

U' Chart

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Summary

The **U' Chart** procedure creates a control chart for data that describe the number of nonconformities per unit recorded as the result of inspecting m samples. Each sample could represent a group of items, a period of time, an area of fabric, or some other sampling unit. Unlike the data plotted using a p chart or np chart, the inspection of an item within the sample does not result in simply an acceptable or unacceptable rating. Rather, it results in a count of the total number of defects or problems observed with that item.

Unlike the U chart, the U' chart does not assume that the samples all come from a Poisson distribution with the same mean. It instead allows the rate to vary over the sampling period, a phenomenon known as *overdispersion*. The methodology used in constructing the chart is due to Laney (2002).

The chart may be constructed in either *Initial Study* (Phase 1) mode, where the current data determine the control limits, or in *Control to Standard* (Phase 2) mode, where the limits come from either a known standard or from prior data.

The data for this analysis consist of m samples from a population detailing:

n_j = size of sample j . This could be expressed as the number of items inspected or the amount of product or time that the sample represents.

c_j = number of nonconformities observed in sample j

Many of the options for the U' chart are the same as those of the P' chart. This document highlights the differences between the 2 charts. For information on options such as runs, consult the *P' Chart* documentation.

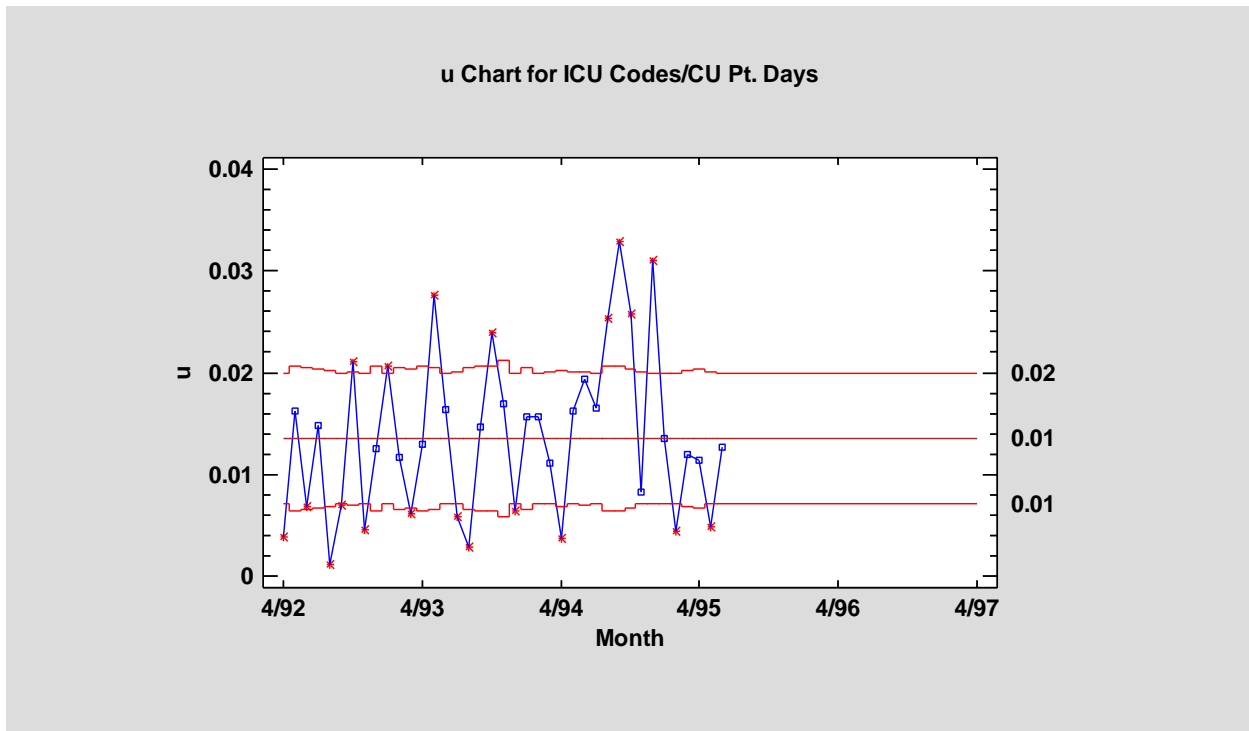
Sample StatFolio: *uprimechart.sgp*

Sample Data

The file *ICU codes.sgd* contains data on $m = 39$ months in an ICU, similar to data discussed by Carey (2003). It records the number of ICU codes that occurred each month, together with the number of patient days. The first several rows of that file are shown below:

Month	CU Pt. Days	ICU Codes
4/92	2900	11
5/92	2400	39
6/92	2500	17
7/92	2560	38
8/92	2700	3
9/92	2900	20

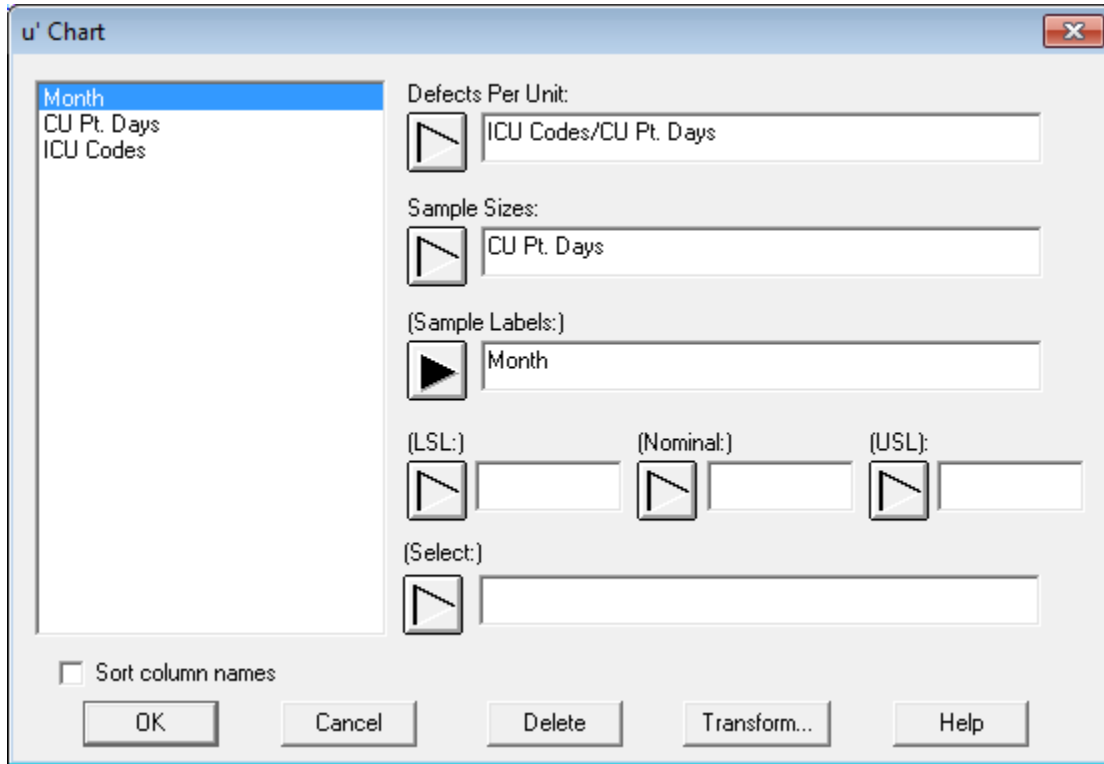
As is typical with very large samples, the variability between days greatly exceeds that expected from a Poisson distribution. The limits on a standard U chart would therefore be much too tight and of little use in separating special variation from common cause variation:



The U' Chart widens the limits to allow for changes in the Poisson mean.

Data Input

The data required by this procedure is a numeric column containing the number of defects or nonconformities in each of m samples, together with the sample sizes.



