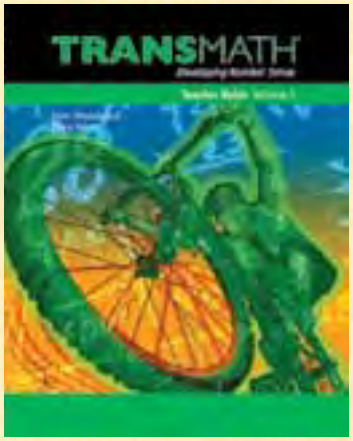
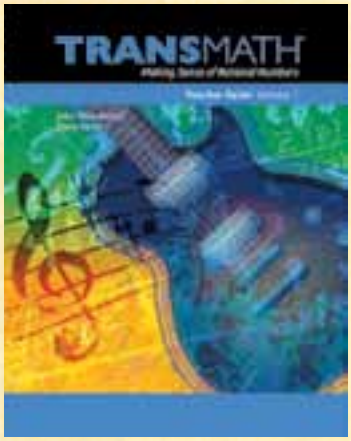
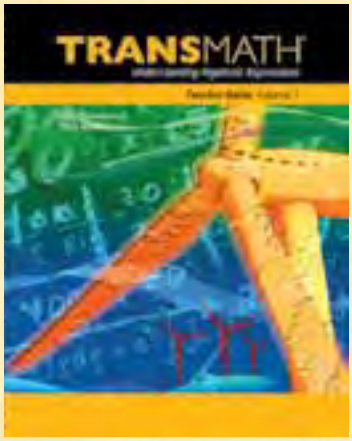
TransMath® Overview

Grades 5–10







TRANSMATH®

Accelerates struggling students to readiness through a conceptual approach focusing on foundational skills

Cambium LEARNING® Group | Voyager

Strategic and Intensive Interventions

Voyager—your RtI partner

Dedicated to the success of every student, Voyager provides strategic and intensive interventions designed to accelerate skill acquisition for all struggling students.

Voyager is a member of Cambium Learning® Group, the leading educational company focused primarily on serving the needs of at-risk and special student populations. Voyager's powerful and effective support interventions, services, and educational technology help accelerate all struggling students to grade-level proficiency. English language learners (ELLs) and students with disabilities derive particular benefits from our interventions and make dramatic gains.

Partnering for RtI solutions

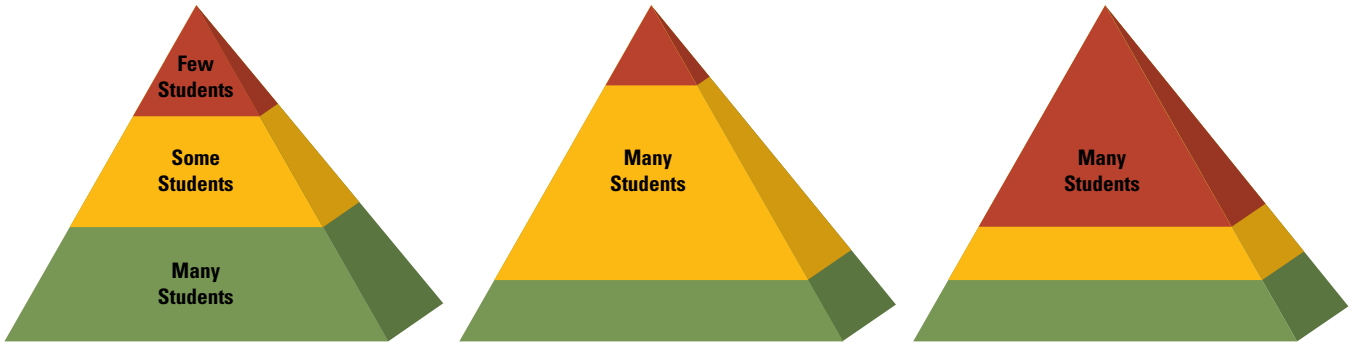
As your intervention partner, we will work with you to develop a customized and integrated solution to meet your Response to Intervention (RtI) needs.

- Our math and literacy interventions are based on and validated by research
- Experienced consultants and practitioners will work with you to develop a customized intervention plan to meet your unique systemwide needs and goals
- Our unparalleled implementation support team will provide onsite and online staff development to ensure fidelity of implementation

Effective RtI key features and benefits

There is no one-size-fits-all solution for struggling learners. Each system and student has specific needs—some only require occasional additional instruction, while others require more comprehensive, long-term support.

What does your school look like?



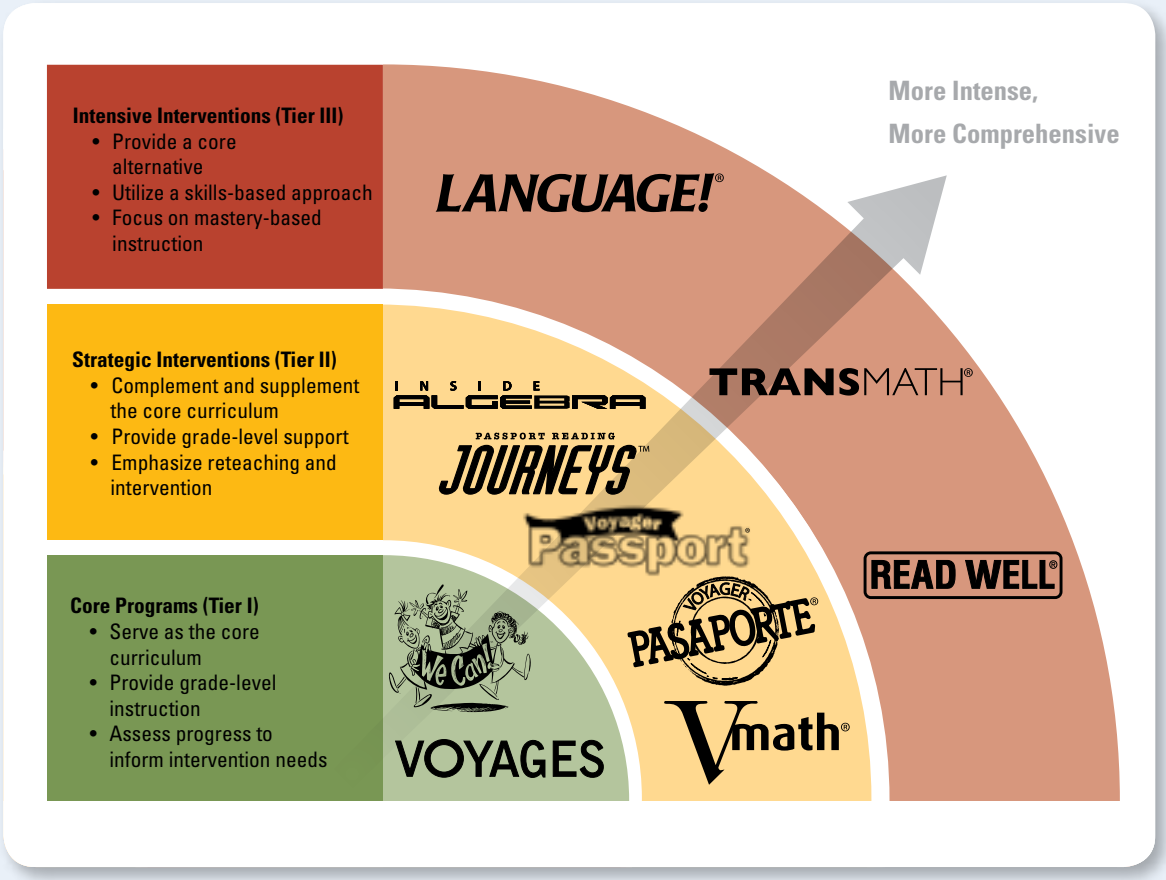
Our interventions and support services are designed to meet the needs of all struggling students by providing multitier instructional interventions aligned to content standards and benchmarks, including the Common Standards.

Key Features of Voyager Interventions	Benefit to Your School/District
Multitier, systematic, scalable approach with supports and tools for differentiated instruction	**Implement an effective and comprehensive intervention plan**
Universal screening and embedded progress monitoring along with a comprehensive Web-based data-management system	**Easily access RtI documentation online and regularly assess and monitor every student's progress**
Based on and validated by research	**Deploy interventions that are proven to work and to turn around low-performing schools**
Onsite and online professional development to increase fidelity of implementation	**Build teacher capacity to ensure interventions are implemented as intended and increase struggling students' academic achievement**

A continuum of interventions for your struggling students

Voyager's interventions are intended to identify struggling students early—before they fall behind—and provide the support they need to be successful.

We provide a continuum of academic interventions in reading and math designed to address the needs of your struggling learners—from strategic interventions that support core curricula to intensive interventions for students who need a completely different approach.



	Literacy	Grades	Description
Core		Early Childhood (PreK)	Comprehensive curriculum includes instruction in math, social studies, reading, pre-writing, music, dramatic arts, and more • Flexible instructional time
Strategic		K–5	Small group reading intervention designed to wrap around core curriculum and accelerate students to grade level using a blended approach of teacher-led instruction and online interactive practice • 30–45 minutes daily
Strategic		K–3	Small group Spanish-language reading intervention designed to build a strong foundation in students' native language to help strengthen their transition to English • 30 or 40 minutes daily
Intensive		K–3	Mastery-based, intensive reading and language arts intervention with focus on decoding, comprehension, spelling, and writing • 60–90 minutes daily
Strategic		6–9	High-interest reading intervention course for middle and high school students that builds academic vocabulary, comprehension, and fluency through motivating topics, teacher-led instruction, and student-centered technology • 50 minutes daily
Intensive		3–12	Mastery-based, intensive reading and language arts intervention that targets the needs of non-readers, struggling readers, and English learners • 90 or 120 minutes daily

	Mathematics	Grades	Description
Core		K–5	Flexible elementary curriculum, organized by grade-level content and broken into two components: *Anchors* and *Excursions* • 50–60 minutes daily
Strategic		2–8	Strategic intervention with a modular approach for targeted skill intervention to reach grade-level expectations • 40–45 minutes daily
Intensive		5–10	Mastery-based, intensive intervention that focuses on the foundational concepts and problem- solving strategies needed for successful entry into algebra • 50–60 minutes daily
Strategic		8–12	Mastery-based, strategic intervention that provides additional strategies for algebra success • 50–60 minutes daily

	Extended Day	Grades	Description
Strategic		K–9	High-interest, adventure-based lessons that help prevent summer learning loss and improve reading skills for students who did not meet proficiency on high-stakes tests • 80 hours of instruction delivered in 3- to 4-hour blocks
Strategic		K–8	Targeted instruction on essential concepts and skills that struggling math students may have missed during the regular school year • Levels A–C include 20 lessons for 40 hours of instruction • Levels D–I offer prepackaged bundles for either 30 or 60 hours of instruction
Strategic		K–8	Adventure-based lessons that integrate cross-curriculum topics to enhance learning and engage learners • More than 60 hours of instruction delivered in a flexible format

	Technology	Grades	Description
Strategic		K–6	Interactive, Web-based reading component that provides engaging, systematic, structured practice and application of the five critical areas of reading • Online component in *Read Well*, *Voyager Passport,* and *TimeWarp Plus*
Strategic		2–8	Interactive, Web-based program that motivates students as they practice math skills and concepts, prepare for high-stakes tests, and play in real-time competitions of mental math with other students around the world • Online component in *TransMath*, *Vmath*, and *Voyages*

TRANSMATH®

What is TRANSMATH®?

