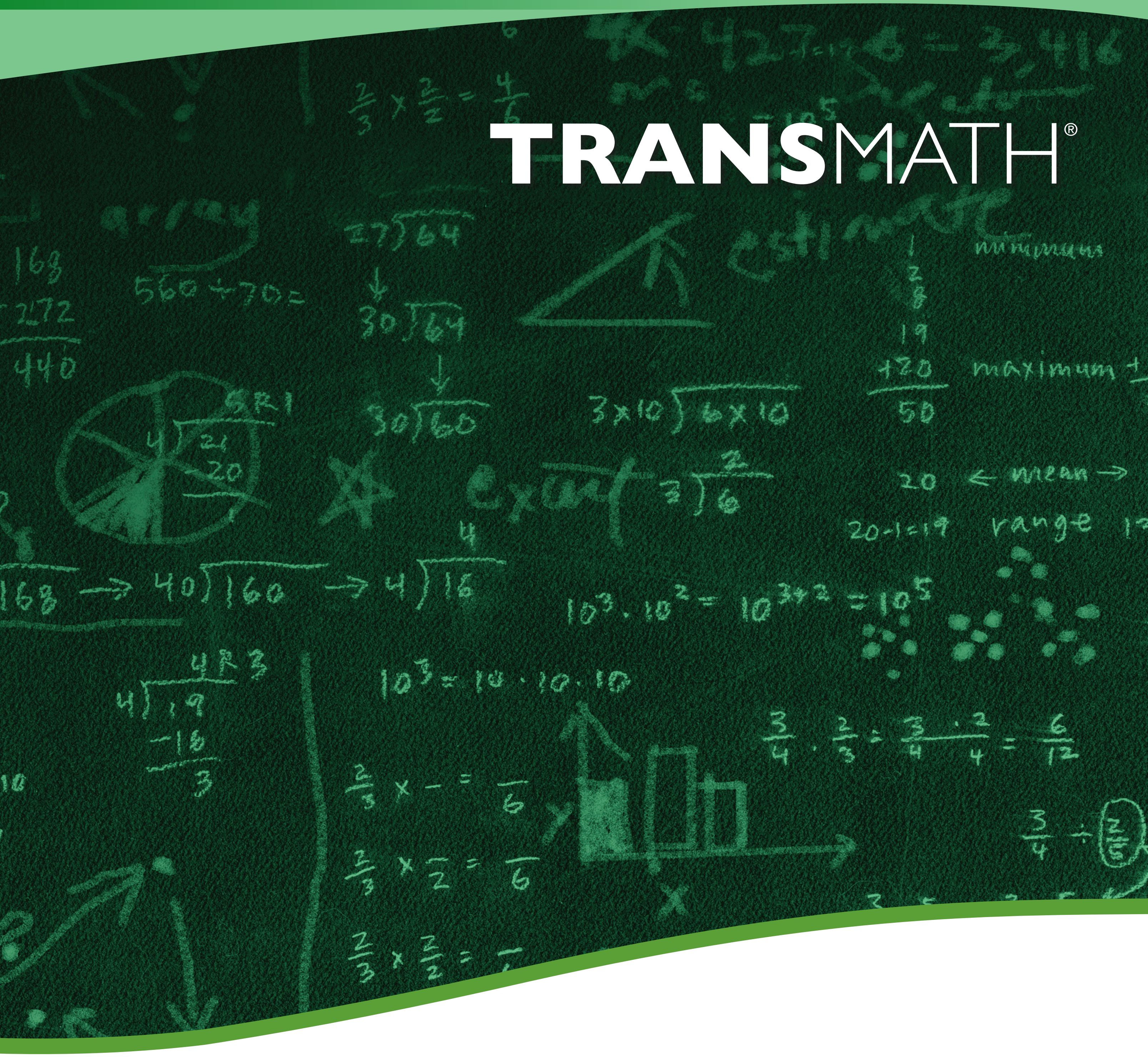
TransMath® Research Base and Effectiveness Data

Grades 5–10



TRANSMATH®

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TransMath Research Base

Recent research in education and economics indicates that mathematics knowledge has important implications for students' future success, whether personal, academic, or job-related. Studies show that secondary school study of mathematics leads to substantial increases in both schooling and income and that poor numeracy is more detrimental than low levels of literacy in obtaining jobs and earning promotions. Research also demonstrates that our information economy requires a more sophisticated, conceptual knowledge of mathematics than that which has been required in the past to achieve success in the labor market (members of the 2005 "Rising Above the Gathering Storm" Committee, 2010; National Center on Education and the Economy, 2006; Uhalde & Strohl, 2006). Increasingly, workers are asked to analyze and graphically represent data, as well as to solve a wide range of ill-defined and multiple-step problems. Technology has become central to these tasks, hence jobs require significant technical skills, with computers performing procedural calculations that have traditionally been solved by hand.

