Design of Experiments Wizard – Robust Parameter Designs

Summary

The DOE Wizard can create experimental designs for use in robust parameter design (RPD). In such experiments, two types of factors are varied:

- *controllable factors* that the experimenter can manipulate both during the experiment and during production.

- *noise factors* that can be manipulated during the experiment but are normally uncontrollable.

The goal of RPD is to find levels of the controllable factors where the response variables are relatively insensitive to changes in the noise factors.

In STATGRAPHICS, robust parameter designs can be constructed in 2 ways:

1. *Crossed approach* – Two separate designs can be created, one for the controllable factors and one for the noise factors. These 2 designs can then be merged by creating runs with all combinations of runs from both designs. This is the method first suggested by Genichi Taguchi and is described in the STATGRAPHICS document titled *DOE Wizard – Inner/Outer Arrays*.

2. *Combined approach* – Both the controllable factors and the noise factors can be studied in a single design. This approach is described by Myers, Montgomery and Anderson-Cook (2009) and has several advantages, including fewer total runs and more insight into the effects of the factors on both the mean and variance of the response.

Example

This document describes the combined approach to robust parameter design, using an example from Myers et al. They describe an experiment performed to study the quality of the images transmitted by color television signals. In that experiment, there were 2 controllable factors:

Controllable Factor	*Low*	*High*	*Units*
filter tabs	5	21	count
sampling frequency	6.25	13.5	MHz

and 2 normally uncontrollable noise factors:

Noise Factor	*Low*	*High*	*Units*
image bits	256	512	count
voltage	100	200	volts

Sample StatFolio: doewiz rpd.sgp

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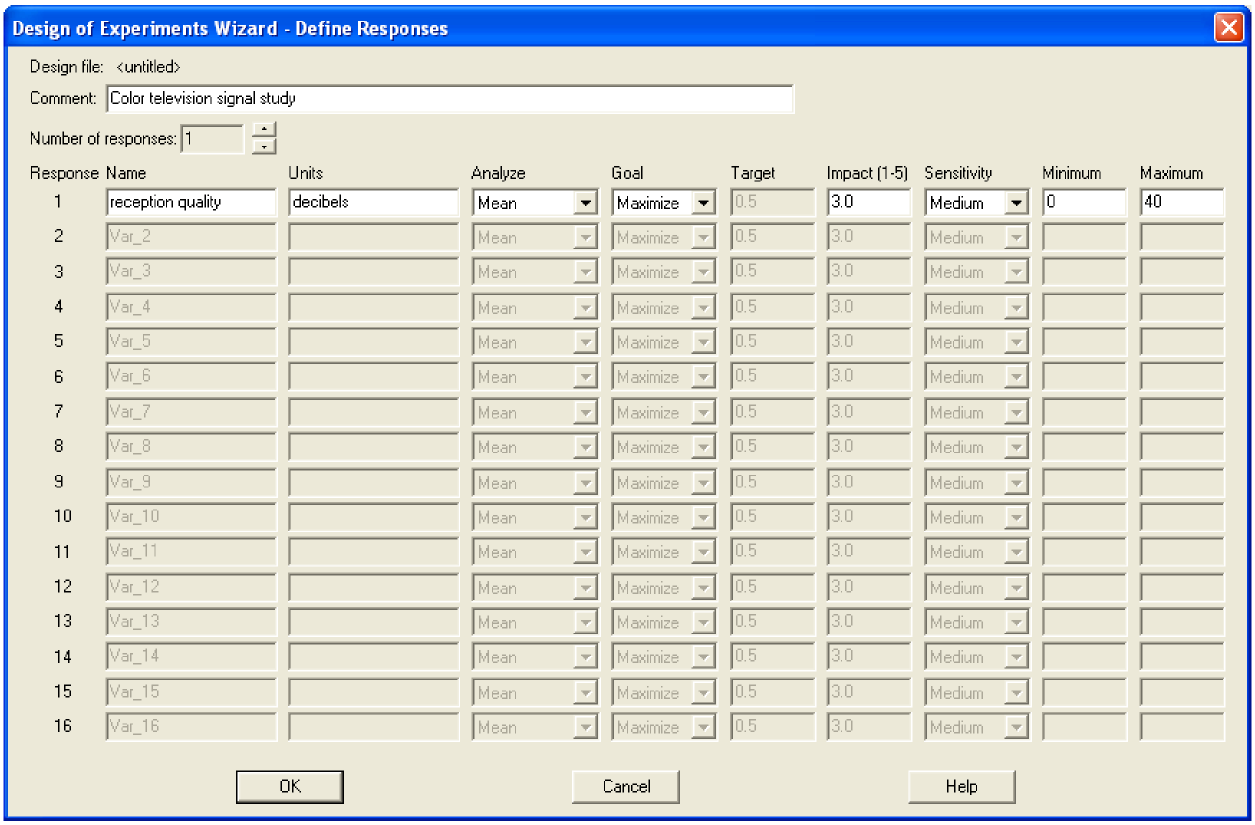
Design Creation

To begin the design creation process, start with an empty StatFolio. Select *DOE – Experimental Design Wizard* to load the DOE Wizard's main window. Then push each button in sequence to create the design.

Step #1 – Define Responses

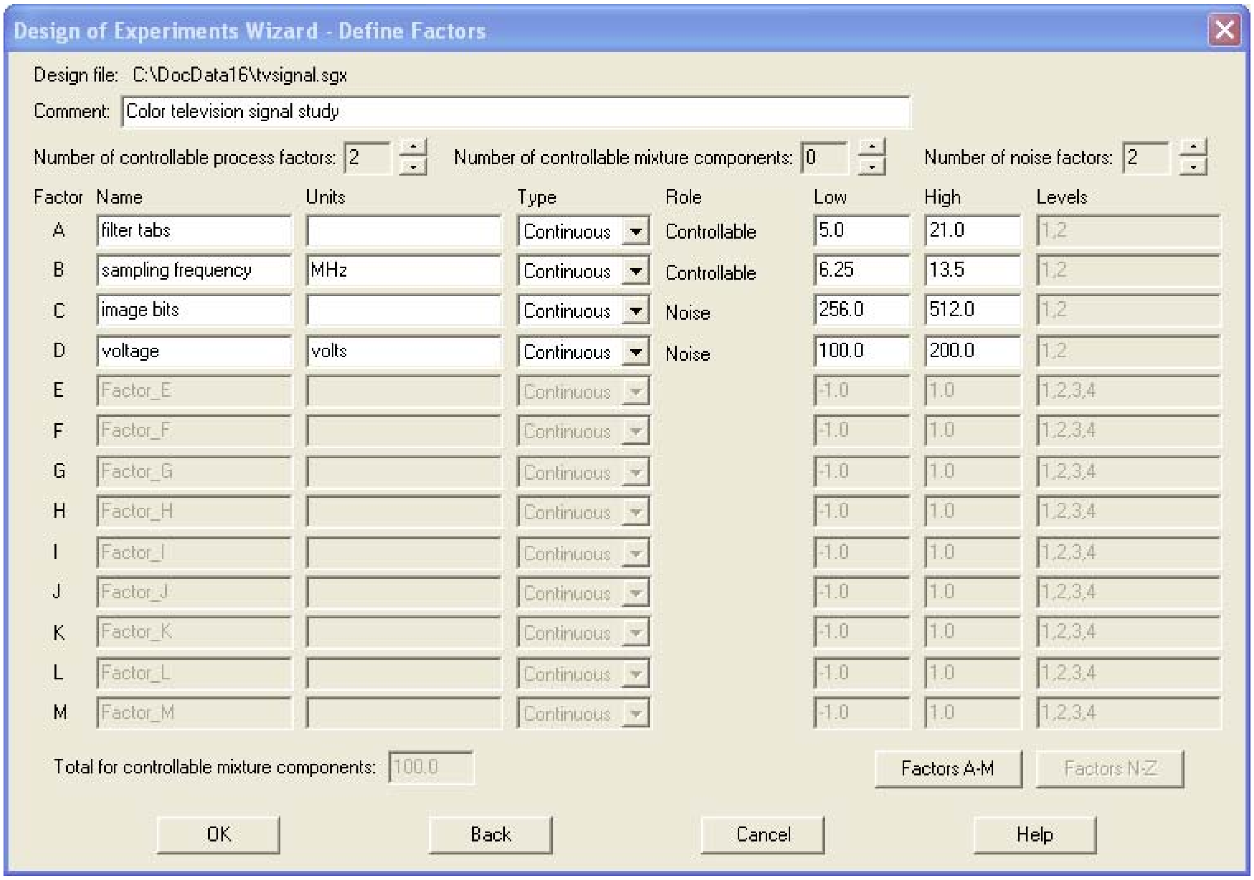
The first step of the design creation process displays a dialog box used to specify the response variables. The response of interest in the current experiment is the quality of the received signal, measured in decibels:



In an RPD study, you should set the *Analyze* field to *Mean*, although both the mean and variance will be modeled. Note that the specified range for *reception quality* is 0 to 40. This range is used to create a desirability function during later optimization procedures, with 0 decibels having desirability equal to 0, while 40 decibels or higher has desirability equal to 1. Since *Sensitivity* is set to *Medium*, the desirability of the response increases linearly between 0 and 40.