TransMath® is the comprehensive mathematics intervention that provides key foundational skill-building and problem-solving experiences through multisensory strategies.

***TransMath*:**

- Affords **ample time** for practice and appropriate pacing
- Teaches **multistep problems** gradually and in a meaningful context
- Utilizes **numerous visual representations** to build conceptual understanding
- Uses **fewer steps**, decreasing mistakes in routine procedures
- Offers a **dual-topic approach**—students experience a numbers strand and an applications strand in every lesson
- Includes an integrated **online data-management system** (**VPORT**,® see page 19 for details)
- Includes **interactive whiteboard activities** to enhance instruction and engage learners

Endorsed by:

CASE
COUNCIL OF ADMINISTRATORS OF SPECIAL EDUCATION

In my 23 years of teaching, TransMath *is the first program where I can fill in the math gaps that my students have. My students kept asking me, 'Why has no one ever shown us how to do it this way before?'*

—James R. McGhee, Middle School Teacher
Albuquerque, New Mexico

Motivates struggling learners to achieve mastery

TransMath achieves results by targeting fewer topics, taught in greater depth, to ensure mastery of skills for successful entry into algebra.

Explicit instruction:

- Teaches skills hierarchically, from number sense to rational numbers to understanding algebra
- Provides differentiation and remediation support through online resources
- Avoids cognitive overload for struggling learners through a dual-topic approach
- Utilizes a skills-based approach with multiple entry points via a Placement Test

TRANSMATH®

At-A-Glance

Who Is *TransMath* for?

TransMath targets the **specific learning needs** of students who:

- Need immediate support
- Lack foundational skills necessary for successful entry into algebra
- Score two or more years below grade level on state standardized tests

Pages 10–11

Research

TransMath was cited in the National Mathematics Advisory Panel findings numerous times. The findings validated the pedagogy and philosophy behind *TransMath*, including the need:

- For **fewer topics in greater depth**
- To focus on the **critical foundations of algebra**—computational fluency, fluency with rational numbers, and aspects of geometry and measurement

Pages 14–17

Scope and Sequence

The *TransMath* program is built around a **dual-strand model.** The Building Number Concepts and Problem Solving topics provide the balance between computational fluency and problem-solving skills, addressing probability, statistics, and particular aspects of geometry and measurement.

Pages 20–23

What Makes *TransMath* Work?

TransMath is the comprehensive mathematics intervention that provides key foundational skill-building and problem-solving experiences by targeting instruction with **fewer topics, taught in greater depth.**

Pages 12–13

Components

The *TransMath* program contains a **wide array of components** to support student learning and teacher implementation.

Page 18–19

The *TransMath* Curriculum

TransMath establishes a strong math foundation with the **Building Number Concepts strand.** Students are taught concepts and skills in the order in which they need to learn them, from developing number sense to thinking algebraically. At the same time, the **Problem Solving** strand provides rich, grade-level experiences.

Pages 24–27

Cambium Learning Group is the leading educational company focused exclusively on at-risk and special student populations.

Balanced Assessment

The *TransMath* assessment system provides teachers with the measures they need to accurately **place students into the curriculum** and to **monitor their progress** through the curriculum.

Pages 28–35

Teacher Support

TransMath provides everything needed for successful implementation:

- **Through technology …** The *TransMath mBook™ Teacher Edition* provides powerful online resources that support teachers in the successful daily implementation of this comprehensive curriculum
- **Through print …** Articulate point-of-use lesson plans and a modified wrap-around format provide everything a teacher needs to implement *TransMath* with fidelity

Pages 36–49

Student Support

TransMath provides daily support for students and parents:

- **Through technology …** *TransMath* is accessible to both students and parents anytime, anywhere, through the *TransMath mBook Study Guide*
- **Through print …** A complete student text includes multiple models and strategies for understanding lesson concepts

Pages 50–55

Cambium Learning Solutions

At Cambium Learning Group, we understand that intervention solutions don't come from programs alone. Cambium Learning Solutions is our division focused on helping educators **effectively implement programs, including *TransMath*, through professional development services.**

Pages 56–57

Sample Pages

Review a sample lesson from both the teacher and student books.

Pages 58–79

Who Is TRANSMATH® for?

TRANSMATH® Targets the Specific Learning Needs of Students Who Need Immediate Support.

- Students lacking the foundational skills necessary for successful entry into algebra
- Students scoring two or more years below grade level on state standardized tests

Extensively cited by the Task Group on Instructional Practices for the National Mathematics Advisory Panel's Final Report, *TransMath* (*Transitional Mathematics*) provides comprehensive skill building by targeting instruction with fewer topics, taught in greater depth. This approach was cited as a key finding for mathematical success by the Trends in International Mathematics and Science Studies (TIMSS) and is supported by the National Council of Teachers of Mathematics (NCTM) Curriculum Focal Points.

Can Your Students Solve This Equation?
10(x + 5) = 2x + 56

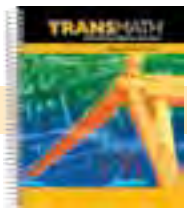
Prerequisite Skills for Algebra Proficiency

$10(x + 5) = 2x + 56$ ←	Recognize that the equation is balanced Recognize that unlike terms cannot be combined Recognize that 2 is a coefficient Be able to use the Distributive Property to delete the parentheses Know basic multiplication
$10x + 50 = 2x + 56$ ←	Recognize that unlike terms cannot be combined
$-50 + 10x + 50 = 2x + 56 + -50$ ←	Understand the need to maintain a balanced equation Know the property of opposites (i.e., 50 and –50)
$10x + -50 + 50 = 2x + 56 + -50$ ←	Be able to use the Commutative Property to combine like terms Know how to add integers Know basic subtraction
$10x = 2x + 6$ ←	Recognize that the equation is balanced
$-2x + 10x = 2x + 6 + -2x$ ←	Understand the need to maintain a balanced equation Know the property of opposites (i.e., 2x and –2x)
$-2x + 10x = 2x + -2x + 6$ ←	Be able to use the Commutative Property to combine like terms Know how to add integers
$8x = 0 + 6$ ←	Recognize that the equation is balanced Know basic addition
$\frac{1}{8} \cdot 8x = 6 \cdot \frac{1}{8}$ ←	Be able to use reciprocals Know how to multiply fractions
$\frac{8}{8}x = \frac{6}{8}$ ←	Know that $1x = x$ (the "invisible coefficient") Know about fractions equal to one (i.e., $\frac{8}{8}$) Know basic multiplication Know how to simplify fractions Know about greatest common factors
$x = \frac{3}{4}$	

Prerequisite Skills Taught to Mastery in:

Level 1

Level 2



Level 3

What Makes TRANSMATH® Work?

The balanced, systematic approach of *TransMath* ensures that students learn the essential skills and strategies to build a strong foundation in mathematics.

Teacher-Led Instruction

Research shows that **explicit instruction** positively affects math achievement for students who have difficulty learning math. The *TransMath* curriculum features:

- Teacher-led, explicit instruction that follows an effective four-part lesson design
- Dual topics to avoid cognitive overload
- Engagement strategies that provide varied and continuous communication opportunities
- Visual models illustrate difficult math concepts
- Distributed practice in every lesson to provide continued practice of previously learned skills

Integrated Technology

TransMath mBook

Students and teachers can access the *mBook*,™ the online component of *TransMath*, anytime, anywhere. *TransMath* is accessible to both students and parents through the *TransMath mBook Study Guide*, which includes:

- The entire *Student Text* to review missed concepts
- Teacher-Talk Tutorials to reinforce difficult concepts with animated instruction

The *TransMath mBook Teacher Edition* provides powerful online resources that support teachers through:

- Downloadable Interactive Click-Thru Slideshow
- Whiteboard activities for both Building Number Concept and Problem-Solving strands of each lesson
- Downloadable Form B Retests for Quizzes and End-of-Unit Assessments
- Online instructional support for students and parents (anytime, anywhere) through Teacher-Talk Tutorials

Independent Technology

VmathLive®

VmathLive is the program's online, independent learning component that works in conjunction with *TransMath*. Through 24/7 access from any Internet-enabled computer, students spend more time on task practicing math.

VmathLive:

- Supports *TransMath* lessons
- Provides computational practice and tutorials of key math concepts
- Reinforces essential math skills and strategies
- Provides real-time math skills competitions with learners around the world

Effective Online Data-Management System

The key to effective instruction is real-time data that track student progress throughout the year. VPORT, Voyager's online data-management system, collects and reports student assessment data, enabling district leaders and teachers to make effective instructional decisions. (See page 19 for details.)

Instructional Principles Are Supported by Research.

A conceptually guided approach to mathematics instruction is essential to the learning needs of struggling students. The five instructional principles that best serve this population of students are the foundation for the pedagogical structure of *TransMath*.

