

Capability Analysis Using Statgraphics Centurion

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Outline

- Definition of process capability analysis
- Examples
 1. Capability analysis for attributes
 2. Estimating capability for variable data
 3. Capability indices
 4. Statistical tolerance limits
 5. Multivariate capability analysis
- Sample size determination



Capability Analysis

Determination based on data of a process's ability to meet established specifications.

Specifications may be stated in terms of **variables** (such as the tolerance on the diameter of a part) or in terms of **attributes** (such as the frequency of customer complaints).



Capability Measurements

The essential measure of process capability is **DPM** (defects per million) or **DPMO** (defects per million opportunities), defined as the number of times that a process does not meet the specifications out of every million possibilities.

DPM may be estimated directly or inferred from statistics such as a **capability index** or **statistical tolerance limit**.



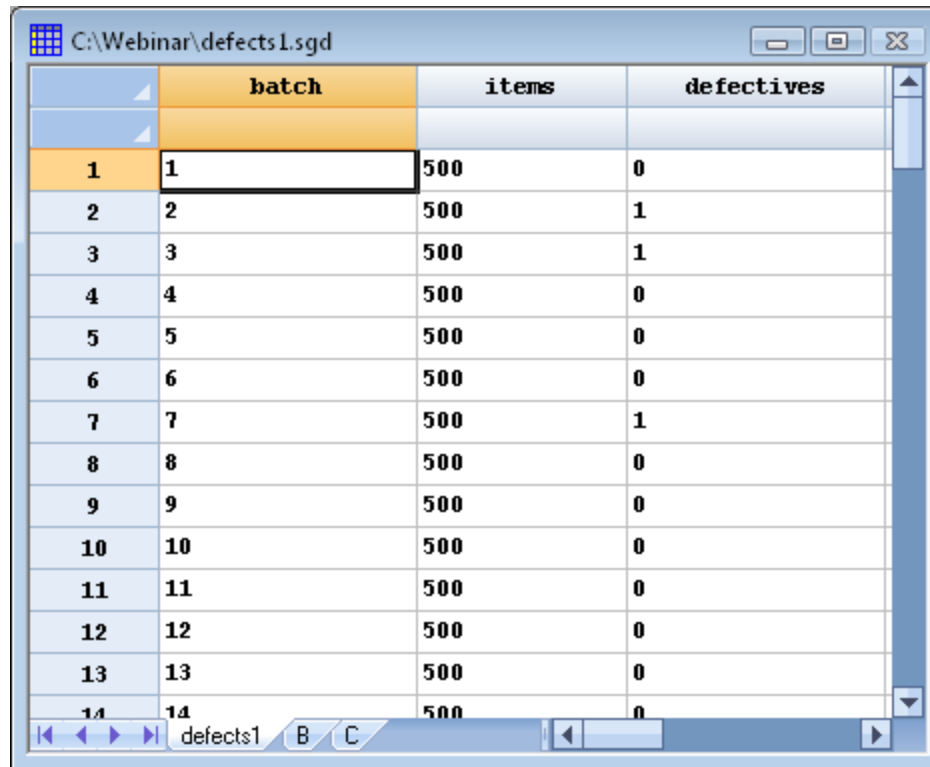
How can we estimate capability using Statgraphics?

- Direct **counting** of defects
- Estimation of DPM from a **fitted distribution**
- Indirect inference about DPM from a **capability index**
- Demonstration of required capability through a **statistical tolerance interval or bound**



Example #1 - defects1.sgd

Inspected $k=30$ batches of $n=500$ items each. Counted number of defective items.



	batch	items	defectives
1	1	500	0
2	2	500	1
3	3	500	1
4	4	500	0
5	5	500	0
6	6	500	0
7	7	500	1
8	8	500	0
9	9	500	0
10	10	500	0
11	11	500	0
12	12	500	0
13	13	500	0
14	14	500	0

Procedure Capability Analysis - Percent Defective

Process Capability Analysis - Percent Defective

batch
items
defectives

Number of Defectives:
▶ defectives

Sample Sizes:
▶ items

(Target % defective:)
▶

(LSL:) (Nominal:) (USL:)
▶ ▶ ▶

(Select:)
▶

Sort column names

OK Cancel Delete Transform... Help

Output

Process Capability Analysis (Percent Defective) - defectives

Process Capability Analysis (Percent Defective) - defectives

Data variable: defectives

Distribution: Binomial

number of samples = 30

average sample size = 500.0

mean percent defective = 0.0866667

	<i>Estimate</i>	<i>Upper 95% Bound</i>
Mean percent defective	0.0866667	0.13772
Defects per million	866.667	1377.2
Process Z	3.13249	2.9939
Tolerance limits (average size sample)		2

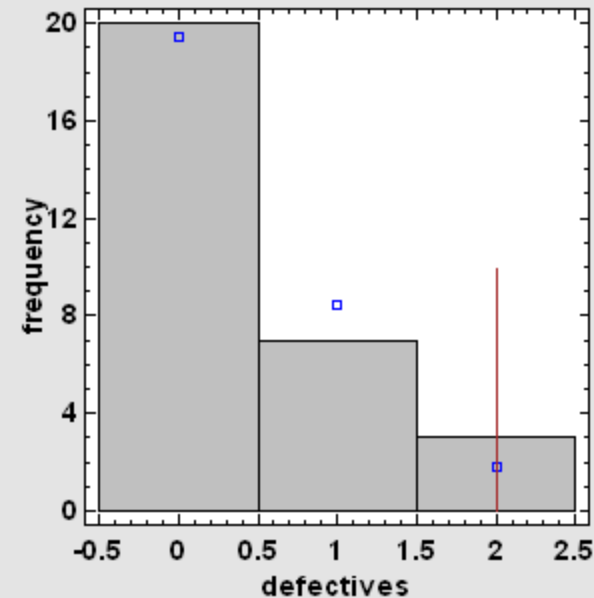
The StatAdvisor

The 30 values of defectives have an average of 0.0866667% defective items. This equi-confidence bound indicates that the mean percent of defective items in the population

The process Z value converts the estimated mean percent defective to a capability and capability of variable data. In most cases, a Z value of at least 4 is desirable.

The tolerance limits show the likely variability amongst samples in the population. In expected to have no more than 2.0 defective items.