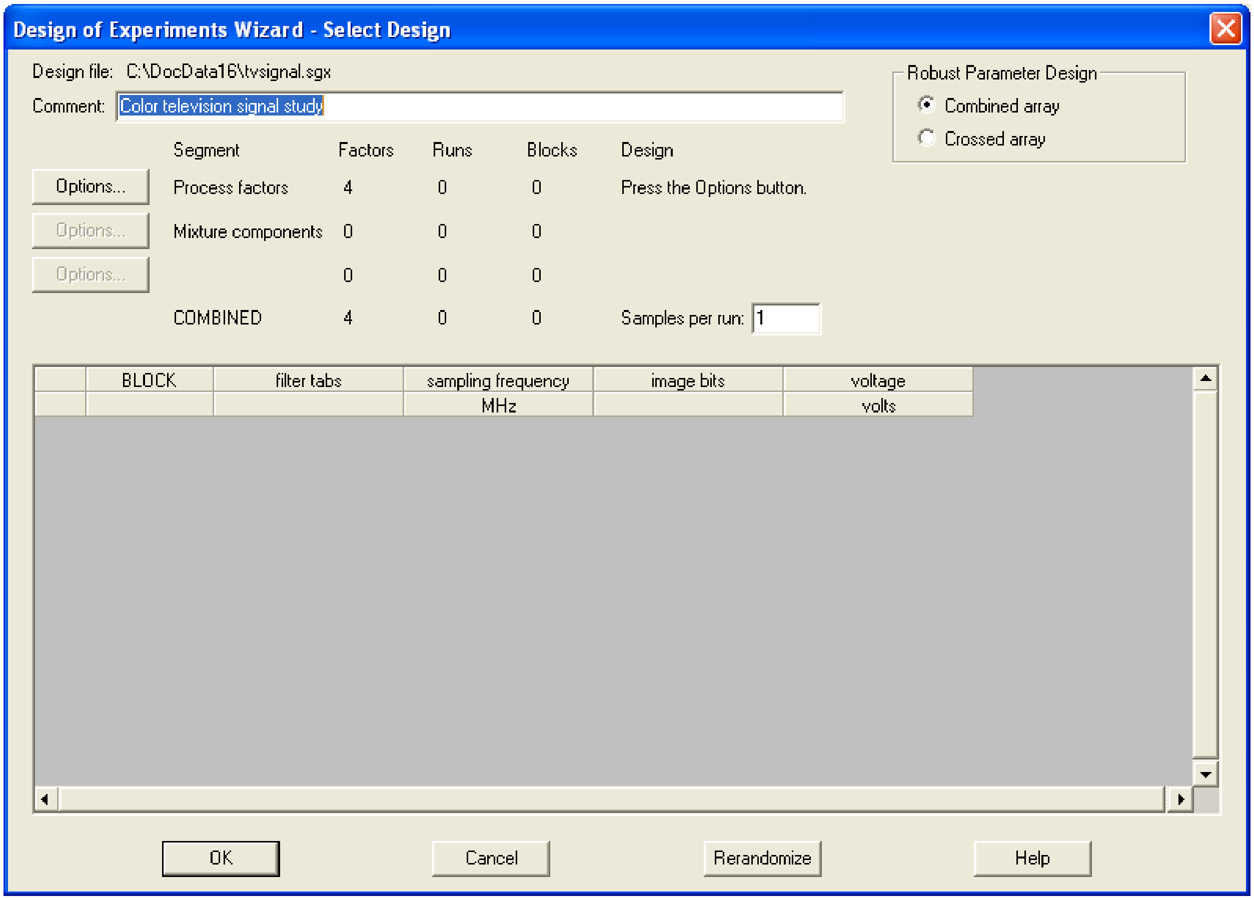
Step #2 – Define Experimental Factors

The second step displays a dialog box used to specify the factors that will be varied. In the current example, there are 2 controllable process factors and 2 noise factors:



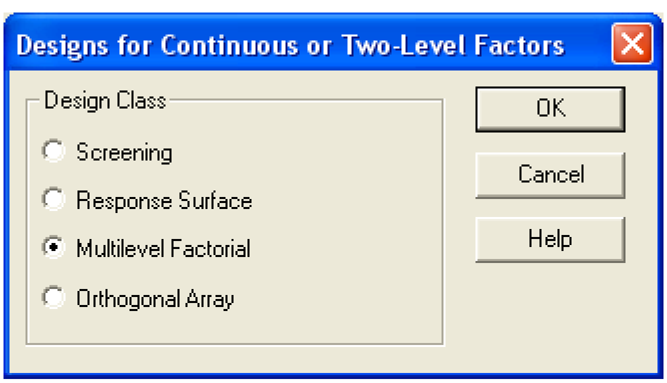
Because of the manner in which the standard deviation is estimated in an RPD study with combined factors, all of the noise factors must be *continuous*.

Step #3 – Select Design
The third step begins by displaying the dialog box shown below:



The first choice that must be made is whether to combine the controllable and noise factors into a single design, or whether to create separate designs for each set and then run all combinations of runs in both sets. To use the combined approach described in this document, set the checkbox under *Robust Parameter Design* to *Combined array*.

Next, press the topmost *Options* button to select a design for all 4 factors. This displays the following dialog box:



Since all of the controllable factors are continuous, four types of designs are available:

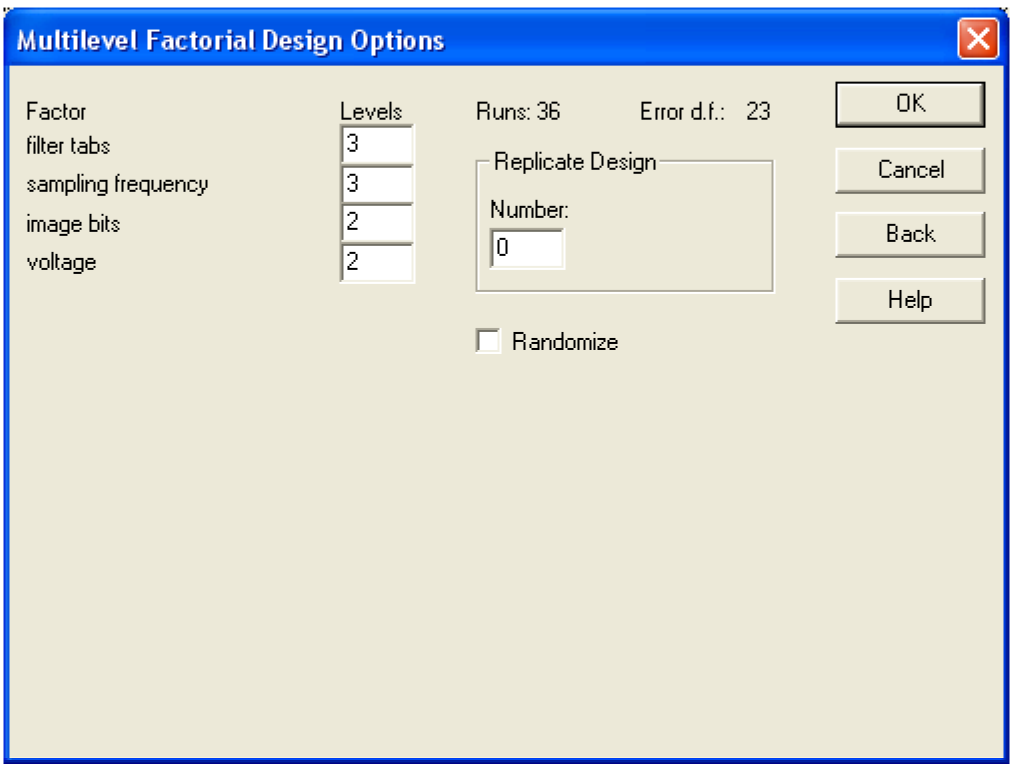
1. *Screening* - designs intended to select the most important factors affecting a response. Most of the designs involve only 2 levels of each factor. The factors may be quantitative or categorical.

