

Sample Size Determination Using Statgraphics Centurion

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Preliminaries

One of the most important questions asked of statisticians is:
“How much data do I need?”

In general:

1. More data is better than less data (from a statistical perspective).
2. On the other hand, more data costs more money and takes longer to collect.
3. The information gained by each additional data value decreases as the sample size increases.
4. All data are not created equal.

Statgraphics Procedures

Sample size determination procedures are available for:

1. Estimating or testing a **single population parameter**
2. Comparing estimates from **2 or more samples**
3. Estimating **capability indices**
4. Constructing **control charts**
5. Developing **statistical tolerance limits**
6. Creating a **screening experiment**
7. Developing an **acceptance sampling plan**

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Two Basic Methods for Determining Sample Size

Method #1: based on the parameter to be estimated, find a sample size n such that the **margin of error** for the estimate is acceptably small.

Method #2: based on a statistical test to be performed, find a sample size n such that the **power** of the test against a specific alternative hypothesis is acceptably large.

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Example #1 – estimating a population proportion using the margin of error method

Suppose we wish to estimate the proportion of voters p who would select Mitt Romney over Barack Obama in a head-to-head election for U.S. president. We could ask that the sample size n be large enough so that the 95% confidence interval for p is no wider than:

$$\hat{p} \pm 0.03$$

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Tools – Sample Size Determination – One Sample

- Select the parameter to be estimated.
- Specified the expected value of the parameter.

Sample-Size Determination

Parameter

Normal Mean

Normal Sigma

Binomial Proportion

Poisson Rate

Hypothesized Mean: 0.0

Hypothesized Sigma: 1.0

Hypothesized Proportion: 0.5

Hypothesized Rate: 1.0

OK

Cancel

Help

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Second Dialog Box

- Specify the desired width of the confidence interval and the level of confidence.
- “Alternative hypothesis” controls whether a two-sided confidence interval or a one-sided confidence bound will be constructed.

Sample-Size Determination Options

Control

Absolute Error
+ 0.03

Relative Error
+ 10.0 %

Power
95.0 %

Difference to Detect:
0.05

Sample Size
30

Confidence Level: 95.0 %

Alternative Hypothesis

Not Equal
 Less Than
 Greater Than

Sigma

To Be Estimated
 Known

OK
Cancel
Help

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Solution:

Sample-Size Determination

Sample-Size Determination

Parameter to be estimated: binomial parameter
Desired tolerance: +- 0.03 when proportion = 0.5
Confidence level: 95.0%

The required sample size is n=1098 observations.

The StatAdvisor

This procedure determines the sample size required when estimating the proportion of a binomial distribution. 1098 observations are required to estimate theta to within +/-0.03 (assuming theta is around 0.5) with 95.0% confidence.

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Example #2 – comparing two means using the power method

Suppose we wish to compare a **new** process to an **old** process. We intend to obtain n_1 measurements from the new process and n_2 measurements from the old process. We wish to be 90% certain that we will obtain a statistically significant result if the mean of the new process is at least 20% larger than the mean of the old process.

Null hypothesis: $\mu_{\text{new}} \leq \mu_{\text{old}}$

Alternative hypothesis: $\mu_{\text{new}} > \mu_{\text{old}}$

We desire large enough samples such that the probability of rejecting the null hypothesis is at least 0.9 when $\mu_{\text{new}} / \mu_{\text{old}} = 1.2$.

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Tools – Sample Size Determination – Two or More Samples

- Specify the parameters to be compared.
- Specify the difference indicated by the null hypothesis.
- Enter assumed means and common with-in group standard deviation.
- Enter percent of data in the first sample (usually 50%).

| Sample-Size Determination (Compare) | |
|---|-----------------------------------|
| Compare | |
| <input checked="" type="radio"/> Two Normal Means | OK |
| <input type="radio"/> Two Normal Sigmas | Cancel |
| <input type="radio"/> Two Binomial Proportions | Help |
| <input type="radio"/> Two Poisson Rates | |
| <input type="radio"/> Several Normal Means | |
| Hypothesized Difference: | Hypothesized Within-Group Sigmas: |
| 0.0 | 5.0 |
| Hypothesized Means: | Hypothesized Rates: |
| 30.0 | 1.0 |
| Hypothesized Proportions: | Number of Means: |
| 0.5 | 3 |
| Percent of Data in First Sample: | |
| 50.0 % | |

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Second Dialog Box

- Specify power (probability of rejecting the null hypothesis).
- Specify the difference to detect (alternative hypothesis).
- Specify confidence level as $1-\alpha$.
- Select a two-sided or one-sided test.
- Indicate whether sigma is known (will use z instead of t).