Process Capability for defectives



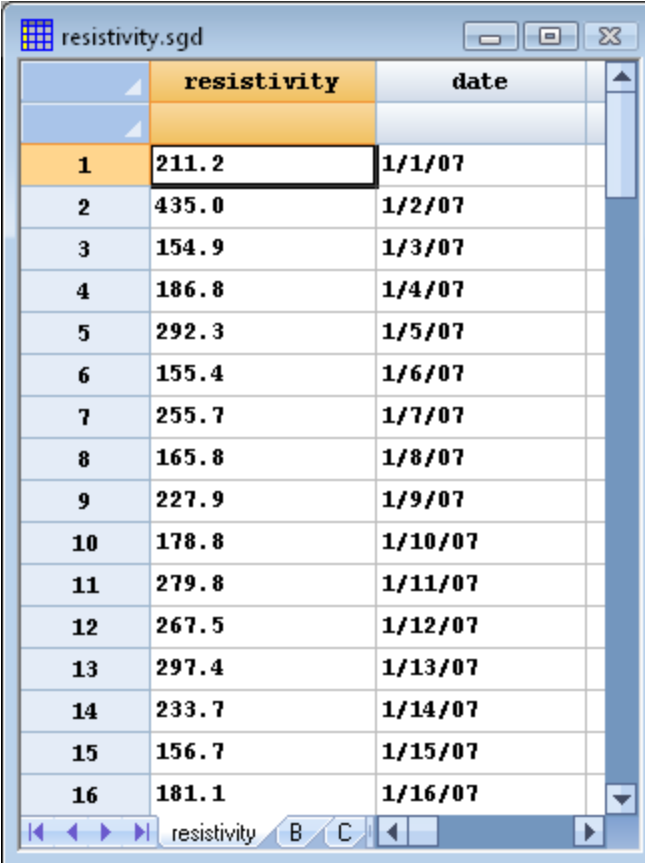
Conclusions

- Best estimate for DPM = 866.7
- With 95% confidence, DPM is no greater than 1,377.2
- Tolerance limit: 95% of all batches of $n=500$ items will have no more than 2 defectives
- Equivalent $Z = 3.13$



Example #2 - resistivity.sgd

Measured resistivity of n=100 electronic components



The screenshot shows a spreadsheet window titled "resistivity.sgd". The spreadsheet contains a table with two columns: "resistivity" and "date". The first row is highlighted in orange. The data is as follows:

	resistivity	date
1	211.2	1/1/07
2	435.0	1/2/07
3	154.9	1/3/07
4	186.8	1/4/07
5	292.3	1/5/07
6	155.4	1/6/07
7	255.7	1/7/07
8	165.8	1/8/07
9	227.9	1/9/07
10	178.8	1/10/07
11	279.8	1/11/07
12	267.5	1/12/07
13	297.4	1/13/07
14	233.7	1/14/07
15	156.7	1/15/07
16	181.1	1/16/07

Procedure Capability Analysis - Variable Data - Individuals

Process Capability Analysis (Individuals)

resistivity
date

Data:
resistivity

(Date/Time/Labels):
date

(LSL:) (Nominal:) (USL):
500

(Select):

Sort column names

OK Cancel Delete Transform... Help

Selecting Proper Distribution

Tests for Normality for resistivity

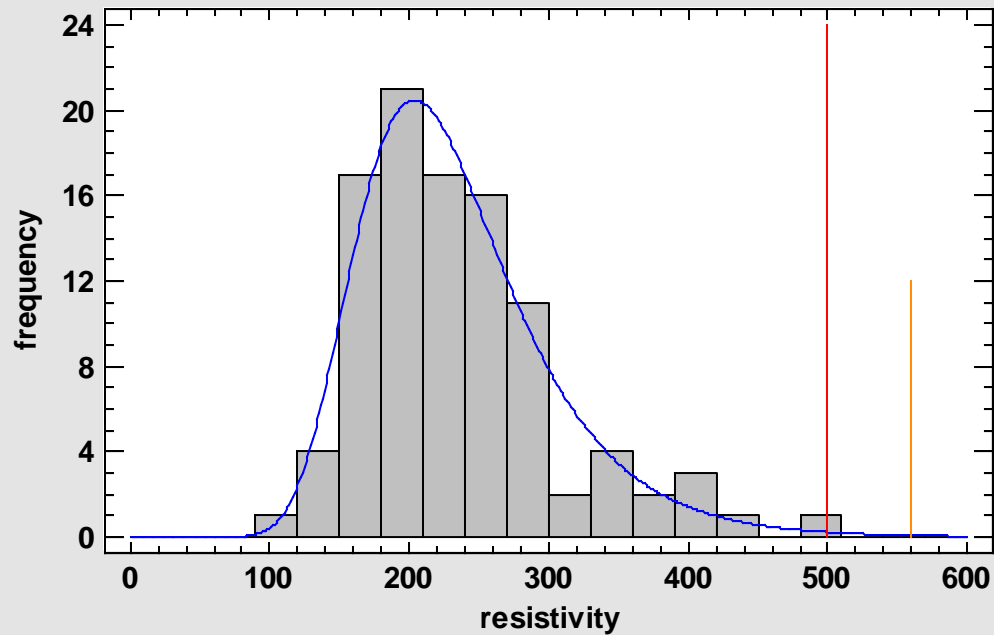
Test	Statistic	P-Value
Shapiro-Wilk <i>W</i>	0.924904	0.00000550813

Comparison of Alternative Distributions

Distribution	Est. Parameters	Log Likelihood	KSD
Largest Extreme Value	2	-557.918	0.0422297
Lognormal	2	-558.632	0.0444956
Loglogistic	2	-559.663	0.0491967
Gamma	2	-560.767	0.0559315
Logistic	2	-566.28	0.0717518
Laplace	2	-568.183	0.0830668
Normal	2	-568.981	0.0891378
Weibull	2	-570.627	0.101169
Smallest Extreme Value	2	-593.519	0.174701
Exponential	1	-646.066	0.423526
Pareto	1	-810.833	0.585075

Capability Plot

Process Capability for resistivity
USL = 500.0



Largest Extreme Value
Mode=203.355
Scale=53.9342

Cpk = 0.85
Ppk = 0.88

Estimate of DPM

Transformation: none

Distribution: Largest Extreme Value

sample size = 100

mode = 203.355

scale = 53.9342

(mean = 234.486)

(sigma = 69.1733)

Equivalent 6.0 Sigma Limits

99.865 percentile = 559.698

median = 223.122

0.134996 percentile = 101.514

	<i>Observed</i>		<i>Estimated</i>	<i>Defects</i>
<i>Specifications</i>	<i>Beyond Spec.</i>	<i>Z-Score</i>	<i>Beyond Spec.</i>	<i>Per Million</i>
USL = 500.0	1.000000%	2.65	0.407788%	4077.88
Total	1.000000%		0.407788%	4077.88

Capability Indices

$$C_P = \frac{USL - LSL}{6\hat{\sigma}}$$

$$C_{PK} = \min \left[\frac{\hat{\mu} - LSL}{3\hat{\sigma}}, \frac{USL - \hat{\mu}}{3\hat{\sigma}} \right]$$

$$Z = \min \left[\frac{\hat{\mu} - LSL}{\hat{\sigma}}, \frac{USL - \hat{\mu}}{\hat{\sigma}} \right]$$

Long-term and Short-term

Capability Indices for resistivity

Specifications

USL = 500.0

	<i>Short-Term</i>	<i>Long-Term</i>
	<i>Capability</i>	<i>Performance</i>
Sigma (after normalization)	1.03374	1.0
Zmin	2.5592	2.64556
Cpk/Ppk	0.853067	0.881852
DPM	5245.67	4077.88

Based on 6.0 sigma limits in the normalized metric. Short-term sigma estimated from average moving range.

