

Capability Analysis (Variable Data)



Revised: 10/10/2017



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Summary

The **Capability Analysis** procedures for variable data are designed to compare a sample of measurements collected from a process to established specification limits for that variable. An estimate is derived of the percentage of items likely to be out of spec. Also calculated are a variety of capability indices that compare the observed performance to the specification limits. Methods are available for handling data from both normal and non-normal distributions.

Sample StatFolio: *capability.sgp*

Sample Data

The file *bottles.sgd* contains the measured bursting strength of $n = 100$ glass bottles, similar to a dataset contained in Montgomery (2005). Each row consists of a sample tested at 10 minute intervals. The table below shows a partial list of the data from that file:

<i>strength</i>	<i>time</i>
255	0:10
232	0:20
282	0:30
260	0:40
255	0:50
233	1:00
240	1:10
255	1:20

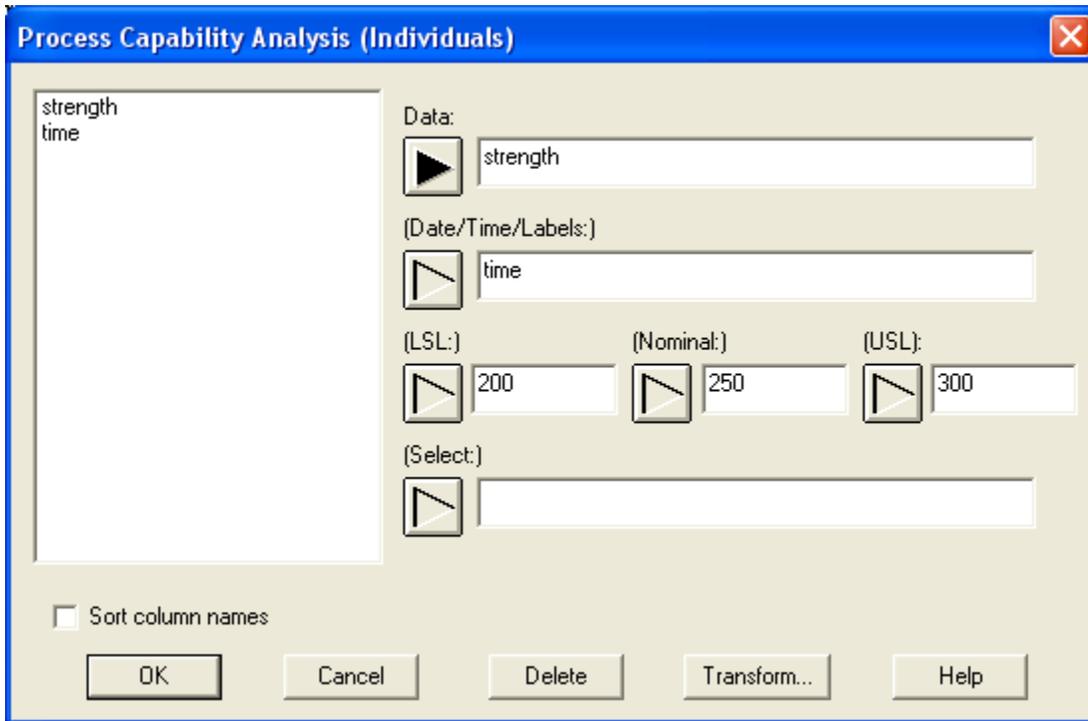
Bottles are required to have a bursting strength between 200 psi and 300 psi.

Data Input

There are two menu selections that perform a capability analysis for variables data, one for individuals data and one for grouped data.

Case #1: Individuals

The data to be analyzed consist of a single numeric column containing n observations. The data are assumed to have been taken one at a time.

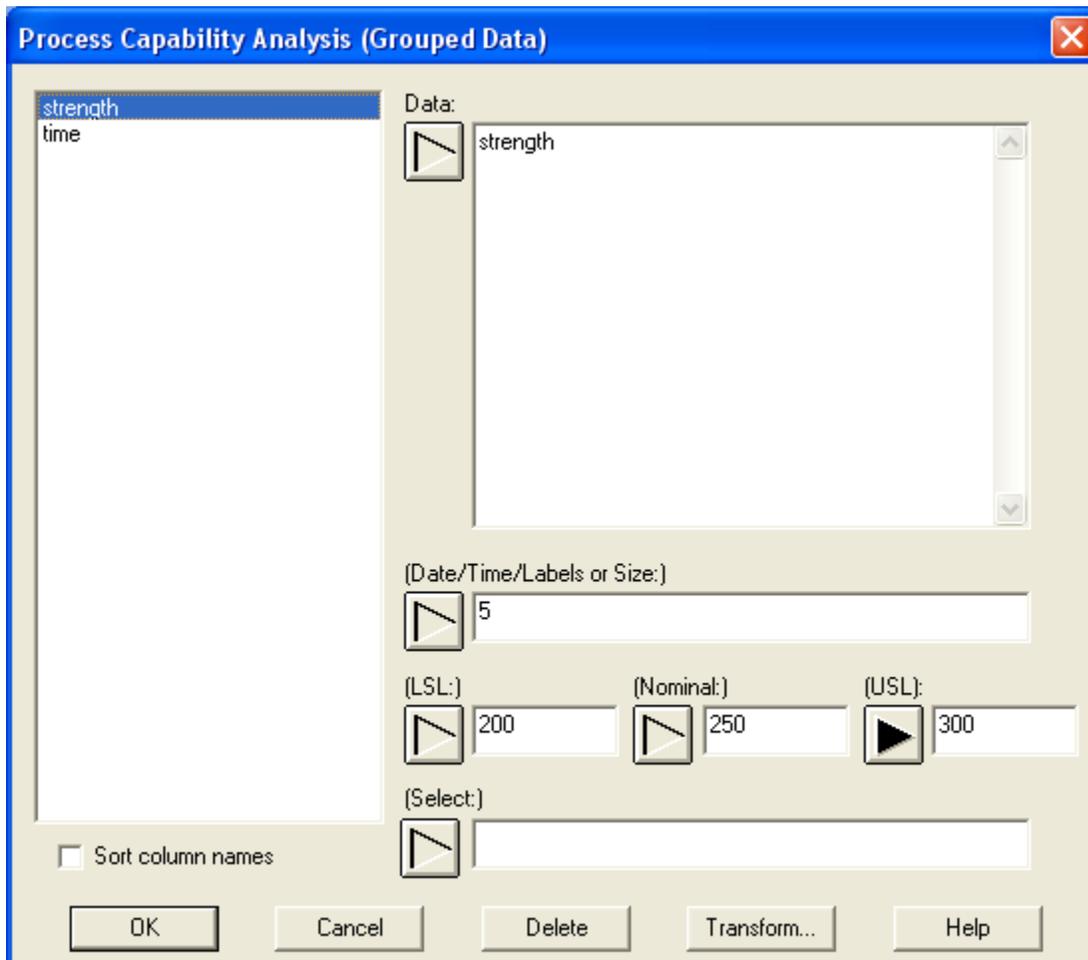


- **Data:** numeric column containing the data to be analyzed.
- **Date/Time/Labels:** optional labels for each observation.
- **USL:** upper specification limit, if any.
- **Nominal:** optional nominal or target value for the variable. If not supplied, certain capability indices will not be calculated.
- **LSL:** lower specification limit, if any.
- **Select:** subset selection.

At least one of the specification limits must be entered.

Case #2: Grouped Data

The data to be analyzed consist of one or more numeric columns. The data are assumed to have been taken in groups, in sequential order by rows.

