The geometry of Shahyad Ariamehr

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Introduction

The monument of Shahyad Ariamehr is being built near Teheran to celebrate the 25th centenary of the foundation of the Iranian Empire, and of the Declaration of Human Rights by Cyrus the Great. As is fitting for such an occasion, it is a monument to the past—its inspiration clearly coming from traditional design. But it has another purpose concerned very much with today. Iran is not advanced in the modern techniques of building and the monument is seen as an opportunity of introducing to that country some of the sophisticated methods of construction available today—a stepping stone to the future, perhaps.

Outer surface geometry

The monument is essentially an external visual experience and the external surface geometry is thus of the greatest importance.

Although the monument has the qualities of a piece of sculpture, considerable rationalization of the details of the geometry has occurred during its development, with no loss of free form effect.

The final geometry is controlled by a 3 m (10 ft.) square module in plan and elevation, with overall proportions governed by a 21 m (69 ft.) square grid.

The external surface is defined by four separate surface geometries and the areas to which these different geometries apply are shown in Fig. 1.

Surfaces governed by the Defining Curve

Most of the external surface is defined by a family of curves related to the Defining Curve which is the projection in the y-z plane of the curve shown in Fig. 1. This family of curves is formed from the simple relationship given in Fig. 2. All the members of this family lie in vertical planes. A surface is formed as soon as plan sections at level 0 m and 45 m (148 ft.) are specified. The plan at level 45 m (148 ft.) is formed entirely of horizontal straight lines, and that at level 0 m generally so, with the exception of the curved portion AB which is represented by a hyperbola. (See Fig. 3.)

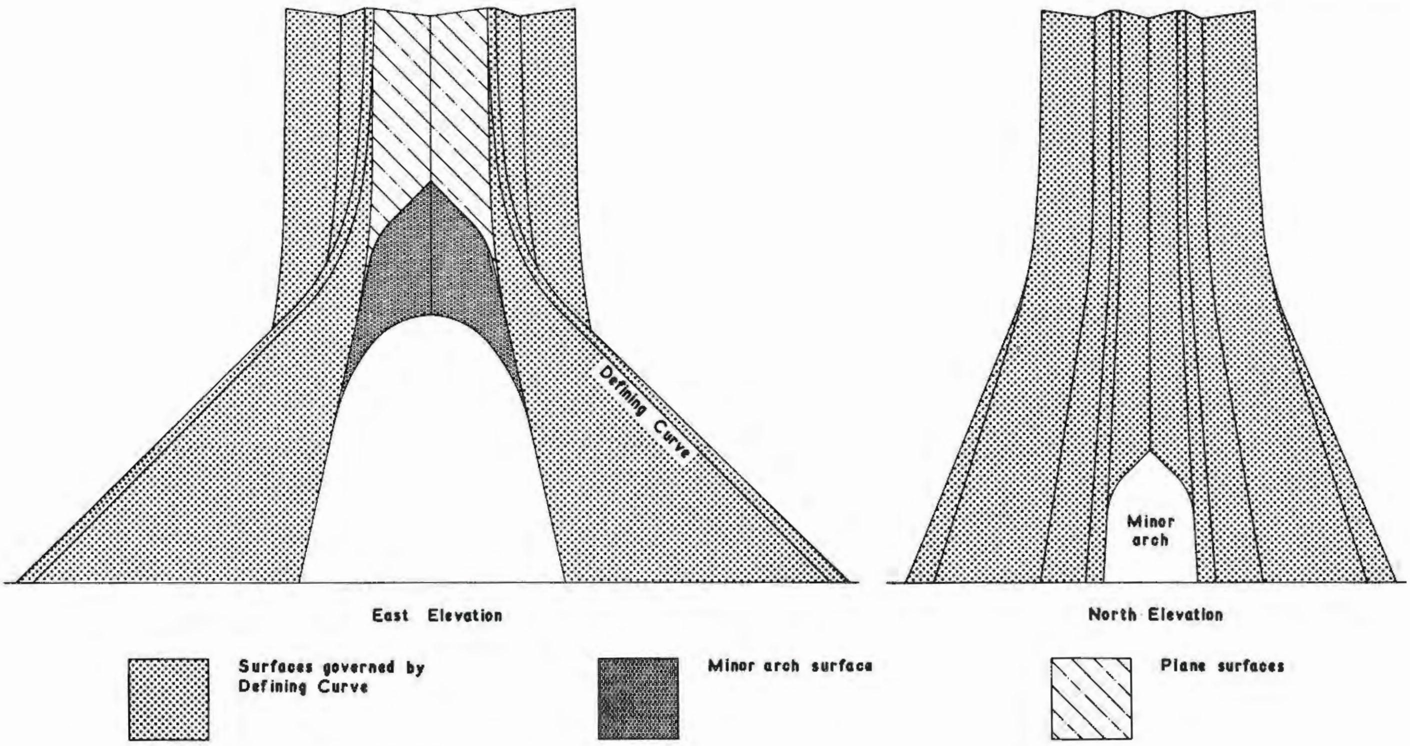
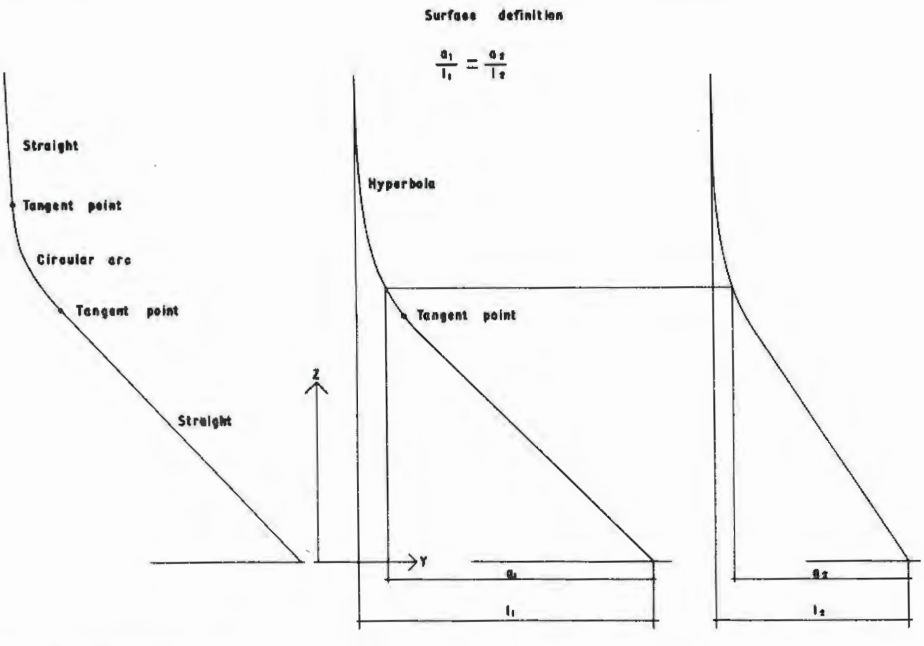


