DOE Wizard – Computer Generated Designs

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Summary

The *Computer Generated* designs allow you to create experimental designs which have optimal properties with respect to the estimation of specific statistical models. Given the definition of an experimental region, a model to be estimated, and the number of experimental runs that can be performed, the program searches for a set of runs that maximizes a selected design optimality criteria.

Computer generated designs are included as an option in the Experimental Design Wizard when constructing several types of designs, including screening designs, response surface designs, and mixture experiments. In addition to lower and upper limits for each experimental factor, constraints may be specified based on linear combinations of the factors. The optimality criteria offered include D-optimality, A-optimality, G-optimality, and I-optimality.

The results of previous experimental runs may be included in the design by entering them into the experiment datasheet before requesting that the computer generate new runs. Additional runs will then be added to those already performed to maximize the specified optimality criterion. This allows the procedure to improve a partial design that has already been performed.

Example #1: Optimized Response Surface Experiment

In the book titled <u>Optimal Design of Experiments: A Case Study Approach</u> (2011), Goos and Jones describe an experiment in which the experimenters wished to model the effect of 4 experimental factors on the peel strength of a package. The factors of interest were:

Factor	Туре	Low	High	Units
temperature	continuous	193	230	degrees C
pressure	continuous	2.2	3.2	bar
speed	continuous	32	50	cpm
supplier	categorical	1	3	

They wished to estimate a model containing:

- 1. Main effects of all 4 factors.
- 2. Two-factor interactions amongst all of the factors.
- 3. Quadratic effects for the 3 continuous factors.

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This document will demonstrate how the DOE Wizard can be used to select a good design for this situation.

Design Creation

To begin the design creation process, start with an empty StatFolio. Select DOE - Experimental Design Wizard to load the DOE Wizard's main window. Then push each button in sequence to create the design.

Step #1 – Define Responses

The first step of the design creation process displays a dialog box used to specify the response variables. For the current example, there is a single response variable:

Comment:	Computer generated									
lumber of	responses: 1	Responses 1-16	Responses 17-	32						
Response		Units	Analyze		Goal	Target	Impact (1-5)	Sensitivity	Minimum	Maximum
1	peel strength		Mean	-	Hit target 💌	4.5	3.0	Medium	▼ 3.0	6.0
2	Var_2		Mean	-	Maximize 💌	0.5	3.0	Medium	<u>_</u>	
3	Var_3		Mean	-	Maximize 💌	0.5	3.0	Medium	-	
4	Var_4		Mean	-	Maximize 💌	0.5	3.0	Medium	-	
5	Var_5		Mean	-	Maximize 💌	0.5	3.0	Medium	-	
6	Var_6		Mean	-	Maximize 💌	0.5	3.0	Medium	-	
7	Var_7		Mean	-	Maximize 💌	0.5	3.0	Medium	v	
8	Var_8		Mean	-	Maximize 💌	0.5	3.0	Medium		
9	Var_9		Mean	-	Maximize 💌	0.5	3.0	Medium	-	
10	Var_10		Mean	-	Maximize 💌	0.5	3.0	Medium	-	
11	Var_11		Mean	-	Maximize 💌	0.5	3.0	Medium	-	
12	Var_12	_	Mean	-	Maximize 👻	0.5	3.0	Medium		
13	Var_13	_	Mean	-	Maximize 👻	0.5	3.0	Medium	- i	
14	, Var_14		Mean	-	Maximize 🖃	0.5	3.0	Medium		
15	, Var_15		Mean	-	Maximize 🖃	0.5	3.0	Medium		
16	Var 16		Mean		Maximize 🖃	0.5	3.0	Medium		

- Name: The name for the variable is *peel strength*.
- Units: The units are not relevant in this example.
- Analyze: The parameter of interest is the *mean* peel strength.
- Goal: The goal of the experiment is to hit a target peel strength close to 4.5.
- **Impact**: This is not relevant since there is only one response.

- **Sensitivity**: The importance of being close to the best desired value has little impact on the optimization result when there is only one response.
- Minimum and Maximum: The range of acceptable peel strength was 3.0 to 6.0.

Step #2 – Define Experimental Factors

The second step displays a dialog box on which to specify the factors that will be varied. In the current example, there are 4 controllable process factors:

Comme	nt: Computer genera	ated design					
Number	r of controllable proce	ess factors: 4 📕 Nur	mber of controllable mi	kture components	s: 0 ·	Number of r	noise factors: 0
Factor	Name	Units	Туре	Role	Low	High	Levels
А	temperature	degrees C	Continuous 💌	Controllable	193.0	230.0	1,2,3,4
В	pressure	bar	Continuous 💌	Controllable	2.2	3.2	1,2,3,4
С	speed	cpm	Continuous 💌	Controllable	32.0	50.0	1,2,3,4
D	supplier		Categorical 💌	Controllable	-1.0	1.0	1,2,3
Е	Factor_E		Continuous 💌		-1.0	1.0	1,2,3,4
F	Factor_F		Continuous 💌		-1.0	1.0	1.2.3.4
G	Factor_G		Continuous 💌		-1.0	1.0	1,2,3,4
Н	Factor_H		Continuous 💌		-1.0	1.0	1.2.3,4
I	Factor_I		Continuous 💌		-1.0	1.0	1,2,3,4
J	Factor_J		Continuous 💌		-1.0	1.0	1.2.3,4
К	Factor_K		Continuous 💌		-1.0	1.0	1.2.3,4
L	Factor_L		Continuous 💌		-1.0	1.0	1,2,3,4
м	Factor_M		Continuous 💌		-1.0	1.0	1,2,3,4
Total	for controllable minute	re components: 1.0				Factors A-M	Factors N-Z

- Name Each factor must be assigned a unique name.
- Units Units are optional.
- **Type** Set the type of the first 3 factors to *Continuous*, since they can be set at any value within a continuous interval. The fourth factor is categorical.
- **Role** The four factors are all *Controllable*.
- **Low** the lower level L_j for the continuous factors.
- **High** the upper level U_j for the continuous factors.

• **Levels** – the allowable levels of the categorical factor *supplier*.

Step #3 - Select Design

Design of Experiments Wizard - Select Design Design file: doe1.sgx Robust Parameter Design Combined array Comment: Compute C Crossed array Segment Factors Runs Blocks Design Options. Process factors 4 0 0 Press the Options button. 0 0 Mixture components Ω Options. 0 0 0 COMBINED 0 0 4 Samples per run: 1 BLOCK temperature pressure speed supplier * bar degrees C cpm ΟK Rerandomize Cancel Constraints Help

The third step begins by displaying the dialog box shown below:

Since all of the factors are controllable process factors, only one *Options* button is enabled. Pressing that button displays a second dialog box:

Designs with Categorical F	actors ×
Design Class	OK
Multilevel Factorial Computer Generated	Cancel
	Help

Since the experiment involves a categorical factor, two types of designs are offered:

1. *Multilevel Factorial* - designs involving all combinations of selected levels of each experimental factor. For example, a 3x3x3x3 factorial design would have 81 runs consisting of all combinations of 3 levels of each factor.

2. *Computer Generated* – If this type of design is selected, the computer will select a set of runs during Step 5 that are optimal for the model to be fit.

Select *Computer Generated* and press *OK*, which will return you to the *Select Design* dialog box:

If satisfied, press *OK* to save the design selection and return to the DOE Wizard's main window, which will now contain a summary of the design:

Step #4: Specify Model

The next step in the design selection process specifies the model that will be fit to the response data. Pressing the fourth button on the DOE Wizard's toolbar displays a dialog box to make that choice:

DOE W	/izard Model Options	×
Process Factors Model	Mixture Components Model	ОК
C Mean	Mean	Cancel
C Linear (Main Effects)	C Linear	
C 2-Factor Interactions	C Quadratic	Help
Quadratic	C Special Cubic	
C Cubic	C Cubic	
Include: A:temperature B:pressure C:speed D:supplier AA AB AC AD BB BC BD CC CD	Exclude:	

The *quadratic* model includes main effects for each of the 4 experimental factors, 6 two-factor interactions (shown as two-letter combinations with different factors), and 3 quadratic terms (shown as two-letter combinations with identical factors). Selected terms could be excluded by double-clicking on them with the left mouse button.

Step #5: Select runs

The next step selects the runs to be performed. Press the *Step 5: Select Runs* button on the DOE Wizard toolbar to display the following dialog box:

			Computer	Generated Desi	gns	×
	BLOCK	temperature	pressure	speed	supplier	
	02001	degrees C	bar	cpm	ouppion	
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						
22						
						•
_ Optimize	e	Display	Number of coeffi	cients: 18	Average prediction variance	e:
I-effi	iciency	Original units	Number of base	runs: 24	D-efficiency:	ок
O D-ef	fficiency	C Coded units		<u> </u>	-	- Com L
	-		Number of replic	ates: 0	A-efficiency:	Cancel
C A-ef	ficiency	🔽 Randomize run order	Number of cente	rpoints: 0	G-efficiency:	Help
O G-ef	fficiency			h blocks of size: 1000	Create Ad	vanced

The dialog box shows the number of coefficients in the model to be estimated. At least as many runs must be selected as there are coefficients. Typically, at least 3 additional runs will be added to that number in order to estimate the experimental error.