Due to changes in our economic reality, the future success of our young people lies largely in their ability to grasp not only procedural aspects of mathematics knowledge, but conceptual aspects as well; the values we hold in mathematics education are changing. The significance of values in education is articulated in the highly cited report from the National Research Council (2002), *Scientific Research in Education*, and by prominent mathematics researchers such as Hiebert (1999), who asked, "What mathematical knowledge is most important?" The topics and approaches represented in *TransMath* are the result of careful consideration of Hiebert's question, and they are consistent with the values and conceptions of mathematics reflected in the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000). The *TransMath* approach is supported by comparative international research, instructional research practices based on principles of cognitive psychology, and numerous syntheses of research in mathematics. The program is also founded on the research in mathematics education for both low-achieving and general education students and on both reform-based and traditional approaches to instruction.

Problems with Traditional Math Instructional Methods and Materials

Studies conducted in the U.S. for more than two decades, along with international comparative studies done in the 1990s, point to significant problems in curriculum and pedagogy in our mathematics classrooms. A large part of the problem has to do with what we value, particularly our emphasis on rote proficiency in calculations and basic skills at the expense of conceptual understanding and the ability to apply mathematical knowledge in varied contexts. Additionally, the choice of problems that are emphasized in our classrooms impacts how students view mathematics as well as their ability to translate ideas to more complex problems in different contexts. For example, a focus on one- and two-step word problems communicates to students that, if a problem cannot be solved in five minutes or less, it is either an illegitimate problem or one that simply cannot be solved (National Research Council, 2005).

A related pedagogical problem in our classrooms involves teachers' daily instructional routines. Teachers in U.S. classrooms tend to begin class with a brief demonstration of a new skill or concept that is followed by a considerable portion of time devoted to independent seatwork. Moreover, secondary teachers devote a significant block of time at the beginning of class to homework review.

In remedial and special education classrooms, the preoccupation with calculations and basic skills is exacerbated, and an increased amount of time is devoted to independent seatwork (Woodward, Takahira, & Baxter, 2002). Researchers have noted that academically low-achieving students spend most of their time learning a narrow hierarchy of basic skills. For example, it has been demonstrated that, on far too many occasions, students with learning disabilities spend a disproportionate amount of time involved in low-level practice built around excessive amounts of independent seatwork (Parmar & Cawley, 1991). Rather than just focusing on the basic skills, low-achieving students need a balanced approach that develops basic skills in conjunction with an understanding of concepts (National Mathematics Advisory Panel, 2008; Woodard, 2006).

Mathematics Reform

The 1989 National Council of Teachers of Mathematics (NCTM) Standards were instrumental in documenting and supporting an important shift in our thinking about mathematics education in the U.S. Put simply, the NCTM Standards embody a new set of values where equity among students, problem solving, communication, conceptual understanding, and use of technology are critical features of the mathematics classroom. Competence in calculations is important, but many mathematical ideas, such as knowing how to multiply fractions, need to be conceptually guided. Moreover, it is recognized that students need to work effectively with a range of tools and manipulatives in order to demonstrate their understanding.

There is evidence that reform-based curricula that were developed in response to the NCTM Standards are positively impacting mathematics achievement in this country (e.g., Senk & Thompson, 2003), but it is less clear that student success is equally distributed. In 2009, 61 percent of fourth grade students and 66 percent of eighth grade students still failed to achieve proficiency in mathematics (U.S. Department of Education, 2010). A series of research studies (Baxter, Woodward, & Olson, 2001; Baxter, Woodward, Voorhies, & Wong, 2002; King, 1993; Mulryan, 1995; Woodward & Baxter, 1997) indicate that students who tend to score below the 40th percentile on standardized tests are less likely to benefit from reform-based programs. The studies also indicate that these students tend not to participate in class, and are less able to stay on task for extended periods of time. Furthermore, the cognitive load of the curriculum is often too great for these students.