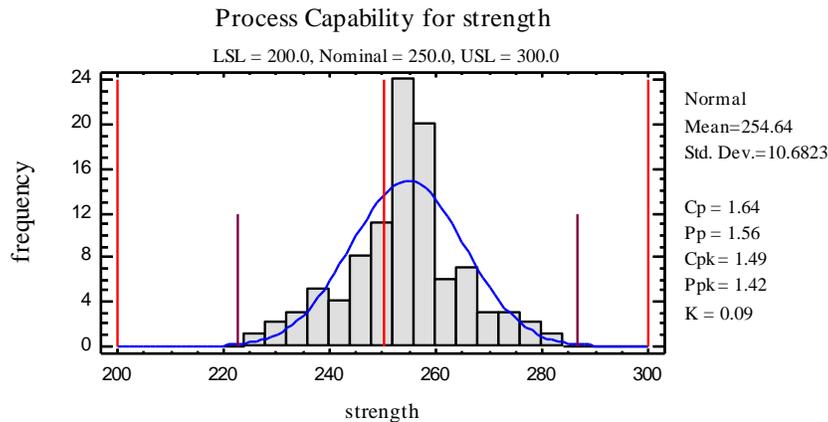


- **Data:** one or more numeric columns. If more than one column is entered, each row of the file is assumed to represent a subgroup with subgroup size m equal to the number of columns entered. If only one column is entered, then the *Date/Time/Labels or Size* field is used to form the groups.
- **Date/Time/Labels or Size:** If each set of m rows represents a group, enter the single value m . For example, entering a 5 as in the example above implies that the data in rows 1-5 form the first group, rows 6-10 form the second group, and so on. If the subgroup sizes are not equal, enter the name of an additional numeric or non-numeric column containing group identifiers. The program will scan this column and place sequential rows with identical codes into the same group.
- **USL:** upper specification limit, if any.
- **Nominal:** optional nominal or target value for the variable. If not supplied, certain capability indices will not be calculated.
- **LSL:** lower specification limit, if any.
- **Select:** subset selection.

At least one of the specification limits must be entered.

Capability Plot

The best picture of the analysis performed by this procedure is given by the *Capability Plot*.

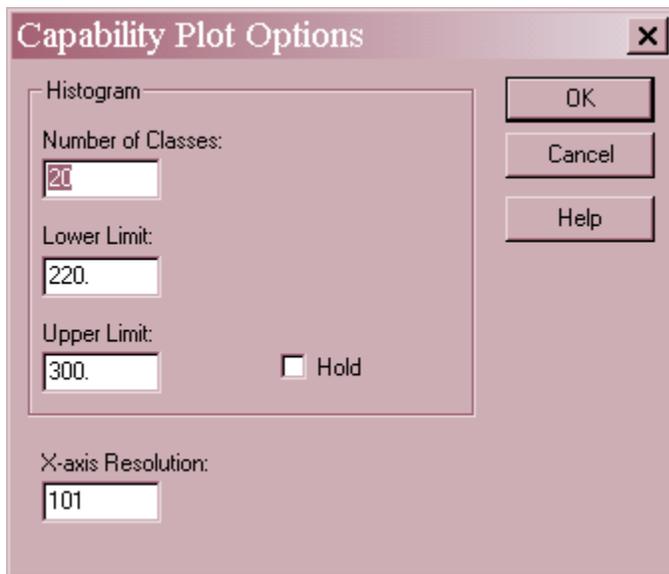


The Capability Plot shows:

1. A *frequency histogram* for the sample data.
2. Tall vertical lines at the specification limits and the nominal value.
3. A *probability density function* that has been fit to the data. By default, a normal distribution is fit, although this may be changed using *Analysis Options*.
4. Shorter vertical lines at specified percentiles of the fitted distribution. By default, the percentiles are positioned to cover 99.73% of the fitted distribution, which for a normal distribution covers a *6-sigma interval* centered at the sample mean.

For a “capable” process, the percentiles will be inside the specification limits.

Pane Options



- **Number of classes:** the number of intervals into which the data will be divided. Intervals are adjacent to each other and of equal width. The number of intervals into which the data is grouped by default is set by the rule specified on the *EDA* tab of the *Preferences* dialog box on the *Edit* menu.
- **Lower Limit:** lower limit of the first interval.
- **Upper Limit:** upper limit of the last interval.
- **Hold:** maintains the selected number of intervals and limits even if the source data changes. By default, the number of classes and the limits are recalculated whenever the data changes. This is necessary so that all observations are displayed even if some of the updated data fall beyond the original limits.

Analysis Summary

The *Analysis Summary* summarizes the input data and displays several important results.

Process Capability Analysis - strength

Data variable: strength

Transformation: none

Distribution: Normal
 sample size = 100
 mean = 254.64
 std. dev. = 10.6823

6.0 Sigma Limits
 +3.0 sigma = 286.687
 mean = 254.64
 -3.0 sigma = 222.593

	<i>Observed</i>		<i>Estimated</i>	<i>Defects</i>
<i>Specifications</i>	<i>Beyond Spec.</i>	<i>Z-Score</i>	<i>Beyond Spec.</i>	<i>Per Million</i>
USL = 300.0	0.000000%	4.25	0.001087%	10.87
Nominal = 250.0		-0.43		
LSL = 200.0	0.000000%	-5.12	0.000016%	0.16
Total	0.000000%		0.001103%	11.03

- **Data variable** – the column or columns containing the data.
- **Transformation** – if a selection was made in the *Data Transformation* field on the *Analysis Options* dialog box, the manner in which the data was transformed is summarized.
- **Distribution** – the assumed distribution for the data and the estimated or user-specified parameters. By default, a normal distribution is assumed unless changed using *Analysis Options*. Distributions are fit using maximum likelihood estimation (MLE) on the combined data from all of the subgroups.
- **Sigma Limits** – for a normal distribution, this displays the sample mean plus and minus a multiple of sigma. Unless changed using *Analysis Options*, the range of values displayed covers 6 times the estimated standard deviation, which corresponds to 99.73% of a normal distribution. If a distribution other than the normal is selected, the output shows *Equivalent Sigma Limits* covering the same percent of the population as the normal limits. For example, the output when fitting a Laplace distribution is shown below:

Equivalent 6.0 Sigma Limits
99.865 percentile = 301.488
median = 255.0
0.134996 percentile = 208.512

Note that the interval for the Laplace distribution is 208.5 to 301.5, which is considerably wider than the normal interval of 222.6 to 286.7. This is because the Laplace distribution has considerably longer tails. A correct analysis depends on selecting the proper distribution, which is discussed in the section on *Comparison of Alternative Distributions*.

- **Specifications** – this table shows the specifications for the data and several important statistics:

Observed Beyond Spec. – the percentage of the input data that is beyond the specification limits.

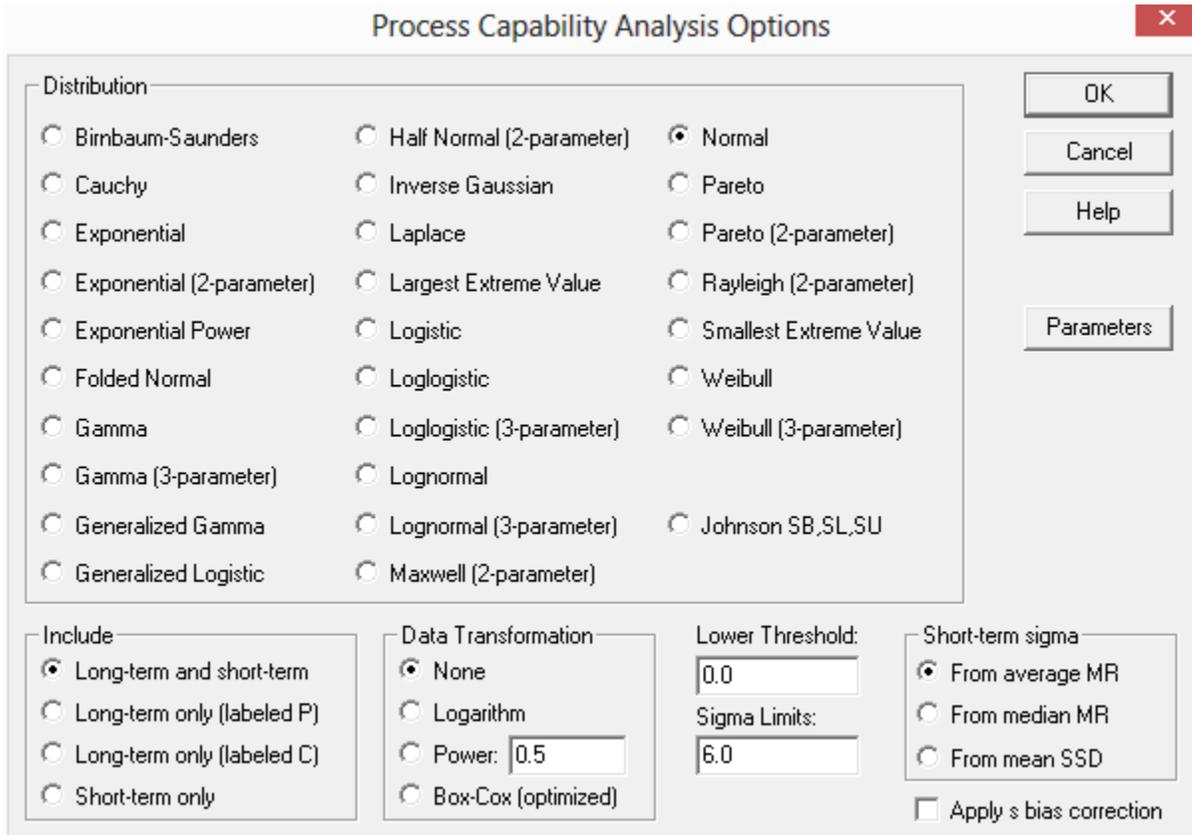
Z-score – for a normal distribution, the distance from the specification limit to the sample mean, divided by the sample standard deviation. For non-normal distributions, an equivalent Z-score is displayed based on the percentage of the fitted distribution that is beyond the specification limit. Consequently, a Z-score of 3, which corresponds to a specification limit that is 3 standard deviations away from the mean in the case of a normal distribution, corresponds to the identical 0.14% beyond the specification limit regardless of what distribution is selected.