Sample-Size Determination Options

Control

Absolute Error
+ 5.0

Relative Error
+ 10.0 %

Power
90.0 %

Difference to Detect:
6.0

Sample Size
30

Confidence Level: 95.0 %

Alternative Hypothesis

Not Equal
 Less Than
 Greater Than

Sigma

To Be Estimated
 Known

OK
Cancel
Help

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Solution:

Sample-Size Determination

Sample-Size Determination

Parameter to be estimated: difference between two normal means
Desired power: 90.0% for difference = 0.0 versus difference = 6.0
Type of alternative: greater than
Alpha risk: 5.0%
Sigma: 5.0 (to be estimated)

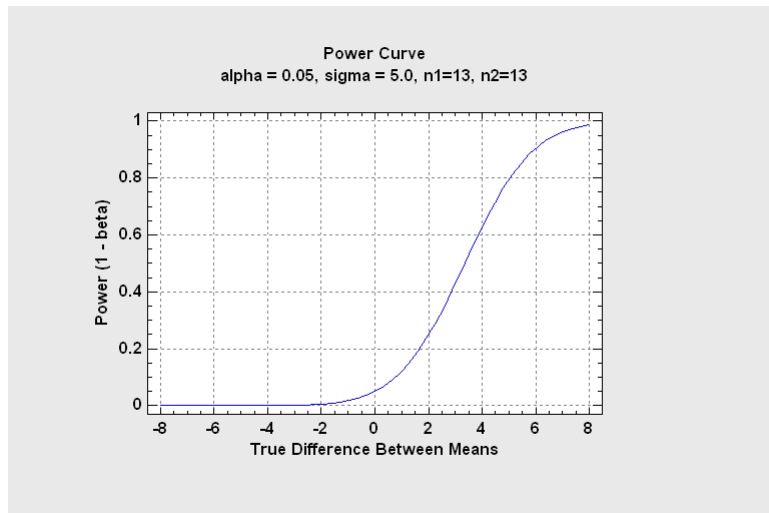
The required sample size is 13 observations from sample 1 and 13 observations from sample 2.

The StatAdvisor

This procedure determines the sample size required when comparing the means of two normal distributions. Assuming that the standard deviations of the normal distributions equal 5.0, 13 observations are required in sample 1 and 13 in sample 2 to have a 90.0% chance of rejecting the hypothesis that $\mu_1 - \mu_2 = 0.0$ when the true $\mu_1 - \mu_2 = 6.0$ (using a one-sided test).

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Power Curve



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Example #3: Capability Indices

A capability index measures how “capable” a process is of meeting established specification limits.

A popular index is

$$C_{pk} = \min\left(\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\right)$$

Often, a process is deemed “capable” if $C_{pk} \geq 1.33$

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Tools – Sample Size Determination – Capability Indices

- Select the index to be estimated.
- Specify the expected value of the index.
- Indicate the desired relative error (distance from estimated C_{pk} to its lower confidence bound).

Sample Size Determination (Capability Indices)

Index:

Cp

Cpk

Cpm

OK

Cancel

Help

Estimated index: 1.33

Mean minus target: 1.0 sigma

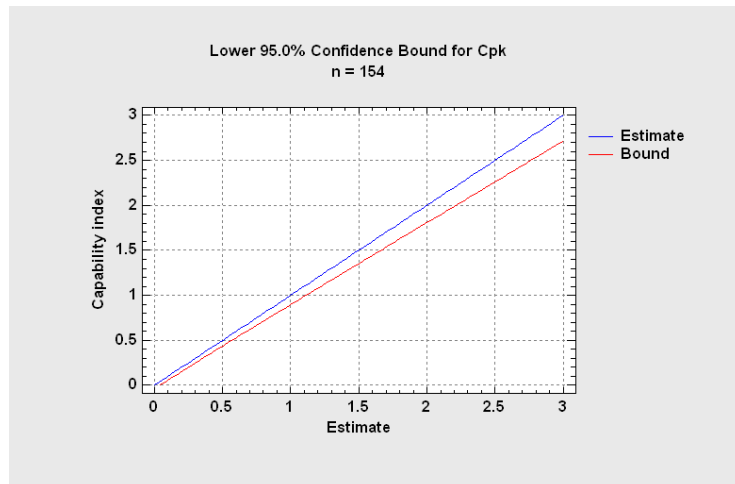
Relative error: 10.0 %

Confidence Level: 95.0 %

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Solution:



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Example #4: Control Charts

For control charts based on grouped data (such as taking k samples from repeated batches and constructing \bar{X} -bar and R charts), determines a suitable value for the group size k .