TransMath: Tailoring Reform to Meet the Needs of Academically Low-Achieving Students

John Woodward, Ph.D., and Mary Stroh, B.S., developed *TransMath Second Edition* (Woodward & Stroh, 2010) to incorporate the key tenets of mathematics reform while also responding to the needs of academically low-achieving students. A cornerstone of the program is that mathematics should be conceptually guided, and all computational practice within *TransMath* is organized around this principle. The design of *TransMath* is also based on one of the key findings from the Trends in International Mathematics and Science Studies (TIMSS), and from recommendations in the recent Institute for Education Sciences practice guide that reviews the evidence for best teaching practices in assisting struggling students in elementary and middle school mathematics (Schmidt, McKnight, & Raizen, 1997; Gersten, Beckmann, Clarke, et al., 2009). Specifically, it is based on the finding that students should explore fewer topics in greater depth. In addition, the topics in *TransMath* correlate with the topics that are recommended in the NCTM Standards (2000) as well as the Common Core Standards (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) and clearly exceed the narrow range of skills that secondary remedial and special education students tend to practice.

TransMath is a comprehensive mathematics intervention that provides key foundational skill building as well as rich problem-solving experiences. *TransMath* simultaneously teaches foundational computation skills while providing the grade-level problem-solving experiences necessary for high-stakes assessments. The curriculum is designed as a middle school core replacement math program specifically attuned to the strengths and weaknesses of struggling math learners.

In particular, *TransMath* serves the needs of students lacking the foundational skills necessary for successful entry into Algebra, or students scoring two or more years below standardized grade level on state tests. These struggling students are often those who are at risk for special education or who have been identified as having a learning disability.

TransMath is divided into three levels, with each level covering a range of topics.

- Level 1, *Developing Number Sense*, covers basic topics such as whole numbers, place value, and operations, as well as fractions, multistep problems, and measurement.
- Level 2, *Making Sense of Rational Numbers*, covers topics such as percentages and exponents, as well as probability.
- Level 3, *Understanding Algebraic Expressions*, covers topics such as inequalities, ratio and proportion, and graphs, as well as three-dimensional geometry.

Throughout each level of the curriculum, conceptual understanding develops and becomes fluent, given sufficient practice and application, to the point where, upon exit of *TransMath,* struggling math learners become proficient and are ready to successfully re-enter grade-level math curricula at the level of Algebra 1.

TransMath Curricular Design Principles

There is a series of curricular design and teaching principles, supported by previous research in the mathematics education literature, that distinguishes *TransMath* from other reform-based programs. The guiding curricular design principles convey the philosophy used throughout the program, and can be seen in each unit across every level. The teaching principles refer to best practices in the classroom that utilize approaches and activities proven to be effective with students who are struggling in mathematics.

Design Principle 1: Ensure That Students Have Relevant Background Knowledge.

Most middle school reform curricula are designed on the assumption that students are competent in whole numbers and operations when they enter the middle grades. This assumption is generally appropriate for average and above-average students, but it does not hold for many struggling math learners. Many of these students still struggle with multiplication and division when they enter middle school, others are weak on basic facts, and most only know how to compute answers by using paper and pencil. They are exceedingly weak in number sense.

TransMath ensures that students have relevant background knowledge by:

- Helping students understand operations on whole numbers conceptually. The first four units of Level 1, *Developing Number Sense*, focus on these concepts. For example, students learn alternative algorithms and compare them to traditional algorithms. This approach enables students to better understand place value, which is critical in mathematics (Hiebert, 1986; Ma, 1999).
- Encouraging the development of number sense related to whole numbers and rational numbers. Students use their understanding of numbers to determine if answers are reasonable. They also learn important strategies for approximating answers to problems and using estimates to solve problems and verify solutions.

Design Principle 2: Provide a Distributed and Balanced Approach to Computational Practice

An underused principle of learning that can enhance curricular materials is controlled distributed practice (Cepeda, et al., 2009; National Mathematics Advisory Panel, 2008). Too often textbooks move from one topic or skill to the next without allowing sufficient opportunities for students to become proficient. As sensible as this idea may seem, it can also be carried to extremes. Traditional task analyses have identified hierarchies of skills that need to be distributed over time (Coyne, Kame'enui, & Carnine, 2007). For students with math difficulties, this might include, for example, four-digit by three-digit multiplication, which would require students to first demonstrate proficiency in two-digit by two-digit problems. Simply stated, distributed practice should include a sensible distribution of skills practice.

In addition to being sensible about distributing the practice of skills over time, it is also important to balance conceptual instruction and computational drills. Traditional mathematics instruction, especially for struggling math learners, focuses excessively on computational drills. In contrast, reform based programs typically provide practice opportunities when a concept is introduced, but do not subsequently offer sufficient distributed computational practice. *TransMath* provides conceptual instruction paired with computational practice and application within each lesson, and revisits these topics in subsequent lessons to provide sufficient practice over time in alternate contexts.