***TransMath* Instructional Principles**	**The Supporting Research**
Visual Representations	Visual representations include models, diagrams, and drawings as well as physical manipulatives. Paivio (1990) is one of many cognitive psychologists whose research supports the fact that information is stored visually as well as textually. The National Mathematics Panel Report (2008a) endorses the use of models and visual images as important ways to promote conceptual understanding, particularly in students with learning difficulties. Well-chosen visual models, in conjunction with conceptual explanations, can help students understand and remember key math concepts.
Controlling Cognitive Load	Many standards-based curricula have lessons that require extensive reading and contain a significant amount of mathematical as well as nonmathematical vocabulary. This is because developers want to provide authentic, or "real-world," problems for students to solve. These curricula also tend to move at a pace that is too fast for struggling students. These factors need to be addressed when standards-based curricula are adapted for struggling students (National Mathematics Panel Report 2008b). Teachers might need to summarize textual materials, and the key mathematical vocabulary needs to be highlighted and reviewed systematically (Baxter, Woodward, and Olson 2001).
Distributed Practice	Psychological and educational research emphasizes the role controlled distributed practice plays in enhancing retention as well as the overall design of curricular materials (e.g., Coyne, Kameenui, and Carnine 2007; Donovan and Radosevich 1999; Pashler, Rohrer, Cepeda, and Carpenter 2007). Too often, textbooks move from one topic or skill to the next without allowing sufficient opportunities for students to become proficient. Distributed practice on a sensible range of skills and concepts is essential to a struggling student's success in mathematics.
Varied Opportunities for Communication	Mathematical discussions can be challenging for teachers as well as students. Teachers sometimes find it difficult to interpret what some students are saying, thus interrupting the flow of the discussion. Teachers might also unintentionally favor those students who contribute the most to a discussion, leaving students with math difficulties ignored. Nonetheless, these students—like their more verbal peers—need structured opportunities to ask questions and explain their thinking. Recent research (e.g., Chapin, O'Connor, and Anderson 2003) offers important principles for conducting whole-group discussions in math classrooms.
Multiple Forms of Assessment	Ongoing or frequent assessment of students with math difficulties is a major concern in special education. This type of assessment can be seen as part of the Response to Treatment Intervention or RtI movement. At the same time, students should also experience other forms of assessment such as performance assessment and daily informal assessment (Lampert 2001; Stiggins 2005; Wiggins and McTighe 2005).

Standard Score Gains for Students Receiving Special Education Services in a Georgia School District

During the 2008–2010 school years, *TransMath* was used as a core replacement math curriculum for low-performing students with special needs. *TransMath* was taught for 360 minutes a week over a two-year period. The KeyMath3,™ an individually administered, norm-referenced test of mathematics concepts, was administered prior to instruction, and again at the end of the school year. Results are reported on a standard score scale with a mean score of 100, representing the national average, and a standard deviation of 15.

If a low-performing student:

- Makes developmentally expected gain on the test, his or her standard score should remain the same from one test period to another
- Improves his or her standard score, the student is bringing their performance closer to the national average
- Accelerates his or her growth—that is, the student makes more than the gain that is developmentally expected—his or her standard score will increase

Figure 1. KeyMath3 results for students with special needs with *TransMath* instruction over two school years: Fall 2008 to spring 2010.

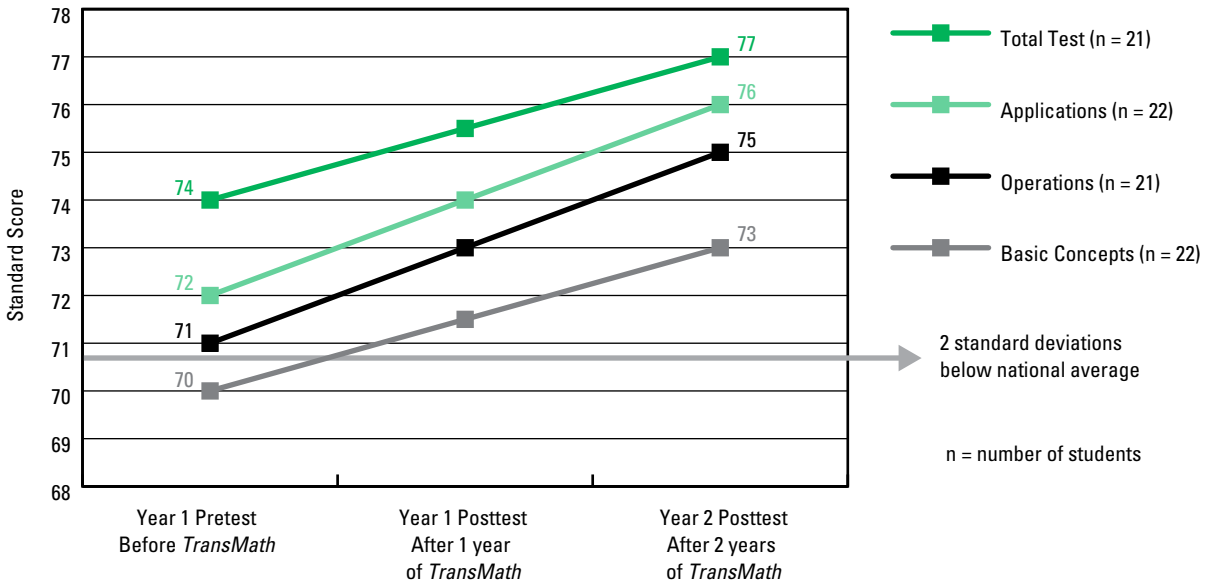


Figure 1 shows that, on average, *TransMath* students who had scored nearly two standard deviations below the national average at pretest were able to improve their standard score by nearly four points or nearly one-third of a standard deviation; that is, the *TransMath* group brought its performance closer to the national average.

Two-Year Gains on the State Assessment for Lee County Public Schools, Florida

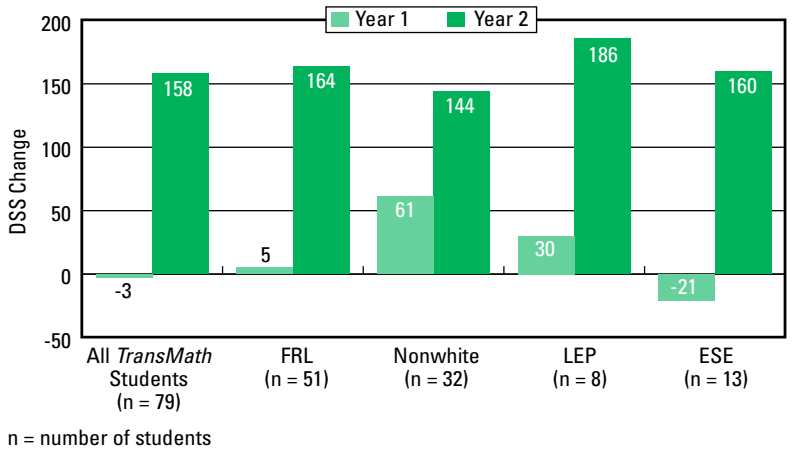
During the school years ending in 2007 and 2008, two middle schools in Florida's Lee County Public Schools began implementing *TransMath* to support students who needed systematic help in building a solid foundation of basic skills and mathematical reasoning. Participants had scored in Level 1 on the Florida Comprehensive Assessment Test® (FCAT), suggesting performance of at least two years below grade level. Across the two schools, *TransMath* instruction was provided either 45 or 90 minutes a day.

Figure 2 shows FCAT developmental scale score (DSS) gains in math among *TransMath* students. Intervention programs targeting students struggling in math often take more than one year of implementation to effect positive change, which makes the results in Year 2 of particular interest.

- In Year 1 of the *TransMath* implementation, the *TransMath* students showed no significant gains in FCAT developmental scale score
- In Year 2, by contrast, the *TransMath* students made statistically significant growth, gaining, on average, 158 DSS points[1]

FCAT performance of *TransMath* students was also disaggregated by ethnic minority status and special program eligibility. All subgroups—including Free/Reduced Lunch (FRL), ethnic minority (i.e., nonwhite), Limited English Proficient (LEP), and Exceptional Special Education (ESE)—made statistically significant gains on the FCAT Math in Year 2, ranging from 144 to 186 points (see Figure 2).[2]

Figure 2. Change in FCAT Math developmental scale scores (DSS) among *TransMath* students in Year 1 and Year 2.



n = number of students

1. Year 1: $F(1,78) < 1$; Year 2: $F(1,78) = 145.20$, $p < .001$, $MSE = 988291$
2. FRL: $F(1,50) = 117.65$, $p < .001$, $MSE = 683881$; Nonwhite: $F(1,31) = 90.89$, $p < .001$, $MSE = 331776$; LEP: $F(1,7) = 41.97$, $p < .001$, $MSE = 137270$; ESE: $F(1,12) = 37.01$, $p < .001$, $MSE = 167521$

Higher Academic Outcomes for *TransMath* Students in Two Bremerton, Washington Schools

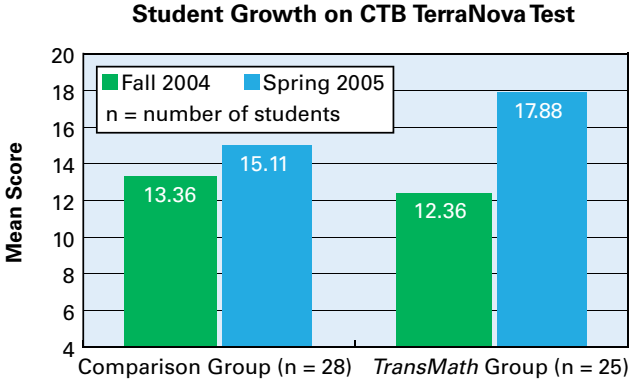
To evaluate the effectiveness of *TransMath* relative to a comparison curriculum, a quasiexperimental study was conducted in two comparable middle schools in Bremerton, Washington, during the 2004–2005 school year.[3]

All participating special education students had been identified for intense remedial instruction in mathematics. At the start of the study, there were no significant differences between the *TransMath* group and the comparison group on the mathematics portion of the CTB TerraNova[4] Test.

The results indicated that *TransMath* students achieved higher academic outcomes than did the students in the comparison group (see Figures 3 and 4). These results are noteworthy considering comparison students received 80 minutes of instruction per day, which was 25 minutes more instructional time per day than the *TransMath* group. In addition, students in the *TransMath* group demonstrated a more positive outlook on mathematics (see Figure 5).