There are several important fields to be completed:

- *Optimize* the criterion to be used to select the experimental runs. Briefly, D-efficiency measures the information generated by the experiment about the model parameters. A-efficiency measures the average variance of the estimates of the model parameters. G-efficiency measures the maximum variance of the predicted response at the design points. I-efficiency measures the average prediction variance over the design space. It is often recommended that D-efficiency be selected when creating a screening design and I-efficiency selected when creating a response surface design.
- *Display* whether the runs should be displayed in their original units or coded units.
- *Randomize run order* whether the order of the runs should be randomized.
- *Number of base runs* the number of different combinations of the factors that should be generated. This number must be greater than or equal to the number of coefficients in the model that will be fit to the data.

- *Number of replicates* the number of additional runs to be added that are replicates of one or more of the base runs.
- *Number of centerpoints* the number of additional runs to be added at the center of the experimental region.
- *Group runs in blocks of size* If desired, the new runs may be placed in blocks of the specified size. Additional terms will be added to the model to allow for differences between the blocks. Selecting this option also places any experimental runs that have already been performed in a different block than the new runs.
- *Advanced* button displays a dialog box for changing the search options:

Computer Generated Designs - Search Options	х
Number of random starts: 10	
Maximum iterations per start:	
Number of continuous factor levels to consider: 3 Set by factor	
Mixture increment between levels:	
OK Cancel Help	

Number of random starts – number of times the algorithm will search for an optimal design from a different random start.

Maximum iterations per start – maximum number of times that the algorithm will try exchanging runs before a solution is accepted.

Number of factor levels to consider - number of levels at which experimental runs may be performed for continuous factors, ranging from the low level of the factor to the high level. Specifying a larger number increases the time required to create the design but may improve the design efficiency. If different numbers of levels are desired for different factors, they may be set by pressing the *Set by factor* button.

Mixture increment between levels – for experiments containing mixture components, the amount by which each component will be changed when attempting to find an optimal set of runs.

Calculate G-efficiency - whether the G-efficiency of the design should be calculated and displayed. This statistic requires calculating the prediction variance at every candidate point considered when constructing the design, which can be very large if there are many factors or factor levels to consider.

In this case, the experimenters decided that they could afford to perform 24 runs, all of which will be allocated to base runs (different combinations of the factors). They also specified that the experiment should include runs at only 3 levels of each continuous factor (the low level, the center level, and the high level).

To generate an I-optimal design, set *Optimize* to *I-efficiency* and press the *Create* button. When the algorithm is complete, the selected experimental runs will be added to the dialog box:

BLOCK	temperature	pressure	speed	supplier	
4	degrees C	bar	cpm		
1 1	193.0	2.7	32.0	3	
2 1	230.0	3.2	50.0	1	
3 1	230.0	3.2	50.0	2	
4 1	211.5	2.7	41.0	2	
5 1	230.0	2.7	50.0	3	
6 1	193.0	3.2	50.0	3	
7 1	211.5	2.7	41.0	2	
8 1	230.0	2.2	41.0	1	
9 1	193.0	3.2	32.0	2	
10 1	193.0	2.2	32.0	2	
11 1	230.0	2.7	32.0	2	
12 1	211.5	2.7	41.0	1	
13 1	211.5	3.2	32.0	3	
4 1	193.0	2.2	50.0	1	
15 1	230.0	3.2	32.0	1	
16 1	193.0	2.2	32.0	1	
17 1	230.0	2.2	50.0	2	
18 1	193.0	2.7	50.0	2	
19 1	193.0	2.2	41.0	3	
20 1	193.0	3.2	41.0	1	
21 1	211.5	2.7	41.0	1	
22 1	230.0	2.2	32.0	3	
				· · · · · · · · · · · · · · · · · · ·	•
Iptimize	Display	Number of cos	efficients: 18	Average prediction variance: 0.	483778
I-efficiency	Original units	Number (1			OK
	_	Number of bas		D-efficiency: 42.73%	
D-efficiency	C Coded units	Number of rep	licates: 0	A-efficiency: 30.37%	Cancel
A-efficiency	🔽 Randomize run ord	der Numberofcer	nterpoints: 0	G-efficiency: 76.23%	Help
G-efficiency			s in blocks of size: 1000	Create Advar	

The efficiencies of the selected design and the average prediction variance are also displayed.

Note: The efficiency values should only be used to compare alternative experiments containing the same number of runs. Their absolute magnitude is usually not important.

If you uncheck the button labeled *Randomize run order*, the runs will be displayed in standard order:

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			Computer	Generated Desig	gns	×
	BLOCK	temperature	pressure	speed	supplier	-
_	1	degrees C	bar	cpm		
1	1	193.0	2.2	32.0	1	
2	1	230.0	3.2	32.0	1	
3	1	230.0	2.2	41.0	1	
4	1	211.5	2.7	41.0	1	
5	1	211.5	2.7	41.0	1	
6	1	193.0	3.2	41.0	1	
7	1	193.0	2.2	50.0	1	
8	1	230.0	3.2	50.0	1	
9	1	193.0	2.2	32.0	2	
10	1	230.0	2.7	32.0	2	
11	1	193.0	3.2	32.0	2	
12	1	211.5	2.7	41.0	2	
13	1	211.5	2.7	41.0	2	
14	1	230.0	2.2	50.0	2	
15	1	193.0	2.7	50.0	2	
16	1	230.0	3.2	50.0	2	
17	1	230.0	2.2	32.0	3	
18	1	193.0	2.7	32.0	3	
19	1	211.5	3.2	32.0	3	
20	1	193.0	2.2	41.0	3	
21	1	230.0	3.2	41.0	3	
22	1	211.5	2.2	50.0	3	-
						•
_ Opti	mize	Display	Number of coeffi	cients: 18	Average prediction variance: 0.483	778
•	-efficiency	Original units	Number of base	runs: 24	D-efficiency: 42.73%	OK
0	D-efficiency	C Coded units	Number of replica	ates: 0	A-efficiency: 30.37%	Cancel
07	A-efficiency	🔲 Randomize run orde	r Number of cente	rpoints: 0	G-efficiency: 76.23%	Help
0.0	G-efficiency		🔽 Group runs in	h blocks of size: 1000	Create Advanced	i

Note that 8 runs are performed for each supplier.

You can change some settings and try again or press *OK* to accept the selection, at which point the selected runs will be placed in datasheet A:

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	BLOCK	temperature	pressure	speed	supplier	peel strength
		degrees C	bar	cpm		
1	1	193.0	2.7	32.0	3	
2	1	230.0	3.2	50.0	1	
3	1	230.0	3.2	50.0	2	
4	1	211.5	2.7	41.0	2	
5	1	230.0	2.7	50.0	3	
6	1	193.0	3.2	50.0	3	
7	1	211.5	2.7	41.0	2	
8	1	230.0	2.2	41.0	1	
9	1	193.0	3.2	32.0	2	
10	1	193.0	2.2	32.0	2	
11	1	230.0	2.7	32.0	2	
12	1	211.5	2.7	41.0	1	
13	1	211.5	3.2	32.0	3	
14	1	193.0	2.2	50.0	1	
15	1	230.0	3.2	32.0	1	
16	1	193.0	2.2	32.0	1	
17	1	230.0	2.2	50.0	2	
18	1	193.0	2.7	50.0	2	
19	1	193.0	2.2	41.0	3	
20	1	193.0	3.2	41.0	1	
21	1	211.5	2.7	41.0	1	
22	1	230.0	2.2	32.0	3	
23	1	230.0	3.2	41.0	3	
24	1	211.5	2.2	50.0	3	

The main DOE Wizard window will reflect the design:

Experimental Design Wizard	
Step 1:Define responses Step 3:Specify model Step 5:Select runs Step 7	Save experiment Step 9:Optimize responses Step 11:Augment design
Step 2:Define exp. factors Step 4:Select design Step 6:Evaluate design Step	8:Analyze data Step 10: Save results Step 12:Extrapolate
Experimental Design Wizard	^
Step 1: Define the response variables to be measured	
	High
	6.0
Step 2: Define the experimental factors to be varied	
Name Units Type Role Low High Levels	
A:temperature degrees C Continuous Controllable 193.0 230.0	
B:pressure bar Continuous Controllable 2.2 3.2	
C:speed cpm Continuous Controllable 32.0 50.0	
D:supplier Categorical Controllable 1,2,3	
Step 3: Specify the initial model to be fit to the experimental results	
Factors Model Coefficients Excluded effects	
Process quadratic 18	
Step 4: Select the experimental design	
Type of Design Centerpoints Centerpoint Design is	Number of Total Total Error
Factors Type Per Block Placement Randomize	nd Replicates Runs Blocks D.F.
Process Computer generated design	
Number of samples per run: 1	
Step 5: Select an optimal subset of the runs (optional) 24 runs selected	~

If the selection is acceptable, press Step 7: Save experiment to save the design.