TransMath provides distributed practice and balances computational drills with conceptually driven problem solving by:

- Limiting the computational drills early in the curriculum that tend to impede a student's progress toward higher mathematics. For example, students first learn concepts like factors, primes, and multiples before they are asked to work through complex long-division problems. Highly procedural tasks like long division, without the foundational conceptual understanding, can often lead to difficulties with more complex topics.
- Utilizing a dual-topic lesson format within each unit. For example, one unit in Level 2, *Making Sense of Rational Numbers*, teaches students about decimals as well as two-dimensional geometry. This structure, along with the larger commitment to teach fewer topics in greater depth, enables practice on concepts to be distributed over time. The concept of distributed practice is central to the academic success of struggling math learners.

Design Principle 3: Use Effective Time Management

For two decades, research has demonstrated that struggling math learners spend too much classroom time completing low-level tasks. Alternatively, struggling students in reform classrooms may be asked to work for an entire period on one problem. Both of these approaches rob students of the opportunity for engagement in challenging mathematics and classroom discussions. Research on struggling math learners indicates that these students are more successful when the instructional period is broken up and they are given the opportunity to engage in varied activities (Kame'enui, 2002).

TransMath utilizes effective time management by:

- Incorporating a four-part structure in almost every lesson: (1) warmup, (2) guided practice, (3) problem solving, and (4) homework. Students begin with a Skills Maintenance warm-up activity that provides distributed practice and the opportunity to rehearse relevant background knowledge. The guided practice portion of the lesson, which has its foundation in the effective teaching literature (Gersten, et al., 2009; Brophy & Good, 1986), allows teachers to present new concepts and probe students' understanding through discussion. The probing of student understanding, particularly as a method for detecting misconceptions, is an important technique found in successful mathematics instruction (Stigler & Hiebert, 1999). Finally, problem-solving activities during the last 20 to 30 minutes of class enable students to work individually, in pairs, or in small groups on challenging problems.

TransMath Research Base

TransMath Best-Practice Teaching Principles for Struggling Math Learners

Recent remedial and special education literature on mathematics indicates an important shift in thinking about classroom practice. The National Mathematics Advisory Panel subreport on Instructional Practices (2008), for example, outlines a broad compromise position on teacher-centered and student-centered approaches to teaching mathematics. This report explicitly recommends a middle ground between these extremes, and it signals a change that has widespread implications for practitioners.

While the student-centered instruction has received little attention in the literature—or, for that matter, in advice to remedial and special educators—this recommendation is an important counterbalance to those in special education who have long advocated highly teacher-centered approaches such as scripted forms of direct instruction. A balance between teacher- and student-centered instruction is much more commensurate with the kind of mathematics instruction that struggling students need (i.e., instruction that goes beyond drill and the development of basic skills).

A broad re-examination of pedagogy is just a first step. The details of teaching practices should also be considered. These details involve specific principles that guide instruction as well as the selection or modification of curricular materials. Four principles are outlined below that have research support from mathematics education, cognitive psychology, and the remedial and special education literature (Anderson, 2005; Baroody & Dwoker, 2003; Hiebert & Carpenter, 1992; Pressley & McCormick, 1995; van Garderen, 2006). Put another way, these are not principles that are unique to the needs of students with mathematics difficulties, but also hold the potential to support their particular academic needs.

Teaching Principle 1: Use of Visual Representations

This principle attends to the use of models and diagrams as well as physical manipulatives. It is widely accepted in cognitive psychology that information is stored visually as well as textually (Paivio, 1990). Models and visual images are particularly important as vehicles for promoting conceptual understanding in students with learning disabilities (van Garderen, 2006) and those struggling with learning math concepts in particular (Gersten, et al., 2009). For example, most students struggle with the rules for operations on fractions. For students with mathematics difficulties, different models in conjunction with conceptual explanations can help them remember why certain procedures have to be performed (e.g., changing denominators in addition and subtraction because of the need for fair shares). They also help show how different rational number concepts like fractions and decimals are related (Woodward, Baxter, & Robinson, 1999).

Visual representations—like words and symbols—can be a vehicle for occasional review of topics at a conceptual level. This point varies slightly from the limited body of special education research that recommends a more linear, "concrete to abstract" continuum of instruction (Butler, Miller, Crehan, Babbitt, & Pierce, 2003). Instead, learning theory that emphasizes schema development (Baroody, 2003; Jitendra, Star, Rodriguez, Lindell, & Someki, 2010; Jitendra, Star, Starosta, et al., 2009) suggests students need to revisit topics periodically at the conceptual level in order to achieve a deeper and more integrated understanding of mathematical concepts.

