Six Sigma Calculator

The *Tools* menu provides access to a *Six Sigma Calculator*. The calculator facilitates the conversion between different metrics used in *Six Sigma* quality improvement projects. It is controlled by the following dialog box:

19 Statgraph	ics (× ₹ 10:54
Z-Score:	3.5	Calculate
	10	
_ ◯ Defects (%)	0.1	=
- Vield (%):	99.9	4
O Cpk:	1.5	Ŧ.
🔿 Sigma level:	6	ī
Sigma shift:	1.5	utwo-sideo

To use the calculator, select one of the six radio buttons, enter a value in the corresponding edit field, and press the *Calculate* button. The corresponding values of the other quality performance metrics will be displayed to the right of the edit fields.

The important input fields on the calculator are:

- Sigma shift: a multiple Δ by which the process mean is assumed to shift over time above and below its long-term mean.
- **Two-sided**: whether to calculate two-sided probabilities. This box should be checked if the process has both upper and lower specification limits that are located at equal distances from the process mean. Note: for asymmetrically placed upper and lower limits, treat the process as having two one-sided limits and combine the results.

As an example, suppose a process creates widgets with a mean length of $\mu = 10.5$ cm and a standard deviation of $\sigma = 0.02$ cm. Suppose also that the specification limits are 10.5 ± 0.1 cm. Assume that the process shifts from its long-term mean by as much as $\Delta = 1.5$ times σ over a long period of time, ranging between 10.47 and 10.53.

Assuming the worst possible scenario, the distance from the process mean to the specification limits may be as small as 3.5σ . This corresponds to a Z-score of

$$Z = \frac{USL - \mu}{\sigma} - \Delta = \frac{10.6 - 10.5}{0.02} - 1.5 = 3.5$$
(1)

Entering 3.5 into the Z-Score input field, selecting *two-sided*, and pressing *Calculate* generates the following results:

Statgraph	ics (Calculate
Z-Score:	3.5	3.5
O DPM:	10	232.673
O Defects (%)	0,1	0.0232673
◯ Yield (%):	99.9	99.9767
🔿 Cpk:	1.5	1.16667
🔘 Sigma level:	6	5
Sigma shift:	1.5	🖌 two-sided

The results displayed are:

- **Z-Score**: the number of standard deviations *Z* between the process mean and the specification limit or limits.
- **DPM (defects per million):** the average number of widgets out of every million that are expected to be outside of the specification limits. If p(d) is the proportion of defective widgets, then

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$$DPM = 1,000,000 \ p(d) \tag{2}$$

• **Defects (%):** the average percent of the widgets that are expected to be outside of the specification limits, calculated from

$$Defects (\%) = 100 p(d) \%$$
 (3)

• Yield (%): the average percent of the widgets that are expected to be within the specification limits, equal to

$$Yield\,(\%) = 100\,(1 - p(d))\%\tag{4}$$

• Cpk: the one-sided capability index, calculated from

$$Cpk = \frac{USL - \mu}{3\sigma} = \frac{Z}{3}$$
⁽⁵⁾

• Sigma level: the process sigma level, calculated from

$$Sigma \ level = Z + \Delta \tag{6}$$

In the example, the process would be estimated to generate approximately 233 defects out of every million produced, which equates to a *Sigma Level* of 5.0.

Calculations

Percent Defective

One-sided

$$p(d) = 1 - \Phi(Z) \tag{7}$$

Two-sided

$$p(d) = 1 - \Phi(Z) + \Phi(-Z - 2\Delta) \tag{8}$$

where $\Phi(Z)$ is the standard normal cumulative distribution function.

Z-Score

One-Sided

$$Z = \Phi^{-1} \left(1 - p(d) \right) \tag{9}$$

Two-sided

Solve (8) numerically for *Z* given p(d).