Evaluate Design

After the design has been created, press the button labeled *Step 6: Evaluate Design* on the DOE Wizard toolbar to display various design diagnostics:

	Tables and Graphs	×
TABLES ▼ Analysis Summary	GRAPHS	OK Cancel
Design Worksheet ANOVA Table	Prediction Variance Plot Prediction Profile	All
Model Coefficients Alias Matrix	 Variance Dispersion Graph Fraction of Design Space Plot 	Help
Correlation Matrix	Power Curve Desirability Plot	
🗖 Desirability	Overlaid Contour Plots	

The *Model Coefficients* table shows the relative standard error of each coefficient in the model to be estimated:

				Power at	Power at	Power at
Coefficient	Standard Error	VIF	Ri-Squared	SN = 0.5	SN = 1.0	SN = 2.0
constant	0.466817		-	7.41%	14.85%	43.68%
А	0.245327	1.08333	0.0769231	13.90%	40.32%	92.11%
В	0.259094	1.07407	0.0689655	12.96%	36.90%	89.23%
С	0.259094	1.07407	0.0689655	12.96%	36.90%	89.23%
D	0.311438	1.5519	0.355627	10.47%	27.32%	76.28%
D	0.296604	1.40759	0.289564	11.04%	29.58%	80.10%
AA	0.584816	1.53904	0.350246	6.53%	11.22%	30.27%
AB	0.280009	1.0846	0.0780028	11.79%	32.50%	84.30%
AC	0.280009	1.0846	0.0780028	11.79%	32.50%	84.30%
AD	0.350264	1.47222	0.320755	9.31%	22.65%	66.48%
AD	0.340207	1.38889	0.28	9.57%	23.71%	68.95%
BB	0.505853	1.36473	0.267254	7.05%	13.36%	38.37%
BC	0.312182	1.16949	0.144928	10.44%	27.21%	76.08%
BD	0.387896	1.50463	0.335385	8.50%	19.36%	57.89%
BD	0.350264	1.47222	0.320755	9.31%	22.65%	66.48%
CC	0.505853	1.36473	0.267254	7.05%	13.36%	38.37%
CD	0.360041	1.2963	0.228571	9.07%	21.69%	64.14%
CD	0.350264	1.22685	0.184906	9.31%	22.65%	66.48%

The standard error is relative in the sense that it is the multiple of the residual standard error, which is not known until the experiment has been performed. Of particular interest are the VIFs (Variance Inflation Factors), which show how much the variance of each coefficient has been

increased relative to a perfectly orthogonal design. Since the largest VIF is less than 1.4, there has been relatively little variance inflation.

Example #2: Adding Runs to an Existing Experiment

After running a set of experimental runs, it is not uncommon to desire that additional runs be added to the original design. This may happen because of various reasons:

- 1. The initial design might have been constructed poorly. It is not uncommon to find experimenters who begin with a haphazard approach to factor level selection and only realize later that the interpretation of their results would be much easier with a designed experiment.
- 2. In other cases, unexpected effects may be observed in the first design and additional runs may be desired to follow up on the initial discoveries.
- 3. In the case of screening designs involving many factors, there may be considerable confounding amongst the effects in the initial design. Breaking apart specific confounding patterns then requires additional runs.
- 4. Conversion of a screening design to a response design may require additional runs to add higher-order terms to the statistical model.

Goos and Jones (2011) describe a situation where a 12-run experiment was performed involving 6 factors, each at 2 levels. The design was constructed to estimate a model involving all 6 main effects and 1 specific 2-factor interaction. After analyzing the results of the first experiment, they wished to augment the design so that it could estimate 5 additional interactions.

To reconstruct their experiment, start with a StatFolio and select DOE - Experimental Design Wizard from the main menu. In Step 1, enter the following information about the response variable:

		Design	of Experiment	s Wizard - De	fine Respo	onses			×
Design fil	e: <untitled></untitled>								
Comment	computer augmented des	ign							
Number o	f responses: 1	Responses 1-16 R	esponses 17-32						
Respons		Units	Analyze	Goal	Target	Impact (1-5)	Sensitivity	Minimum	Maximum
1	yield	mg	Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
2	Var_2		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
3	Var_3		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
4	Var_4		Mean 💌	Maximize 👻	0.5	3.0	Medium 💌		
5	Var_5		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
6	Var_6		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
7	Var_7		Mean 💌	Maximize 👻	0.5	3.0	Medium 💌		
8	Var_8		Mean 💌	Maximize 👻	0.5	3.0	Medium 💌		
9	Var_9		Mean 💌	Maximize 💌	0.5	3.0	Medium		
10	Var_10		Mean 💌	Maximize 👻	0.5	3.0	Medium 💌		
11	Var_11		Mean 💌	Maximize 👻	0.5	3.0	Medium 💌		
12	Var_12		Mean 💌	Maximize 💌	0.5	3.0	Medium		
13	Var_13		Mean 💌	Maximize 👻	0.5	3.0	Medium 💌		
14	Var_14		Mean 💌	Maximize 👻	0.5	3.0	Medium 💌		
15	Var_15		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
16	Var_16		Mean 💌	Maximize 💌	0.5	3.0	Medium		
	(эк		Cancel			Help		

In step 2, enter the following information about the experimental factors:

Comme	nt: computer augm	ented design					
lumbei	r of controllable proc	ess factors: 📙 📩	Number of contr	ollable mixture compo	nents: 0	Number of	noise factors: 0
actor	Name	Units	Туре	Role	Low	High	Levels
А	methanol		Continu	ious 💌 Controllabl	e 0.0	10.0	1,2,3,4
В	ethanol		Continu	ious 💌 Controllabl	e 0.0	10.0	1,2,3,4
С	propanol		Continu	ious 💌 Controllabl	e 0.0	10.0	1,2,3,4
D	butanol		Continu	ious 💌 Controllabl	e 0.0	10.0	1,2,3,4
Е	рН		Continu	ious 💌 Controllabl	e 6.0	9.0	1,2,3,4
F	time		Continu	ious 💌 Controllabl	e 1.0	2.0	1,2,3,4
G	Factor_G		Continu	ious 💌	-1.0	1.0	1,2,3,4
Н	Factor_H		Continu	ious 💌	-1.0	1.0	1,2,3,4
I	Factor_I		Continu	ious 👻	-1.0	1.0	1,2,3,4
J	, Factor_J	— í	Continu	ious 👻	-1.0	1.0	1,2,3,4
к	, Factor_K	— (Continu	ious 👻	-1.0	1.0	1,2,3,4
L	, Factor_L	— í	Continu	ious 👻	-1.0	1.0	1,2,3,4
м	, Factor_M	— í	, Continu	ious 🔻	-1.0	1.0	1,2,3,4
	, –	,			1	,	

In Step 3, select a Computer Generated design:

Designs for Continuous or Two-Lev	vel Fact 💌
Design Class	OK
C Screening	Cancel
Response Surface	
C Multilevel Factorial	Help
C Orthogonal Array	
 Computer Generated 	

In Step 4, select a model involving all 6 main effects and 6 two-factor interactions:

DOE W	vizard Model Options	×
Process Factors Model	Mixture Components Model	OK
 Mean Linear (Main Effects) 2-Factor Interactions Quadratic Cubic 	 Mean Linear Quadratic Special Cubic Cubic 	Cancel Help
Include: A:methanol B:ethanol C:propanol D:butanol E:pH F:time AD AF BC BF CF DF	Exclude: AA AB AC AE BB BD BE CC CD CC CD CE DD DE EE CC	

Before selecting the experimental runs, go to the DataBook and put in information about the 12 runs that have already been performed:

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			с	hapter3.sgx					83
	BLOCK	methanol	ethanol	propanol	butanol	pH	time	yield	4
								mg	
1	1	0	0	0	10	6	1	10.94	
2	1	0	10	0	0	9	1	15.79	
3	1	0	10	0	10	9	2	25.96	
4	1	10	10	10	0	6	1	35.92	
5	1	0	0	10	0	6	2	22.92	
6	1	0	10	10	10	6	1	23.54	
7	1	10	10	0	0	6	2	47.44	
8	1	10	0	0	0	9	1	19.80	
9	1	10	0	10	10	9	1	29.48	
10	1	0	0	10	0	9	2	17.13	
11	1	10	10	10	10	9	2	43.75	
12	1	10	0	0	10	6	2	40.86	
13									
14									
15									
4 4)	l chapter3 E	3 C							

Be sure to include the values for *Yield*, so that the algorithm will know that those runs have already been performed.