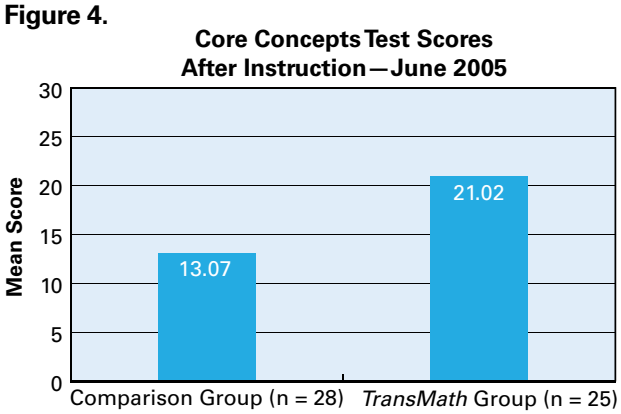
3. The Complete published report in the *Journal of Special Education* (Woodward and Brown 2006) can be viewed at www.voyagerlearning.com/transmath
4. McGraw-Hill. 2002. CTB TerraNova. Monterrey, Calif.: CTB McGraw-Hill

Figure 3.



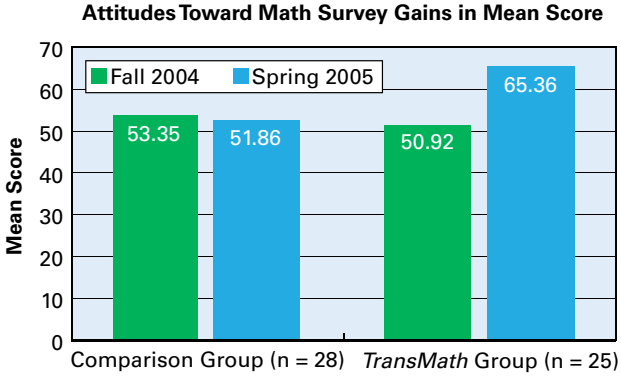
The data indicate that *TransMath* students showed significantly greater gains by the end of the year than comparison students.

Figure 4.



On this cumulative measure of core math concepts taught across the entire year, data show that the *TransMath* group outperformed the comparison group in mathematics achievement.

Figure 5.



TransMath students developed a more positive attitude toward math, which included their self-perceptions as problem solvers.

What Are the TRANSMATH® Components?

Teacher Materials

Teacher Guides—3 Levels
- Two-Volume Set at each level
- Level 1: 9 units
- Level 2: 9 units
- Level 3: 10 units

Teacher Placement Guide
- Guides teachers in administering and scoring the placement test

Transparencies and Manipulatives

VPORT Online Assessment System
- Comprehensive data-management system guides instruction and monitors change

mBook Teacher Edition
- Provides online access to all teacher and student components and tools for Professional Development, Concept Modeling, and Reinforcement

Student Materials

Student Text—3 Levels
- Level 1: 9 units
- Level 2: 9 units
- Level 3: 10 units

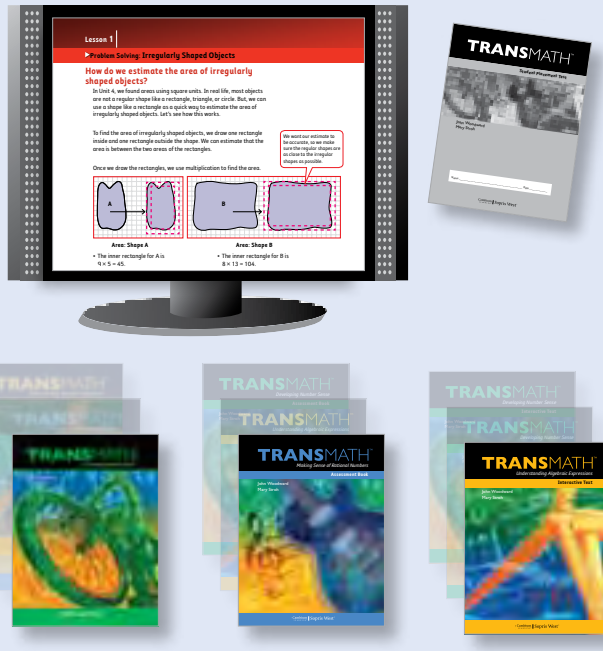
Student Placement Test
- Accurately places students into the curriculum

Interactive Text
- Provides the in-class activities for application of skills

Assessment Book
- Contains all Quizzes, End-of-Unit Assessments, and Performance Assessments

mBook Study Guide
- Provides online access to Student Text and Interactive Reinforcement Exercises

VmathLive
- Provides interactive online practice and tutorials



Simplify data management with VPORT

VPORT is a Web-based application that houses the data-management system and *mBook*.

It is a powerful tool that serves as the management, delivery, scoring, and reporting mechanism for *TransMath*. Teachers and administrators can instantly access reports and tools to measure student progress and track progress monitoring assessments using real-time data to:

- Identify individual instructional needs and goals
- Adjust instruction based on skills level
- Monitor progress against goals
- Communicate progress to the instructional team

VPORT®

VPORT Reports

TransMath assessments are processed online, and the real-time data allow educators instant access to multiple reports showing student performance at various levels.

Key Measures Report

Dashboard

mBook

Scope and Sequence

Dual Topics Provide a Balance of Conceptual Learning and Problem-Solving Applications.

The dual-topic approach:

- Breaks learning into smaller parts
- Increases student engagement
- Addresses the issue of cognitive overload for struggling students

		Building Number Concepts	Problem Solving
Level 1: Developing Number Sense	Unit 1	Addition	Working With Data
	Unit 2	Subtraction	Working With Data
	Unit 3	Multiplication	Introduction to Measurement
	Unit 4	Division	Measuring Two-Dimensional Objects
	Unit 5	Factors, Primes, Composites	Area and Perimeter
	Unit 6	Common Factors and Number Patterns	Properties of Shapes
	Unit 7	More Number Patterns and Common Multiples	Slides, Flips, Turns, and Symmetry
	Unit 8	Concept of Fractions	Introduction to Statistics
	Unit 9	Adding and Subtracting Fractions	Converting Units of Measurement
Level 2: Making Sense of Rational Numbers	Unit 1	Review of Whole Numbers and Fractions	Working With Data
	Unit 2	Multiplication and Division of Fractions	Tools for Measurement and Construction
	Unit 3	Working With Mixed Numbers	Tessellations, Geometry, and Measurement
	Unit 4	The Concept of Decimal Numbers	Triangles and Quadrilaterals
	Unit 5	Operations on Decimal Numbers	Area of Two-Dimensional Shapes
	Unit 6	Understanding Percents	Percents in Word Problems and Graphs
	Unit 7	Scientific Notation	Probability
	Unit 8	Integers	Finding Points on a Graph
	Unit 9	Operations on Integers	Coordinate Graphs and Transformations
Level 3: Understanding Algebraic Expressions	Unit 1	Fractions and Decimal Numbers	Statistics
	Unit 2	Variables	Ratios and Proportions
	Unit 3	Inequalities	Working With Rates
	Unit 4	Algebraic Patterns	Ratios
	Unit 5	Algebraic Expressions	Surface Area of Three-Dimensional Shapes
	Unit 6	Algebraic Rules and Properties	Volume of Three-Dimensional Shapes
	Unit 7	Introduction to Algebraic Equations	Geometric Construction and Angle Measurement
	Unit 8	Solving Different Kinds of Algebraic Equations	Links and Angles
	Unit 9	Introduction to Functions	Working With Coordinate Graphs
	Unit 10	Square Roots and Irrational Numbers	Nonlinear Functions

Level 1: Developing Number Sense

Level 1: Developing Number Sense

	Building Number Concepts	Problem Solving
Unit 1	**Addition** • Determine the place value of digits in a whole number. • Find sums of whole numbers with and without regrouping. • Round and estimate with whole numbers.	**Working With Data** • Read and interpret word problems. • Create, read, and interpret bar graphs. • Create pictographs.
Unit 2	**Subtraction** • Understand the relationship between basic and extended subtraction facts. • Solve whole-number subtraction problems using a variety of strategies. • Estimate the solution to problems by rounding.	**Working With Data** • Identify the question being asked in a word problem. • Read and analyze data in bar graphs and tables. • Solve word problems using whole-number subtraction.
Unit 3	**Multiplication** • Understand the relationship between basic and expanded multiplication. • Recognize and factor out powers of 10 from multiplication problems. • Estimate the solution to whole-number multiplication problems.	**Introduction to Measurement** • Measure using common objects. • Measure objects using inches and metric units. • Use a variety of measurement strategies in real-world problems.
Unit 4	**Division** • Understand the relationship between multiplication and division. • Solve problems using basic and extended division facts. • Represent whole-number division problems in a variety of ways.	**Measuring Two-Dimensional Objects** • Use square units to measure the area of shapes. • Apply the concept of area to real-world situations. • Solve word problems using whole-number division.
Unit 5	**Factors, Primes, Composites** • Factor whole numbers using a variety of methods. • Determine if a given number is prime or composite. • Find the prime factorization of a whole number.	**Area and Perimeter** • Explore the relationship between perimeter and area of shapes. • Discover and use area formulas for triangles and parallelograms. • Find the area of irregularly shaped objects.
Unit 6	**Common Factors and Number Patterns** • Find common factors for whole numbers using a variety of methods. • Identify the greatest common factor for two or more whole numbers. • Explore patterns in odd, even, and square numbers.	**Properties of Shapes** • Group shapes based on common properties. • Explore congruence and similarity of shapes. • Expand and contract shapes on a grid.
Unit 7	**More Number Patterns and Common Multiples** • Understand the relationships between triangular and square numbers. • Use exponents to show repeated multiplication. • Identify common multiples of two or more whole numbers.	**Slides, Flips, Turns, and Symmetry** • Recognize slides, flips, and turns in shapes. • Use tangrams to explore the properties of shapes. • Understand reflection and rotational symmetry.
Unit 8	**Concept of Fractions** • Recognize common fractions between whole numbers. • Represent fractions using shapes and fraction bars. • Find equivalent fractions.	**Introduction to Statistics** • Find the mean, median, and range of a set of data. • Use tables to organize data. • Read and create line plots and stem-and-leaf plots.
Unit 9	**Adding and Subtracting Fractions** • Add and subtract fractions with like and unlike denominators. • Find the least common multiple of two or more whole numbers. • Use least common multiples to find common denominators.	**Converting Units of Measurement** • Understand common units of measurement. • Convert units using a conversion table. • Measure objects to the nearest ¼ inch.