Fig. 1 Elevations of the monument showing breakdown of surface definition



a The original form of the Defining Curve

b The final form of the Defining Curve

c A typical member of the family of defining curves

Fig. 2 The Defining Curve

This figure also shows the simple relationship which governs the position of a defining curve of characteristic length l_2. An outcome of this definition of geometry is that at any horizontal section, the wall profile will consist of straight lines, with the exception of the length A'B'.

The surfaces are formed by slabs of marble as external cladding/permanent shuttering and the family of defining curves is expressed in the form of grooves running from top to bottom. The depth and width of these grooves vary in such a way that the ratios of depth and width to the horizontal extent of the surface at any level are constant. Typical shapes used for the marble slabs are shown in Fig. 4. The vertical projections of the heights of these slabs decrease in an arithmetic progression as the level increases.

The Defining Curve can be considered as being made up of two parts.

The lower straight portion below level 21 m (69 ft.) is the diagonal of a basic 21 m (69 ft.) setting out square, while the upper portion is curved and lies within a 6 m (20 ft.) wide by 24 m (79 ft.) high rectangle.

The Defining Curve was originally envisaged by the architect as being made up of two straight lines joined by a circular arc which was tangential to both. (See Fig. 2a.) This was

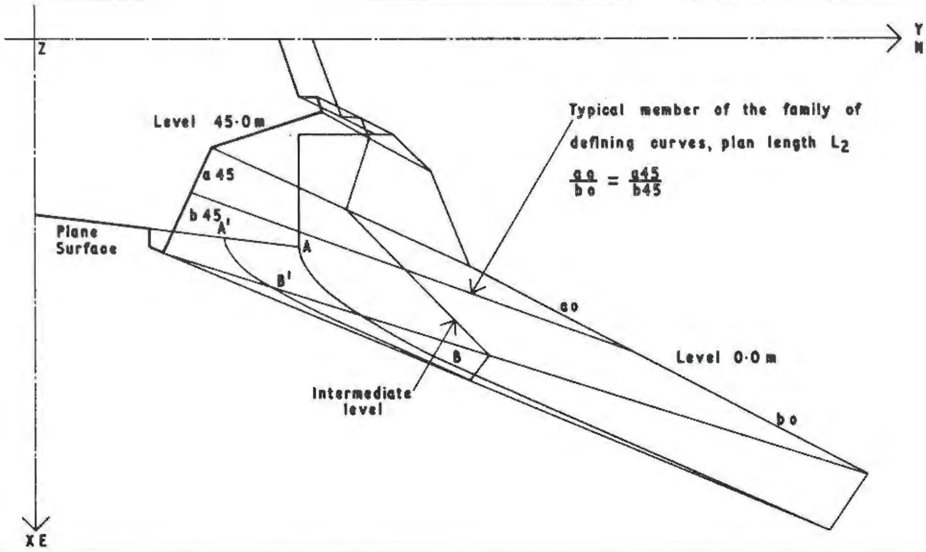


Fig. 3 Surface definition by plan sections

not a positive requirement, but instead represented a statement of the general profile needed in terms of simple geometrical forms. It also had a number of disadvantages. Although easy to draw, it was not easy to represent in a continuous form algebraically, and the lack of transition in curvature between straights and the circular arc could read badly on the large scale of the finished monument. The circular arc only applied in the case of the Defining Curve itself, the other members of the family being made up from straights and a second order curve. Because of these disadvantages, it was decided to replace this compound definition with a smooth curve. A second order curve was obviously required to eliminate the possibility of reversals of curvature occurring between control points, which might occur with a higher order curve. The general second order curve:

$$Ax^2 + By^2 + Cxy + Dx + Ey + 1 = 0$$

has five degrees of freedom to be controlled. To represent the complete Defining Curve in this way proved impossible as the architect required a near perfect straight line from level 0 m to level 21 m (69 ft.). A full height curve could have been used if a maximum chord offset between levels 0 m and 21 m (69 ft.) of about 100 mm (4 in.) was acceptable, but the architect was not satisfied with this. When this offset was reduced by adjusting the position of a controlling point, a sudden change in the form of the hyperbola resulted and two separate branches with a gap in the middle were obtained.

It was then decided to keep the straight line definition to level 21 m (69 ft.) and introduce a second order curve from level 21 m (69 ft.) to the top. Four controlling points were selected from the original Defining Curve, together with the condition that it should be tangential to the straight portion at level 21 m (69 ft.). A close approximation to the original was obtained. The architect realized the infinite possibilities of this procedure, and a number of further modifications was made to achieve the final profile.

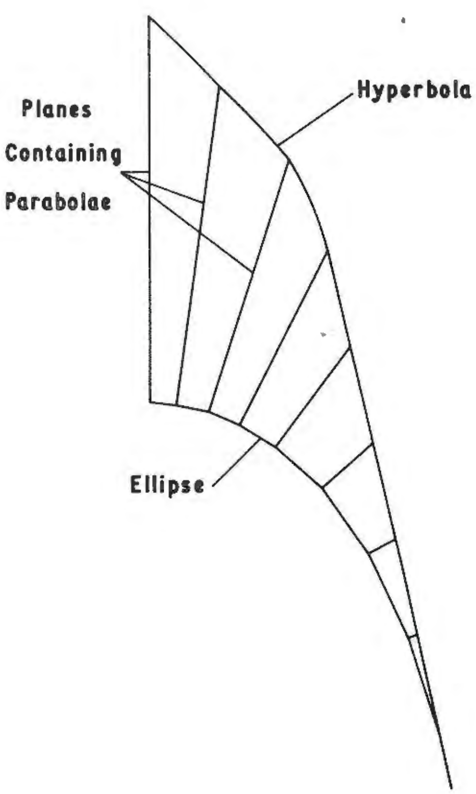
Considerable attempts were made to establish a general algebraic expression for the surface, and, for the portion below level 21 m (69 ft.), it proved to be of the form:

$$Az^2 + Bzx + Czy + Dx + Ey + Fz + 1 = 0$$

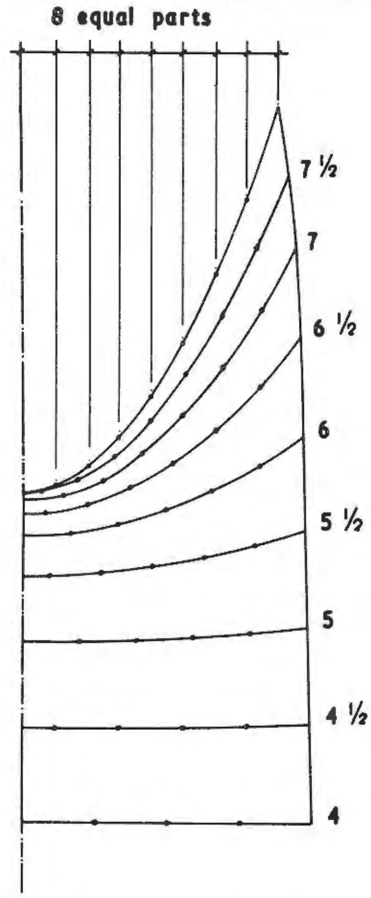
However the surfaces above level 21 m (69 ft.) proved too difficult, as did the obtaining of general expressions for normals and parallel surfaces and, as these were the main reasons for wanting the surface equations, no further effort was put into this approach.

At the same time, a parallel line of attack produced a numerical solution to the problem of surface definition by developing computer programs based on the fundamental synthesis of the geometry. This did not, of course, eliminate the problems of normals and parallel surfaces (which still remain unsolved), but enabled the global co-ordinates of any surface to be obtained at any level, or at a number of levels at equal intervals or at

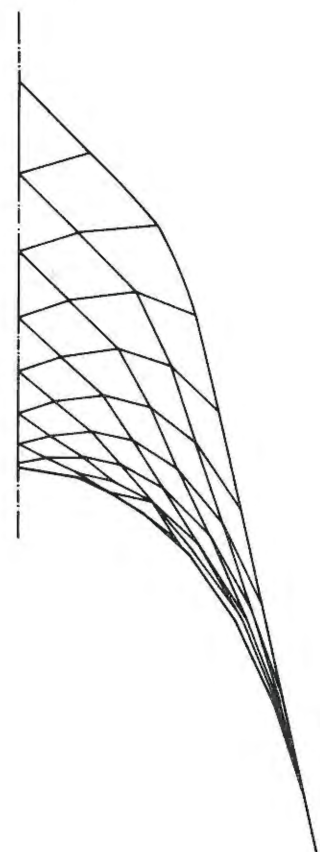
Fig. 4 Main arch



a Half of east elevation of main arch showing typical members of the family of defining parabolae



b Half of east-west section of main arch showing the division of parabolae



c Half of east elevation of main arch showing basis of stone rib pattern intervals equal to the characteristic stone heights.

Plane surfaces

The plane surfaces are vertical and inclined slightly to one another as shown in Fig. 3. They rise from the boundary of the main arch to the top of the monument.

Surface of the main arch

The main arch surface takes the form of a saddle. It is bounded below by inclined walls at level 12 m (39 ft.) and above by the vertical plane walls. The lower boundary is a horizontal straight line. The arch base curve is elliptical, tangential to the walls at level 12 m (39 ft.) with its crown at level 21 m (69 ft.).

The form of the upper boundary is very much governed by the form of the Defining Curve. As with the latter, it is defined in its projection in the y-z plane. It was originally drawn by the architect in three parts:

1 a straight portion from level 12 m (39 ft.) to level 23 m (75 ft.), which was coincident with the inclined springing walls below level 12 m (39 ft.).

2 an upper straight portion which coincided with the continuation of the lower straight portion of the Defining Curve and

3 a circular arc tangential to both of these straights. In order to remove the visual effects of abrupt changes in curvature which would result from this compound curve, the original definition was replaced with a single second order curve of the same general form as that used for the Defining Curve. This straightforward algebraic form would also help to simplify the overall surface definition. Four points were used as controlling conditions for determining the equation, together with the condition that it should be tangential to the inclined walls at level 12 m (39 ft.).

The surface is formed by a family of parabolae. These lie in planes which are normal to the ellipse where they meet it, and intersect the boundary hyperbola in such a way as to divide its projection on the y-z plane into equal arc lengths. This is shown in Fig. 4a. The parabolae are in turn divided up as shown in Fig. 4b. The nodes so formed are then joined to form the marble rib pattern which is required (Fig. 4c). The final form of the ribs which are curved is shown in Fig. 6. The surface defined above corresponds to the outer surface of the ribs.

Much effort was expended in an attempt to define the surface algebraically but the rigid constraints in form and boundary prevented this, or perhaps experience of a similar unavailing task performed for the Defining Curve surfaces made surrender too easy. The geometry was, therefore, again evaluated by numerical techniques. Computer programs were used to determine the actual boundary intercepts; the co-ordinates of the curved ribs at increments along their length and horizontal section co-ordinates of the outer surface.

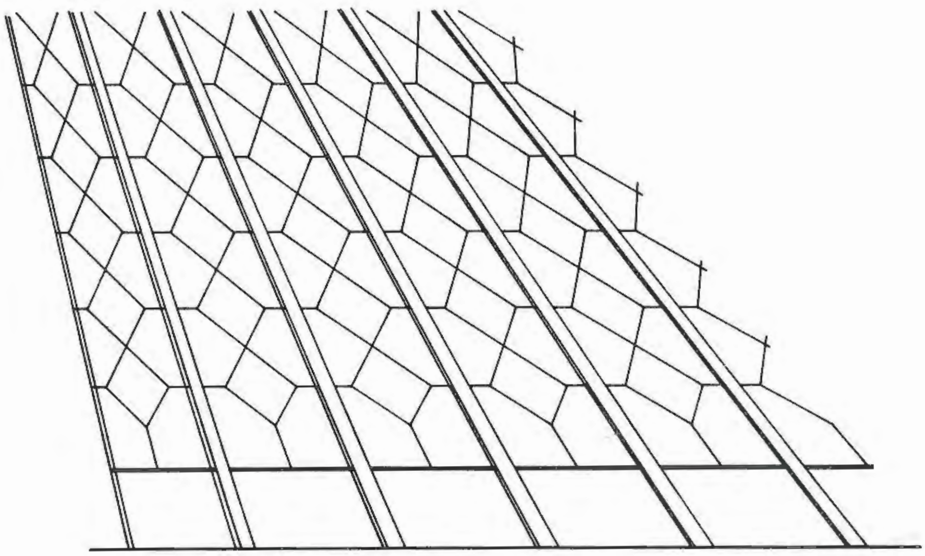
Surface of the minor arch

This arch forms a side entrance to the main arch area. It has a constant section of a form similar to the upper boundary of the main arch.