Now return to the DOE Wizard and press the button labeled *Step 5: Select runs*. The dialog box will show the 12 runs that have already been performed:

1 1 0.0 0.0 0.0 10.0 6.0 1.0 2 1 0.0 10.0 0.0 0.0 3.0 1.0 3 1 0.0 10.0 0.0 10.0 3.0 1.0 3 1 0.0 10.0 0.0 3.0 2.0 4 1 10.0 10.0 0.0 6.0 1.0 5 1 0.0 10.0 10.0 0.0 6.0 2.0 6 1 0.0 10.0 10.0 10.0 6.0 2.0 6 1 0.0 10.0 10.0 10.0 1.0 3.0 2.0 7 1 10.0 0.0 0.0 3.0 2.0 3	4	BLOCK	methanol	ethanol	propanol	butanol	pН	time	
2 1 0.0 10.0 0.0 0.0 9.0 1.0 3 1 0.0 10.0 0.0 10.0 9.0 2.0 4 1 10.0 10.0 10.0 0.0 6.0 1.0 5 1 0.0 0.0 10.0 0.0 6.0 2.0 6 1 0.0 10.0 10.0 0.0 6.0 2.0 6 1 0.0 10.0 10.0 10.0 6.0 2.0 6 1 0.0 10.0 10.0 10.0 6.0 2.0 7 1 10.0 0.0 0.0 0.0 9.0 1.0 9 1 10.0 0.0 10.0 10.0 9.0 2.0 10 1 0.0 0.0 10.0 9.0 2.0 1.0 12 1 10.0 0.0 0.0 10.0 6.0 2.0 <tr< td=""><td>1</td><td>1</td><td>0.0</td><td>0.0</td><td>0.0</td><td>10.0</td><td>60</td><td>10</td><td></td></tr<>	1	1	0.0	0.0	0.0	10.0	60	10	
3 1 0.0 10.0 0.0 10.0 9.0 2.0 4 1 10.0 10.0 10.0 0.0 6.0 1.0 5 1 0.0 0.0 10.0 0.0 6.0 2.0 6 1 0.0 10.0 10.0 0.0 6.0 2.0 6 1 0.0 10.0 10.0 0.0 6.0 2.0 6 1 0.0 10.0 10.0 10.0 6.0 2.0 7 1 10.0 0.0 0.0 0.0 9.0 1.0 9 1 10.0 0.0 10.0 10.0 9.0 2.0 10 1 0.0 0.0 10.0 9.0 2.0 2.0 11 1 10.0 10.0 10.0 9.0 2.0 2.0 12 1 10.0 0.0 0.0 10.0 6.0 2.0 <		•							
4 1 10.0 10.0 10.0 0.0 6.0 1.0 5 1 0.0 0.0 10.0 0.0 6.0 2.0 6 1 0.0 10.0 10.0 10.0 6.0 1.0 7 1 10.0 10.0 0.0 6.0 2.0 8 1 10.0 0.0 0.0 6.0 2.0 8 1 10.0 0.0 0.0 9.0 1.0 9 1 10.0 0.0 10.0 9.0 1.0 9 1 0.0 0.0 10.0 9.0 2.0 1 1 0.0 0.0 10.0 9.0 2.0 1 1 0.0 0.0 10.0 10.0 9.0 2.0 1 1 1 0.0 0.0 0.0 10.0 9.0 2.0 1 1 0.0 0.0 0.0 <t< td=""><td>-</td><td>•</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	-	•							
5 1 0.0 0.0 10.0 0.0 6.0 2.0 6 1 0.0 10.0 10.0 10.0 6.0 1.0 7 1 10.0 10.0 0.0 6.0 2.0 8 1 10.0 0.0 0.0 6.0 2.0 9 1 10.0 0.0 0.0 0.0 9.0 1.0 9 1 10.0 0.0 10.0 10.0 9.0 1.0 0 1 0.0 0.0 10.0 9.0 2.0 1.0 0 1 0.0 0.0 10.0 9.0 2.0 2.0 1 1 10.0 10.0 10.0 9.0 2.0 2.0 2 1 10.0 0.0 0.0 10.0 9.0 2.0 3 - - - - - - 4 - - -	-	•							
6 1 0.0 10.0 10.0 10.0 6.0 1.0 7 1 10.0 10.0 0.0 0.0 6.0 2.0 8 1 10.0 0.0 0.0 0.0 9.0 1.0 9 1 10.0 0.0 10.0 10.0 9.0 1.0 9 1 0.0 0.0 10.0 9.0 1.0 0 1 0.0 0.0 10.0 9.0 2.0 1 1 0.0 0.0 10.0 9.0 2.0 1 1 0.0 0.0 10.0 9.0 2.0 2 1 10.0 0.0 0.0 10.0 9.0 2.0 3 - - - - - - - 4 - - - - - - - - 5 - - - -		•							
7 1 10.0 10.0 0.0 0.0 6.0 2.0 8 1 10.0 0.0 0.0 0.0 9.0 1.0 9 1 10.0 0.0 10.0 9.0 1.0 9 1 10.0 0.0 10.0 9.0 1.0 0 1 0.0 0.0 10.0 9.0 2.0 1 1 0.0 0.0 10.0 9.0 2.0 1 1 0.0 0.0 10.0 9.0 2.0 2 1 10.0 0.0 0.0 10.0 9.0 2.0 3 - - - - - - - 4 - - - - - - - - 5 - - - - - - - - 6 - - - - -	-	•							
8 1 10.0 0.0 0.0 0.0 9.0 1.0 9 1 10.0 0.0 10.0 10.0 9.0 1.0 9 1 0.0 0.0 10.0 10.0 9.0 1.0 0 1 0.0 0.0 10.0 10.0 9.0 2.0 1 1 10.0 10.0 10.0 10.0 9.0 2.0 2 1 10.0 0.0 0.0 10.0 9.0 2.0 3 4 5 5 6 7	~	•							
9 1 10.0 0.0 10.0 10.0 9.0 1.0 0 1 0.0 0.0 10.0 0.0 9.0 2.0 1 1 10.0 10.0 10.0 9.0 2.0 2 1 10.0 0.0 0.0 10.0 9.0 2.0 2 1 10.0 0.0 0.0 10.0 9.0 2.0 3 - - - - - - - 4 - <									
1 0.0 0.0 10.0 0.0 9.0 2.0 1 10.0 10.0 10.0 9.0 2.0 2 1 10.0 0.0 0.0 10.0 9.0 2.0 3 - - - - - - - 4 - - - - - - - - 5 - - - - - - - - - - - 6 -		1	10.0	0.0	10.0	10.0			
2 1 10.0 0.0 0.0 10.0 6.0 2.0 3 -	-	1	0.0	0.0	10.0	0.0	9.0	2.0	
2 1 10.0 0.0 10.0 6.0 2.0 3 - - - - - 4 - - - - - 5 - - - - - 6 - - - - - 7 - - - - - 8 - - - - - 9 - - - - - 100 - - - - -	1	1	10.0	10.0	10.0	10.0	9.0	2.0	
4	-	1	10.0	0.0	0.0	10.0	6.0	2.0	
5	3								
6	4								
7	5								
8	6								
9	7								
0	8								
	9								
	20								
	21								
22	22								
									•
ptimize Display Number of coefficients: 14 D-efficiency:	ptimi	ze	- Display-		Number of coe	fficients: 14	D-effici	encur	
Defficiency Deficiency	•			units			_	-	OK
Number of base runs. 20 G-emclency:		-			Number of basi	e runs: 20	G-ethci	ency:	
G-efficiency C Coded units Number of replicates: A-efficiency:	G-	efficiency	Coded	units	Number of repli	icates: 0	A-effici	ency:	Cancel

Now:

- 1. Set *Optimize* to *D-efficiency* to create a D-optimal design.
- 2. Set *Number of base runs* to 20 to add 8 additional runs to the original 12 runs.
- 3. Push the *Advanced* button and set *Number of factor levels to consider* to 2 to consider only runs involving the low and high levels of each factor:

Computer Generated Designs - Search Options	×
Number of random starts:	
10 Maximum iterations per start:	
Number of continuous factor levels to consider:	
Mixture increment between levels:	
OK Cancel Help	

4. Push the *Create* button to initiate the design creation process. When the alogorithm is complete, 8 additional runs will be added to the dialog box:

Computer Generated Designs							
	BLOCK	methanol	ethanol	propanol	butanol	pH	A
1	1	0.0	0.0	0.0	10.0	6.0	1.0
2	1	0.0	10.0	0.0	0.0	9.0	1.0
3	1	0.0	10.0	0.0	10.0	9.0	2.0
4	1	10.0	10.0	10.0	0.0	6.0	1.0
5	1	0.0	0.0	10.0	0.0	6.0	2.0
6	1	0.0	10.0	10.0	10.0	6.0	1.0
7	1	10.0	10.0	0.0	0.0	6.0	2.0
8	1	10.0	0.0	0.0	0.0	9.0	1.0
9	1	10.0	0.0	10.0	10.0	9.0	1.0
10	1	0.0	0.0	10.0	0.0	9.0	2.0
11	1	10.0	10.0	10.0	10.0	9.0	2.0
12	1	10.0	0.0	0.0	10.0	6.0	2.0
13	2	0.0	10.0	10.0	0.0	6.0	2.0
14	2	0.0	0.0	0.0	0.0	9.0	2.0
15	2	0.0	10.0	10.0	10.0	9.0	1.0
16	2	0.0	0.0	10.0	0.0	6.0	1.0
17	2	10.0	10.0	0.0	10.0	6.0	1.0
18	2	10.0	0.0	0.0	0.0	6.0	1.0
19	2	0.0	0.0	10.0	10.0	6.0	2.0
20	2	10.0	0.0	10.0	0.0	9.0	2.0
21							
22							
•							•
Optin	nize	Display	Number of coeffic	ients: 14	Average prediction variance	e: 0.296409	
Οŀ	efficiency	Original units	Number of base ru	uns: 20	D-efficiency: 94.03%		OK
ΘD	-efficiency	C Coded units	Number of replica	tes: 0	A-efficiency: 89.52%		Cancel
ΟA	-efficiency	Randomize run orde	r Number of center	points: 0	G-efficiency: 92.19%		Help
ΟG	-efficiency			blocks of size: 1000	Create Ad	lvanced	