2. *Response Surface* - designs intended to select the optimal settings of a set of experimental factors. The designs involve at least 3 levels of the experimental factors, which must be quantitative.

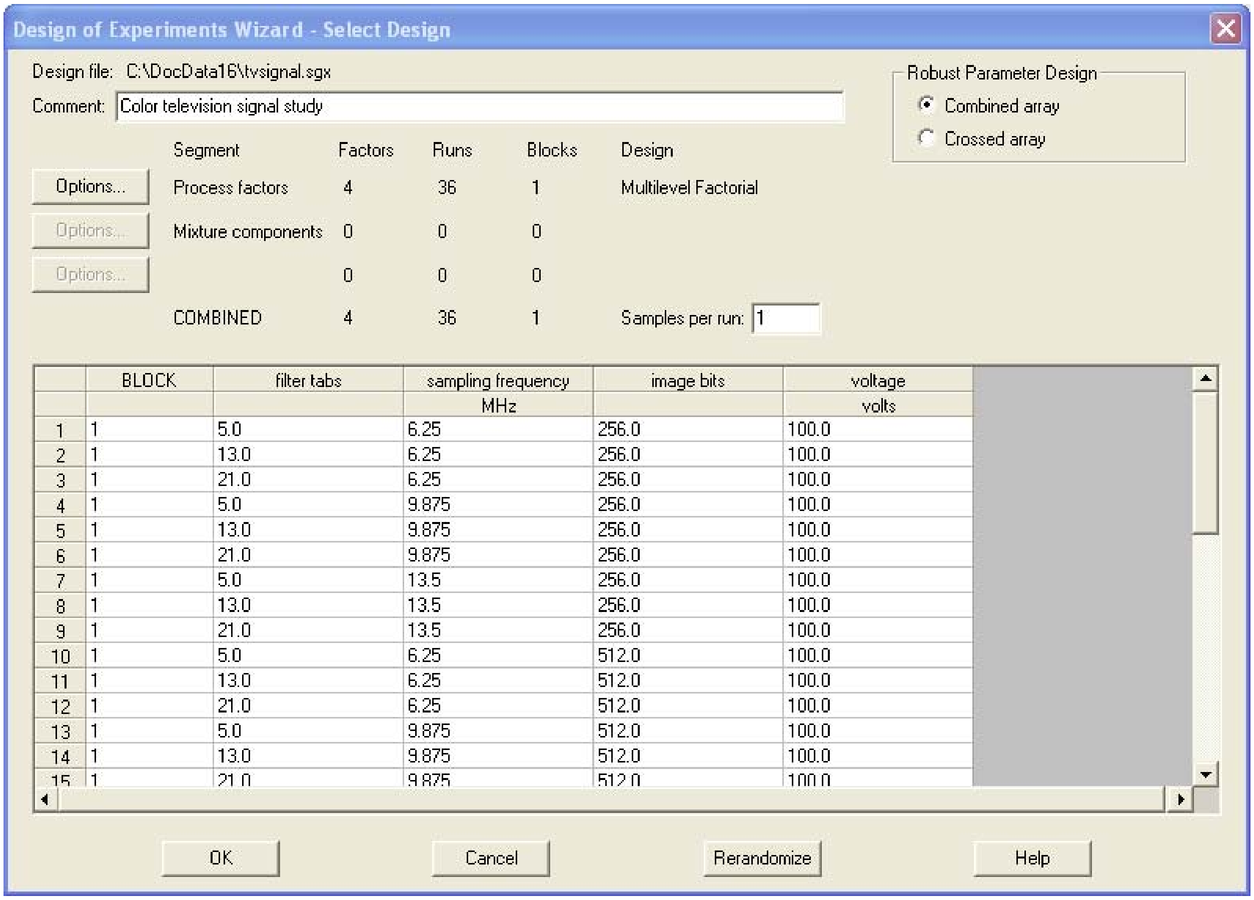
3. *Multilevel Factorial* - designs involving different numbers of levels for each experimental factor. The factors must be quantitative.

4. *Orthogonal Array* – a general class of designs developed by Genichi Taguchi. The factors may be quantitative or categorical.

Selecting the *Multilevel Factorial* option and pressing OK displays another dialog box:



In this example, the experimenters choose to run 3 levels of each of the controllable factors and 2 levels of each of the noise factors, resulting in a total of 36 runs. Complete the dialog box as shown above and press *OK* to put the design in the *Select Design* window:



Design of Experiments Wizard - Select Design

Design file: C:\DocData16\tvsignal.sgx

Comment: Color television signal study

Robust Parameter Design

Combined array

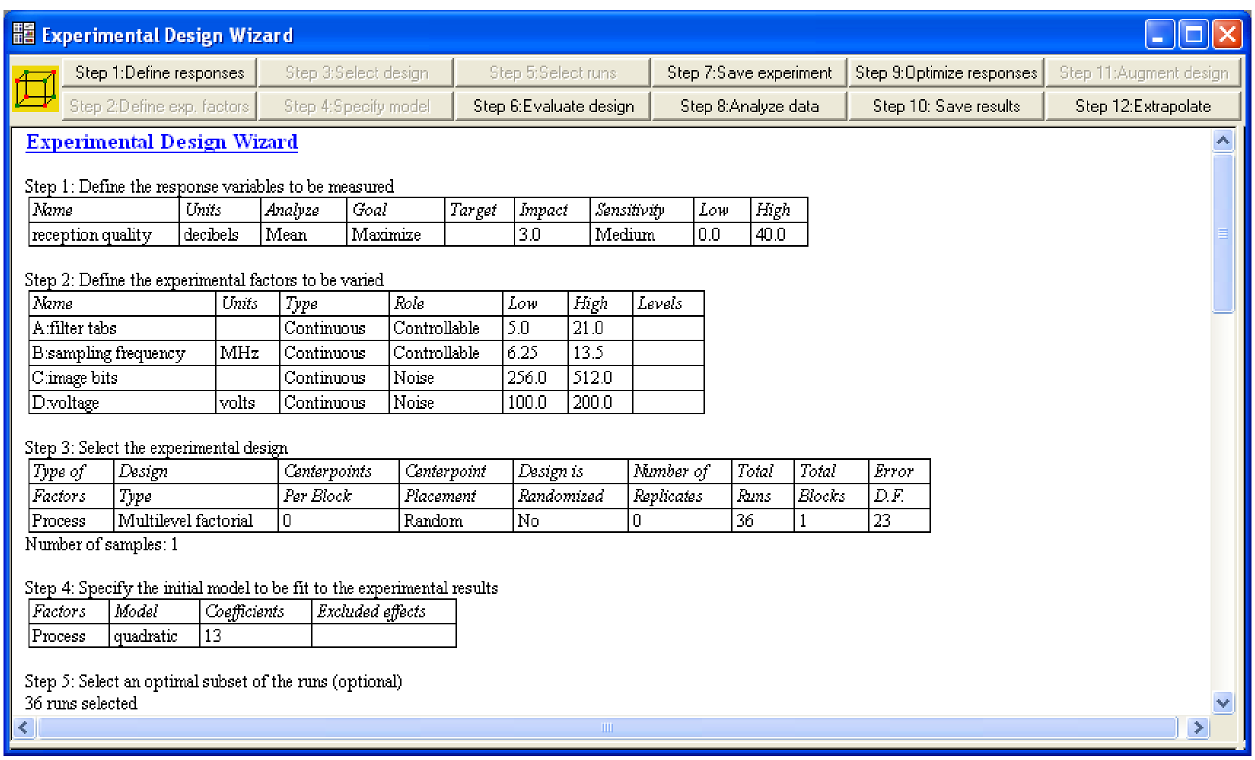
Crossed array

	Segment	Factors	Runs	Blocks	Design
Options...	Process factors	4	36	1	Multilevel Factorial
Options...	Mixture components	0	0	0	
Options...		0	0	0	
	COMBINED	4	36	1	Samples per run: 1

	BLOCK	filter tabs	sampling frequency MHz	image bits	voltage volts
1	1	5.0	6.25	256.0	100.0
2	1	13.0	6.25	256.0	100.0
3	1	21.0	6.25	256.0	100.0
4	1	5.0	9.875	256.0	100.0
5	1	13.0	9.875	256.0	100.0
6	1	21.0	9.875	256.0	100.0
7	1	5.0	13.5	256.0	100.0
8	1	13.0	13.5	256.0	100.0
9	1	21.0	13.5	256.0	100.0
10	1	5.0	6.25	512.0	100.0
11	1	13.0	6.25	512.0	100.0
12	1	21.0	6.25	512.0	100.0
13	1	5.0	9.875	512.0	100.0
14	1	13.0	9.875	512.0	100.0
15	1	21.0	9.875	512.0	100.0

