

Design of Experiments – Single Factor Categorical Designs

Summary

The *Single Factor Categorical Designs* selection on the *Create Design* menu creates experimental designs for situations where primary interest centers on comparing levels of a single categorical factor. The procedure will generate runs at each level of that factor. In addition, one or more blocking factors may be included in the design. The procedure is capable of generating:

1. completely randomized designs
2. randomized block designs
3. balanced incomplete block (BIB) designs
4. Latin squares
5. Graeco-Latin squares
6. Hyper-Graeco-Latin squares

Example

The example described in this documentation comes from Box, Hunter and Hunter (1978). They describe an experiment performed to compare the effect of 7 treatments (A, B, C, D, E, F, and G) on the wearing quality of a particular material. Unfortunately, the machine used to measure wear could only accommodate 4 samples during any one run. Expecting potential differences between runs of the machine, they wished to treat the runs as a blocking variable in order to reduce any potential confounding of run-to-run differences with differences between the treatments.

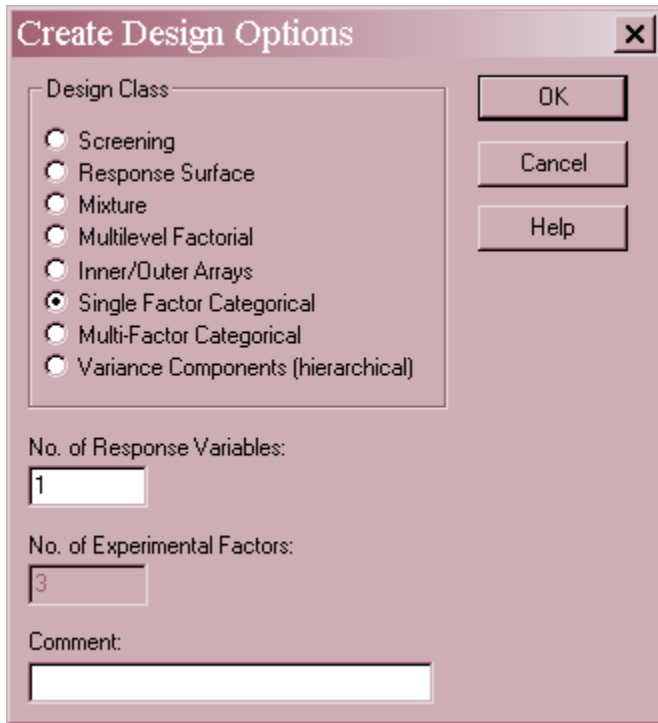
Sample StatFolio: doe singlecat.sgp

Design Creation

To create an experiment in which the primary goal is to compare q levels of a single categorical factor, select *Create Design* from the design of experiments menu and complete the dialog boxes as described below.

Dialog #1 – Design Type

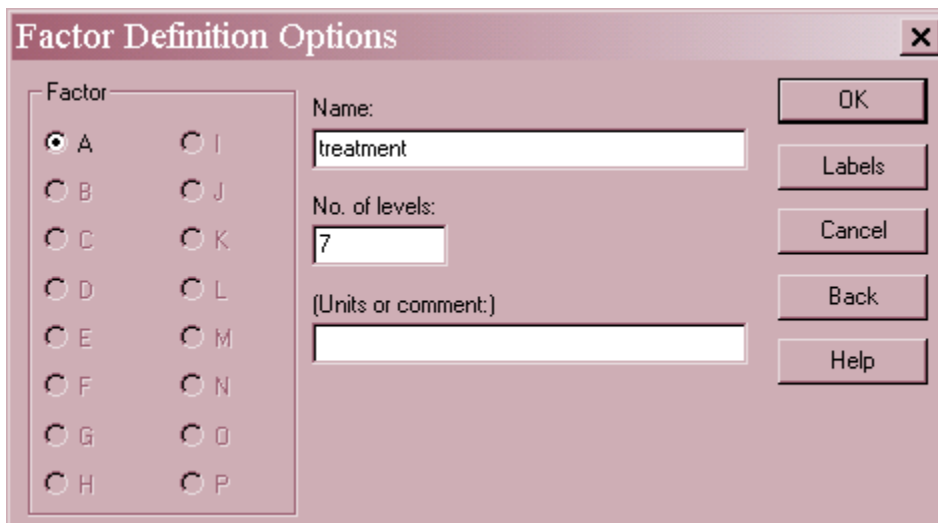
The first dialog box displayed during design creation specifies the type of design to be created:



- **Design Class:** type of design to be created.
- **No. of Response Variables:** the number of response variables Y that will be measured during each experimental run. This number may range from 1 to 16.
- **Comment:** a comment to appear on the output from the analysis procedures.