Estimated Beyond Spec. – the percentage of the fitted distribution that is beyond the specification limits. This estimates the percentage of non-conforming product being produced.

Defects Per Million – the *Estimated Beyond Spec.* expressed in terms of the number of non-conforming items out of every million produced.

Assuming a normal distribution for bottle bursting strengths, it is estimated that 11 bottles out of every million produced will be outside the specification limits.

Analysis Options



Process Capability Analysis Options

Distribution

- Birnbaum-Saunders
- Cauchy
- Exponential
- Exponential (2-parameter)
- Exponential Power
- Folded Normal
- Gamma
- Gamma (3-parameter)
- Generalized Gamma
- Generalized Logistic
- Half Normal (2-parameter)
- Inverse Gaussian
- Laplace
- Largest Extreme Value
- Logistic
- Loglogistic
- Loglogistic (3-parameter)
- Lognormal
- Lognormal (3-parameter)
- Maxwell (2-parameter)
- Normal
- Pareto
- Pareto (2-parameter)
- Rayleigh (2-parameter)
- Smallest Extreme Value
- Weibull
- Weibull (3-parameter)
- Johnson SB,SL,SU

Include

- Long-term and short-term
- Long-term only (labeled P)
- Long-term only (labeled C)
- Short-term only

Data Transformation

- None
- Logarithm
- Power:
- Box-Cox (optimized)

Lower Threshold:

Sigma Limits:

Short-term sigma

- From average MR
- From median MR
- From mean SSD

Apply s bias correction

Buttons: OK, Cancel, Help, Parameters

- **Distribution** – the assumed distribution for the data. Select *Comparison of Alternative Distributions* to compare the goodness-of-fit for various distributions.
- **Include** – the type of capability indices to be calculated and displayed on the *Capability Plot*. The choices are:

Long-term and short-term – calculate both long-term and short-term indices.

Long-term only (labeled P) – calculate only long-term indices and label them with the letter P, as in Ppk.

Long-term only (labeled C) – calculate only long-term indices and label them with the letter C, as in Cpk.

Short-term only – calculate only short-term indices. These are always labeled with the letter C, as in Cpk.

- **Data Transformation** – the type of transformation applied to the data before fitting the selected distribution. Often, if a normal distribution does not fit the original data well, it will fit some transformation of the data. The choices are:

None – fits the original data.

Logarithm – fits the natural logarithms of the data.

Power – fits the data after raising each value to the specified power.

Box-Cox (optimized) – fits the data after raising each value to a power determined using the Box-Cox procedure.

Note that even when a transformation is selected for analysis, most graphs and tables display results in the original (not the transformed) metric.

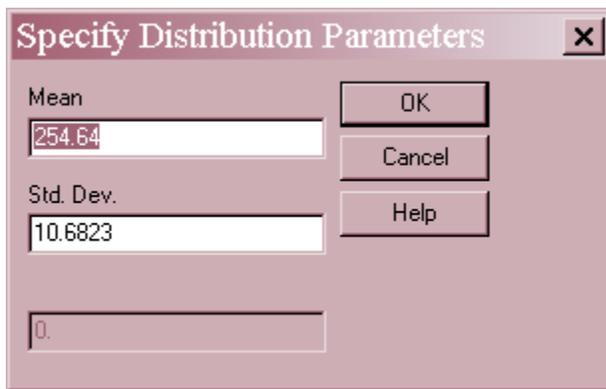
Lower Threshold – for distributions defined by a lower threshold, the value of that lower limit. This includes all distributions that indicate the number of parameters after their name. Note that the "Lower Threshold" corresponds to -1 times the "Addend" determined by the Box-Cox transformation in several other procedures such as the *Process Capability Analysis (Variables) Statlet*.

- **Sigma Limits** – the sigma spread used to plot the limits on the *Capability Plot*. This value is usually set to 6.
- **Short-term Sigma** – the method used to estimate the process standard deviation. For grouped data, the choices are:
 - *From average range* – estimate sigma from a weighted average of the subgroup ranges.
 - *From average s* – estimate sigma from a weighted average of the subgroup standard deviations.
 - *From pooled s* – estimate sigma from the within-group mean squared error as in a oneway analysis of variance.

For individuals data, the choices are:

- *From average MR* – estimate sigma from the average moving range of consecutive observations.

- *From median MR* – estimate sigma from the median moving range of consecutive observations.
- *From mean SSD* – estimate sigma from the squared successive differences (the squared differences between consecutive observations).
- **Apply s bias correction** - if checked, corrections are applied to the estimates of σ to remove any bias. The formulas affected include the grouped estimates based on the average and pooled s, and the individuals estimate based on the mean SSD. This setting also affects the long-term estimate of sigma.
- **Parameters** – Push this button to specify values for the distribution parameters. Normally, the parameters will be estimated from the data. However, this option allows you to fix the values of those parameters.



The image shows a dialog box titled "Specify Distribution Parameters" with a close button (X) in the top right corner. It contains three input fields and three buttons. The first input field is labeled "Mean" and contains the value "254.64". The second input field is labeled "Std. Dev." and contains the value "10.6823". The third input field is empty and contains the value "0.". To the right of the input fields are three buttons: "OK", "Cancel", and "Help".

Capability Indices

Capability indices summarize the performance of a process in comparison to the specification limits. Depending on the selections on the *Pane Options* dialog box, STATGRAPHICS will calculate and display a large number of indices.

Capability Indices for strength		
Specifications		
USL = 300.0		
Nom = 250.0		
LSL = 200.0		
	<i>Short-Term</i>	<i>Long-Term</i>
	<i>Capability</i>	<i>Performance</i>
Sigma	10.1637	10.6823
Cp/Pp	1.63982	1.56021
CR/PR	60.9822	64.0938
CM/PM	1.22987	1.17016
Zusl	4.46294	4.24628
Zlsl	5.376	5.115
Zmin	4.46294	4.24628
Cpk/Ppk	1.48765	1.41543
Cpk/Ppk (upper)	1.48765	1.41543
Cpk/Ppk (lower)	1.792	1.705
CCpk	1.63982	
Cpm		1.4299
K		0.0928
% beyond spec	0.000408377	0.0011032
DPM	4.08377	11.032
Sigma Quality Level	5.96075	5.74292

Based on 6 sigma limits. Short-term sigma estimated from average moving range. The Sigma Quality Level includes a 1.5 sigma drift in the mean.

The above table shows two columns of indices, one labeled “short-term” and the other labeled “long-term”. Short-term indices are calculated by looking at the variation within subgroups (if the data are grouped) or between consecutive observations (if the data are collected as individuals). Long-term indices are calculated by looking at the variation over the entire sampling period. Some analysts prefer to call the short-term variability the “within” variability and the long-term variability the “total” variability. Note: short-term capability is only estimated for data assumed to come from a normal distribution.

The values in the table are described below.

- Sigma** – The key distinction between short-term and long-term indices is the method used to estimate σ , the standard deviation of the process. The long-term sigma is estimated from the sample standard deviation of the entire data set. This could potentially include variation caused by drifts in the process during the period in which the data was collected, so that a large estimate could be a sign of either large inherent variation or poor process control. The short-term estimate is obtained from either the moving range, the mean squared successive differences, or the subgroup standard deviations, depending on the settings on the *Capability* tab of the *Preferences* dialog box, accessible from the *Edit* menu. Since this estimate is taken from observations close together in time, it is much less influenced by lack of control over

the process. However, it does not necessarily represent the current performance of the process.