Six Sigma Calculator

Six Sigma Indices

Input

Z-Score: 3.0

DPM: 10.0

Defects (%): 0.01

Yield (%): 99.99

Cpk: 0.83

Sigma level: 6.0

Sigma shift: 1.5

Specifications

Two-sided

Lower limit only

Upper limit only

OK

Cancel

Help

Six Sigma Calculator

Input: Cpk = 0.83

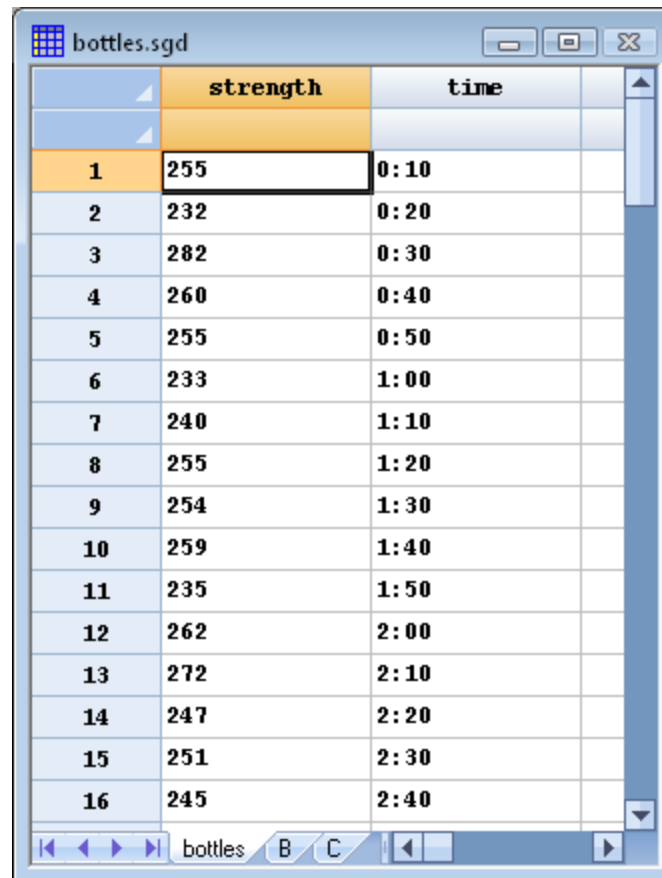
Sigma shift = 1.5

Equivalent values:

Index	Value
Z-Score	2.49
DPM	6387.15
Defects	0.638715
Yield	99.3613
Cpk	0.83
SQL	3.99

Example #3 - bottles.sgd

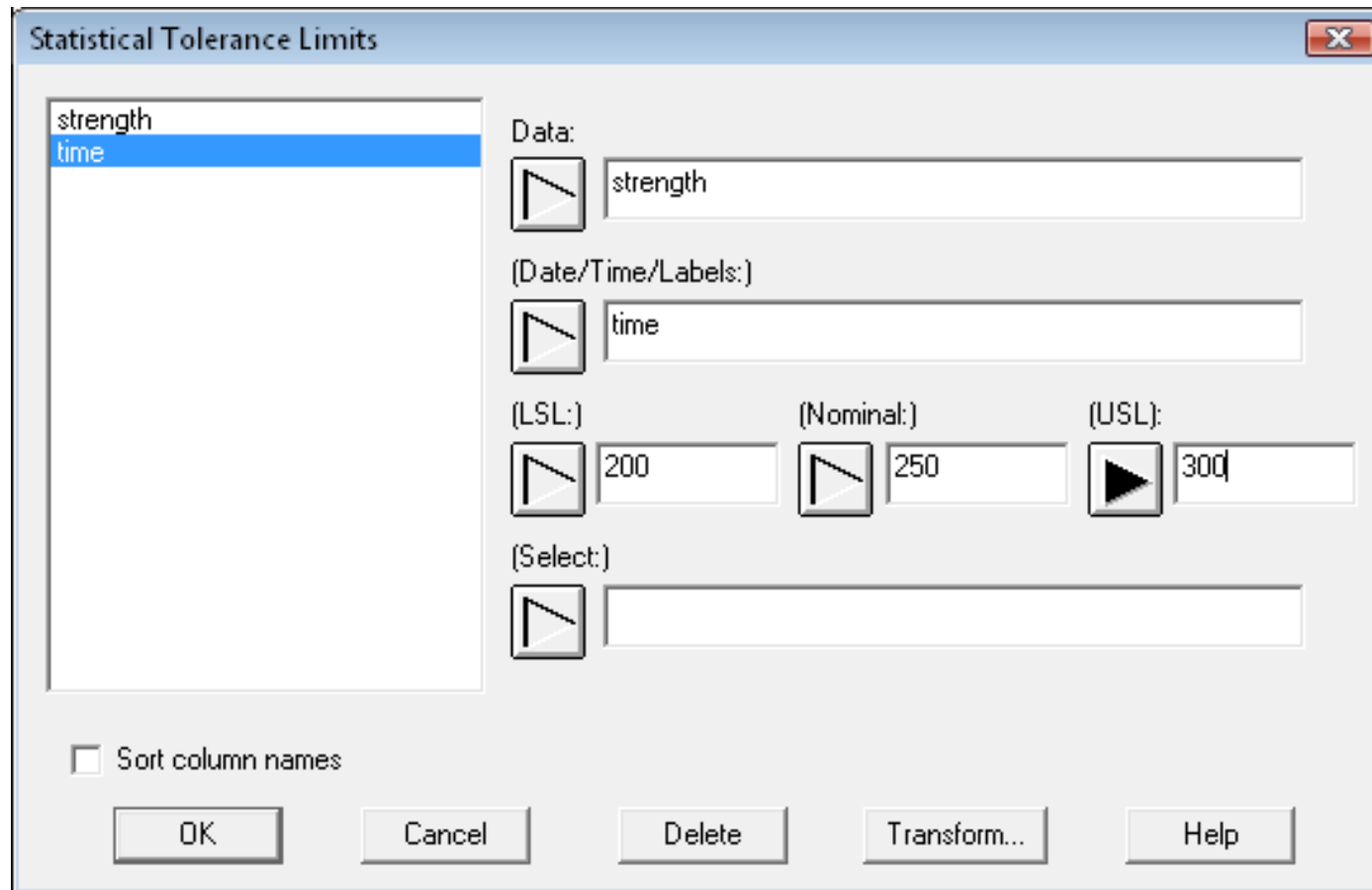
Measured breaking strength of n=100 glass bottles



The image shows a screenshot of a spreadsheet application window titled "bottles.sgd". The spreadsheet contains a table with two columns: "strength" and "time". The rows are numbered 1 through 16. The "strength" column contains numerical values, and the "time" column contains time values in HH:MM format. The first row (row 1) is highlighted in orange, and the cell containing "255" in the "strength" column is also highlighted with a black border.

