Toolwear Charts

Summary
The Toolwear Charts procedure creates control charts for a single numeric variable where the measurements are expected to change in a linear fashion due to the wear of a tool. Unlike most other control charts, the control limits are not horizontal, but instead follow the expected trend due to the tool wear. Normally, specification limits are added to the chart to help determine when it is time to change the tool.

There are two Toolwear menu selections, one designed for data collected in subgroups and the other for individuals. The procedure creates both a toolwear chart and an R chart, S chart, or MR(2) chart. The charts may be constructed in either Initial Study (Phase 1) mode, where the current data determine the slope and spacing of the control limits, or in Control to Standard (Phase 2) mode, where the limits come from either a known standard or from prior data.

Sample StatFolio: toolwear chart.sgp

Sample Data:

The file toolwear.sgd contains 7 samples of 5 measurements each, taken once every hour from the use of a cutting tool. The data are from Duncan (1986). The data in that file are shown below:

<table>
<thead>
<tr>
<th>Time</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00</td>
<td>0.255</td>
<td>0.256</td>
<td>0.256</td>
<td>0.257</td>
<td>0.258</td>
</tr>
<tr>
<td>10:00</td>
<td>0.255</td>
<td>0.256</td>
<td>0.257</td>
<td>0.258</td>
<td>0.258</td>
</tr>
<tr>
<td>11:00</td>
<td>0.257</td>
<td>0.257</td>
<td>0.258</td>
<td>0.258</td>
<td>0.259</td>
</tr>
<tr>
<td>12:00</td>
<td>0.258</td>
<td>0.258</td>
<td>0.259</td>
<td>0.260</td>
<td>0.261</td>
</tr>
<tr>
<td>1:00</td>
<td>0.259</td>
<td>0.260</td>
<td>0.260</td>
<td>0.260</td>
<td>0.262</td>
</tr>
<tr>
<td>2:00</td>
<td>0.260</td>
<td>0.262</td>
<td>0.262</td>
<td>0.263</td>
<td>0.264</td>
</tr>
<tr>
<td>3:00</td>
<td>0.263</td>
<td>0.263</td>
<td>0.264</td>
<td>0.264</td>
<td>0.265</td>
</tr>
</tbody>
</table>

The specification limits for the dimension of the part are 0.255 to 0.265 inches. Note the increase in the measurements from hour to hour as the tool wears.
**Data Input**

There are two menu selections that create toolwear charts, one for individuals data and one for grouped data. In the case of grouped data, the original observations may be entered, or subgroup statistics may be entered instead.

**Case #1: Individuals**
The data to be analyzed consist of a single numeric column containing \( n \) observations. The data are assumed to have been taken one at a time.

- **Observations**: numeric column containing the data to be analyzed.

- **Date/Time/Labels**: optional labels for each observation.

- **LSL, Nominal, USL**: optional lower specification limit, nominal (target) value, and upper specification limit.

- **Select**: subset selection.

**Case #2: Grouped Data – Original Observations**
The data to be analyzed consist of one or more numeric columns. The data are assumed to have been taken in groups, in sequential order by rows.
• **Observations:** one or more numeric columns. If more than one column is entered, each row of the file is assumed to represent a subgroup with subgroup size \( m \) equal to the number of columns entered. If only one column is entered, then the *Date/Time/Labels or Size* field is used to form the groups.

• **Date/Time/Labels or Size:** If each set of \( m \) rows represents a group, enter the single value \( m \). For example, entering a 5 implies that the data in rows 1-5 form the first group, rows 6-10 form the second group, and so on. If the subgroup sizes are not equal, enter the name of an additional numeric or non-numeric column containing group identifiers. The program will scan this column and place sequential rows with identical codes into the same group.

• **LSL, Nominal, USL:** optional lower specification limit, nominal (target) value, and upper specification limit.

• **Select:** subset selection.
Case #3: Grouped Data – Subgroup Statistics
In this case, the statistics for each subgroup have been computed elsewhere and entered into the datasheet, as in the table below:

<table>
<thead>
<tr>
<th>Sample</th>
<th>Means</th>
<th>Ranges</th>
<th>Sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2564</td>
<td>0.003</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>0.2568</td>
<td>0.003</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>0.2578</td>
<td>0.002</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>0.2592</td>
<td>0.003</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>0.2602</td>
<td>0.003</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>0.2622</td>
<td>0.004</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>0.2638</td>
<td>0.002</td>
<td>5</td>
</tr>
</tbody>
</table>
• **Subgroup Statistics**: the names of the column containing the subgroup means, subgroup ranges, and subgroup sizes.

• **Date/Time/Labels**: optional labels for each subgroup.

• **LSL, Nominal, USL**: optional lower specification limit, nominal (target) value, and upper specification limit.

• **Select**: subset selection.