- **C_p/P_p** – This two-sided capability index compares the distance between the specification limits to k -sigma:

$$C_p = \frac{USL - LSL}{k\hat{\sigma}} \quad (1)$$

Note: k is normally set to 6, although it may be changed using *Pane Options*. Many companies require that C_p be at least 1.33.

- **CR/PR** – The capability ratio, defined by:

$$C_R = 100 \frac{k\hat{\sigma}}{USL - LSL} \% \quad (2)$$

This ratio is the reciprocal of C_p .

- **CM/PM** – The machine capability index, defined by:

$$C_M = \frac{USL - LSL}{8\hat{\sigma}} \quad (3)$$

The denominator of this index is fixed at 8-sigma.

- **Z_{usl}** – A Z-score for the upper specification limit:

$$Z_{USL} = \frac{USL - \hat{\mu}}{\hat{\sigma}} \quad (4)$$

- **Z_{lsl}** – A Z-score for the lower specification limit:

$$Z_{LSL} = \frac{\hat{\mu} - LSL}{\hat{\sigma}} \quad (5)$$

- **Z_{min}** – The smaller of the calculated Z-scores:

$$Z_{MIN} = \min(Z_{LSL}, Z_{USL}) \quad (6)$$

A Z-score of 4 corresponds to a C_{pk} of 1.33 if the data come from a normal distribution.

- **C_{pk(upper)}** – a one-sided capability index based on the upper specification limit:

$$C_{PK(upper)} = \frac{USL - \hat{\mu}}{(k/2)\hat{\sigma}} \quad (7)$$

- **Cpk(lower)** – a one-sided capability index based on the lower specification limit:

$$C_{PK(lower)} = \frac{\hat{\mu} - LSL}{(k/2)\hat{\sigma}} \quad (8)$$

- **Cpk** – The smaller of the two one-sided indices:

$$C_{PK} = \min(C_{PK(upper)}, C_{PK(lower)}) \quad (9)$$

Note: $k/2$ is normally set to 3, although it may be changed using *Pane Options*. Many companies require that Cpk be at least 1.33.

- **CCpk** – A modified version of Cpk based on the target or nominal value T , rather than the estimated process mean:

$$CC_{PK} = \min\left(\frac{USL - T}{(k/2)\hat{\sigma}}, \frac{T - LSL}{(k/2)\hat{\sigma}}\right) \quad (10)$$

If the nominal value is not specified, then T is set halfway between the upper and lower specification limits.

- **Cpm** – A modified version of Cp measuring variation around the target or nominal value T rather than the estimated process mean:

$$C_{PM} = \frac{C_P}{\sqrt{1 + \frac{(\hat{\mu} - T)^2}{\hat{\sigma}^2}}} \quad (11)$$

Cpm may be substantially less than Cp if the process is significantly off-center.

- **K** – a measure of the distance from the target to the estimated process mean, scaled by the distance between the specification limits:

$$\text{If } \hat{\mu} > T: K = \frac{\hat{\mu} - T}{USL - T}, \text{ else: } K = \frac{\hat{\mu} - T}{T - LSL} \quad (12)$$

- **% beyond spec.** – the estimated percent of the population beyond the specification limits, based on the fitted distribution.
- **DPM** – the estimated defects per million, based on the estimated % beyond spec.
- **Sigma Quality Level** – an index of the level of quality for the process developed as part of the Six Sigma process. If there is only one specification limit, the SQL equals either $Zmin$ or $(Zmin+1.5)$, depending on the *1.5 Sigma Shift* setting on the *Pane Options* dialog box.

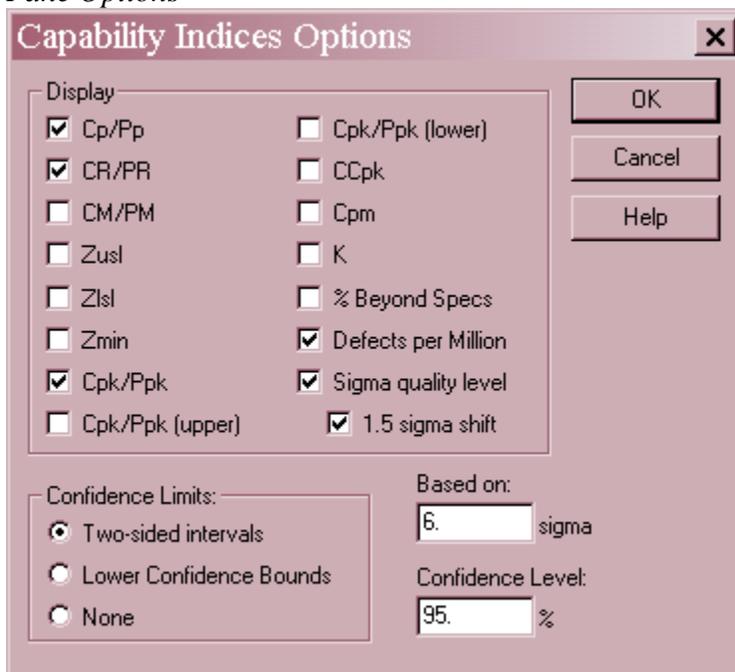
For the bottle bursting strength, note that both Cpk and Ppk are greater than 1.33, which would normally be considered acceptable performance. The value of $K = 0.09$ indicates that the estimated process mean exceeds the target value T by approximately 9% of the distance from the target value to the upper specification limit.

STATGRAPHICS also calculates confidence intervals or bounds for several of the capability indices:

95.0% Confidence Intervals		
Index	Lower Limit	Upper Limit
Cp	1.4116	1.86767
Pp	1.34307	1.77699
Cpk	1.27038	1.70492
Ppk	1.20773	1.62312
Cpm	1.23435	1.6251

Since the indices are statistics calculated from data, there is a margin of error in their ability to estimate the true process capability. For small samples, that margin of error can be substantial. Based on the above table, we may state with 95% confidence that the true Cpk for the bottle bursting strengths is somewhere between 1.27 and 1.70.

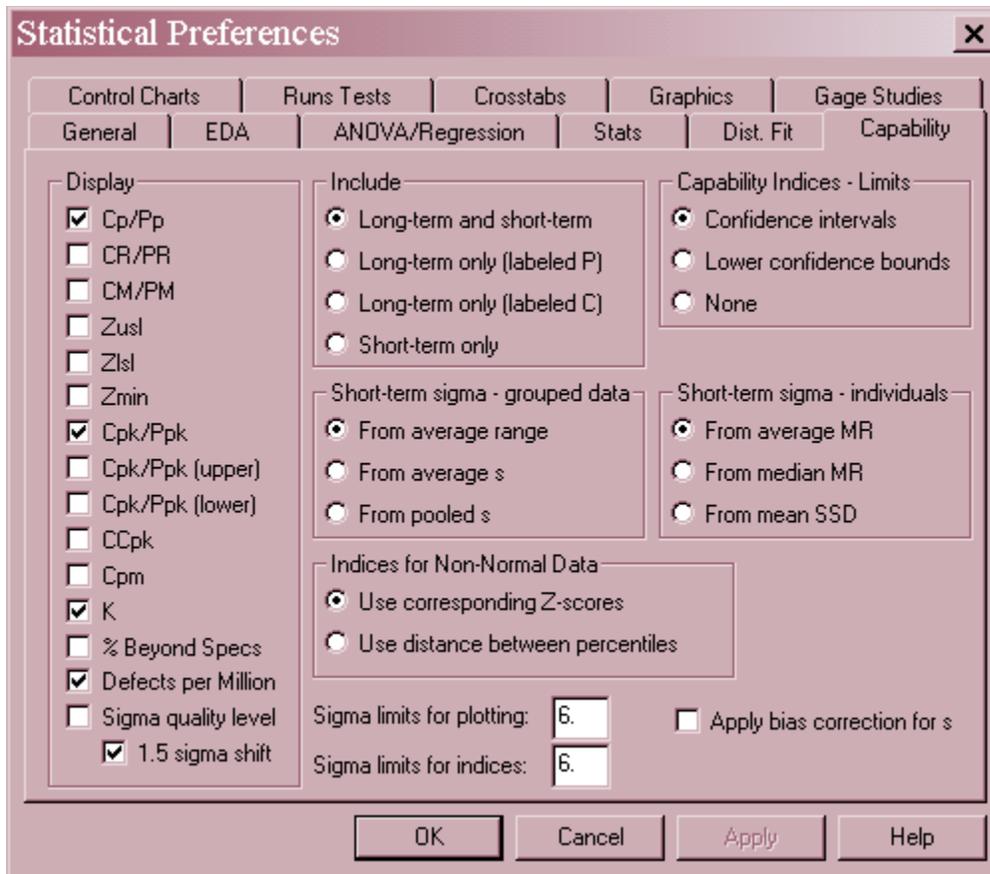
Pane Options



- **Display** – select the indices to display. The selection on this dialog box affects both the tabulated indices and the *Capability Plot*. To change the default selections, use the *Capability* tab on the *Preferences* dialog box accessible through the *Edit* menu.
- **Confidence Limits** – the type of limits to be displayed for the capability indices. Either a two-sided confidence interval or a one-sided lower confidence bound may be calculated.
- **Based on** – k , the multiple of sigma used to calculate indices such as Cp and Cpk . This is usually set to 6.
- **Confidence Level** – the level of confidence for the confidence limits.