OK Cancel Rerandomize Help

If the design is acceptable, press OK to save it to the STATGRAPHICS DataBook and return to the DOE Wizard's main window, which should now contain a summary of the design:



Experimental Design Wizard

Step 1:Define responses | Step 3:Select design | Step 5:Select runs | Step 7:Save experiment | Step 9:Optimize responses | Step 11:Augment design

Step 2:Define exp. factors | Step 4:Specify model | Step 6:Evaluate design | Step 8:Analyze data | Step 10: Save results | Step 12:Extrapolate

Experimental Design Wizard

Step 1: Define the response variables to be measured

Name	Units	Analyze	Goal	Target	Impact	Sensitivity	Low	High
reception quality	decibels	Mean	Maximize		3.0	Medium	0.0	40.0

Step 2: Define the experimental factors to be varied

Name	Units	Type	Role	Low	High	Levels
A:filter tabs		Continuous	Controllable	5.0	21.0	
B:sampling frequency	MHz	Continuous	Controllable	6.25	13.5	
C:image bits		Continuous	Noise	256.0	512.0	
D:voltage	volts	Continuous	Noise	100.0	200.0	

Step 3: Select the experimental design

Type of Factors	Design Type	Centerpoints Per Block	Centerpoint Placement	Design is Randomized	Number of Replicates	Total Runs	Total Blocks	Error D.F.
Process	Multilevel factorial	0	Random	No	0	36	1	23

Number of samples: 1

Step 4: Specify the initial model to be fit to the experimental results

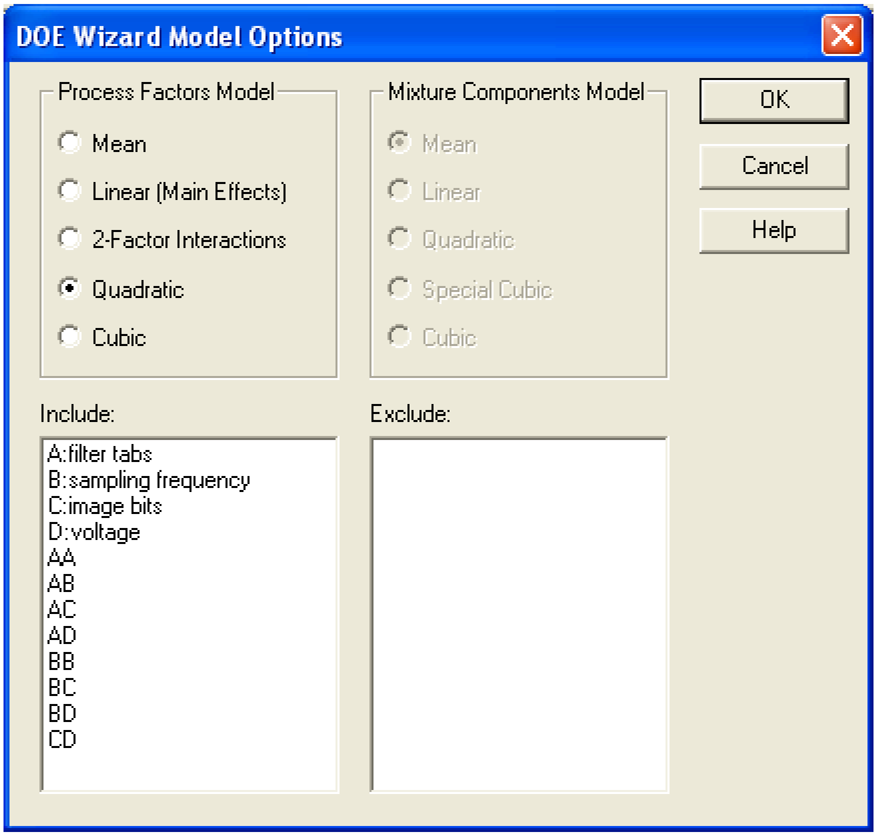
Factors	Model	Coefficients	Excluded effects
Process	quadratic	13	

Step 5: Select an optimal subset of the runs (optional)

36 runs selected

Step #4: Specify Model

Before evaluating the properties of the design, a tentative model must be specified. Pressing the fourth button on the DOE Wizard's toolbar displays a dialog box to make that choice. Initially, the dialog box displays the following selections:



The default model includes main effects for each of the 4 factors, together with the two-factor interactions and quadratic effects for the two 3-level factors.

Step #5: Select Runs

Since we intend to run all of the runs in the base design, this step can be omitted.

Design Properties

Step #6: Evaluate Design

Several of the selections presented when pressing button #6 are helpful in evaluating the selected design:

Design Worksheet

The design worksheet shows the 36 runs that have been created, in the order they are to be run. A section of that worksheet is shown below:

Worksheet for C:\DocData16\tvsignal.sgx - Color television signal study

run	*filter tabs*	*sampling frequency*	*image bits*	*voltage*	*reception quality*
		MHz		*volts*	*decibels*
1	5.0	6.25	256.0	100.0	
2	13.0	6.25	256.0	100.0	
3	21.0	6.25	256.0	100.0	
4	5.0	9.875	256.0	100.0	
5	13.0	9.875	256.0	100.0	
6	21.0	9.875	256.0	100.0	
7	5.0	13.5	256.0	100.0	
8	13.0	13.5	256.0	100.0	
9	21.0	13.5	256.0	100.0	
10	5.0	6.25	512.0	100.0	

ANOVA Table
The ANOVA table shows the breakdown of the degrees of freedom in the design:

ANOVA Table

Source	D.F.
Model	12
Total Error	23
Lack-of-fit	23
Pure error	0
Total (corr.)	35

Since there are no replicates, all of the error degrees of freedom correspond to lack-of-fit.

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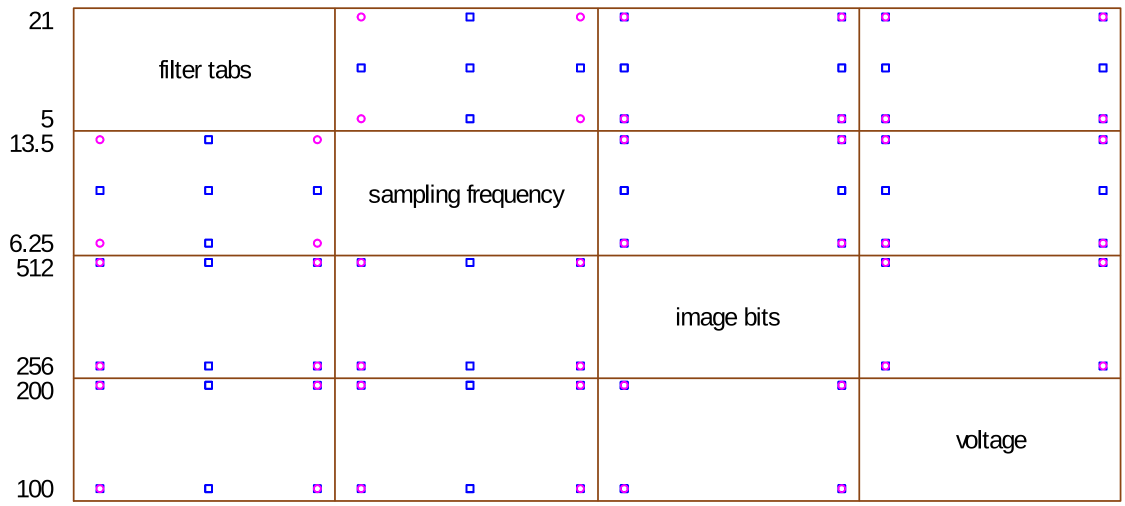
Display Design