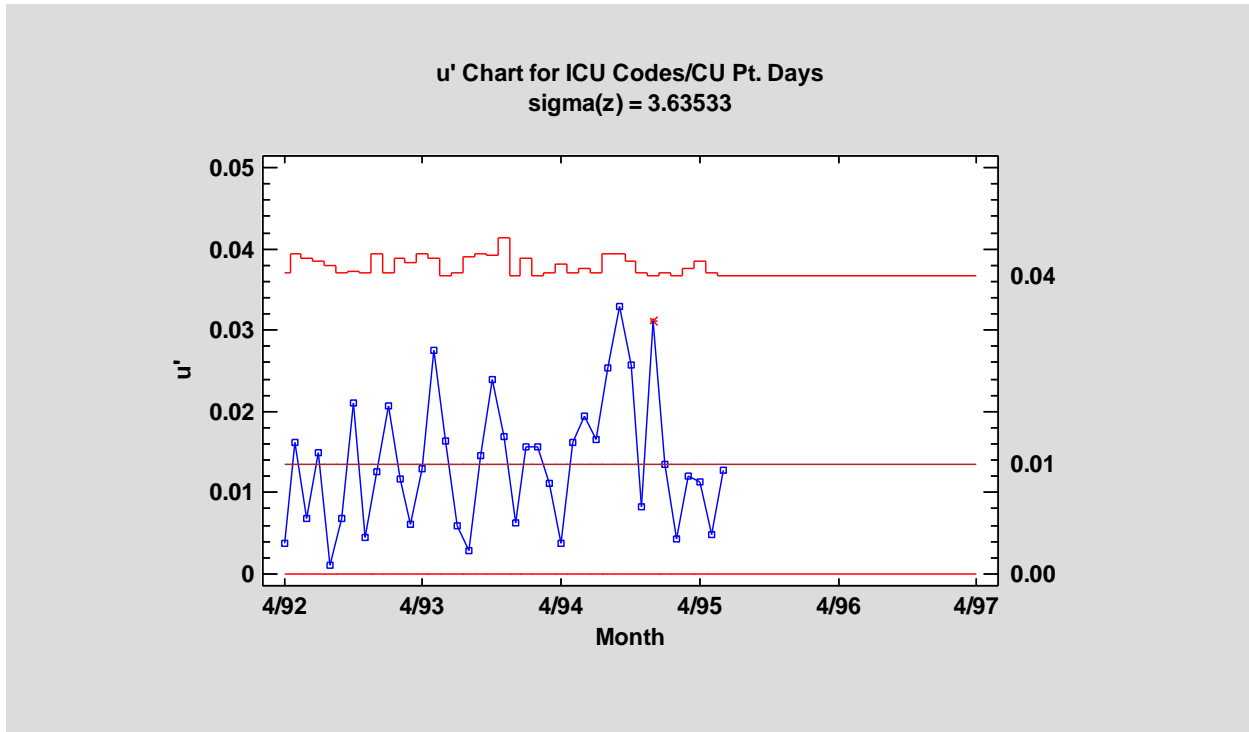
- **Counts:** a column containing the number of nonconformities in each sample.
- **Sample Sizes:** the sizes of the samples. If all sample sizes are equal, you can enter a single value such as “500”. Otherwise, enter a column with the sizes corresponding to each count.
- **Sample Labels:** optional labels for each sample.
- **LSL, Nominal, USL:** optional lower specification limit, nominal (target) value, and upper specification limit.
- **Select:** subset selection.

In entering the data, the above dialog box calculates the rate of ICU codes per patient day “on-the-fly” by dividing the counts by the sample sizes:

$$u_j = c_j / n_j \tag{1}$$

U' Chart

The *U' Chart* plots the rates of nonconformities per unit within each sample.



In Phase 1 (*Initial Studies*) mode, the centerline and control limits are determined from the data. The centerline is located at:

$$\bar{u} = \frac{\sum_{j=1}^m n_j u_j}{\sum_{j=1}^m n_j} \quad (2)$$

The control limits are placed above and below the centerline at:

$$\bar{u} \pm 3\sqrt{\bar{u} / n} \hat{\sigma}_z \quad (3)$$

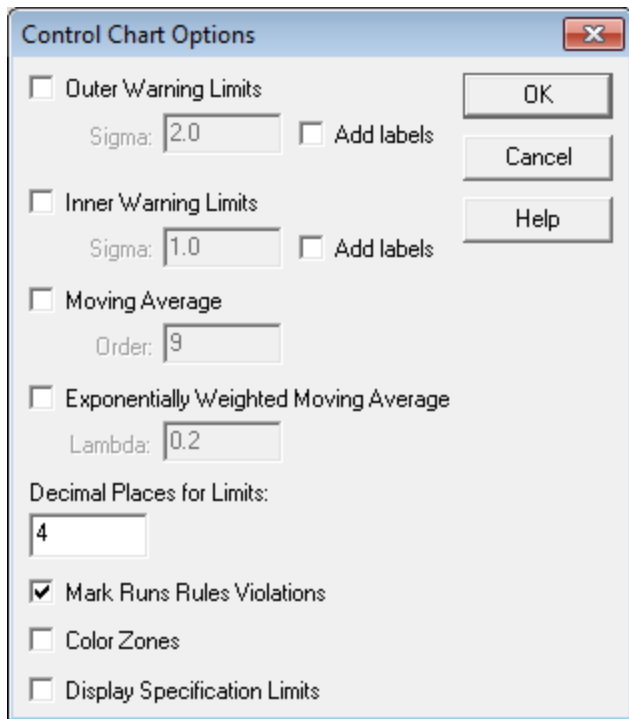
where k is the sigma multiple specified on the *Control Charts* tab of the *Preferences* dialog box ($k = 3$ except in rare cases), n is the sample size, and $\hat{\sigma}_z$ is an estimate of the relative amount of process variation not explained by the Poisson distribution alone.

If the sample sizes are not equal, then depending on *Analysis Options*, n is replaced by either:

- (1) \bar{n} , the average sample size. In this case, the control limits are the same for all samples.
- (2) n_j , the individual sample sizes. In this case, the control limits are step functions.

Any points beyond the control limits will be flagged using a special point symbol. Any point excluded from the analysis, usually by clicking on a point on the chart and pressing the *Exclude/Include* button, will be indicated by an X. If so indicated on the *Pane Options* dialog box, unusual sequences of points may also be flagged. In the current chart, all samples are within the control limits, although one point is flagged because it is the fourth point out of 5 more than one sigma above the centerline.

Pane Options



- **Outer Warning Limits:** check this box to add warning limits at the specified multiple of sigma, usually at 2 sigma.
- **Inner Warning Limits:** check this box to add warning limits at the specified multiple of sigma, usually at 1 sigma.
- **Moving Average:** check this box to add a moving average smoother to the chart. In addition to the sample data, the average of the most recent q points will also be displayed, where q is the *order* of the moving average. The default value $q = 9$ since the 1-sigma inner warning limits for the original data are equivalent to the 3-sigma control limits for that order moving average.
- **Exponentially Weighted Moving Average:** check this box to add an EWMA smoother to the chart. In addition to the sample data, an exponentially weighted moving average of the sample data will also be displayed, where λ is the smoothing parameter of the EWMA. The default value $\lambda = 0.2$ since the 1-sigma inner warning limits for the original rates are equivalent to the 3-sigma control limits for that EWMA.
- **Decimal Places for Limits:** the number of decimal places used to display the control limits.

- **Mark Runs Rules Violations:** flags with a special point symbol any unusual sequences or runs. The runs rules applied by default are specified on the *Runs Tests* tab of the *Preferences* dialog box.
- **Color Zones:** check this box to display green, yellow and red zones.
- **Display Specification Limits:** whether to add horizontal lines to the chart displaying the location of the specification limits (if any).

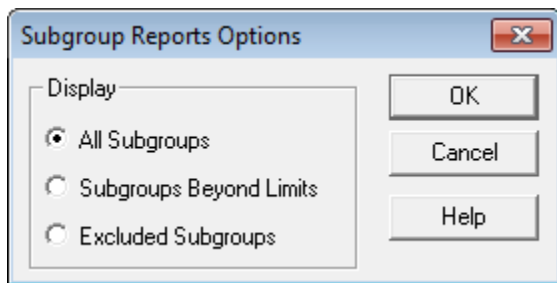
U' Chart Report

This pane tabulates the values plotted on the control chart:

u' Chart Report		
All Samples		
X = Excluded * = Beyond Limits		
Sample	Size	u
4/92	2900	0.0037931
5/92	2400	0.01625
6/92	2500	0.0068
7/92	2560	0.0148437
8/92	2700	0.00111111
9/92	2900	0.00689655
10/92	2850	0.0210526
11/92	2900	0.00448276
12/92	2400	0.0125
1/93	2900	0.0206897
2/93	2500	0.0116
3/93	2600	0.00615385
4/93	2400	0.0129167
5/93	2500	0.0276
6/93	3000	0.0163333
7/93	2890	0.00588235
8/93	2450	0.00285714
9/93	2400	0.0145833
10/93	2430	0.0238683
11/93	2070	0.0169082
12/93	3000	0.00633333
1/94	2500	0.0156

Out-of-control points are indicated by an asterisk. Points excluded from the calculations are indicated by an X.