Preferences

The *Capability* tab of the *Preferences* dialog box, accessible from the *Edit* menu, sets various defaults that affect the manner in which the capability analysis is performed.



- **Display** – the indices displayed by default in the capability analysis procedures.
- **Include** – the type of capability indices to be calculated and how they will be labeled. The choices are:

Long-term and short-term – calculate both long-term and short-term indices.

Long-term only (labeled P) – calculate only long-term indices and label them with the letter P, as in Ppk.

Long-term only (labeled C) – calculate only long-term indices and label them with the letter C, as in Cpk.

Short-term only – calculate only short-term indices. These are always labeled with the letter C, as in Cpk.

- **Confidence Limits** – the type of limits to be displayed for the capability indices.
- **Short-term sigma – grouped data** – the method used to estimate the process standard deviation for grouped data. The choices are:

From average range – estimate sigma from a weighted average of the subgroup ranges.

From average s – estimate sigma from a weighted average of the subgroup standard deviations.

From pooled s – estimate sigma from the within-group mean squared error as in a oneway analysis of variance.

- **Short-term sigma – individuals** – the method used to estimate the process standard deviation when the data are not grouped. The choices are:

From average MR – estimate sigma from the average moving range of consecutive observations.

From median MR – estimate sigma from the median moving range of consecutive observations.

From mean SSD – estimate sigma from the squared successive differences (the squared differences between consecutive observations).

- **Apply bias correction for s** – if checked, corrections are applied to the estimates of σ to remove any bias. The formulas affected include the grouped estimates based on the average and pooled s, and the individuals estimate based on the mean SSD. This setting also affects the long-term estimate of sigma.
- **Indices for non-normal data** – when constructing capability indices for non-normal distributions, controls whether the index is based on corresponding Z-scores or the distance between percentiles. If *Corresponding Z-Scores* is selected, then the relationship between the capability indices and the percentage of the population beyond the specification limits is the same for all distributions. If *Use Distance between Percentiles* is selected, then the definition of the capability indices as ratios of distances is retained, but a Z-score of 4 equates to different percentages of non-conforming items for different distributions.
- **Sigma limits for plotting** – defines the distance between the tolerance limits shown on the capability plots.
- **Sigma limits for indices** – defines the number of standard deviations used in the denominator of C_p and related capability indices. This value is usually set to 6.

Tests for Normality

The estimated process capability shown above is highly dependent on the assumed distribution of the observations. By default, it is usually assumed that the data follow a normal distribution. The *Tests for Normality* pane performs one or more tests to determine whether or not this is a reasonable assumption. For each test, the hypotheses of interest are:

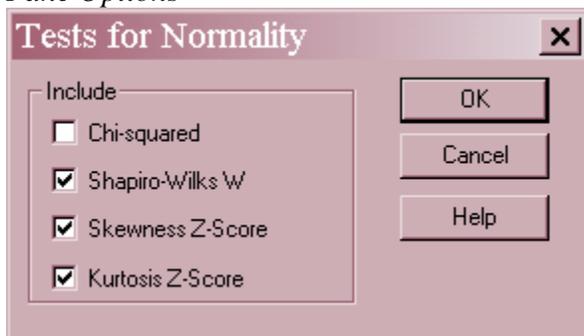
- Null hypothesis: data are independent samples from a normal distribution
- Alt. hypothesis: data are not independent samples from a normal distribution

Tests for Normality for strength		
Test	Statistic	P-Value
Shapiro-Wilks W	0.97781	0.416181
Skewness Z-score	0.399024	0.689872
Kurtosis Z-score	1.18663	0.235371

The tests to be run are selected using *Pane Options*. Each test is displayed with its associated test statistic and *P-Value*. Small P-values (below 0.05 if operating at the 5% significance level) lead to a rejection of the null hypothesis and thus to a rejection of the normal distribution. In the above table, the P-Values are all well above 0.05, so there is not statistically significant non-normality in the data.

For a detailed description of the tests, see the documentation for *Distribution Fitting (Uncensored Data)*.

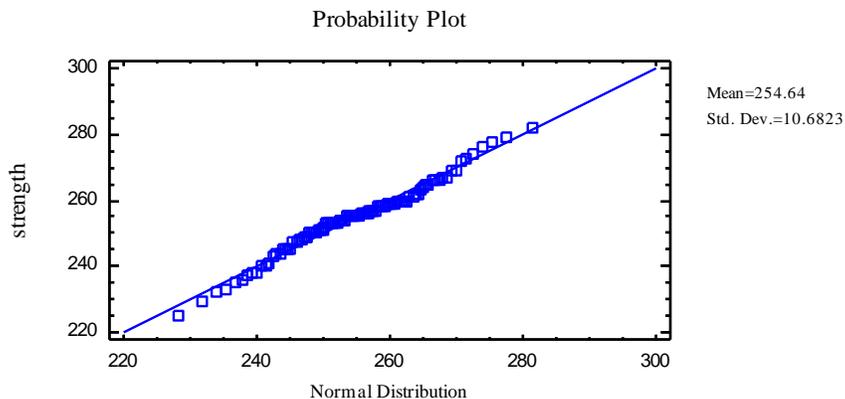
Pane Options



Include – select the tests to include in the output. The default tests are defined on the *Dist. Fit* tab of the *Preferences* dialog box accessed from the *Edit* menu.

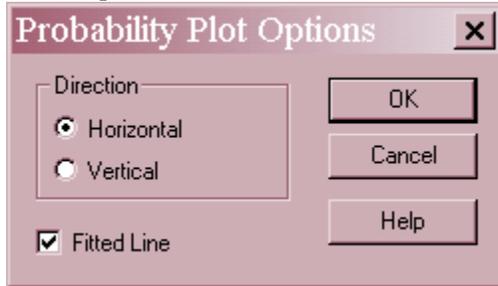
Probability Plot

The *Probability Plot* is another method by which one can judge whether or not the currently selected distribution adequately describes the data.



The plot shows the data values, sorted from smallest to largest, plotted against equivalent percentiles of the fitted distribution. If the fitted distribution is a reasonable model for the data, they will fall approximately along a straight line. For the bottle strengths, the normal distribution does a reasonable job.

Pane Options



- **Direction:** the axis along which the percentiles of the fitted distribution will be plotted.
- **Fitted line:** check this box to include a diagonal line on the plot.

Comparison of Alternative Distributions

If there were reason to doubt the adequacy of the normal distribution, a different distribution could be selected using *Analysis Options*. To help determine a reasonable alternative, the *Comparison of Alternative Distributions* pane fits a variety of different distributions and sorts them by how well they fit the data.