Null hypothesis: process mean = target value

Alt. hypothesis: process mean = target value + Δ

The value of k is based on the average run length (ARL), which is the average number of batches that will need to be inspected before getting an out-of-control signal if the mean shifts from the null hypothesis to the alternative.

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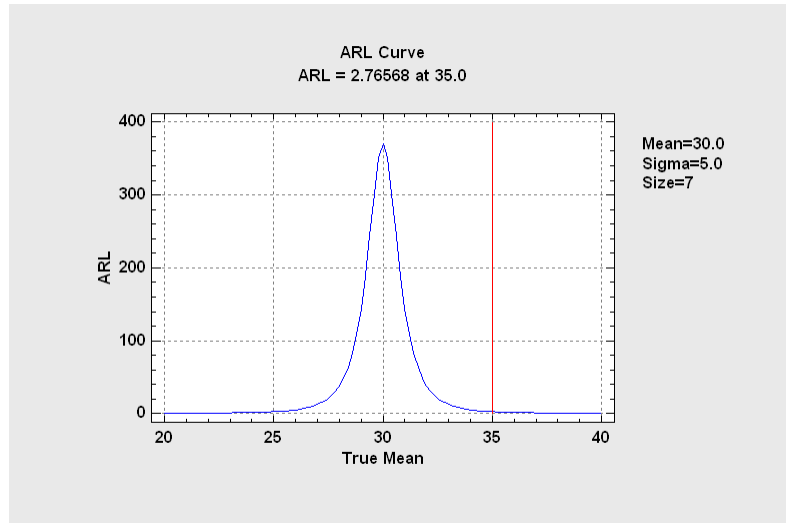
Tools – Sample Size Determination – Control Charts

- Select type of chart
- Enter in-control process mean and sigma
- Enter target out-of-control mean
- Enter desired ARL at the target mean

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ARL Curve for $k = 7$



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Example #5: Statistical Tolerance Limits

Statistical tolerance limits bound a specified proportion of a population at a specified level of confidence.

Suppose I wish to demonstrate that I am 95% confident that at least 99% of measurements obtained from items I am producing will fall within my specification limits.

I could ask for a sufficiently large sample size n such that the expected width of my tolerance interval is no more than 50% of the distance between the specification limits.

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Tools – Sample Size Determination – Statistical Tolerance Limits

- Select assumed distribution
- Specify parameters
- Select type of limits
- Enter confidence level and population proportion
- Enter specification limits
- Enter allowance (percentage of distance between spec limits)

Sample Size Determination - Statistical Tolerance Limits

Distribution

Normal
 Mean: 50.0 Sigma: 5.0

Lognormal
 Mean: 50.0 Sigma: 10.0 Threshold: 0.0

Weibull
 Shape: 5.0 Scale: 50.0 Threshold: 0.0

Nonparametric

Type of Limits

Two-sided
 Confidence Level: 95.0 % Population Proportion: 99.0 %

Lower limit only
 Lower Spec. Limit: 20.0 Upper Spec. Limit: 80.0 Allowance: 50.0 %

Upper limit only

OK Cancel Help

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Solution:

Sample Size Determination (Statistical Tolerance Intervals)

Sample Size Determination (Statistical Tolerance Intervals)

| Conf. Level | Pop. Percentage | Distribution | Mean | Std. dev. | Lower spec. |
|-------------|-----------------|--------------|------|-----------|-------------|
| 95.0% | 99.0% | Normal | 50.0 | 5.0 | 20.0 |