Internal structures

Although the external surface is of the greatest importance there is a large volume of space inside the monument, and the architect has chosen to fill this with a number of interesting reinforced concrete structures.

Fig. 5 Typical pattern of marble cladding



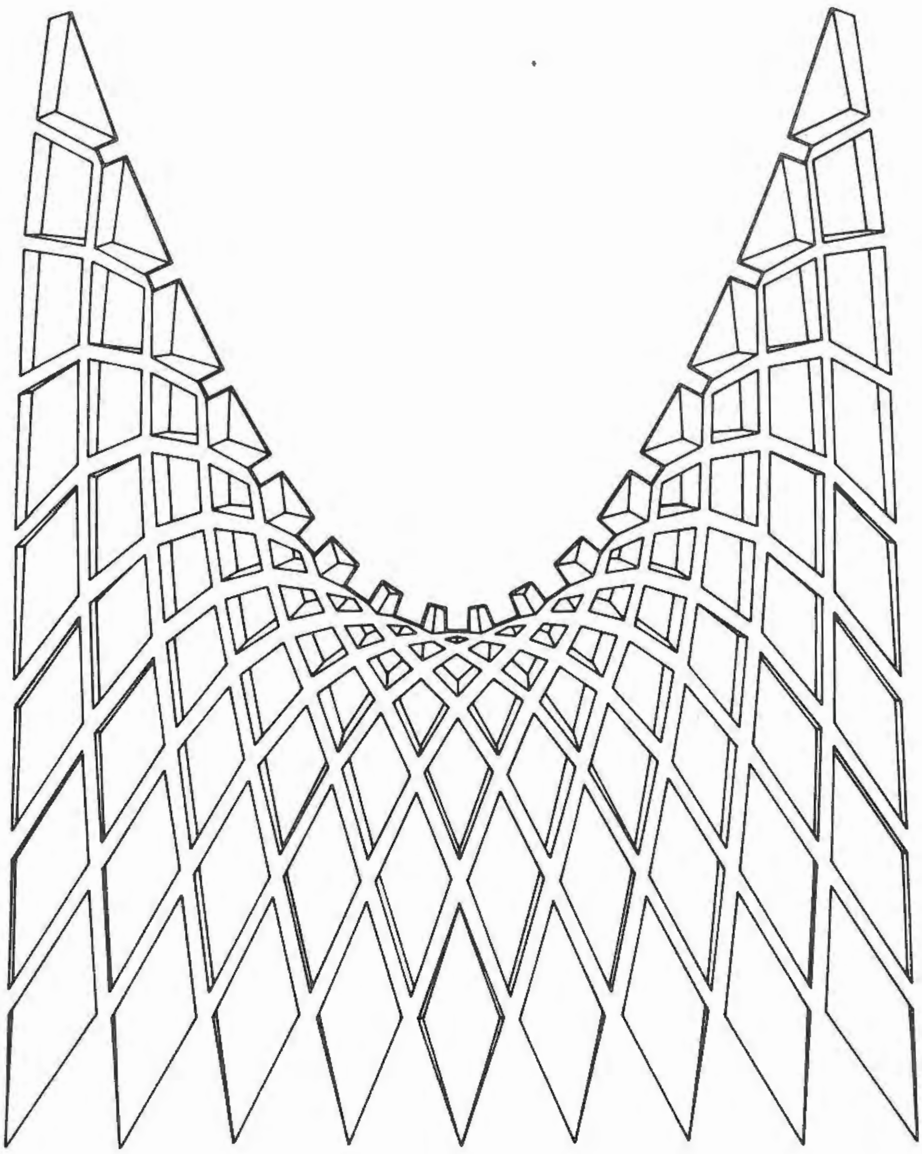


Fig. 6
East-west section through main arch showing stone rib pattern

Dome

Situated near the top of the monument is a dome about 10 m (33 ft.) high. The geometry of the dome is based very much on traditional Iranian architecture, the dome being the standard method in Iran of enclosing large spaces. They are commonly made of mud bricks and lined internally with coloured ceramics.

The dome in the monument is a multi-facet surface made up of triangular planes, of which there is considerable repetition. The layout of these elements and a section are shown in Fig. 7.

As can be seen from the plan, the geometry of the dome is governed very much by the external wall profile of the monument at the level at which it occurs.

The triangular elements will be of precast white bush-hammered concrete with ceramics and coloured glass lights as decoration.