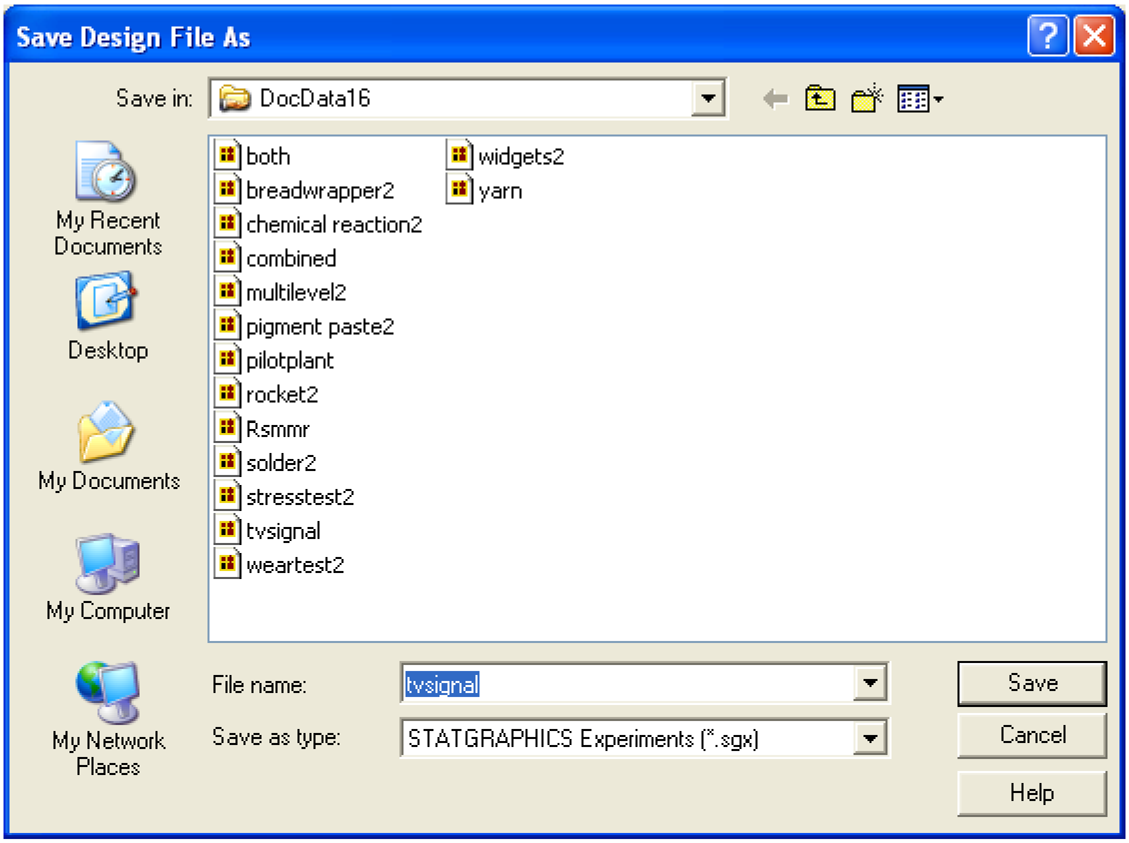
The combined design is shown below using a matrix plot:



Saving the Design File

Step #7: Save experiment

Once the experiment has been created and any additional runs entered, it must be saved on disk. Press the button labeled *Step 7* and select a name for the experiment file:



Design files are extended data files and have the extension *.sgx*. They include the data together with other information that was entered on the input dialog boxes.

To reopen an experiment file, select *Open Data File* from the *File* menu. The data will be loaded into the datasheet, and the *Experimental Design Wizard* window will be displayed.

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Analyzing the Results

After the design file has been created and saved, the experiments would be performed. At a later date, once the results have been collected, the experimenter would return to STATGRAPHICS and reopen the saved design file using the *Open Data Source* selection on the main *File* menu. The results can then be typed into the response columns. The results for the example are displayed below:

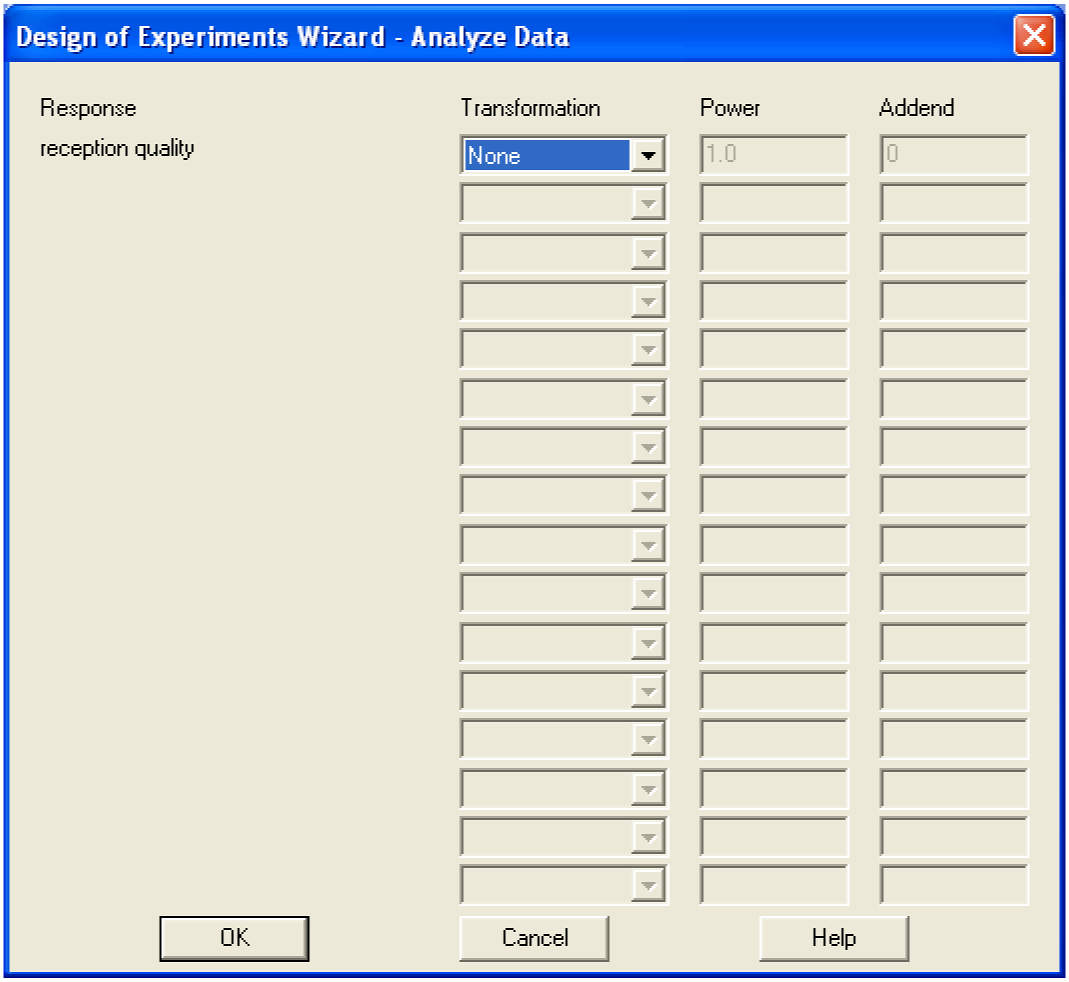
Worksheet for tvsignal.sgx - Color television signal study

run	*filter tabs*	*sampling frequency*	*image bits*	*voltage*	*reception quality*
		MHz		*volts*	*decibels*
1	5.0	6.25	256.0	100.0	33.5021
2	13.0	6.25	256.0	100.0	30.4481
3	21.0	6.25	256.0	100.0	21.1553
4	5.0	9.875	256.0	100.0	35.8234
5	13.0	9.875	256.0	100.0	34.8679
6	21.0	9.875	256.0	100.0	26.6736
7	5.0	13.5	256.0	100.0	33.0773
8	13.0	13.5	256.0	100.0	35.2202
9	21.0	13.5	256.0	100.0	32.1245
10	5.0	6.25	512.0	100.0	25.2683
11	13.0	6.25	512.0	100.0	15.1493
12	21.0	6.25	512.0	100.0	0.7917
13	5.0	9.875	512.0	100.0	32.7928
14	13.0	9.875	512.0	100.0	27.7724
15	21.0	9.875	512.0	100.0	15.5132
16	5.0	13.5	512.0	100.0	36.2500
17	13.0	13.5	512.0	100.0	33.3280
18	21.0	13.5	512.0	100.0	26.1673
19	5.0	6.25	256.0	200.0	41.2268
20	13.0	6.25	256.0	200.0	41.2870
21	21.0	6.25	256.0	200.0	34.1086
22	5.0	9.875	256.0	200.0	38.0689
23	13.0	9.875	256.0	200.0	40.2276
24	21.0	9.875	256.0	200.0	38.1477
25	5.0	13.5	256.0	200.0	31.8435
26	13.0	13.5	256.0	200.0	37.1008
27	21.0	13.5	256.0	200.0	38.1193
28	5.0	6.25	512.0	200.0	31.9930
29	13.0	6.25	512.0	200.0	23.9883
30	21.0	6.25	512.0	200.0	15.7450
31	5.0	9.875	512.0	200.0	34.0383
32	13.0	9.875	512.0	200.0	31.1321
33	21.0	9.875	512.0	200.0	25.9873
34	5.0	13.5	512.0	200.0	34.0162
35	13.0	13.5	512.0	200.0	35.2085
36	21.0	13.5	512.0	200.0	32.1622

For convenience, the data are included with STATGRAPHICS Centurion in a file named *tvsignal.sgx*.

