Acceptance Sampling (Attributes)

When lots containing a relatively large number of items require inspection, acceptance sampling plans can provide reasonable protection against shipping or receiving an unacceptable fraction of non-conforming items without inspecting 100% of the lot. The **Acceptance Sampling (Attributes)** procedure generates acceptance sampling plans for situations when items cannot be measured, only classified as conforming or non-conforming. In such plans, a sample of size n is drawn from a lot of N items and the lot is accepted if the number of non-conforming items in the lot is less than or equal to c.

STATGRAPHICS generates three types of acceptance sampling plans:

- **OC Plans** - plans that control the alpha and beta risks, i.e., the probability of accepting a bad lot and the probability of rejecting a good lot. For such a plan, "good" and "bad" must be well-defined.

- **AOQL Plans** - plans that minimize the average outgoing quality limit, i.e., the maximum fraction of non-conforming items accepted on average. Such a plan requires 100% inspection and rectification of all rejected lots.

- **LTPD Plans** - plans that minimize total inspection while controlling the risk of rejecting a bad lot, where "bad" must again be well-defined. Such a plan also requires 100% inspection and rectification of all rejected lots.

Sample StatFolio: *acceptattributes.sgp*

Sample data:
None.

Acceptance Sampling Plans for Attributes

In an attributes acceptance sampling plan, n items are selected from a lot of size N. Each item is inspected and the number of unacceptable or "non-conforming" items is recorded. The lot is then accepted or rejected according to the following rules:

- If the number of non-conforming items in the sample is less than or equal to c, the lot is accepted and no further inspection is performed.

- If the number of non-conforming items in the sample is greater than c, one of two actions is taken:

    1. If the lot is *rectifiable*, then all remaining items in the lot are inspected. Any non-conforming items are replaced by conforming ones to yield N conforming items.

    2. If the lot is not rectifiable, the lot is rejected without further inspection and returned to the producer.

In the discussion that follows, several terms are important:

1. AQL = *acceptable quality level*, defined as the poorest level of quality that the consumer finds acceptable *on average*.

2. LTPD = *lot tolerance percent defective*, defined as the poorest level of quality that the consumer is willing to tolerate in any given lot.

3. OC(θ) = *operating characteristic*, defined as the probability that a sampling plan will accept a lot when the lot contains a fraction θ of non-conforming items.

4. AOQL = *average outgoing quality limit*, defined as the maximum percent of defective items accepted by a given sampling plan assuming that all rejected lots are subjected to 100% inspection and all non-conforming items in such lots are replaced with conforming items.

Data Input

The data input dialog box defines the desired features of the sampling plan.

- **Action** - defines the type of action desired:

  1. *Create OC Plan* - creates a plan that controls the probability of accepting a lot at both the AQL and the LTPD. Rejected lots are sent back to the producer without being rectified.

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2. *Create AOQL Plan* - creates a plan that minimizes the total number of inspected units at a selected fraction of non-conforming items while insuring that the maximum percentage of non-conforming items accepted does not exceed a specified value. Rejected lots are subjected to 100% inspection and rectified.

3. *Create LTPD Plan* - creates a plan that minimizes the total number of inspected units at a selected fraction of non-conforming items while controlling the probability of accepting a lot at the LTPD. Rejected lots are subjected to 100% inspection and rectified.

4. *Analyze Existing Plan* - computes the operating characteristic curve for a sampling plan specified by the user.

- **Quality Levels** - defines the percentage defective in "good" and "bad" lots:

  1. AQL (*acceptable quality level*) - the poorest level of quality that the consumer finds acceptable *on average*.

  2. LTPD (*lot tolerance percent defective*) - the poorest level of quality that the consumer is willing to tolerate in any given lot.

- **Lot Size** - the number of items N in the lot.