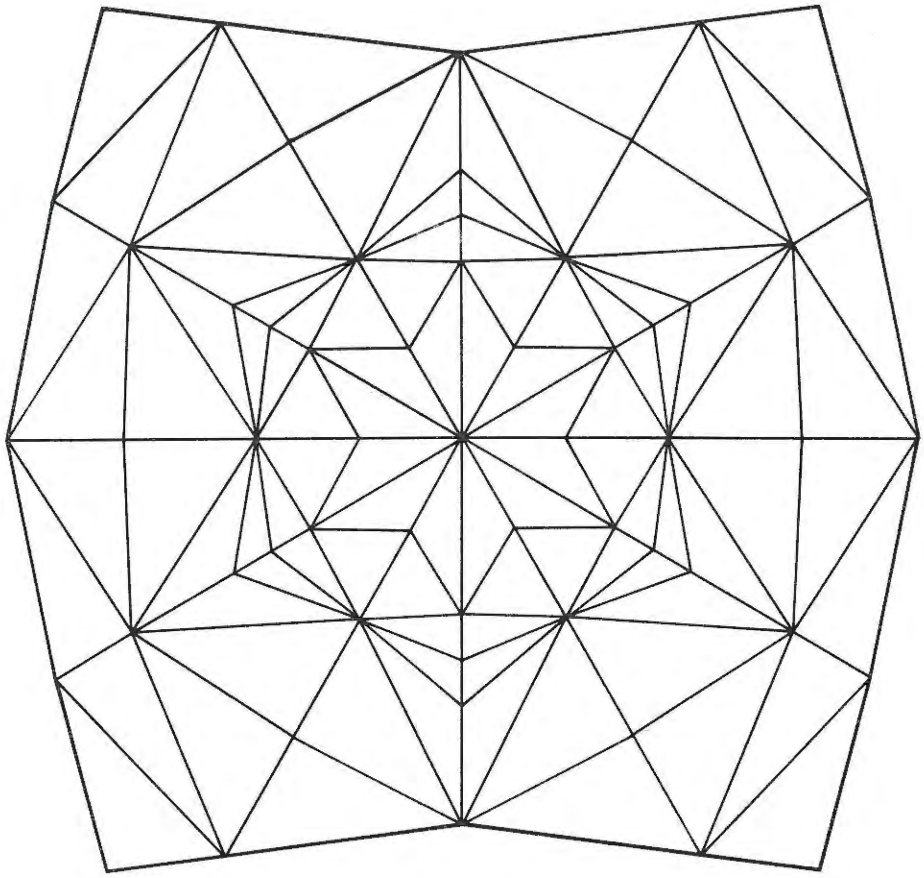


Fig. 7a above right
Plan of the dome

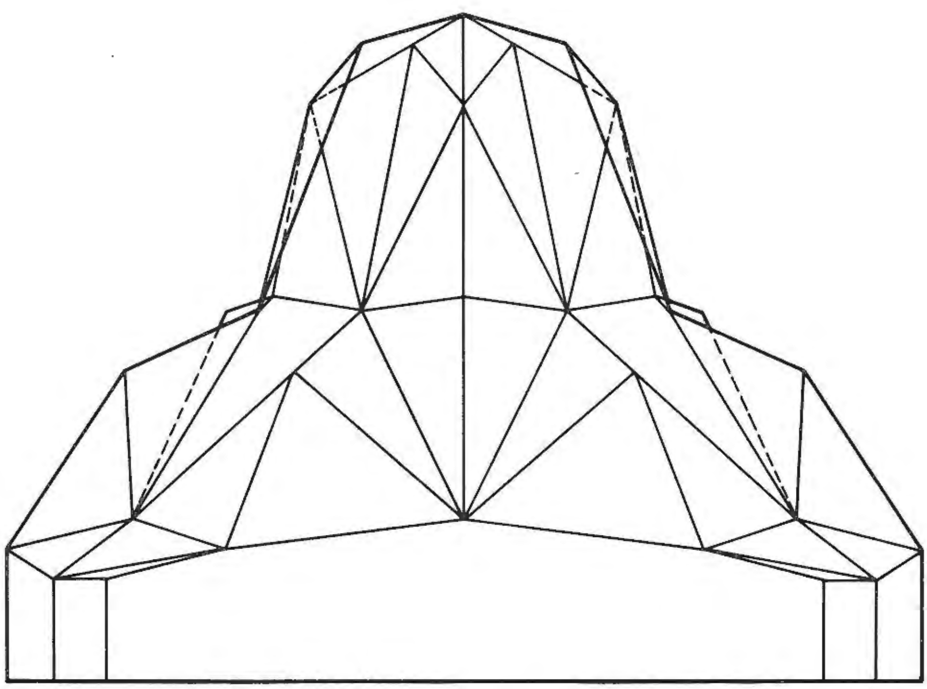


Fig. 7b below right
Section through dome

Floors

The geometry of the major floors which occur above level 21 m (69 ft.) is a function of the internal profile of the monument at their respective levels.

Floor at 23.4 m (77 ft.)

A section through the monument at this level includes a profile of the main arch, presenting an internal space in the shape of an egg timer (Fig. 8). The floor at this level spans by means of ribs from wall to wall, with a support on the crown of the arch. The ribs have profiles similar to that of the main arch section at this level. The internal profile at level 23.4 m (77 ft.) was obtained from the external one, and a best fit second order curve found to represent this. The rib profiles were then obtained at increments along their length to give the profiles shown in Fig. 8.

Floor at 33 m (108 ft.)

The main arch surface ends below this level and the section at level 33 m (108 ft.) is typical of the upper levels of the monument. The geometry of the ribs of this floor is based on the boundary wall profile at soffit level. The rib profiles are set out in a floral pattern from circular arcs as shown in Fig. 9a. The ribs corresponding to this setting-out are shown in Fig. 9b.