Step #8: Analyze data

Once the data have been entered, press the button labeled *Step #8* on the Experiment Design Wizard toolbar. This will display a dialog box listing each of the response variables:



The single response variable to be analyzed is the reception quality. No transformation is necessary. Press *OK* to perform the analysis.

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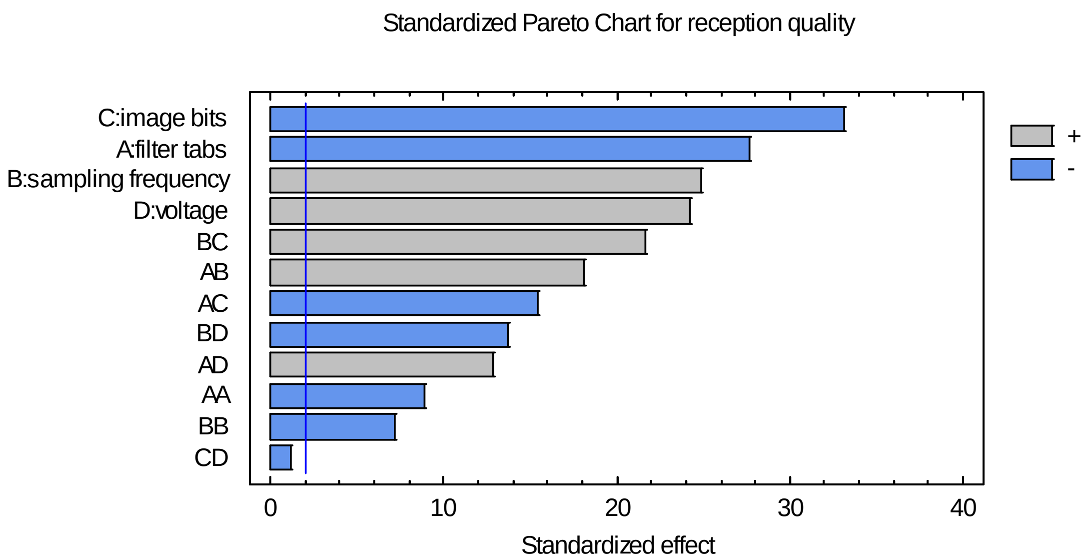
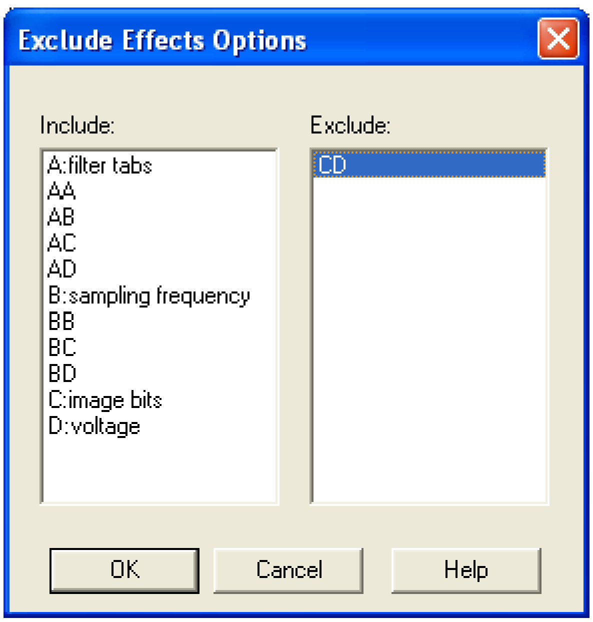
Analysis Output

For a robust parameter design with a combined structure, the statistical model includes both the controllable factors and the noise factors. For the current design, the standardized Pareto chart of effects is shown below:



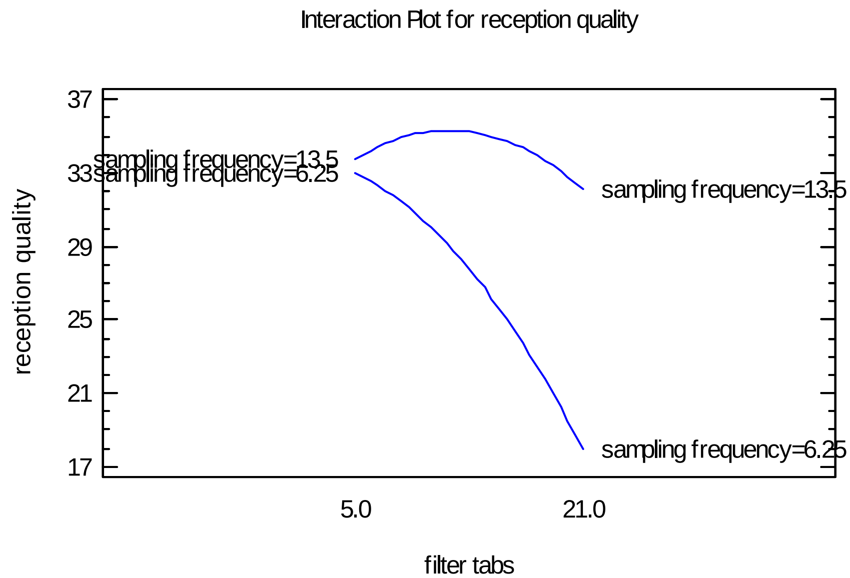
All of the main effects and two-factor interactions are statistically significant, with the exception of the interaction between the two noise variables. Using the *Exclude* button on the *Analysis Options* dialog box, that interaction can be removed:



The *Main Effects Plot* shows the impact of each factor on the response:

On average, the noise factors have a large impact on reception quality.

Since the 2 controllable factors interact, the *Interaction Plot* for those factors is also of interest:



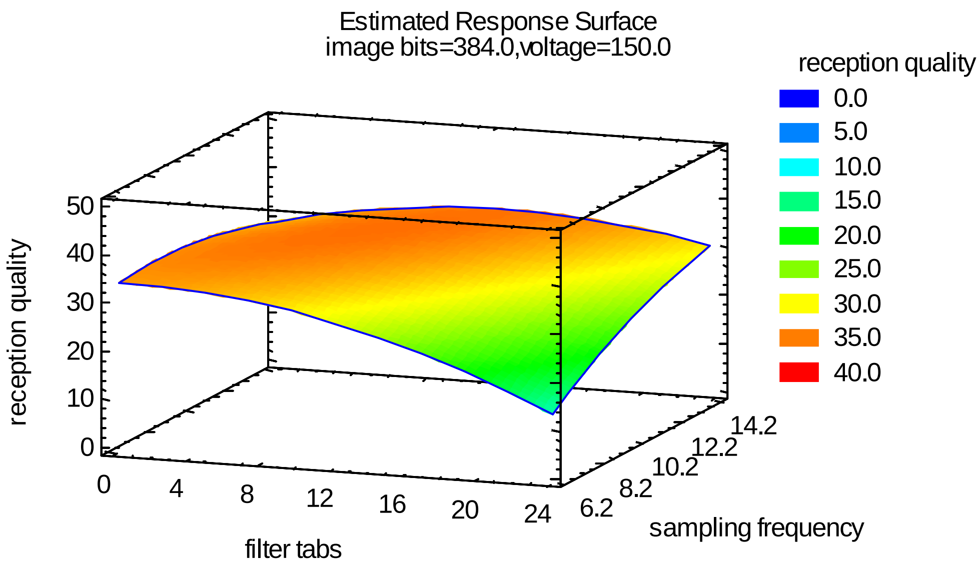
Best *reception quality* is achieved at high *sampling frequency* and a medium level of *filter tabs*, although *quality* is not very sensitive to *filter tabs* when *sampling frequency* is high.

The fitted regression model as displayed by the StatAdvisor is:

reception quality = 34.6627 - 0.569517*filter tabs + 1.34978*sampling frequency - 0.0686362*image bits + 0.118187*voltage - 0.0363699*filter tabs^2 + 0.115465*filter tabs*sampling frequency - 0.00226965*filter tabs*image bits + 0.00483039*filter tabs*voltage - 0.142082*sampling frequency^2 + 0.00704372*sampling frequency*image bits - 0.0114369*sampling frequency*voltage - 0.000021701*image bits*voltage

The above model is given in the original units of the factors.