TransMath incorporates the use of visual representations by:

- Approaching instruction with visual models that help students see the concept being taught. For example, place-value coins can be used to help students visualize what is happening when numbers are regrouped, or the use of a number line can help students understand the addition and subtraction of negative and positive integers. *TransMath* also uses visual representations to link related concepts, such as fractions and decimals, so that students can grasp a deeper understanding of their similarities and differences. Identifying key similarities and differences is essential for effective conceptual instruction (Marzano, Pickering, & Pollock, 2001).

Teaching Principle 2: Controlling Cognitive Load

In the effort to provide naturalistic contexts for mathematics, some standards-based curricula have created lessons that require extensive reading and contain a significant number of mathematical as well as nonmathematical vocabulary. Furthermore, units or modules in a year's curriculum often move at a pace that is too fast for students with mathematics difficulties. These factors need to be addressed as many standards-based curricula are adapted for this population (National Mathematics Advisory Panel, 2008). To reach struggling math learners, teachers may need to summarize traditional textual materials, and the key mathematical vocabulary needs to be highlighted and reviewed systematically (Baxter, Woodward, & Olson, 2001; Clark, Nguyen, & Sweller, 2006). The challenge to modify standards-based curricula can be considerable for certain topics. However, the richness of these materials in terms of content and process strands clearly surpasses the kinds of materials commonly used in remedial and special education settings.

TransMath lessons control the cognitive load placed on students by:

- Separating lessons into two strands or activities (concepts and problem solving) to help reduce the amount of cognitive overload from staying on one concept for an entire class period. Changing content keeps students engaged and signals to students a shift in the nature of the activities. A typical *TransMath* lesson moves from conceptual and procedural practice to concept application through problem solving. The cognitive load of learning is modulated because students move from individualized learning in the Building Number Concepts strand to paired or small group work in the Problem Solving strand.

Teaching Principle 3: Varied Opportunities for Communication

Substantive mathematical discussions can be challenging for teachers as well as students (Chapin, O'Connor, & Anderson, 2003). It is difficult to interpret what some students are saying, whom to call on next, and how to generate a deeper level of understanding. Unintentionally, teachers may favor those students who contribute the most to a discussion, leaving students with math difficulties ignored and/or feeling intimidated by the abstract level of the talk. Nonetheless, these students, like their more verbal peers, need to verbalize their thought processes, ask questions, and explain or defend their thinking. Teachers need to structure and promote this discussion as well as provide corrective feedback (Gersten et al., 2009; Common Core Standards, 2010). As Stone (1998) has argued, scaffolding a student's thinking is difficult to do in a large group, particularly for students with math difficulties. At times, small, ad hoc groups can create the opportunity for students with math difficulties to talk. Structured peer interactions provide a forum for one-to-one communication.

TransMath provides varied opportunities for communication by:

- Helping teachers engage in a brief dialog with students during the practice phase of each lesson to probe student understanding. Teachers can then have a better idea of why or how a mistake may have occurred. Discussions during the Problem Solving strand allow for more extensive probing. Teachers are guided to encourage students to explain and even defend their methods for solving the problem. Throughout the curriculum, teachers are guided to encourage students to present different solutions to the same problem so that others in the class have the opportunity to see and hear several strategies. This cooperative learning method departs from relying on teacher-led activities and can be effective at promoting student achievement (Marzano, et al., 2001).

Teaching Principle 4: Multiple Forms of Assessment

There is considerable interest in special education regarding the role of ongoing assessment in mathematics classrooms for students with math difficulties. This focus on administering different types of assessment frequently can be seen as part of the Response to Intervention, or RtI, movement in special education and is a recommended best practice in the classroom with struggling math learners (Gersten et al., 2009). Advocacy for RtI has increased over the past few years, and it is seen as a way to increase research-based instructional support for struggling students before formal placement in special education.

As a part of any effective instruction, the teacher should be conducting formative assessments. These frequent, easy-to-administer assessments determine if students have retained the foundational skills and concepts taught in the lesson (or series of lessons). Typically, these assessments are administered frequently, where natural transitions in the curriculum occur, and are used by the teacher to adjust instruction. While this type of informal assessment based on the lesson content is important, we know that: (a) struggling math learners need to be able to demonstrate their knowledge and skills in ways beyond traditional types of assessment and (b) it is important to determine if the content and skills in the lessons taught generalize to alternate contexts. For these reasons, it is reasonable to propose that some kind of frequent assessment of basic skills should be complemented by other kinds of assessment such as performance and informal assessments.