5. Push *OK* to save the design. © 2013 by StatPoint Technologies, Inc.

	BLOCK	methanol	ethanol	propanol	butanol	pH	time	yield
								mg
1	1	0.0	0.0	0.0	10.0	6.0	1.0	10.94
2	1	0.0	10.0	0.0	0.0	9.0	1.0	15.79
3	1	0.0	10.0	0.0	10.0	9.0	2.0	25.96
4	1	10.0	10.0	10.0	0.0	6.0	1.0	35.92
5	1	0.0	0.0	10.0	0.0	6.0	2.0	22.92
6	1	0.0	10.0	10.0	10.0	6.0	1.0	23.54
7	1	10.0	10.0	0.0	0.0	6.0	2.0	47.44
8	1	10.0	0.0	0.0	0.0	9.0	1.0	19.80
9	1	10.0	0.0	10.0	10.0	9.0	1.0	29.48
10	1	0.0	0.0	10.0	0.0	9.0	2.0	17.13
11	1	10.0	10.0	10.0	10.0	9.0	2.0	43.75
12	1	10.0	0.0	0.0	10.0	6.0	2.0	40.86
13	2	0.0	10.0	10.0	0.0	6.0	2.0	
14	2	0.0	0.0	0.0	0.0	9.0	2.0	
15	2	0.0	10.0	10.0	10.0	9.0	1.0	
16	2	0.0	0.0	10.0	0.0	6.0	1.0	
17	2	10.0	10.0	0.0	10.0	6.0	1.0	
18	2	10.0	0.0	0.0	0.0	6.0	1.0	
19	2	0.0	0.0	10.0	10.0	6.0	2.0	
20	2	10.0	0.0	10.0	0.0	9.0	2.0	

The new runs have now been added to the DataBook:

Example #3: Incorporating Design Region Constraints

It is not infrequent to find when designing an experiment that the feasible region in which design points may be placed is not cuboidal. In other words, having set low and high ranges for each of the factors, certain combinations of factor levels falling between the lows and the highs may not correspond to acceptable locations at which to perform an experiment. Statgraphics allows you to specify constraints on linear combinations of the factors that limit the locations at which experiments may be run.

Goos and Jones (2011) describe an experiment intended to maximize the yield of a chemical process. The first factor (Time) was allowed to vary between 430 and 640 seconds. The second factor (Temperature) was allowed to vary between 500 and 550 degrees Kelvin. However, experiments could only be run in regions where two additional constraints were satisfied:

0.3 Time + Temperature \geq 539.8

0.09 Time + Temperature ≤ 587.8

Outside that constrained region, it was thought that the process would behave much differently than within the region, so that any experimental points outside the region could potentially distort the estimated statistical model.

To reconstruct their experiment, start with an StatFolio and select DOE - Experimental Design Wizard from the main menu. In Step 1, enter the following information about the response variable:

Comment:	rsm with constraints								
Number of	responses: 1	- Responses 1-1	6 Responses 17-32						
Response	Name	Units	Analyze	Goal	Target	Impact (1-5)	Sensitivity	Minimum	Maximum
1	yield	*	Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
2	Var_2		Mean 👻	Maximize 💌	0.5	3.0	Medium 💌		
3	Var_3		Mean 👻	Maximize 💌	0.5	3.0	Medium 💌		
4	Var_4		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
5	Var_5		Mean 👻	Maximize 💌	0.5	3.0	Medium 💌		
6	Var_6		Mean 👻	Maximize 💌	0.5	3.0	Medium 💌		
7	Var_7		Mean 👻	Maximize 💌	0.5	3.0	Medium 💌		
8	Var_8		Mean 👻	Maximize 💌	0.5	3.0	Medium 💌		
9	Var_9		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
10	Var_10		Mean 👻	Maximize 💌	0.5	3.0	Medium 💌		
11	Var_11		Mean 👻	Maximize 💌	0.5	3.0	Medium 💌		
12	Var_12		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
13	Var_13		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
14	Var_14		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
15	Var_15		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
16	Var_16		Mean 👻	Maximize 🔻	0.5	3.0	Medium 👻		

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lumbe	ent: rsm with constra		Number of controllable mi	xture component:	s: 0	Number of r	noise factors: 0
actor	Name	Units	Туре	Role	Low	High	Levels
А	time	seconds	Continuous 💌	Controllable	430	650	1,2,3,4
В	temperature	degrees K	Continuous 💌	Controllable	500	550	1,2,3,4
С	Factor_C		Continuous 💌		-1.0	1.0	1,2,3,4
D	Factor_D		Continuous 💌		-1.0	1.0	1,2,3,4
Е	Factor_E		Continuous 💌		-1.0	1.0	1,2,3,4
F	Factor_F		Continuous 💌		-1.0	1.0	1,2,3,4
G	Factor_G		Continuous 💌		-1.0	1.0	1,2,3,4
Н	Factor_H		Continuous 💌		-1.0	1.0	1,2,3,4
I	Factor_I		Continuous 💌		-1.0	1.0	1,2,3,4
J	Factor_J		Continuous 💌		-1.0	1.0	1,2,3,4
к	Factor_K		Continuous 💌		-1.0	1.0	1,2,3,4
L	Factor_L		Continuous 💌		-1.0	1.0	1,2,3,4
м	, Factor_M		Continuous 💌		-1.0	1.0	1,2,3,4

In step 2, enter the following information about the experimental factors:

In Step 3, select a *Computer Generated* design on the first dialog box:

Designs for Continuous or Two-Level Fact								
Design Class	ОК							
C Screening C Response Surface	Cancel							
C Multilevel Factorial	Help							
O thogonal Array								
 Computer Generated 								

Press OK to return to the Select Design dialog box.

After *Computer Generated* is selected, the *Constraints* button at the bottom of the *Select Design* dialog box will be enabled. Pressing that button displays a dialog box in which up to 20 constraints involving the experimental factors may be entered:

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	Computer Generated Designs	
Factors A:temperature B:pressure C:speed D:supplier E: F: G: H: J: J: K: L: M: N: O: P: Q: R: S: T: U: V: V: V: V: Y: Z:	Constraints	OK Cancel Help
Add constraint	t Edit constraint	Delete constraint

You may enter constraints of the form

 $C_1 X_1 + C_2 X_2 + C_3 X_3 \leq d$

or

$$C_1X_1 + C_2X_2 + C_3X_3 \ge d$$

by pressing the *Add constraint* button and entering the values of C_1 , C_2 and C_3 . For the current example, the first constraint should be added by entering the following:

				STATGRAPHICS – Rev. 3/13/20)15
		Design Fa	actor Constraints	×	
0.3	A+ 1.0	B+ 0	C+0	D 🔲 Coded units	
+0	E+0	F+ 0	G+0	Н	
+0	I+ 0	J+ 0	K+ 0	L	
+0	M+0	N+0	0+0	P >= ▼ 539.8	
	ОК		Cancel	Help	

Press OK and then Add constraint again to add the second constraint:

		Design Fa	ctor Constraints	×
0.09	A+ 1.0	B+0	C+0	D 🗌 Coded units
+0	E+ 0	F+ 0	G+0	Н
+0	I+ 0	J+ 0	K+0	L
+0	M+0	N+0	0+0	P <= ▼ 587.8
	ОК		Cancel	Help

Press OK again and review the entered constraints:

C	omputer Generated Designs	×
Factors A:time B:temperature C: D: E: F: G: H: J: J: K: L: M: N: D:	Constraints 0.3A+1.0B>=539.8 0.09A+1.0B<=587.8	OK Cancel Help
Add constraint	Edit constraint De	elete constraint

Note: if your design contains both process factors and mixture components, each constraint may involve only one type of factor.