- **Desired Features** - characteristics desired of the sampling plan, depending upon the type of plan selected:

Type of Plan	Feature #1	Feature #2
OC Plan	Producer's risk α - the probability of rejecting a lot with a percent defective equal to the AQL.	Consumer's risk β - the probability of accepting a lot with a percent defective equal to the LTPD.
AOQL Plan	Average percent defective at which inspection will be minimized.	The AOQL or maximum % of non-conforming items accepted after rectification
LTPD Plan	Average percent defective at which inspection will be minimized.	Consumer's risk β - the probability of accepting a lot with a percent defective equal to the LTPD.

- **Current Plan** - If *Analyze Existing Plan* is selected, the sample size n and acceptance number c of the plan to be analyzed.

Analysis Summary

The *Analysis Summary* displays the generated plan:

Acceptance Sampling for Attributes
Lot size: 10000

Desired features
Producer's risk (alpha): 5.0%
Consumer's risk (beta): 10.0%

Generated plan
Sample size (n) = 1948
Acceptance number (c) = 14

Plan attributes
Acceptable quality level (AQL): 0.5%
Producer's risk (alpha) = 4.94005%
Lot tolerance percent defective (LTPD): 1.0%
Consumer's risk (beta) = 9.98779%

Average Outgoing Quality Limit (AOQL) = 0.399899% at 0.573387% defective
Average Total Inspection (ATI) =
  2345.77 units per lot at the AQL
  3025.67 units per lot at the AOQL
  9195.78 units per lot at the LTPD

There are several important sections of the output:

- **Desired Features** - summarizes the user-specified features upon which the plan is based. In the above example, the plan was constructed so as to have a producer's risk of no more than 5% and a consumer's risk of no more than 10%.

- **Generated plan** - shows the smallest sampling plan that has the desired features. In the example, $n = 1,948$ items are to be sampled from the lot of $N = 10,000$ and the lot accepted if no more than $c = 14$ items are non-conforming.

- **Plan Attributes** - exact results for the generated plan. This includes:

  Producer's risk at the AQL - probability of rejecting a "good" lot.

  Consumer's risk at the LTPD - probability of accepting a "bad" lot.

  Average Outgoing Quality Limit - Assuming that rejected lots are 100% inspected and that any non-conforming items are replaced by good items, this is the maximum fraction of non-conforming items that are accepted.

  Average Total Inspection - assuming that rejected lots are 100% inspected and that any non-conforming items are replaced by good items, this is the average percentage of items in a lot that will be inspected.

For the current plan, the alpha and beta risks are slightly smaller than requested, since the sample size must be an integer. If rejected lots are rectified, the maximum average percent of non-conforming items that will be accepted is approximately 0.4% which would occur if the

incoming lots contain 0.573% non-conforming items. For lots containing exactly 0.5% defective items ("good" lots), approximately 2,346 items out of each lot of 10,000 will be inspected on average.

OC Curve

The *OC Curve* shows the probability that a lot with a percentage of non-conforming items equal to 100θ% will be rejected by the current sampling plan:

Operating Characteristic (OC) Curve
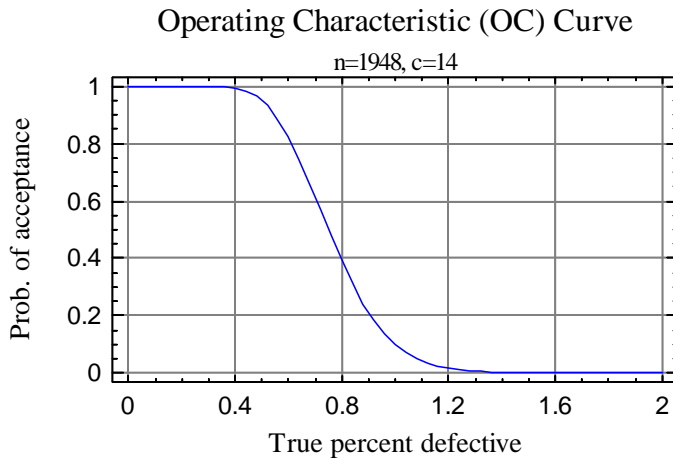
For an *OC Plan* such as was generated for the current example, the OC Curve passes through *(1-α)* at the AQL and β at the LTPD.