Scope and Sequence

Level 2: Making Sense of Rational Numbers

		Building Number Concepts	Problem Solving
Level 2: Making Sense of Rational Numbers	Unit 1	**Review of Whole Numbers and Fractions** • Use place-value concepts to add and subtract whole numbers. • Use a variety of representations for fractions and decimal numbers. • Find the least common multiple of two or more whole numbers.	**Working With Data** • Read, create, and interpret bar graphs, pictographs, stem-and-leaf plots, and line graphs. • Use a bar graph to find the average of a set of data.
	Unit 2	**Multiplication and Division of Fractions** • Use models to show multiplication and division of fractions. • Understand how multiplication and division of fractions is different from whole numbers. • Use the traditional methods to multiply and divide fractions.	**Tools for Measurement and Construction** • Develop an understanding of basic geometric terms. • Measure lengths and angles using a variety of tools and units. • Use a compass to complete basic geometric constructions.
	Unit 3	**Working With Mixed Numbers** • Use the LAPS strategy to add, subtract, multiply, and divide mixed numbers. • Use approximations to estimate answers to problems involving fractions and mixed numbers.	**Tessellations, Geometry, and Measurement** • Recognize and use translations, reflections, and rotations of shapes. • Create and analyze tessellations.
	Unit 4	**The Concept of Decimal Numbers** • Understand the relationship between fractions and decimal numbers. • Convert fractions to decimal numbers and decimal numbers to fractions. • Use strategies to round decimal numbers.	**Triangles and Quadrilaterals** • Classify triangles based on their properties. • Classify quadrilaterals based on their properties. • Understand the result of changing the dimensions of a shape.
	Unit 5	**Operations on Decimal Numbers** • Demonstrate addition and subtraction of decimal numbers. • Use models to show multiplication and division of decimal numbers. • Use rounding strategies when working with decimal numbers.	**Area of Two-Dimensional Shapes** • Use formulas to find the area of rectangles, triangles, and other quadrilaterals. • Develop an understanding of the parts of a circle. • Find the circumference and area of a circle.
	Unit 6	**Understanding Percents** • Understand the relationship between fractions, decimal numbers, and percents. • Convert between fractions, decimal numbers, and percents. • Use models to represent and understand percents.	**Percents in Word Problems and Graphs** • Read, create, and interpret circle graphs. • Use graphs to show percent increase or decrease. • Solve problems involving percent increase or decrease.
	Unit 7	**Scientific Notation** • Understand the use of standard notation and scientific notation. • Use scientific notation to write very large and very small numbers.	**Probability** • Use fractions, decimal numbers, and percents to show probabilities. • Use models to find the probability of a single event. • Find the probability of independent and dependent events.
	Unit 8	**Integers** • Use integers to represent values greater than and less than zero. • Use a number line to order and compare integers. • Use models to add and subtract integers.	**Finding Points on a Graph** • Read, create, and interpret dot graphs. • Use a coordinate grid to graph *x* and *y* coordinates. • Recognize and describe symmetry on a coordinate graph.
	Unit 9	**Operations on Integers** • Use rules for integer operations to solve problems. • Use models to show multiplication and division of integers. • Use the PASS rule to multiply and divide integers.	**Coordinate Graphs and Transformations** • Use a coordinate graph to show translated and reflected shapes. • Use a coordinate graph to tell the difference between a translation and a reflection. • Use a table to show translated and reflected shapes.

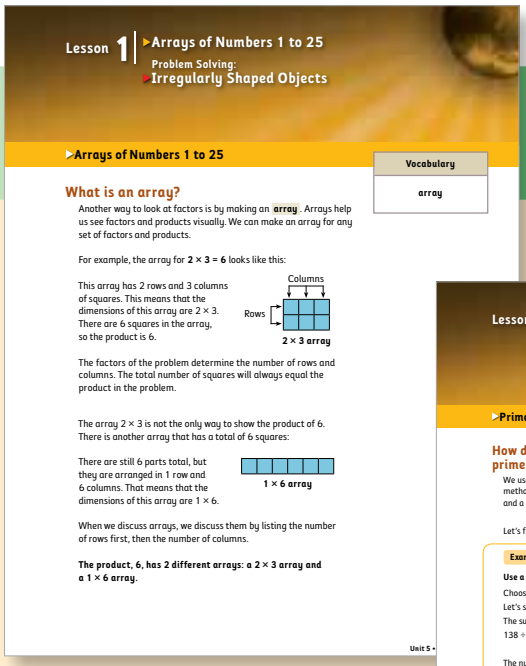
Level 3: Understanding Algebraic Expressions

Level 3: Understanding Algebraic Expressions

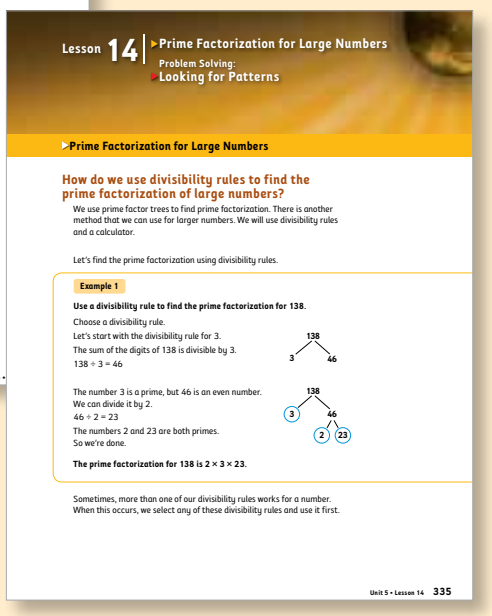
	Building Number Concepts	Problem Solving
Unit 1	**Fractions and Decimal Numbers** • Use models to show the relationship between fractions and decimal numbers. • Use a variety of methods to add, subtract, multiply, and divide rational numbers. • Use rounding and estimation strategies with rational numbers.	**Statistics** • Find the mean, median, mode, and range of a set of data. • Read, create, and interpret box-and-whisker plots and scatter plots. • Identify direct and indirect relationships in data using a scatter plot.
Unit 2	**Variables** • Use variables to describe patterns. • Use variables to represent unknown values in formulas and equations. • Convert between equations and statements using words.	**Ratios and Proportions** • Represent part-to-whole and part-to-part relationships using ratios. • Recognize and represent proportional relationships. • Use proportions to identify similar shapes.
Unit 3	**Inequalities** • Represent inequalities using symbols and number lines. • Represent written statements using inequalities. • Create written statements from inequalities.	**Working With Rates** • Solve rate problems using proportions. • Find unit rates using proportions. • Compare two rates using proportions.
Unit 4	**Algebraic Patterns** • Use variables to represent numeric patterns. • Use variables to analyze patterns and make predictions. • Represent even and odd numbers and divisibility rules using algebraic equations.	**Ratios** • Represent part-to-whole and part-to-part relationships using ratios. • Solve real-world problems involving ratios. • Use percents to make comparisons.
Unit 5	**Algebraic Expressions** • Evaluate numeric expressions using order of operations rules. • Recognize like and unlike terms in an algebraic expression. • Simplify algebraic expressions using the properties of numbers.	**Surface Area of Three-Dimensional Shapes** • Identify the attributes of three-dimensional shapes. • Use formulas to find the surface area of cylinders and prisms. • Find the surface area of pyramids and polyhedrons by breaking the shapes into familiar parts.
Unit 6	**Algebraic Rules and Properties** • Use order of operations rules to evaluate algebraic and numeric expressions. • Use substitution to evaluate algebraic expressions. • Apply the distributive property to algebraic expressions.	**Volume of Three-Dimensional Shapes** • Use formulas to find the volume of cylinders and prisms. • Find the volume of pyramids and cones by comparing them to prisms and cylinders. • Use a formula to find the volume of a sphere.
Unit 7	**Introduction to Algebraic Equations** • Understand the basic properties of algebraic equations. • Balance equations involving symbols or variables. • Solve problems involving algebraic equations.	**Geometric Construction and Angle Measurement** • Use a compass and straightedge to construct basic figures. • Use algebraic reasoning to find missing angle measures. • Explore the properties of triangles with congruent angles.
Unit 8	**Solving Different Kinds of Algebraic Equations** • Use a variety of rules and properties to solve algebraic equations. • Use algebraic equations to describe a given situation. • Solve word problems involving algebraic equations using models, and check answers for how reasonable they are.	**Lines and Angles** • Use algebra to find the measures of interior angles in a polygon. • Use angle rules to solve problems involving related angles (vertical, corresponding, right, and supplementary). • Complete simple proofs involving angle measures.
Unit 9	**Introduction to Functions** • Use word problems and tables to think about functional relationships. • Interpret the slope and *y*-intercept of a function in a real-world situation. • Use a function to make predictions in a real-world situation.	**Working With Coordinate Graphs** • Graph linear functions on a coordinate graph. • Convert functions between representations (tables, graphs, and equations). • Interpret the intersection of two functions in a real-world situation.
Unit 10	**Square Roots and Irrational Numbers** • Solve algebraic equations and estimate answers involving square roots. • Use the Pythagorean theorem to find the lengths of sides of right triangles. • Identify and use irrational numbers.	**Nonlinear Functions** • Tell whether a function is linear or nonlinear given a table, equation, or graph. • Graph nonlinear functions on a coordinate graph. • Understand the role of the coefficient in a nonlinear function.

Establishes a Strong Math Foundation With the Building Number Concepts Strand.

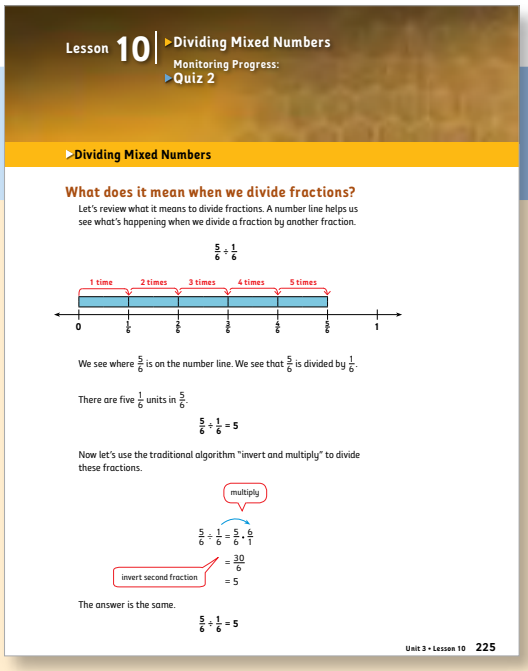
Students are taught concepts and skills in the order in which they need to learn them—from developing number sense to thinking algebraically.