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Press *OK* twice to return to the main DOE Wizard window, which will display the entered constraints:

		ic responses	Step 1:Define responses Step 3:Select design Step 5:Select runs							Step 7:Save experiment Step 9:Optimize responses		Step 11:Augment design		
	2:Define		<u> </u>						- <u></u>	· ·				
	Step 2:Define exp. factors Step 4:Specify model			model	Step 6	5:Evaluat	te design	Step 8:Ar	halyze data	Step 10): Save res	ults	Step 12:Extrapolate	
xperime	ntal D	esign W	izard											
Vame Uni			ables to be n <i>Toal</i>	Target		Sensiti	ivite	Low	High					
vield %		~	/aximize	1 co gei	3.0	Mediu		2011	mgn					
/iciu /o	111		/laxiiiii2e		5.0	Ivieut	ш							
ep 2: Defin	e the ext	perimental f	factors to be	varied										
Vame		Units	Type		Role		Low	High	Levels					
A:time		seconds	Continu	ous	Controllal	ble	430.0	650.0						
3:temperatu	ire	degrees K	Continu	ous	Controlla	ble	500.0	550.0						
		perimental d	esign											_
	Design				Centerpoint		Centerpo		Design is	Number of		Total	Error	•
	Туре			j	Per Block		Placeme	nt	Randomized	Replicates	Runs	Blocks	D.F.	
rocess	Comput	ter generated	d design											

In Step 4, select a cubic model:

DOE W	/izard Model Options	×
Process Factors Model	Mixture Components Model	ОК
C Mean	🖲 Mean	Cancel
C Linear (Main Effects)	C Linear	
C 2-Factor Interactions	C Quadratic	Help
C Quadratic	C Special Cubic	
Cubic	C Cubic	
Include: A:time B:temperature AA AB BB AAA AAB ABB BBB	Exclude:	

This model involves 2 main effects, 1 two-factor interaction, 2 quadratic effects, 2 cubic terms of the form x_j^3 , and 2 mixed cubic terms of the form $x_i x_j^2$.

		Computer G	enerated De	signs		
В	LOCK time seconds	temperature degrees K				A
1	seconds	degrees it				
2						
3						
4						
5						
6 7						
8						
9						
10						
11						
12						
13						
14						
15 16						
17						
18						
19						
20						
21						
22						
						•
Optimize —	Display	Number of coefficie	nts: 10	D-efficiency:		
D-efficie	ncy 💿 Original units	Number of base rur	s: 13	G-efficiency:		OK
C G-efficie	ncy Coded units	Number of replicate	1	A-efficiency:		Cancel
C A-efficie	ncy 🔽 Randomize run or	der Number of centerpo	ints: 0	Average prediction	on variance:	Help
C I-efficier	су	💌 Block old runs v		Create	Advanced	

Now press the button labeled *Step 5: Select runs*. Complete the dialog box as shown below:

The settings request a D-optimal design with a total of 15 runs, 2 of which are to be replicates of 2 of the 13 base runs. Press the *Create* button to generate the design:

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				a	STATORAFILCS - N	x v. 5/15/201
			Computer	Generated Desig	ins	
	BLOCK	time	temperature			
		seconds	degrees K			
1	1	720.0	523.0			
2	1	605.0	527.5			
3	1	360.0	529.0			
4	1	475.838	525.525			
5	1	499.363	542.857			
6	1	360.0	535.0			
7	1	360.0	529.0			
8	1	590.544	534.651			
9	1	360.0	544.75			
10	1	643.83	520.485			
11	1	457.222	537.9			
12	1	720.0	520.0			
13	1	360.0	550.0			
14	1	475.838	525.525			
15	1	420.0	550.0			
16						
17						
18						
19						
20						
21						
22						-
						•
– Optir	mize	Display	Number of coeffic	cients: 10	Average prediction variance: 0.618091	
O I	-efficiency	 Original units 	Number of base r	runs: 13	D-efficiency: 5.46%	OK
• 0)-efficiency	C Coded units	Number of replica	ates: 2	A-efficiency: 0.27%	Cancel
C A	\-efficiency	🔽 Randomize run orde	Number of center	rpoints: 0		Help
0.6	à-efficiency			blocks of size: 1000	Create Advanced	

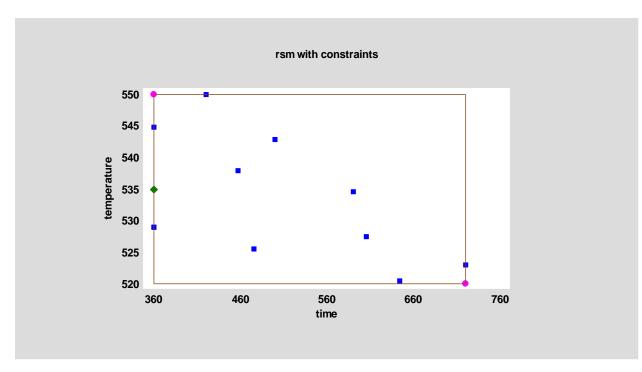
Push *OK* to save the design.

Once the design has been saved to the datasheet, you may round off the levels if desired:

C:\Data\Version17 Tests\doewiz compgen constraints.sgx								
	BLOCK	time	temperature	yield				
		seconds	degrees K	8				
1	1	720.0	523.0					
2	1	605.0	527.5					
3	1	360.0	529.0					
4	1	476	526					
5	1	499	543					
6	1	360.0	535.0					
7	1	360.0	529.0					
8	1	591	535					
9	1	360.0	545					
10	1	644	520					
11	1	457	538					
12	1	720.0	520.0					
13	1	360.0	550.0					
14	1	476	526					
15	1	420.0	550.0					
16					-			
$ \bullet \bullet \bullet \bullet \bullet$	ABC			•				

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To display the design points, select *Step 6: Evaluate design* from the DOE Wizard toolbar. The *Design Points* graph displays the data as shown below:



Original vertices are shown as round point symbols.

Example #4: Designs with Mixture Variables

Many experiments involve determining tradeoffs between components which must sum to a fixed value. A typical example is a formulation which consists of 3 components. While the experimenter may be able to tradeoff one component for another, the sum of the percentages of each component must equal 100%. Such "mixture" experiments may or may not include additional process factors.

A typical example of a mixture problem, discussed by Myers and Montgomery (2002), involves the formulation of a rocket propellant. The propellant is a mixture of three components: a fuel, an oxidizer, and a binder. The researcher wished to find a combination of these three components which achieved a satisfactory burn rate. Since an inert component made up 10% of the propellant, the factors were constrained by:

fuel + oxidizer + binder = 90%.

In addition, there were lower bounds for each of the three components:

 $\begin{array}{l} 30\% \leq fuel \\ 20\% \leq oxidizer \\ 20\% \leq binder \end{array}$

Given the constraints, there is a remaining 20% of the mixture that can be any combination of *fuel*, *oxidizer*, and/or *binder*.

To reconstruct their experiment, start with an StatFolio and select DOE - Experimental Design Wizard from the main menu. In Step 1, enter the following information about the response variable:

		Design	of Experiment	s Wizard - De	fine Resp	onses			×
Design fil	e: propellant.sgx								
Comment	Rocket propellant examp	le							
Number o	of responses: 1	Responses 1-16 R	esponses 17-32						
Respons	e Name	Units	Analyze	Goal	Target	Impact (1-5)	Sensitivity	Minimum	Maximum
1	burn rate	cm per second	Mean 💌	Hit target 💌	85.0	3.0	Medium 💌		
2	Var_2		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
3	Var_3		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
4	Var_4		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
5	Var_5		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
6	Var_6		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
7	Var_7		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
8	Var_8		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
9	Var_9		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
10	Var_10		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
11	Var_11		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
12	Var_12		Mean	Maximize 💌	0.5	3.0	Medium 💌		
13	Var_13		Mean	Maximize 💌	0.5	3.0	Medium 💌		
14	Var_14		Mean 💌	Maximize 💌	0.5	3.0	Medium 💌		
15	Var_15		Mean	Maximize 💌	0.5	3.0	Medium 💌		
16	Var_16		Mean	Maximize 💌	0.5	3.0	Medium 💌		
		ОК		Cancel			Help		

The goal of the experiment was to achieve a burn rate close to 85 cm per second.

In step 2, enter the following information about the experimental factors:

Design	file: propellant.sgx	-	f Experiments W				
Comme	nt: Rocket propellar	nt example					
Numbe	r of controllable proce	Number of r	noise factors: 0				
Factor	Name	Units	Туре	Role	Low	High	Levels
А	fuel	percent	Mixture	Controllable	30.0	50.0	30.0,50.0
В	oxidizer	percent	Mixture	Controllable	20.0	40.0	20.0,40.0
С	binder	percent	Mixture	Controllable	20.0	40.0	20.0,40.0
D	Factor_D		Continuous 📼]	-1.0	1.0	1,2,3,4
Е	Factor_E		Continuous 📼]	-1.0	1.0	1,2,3,4
F	Factor_F		Continuous 📼]	-1.0	1.0	1.2.3,4
G	Factor_G		Continuous 📼]	-1.0	1.0	1,2,3,4
Н	Factor_H		Continuous 📼]	-1.0	1.0	1,2,3,4
T	Factor_I		Continuous 📼]	-1.0	1.0	1,2,3,4
J	Factor_J		Continuous 📼]	-1.0	1.0	1,2,3,4
к	Factor_K		Continuous 📼]	-1.0	1.0	1.2.3,4
L	Factor_L		Continuous 📼]	-1.0	1.0	1,2,3,4
м	Factor_M		Continuous 📼]	-1.0	1.0	1,2,3,4
Tota	for controllable mixtu	re components: 90.0				Factors A-M	Factors N-Z
	OK		Back	Cance	1		Help

There are 3 mixture components which may sum to 90%.