**Toolwear Chart**
The Toolwear chart plots the observations or subgroup means together with a trend line and control limits.

![Toolwear Chart for X1-X5](image)

The centerline of the chart is defined by

\[ \mu_j = A + B \, j \]  

where \( B \) is the slope of the line and \( A \) is the Y-intercept. The control limits are drawn around the centerline at

\[ \hat{\mu} \pm k \frac{\hat{\sigma}}{\sqrt{n}} \]  

where \( k \) is the multiple (usually equal to 3) specified on the Control Charts tab of the Preferences dialog box, accessible from the Edit menu. This allows for the natural variability around the regression line, but does not account for any error in estimating the line. In Initial Studies mode, the intercept and slope of the line are obtained from the least squares regression line of the plotted points against \( j \). The analyst may choose to include the model estimation error using the Analysis Options dialog box, in which case the limits are placed at
\[
\hat{A} + \hat{B}j \pm k\hat{\sigma} \sqrt{\frac{1}{n} \left( 1 + \frac{(j - \bar{j})^2}{\sum_{i=1}^{m} n_i (i - \bar{j})^2} \right)}
\]

where \( \bar{j} \) is the average of the period numbers used to fit the line. The standard deviation around the line may be estimated using either the mean squared error (MSE) of the fitted regression model or through the average range or standard deviation, as described below.

In the current example, all points remain well inside the control limits, indicating no significant departure from the estimated trend line at any point in time.

**Analysis Summary**

This pane displays the fitted trend model and summarizes the control charts.

**Toolwear Chart - X1-X5**
- Number of subgroups = 7
- Subgroup size = 5.0
- 0 subgroups excluded
  - Upper spec. limit: 0.265
  - Lower spec. limit: 0.255
- Distribution: Normal
- Transformation: none

<table>
<thead>
<tr>
<th>Toolwear Chart</th>
<th>#1-7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>#1-7</td>
</tr>
<tr>
<td>UCL: +3.0 sigma</td>
<td>0.264927</td>
</tr>
<tr>
<td>Centerline</td>
<td>0.263279</td>
</tr>
<tr>
<td>LCL: -3.0 sigma</td>
<td>0.261631</td>
</tr>
</tbody>
</table>

0 beyond limits

<table>
<thead>
<tr>
<th>Range Chart</th>
<th>#1-7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>#1-7</td>
</tr>
<tr>
<td>UCL: +3.0 sigma</td>
<td>0.0060414</td>
</tr>
<tr>
<td>Centerline</td>
<td>0.00285714</td>
</tr>
<tr>
<td>LCL: -3.0 sigma</td>
<td>0.0</td>
</tr>
</tbody>
</table>

0 beyond limits

<table>
<thead>
<tr>
<th>Estimates</th>
<th>#1-7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>#1-7</td>
</tr>
<tr>
<td>Process mean</td>
<td>0.254429+0.00126429*t</td>
</tr>
<tr>
<td>Process sigma</td>
<td>0.00122835</td>
</tr>
<tr>
<td>Average range</td>
<td>0.00285714</td>
</tr>
<tr>
<td>Sigma estimated from average range</td>
<td></td>
</tr>
</tbody>
</table>

Included in the table are:

- **Subgroup Information**: the number of observations or subgroups and the average subgroup size.

- **Specification Limits**: the specification limits for the variable of interest.
• **Distribution:** the assumed distribution for the data. By default, the data are assumed to follow a normal distribution. However, one of 26 other distributions may be selected using *Analysis Options*.

• **Transformation:** any transformation that has been applied to the data. Using *Analysis Options*, you may elect to transform the data using either a common transformation such as a square root or optimize the transformation using the Box-Cox method.

• **Toolwear Chart:** a summary of the centerline and control limits for the toolwear chart, together with a count of any points beyond the control limits.

• **MR(2)/R/S Chart:** a summary of the centerline and control limits for the dispersion chart.

• **Estimates:** estimates of the fitted trend line and the process standard deviation $\sigma$. The trend line assumes that the first point was plotted at $j = 1$. The method for estimating the process sigma depends on the settings on the *Analysis Options* dialog box, described below.

• **Mean residual MR(2), Average Range, or Average S:** the average of the values plotted on the dispersion chart.

### Analysis Options

![Toolwear Chart Options](image)

- **Type of Study:** determines how the control limits are set. For an *Initial Study* (Phase 1) chart, the limits are estimated from the current data. For a *Control to Standard* (Phase 2) chart, the control limits are determined from the information in the *Control to Standard* section of the dialog box.
- **Estimate sigma from**: specifies whether the process sigma should be estimated from the dispersion chart, or whether it should be estimated from the residual mean squared error of the fitted regression model.

- **Control to Standard**: To perform a Phase 2 analysis, select *Control to Standard* for the *Type of Study* and then enter the established standard process sigma (or other parameters if not assuming a normal distribution) and the intercept and slope of the trend line.