Performance assessments are crucial because they measure the extent to which students can generalize their understanding to novel contexts. Performance assessments call on students to communicate their thinking using written or oral explanations as well as symbols and diagrams. It also affords students the opportunity to see and compare other strategies for solving problems. This type of assessment is particularly important for teachers who have tended to focus exclusively on basic skills. Performance assessment underscores how well a student has understood the big ideas or enduring understandings of a unit of instruction (Wiggins & McTighe, 2005). Helping ensure that students understand big ideas as well as supporting knowledge is crucial for struggling students (Coyne, et al., 2007). In addition, performance assessments can often prepare students for what they are likely to encounter on state standardized tests, as well as measure the degree to which students have been able to communicate their knowledge and demonstrate their skills in a generalized fashion (Common Core Standards, 2010).

A second form of assessment is informal assessment, or what is a major part of what Stiggins (2000) calls assessing personal communication. Teachers need to look closely at student work and listen carefully to their talk as a way to understand the extent to which concepts are understood. Lampert (2001) is equally instructive in her description of the array of information that expert teachers use to assess mathematical understanding. Given the importance of conceptual understanding in today's mathematics classroom, informal assessment is a critical tool for teachers. For students who struggle in mathematics, it is essential to determine and then address misconceptions as they arise in instruction.

TransMath incorporates multiple forms of assessment by:

- Assessing understanding using informal assessment on a daily basis. For example, using the "Check for Understanding" feature, teachers are guided to listen carefully to how students answer questions.
- Utilizing performance and formative assessments such as baseline and end-of-level progress indicators, daily quizzes, and end-of-unit assessments to determine how well students can transfer their knowledge to varied contexts.

These best-practice teaching principles have a broad foundation in the mathematics education and cognitive psychology research literature as well as more recent research in remedial and special education. The principles offer practitioners ways to support students who are struggling in their math classes in the same way that they simultaneously afford opportunities to students who are faring better.

Use of Technology for Struggling Students: Interactive White Boards

The types of technological support in the modern classroom are both varied and widespread. As teacher-led instruction becomes balanced with student-led activities and student-to-student interactions, the learning environment has shifted to one where the student is an active and engaged participant in the learning process. The use of technology has certainly shaped and enriched this learning environment. While teachers, like anyone in the general population, can often rush to embrace a new technology, it is often most effective to carefully plan how the sequence of the lesson will unfold and how the concepts will be taught with the intended technology. Planning of the lesson sequence and concept instruction is also essential when using a relatively new technology called Interactive White Boards (IWB; Miller, Glover, & Averis, 2005; Marzano, 2009).

An IWB acts as an interface between the user and the computer. The board looks like a traditional white board one would use with dry-erase markers. It differs from a traditional white board in that the teacher or student can stand in front of the board and, by using a pointer or finger, drive computer software. As one can imagine, when the board is placed in front of the class, a new range of whole class activities becomes possible.

As with any technology, the IWB has appropriate usage during mathematics instruction. When used well, IWBs can be effective in increasing student learning. IWB technology provides the opportunity for the teacher to more easily modulate the frequency and duration of teacher-student interactivity through lessons that are both teacher-led and student-led. During mathematics instruction, IWBs can be a powerful tool that allows for student engagement with visual models that are constrained by mathematical axioms (Clark-Jeavons, 2005). For example, with dynamic geometry software students can explore Euclidian geometric space through touching and rotating visual models on the IWB. The use of both interactive and noninteractive visual models in an IWB environment can be effective in raising student achievement (Marzano, 2009). Finally, IWBs can improve the clarity of the presentation, engage student interest, and help with student note taking. All of these aspects have been proven essential with struggling math learners (Office for Standards in Education, 2002). In sum, IWBs can enhance classroom instruction by providing a rich, exploratory environment for student learning and engagement.

TransMath incorporates IWBs by:

- Helping students visualize lesson concepts. The instruction in each *TransMath* lesson is divided into individual steps that the teacher can use to demonstrate key concepts using the IWB, developing understanding and building foundational knowledge for students. Students are given the opportunity to see and interact with visual models, promoting conceptual understanding.
- Providing opportunities for informal and formative assessment. The "Check for Understanding" features are incorporated into the IWB lessons, allowing teachers to immediately receive feedback on student progress. When used in conjunction with an integrated classroom response (clicker) system, results can be recorded and tracked for many of these assessments.

Conclusion

While our understanding of what constitutes effective teaching and learning is constantly evolving as a result of emerging research and technology, all students, including those who are struggling, must have the opportunity to learn mathematics through a coherent and challenging curriculum that supports the development of higher mathematics. Woodward and Stroh developed *TransMath* utilizing special education and general education research on both traditional and reform-based mathematics instruction in order to assist teachers in meeting the needs of students who are at risk of failure.