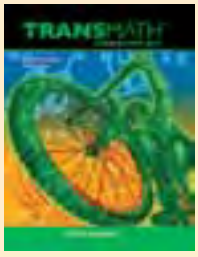
Level 1, Unit 5, Lesson 1



Level 1, Unit 5, Lesson 14



Level 2, Unit 3, Lesson 10



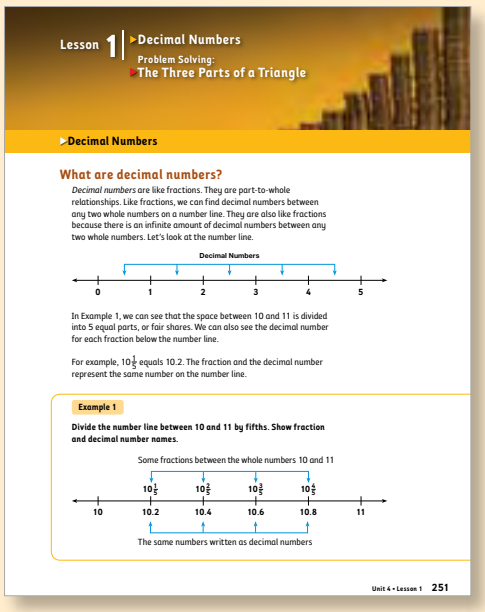
Developing Number Sense

Making Sense of Rational Numbers

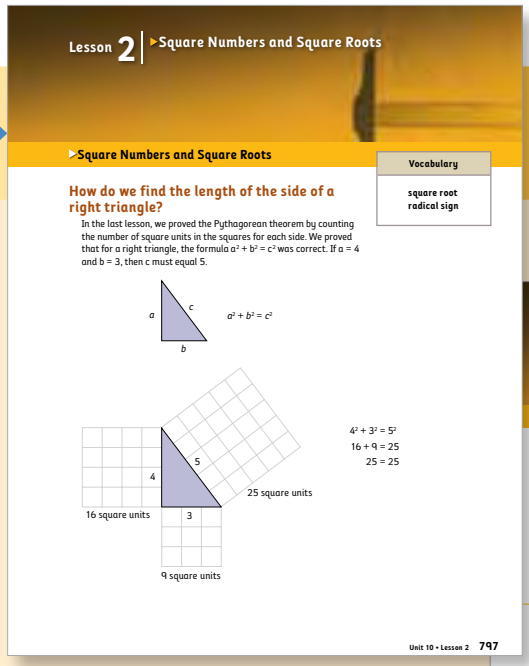
The Building Number Concepts strand encompasses:

- Whole-number computation
- Factors, primes, and composites
- Rational-number computation
- Comparison of fractions, decimal numbers, and percents
- Exponents and integers
- Variables and algebraic equations
- Inequalities and functions

"The National Mathematics Advisory Panel's Final Report cites the three critical foundations of algebra—fluency with whole numbers, fluency with fractions, and particular aspects of geometry and measurement—as being 'the most essential mathematics for students to learn thoroughly prior to algebra course work' (page 17)."

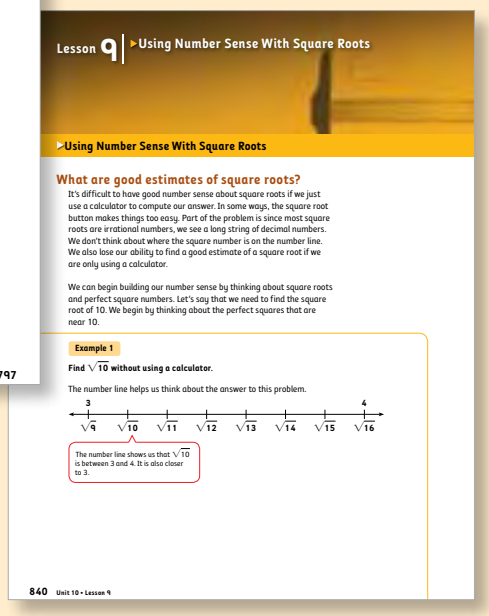


Level 2, Unit 4, Lesson 1



Level 3, Unit 10, Lesson 2

Understanding Algebraic Expressions

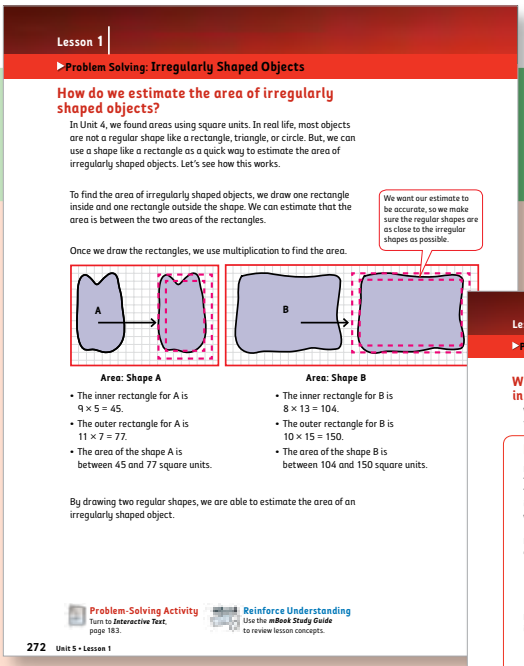


Level 3, Unit 10, Lesson 9

Successful entry into algebra

Provides Rich, Grade-Level, Problem-Solving Experiences With the Problem-Solving Strand.

With *TransMath*, students apply previously learned concepts and engage in critical thinking to solve multistep problems needed for higher mathematics or the working world.


Lesson 1

Problem Solving: Irregularly Shaped Objects

How do we estimate the area of irregularly shaped objects?

In Unit 4, we found areas using square units. In real life, most objects are not a regular shape like a rectangle, triangle, or circle. But, we can use a shape like a rectangle as a quick way to estimate the area of irregularly shaped objects. Let's see how this works.

To find the area of irregularly shaped objects, we draw one rectangle inside and one rectangle outside the shape. We can estimate that the area is between the two areas of the rectangles.

We want our estimate to be accurate, so we make sure the regular shapes are as close to the irregular shapes as possible.

Once we draw the rectangles, we use multiplication to find the area.

A

B

Area: Shape A

- The inner rectangle for A is 9 × 5 = 45.
- The outer rectangle for A is 11 × 7 = 77.
- The area of the shape A is between 45 and 77 square units.

Area: Shape B

- The inner rectangle for B is 8 × 13 = 104.
- The outer rectangle for B is 10 × 15 = 150.
- The area of the shape B is between 104 and 150 square units.

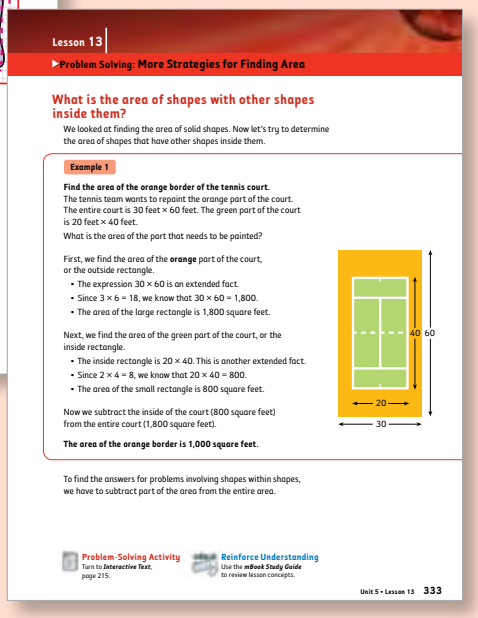
By drawing two regular shapes, we are able to estimate the area of an irregularly shaped object.

Problem-Solving Activity
Turn to *Interactive Text*, page 183.

Reinforce Understanding
Use the *mBook Study Guide* to review lesson concepts.

272 Unit 5 • Lesson 1

Level 1, Unit 5, Lesson 1


Lesson 13

Problem Solving: More Strategies for Finding Area

What is the area of shapes with other shapes inside them?

We looked at finding the area of solid shapes. Now let's try to determine the area of shapes that have other shapes inside them.

Example 1

Find the area of the orange border of the tennis court.
The tennis team wants to repaint the orange part of the court. The entire court is 30 feet × 60 feet. The green part of the court is 20 feet × 40 feet.

What is the area of the part that needs to be painted?

First, we find the area of the **orange** part of the court, or the outside rectangle.

- The expression 30 × 60 is an extended fact.
- Since 3 × 6 = 18, we know that 30 × 60 = 1,800.
- The area of the large rectangle is 1,800 square feet.

Next, we find the area of the green part of the court, or the inside rectangle.

- The inside rectangle is 20 × 40. This is another extended fact.
- Since 2 × 4 = 8, we know that 20 × 40 = 800.
- The area of the small rectangle is 800 square feet.

Now we subtract the inside of the court (800 square feet) from the entire court (1,800 square feet).

The area of the orange border is 1,000 square feet.

40 60 20 30

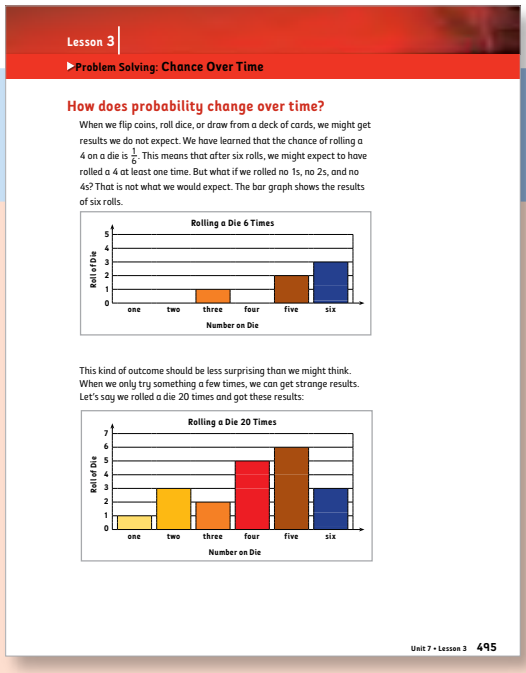
To find the answers for problems involving shapes within shapes, we have to subtract part of the area from the entire area.

