Acceptance Charts

Summary

The Acceptance Charts procedure creates control charts with modified control limits based on both the standard deviation of the process and on specification limits for the variable being plotted. These charts may be used to monitor processes with a very high Cpk, where it not necessary to achieve a perfectly stable mean. In such a chart, the control limits would be placed far enough inside the specification limits to insure that a signal is generated whenever the process comes too close to the specs, but otherwise the process would be allowed to vary as it pleases around the long-term process mean.

There are two Acceptance Chart menu selections, one designed for data collected in subgroups and the other for individuals. The procedure creates both a modified X or X-Bar chart and an R chart, S chart, or MR(2) chart. The charts may be constructed in either *Initial Study* (Phase 1) mode, where the current data determine the slope and spacing of the control limits, or in *Control to Standard* (Phase 2) mode, where the limits come from either a known standard or from prior data.

Sample StatFolio: acceptance chart.sgp

Sample Data:

The file *bottles.sgd* contains the measured bursting strength of n = 100 glass bottles, similar to a dataset contained in Montgomery (2005). Each row consists of a sample tested at 10 minute intervals. The table below shows a partial list of the data from that file:

strength	time
255	0:10
232	0:20
282	0:30
260	0:40
255	0:50
233	1:00
240	1:10
255	1:20
254	1:30
259	1:40
235	1:50
262	2:00

Bottles are required to have a bursting strength between 200 psi and 300 psi.

Data Input

There are two menu selections that create acceptance charts, one for individuals data and one for grouped data. In the case of grouped data, the original observations may be entered, or subgroup statistics may be entered instead.

Case #1: Individuals

The data to be analyzed consist of a single numeric column containing n observations. The data are assumed to have been taken one at a time.

Acceptance Individuals Chart		X
strength time	Observations: Image: Strength (Date/Time/Labels:) Image: Image	
🧮 Sort column names		
OK Cancel	Delete Transform Help	

- **Observations:** numeric column containing the data to be analyzed.
- Date/Time/Labels: optional labels for each observation.
- LSL, Nominal, USL: optional lower specification limit, nominal (target) value, and upper specification limit. Enter at least one specification limit.
- **Select:** subset selection.

<u>Case #2: Grouped Data – Original Observations</u>

The data to be analyzed consist of one or more numeric columns. The data are assumed to have been taken in groups, in sequential order by rows.

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Acceptance Chart	×
strength time	Data © Observations Strength
	Subgroup Statistics Means: Ranges: Sizes:
	Date/Time/Labels or Size:
Sort column names	Delete Transform Help

- **Observations:** one or more numeric columns. If more than one column is entered, each row of the file is assumed to represent a subgroup with subgroup size *m* equal to the number of columns entered. If only one column is entered, then the *Date/Time/Labels or Size* field is used to form the groups.
- **Date/Time/Labels or Size**: If each set of *m* rows represents a group, enter the single value *m*. For example, entering a 5 implies that the data in rows 1-5 form the first group, rows 6-10 form the second group, and so on. If the subgroup sizes are not equal, enter the name of an additional numeric or non-numeric column containing group identifiers. The program will scan this column and place sequential rows with identical codes into the same group.
- LSL, Nominal, USL: optional lower specification limit, nominal (target) value, and upper specification limit. Enter at least one specification limit.
- **Select:** subset selection.

Case #3: Grouped Data – Subgroup Statistics

In this case, the statistics for each subgroup have been computed elsewhere and entered into the datasheet, as in the table below:

Sample	Means	Ranges	Sizes
1	256.8	50	5
2	248.2	26	5
3	253.4	37	5
4	244.6	29	5
5	254.4	13	5

Acceptance Chart	
MEANS SIZES RANGES	Data Observations
	 Subgroup Statistics Means: MEANS Ranges: RANGES Sizes: SIZES
	(Date/Time/Labels:) (LSL:) (Nominal:) (USL): (Select:)
Sort column names	Delete Transform Help

• **Subgroup Statistics**: the names of the column containing the subgroup means, subgroup ranges, and subgroup sizes.