Dialog #2 – Experimental Factor

The second dialog box requests information about the experimental factor to be tested:



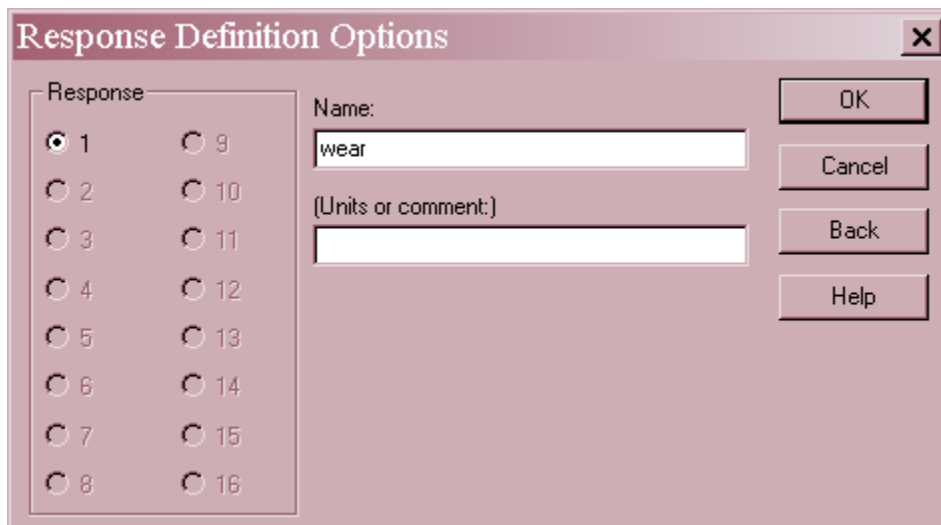
- **Name:** enter a name for the factor containing up to 32 characters. A column will be created in the datasheet with the indicated name.
- **No. of levels:** number of different levels of the factor at which experiments will be performed.
- **Units or comment** - an optional label or comment of up to 64 characters to be included on the experiment worksheet.
- **Labels button:** Press this button to enter identifiers for each level:



If labels are not specified, the factor levels will be numbered from 1 through q .

Dialog #3 – Response Variables

The third dialog box requests information about each of the response variables:

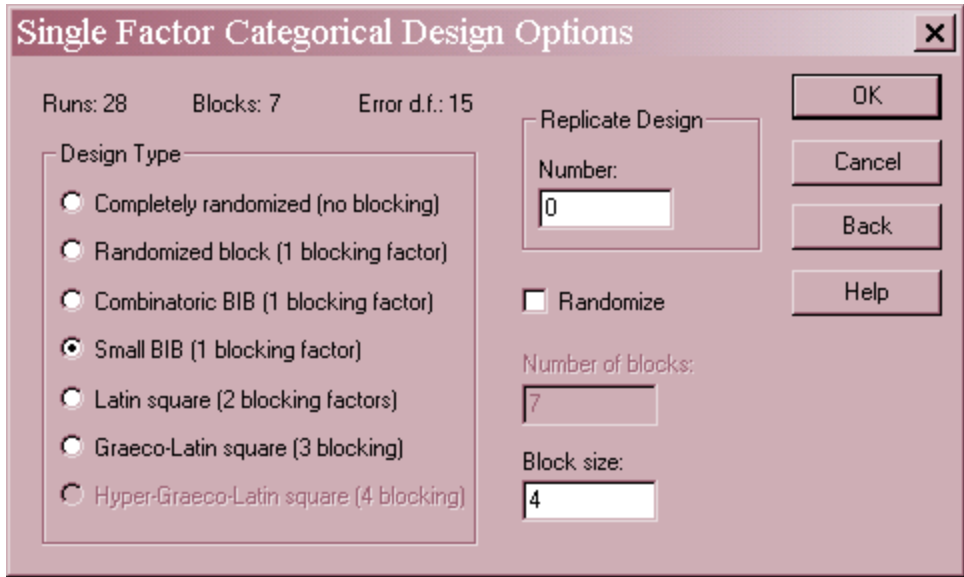


Click on the numbers 1, 2, 3, ..., one at a time and enter the following information for each response variable in the experiment:

- **Name** - a name for each response containing up to 32 characters.
- **Units or comment** - an optional label or comment of up to 64 characters to be included on the experiment worksheet.