- **Include Model Estimation Error**: for initial studies mode, whether the estimation error of the regression line should be added to the estimate of the process sigma. In small samples, this could increase the width of the control limits substantially.

- **X-Bar Control Limits**: specify the multiple $k$ to use in determining the upper and lower control limits on the toolwear chart. To suppress a limit completely, enter 0.

- **R or S Control Limits**: specify the multiple $k$ to use in determining the upper and lower control limits on the MR(2), R, or S chart. To suppress a limit completely, enter 0.

- **Exclude button**: Use this button to exclude specific subgroups from the calculations.

- **Transform Button**: Use this button to specify a transformation or non-normal distribution.

For a discussion of the *Exclude* and *Transform* features, see the documentation for *Individuals Control Charts*.

**MR(2)/R/S Chart**

A second chart is also included to monitor the process variability.

![Range Chart for X1-X5](image)

For individuals data, the chart displayed is an MR(2) chart, described in the *Individuals Control Charts* documentation. For grouped data, either an R chart or an S chart is plotted, depending on the setting on the *Control Charts* tab of the *Preferences* dialog box.
These charts are described in the *X-Bar and R Charts* and the *X-Bar and S Charts* documents.

**Pane Options**

- **Outer Warning Limits**: check this box to add warning limits at the specified multiple of sigma, usually at 2 sigma.

- **Inner Warning Limits**: check this box to add warning limits at the specified multiple of sigma, usually at 1 sigma.

- **Moving Average**: check this box to add a moving average smoother to the chart. In addition to the subgroup means, the average of the most recent \( q \) points will also be displayed, where \( q \) is the order of the moving average. The default value \( q = 9 \) since the 1-sigma inner warning limits for the original subgroup means are equivalent to the 3-sigma control limits for that order moving average.

- **Exponentially Weighted Moving Average**: check this box to add an EWMA smoother to the chart. In addition to the subgroup means, an exponentially weighted moving average of the subgroup means will also be displayed, where \( \lambda \) is the smoothing parameter of the EWMA. The default value \( \lambda = 0.2 \) since the 1-sigma inner warning limits for the original subgroup means are equivalent to the 3-sigma control limits for that EWMA.

- **Decimal Places for Limits**: the number of decimal places used to display the control limits.
• **Mark Runs Rules Violations**: flags with a special point symbol any unusual sequences or runs. The runs rules applied by default are specified on the *Runs Tests* tab of the *Preferences* dialog box.

• **Color Zones**: check this box to display green, yellow and red zones.

• **Display Specification Limits**: whether to add horizontal lines to the chart displaying the location of the specification limits (if any).

### Toolwear Chart Report

This pane tabulates the values plotted on the control charts:

<table>
<thead>
<tr>
<th>Subgroup</th>
<th>Size</th>
<th>X-bar</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00</td>
<td>5</td>
<td>0.2564</td>
<td>0.003</td>
</tr>
<tr>
<td>10:00</td>
<td>5</td>
<td>0.2568</td>
<td>0.003</td>
</tr>
<tr>
<td>11:00</td>
<td>5</td>
<td>0.2578</td>
<td>0.002</td>
</tr>
<tr>
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</tr>
<tr>
<td>1:00</td>
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<td>0.2602</td>
<td>0.003</td>
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<td>0.2622</td>
<td>0.004</td>
</tr>
<tr>
<td>3:00</td>
<td>5</td>
<td>0.2638</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Out-of-control points are indicated by an asterisk. Points excluded from the calculations are indicated by an X.

### Pane Options

- **Display**: specify the observations or subgroups to display in the report.

### Runs Tests

The *Runs Tests* pane displays the results of standard tests designed to look for unusual sequences of points.

- **Rules**: (A) runs above or below centerline of length 8 or greater.
For a detailed description of these tests, see the *X-Bar and R Charts* documentation.

**Runs Options**

Select the runs tests to be applied and the parameters that define those tests. For example, some practitioners prefer to test for runs of length 7 rather than 8.

**Tolerance Chart**

The Tolerance Chart plots the data together with the specification limits.
- **Nominal**: enter a nominal or target value, if any.

- **Decimal Places for Limits**: the number of decimal places to use for the limits.

- **Plot Points**: select to display each observation. Otherwise, only the ranges will be plotted.

- **Plot Nominal**: select to draw a line at the nominal value.

**Save Results**
The following results can be saved to the datasheet, depending on whether the data are individuals or grouped:

1. *Observations or means* – the original observations or subgroup means.
2. *Ranges, sigmas, or moving ranges* – the values plotted on the dispersion chart.
3. *Sizes* – the subgroup sizes.
4. *Labels* – the subgroup labels.
5. *Process Mean* – the estimated process mean.
7. *Residuals* – the residuals from the trend model.