Comparison of Alternative Distributions				
Distribution	Est. Parameters	Log Likelihood	KS D	A ²
Laplace	2	-375.493	0.05	0.246061
Exponential Power	3	-375.389	0.0546134	0.249059
Logistic	2	-377.021	0.0821372	0.440957
Loglogistic	2	-377.324	0.0854092	0.499323
Normal	2	-378.253	0.0989897	0.788306
Gamma	2	-378.51	0.104262	0.834605
Birnbaum-Saunders	2	-378.695	0.107108	0.872569
Inverse Gaussian	2	-378.695	0.107111	0.872599
Lognormal	2	-378.694	0.107325	0.88292
Weibull	2	-383.901	0.137322	1.82599
Smallest Extreme Value	2	-385.804	0.145066	2.17477
Largest Extreme Value	2	-389.32	0.15057	2.9171
Exponential	1	-653.985	0.586708	42.309
Pareto	1	-825.078	0.623869	45.212

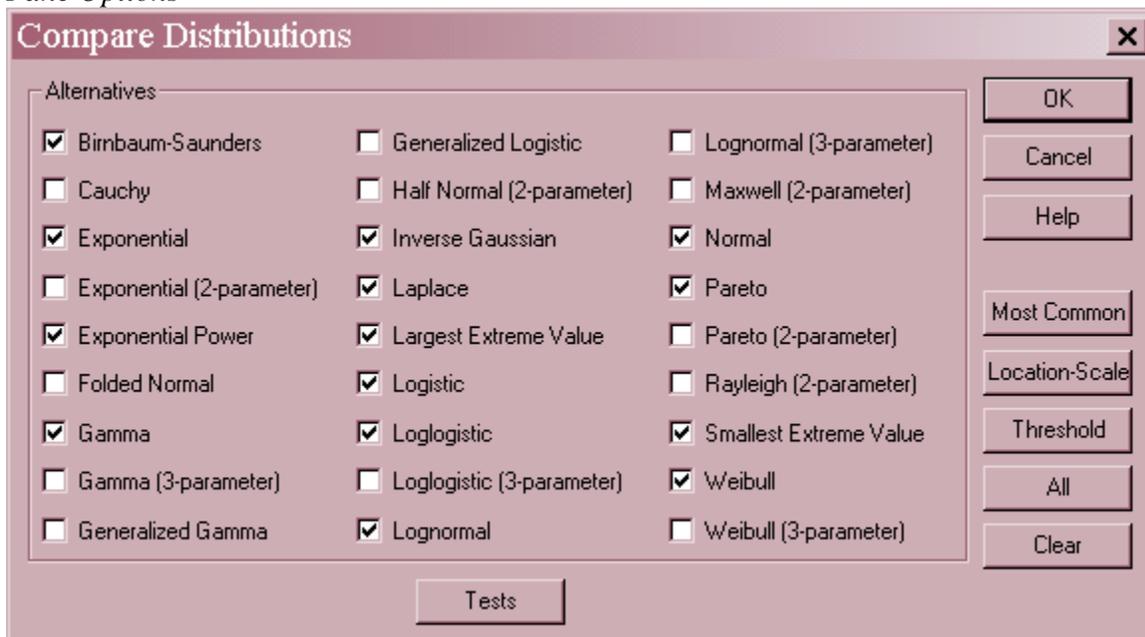
The table shows:

- **Distribution** – the name of the distribution fit. You may select additional distributions using *Pane Options*.
- **Est. Parameters** – the number of estimated parameters for that distribution.

- **Log Likelihood** – the natural logarithm of the likelihood function. Larger values tend to indicate better fitting distributions.
- **KS D, A², and other statistics** – values of various goodness-of-fit statistics, selected using the *Tests* button on the *Pane Options* dialog box. Smaller values tend to indicate better fitting distributions.

The distributions are sorted from best to worst according to one of the goodness-of-fit columns. That column is selected using the *Tests* button on the *Pane Options* dialog box. The above table shows the distributions sorted according to the value of the Anderson-Darling A² statistic. According to that statistic, the Laplace distribution fits best.

Pane Options



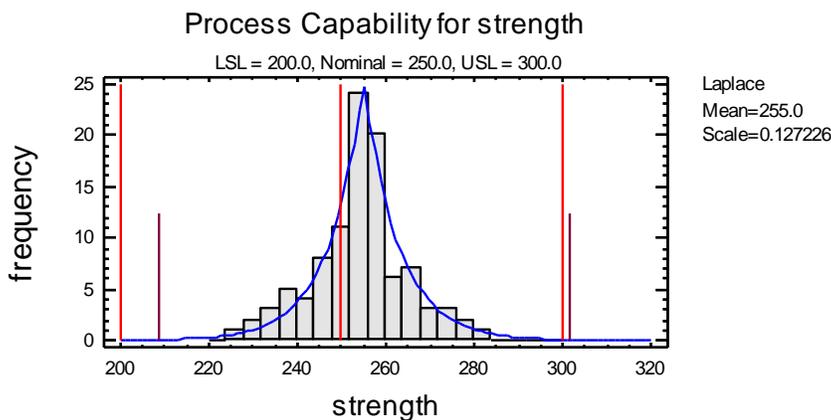
- **Alternatives**: select the distributions to be fit to the data.
- **Most Common** – push this button to select the most commonly used distributions.
- **Location-Scale** – push this button to select all distributions that are parameterized by a location parameter (such as a mean) and a scale parameter (such as a standard deviation).
- **Threshold** – push this button to select all distributions that contain a lower threshold parameter.
- **All** – push this button to select all distributions.
- **Clear** – push this button to deselect all distributions.
- **Tests** – push this button to display the dialog box used to specify the desired goodness-of-fit statistics:



- **Include** – select the goodness-of-fit statistics to be included in the table. The list includes the likelihood function and various statistics displayed on the *Goodness-of-Fit* pane.
- **Sort By** – select one of the included statistics to use to sort the distributions from best to worst.

Example – Fitting a Laplace Distribution

It is interesting to examine the differences if a Laplace distribution is fit to the data rather than a normal distribution. A Laplace distribution is very peaked in the center and has relatively long tails.



The shorter vertical lines are now placed at positions that cover the center 99.73% of the Laplace distribution, rather than plus and minus 3 sigma. This turns out to be a substantially wider range than before.

Some other interesting comparisons are shown in the following table:

	Normal Distribution	Laplace Distribution
DPM – long-term	11.03	2088.47

Ppk	1.415	0.981
Sigma Quality Level	5.74	4.36

Clearly, the estimated performance of the process is heavily dependent on which distribution is used.

Goodness-of-Fit Tests

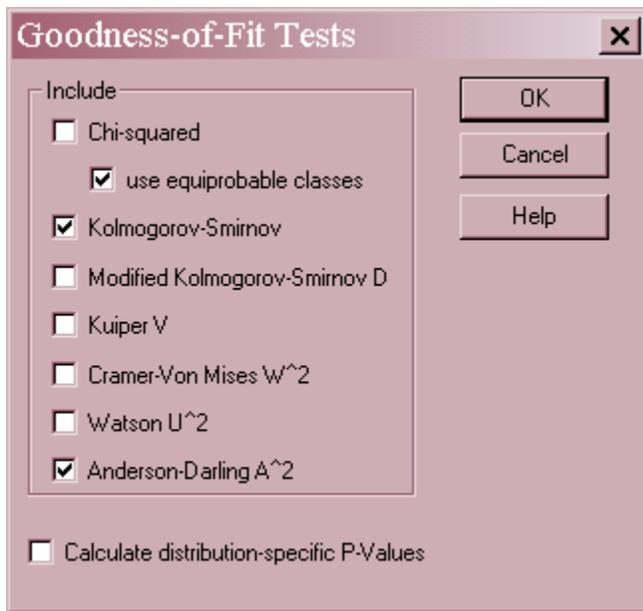
If a distribution other than the normal is fit to the data, its adequacy as a model for the data can be tested using the *Goodness-of-Fit Tests*. Up to 7 different tests may be performed. For all tests, the hypotheses of interest are:

- Null hypothesis: data are independent samples from the specified distribution
- Alt. hypothesis: data are not independent samples from the specified distribution

Goodness-of-Fit Tests for strength			
Kolmogorov-Smirnov Test			
	<i>Laplace Distribution</i>		
DPLUS	0.0499018		
DMINUS	0.05		
DN	0.05		
P-Value	0.963945		
<i>EDF Statistic</i>	<i>Value</i>	<i>Modified Form</i>	<i>P-Value</i>
Anderson-Darling A ²	0.246061	0.246061	>=0.10
*Indicates that the P-Value has been compared to tables of critical values specially constructed for fitting the currently selected distribution. Other P-values are based on general tables and may be very conservative.			

The tests to be run are selected using *Pane Options*. Two common tests are shown above. In each case, if the P-Value is large (greater or equal to 0.05 if operating at the 5% significance level), then the distribution is a reasonable model for the data. For a detailed description of the tests, see the documentation for *Distribution Fitting (Uncensored Data)*.