Pane Options



- **Display:** specify the samples (subgroups) to display in the report.

Analysis Summary

The *Analysis Summary* summarizes the data and the control chart:

<u>u' Chart - ICU Codes/CU Pt. Days</u>	
Number of subgroups = 39	
Average subgroup size = 2697.95	
0 subgroups excluded	
u' Chart	
Period	#1-39
UCL: +3.0 sigma	0.0367008
Centerline	0.0135145
LCL: -3.0 sigma	0.0
0 beyond limits	
Estimates	
Period	#1-39
Process mean	0.0135145
Sigma	0.00813631
Sigma(z)	3.63533

Included in the table are:

- **Sample Information:** the number of samples m and the average sample size

$$\bar{n} = \frac{\sum_{j=1}^m n_j}{m} \tag{4}$$

If any samples have been excluded from the calculations, that number is also displayed.

- **u' Chart:** a summary of the centerline and control limits for the u' chart. Using *Analysis Options*, separate limits may be calculated for different periods (sets of samples).
- **Estimates:** estimates of the process mean rate

$$\hat{\mu} = \bar{u} \tag{5}$$

and the process sigma

$$\hat{\sigma} = \sqrt{\bar{u} / \bar{n}} \hat{\sigma}_z \tag{6}$$

The estimated relative amount of additional process variation $\hat{\sigma}_z$ is also displayed.

Analysis Options

- **Type of Study:** determines how the control limits are set. For an *Initial Study* (Phase 1) chart, the limits are estimated from the current data. For a *Control to Standard* (Phase 2) chart, the control limits are determined from the information in the *Control to Standard* section of the dialog box.
- **Normalize:** if selected, all statistics will be normalized by calculating Z-scores and the Z-scores plotted on the charts. The centerline on such a chart is always located at 0, and the control limits are always located at $\pm k$.
- **Avg. Subgroup Size:** if checked, the control limits will be horizontal lines based on the average sample size. If not checked, the individual sample sizes will be used, resulting in step function control limits if the sample sizes are not all equal.
- **Use Zone Format:** if checked, the statistics will be plotted using a zone chart rather than the usual format.
- **Recalculate at:** the control limits of the chart may be recalculated at up to 4 locations along the X axis by specifying the sample numbers at which new estimates are to be initiated. Separate estimates of the mean number of nonconforming items per unit will be obtained using the data in the different sections. In such cases, the control limits will be adjusted at the start of each new period.
- **U' Chart Control Limits:** specify the multiple k to use in determining the upper and lower control limits on the u' chart. To suppress a limit completely, enter 0.
- **Control to Standard:** to perform a Phase 2 analysis, select *Control to Standard* for the *Type of Study* and then enter either of the following:

Specify Parameters: specify the established standard mean number of nonconforming items per unit and the assumed value for σ_z .

Specify Control Limits: specify the location of the centerline and control limits exactly where you wish them to be placed.

- **Exclude:** use this button to exclude specific samples from the calculations.

Save Results

The following results can be saved to the datasheet:

1. *us* – the sample rates.
2. *Sizes* – the sample sizes.
3. *Labels* – the sample labels.
4. *Mean u* – the estimated mean rate of nonconformities per unit.
5. *Sigma* – the estimated standard deviation of the rates for a sample of average size.

Calculations

Estimate of process variability for sample j :
$$\hat{\sigma}_{u_j} = \sqrt{\frac{\bar{u}}{n_j}} \quad (7)$$

Normalized value for sample j :
$$z_j = \frac{u_j - \bar{u}}{\hat{\sigma}_{u_j}} \quad (8)$$

Average moving range:
$$\bar{R} = \frac{1}{m-1} \sum_{j=2}^m |z_j - z_{j-1}| \quad (9)$$

Estimated relative amount of additional process variation:
$$\hat{\sigma}_z = \frac{\bar{R}}{1.128} \quad (10)$$

References

Laney, David B. (2002) *Improved Control Charts for Attributes*. ASQ and Marcel Dekker Inc.

Carey, Raymond G. (2003) *Improving Healthcare with Control Charts: Basic and Advanced SPC Methods and Case Studies*. ASQ Quality Press.