	strength	time
1	255	0:10
2	232	0:20
3	282	0:30
4	260	0:40
5	255	0:50
6	233	1:00
7	240	1:10
8	255	1:20
9	254	1:30
10	259	1:40
11	235	1:50
12	262	2:00
13	272	2:10
14	247	2:20
15	251	2:30
16	245	2:40

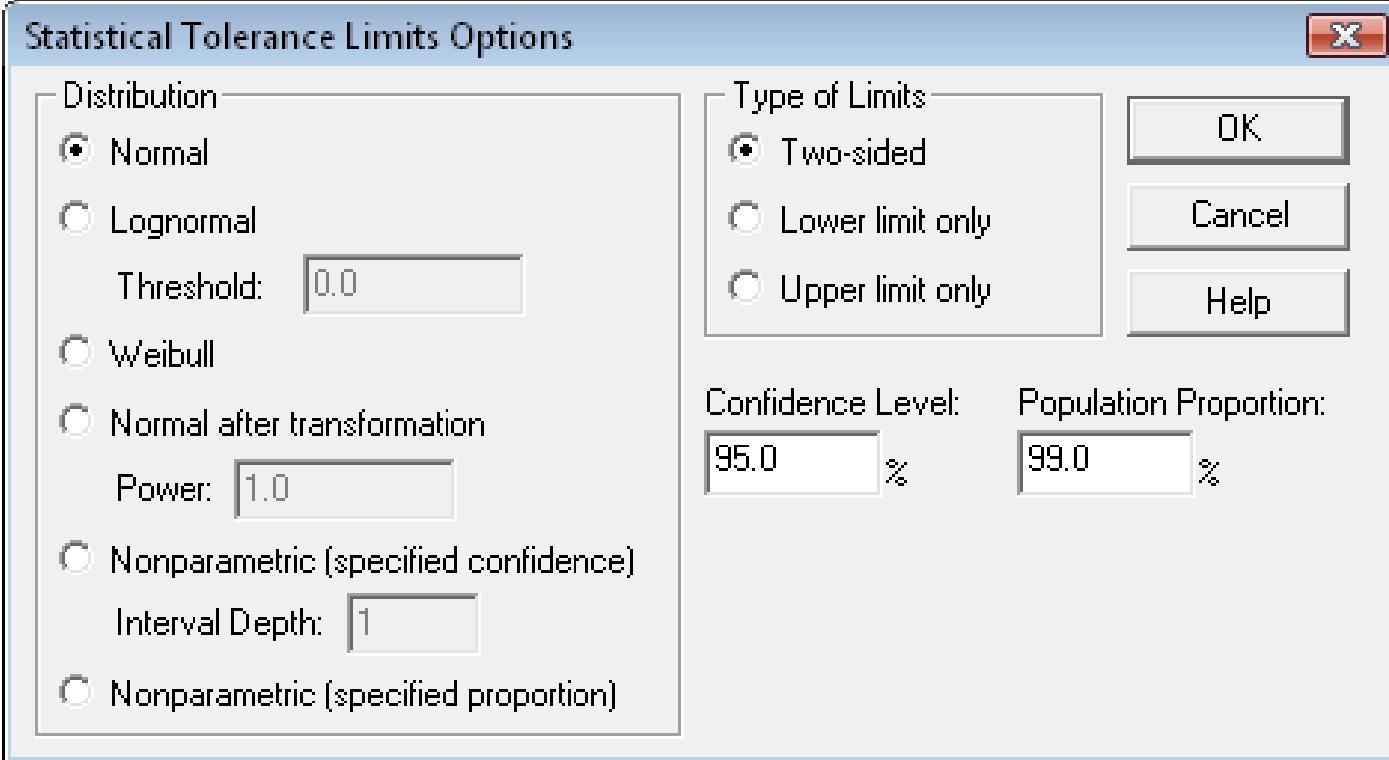
Statistical Tolerance Limits



The dialog box titled "Statistical Tolerance Limits" contains the following elements:

- A list box on the left containing "strength" and "time", with "time" selected.
- A "Data:" label followed by a text box containing "strength".
- A "(Date/Time/Labels:)" label followed by a text box containing "time".
- Three input fields for tolerance limits: "(LSL:)" with "200", "(Nominal:)" with "250", and "(USL:)" with "300".
- A "(Select:)" label followed by an empty text box.
- An unchecked checkbox labeled "Sort column names".
- Buttons for "OK", "Cancel", "Delete", "Transform...", and "Help".

