Sample Size Determination (Capability Indices)

This procedure determines a suitable sample size for estimating three capability indices:

- **Cp** – This two-sided capability index compares the distance between the specification limits to 6-sigma:
  \[
  C_p = \frac{USL - LSL}{6\sigma}
  \]
  (1)

- **Cpk** – The smaller of two one-sided indices:
  \[
  C_{pk} = \min\left(\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\right)
  \]
  (2)

- **Cpm** – A modified version of Cp measuring variation around the target or nominal value \( T \) rather than the estimated process mean:
  \[
  C_{pm} = \frac{C_p}{\sqrt{1 + \frac{(\mu - T)^2}{\sigma^2}}}
  \]
  (3)

where LSL and USL are the lower and upper specification limits, respectively, \( \mu \) is the process mean, and \( \sigma \) is the process sigma. The estimates of the indices, which will be labeled \( \hat{C}_p \), \( \hat{C}_{pk} \), and \( \hat{C}_{pm} \) are calculated by replacing the mean and sigma with estimated values.

**Sample StatFolio:** capsize.sgp

**Sample Data:**

None.
Data Input

The first dialog box displayed by this procedure is used to specify the problem of interest to the analyst.

- **Index**: the capability index to be estimated. It is assumed that a random sample of size $n$ will be used to estimate the indicated index. The procedure will determine a suitable value for $n$.

- **Relative error**: the maximum desired difference between the estimate of the capability index and its lower confidence bound.

- **Estimated index**: the anticipated value of the capability index (required for Cpk only). This is the value at which the relative error will be fixed. For Cp and Cpm, the relative error is the same for all values of the index.

- **Confidence level**: the level of confidence for the lower confidence bound.

- **Mean minus target**: the anticipated difference between the estimated mean and the target value (required for Cpm only). This is the value at which the relative error will be fixed.

For example, the above dialog box indicates that the lower 95% confidence bound for Cpk should be no more than 10% below the estimated value when $\hat{C}_{pk}$ equals 1.33.
Analysis Summary
The Analysis Summary displays the required sample size:

### Sample Size Determination (Capability Indices)
- **Capability index:** Cpk
- **Estimate:** 1.33
- **Relative error:** 10.0%
- **Confidence level:** 95.0%

The required sample size is 154.

**The StatAdvisor**
To be 95.0% confident that the true value of Cpk is no less than 10.0% below the estimated value, the required sample size is 154 if the estimate equals 1.33.

In the current example, a sample of \( n = 154 \) observations is required to achieve the desired lower bound.

### Confidence Bounds
The Confidence Bounds plot shows the lower confidence bound as a function of the estimated capability index:

For a sample of \( n = 154 \), the bound is 10% below the estimate when \( \hat{C}_{pk} \) equals 1.33. For Cpk, the relative error increases as the estimate increases.
Calculations

The program finds the smallest $n$ such that the lower confidence bound for the ratio of the true capability index to its estimated value satisfies the condition specified in the relative error field of the dialog box.

Let $\alpha$ equal 1 minus the desired confidence level. Then the equations for the lower confidence bound of that ratio are shown below:

- **Cp**

\[
\frac{\sqrt{X_{\alpha,n-1}^2}}{n - 1}
\]  

(4)

where $X_{\alpha,n-1}^2$ is the value at which the cumulative chi-square distribution with $n$-1 degrees of freedom equals $\alpha$.

- **Cpk**

\[
1 - z_{\alpha} \sqrt{\frac{1}{9n} + \frac{\hat{C}_{pk}^2}{2(n-1)}} / \hat{C}_{pk}
\]  

(5)

where $z_{\alpha}$ is the value at which the cumulative standard normal distribution equals $1-\alpha$ and $\hat{C}_{pk}$ is the value entered in the estimated index field on the dialog box.

- **Cpm**

\[
\sqrt{\frac{nX_{\alpha,f}^2}{(n-1)f}}
\]  

(6)

where

\[
f = \frac{(n + \hat{\lambda})^2}{n + 2\hat{\lambda}}
\]  

(7)

and

\[
\hat{\lambda} = n\delta^2
\]  

(8)

where $\delta$ is the value entered in the mean minus target field on the dialog box.


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