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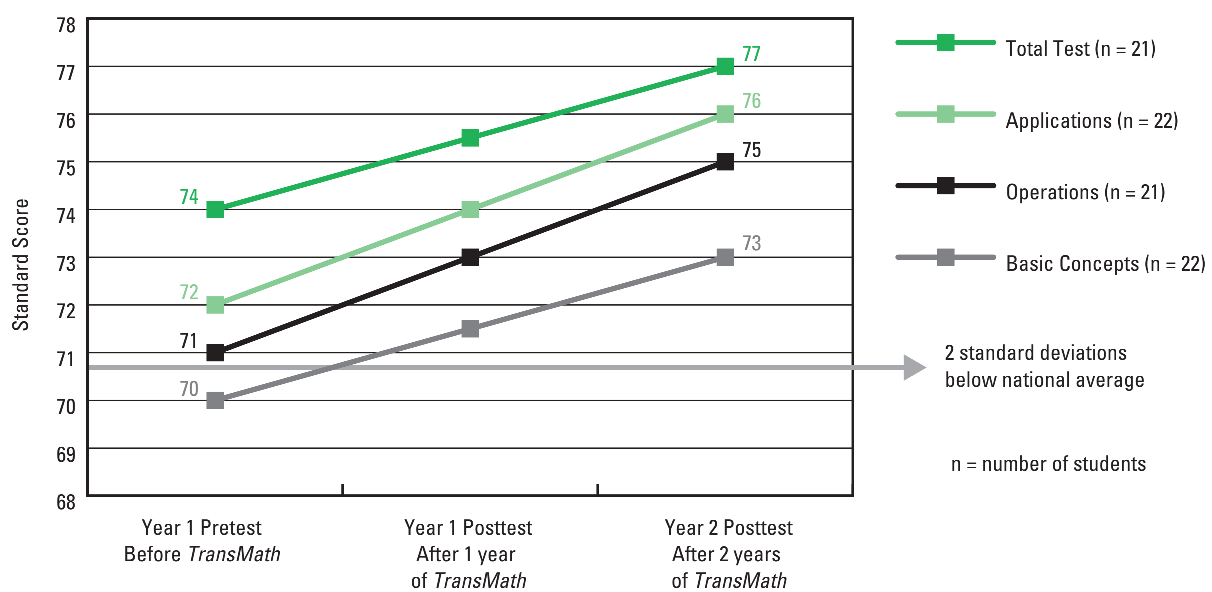
TransMath Effectiveness Data

Standard Score Gains for Students Receiving Special Education Services in a Georgia School District

During the 2008–2010 school years, *TransMath* was used as a core replacement math curriculum for low-performing students with special needs. *TransMath* was taught for 360 minutes a week over a two-year period. The *KeyMath-3*, an individually administered, norm-referenced test of mathematics concepts, was administered prior to instruction, and again at the end of the school year. Results are reported on a standard score scale with a mean score of 100, representing the national average, and a standard deviation of 15. If a low-performing student makes developmentally expected gain on the test, his or her standard score should remain the same from one test period to another.

If, by contrast, a low-performing student accelerates his or her growth—that is, the student makes more than the gain that is developmentally expected—his or her standard score will increase. When low-performing students improve their standard score, they are bringing their performance closer to the national average. *Figure 1* shows that, on average, *TransMath* students who had scored nearly two standard deviations below the national average at pretest were able to improve their standard score by nearly four points or nearly one-third of a standard deviation; that is, the *TransMath* group brought its performance closer to the national average.

Figure 1. *KeyMath-3* results for students with special needs with *TransMath* instruction over two school years: Fall 2008 to spring 2010.

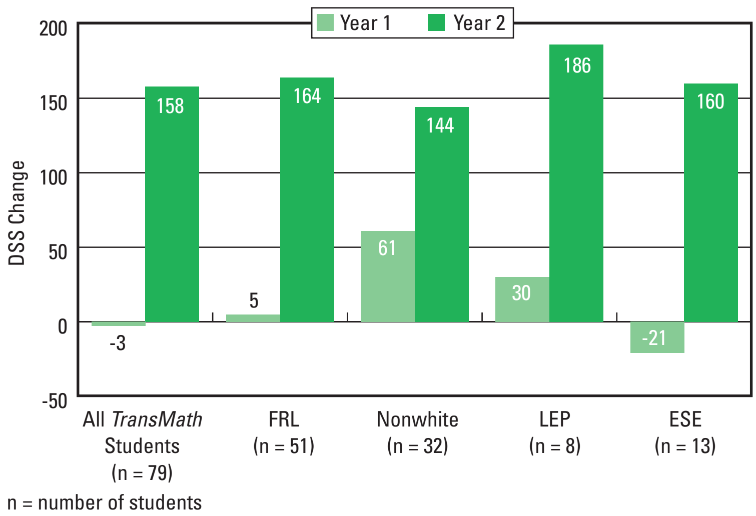


Two-Year Gains on the State Assessment for Lee County Public Schools, Florida

During the school years ending in 2007 and 2008, two middle schools in Lee County Public Schools began implementing *TransMath* to support students who needed systematic help in building a solid foundation of basic skills and mathematical reasoning. Participants had scored in Level 1 on the Florida Comprehensive Assessment Test® (FCAT), suggesting performance of at least two years below grade level. Across the two schools, *TransMath* instruction was provided either 45 or 90 minutes a day.

