th>PC6</th>
<th>PC7</th>
<th>PC8</th>
<th>PC9</th>
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</thead>
<tbody>
<tr>
<td>Zero maturity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-month</td>
<td>0.068</td>
<td>0.530</td>
<td>0.485</td>
<td>-0.205</td>
<td>-0.655</td>
<td>0.074</td>
<td>0.000</td>
<td>-0.042</td>
<td>0.005</td>
</tr>
<tr>
<td>6-month</td>
<td>0.106</td>
<td>0.582</td>
<td>0.327</td>
<td>-0.040</td>
<td>0.734</td>
<td>-0.029</td>
<td>-0.002</td>
<td>0.035</td>
<td>-0.021</td>
</tr>
<tr>
<td>1-year</td>
<td>0.237</td>
<td>0.405</td>
<td>-0.347</td>
<td>0.722</td>
<td>-0.166</td>
<td>-0.315</td>
<td>0.105</td>
<td>0.008</td>
<td>-0.024</td>
</tr>
<tr>
<td>2-year</td>
<td>0.346</td>
<td>0.201</td>
<td>-0.403</td>
<td>-0.064</td>
<td>-0.003</td>
<td>0.643</td>
<td>-0.473</td>
<td>-0.037</td>
<td>0.189</td>
</tr>
<tr>
<td>3-year</td>
<td>0.389</td>
<td>0.056</td>
<td>-0.278</td>
<td>-0.338</td>
<td>-0.021</td>
<td>0.138</td>
<td>0.559</td>
<td>0.291</td>
<td>-0.487</td>
</tr>
<tr>
<td>4-year</td>
<td>0.406</td>
<td>-0.038</td>
<td>-0.105</td>
<td>-0.299</td>
<td>0.025</td>
<td>-0.279</td>
<td>0.314</td>
<td>-0.406</td>
<td>0.626</td>
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<tr>
<td>5-year</td>
<td>0.418</td>
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<td>0.039</td>
<td>-0.179</td>
<td>-0.009</td>
<td>-0.441</td>
<td>-0.525</td>
<td>-0.284</td>
<td>-0.475</td>
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<tr>
<td>7-year</td>
<td>0.408</td>
<td>-0.222</td>
<td>0.262</td>
<td>0.070</td>
<td>-0.028</td>
<td>-0.155</td>
<td>-0.176</td>
<td>0.749</td>
<td>0.305</td>
</tr>
<tr>
<td>10-year</td>
<td>0.389</td>
<td>-0.325</td>
<td>0.466</td>
<td>0.436</td>
<td>0.052</td>
<td>0.407</td>
<td>0.218</td>
<td>-0.323</td>
<td>-0.126</td>
</tr>
</tbody>
</table>

All the principal components corresponding to arbitrarily chosen default date of 2010-05-05.
Positions taken in Hedge Portfolios

Below is a graphical representation of the hedge positions chosen to hedge the corresponding portfolios for the different strategies.

First follows the 3-factor PCA immunization where positions have been taken in three different contracts for each iteration. We see that, generally, the positions taken are taken steadily in the 1-year, 3-year, and 7-year tenors (except for the last 50 iterations where the hedge chooses positions more randomly). The reason for this behavior is that the Matlab function “\" has been applied to an overdetermined system.

Subsequently, the parallel shift immunization positions are presented. Here, the system of equation is even less determined, as only one position is taken to account for the parallel shift. The tendency is to take positions in the long tenor, i.e. the 7-year maturity.
Figure 55, PCA hedging positions, FLP

The plot shows the computed hedging positions (y-axis) in each hedging instrument for all iterations (x-axis).
Figure 56, PCA hedging positions, MLP

The plot shows the computed hedging positions (y-axis) in each hedging instrument for all iterations (x-axis)
Figure 57, PCA hedging positions, ELP

The plot shows the computed hedging positions (y-axis) in each hedging instrument for all iterations (x-axis)
Figure 58, Parallel shift hedging positions, FLP

The plot shows the computed hedging positions (y-axis) in each hedging instrument for all iterations (x-axis)
The plot shows the computed hedging positions (y-axis) in each hedging instrument for all iterations (x-axis)
Figure 60, Parallel shift hedging positions, ELP

The plot shows the computed hedging positions (y-axis) in each hedging instrument for all iterations (x-axis)
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