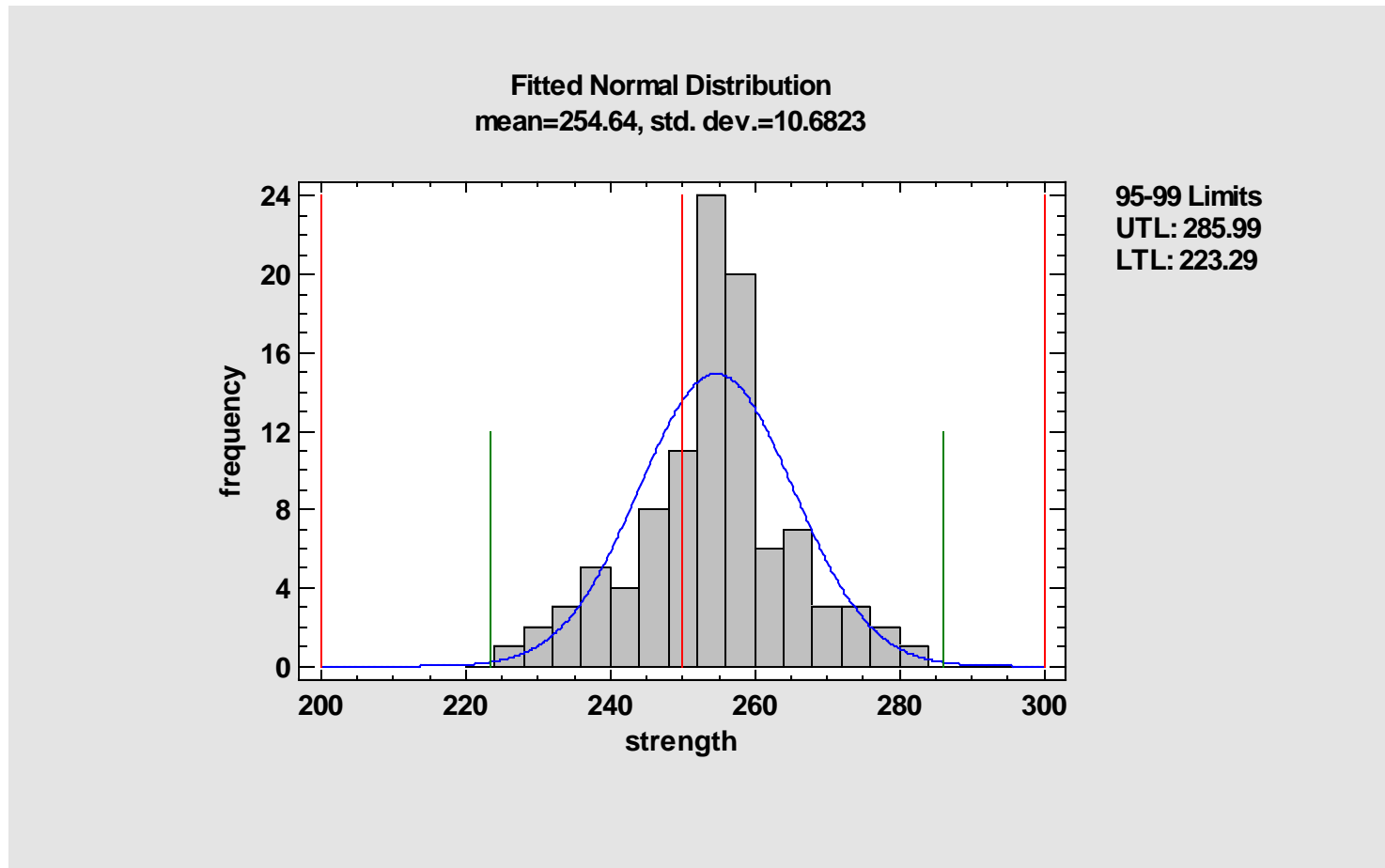
Tolerance Limit Options



The image shows a dialog box titled "Statistical Tolerance Limits Options" with a close button (X) in the top right corner. The dialog is divided into several sections:

- Distribution:** A group box containing radio buttons for "Normal" (selected), "Lognormal", "Weibull", "Normal after transformation", "Nonparametric (specified confidence)", and "Nonparametric (specified proportion)".
 - Under "Lognormal", there is a "Threshold:" label and a text box containing "0.0".
 - Under "Normal after transformation", there is a "Power:" label and a text box containing "1.0".
 - Under "Nonparametric (specified confidence)", there is an "Interval Depth:" label and a text box containing "1".
- Type of Limits:** A group box containing radio buttons for "Two-sided" (selected), "Lower limit only", and "Upper limit only".
- Confidence Level:** A label followed by a text box containing "95.0" and a percent sign (%).
- Population Proportion:** A label followed by a text box containing "99.0" and a percent sign (%).
- Buttons:** Three buttons are located on the right side: "OK", "Cancel", and "Help".

Output



Conclusions

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Assuming that strength comes from a normal distribution, the tolerance limits state that we can be 95.0% confident that 99.0% of the distribution lies between 223.295 and 285.985. This interval is computed by taking the mean of the data ± 2.93431 times the standard deviation.

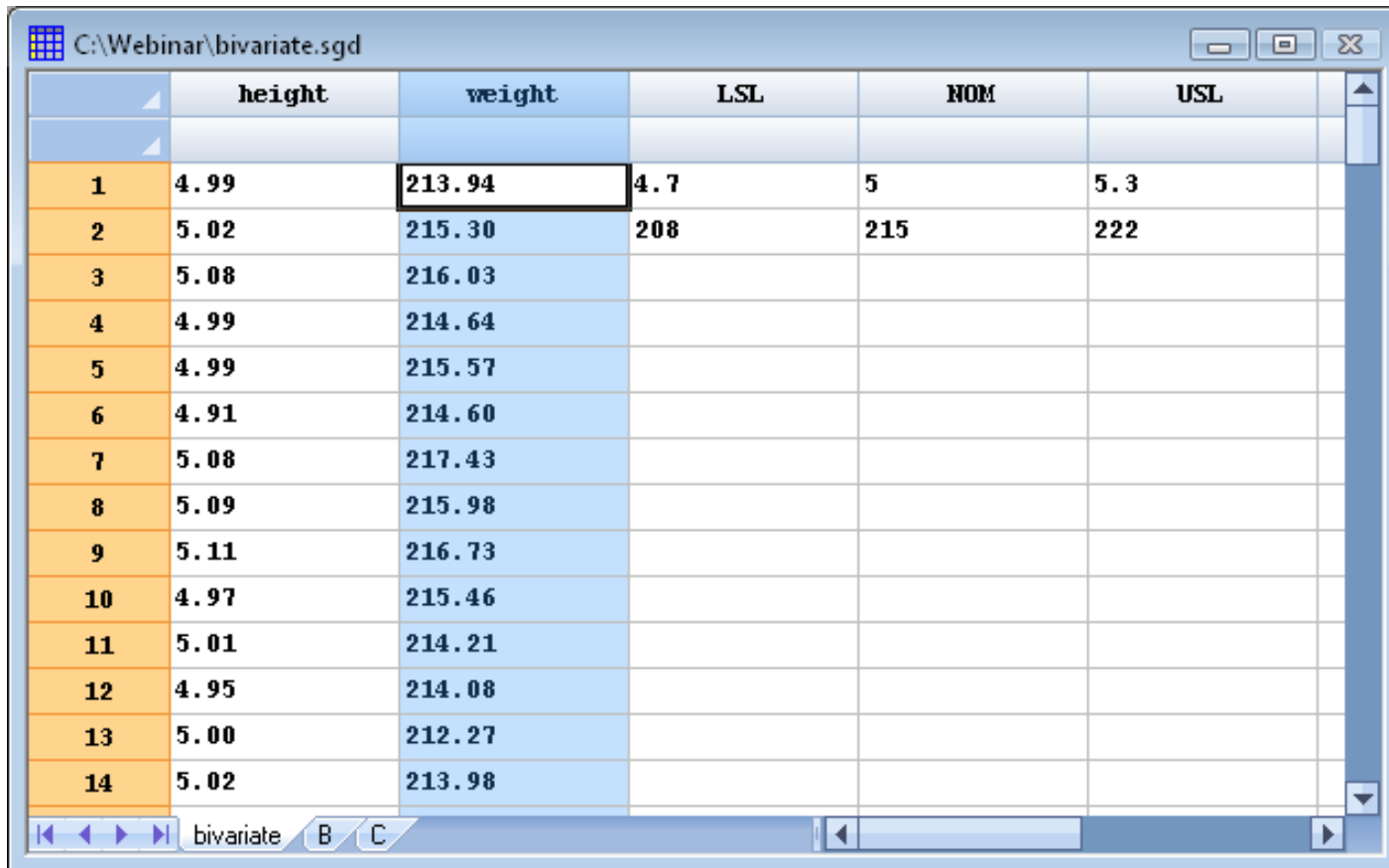
Multivariate Capability Analysis

- For multiple variables, determines the probability that ALL variables meet their established specification limits.
- Important when the variables are strongly correlated.