Figure 2 shows FCAT developmental scale score (DSS) gains in math among *TransMath* students. Intervention programs targeting students struggling in math often take more than one year of implementation to effect positive change, which makes the results in Year 2 of particular interest. In Year 1 of the *TransMath* implementation, the *TransMath* students showed no significant gains in FCAT developmental scale score. In Year 2, by contrast, the *TransMath* students made statistically significant growth, gaining, on average, 158 DSS points.[1] FCAT performance of *TransMath* students was also disaggregated by ethnic minority status and special program eligibility. All subgroups—including Free/Reduced Lunch (FRL), ethnic minority (i.e., nonwhite), Limited English Proficient (LEP), and Exceptional Special Education (ESE)—made statistically significant gains on the FCAT Math in Year 2, ranging from 144 to 186 points (see *Figure 2*).[2]

Figure 2. Change in FCAT Math developmental scale scores (DSS) among *TransMath* students in Year 1 and Year 2.



[1] Year 1: $F(1,78) < 1$; Year 2: $F(1,78) = 145.20$, $p < .001$, $MSE = 988291$

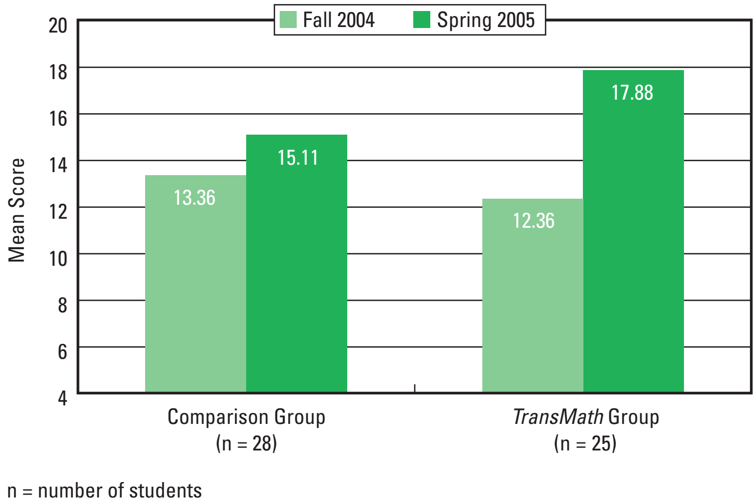
[2] FRL: $F(1,50) = 117.65$, $p < .001$, $MSE = 683881$; Nonwhite: $F(1,31) = 90.89$, $p < .001$, $MSE = 331776$; LEP: $F(1,7) = 41.97$, $p < .001$, $MSE = 137270$; ESE: $F(1,12) = 37.01$, $p < .001$, $MSE = 167521$

TransMath Effectiveness Data

Results From a Quasi-Experimental Study Conducted in Bremerton, Washington

To evaluate the effectiveness of *TransMath* relative to a comparison curriculum, a quasi-experimental study was conducted in two comparable middle schools in Bremerton, Washington, during the 2004–2005 school year.[3] All participating students were in the sixth grade, qualified for special education services, and had been identified for intense, remedial instruction in mathematics. The *TransMath* group received 55 minutes of instruction a day, while the comparison group received 80 minutes of instruction a day. At the start of the study, there were no significant differences between the *TransMath* group and the comparison group on a standardized achievement measure, the mathematics portion of the CTB TerraNova[4] test. By the end of the school year, *TransMath* students achieved higher academic outcomes than did the students in the comparison group (see *Figure 3*). These results are noteworthy considering comparison students received an additional 25 minutes of instruction per day.

Figure 3. CTB TerraNova: Comparison and *TransMath* group student results.



The data indicate that *TransMath* students showed significantly greater gains by the end of the year than comparison students.

[3] The complete published report in the *Journal of Special Education* (Woodward & Brown, 2006) can be viewed at www.voyagerlearning.com/transmath

[4] McGraw-Hill.® (2002). CTB TerraNova.™ Monterrey, CA: CTB McGraw-Hill.

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