Pane Options



- **Include:** select one or more tests to perform. For the chi-squared test, select *use equiprobable classes* to group data into classes with equal expected frequencies. If this option is not checked, classes will be created that match the *Capability Plot*.
- **Calculate distribution-specific P-Values** – if checked, the P-Values will be based on tables or formulas specifically developed for the distribution being tested. Otherwise, the P-Values will be based on a general table or formula that applies to all distributions. The general approach is more conservative (will not reject a distribution as easily) but may be preferred when comparing P-Values amongst different distributions.

Non-Normal Capability Indices

Rather than finding an alternative distribution if the normal distribution does not fit the data, one can instead calculate capability indices specifically designed to describe non-normal data.

STATGRAPHICS calculates two types of non-normal indices:

1. an index referred to as $Cp(q)$ based on the quantiles of a Pearson curve selected to match the skewness and kurtosis of the data.
2. an index called Cpc that is based on the average absolute distance of the data values from their target.

Nonnormal Capability Indices for strength		
Selected Pearson Curve		
Percentage	Percentile	Specification
99.865	287.825	300.0
50	254.833	300.0
0.135	217.316	200.0
<i>Index</i>		<i>Estimate</i>
Pp(q)		1.41825

Ppk(q)	1.36902
Ppk (upper)(q)	1.36902
Ppk (lower)(q)	1.46155
K(q)	0.0966624

Index	Estimate	Lower 95% C.L.	Upper 95% C.L.
Cpc	1.44544	1.25357	1.70666

NOTE: these indices are based on the distance between equivalent 6-sigma limits and correspond to long-term performance.

The indices are calculated as follows:

- **Cp(q)** – To calculate this index:
 1. Calculate the skewness and kurtosis of the data.
 2. If the skewness is less than -4 or greater than +4, or if the kurtosis is less than -1.4 or greater than 36.6, no indices are computed. Otherwise, a Pearson curve is selected to match the sample skewness and kurtosis.
 3. Estimates of the 0.135 percentile, the median, and the 99.865 percentile are obtained from the selected Pearson curve.
 4. Capability indices are then calculated in the usual manner, except that the distance between the percentiles replaces the usual 6σ spread of the normal distribution.

Note the effect on several common indices:

	Assuming normality	Using Cp(q) approach
Pp	1.56	1.42
Ppk	1.42	1.37
K	0.093	0.097

The estimated capability is a little worse than with the standard indices, which is in line with the slightly long tails. However, the effect is not nearly as dramatic as if a Laplace distribution had been selected.

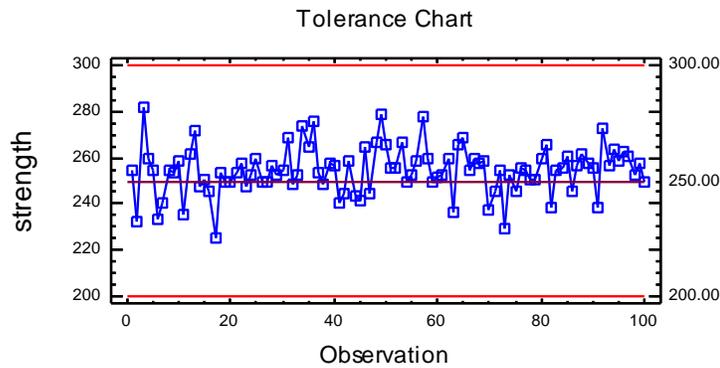
- **Cpc** – To calculate this index, let:

$$C_{PC} = \frac{USL - LSL}{k \sqrt{\frac{\pi}{2} \frac{\sum_{i=1}^n |x_i - T|}{n}}} \quad (13)$$

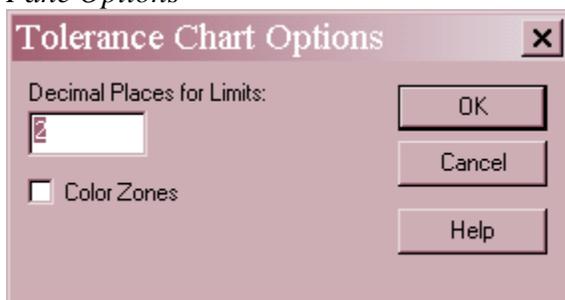
where k is commonly set to 6. The interesting feature of this statistic is that it is based on the average absolute distance of the observations from the target.

Tolerance Chart

The Tolerance Chart plots the data in row order with horizontal lines indicating the target and specification limits.



Pane Options



- **Decimal Places for Limits** – number of decimal places for displaying the values to the right of the graph.
- **Color Zones:** check this box to display green, yellow and red zones.

Normal Tolerance Limits

If the data indicate that the current specification limits cannot be met, it is interesting to ask what limits *could* be met. One approach to establishing new limits is to calculate *Normal Tolerance Limits*. Normal tolerance limits give a range of values for X such that one may be $100(1-\alpha)\%$ confident that P percent of the population from which a data sample comes falls within that range. Assuming that the data come from a normal distribution, a two-sided tolerance limit may be calculated by taking the sample mean plus and minus a multiple of the standard deviation, according to

$$\bar{x} \pm Ks \quad (14)$$

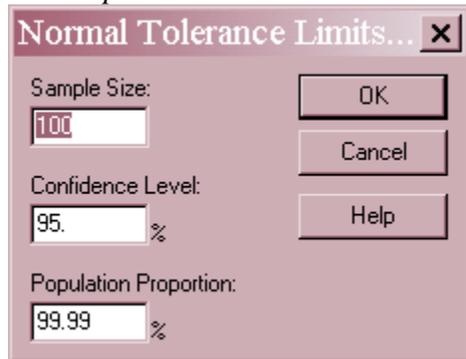
The factor K depends upon the sample size n , the level of confidence $(1-\alpha)$, and the specified percentage P .

Normal Tolerance Limits for strength	
Normal distribution	
Sample size =	100
Mean =	254.64
Sigma =	10.6823
Specifications	
USL =	300.0
Nom =	250.0
LSL =	200.0
95.0% tolerance interval for 99.99% of the population	
Xbar +/-	4.43436 sigma
Upper:	302.009
Lower:	207.271

For example, the above table states that one may be 95% confident that 99.99% of all bottles produced will have bursting strengths between 207 and 301. Consequently, establishing a spec based on these limits should give a defect rate of no more than 1 bottle out of every 100,000.

It is important to note that the above interval is not simply the interval under the fitted normal curve containing an area of 99.99%. It is wider than that since it allows for sampling variability in both the mean and standard deviation.

Pane Options



- **Confidence Level** – specify the level of confidence for the tolerance limits, i.e., $100(1-\alpha)\%$.
- **Population Proportion** – specify the percentage of the population P that the tolerance limits bound.

Distribution-Free Limits

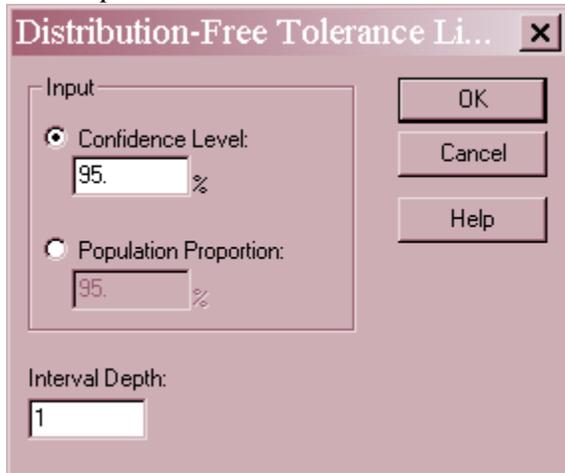
The k -th smallest and k -th largest values in a data sample may be used to construct tolerance limits for the population from which the data come without assuming any specific distribution. The resulting tolerance limits give a range of values for X such that one may be $100(1-\alpha)\%$ confident that **at least** P percent of the population from which a data sample comes falls within that range. The interval can be quite conservative, with the actual percentage being much larger than that stated.

Distribution-Free Tolerance Limits for strength	
Data summary	
Count	= 100
Maximum	= 282.0
Median	= 255.0
Minimum	= 225.0
Specifications	
USL	= 300.0
Nom	= 250.0
LSL	= 200.0
95.0% tolerance interval for 95.3433% of the population	
Upper:	282.0
Lower:	225.0
(Based on an interval depth = 1)	

For example, the above table takes the most extreme values of *strength* and states that one can be 95% confident that at least 95.3433% of all bottles would have bursting strengths between 225 psi and 282 psi.