Dialog #4 – Design Selection

The fourth dialog box is used to specify the type of design to be created:



- **Design Type:** The following types of designs may be available, depending upon the number of levels of the experimental factor:
 1. *Completely randomized design* - a design in which a random sample of measurements is taken from each of the q levels, with no attempt to account for the effects of any other factor.
 2. *Randomized block design* - a design in which an equal number of observations is taken from each treatment at two or more levels of a blocking or nuisance factor. Block effects are included in the model to reduce the magnitude of the experimental error.
 3. *Combinatoric BIB* - a Balanced Incomplete Block design involving a single blocking variable where the number of treatments in each block is less than q . If k treatments can be run in each block, the design requires $\binom{q}{k}$ blocks, which represents the number of ways of choosing k items out of q .
 4. *Small BIB* - a Balanced Incomplete Block design in which the number of blocks is less than that required by a full combinatoric BIB. These designs are only available for certain combinations of the number of factor levels and the block size.

5. *Latin square* - a design in which treatments are balanced across two blocking factors, but where the number of runs is less than would be required by a full multi-factor factorial design.
 6. *Graeco-Latin square* - a design in which treatments are balanced across three blocking factors.
 7. *Hyper-Graeco-Latin square* - a design in which treatments are balanced across four blocking factors.
- **Replicate Design** - the number of additional observations to be taken from each treatment level or treatment-block combination.
 - **Randomize** - whether or not to randomly order the runs in the experiment.
 - **Number of blocks** - for designs involving one or more blocking factors, the number of blocks.
 - **Block size** - for BIB designs, the number of treatments that can be tested in each block.

Based on the selected design, the dialog box calculates and displays the total number of runs (tests) to be performed, the number of blocks, and the degrees of freedom that will be available to estimate the experimental error. Note that the degrees of freedom are calculated assuming that the blocking factors do not interact with the main experimental factor, which is the usual assumption.

The dialog box above requests a small BIB design capable of testing four treatments in a single block. As indicated, a design is available in 7 blocks (runs of the machine). A total of 28 tests will be performed, meaning that each of the 7 materials will be included in 4 blocks.

Design Attributes

Once the dialog boxes have been completed, the *Design Attributes* window will be displayed:

<u>Single Factor Categorical Design Attributes</u>		
Design class: Single Factor Categorical		
File name: <Untitled>		
Base Design		
Number of experimental factors: 1		
Number of blocks: 7		
Number of responses: 1		
Number of runs: 28		
Error degrees of freedom: 15		
Randomized: No		
<i>Factors</i>	<i>Levels</i>	<i>Units</i>
treatment	7	
<i>Responses</i>	<i>Units</i>	
wear		

It contains a summary of the generated design. To save the design, select *Save Design File* from the main *File* menu. The example is saved in the *weartest.sfx* file.

Worksheet

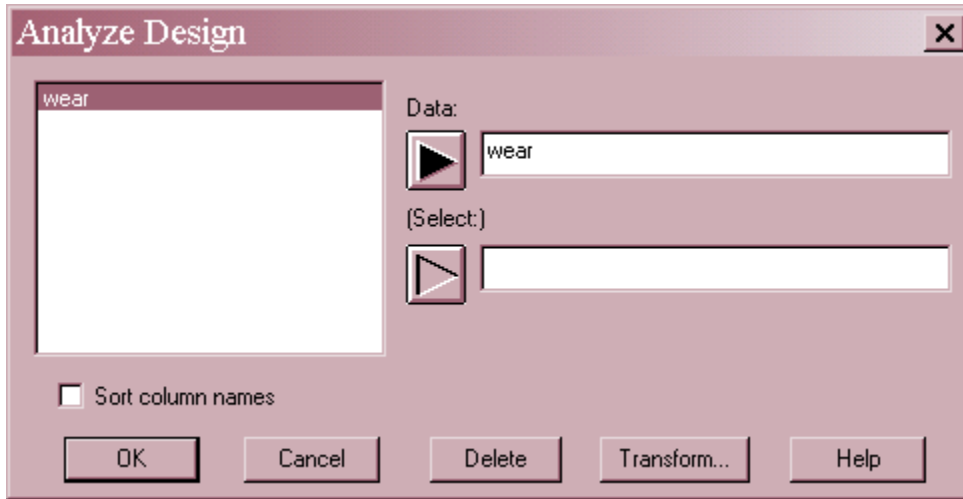
The experimental runs are entered into the datasheet automatically. They may also be displayed in the *Worksheet* shown below:

Worksheet for weartest.sfx			
<i>run</i>	<i>block</i>	<i>treatment</i>	<i>wear</i>
1	1	C	
2	1	D	
3	1	F	
4	1	G	
5	2	A	
6	2	B	
7	2	F	
8	2	G	
9	3	B	
10	3	D	
11	3	E	
12	3	G	
13	4	A	
14	4	C	
15	4	E	
16	4	G	
17	5	B	
18	5	C	
19	5	E	
20	5	F	
21	6	A	
22	6	D	
23	6	E	
24	6	F	
25	7	A	
26	7	B	
27	7	C	
28	7	D	

Examining the worksheet closely, it will be noted that each treatment appears 4 times. In addition, each pair of treatments occurs together in the same block twice. This gives the design as much balance as possible without running every treatment in every block.