Pushing the button labeled *Step 3: Select design* displays the dialog box shown below:

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		D	esign of E	Experime	nts Wizard - Select Design		×
Design file: pro						- Robust Parameter Design	7
Comment: <mark>Roc</mark>	ket propellant example	_	_			 Combined array C Crossed array 	
	Segment	Factors	Runs	Blocks	Design		
Options	Process factors	0	0	0			
Options	Mixture components	3	0	0	Press the Options button.		
Options		0	0	0			
	COMBINED	3	0	0	Samples per run: 1		
BLO	ICK fuel		oxidi	zer	binder		
	percer	nt	perc		percent		
,	ок	Cano		D	erandomize Constrai		_
				-	Consta	nis neip	

Press the *Options* button for the mixture components to display a list of designs appropriate for 3 mixture components:

Mixture Design Selection								
Name	Linear	Quadratic	Special Cubic	Cubic				
Simplex-Lattice	3	6	10	10	-			
Simplex-Lattice	3	6	10	10				
Simplex-Centroid	7	7	7					
Extreme vertices	3							
Computer generated design User-specified design								
Display Blocked Designs	Cancel	Bac	sk	Help				

Select *Computer generated design* and press *OK* to return to the previous dialog box:

STATGRAPHICS - Rev. 3/13/2015

		D	esign of	Experime	nts Wizard - Select Design		×
Design file: prop						Robust Parameter Design	
Comment: Rock	ket propellant exam	 Combined array C Crossed array 					
	Segment	Factors	Runs	Blocks	Design		
Options	Process factors	0	0	0			
Options	Mixture compone	ents 3	0	0	Computer generated design		
Options		0	0	0			
	COMBINED	3	0	0	Samples per run: 1		
BLO	CK .	fuel	ovic	dizer	binder		
		ercent		cent	percent		
•							
	ок	C		D	erandomize	inte de la	
	UK	Can	cei	Re		aints Help	

Press *OK* again to return to the main DOE Wizard window:

St St	Step 1:Define responses Step 3:Select design Step 5:Select runs Step 7:Save experiment Step 9:Optimize responses Step 11											Step 11:	Augment	desigr			
	Step 2:Define exp. factors Step 4:Specify model						Step 6:Evaluate design			Step 8:Analyze data		Step 10: Save results			Step 12:Extrapolate		-
Experin	xperimental Design Wizard																
tep 1: De	fine the respons	e variables	to be mea	sured													
Name	Units			Goal	Tar	rget I	mpact	Sensitivi	ity	Low	High						
burn rate	cm per seco	nd M	lean	Hit target	85.0	0 3	3.0	Medium	1								
step 2: De	fine the experim	ental facto	rs to be va	inea													
A:fuel	Units percent r percent	<i>Type</i> Mixture Mixture	Role Control Control		Low 30.0 20.0	High 50.0 40.0	Levels										
Name A:fuel B:oxidizer C:binder	percent	Mixture	Control	llable	30.0	50.0	Levels										
A:fuel B:oxidizer C:binder Mixture to	percent r percent percent	Mixture Mixture Mixture	Control Control Control	llable	30.0 20.0	50.0 40.0	Levels										
A:fuel B:oxidizer C:binder Vixture to Step 3: Sele	percent percent percent tal = 0.0	Mixture Mixture Mixture	Control Control Control	llable llable	30.0 20.0	50.0 40.0 40.0	Levels		Design	is	Number of	Total	Total	Error	-]		
A:fuel B:oxidizer C:binder Mixture to	percent r percent percent tal = 0.0 ect the experime	Mixture Mixture Mixture	Control Control Control	llable llable	30.0 20.0 20.0	50.0 40.0 40.0		int 1	Design 1 Random		Number of Replicates	Total Runs	Total Blocks	Error D.F.	-		

Press the *Step 4* button to select the desired model:

	STAT	GRAPHICS – Rev. 3/13/2015
DOE Wizard Model Options	×	
Process Factors Model Mean Linear (Main Effects) 2-Factor Interactions Quadratic Quadratic Quadratic Quadratic Cubic	OK Cancel Help	

In the example, the researchers choose the popular *Special Cubic* model which has 7 coefficients that must be estimated.

Now press the button labeled *Step 5: Select runs*. Complete the dialog box as shown below:

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			Computer (Generated Desig	Ins	×
	BLOCK	fuel	oxidizer	binder		-
		percent	percent	percent		
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
16						
17						
18						
19						
20						
21						
22						-
•				· · · · · · · · · · · · · · · · · · ·		•
_ Optim	nize	Display	Number of coeffici	ents: 7	Average prediction variance:	
010	efficiency	Original units	Number of base ru	ins: 10		ОК
I⊙ p	-efficiency	C Coded units			D-efficiency:	Cancel
-			Number of replicat	-	A-efficiency:	Lancel
	-efficiency	💌 Randomize run order	Number of centerp	points: 0	G-efficiency:	Help
G	-efficiency		🔽 Group runs in l	blocks of size: 1000	Create Advanced	

The settings request a D-optimal design with a total of 10 runs.

Before creating the design, press the *Advanced* button and set the *Mixture increment* field to 6.6667:

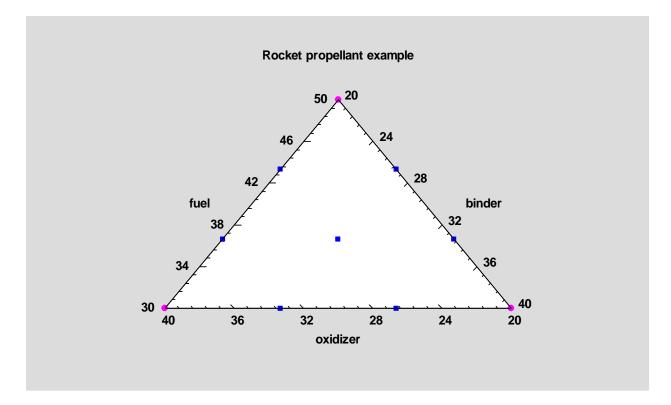
Computer Generated Designs - Search Options	×
Number of random starts: 10	
Maximum iterations per start:	
Set by factor	
Mixture increment between levels:	
6.66667	
Calculate G-efficiency (may be time-consuming)	
OK Cancel Help	

This tells the program to consider experimental runs spaced at increments of 62/3% between the low and high levels of each component.

BLOCI	< fuel	oxidizer	binder		
	percent	percent	percent		
1 1	36.6667	33.3333	20.0		
2 1	30.0	40.0	20.0		
3 1	50.0	20.0	20.0		
4 1	30.0	20.0	40.0		
5 1	43.3333	26.6667	20.0		
6 1	36.6667	20.0	33.3333		
7 1	30.0	33.3333	26.6667		
8 1	30.0	26.6667	33.3333		
9 1	43.3333	20.0	26.6667		
10 1	36.6667	26.6667	26.6667		
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					
21					
22					
					•
)ptimize	Display	Number of coe	efficients: 7	Average prediction variance: 0.522307	
l-efficiency	Original units	Number of bas	se runs: 10	D-efficiency: 13.35%	OK
D-efficiency	C Coded units			•	Cancel
e onloiding		Number of rep	licates: 0	A-efficiency: 1.12%	Lancel

Press the *Create* button to generate the design:

To display the design points, select *Step 6: Evaluate design* from the DOE Wizard toolbar. The *Design Points* graph displays the data as shown below:



The selected runs include the 3 vertices, 2 points on each face, and the centerpoint.

STATGRAPHICS – Rev. 3/13/2015 Example #5: Designs with Both Process and Mixture Variables

Many experiments involve determining tradeoffs between components which must sum to a fixed value. A typical example is a formulation which consists of 3 components. While the experimenter may be able to tradeoff one component for another, the sum of the percentages of each component must equal 100%. Such "mixture" experiments may or may not include additional process factors.

Goos and Jones (2011) describe an experiment intended to study the effects on *reflectivity* in a rolling mill producing sheet aluminum. The experiment involved two process factors (*spray volume* and *oil-water ratio*) and three mixture components (percentage of thickness reduction assigned to each of 3 *rollers*).