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- Date/Time/Labels: optional labels for each subgroup.
- LSL, Nominal, USL: optional lower specification limit, nominal (target) value, and upper specification limit. Enter at least one specification limit.
- Select: subset selection.

Acceptance Chart

The Acceptance chart plots the observations or subgroup means together with both control limits and specification limits.



The centerline is located at the weighted average of the observations or subgroup means:

$$= \frac{\sum_{j=1}^{m} n_j \overline{x}_j}{\sum_{j=1}^{m} n_j}$$
(1)

The control limits are positioned with respect to the specification limits in either of two ways, as indicated on the *Analysis Options* dialog box:

• **Sigma Multiple Method:** This method begins by finding the most extreme values of the process mean that would result in no more than a specified fraction δ of items outside the specification limits. It then positions control limits at the usual *k*-sigma distance around those extreme means instead of around the centerline. The resulting control limits are:

$$UCL = (USL - Z_{\delta}\sigma) + k\sigma / \sqrt{\overline{n}}$$
⁽²⁾

$$LCL = (LSL + Z_{\delta}\sigma) - k\sigma / \sqrt{\overline{n}}$$
(3)

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where Z_{δ} is the point on a standard normal distribution leaving a fraction δ in the upper tail of the curve and k is the multiple (usually equal to 3) specified on the *Control Charts* tab of the *Preferences* dialog box, accessible from the *Edit* menu. The fraction δ is specified in the *X-bar Control Limits* field on the *Analysis Options* dialog box.

• Beta Risk Method: This method begins by specifying β , the probability of not signaling an out-of-control situation when the process mean is such that the proportion of items beyond the specification limits equals δ . It then positions the control limits at

$$UCL = (USL - Z_{\delta}\sigma) - Z_{\beta}\sigma / \sqrt{n}$$
⁽⁴⁾

$$LCL = (LSL + Z_{\delta}\sigma) + Z_{\beta}\sigma / \sqrt{n}$$
(5)

Note that this second method places the control limits inside the extreme values for the process mean rather than outside as is the case when the *Sigma Multiple* method is used.

The method for estimating the process sigma depends on the settings on the *Control Charts* tab of the *Preferences* dialog box, as discussed in the *Analysis Summary* section below.

Analysis Summary

This pane summarizes the control charts.

Acceptance Individuals Control Chart - strength	
Number of observations $= 100$	
0 observations excluded	
Maximum fraction defective: 0.0001 Sigma multiple: 3.0 Upper spec. limit: 300.0 Lower spec. limit: 200.0	
Distribution: Normal	
Transformation: none	
X Chart	
Period #1-100	
UCL: +3.0 sigma 292.81	
Centerline 250.0	
LCL: -3.0 sigma 207.19	
0 beyond limits	
•	
MR(2) Chart	
Period #1-100	
UCL: +3.0 sigma 36.855	
Centerline 11.28	
LCL: -3.0 sigma 0.0	
1 beyond limits	
Estimates	
Period #1-100 Standard	
Process mean 254.64 250.0	
Process sigma 10.1637 10.0	
Mean MR(2) 11.4646	
Sigma estimated from average moving range	

Included in the table are:

- **Subgroup Information**: the number of observations or subgroups and the average subgroup size.
- Maximum Fraction Defective and Sigma Multiple or Beta Risk: the parameters δ and k or δ and β used to define the control limits.
- **Specification Limits**: the specification limits for the variable of interest.
- **Distribution:** the assumed distribution for the data. By default, the data are assumed to follow a normal distribution. However, one of 26 other distributions may be selected using *Analysis Options*.
- **Transformation:** any transformation that has been applied to the data. Using *Analysis Options*, you may elect to transform the data using either a common transformation such as a square root or optimize the transformation using the Box-Cox method.