| Upper spec. | Allowance |
|-------------|-----------|
| 80.0 | 50.0% |

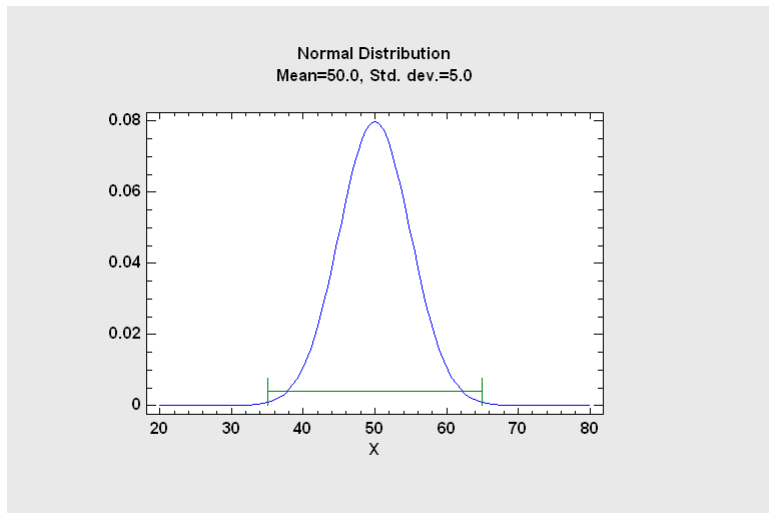
The required sample size is 76.

| Lower tolerance limit | Upper tolerance limit | K-factor |
|-----------------------|-----------------------|----------|
| 35.0055 | 64.9945 | 2.99889 |

The StatAdvisor
 Given a sample from a normal distribution with a mean equal to 50.0 and a standard deviation equal to 5.0, 76 items are required to create a 95.0% tolerance interval for 99.0% of the population which is entirely within the specification limits.

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Tolerance Plot



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Example #6 – Screening Designs

A screening design is a statistically designed experiment in which many factors are varied with the goal of identifying the factors that have the biggest impact on a response variable.

Most common screening designs contain:

- Experiments at 2 levels of each factor (low and high).
- Additional experiments at a central level of all of the factors.

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Tools – Sample Size Determination – Screening Designs

- Select the type of designs to be considered.
- Specify the maximum runs per block.
- Specify the minimum centerpoints per block.
- Enter the assumed sigma for the experimental error.

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Second Dialog Box

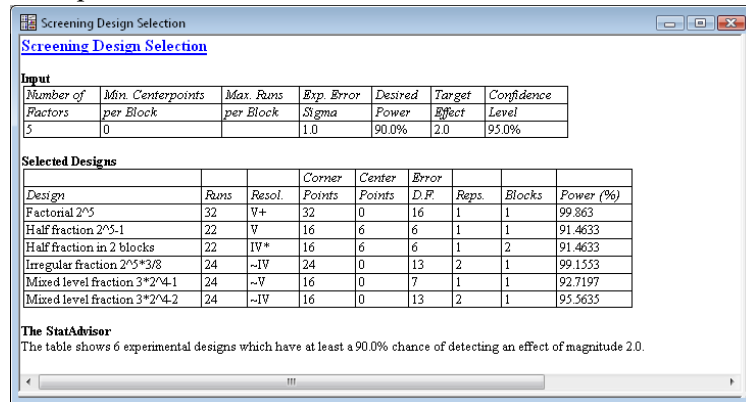
- Specify either the margin of error for estimated effects or the power of t-tests against an effect of a given magnitude.
- “Effect” is defined as the change in the response as a factor is moved from its low level to its high level.

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Output

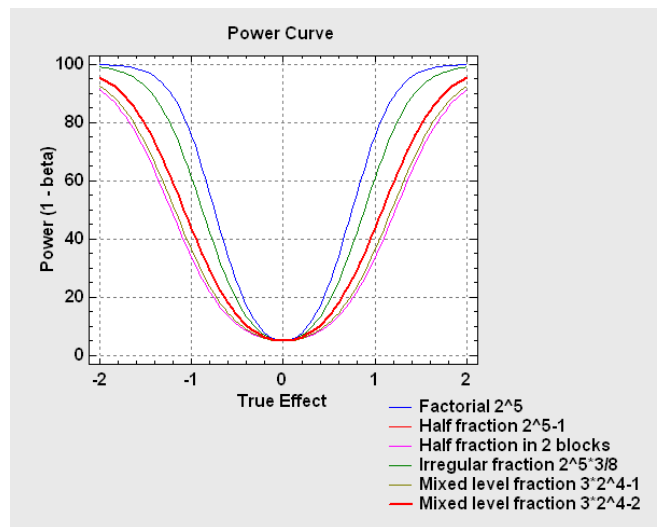
Output shows selected designs that meet the specified requirements.



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Power Curve



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Example #7 – Acceptance Sampling

Sample n items from a lot of size N . Based on the sample, either accept or reject the lot.

Case #1 (**attributes**): Inspect each item and classify it as good or bad. If the number of bad items is less than or equal to c , accept the lot.

Case #2 (**variables**): Measure each item. If the mean of the n items is far enough from the nearest specification limit, accept the lot.