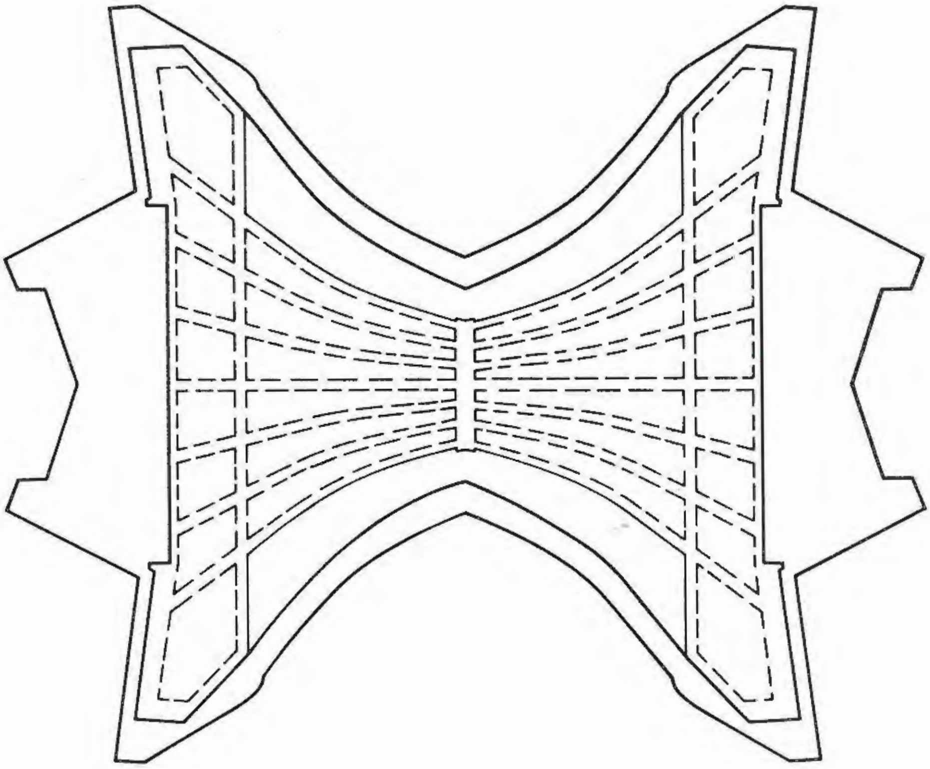


Fig. 8
Plan of the floor at 23.4 m

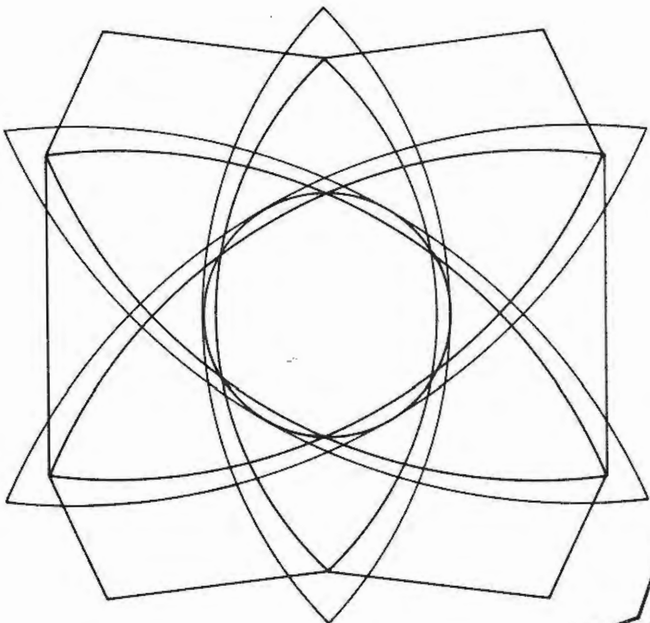


Fig. 9a above
Plan showing the geometry of the rib pattern of the floor at 33 m

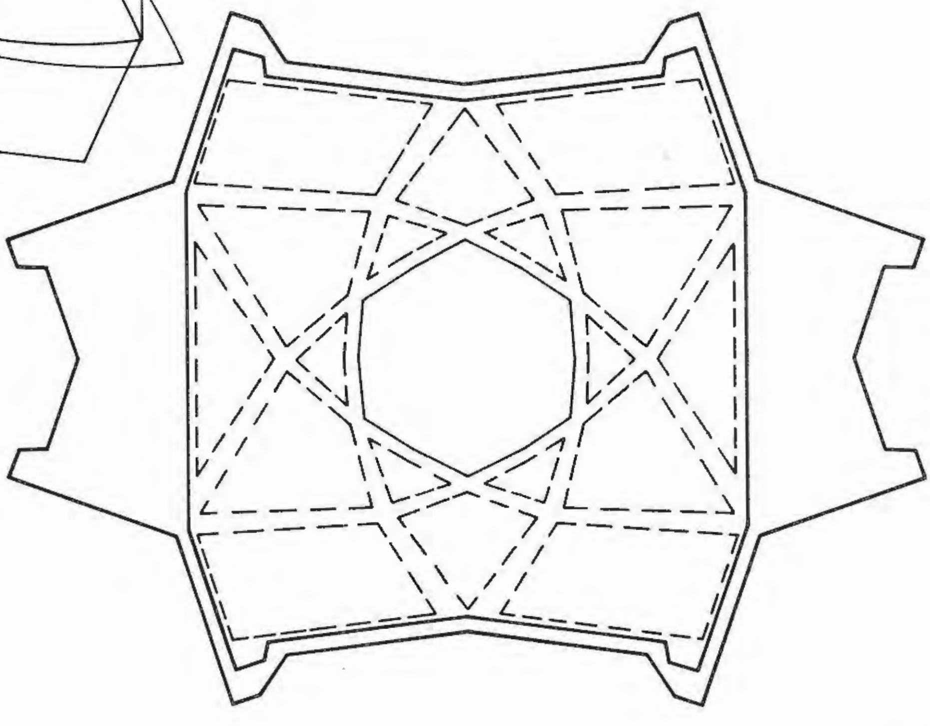


Fig. 9b right
Plan of the floor at 33 m

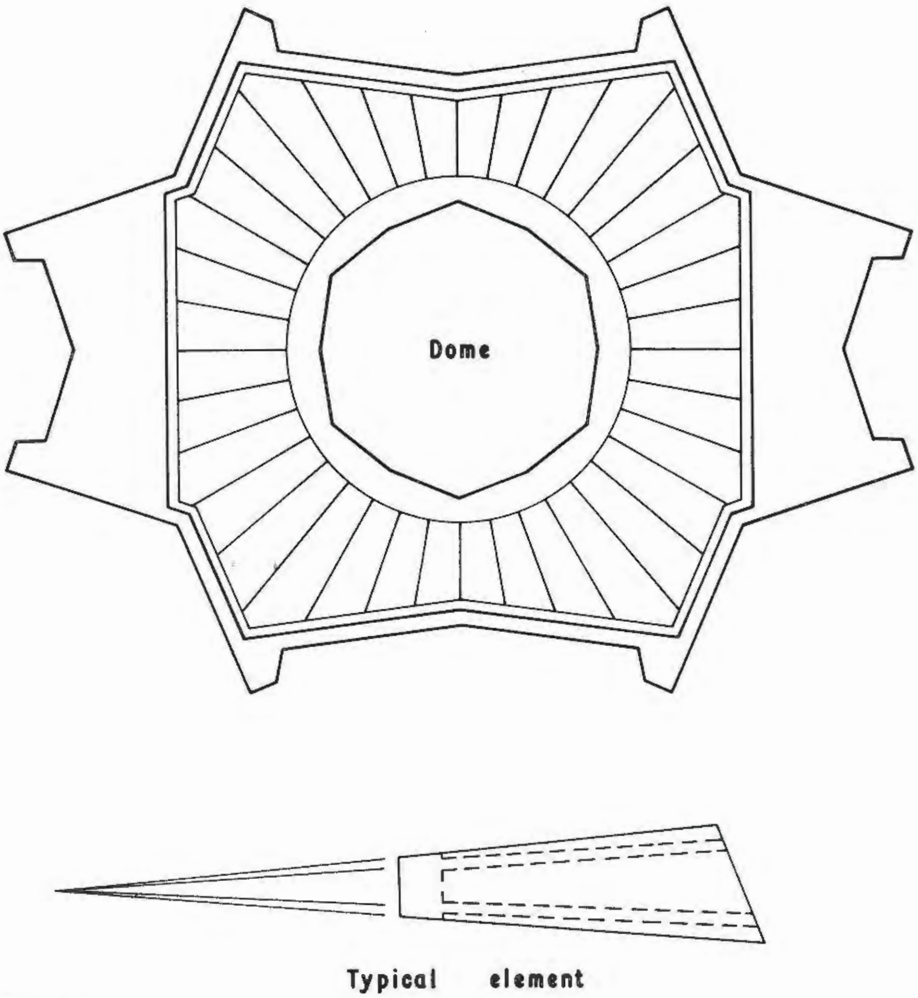


Fig. 10
Plan of the floor at 39.5 m

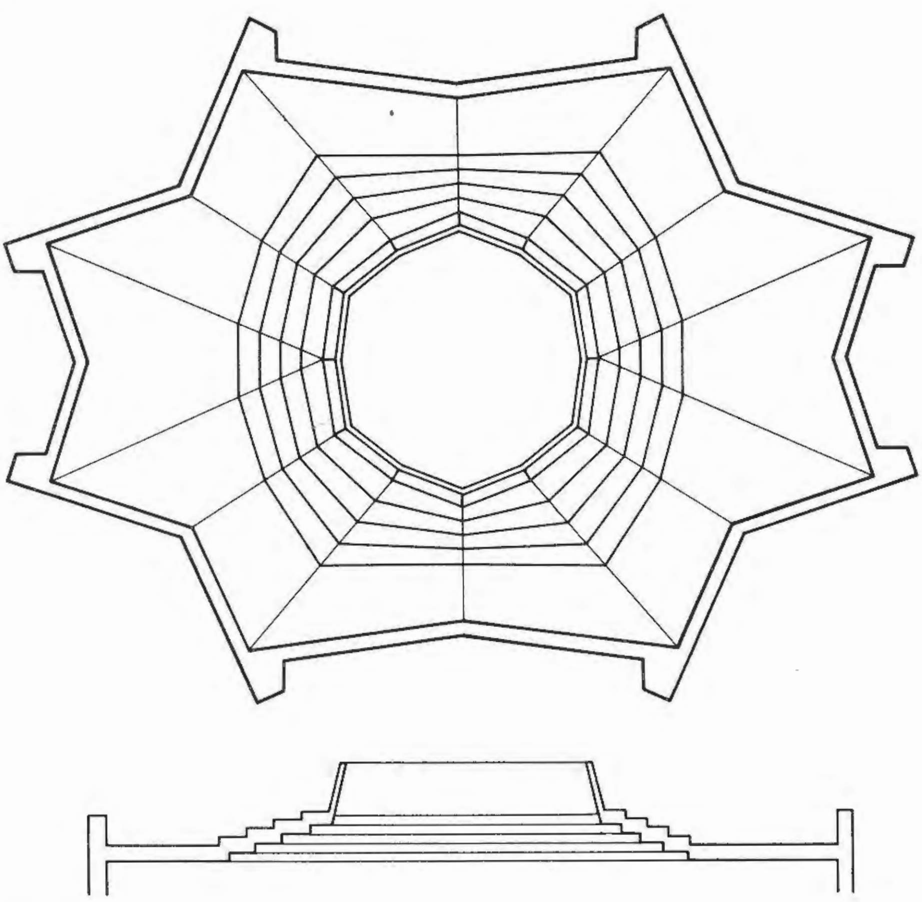


Fig. 11
Plan and section of the floor at 43.5 m

Floor at 39.5 m (130 ft.)

This floor is bounded on the outside by a typical external wall profile and on the inside by the dome, and spans between the two. The soffit of the floor is not visible and a sophisticated expression of the rib geometry was not required for architectural reasons. The geometry chosen, for purely structural reasons, gives a floor structure of radial ribs, enabling economical precasting of 36 elements to be achieved while taking account of the dome boundary and the varying perimeter line, Fig. 10.

Floor at 43.5 m (143 ft.)

This floor comprises two conical surfaces, resting on a flat boundary slab (Fig. 11). The inner cone forms a parapet to the edge of the floor, leaving a gap between itself and the dome to allow light to penetrate to lower floors. The geometry of this cone, which consists of twelve planes, is related to the geometry of the dome at this level. The base of the lower cone is set out from the external wall profile at this level as shown in Fig 11. In order not to introduce warped surfaces, both the top and the soffit of this part of the floor are stepped.

Structures below ground level

In order to remove the need for ground level door openings which would interfere with the surface of the monument, the internal space is entered by means of an entrance tunnel and a basement. Both of these are of reinforced concrete.

Basement

The basement occupies a volume about 21 m (69 ft.) x 16 m (52 ft.) x 5 m (16 ft.) high, occurring between the buttress legs as shown in Fig. 12 and will be used for display purposes. The roof has to be able to support loads caused by heavy vehicles which may be required to pass under the arch on festival occasions, and so the structural form used has to be efficient. To combine this requirement with the need to provide an interesting space architecturally, a two pinned portal structure of varying cross section spanning in the shorter direction, is used. This produces the internal folded effect shown in Fig. 13.

Entrance tunnel

The basement is entered by means of an underground entrance tunnel which begins about 30 m (98 ft.) from the basement with steps down from ground level to underground cloakrooms and a ticket office. The tunnel itself also has a folded form but this has no structural significance. The form is shown in plan and section in Fig. 14.

Conclusion

This article has concentrated on the development of the geometrical forms used in the monument. The further development of the material and structural forms is another story.