AOQ Curve

The AOQ curve shows the average outgoing quality of lots with a percentage of non-conforming items equal to 100θ% when subjected to the current sampling plan:

Average Outgoing Quality (AOQ) Curve
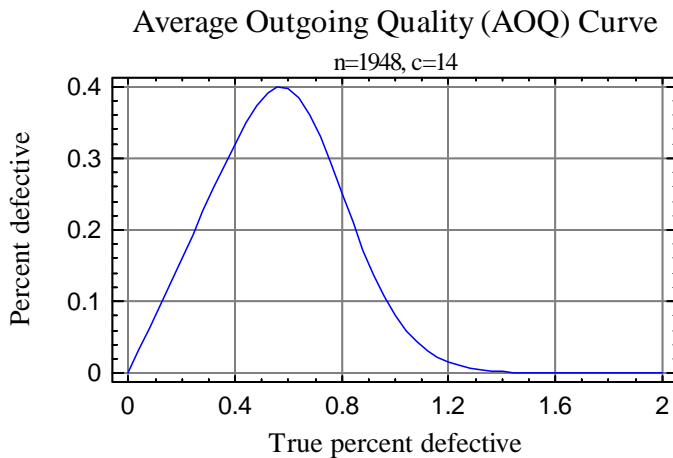
The curve peaks at the AOQL.

ATI Curve

The ATI curve shows the average number of items inspected for lots with a percentage of non-conforming items equal to $1000\theta\%$ when subjected to the current sampling plan:

Average Total Inspection (ATI) Curve
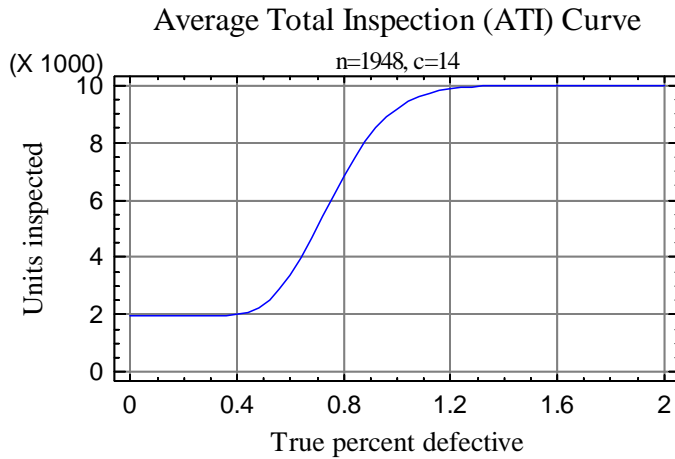
The ATI curve assumes that rejected lots are rectified.

Calculations

Acceptance Probability
The probability of accepting a lot containing a fraction non-conforming items equal to θ is computed from the hypergeometric distribution according to

$$P(accept|\theta) = \sum_{x=0}^{c} P_H(x|\theta, N) = \sum_{x=0}^{c} \frac{\binom{N\theta}{x}\binom{N(1-\theta)}{n-x}}{\binom{N}{n}} \tag{1}$$

Producer's Risk

$$\alpha = P(reject|AQL) = 1 - P(accept|AQL) \tag{2}$$

Consumer's Risk

$$\beta = P(accept|LTPD) \tag{3}$$

Average Outgoing Quality

$$AOQ(\theta) = \theta P(accept \mid \theta)\left(\frac{N-n}{N}\right) \tag{4}$$

Average Outgoing Quality Limit

$$AOQL = \max_{\theta}\left[\theta P(accept \mid \theta)\left(\frac{N-n}{N}\right)\right] \qquad (5)$$

Average Total Inspection

$$ATI(\theta) = n + \left(1 - P(accept|\theta)\right)(N-n) \qquad (6)$$

<u>Reference</u>

Montgomery, D. C. (2005). <u>Introduction to Statistical Quality Control, 5th edition</u>. New York: John Wiley and Sons.