Holding the noise factors constant at their central values, the response surface is shown below:



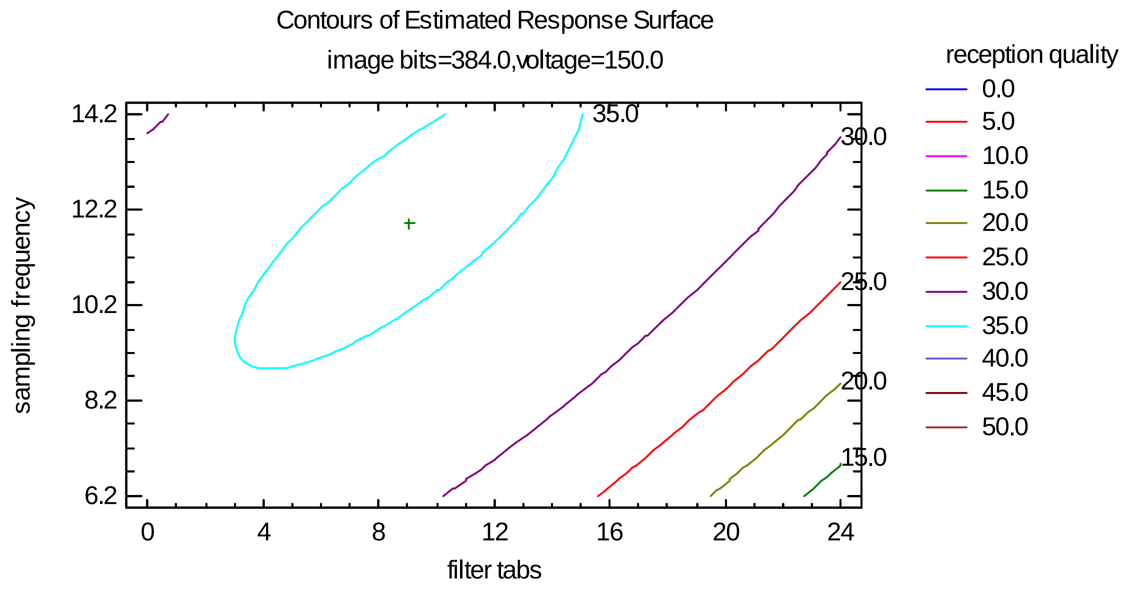
Except for the front right corner, *reception quality* is fairly consistent. Selecting *Optimize Response*, the point of maximum quality in the space of the controllable factors will be displayed:

Optimize Response
Goal: maximize reception quality

Optimum value = 35.4705

Factor	*Low*	*High*	*Optimum*
filter tabs	5.0	21.0	9.05819
sampling frequency	6.25	13.5	11.9111
image bits	256.0	512.0	384.0
voltage	100.0	200.0	150.0

Note that the noise variables are fixed at their central levels. The location of the optimum can also be displayed on a contour plot:

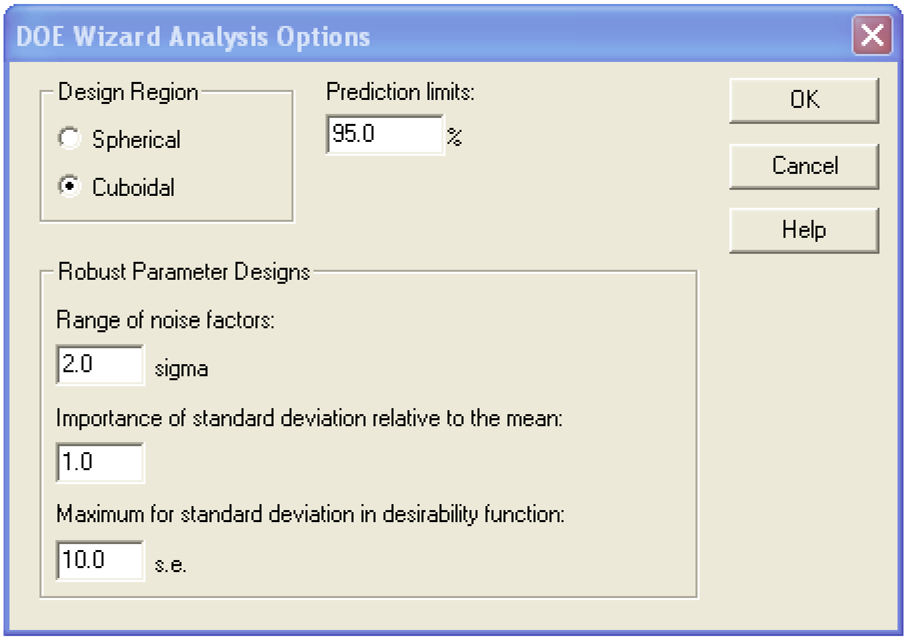


Optimization

Step #9: Optimize responses

Once a statistical model has been developed for each response, the analyst may now determine what combination of factors will yield the best results. For a robust parameter design, this involves finding a combination of the factors that achieves the desired mean result while keeping the standard deviation small. STATGRAPHICS optimizes the results by combining the mean and standard deviation using desirability functions.

The *Analysis Options* dialog box contains three special fields that control how the mean and standard deviation are combined:

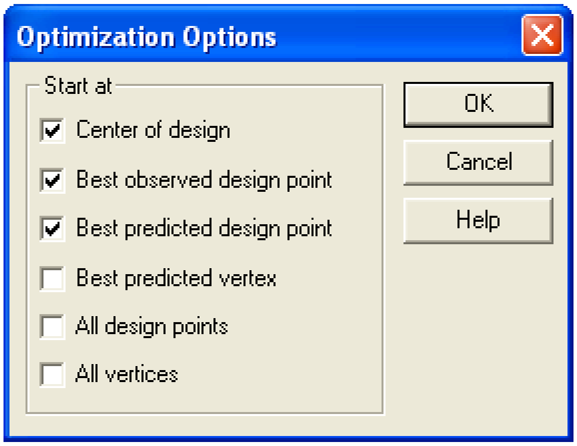


- **Range of noise factors:** This field specifies the number of standard deviations σ_z of the noise factors covered by the distance between the low and high levels of those factors. It is assumed that this multiple is the same for all noise factors. Note that while this entry affects the estimated standard deviation of the response, it does not affect the location of the minimum process variance.

- **Importance of standard deviation relative to mean:** This field specifies the relative weight given to the standard deviation when the desirability function is created. A value of 1 gives the mean and standard deviation equal weight. A value of 0.5 gives one-half as much weight to the standard deviation as to the mean, while a value of 2 gives the standard deviation twice as much weight as the mean.

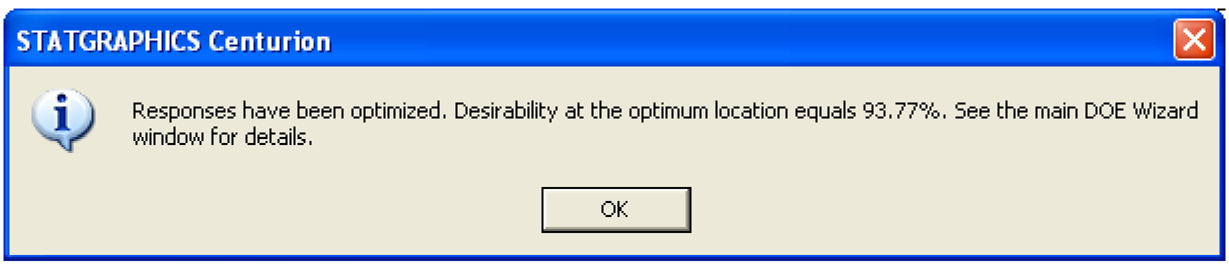
- **Maximum for standard deviation in desirability function:** When constructing the desirability function for the standard deviation, desirability is defined as 1.0 when the noise factors contribute nothing to the overall process variance. This field specifies the location at which desirability falls to 0 as a multiple of the standard error from the fitted response model.

To perform the optimization, press the button labeled *Step #9* on the *Experimental Design* Wizard toolbar. This first displays the dialog box shown below:



Since optimization requires searching for the best conditions throughout the experimental region, it is a good idea to begin that search at many different points in order to avoid finding only a local optimum.

When the optimization is complete, a message similar to that shown below will be displayed:



The dialog box indicates the "Desirability" of the final result, based on a metric designed to balance competing requirements of multiple responses (see the document titled DOE Wizard for full details).