Problem-Solving Activity
Turn to *Interactive Text*, page 215.

Reinforce Understanding
Use the *mBook Study Guide* to review lesson concepts.

Unit 5 • Lesson 13 333

Level 1, Unit 5, Lesson 13


Lesson 3

Problem Solving: Chance Over Time

How does probability change over time?

When we flip coins, roll dice, or draw from a deck of cards, we might get results we do not expect. We have learned that the chance of rolling a 4 on a die is $\frac{1}{6}$. This means that after six rolls, we might expect to have rolled a 4 at least one time. But what if we rolled no 1s, no 2s, and no 4s? That is not what we would expect. The bar graph shows the results of six rolls.

Rolling a Die 6 Times

Roll of Die: 0 1 2 3 4 5

one two three four five six

Number on Die

This kind of outcome should be less surprising than we might think. When we only try something a few times, we can get strange results. Let's say we rolled a die 20 times and got these results:

Rolling a Die 20 Times

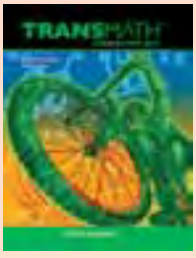
Roll of Die: 0 1 2 3 4 5 6 7

one two three four five six

Number on Die

Unit 7 • Lesson 3 495

Level 2, Unit 7, Lesson 3



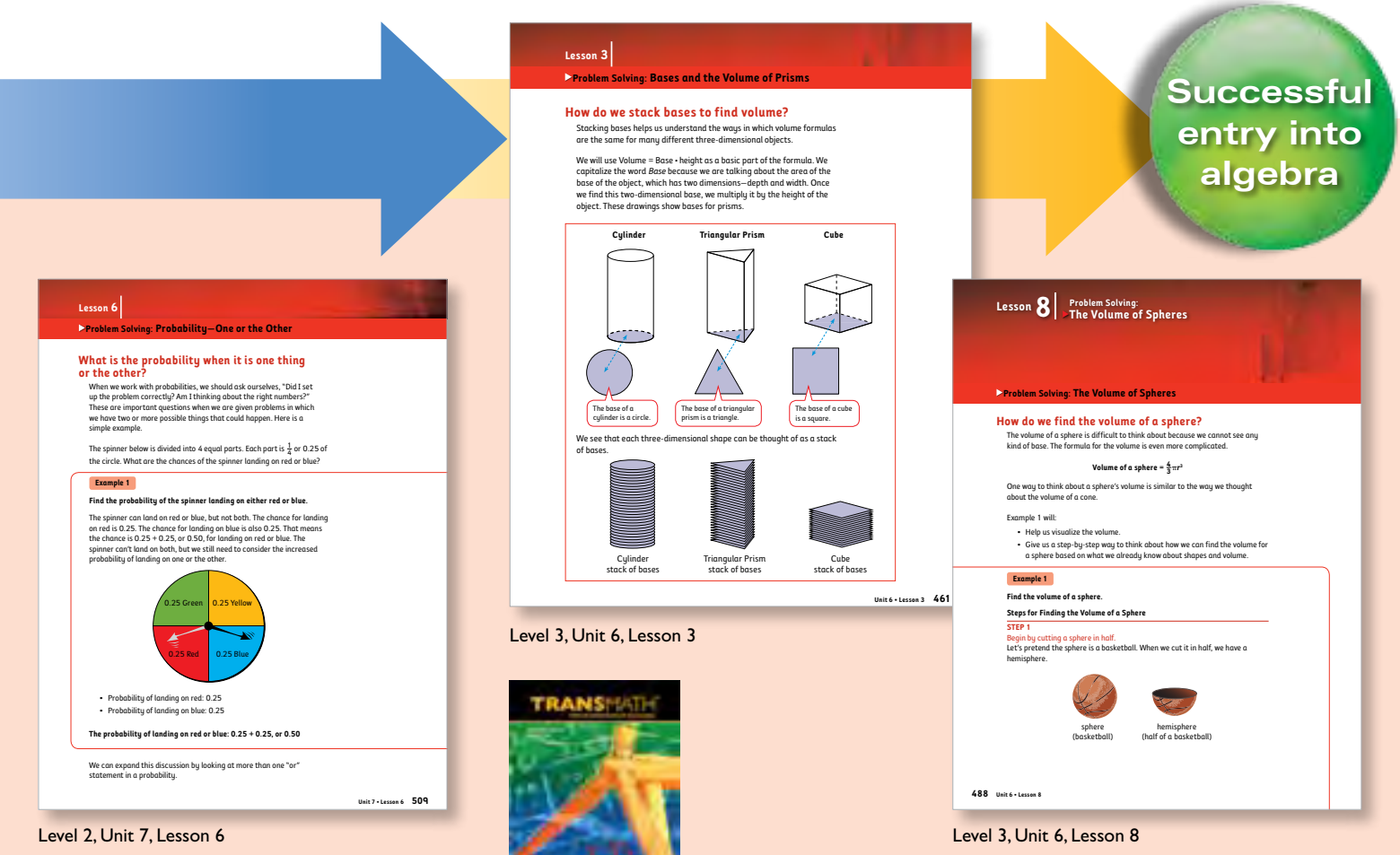
Developing Number Sense

Making Sense of Rational Numbers

The Problem-Solving strand encompasses:

- Work with data
- One-, two-, and three-dimensional objects
- Measurement tools
- Probability
- Proportional thinking
- Properties of shapes
- Angles, transversals, and geometric transformations

> For all content areas, conceptual understanding, computational fluency, and problem-solving skills are each essential and mutually reinforcing, influencing performance on such varied tasks as estimation, word problems, and computation.
>
> —National Mathematics Advisory Panel's Final Report, page 30



Level 2, Unit 7, Lesson 6

Level 3, Unit 6, Lesson 3

Level 3, Unit 6, Lesson 8

Successful entry into algebra

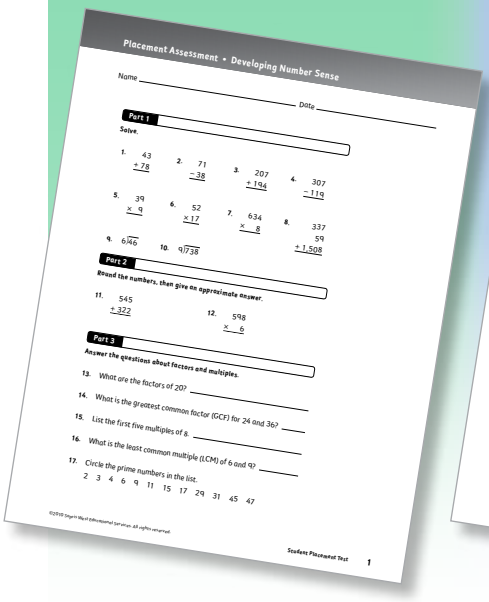
Understanding Algebraic Expressions

Placement

TransMath placement is based on students' skill levels, not grade levels. Students may place into one of these three entry points:

Entry Point 1

Developing Number Sense: For students showing the need for foundational number sense skills


Placement Assessment • Developing Number Sense

Name _____ Date _____

Part 1

Solve.

1. 43 + 78
2. 71 − 38
3. 207 + 194
4. 307 − 119
5. 39 × 9
6. 52 × 17
7. 634 × 8
8. 337 + 1,508
9. 6)46
10. 9)738

Part 2

Round the numbers, then give an approximate answer.

11. 545 + 322
12. 598 × 6

Part 3

Answer the questions about factors and multiples.

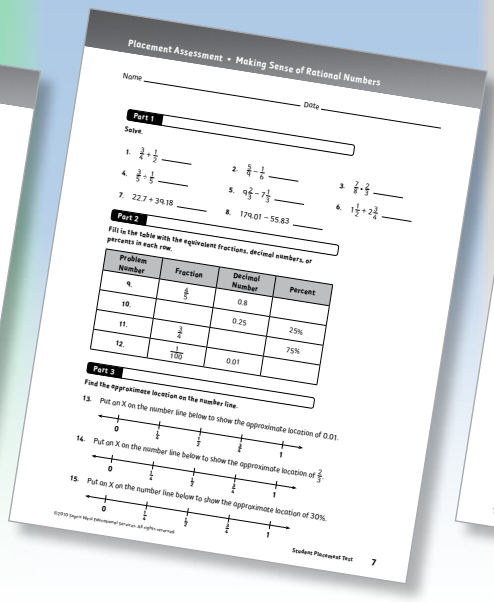
13. What are the factors of 20?
14. What is the greatest common factor (GCF) for 24 and 36?
15. List the first five multiples of 8.
16. What is the least common multiple (LCM) of 6 and 9?
17. Circle the prime numbers in the list.

2 3 4 6 9 11 15 17 29 31 45 47

Student Placement Test 1

Entry Point 2

Making Sense of Rational Numbers: For students showing proficiency in basic number sense skills but lacking the foundational skills for rational numbers


Placement Assessment • Making Sense of Rational Numbers

Name _____ Date _____

Part 1

Solve.

1. $\frac{3}{4} + \frac{1}{2}$
2. $\frac{5}{6} - \frac{1}{6}$
3. $\frac{7}{8} \cdot \frac{2}{3}$
4. $\frac{3}{5} \div \frac{6}{5}$
5. $9\frac{2}{3} - 7\frac{1}{3}$
6. $1\frac{1}{2} \cdot 2\frac{3}{4}$
7. 22.7 + 39.18
8. 174.01 − 55.83

Part 2

Fill in the table with the equivalent fractions, decimal numbers, or percents in each row.