To reconstruct their experiment, start with a StatFolio and select DOE - Experimental Design Wizard from the main menu. In Step 1, enter the following information about the response variable:

		Design	of Experimen	ts Wizard - De	fine Respo	onses			×
Design fil	e: <untitled></untitled>								
Comment	: milling experiment								
Number o	of responses: 1	Responses 1-16	esponses 17-32						
Response	e Name	Units	Analyze	Goal	Target	Impact (1-5)	Sensitivity	Minimum	Maximum
1	reflectivity		Mean	Maximize 💌	0.5	3.0	Medium 💌		
2	Var_2		Mean	Maximize 💌	0.5	3.0	Medium 📼		
3	Var_3		Mean	Maximize 💌	0.5	3.0	Medium 💌		
4	Var_4		Mean	Maximize 💌	0.5	3.0	Medium 💌		
5	Var_5		Mean	Maximize 💌	0.5	3.0	Medium 💌		
6	Var_6		Mean	Maximize 💌	0.5	3.0	Medium 💌		
7	Var_7		Mean	Maximize 💌	0.5	3.0	Medium 💌		
8	Var_8		Mean	Maximize 💌	0.5	3.0	Medium 💌		
9	Var_9		Mean	Maximize 💌	0.5	3.0	Medium 💌		
10	Var_10		Mean	Maximize 💌	0.5	3.0	Medium 💌		
11	Var_11		Mean	Maximize 💌	0.5	3.0	Medium 💌		
12	Var_12		Mean	Maximize 💌	0.5	3.0	Medium 💌		
13	Var_13		Mean	Maximize 💌	0.5	3.0	Medium 💌		
14	Var_14		Mean	Maximize 💌	0.5	3.0	Medium 💌		
15	Var_15		Mean	Maximize 💌	0.5	3.0	Medium 💌		
16	Var_16		Mean	Maximize 💌	0.5	3.0	Medium 💌		
		ок		Cancel			Help		

In step 2, enter the following information about the experimental factors:

		Design of E	xperiments W	izard - Define	Factors	in mos	×
	21						
-	file: <untitled></untitled>					-	
Comme	nt: milling experiment						
Numbe	r of controllable process f	actors: 2 📩 Numb	er of controllable mi	ixture components:	3	Number of n	oise factors: 0
Factor	Name	Units	Туре	Role	Low	High	Levels
А	spray volume		Categorical 💌	Controllable	-1.0	1.0	low,high
В	oil-water		Categorical 💌	Controllable	-1.0	1.0	low,high
С	roller 1	reduction	Mixture	Controllable	0.1	0.8	1,2,3,4
D	roller 2	reduction	Mixture	Controllable	0.1	0.8	1,2,3,4
Е	roller 3	reduction	Mixture	Controllable	0.1	0.8	1,2,3,4
F	Factor_F		Continuous 💌		-1.0	1.0	1,2,3,4
G	Factor_G		Continuous 💌		-1.0	1.0	1,2,3,4
Н	Factor_H		Continuous 💌		-1.0	1.0	1,2,3,4
T	Factor_I		Continuous 💌	[-1.0	1.0	1,2,3,4
J	Factor_J		Continuous 💌	[-1.0	1.0	1,2,3,4
К	Factor_K		Continuous 💌	[-1.0	1.0	1,2,3,4
L	Factor_L		Continuous 💌	[-1.0	1.0	1,2,3,4
м	Factor_M		Continuous 💌		-1.0	1.0	1,2,3,4
Tota	l for controllable mixture c	omponents: 1.0				Factors A-M	Factors N-Z
		,					
	ОК	Bac	:k	Cancel			Help
	OK	Bac	*	Cancel			Help

The two process variables are treated as categorical factors with 2 levels each. The 3 mixture components, which represent the proportion of thickness reduction performed by each roller, are each constrained to range between 0.1 and 0.8, summing to 1.

Pushing the button labeled *Step 3: Select design* displays the dialog box shown below:

					1	STATGR.	APHIC	CS - Rev. 3/	/13/2015
		De	esign of E	xperime	nts Wizard - Select	Design			×
Design file: C:\l Comment: millin	Data\Version17 Tests\c 1g experiment	hapter6.sg:	([Robust Para	ameter Design ned array	
	Segment	Factors	Runs	Blocks	Design		C Crosse	d array	
Options	Process factors	2	0	0	Press the Options butt	on.			
Options	Mixture components	3	0	0	Press the Options butt	on.			
Options		0	0	0					
	COMBINED	5	0	0	Samples per run: 1				
BLO	ICK spray vol	ume	oil-wa	ater	roller 1 reduction	roller 2 reductior	1	roller 3 reduction	
•									
	OK	Cano	el	R	erandomize	Constraints		Help	

Press the Options button for the process factors to display the following choices:

Designs for Continuous or Two-Lev	vel Fact 💌
Design Class	ОК
C Screening	Cancel
C Response Surface	
O Multilevel Factorial	Help
C Orthogonal Array	
 Computer Generated 	

Select *Computer Generated* and press *OK* to return to the *Select Design* dialog box:

					S	STATGE	RAPH	ICS - Rev. 3/	13/2015
		De	esign of E	Experime	nts Wizard - Select I	Design			×
Design file: C:\E Comment: millin	Data\Version17 Tests\c ig experiment	hapter6.sg:	(Parameter Design mbined array	
	Segment	Factors	Runs	Blocks	Design		C Cro	ssed array	
Options	Process factors	2	0	0	Computer generated de	esign			
Options	Mixture components	3	0	0	Computer generated de	esign			
Options		0	0	0					
	COMBINED	5	0	0	Samples per run: 1				
A BLO	CK spray voli	ume	oil-w	ater	roller 1 reduction	roller : reducti		roller 3 reduction	
									F
	ov.		. 1	_	1		1		
	ОК	Cano	el		erandomize	Constraints		Help	

Notice that the *Design* for the mixture components has also been set to *Computer Generated Design*, since a single design will be generated for all 5 factors.

Next, push the button labeled *Step 4: Specify model* to display the dialog box shown below:

		STAT	GRAPHICS – Rev.
DOE Wi	zard Model Options	×	
Process Factors Model Mean Linear (Main Effects) 2-Factor Interactions Quadratic Cubic Include:	Mixture Components Model C Mean C Linear C Quadratic C Special Cubic C Cubic Exclude:	OK Cancel Help	
C:roller 1 AC BC D:roller 2 AD BD E:roller 3 AE BE			

Selecting a linear model for both the process factors and the mixture components results in a model contraining 9 coefficients that must be estimated.

After saving the choice of models, press the button labeled Step 5: Select runs to display the design creation dialog box:

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		Computer (Generated Desi	igns		
⊿ BLOCK	spray volume	oil-water	roller 1	roller 2	roller 3	
4			reduction	reduction	reduction	
1						
2						_
3 4						
4 5						
6						
7						
8						
9						
10						_
11						
12 13						
14						_
15						
16						
17						
18						
19						_
20						_
21						
22						
)ptimize	Display	Number of coeffici	ents: 9	Average prediction varian	ce:	
I-efficiency	 Original units 	Number of base ru	ns: 12	D-efficiency:		OK
D-efficiency	C Coded units	Number of replicat	es: 0	A-efficiency:		Cancel
A-efficiency	🔽 Randomize run order	Number of centerp	oints: 0	G-efficiency:		Help
G-efficiency			blocks of size: 1000	Create A	Advanced	

Be sure that "D-efficiency" is checked and that the requested number of base runs is 12.

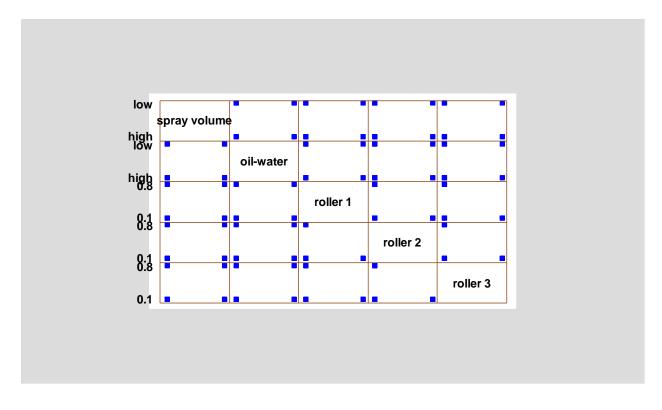
Press the Advanced button and set the *Mixture increment* field to 0.1:

Computer Generated Designs - Search Options	X
Number of random starts: 10	
Maximum iterations per start:	
Set by factor	
Mixture increment between levels:	
Calculate G-efficiency (may be time-consuming)	
OK Cancel Help	

Press *OK* and then *Create* to create the design as shown below:

		Compute	er Generated Desig	INS		
⊿ BLOCK	spray volume	oil-water	roller 1	roller 2	roller 3	
			reduction	reduction	reduction	
1 1	low	high	0.1	0.1	0.8	
2 1	high	high	0.1	0.8	0.1	
3 1	low	low	0.8	0.1	0.1	
4 1	high	low	0.1	0.8	0.1	
5 1	low	low	0.1	0.8	0.1	
6 1	high	low	0.8	0.1	0.1	
7 1	low	high	0.8	0.1	0.1	
8 1	high	high	0.1	0.1	0.8	
9 1	low	low	0.1	0.1	0.8	
0 1	high	high	0.8	0.1	0.1	
1 1	high	low	0.1	0.1	0.8	
2 1	low	high	0.1	0.8	0.1	
3						
4						
5						
6						
7						
8						
9						
20						
21						
22						
						•
ptimize	Display	Number of coe	fficients: 9	Average prediction varia	nce: 0.373462	
I-efficiency	 Original units 	Number of bas		D-efficiency: 83.99%	NGG. 0.010402	ОК
D-efficiency	C Coded units	Number of rep		A-efficiency: 66.67%		Cancel
A-efficiency	Randomize run or	der Numberofcer	iterpoints: 0	G-efficiency: 100.00%		Help
G-efficiency		🔽 Group runs	in blocks of size: 1000	Create	Advanced	

To display the design points, select *Step 6: Evaluate design* from the DOE Wizard toolbar. The *Design Points* graph displays the experimental points as shown below



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