Ove Arup & Partners' contribution to the geometrical form has been described in some detail, but it must be pointed out that the inspiration for all the forms used came from the architect, Hossein Amanat of Teheran. His design was selected from a national competition for a monument for this important celebration. It was because of the architect's tenacity in keeping, and improving where possible, all his original ideas that the problems described here had to be faced and solved.

The contract has been let to an Iranian firm, the MAAP Construction Company and the work up to ground level is now complete. The marble is being cut, and the task of construction of the superstructure has just begun. Robert Afia is our resident engineer for the project.

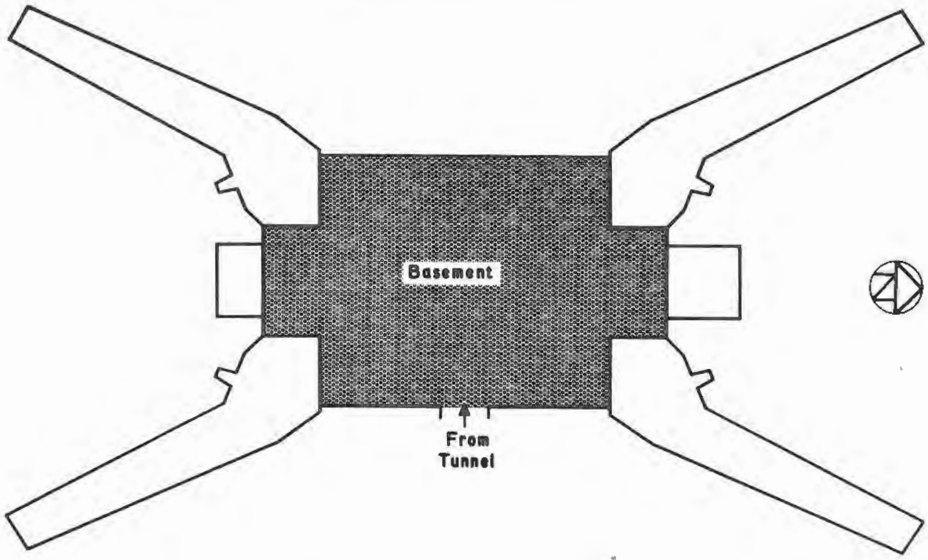
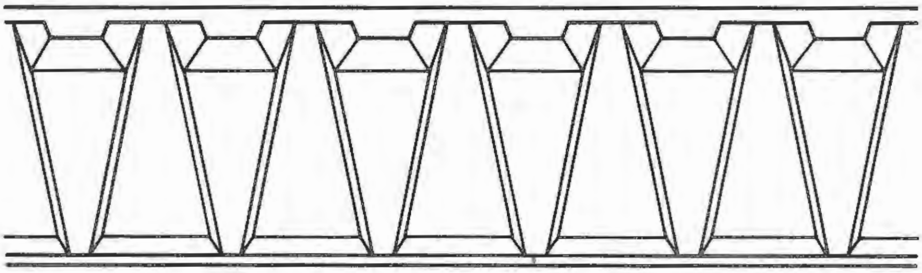
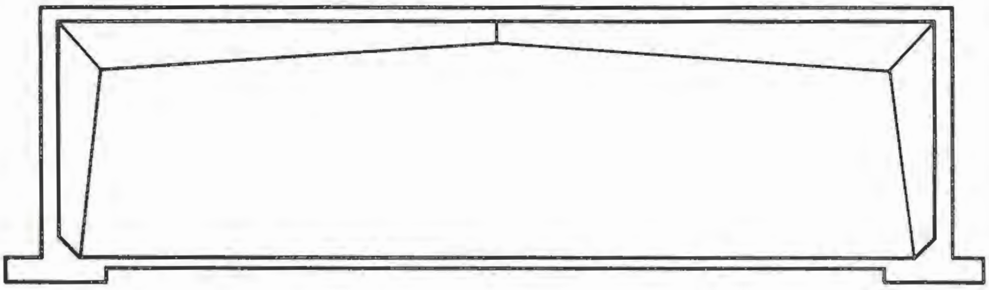


Fig. 12
Plan at −5 m showing the area occupied by the basement

Fig. 13 Basement

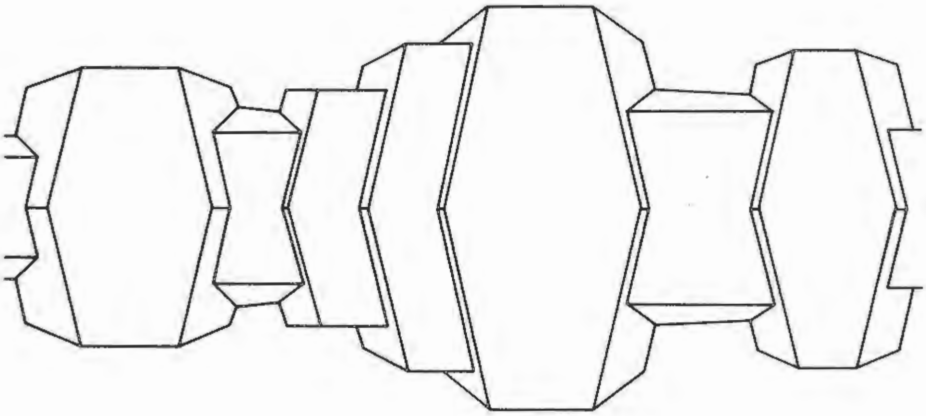


a Elevation of the basement wall

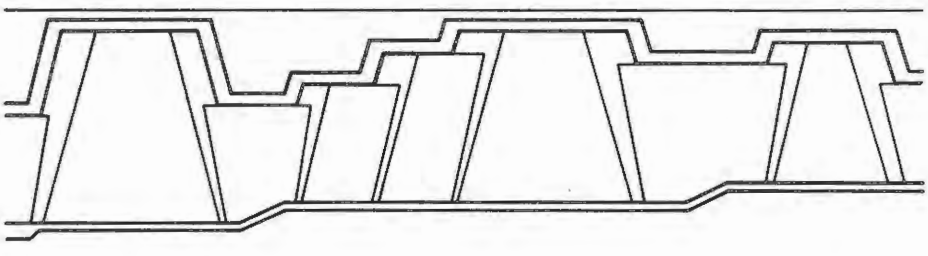


b Cross section of the basement

Fig. 14 Entrance Tunnel



a Plan of the entrance tunnel



b Longitudinal section through the entrance tunnel