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Formulation

1. Null hypothesis: The lot is **good**: % defective items \leq AQL (acceptable quality limit)
2. Alt. hypothesis: The lot is **bad**: % defective items \geq LTPD (lot tolerance percent defective)

- **Producer's risk** α : probability of rejecting a good lot
- **Consumer's risk** β : probability of accepting a bad lot

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SPC – Acceptance Sampling - Attributes

Acceptance Sampling - Attributes

Action:

- Create OC plan
- Create AOQL plan
- Create LTPD plan
- Analyze existing plan

Quality Levels:

Acceptable quality level (AQL): 1.0 %

Lot tolerance percent defective (LTPD): 3.0 %

Lot size: 10000

Desired Features:

Producer's risk (alpha): 5.0 %

Consumer's risk (beta): 5.0 %

Current Plan:

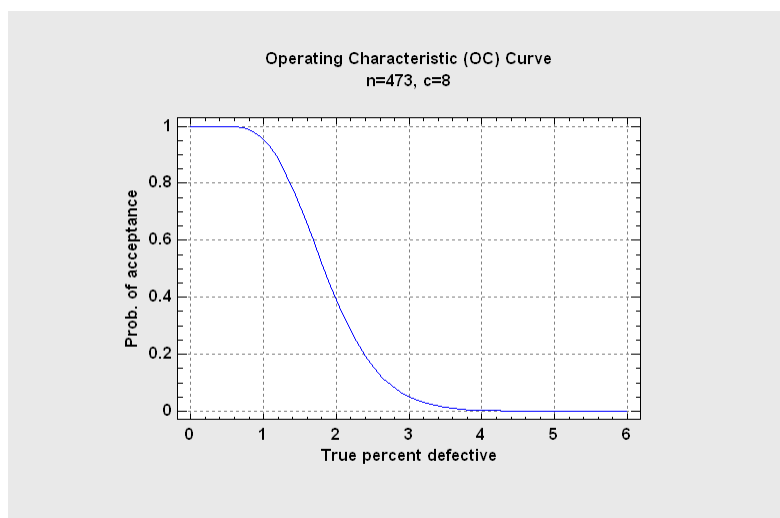
Sample size (n):

Acceptance number (c):

OK Cancel Help

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Solution – accept lot if after inspecting 473 items, no more than 8 items are defective.



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SPC – Acceptance Sampling - Variables

Acceptance Sampling - Variables

Action:

- Create OC plan
- Create AOQL plan
- Create LTPD plan
- Analyze existing plan

Quality Levels:

Acceptable quality level (AQL): 1.0 %

Lot tolerance percent defective (LTPD): 3.0 %

Lot size: 10000

Process Sigma:

- Assume known value
- Use sample estimate

Desired Features:

Producer's risk (alpha): 5.0 %

Consumer's risk (beta): 5.0 %

Current Plan:

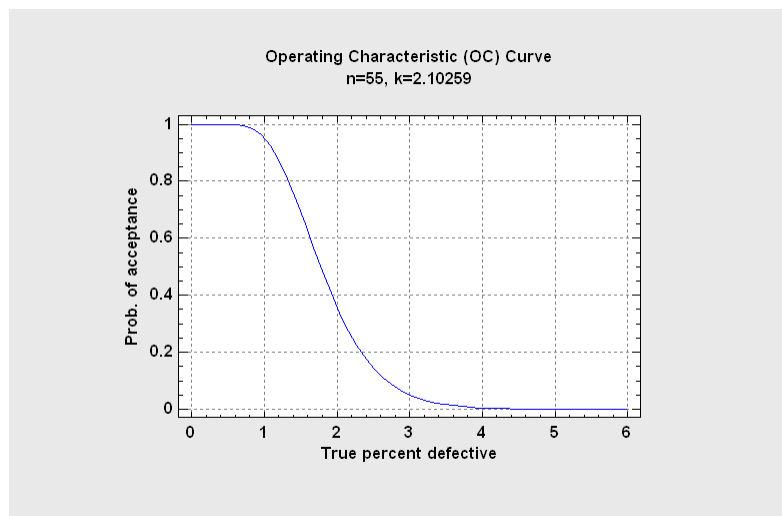
Sample size (n):

Critical distance (k):

OK Cancel Help

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Solution – accept lot if after measuring 55 items, the sample mean is no closer than 2.10259 standard deviations to the nearest spec limit.



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More Information

Go to www.statgraphics.com

Or send e-mail to info@statgraphics.com

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