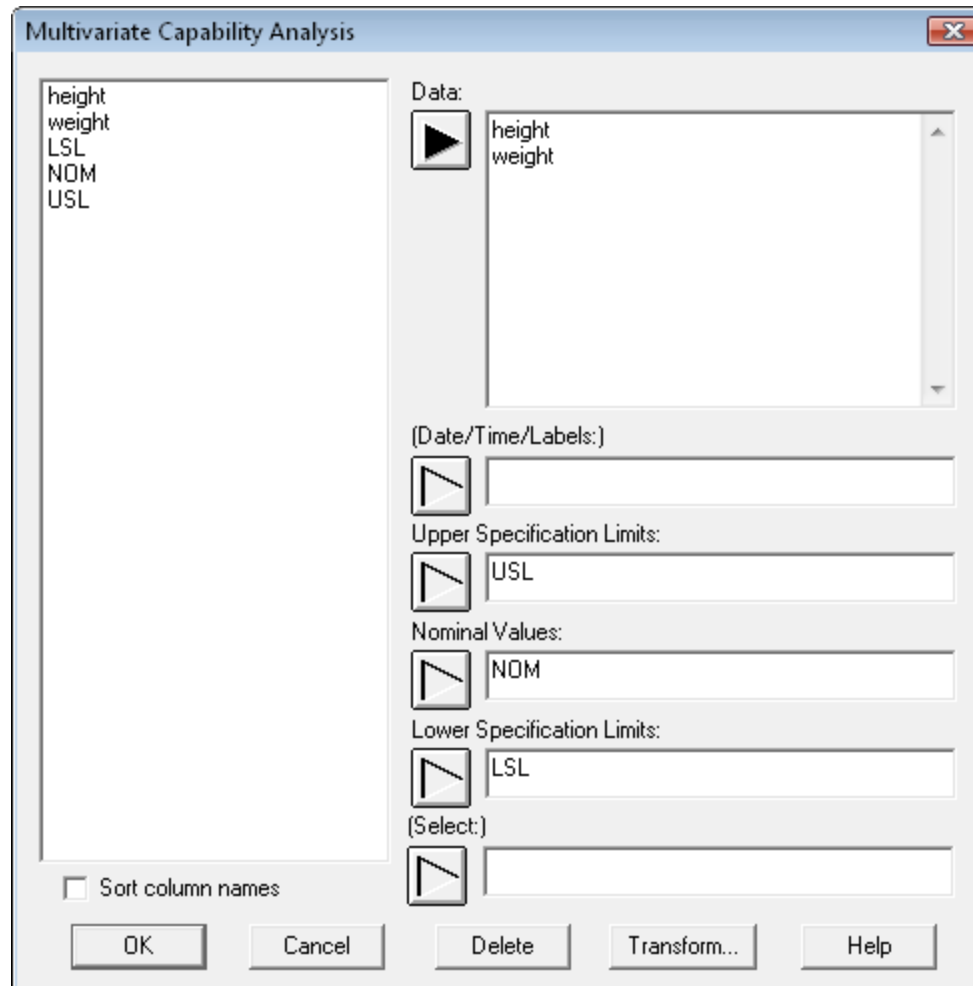
Example #4 - bivariate.sgd

Measurements of height and weight of n=150 items.
Specs: height 5 ± 0.3 , weight 215 ± 7

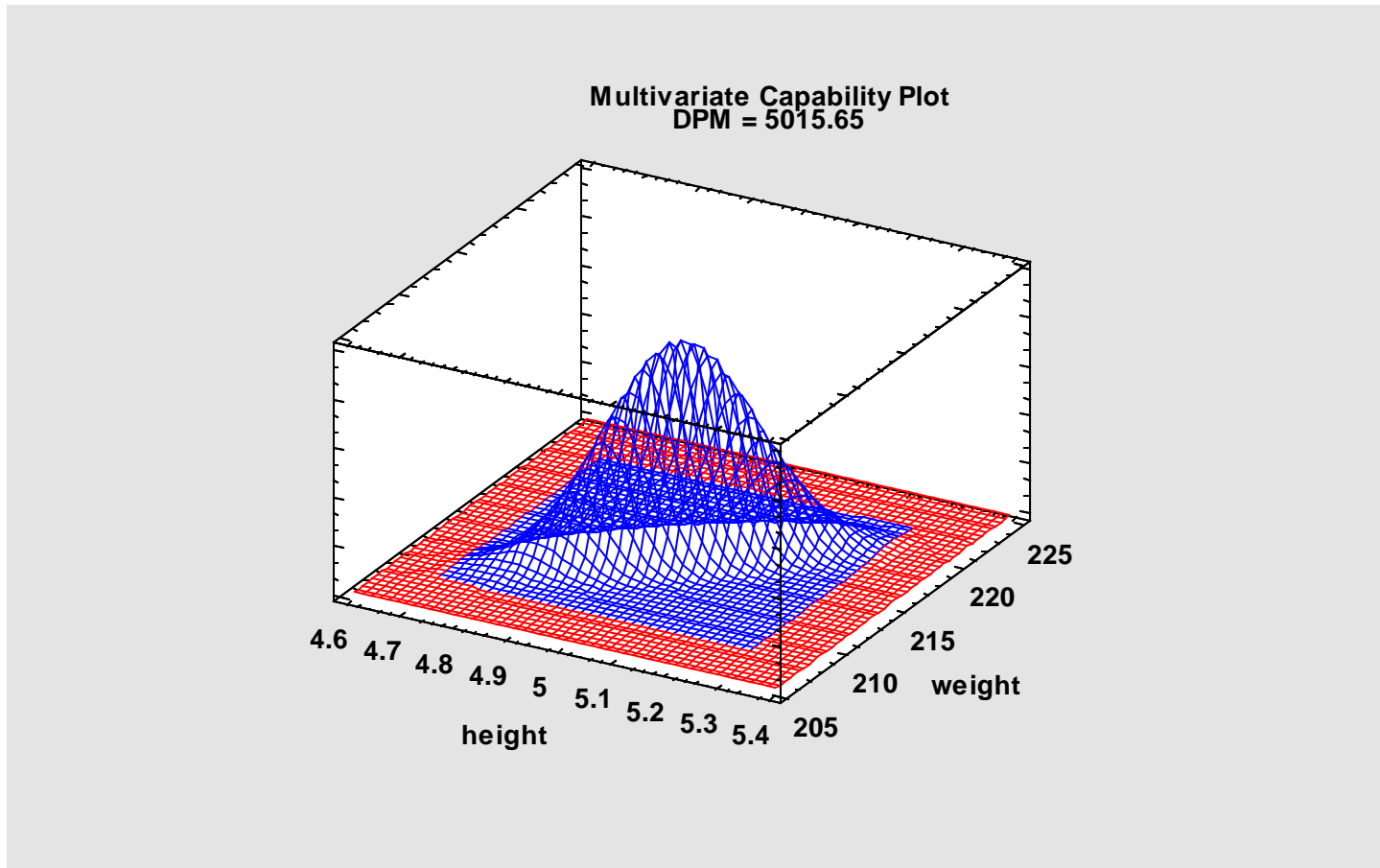


	height	weight	LSL	NOM	USL
1	4.99	213.94	4.7	5	5.3
2	5.02	215.30	208	215	222
3	5.08	216.03			
4	4.99	214.64			
5	4.99	215.57			
6	4.91	214.60			
7	5.08	217.43			
8	5.09	215.98			
9	5.11	216.73			
10	4.97	215.46			
11	5.01	214.21			
12	4.95	214.08			
13	5.00	212.27			
14	5.02	213.98			