- X or X-Bar Chart: a summary of the centerline and control limits for the acceptance chart, together with a count of any points beyond the control limits.
- MR(2)/R/S Chart: a summary of the centerline and control limits for the dispersion chart.
- Estimates: estimates of the process mean and the process standard deviation σ . The method for estimating the process sigma depends on the settings on the *Analysis Options* dialog box, described below.
- Mean residual MR(2), Average Range, or Average S: the average of the values plotted on the dispersion chart.

Analysis Options

Acceptance Individuals	s Chart Options	
Type of Study C Initial Study C Control to Standard	Specify Sigma Multiple Beta Risk	Control to Standard Mean: 250.0 Std. Dev.: 10.0
X Control Limits Fraction Nonconforming: 0.0001 Sigma multiple: 3.0	MR(2) Control Limits Upper: 3.0 Sigma Lower: -3.0 Sigma	
OK Canc	el Exclude	Transform Help

- **Type of Study:** determines how the control limits are set. For an *Initial Study* (Phase 1) chart, the limits are estimated from the current data. For a *Control to Standard* (Phase 2) chart, the control limits are determined from the information in the *Control to Standard* section of the dialog box.
- **Specify**: specifies the method used to position the control limits. *Sigma Multiple* places the X chart control limits outside the most extreme means that correspond to the specified *Fraction Nonconforming*. *Beta Risk* positions the limits so as to miss a shift in the process mean corresponding to the *Fraction Nonconforming* with probability equal to *Beta Risk*.
- **Control to Standard:** To perform a Phase 2 analysis, select *Control to Standard* for the *Type of Study* and then enter the established standard process mean and sigma (or other parameters if not assuming a normal distribution).

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- X or X-Bar Control Limits: The control limits are specified in relationship to the maximum allowable fraction of nonconforming items δ . If the *Specify* field is set to *Sigma Multiple*, then the value of *k* is entered. If the *Specify* field is set to *Beta Risk*, then the value of β is entered.
- MR(2), R or S Control Limits: specify the multiple *k* to use in determining the upper and lower control limits on the MR(2), R, or S chart. To suppress a limit completely, enter 0.
- Exclude button: Use this button to exclude specific subgroups from the calculations.
- Transform Button: Use this button to specify a transformation or non-normal distribution.

For a discussion of the *Exclude* and *Transform* features, see the documentation for *Individuals Control Charts*.

MR(2)/R/S Chart

A second chart is also included to monitor the process variability.



MR(2) Chart for strength

For individuals data, the chart displayed is an MR(2) chart, described in the *Individuals Control Charts* documentation. For grouped data, either an R chart or an S chart is plotted, depending on the setting on the *Control Charts* tab of the *Preferences* dialog box:

 Preferred Dispersion Chart—
Range
🔿 Sigma

These charts are described in the X-Bar and R Charts and the X-Bar and S Charts documents.

Pane Options	
Control Chart Options	
Outer Warning Limits	ОК
Sigma <mark>(</mark> 2.0	Cancel
Inner Warning Limits	Help
Moving Average Order 9 Exponentially Weighted Moving Ave	erage
Lambda: 0.2 Decimal Places for Limits:	
2	
Mark Runs Rules Violations	
Color Zones	
Display specification limits	

- **Outer Warning Limits**: check this box to add warning limits at the specified multiple of sigma, usually at 2 sigma.
- **Inner Warning Limits**: check this box to add warning limits at the specified multiple of sigma, usually at 1 sigma.
- Moving Average: check this box to add a moving average smoother to the chart. In addition to the subgroup means, the average of the most recent q points will also be displayed, where q is the *order* of the moving average. The default value q = 9 since the 1-sigma inner warning limits for the original subgroup means are equivalent to the 3-sigma control limits for that order moving average.
- Exponentially Weighted Moving Average: check this box to add an EWMA smoother to the chart. In addition to the subgroup means, an exponentially weighted moving average of the subgroup means will also be displayed, where λ is the smoothing parameter of the EWMA. The default value λ = 0.2 since the 1-sigma inner warning limits for the original subgroup means are equivalent to the 3-sigma control limits for that EWMA.
- **Decimal Places for Limits:** the number of decimal places used to display the control limits.
- Mark Runs Rules Violations: flags with a special point symbol any unusual sequences or runs. The runs rules applied by default are specified on the *Runs Tests* tab of the *Preferences* dialog box.
- Color Zones: check this box to display green, yellow and red zones.