If you press *OK*, additional information will be added to the main *DOE Wizard* window:

Step 9: Optimize the responses

Response Values at Optimum

Response	*Prediction*	*Lower 95.0% Limit*	*Upper 95.0% Limit*	*Standard Deviation*	*Desirability*
reception quality	35.2262	34.7421	35.7103	0.752495	0.937712

Factor Settings at Optimum

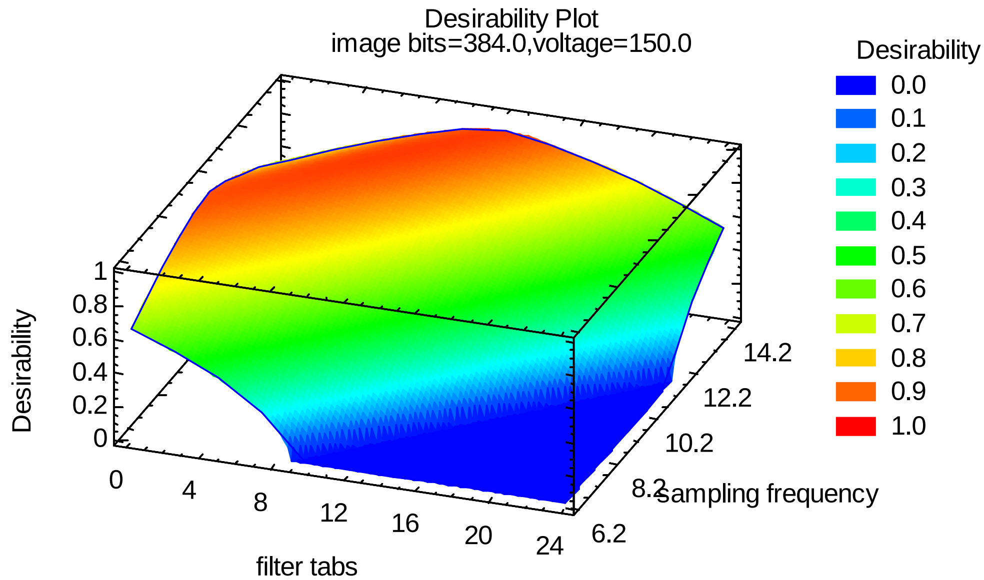
Factor	*Setting*
filter tabs	8.35459
sampling frequency	12.92

The table shows the estimated *reception quality* at the optimal settings of the experimental factors, together with a 95% confidence interval. It also shows the estimated process standard deviation, calculated using the transmission-of-error formula:

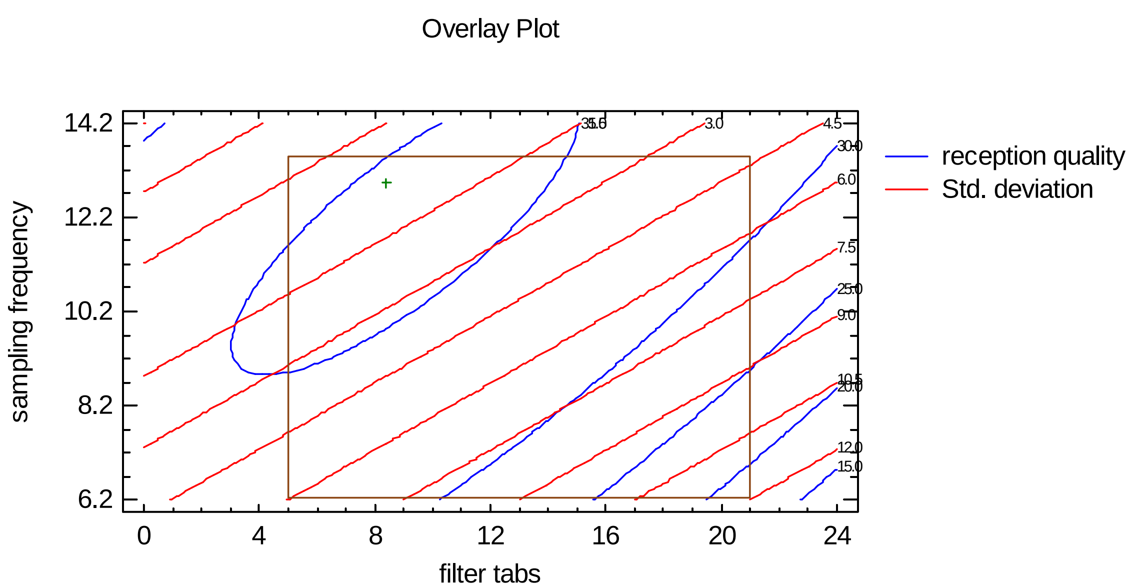
$$se = \sqrt{\sum_{i=1}^{r}\left[\frac{\partial y(x,z)}{\partial z_i}\sigma_z\right]^2 + \sigma^2} \qquad (1)$$

where the summation is over the r noise factors. σ^2 is estimated using the mean squared error of the fitted model, while the variance of the noise factors σ_z^2 is set based on the range specified on the *Analysis Options* dialog box.

If you push the *Tables and Graphs* button on the analysis toolbar, you can display the estimated desirability throughout the experimental region. An interesting type of display is the contoured surface plot shown:



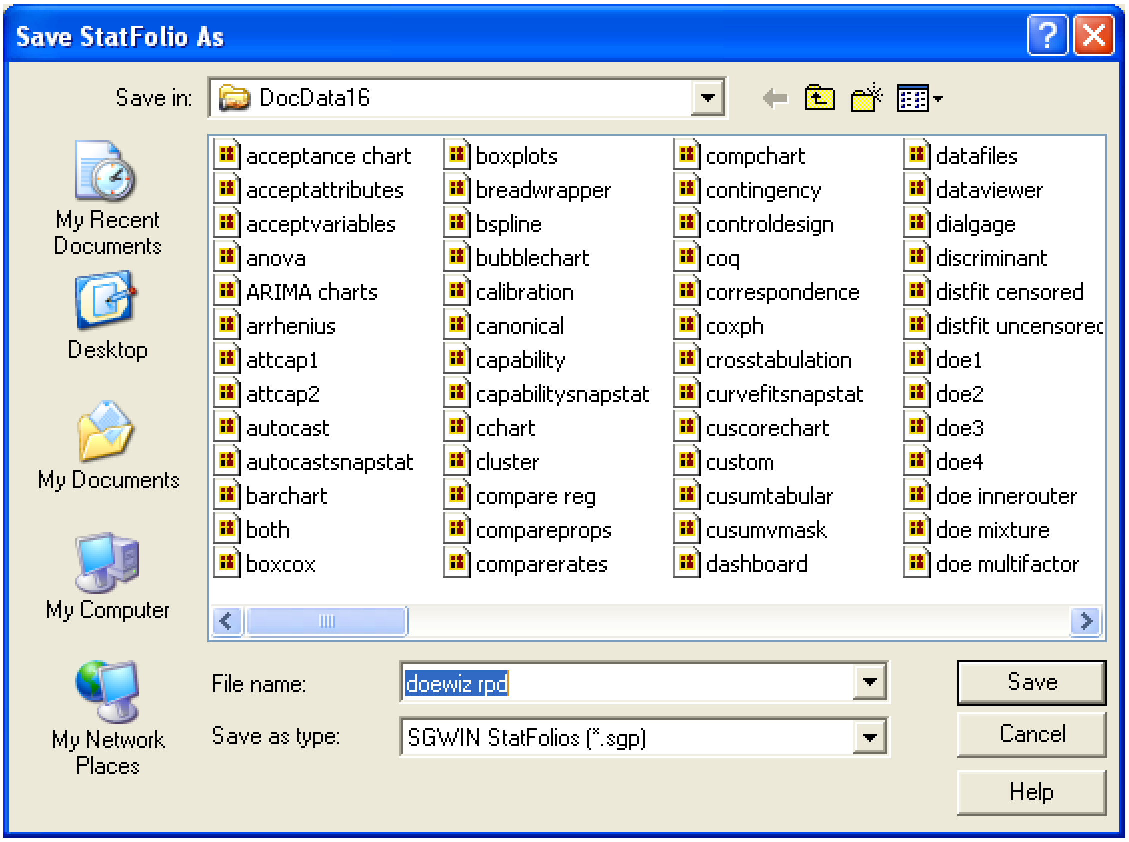
The mean and standard deviation can also be plotted together using the *Overlaid Contour Plots* graph:



The derived optimal location is shown as a small plus sign. It is offset slightly from the point of maximum quality (the center of the ellipse), closer to the location of minimum variance (which is in a valley defined by the straight lines on either side of the optimum).

Step 10: Save results

The button labeled *Step 10* allows you to save the results in a StatFolio:



IMPORTANT: When using the *Experimental Design Wizard*, two files are created:

1. An experiment file with the extension *.sgd* which stores information about the experimental data.

2. A StatFolio with the extension *.sgp* that stores the results of the analysis.

If you move the experiment to another computer, be sure to transfer both files.

Step 11: Augment Design

This option is not available for the current experiment.

Step 12: Extrapolate

Since the optimum location is within the design space, there is no reason to extrapolate the model.

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