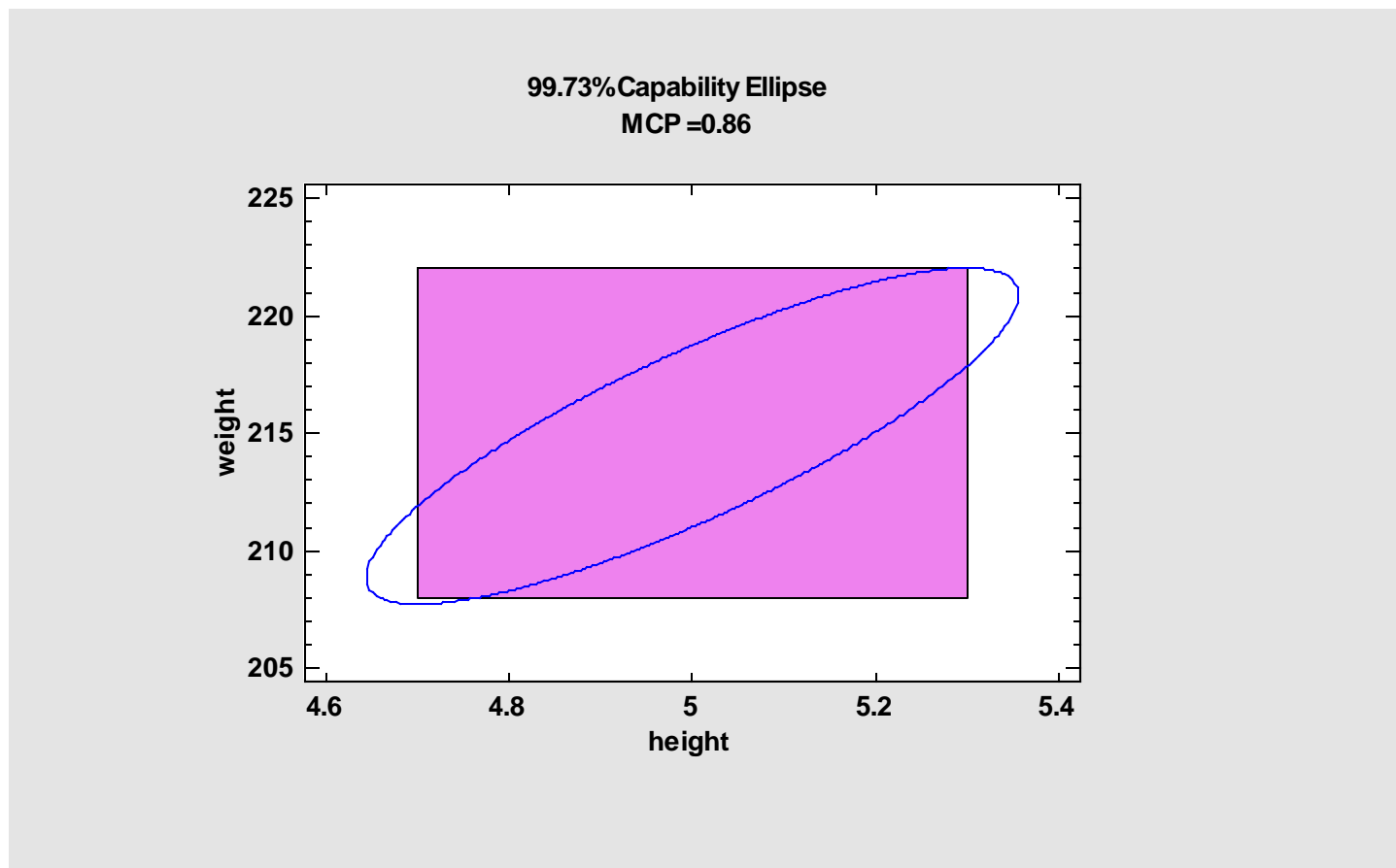
Data Input



Bivariate Normal Distribution



Capability Ellipse



Multivariate Capability

Multivariate Capability Analysis

Data variables:

height

weight

Number of complete cases: 150

	<i>Sample</i>	<i>Sample</i>			
<i>Variable</i>	<i>Mean</i>	<i>Std. Dev.</i>	<i>LSL</i>	<i>Nominal</i>	<i>USL</i>
height	5.0	0.105271	4.7	5.0	5.3
weight	214.883	2.11459	208.0	215.0	222.0

	<i>Observed</i>	<i>Estimated</i>	<i>Estimated</i>
<i>Variable</i>	<i>Beyond Spec.</i>	<i>Beyond Spec.</i>	<i>DPM</i>
height	0.0%	0.437478%	4374.78
weight	0.0%	0.0948817%	948.817
Joint	0.0%	0.501565%	5015.65

Capability Indices

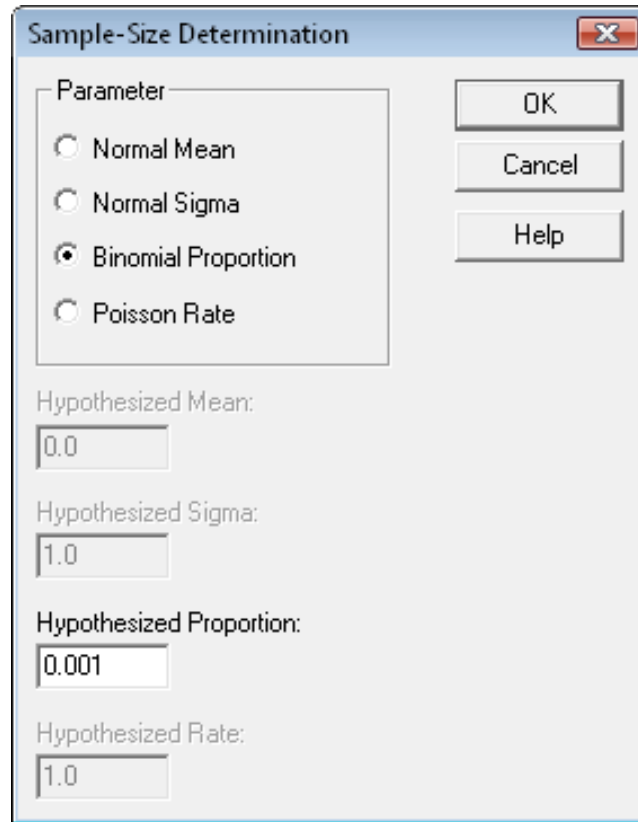
<i>Index</i>	<i>Estimate</i>
MCP	0.86
MCR	116.52
DPM	5015.65
Z	2.57475
SQL	4.07475

Based on 6.0 sigma limits.