• **Display Specification Limits**: whether to add horizontal lines to the chart displaying the location of the specification limits

Acceptance Chart Report

This pane tabulates the values plotted on the control charts:

Acceptance In	ndividual	s Chart Report	
Observations l	Beyond Li	imits	
X = Excluded	* =	Beyond Limits	
Observation	X	MR(2)	
3	282.0	* 50.0	

Out-of-control points are indicated by an asterisk. Points excluded from the calculations are indicated by an X.

Pane Options

Acceptance Individuals	s Chart 🗴
Display	OK
C All Observations	Cancel
Observations Beyond Limits	
C X Beyond Limits	
C MR(2) Beyond Limits	
C Excluded Observations	

• **Display:** specify the observations or subgroups to display in the report.

Capability Indices

The *Capability Indices* pane displays the values of selected indices that measure how well the data conform to the specification limits.

Capability	Indices for st	trength
Specification	Specifications	
USL = 3	USL = 300.0	
Nom = 250.0		
LSL = 2	200.0	
	-	
	Short-Term	Long-Term
	Capability	Performance
Sigma	10.1637	10.6823
Cp/Pp	1.63982	1.56021
CR/PR	60.9822	64.0938
Cpk/Ppk	1.48765	1.41543
Κ		0.0928
Based on 6	sigma limits.	Short-term sigma

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The indices displayed by default depend on the settings of the *Capability* tab on the *Preferences* dialog box. A detailed discussion of these indices may be found in the documentation for *Process Capability* (*Variables*).

Pane	Options

Capability Indices Options		
Display	_	OK
Cp/Pp	Cpk/Ppk (lower)	Cancel
CR/PR	I_ CCpk	
	Cpm	Help
🗖 Zusl	I K	
🗖 ZIsi	🔲 % Beyond Specs	
🗖 Zmin	Defects per Million	
🔽 Cpk/Ppk	🔲 Sigma quality level	
Cpk/Ppk (upper)	🔽 1.5 sigma shift	
L		

• **Display**: select the indices to be displayed.

OC Curve

The *OC* (*Operating Characteristic*) *Curve* is designed to illustrate the properties of a Phase 2 control chart.



The chart displays the probability that a plotted value will be within the control limits on the X or X-bar chart, as a function of the true process mean. For example, if the process mean were to shift to 190 or 310, the probability that the next point on the X chart will be within the control limits is approximately 5%.

ARL Curve

The ARL Curve is another way to view the performance of a Phase 2 chart.



The ARL curve plots the average run length (average number of values plotted up to and including the first point beyond the control limits) as a function of the true process mean. Assuming that the process mean suddenly shifts to a new value, the chart shows how long it takes on average until an out-of-control signal is generated. Note how large the ARL has become around the process mean of 250, implying that the relaxed control limits will almost never result in a false alarm. At a shift to $\mu = 270$, the ARL is approximately 90.

Save Results

The following results can be saved to the datasheet, depending on whether the data are individuals or grouped:

- 1. Observations or averages the values plotted on the X or X-bar chart.
- 2. Ranges, sigmas, or moving ranges the values plotted on the dispersion chart.
- 3. *Sizes* the subgroup sizes.
- 4. *Labels* the subgroup labels.
- 5. Process Mean the estimated process mean.
- 6. Process Sigma the estimated process standard deviation.
- 7. *Included Observations* a column of 0's and 1's for excluded and included observations, respectively. This column can then be used in the *Select* field on other data input dialog boxes.