In this procedure, you can select *Pane Options* to choose either the level of confidence $100(1-\alpha)$ or the population percentage P , but not both.

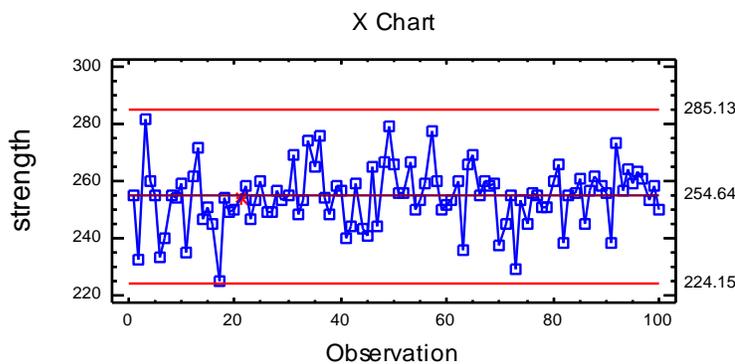
Pane Options



- **Input** - specify either the level of confidence for the interval $100(1-\alpha)$ or the population percentage P .
- **Interval Depth** – specify the value of k used to select the order statistics upon which the limits are based. In creating the interval, the procedure uses the k -th smallest and k -th largest data values.

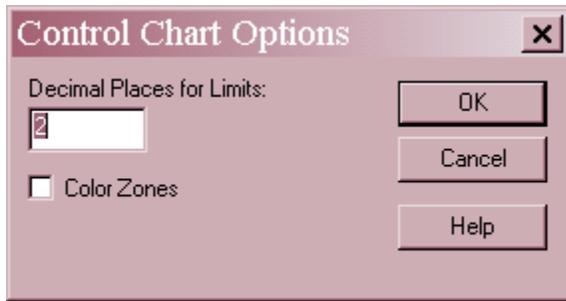
X or X-bar Chart

A standard X chart is created if the data are individuals, while an X-bar chart is created for grouped data.



Out-of-control points will be flagged, as will runs rules violations if your *Control Charts* preferences are to include rules violations on control charts. Any out-of-control signals should be carefully evaluated since they can impact the estimated process capability.

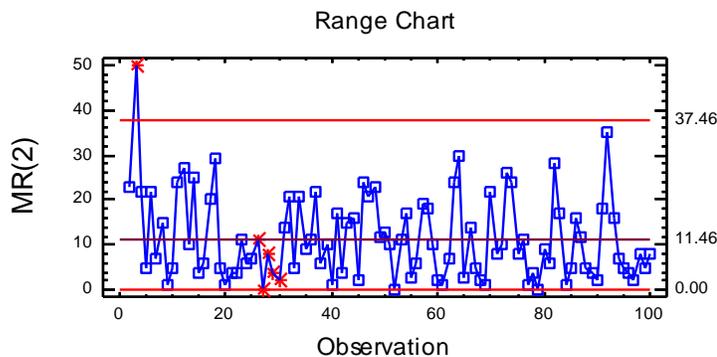
Pane Options



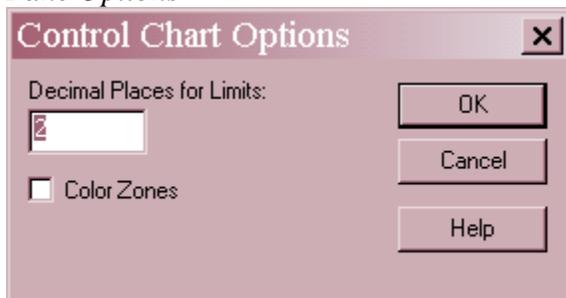
- **Decimal Places for Limits** – number of decimal places for displaying the values to the right of the graph.
- **Color Zones:** check this box to display green, yellow and red zones.

MR(2) or R Chart

An MR(2) chart is created if the data are individuals, while an R chart is created for grouped data.



Pane Options



- **Decimal Places for Limits** – number of decimal places for displaying the values to the right of the graph.
- **Color Zones:** check this box to display green, yellow and red zones.

Calculations

Long-term Sigma

If no bias correction:

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad (15)$$

If bias correction:

$$\hat{\sigma} = \frac{\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}}{c_4(n)} \quad (16)$$

Short-Term Sigma – Individuals

From average MR:

$$\hat{\sigma} = \frac{\bar{R}}{d_2(2)} \quad (17)$$

From median MR:

$$\hat{\sigma} = \frac{\tilde{R}}{d_4(2)} \quad (18)$$

From mean SSD with no bias correction:

$$\hat{\sigma} = \frac{1}{\sqrt{2}} \sqrt{\frac{\sum_{i=2}^n (x_i - x_{i-1})^2}{n-1}} \quad (19)$$

From mean SSD with bias correction:

$$\hat{\sigma} = \frac{\frac{1}{\sqrt{2}} \sqrt{\frac{\sum_{i=2}^n (x_i - x_{i-1})^2}{n-1}}}{c'_4(n)} \quad (20)$$

Short-Term Sigma – Grouped Data

From average range:

$$\hat{\sigma} = \frac{\sum_{j=1}^k \left(\frac{f_j R_j}{d_2(n_j)} \right)}{\sum_{j=1}^k f_j} \quad (21)$$

where

$$f_j = \frac{d_2^2(n_j)}{d_3^2(n_j)} \quad (22)$$

From average s with no bias correction:

$$\hat{\sigma} = \frac{\sum_{j=1}^k n_j s_j}{\sum_{j=1}^k n_j} \quad (23)$$

From average s with bias correction:

$$\hat{\sigma} = \frac{\sum_{j=1}^k \left(\frac{n_j s_j}{c_4(n_j)} \right)}{\sum_{j=1}^k h_j} \quad (24)$$

where

$$h_j = \frac{c_4^2(n_j)}{1 - c_4^2(n_j)} \quad (25)$$

From pooled s with no bias correction:

$$\hat{\sigma} = \frac{\sum_{j=1}^k (n_j - 1) s_j^2}{\sum_{j=1}^k (n_j - 1)} \quad (26)$$

From pooled s with bias correction:

$$\hat{\sigma} = \frac{\sum_{j=1}^k (n_j - 1) s_j^2}{c_4(d) \sum_{j=1}^m (n_j - 1)} \quad (27)$$

where

$$d = 1 + \sum_{j=1}^k (n_j - 1) \quad (28)$$

Confidence Interval for Cp

$$\hat{C}_P \sqrt{\frac{\chi_{1-\alpha/2, n-1}^2}{n-1}} \leq C_P \leq \hat{C}_P \sqrt{\frac{\chi_{\alpha/2, n-1}^2}{n-1}} \quad (29)$$

Confidence Interval for Cpk

$$\hat{C}_{PK} \left[1 - Z_{\alpha/2} \sqrt{\frac{1}{9n\hat{C}_{PK}^2} + \frac{1}{2(n-1)}} \right] \leq C_{PK} \leq \hat{C}_{PK} \left[1 + Z_{\alpha/2} \sqrt{\frac{1}{9n\hat{C}_{PK}^2} + \frac{1}{2(n-1)}} \right] \quad (30)$$

Confidence Interval for Cpc

$$\frac{\hat{C}_{PC}}{1 + t_{\alpha/2, n-1} \left[s_C / (\bar{c} \sqrt{n}) \right]} \leq C_{PC} \leq \frac{\hat{C}_{PC}}{1 - t_{\alpha/2, n-1} \left[s_C / (\bar{c} \sqrt{n}) \right]} \quad (31)$$

where

$$\bar{c} = \frac{\sum_{i=1}^n |x_i - T|}{n}, \quad s_C^2 = \frac{1}{n-1} \left(\sum_{i=1}^n |x_i - T|^2 - n\bar{c}^2 \right) \quad (32)$$

Confidence Interval for Cpm

$$\hat{C}_{PM} \sqrt{\frac{\chi_{1-\alpha/2, \nu}^2}{\nu}} \leq C_{PM} \leq \hat{C}_{PM} \sqrt{\frac{\chi_{\alpha/2, \nu}^2}{\nu}} \quad (33)$$

where

$$\nu = \frac{(n + \lambda)^2}{n + 2\lambda}, \quad \lambda = n \left(\frac{\hat{\mu} - T}{\hat{\sigma}} \right)^2 \quad (34)$$