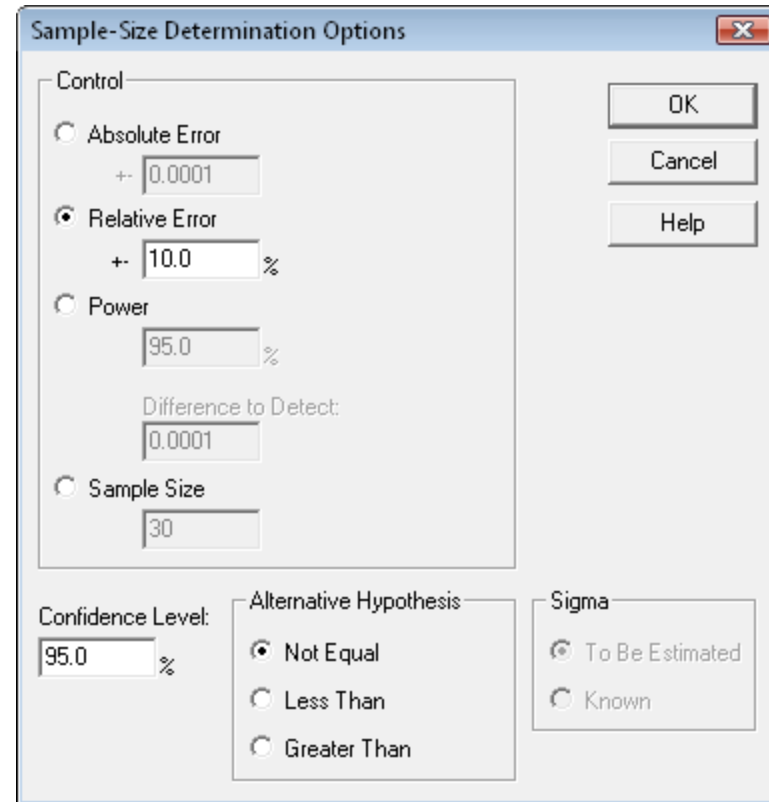
Multivariate capability indices defined to give same relationship with DPM as in univariate case.

Sample Size Determination - Counting

Suppose I wish to estimate DPM to within +/-10% with 95% confidence.



Sample-Size Determination dialog box. The 'Parameter' section has radio buttons for 'Normal Mean', 'Normal Sigma', 'Binomial Proportion' (selected), and 'Poisson Rate'. Below are input fields for 'Hypothesized Mean' (0.0), 'Hypothesized Sigma' (1.0), 'Hypothesized Proportion' (0.001), and 'Hypothesized Rate' (1.0). Buttons for 'OK', 'Cancel', and 'Help' are on the right.



Sample-Size Determination Options dialog box. The 'Control' section has radio buttons for 'Absolute Error' (with input 0.0001), 'Relative Error' (selected, with input 10.0%), 'Power' (with input 95.0%), and 'Sample Size' (with input 30). The 'Difference to Detect' field is 0.0001. The 'Confidence Level' is 95.0%. The 'Alternative Hypothesis' section has radio buttons for 'Not Equal' (selected), 'Less Than', and 'Greater Than'. The 'Sigma' section has radio buttons for 'To Be Estimated' (selected) and 'Known'. Buttons for 'OK', 'Cancel', and 'Help' are on the right.

Sample-Size Determination

Parameter to be estimated: binomial parameter

Desired tolerance: +/- 10.0% when proportion = 0.001

Confidence level: 95.0%

The required sample size is $n=422065$ observations.

Sample Size Determination – Capability Indices

Suppose I wish to estimate Cpk to within +/-10% with 95% confidence.

Sample Size Determination (Capability Indices)

Index

Cp

Cpk

Cpm

OK

Cancel

Help

Estimated index: 1.33

Mean minus target: 1.0 sigma

Relative error: 10.0 %

Confidence Level: 95.0 %

Sample Size Determination (Capability Indices)

Capability index: Cpk

Estimate: 1.33

Relative error: 10.0%

Confidence level: 95.0%

The required sample size is 154.

Requires measuring $n = 154$ items

Sample Size Determination – Statistical Tolerance Limits

Sample Size Determination - Statistical Tolerance Limits

Distribution:

- Normal
- Lognormal
- Weibull
- Nonparametric

Normal Distribution Parameters:

Mean: 250.0 Sigma: 11.0

Lognormal Distribution Parameters:

Mean: 50.0 Sigma: 10.0 Threshold: 0.0

Weibull Distribution Parameters:

Shape: 5.0 Scale: 50.0 Threshold: 0.0

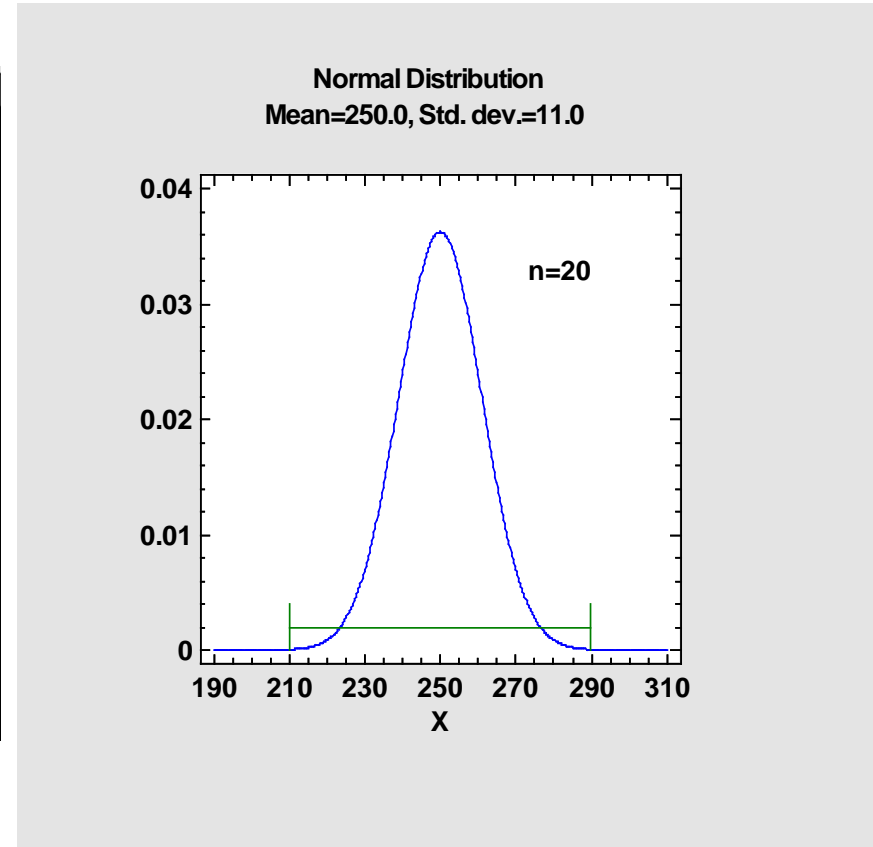
Type of Limits:

- Two-sided
- Lower limit only
- Upper limit only

Confidence Level: 95.0 % Population Proportion: 99.0 %

Lower Spec. Limit: 200.0 Upper Spec. Limit: 300.0 Allowance: 80.0 %

Buttons: OK, Cancel, Help



A 95-99 tolerance interval covering 80% of the distance between the spec limits requires a sample of $n = 20$ items in this case.

More Information

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