Analyzing the Data

After the results of the experimental runs have been entered, *Analyze Data* can be selected from the DOE menu. A dialog box will be displayed requesting the column containing the response to be analyzed:



- **Data:** column containing the values of the response variable to be analyzed.
- **Select:** subset selection.

When OK is pressed, the program will invoke one of two procedures:

1. The *Oneway ANOVA* procedure for a completely randomized design with no blocking variables.
2. The *Multifactor ANOVA* procedure for the designs containing one or more blocking variables.

Full details of those analyses are contained in the corresponding documentation.

Of particular interest in the current example are several tables and graphs:

ANOVA Table

This table is used to judge whether or not there are statistically significant differences between the levels of the experimental factor:

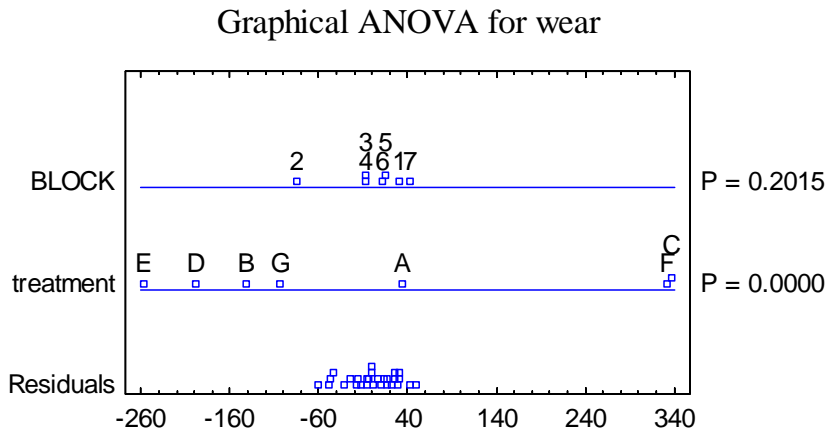
Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
MAIN EFFECTS					
A:treatment	506799.0	6	84466.4	57.40	0.0000
B:BLOCK	14570.1	6	2428.35	1.65	0.2015
RESIDUAL	22071.4	15	1471.43		
TOTAL (CORRECTED)	626265.0	27			

All F-ratios are based on the residual mean square error.

A small P-value for factor A (less than 0.05 if operating at the 5% significance level) indicates that there *are* significant differences between treatments. If the current example, the differences are highly significant. Of secondary interest is the P-value for *BLOCK*. Since the P-value in the above table is greater than 0.05, the block effects are *not* statistically significant, meaning that there were not large differences between runs of the wear testing machine.

Graphical ANOVA

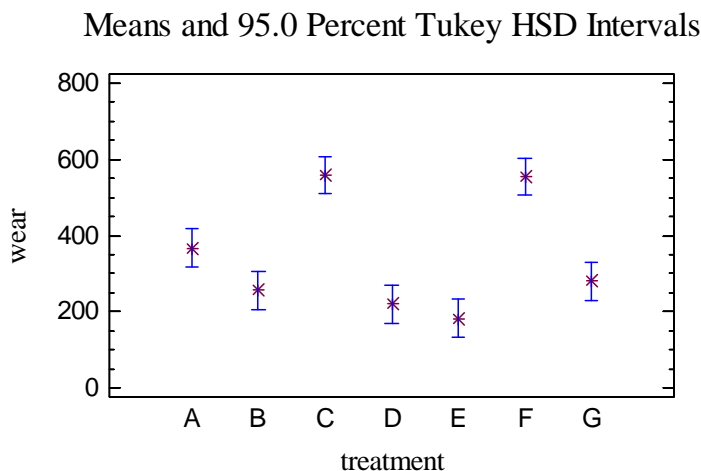
A new method for illustrating the differences between blocks and treatments, from Hunter (2005), is shown below:



The plot shows the scaled deviations of the block and treatment averages from the grand mean, together with the model residuals. Scaling is such that, if a factor has no effect, the variation observed should be comparable to that of the residuals. Note that the variation among blocks is well within that observed for the residuals, with the possible exception of block #2. Note also that treatment E shows the least wear, although other some other treatments are relatively close.

Main Effects Plot

The *Main Effects Plot* can be used to determine which treatments are significantly different from which others. Because of the large number of treatments, the plot below shows the treatment means with Tukey HSD intervals, which allows the experimenter to compare all pairs of treatments with an experiment-wide error rate of 5%:



Treatment E showed the least wear on average. However, since its interval overlaps those of treatments B, D and G, it cannot be declared to be significantly better than any of those 3 other treatments.