Problem Number	Fraction	Decimal Number	Percent
9.	$\frac{4}{5}$		
10.		0.8	
11.	$\frac{3}{4}$	0.25	
12.			75%
	$\frac{1}{100}$	0.01	

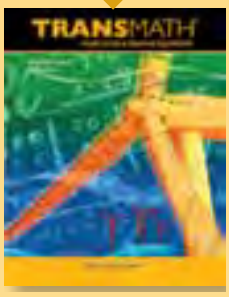
Part 3

Find the approximate location on the number line.

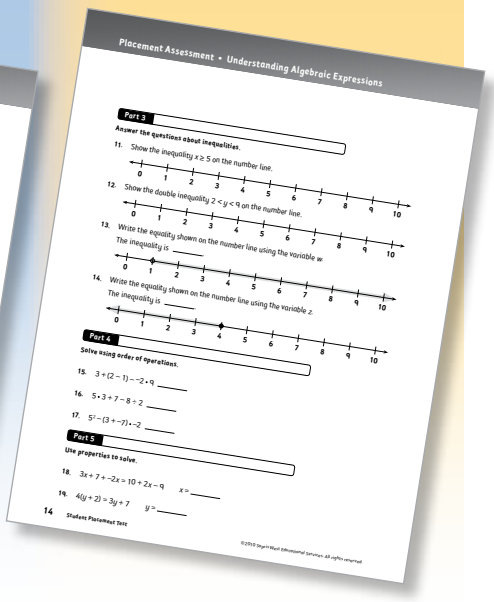
13. Put an X on the number line below to show the approximate location of 0.01.
14. Put an X on the number line below to show the approximate location of $\frac{2}{3}$.
15. Put an X on the number line below to show the approximate location of 30%.

Student Placement Test 7

Entry Point 3



Understanding Algebraic Expressions: For students showing proficiency with rational numbers but lacking the foundational skills for prealgebra


Placement Assessment • Understanding Algebraic Expressions

Part 3

Answer the questions about inequalities.

11. Show the inequality $x \geq 5$ on the number line.
12. Show the double inequality $2 < y < 9$ on the number line.
13. Write the equality shown on the number line using the variable w.
The inequality is _____
14. Write the equality shown on the number line using the variable z.
The inequality is _____

Part 4

Solve using order of operations.

15. $3 + (2 - 1) - -2 \cdot 9$
16. $5 \cdot 3 + 7 - 8 \div 2$
17. $5^2 - (3 + -7) \cdot -2$

Part 5

Use properties to solve.

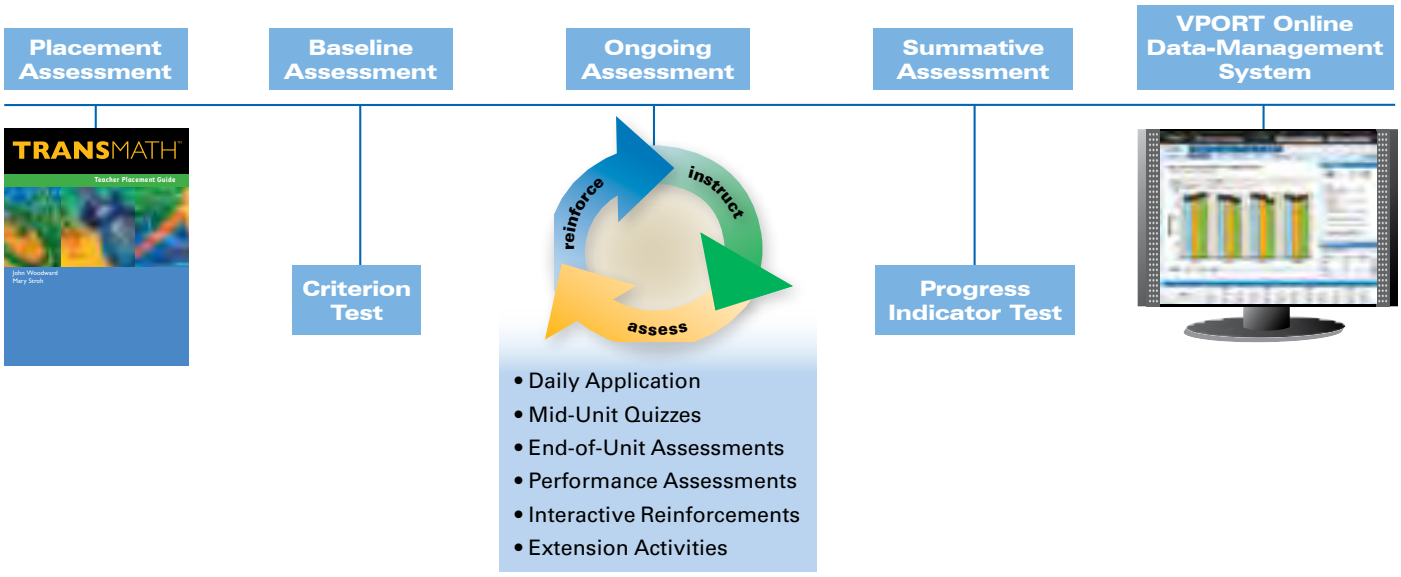
18. $3x + 7 + -2x = 10 + 2x - 9$ $x =$ _____
19. $4(y + 2) = 3y + 7$ $y =$ _____

14

Student Placement Test

The TRANSMATH® Assessment System

This comprehensive assessment system provides teachers with the measures they need to accurately place students into the curriculum and to monitor their progress though the curriculum. It furnishes the teacher with the data necessary to inform instruction to ensure each student meets his or her goals.



Placement

Based on students' demonstrated understanding of key mathematics concepts and skills, data from the *TransMath* placement tests accurately place students at one of the three entry points of the curriculum.

Baseline Assessments

Administered at the beginning of each book level, the Baseline Assessment establishes a starting point for measuring student's progress through the curriculum.

Ongoing Assessments

Regular assessment of student mastery of the concepts and skills taught in the curriculum ensures that teachers can adjust pacing or instruction to meet the needs of individual students.

Summative Assessments

Given at the end of each book level, the Progress Indicators measure the critical skills of mathematics through curriculum-based measures. Comparing Progress Indicators to the Baseline Assessments accurately tracks student's progress through the curriculum.

VPORT Online Data-Management System

This user-friendly database allows teachers and administrators to record, track, and report student test results. Reports can be generated at the individual, class, building, and district levels.

Ongoing Assessment

Informal Assessment

TransMath provides teachers with numerous opportunities to assess student knowledge as concepts and skills are being developed.

Check for Understanding

Check for Understandings informally assess student learning and prescribe solutions for immediate support. Embedded engagement strategies provide varied opportunities for student communication, engaging all students. Check for Understandings occur after the modeling of each major lesson concept.

Apply Skills

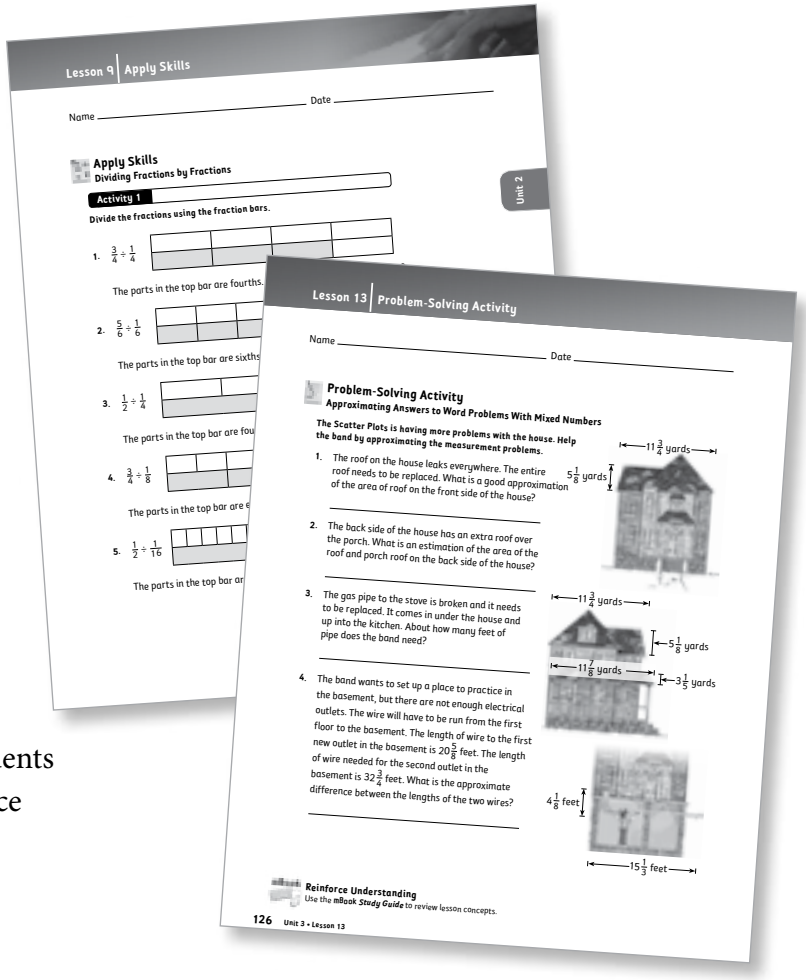
Apply Skills activities allow students to apply the skills they learned in the Building Number Concepts section of each lesson.

Problem-Solving Activity

Problem-Solving activities allow students to apply knowledge of the concepts from the Problem Solving section of each lesson.

Homework

Daily homework activities allow students to practice new concepts and reinforce prior learning.



Formal Assessment

Each unit of *TransMath* contains multiple methods to assess student's reasoning and ability to communicate ideas. Each type of assessment serves a different purpose.

Quiz

Quizzes occur every five lessons to give teachers important feedback on student progress. Results inform instruction for differentiation days and subsequent lessons.

Students	Assess	Differentiate
	Day 1	Day 2
All	Quiz X Form A	Review Quiz
Scored 80% or Above		Extension
Scored Below 80%		Reinforcement

Day 1: Students take quiz during the second half of the lesson.

Day 2: Students work on differentiation activities based on their performance on the quiz.

End-of-Unit Assessment

The End-of-Unit Assessment measures student mastery of skills taught in the unit. Targeted support is then provided on differentiation days to reinforce difficult skills, to help students achieve mastery.

Performance Assessment

The Performance Assessment measures student's ability to reason and communicate. Students practice applying unit concepts in the context of a high-stakes test.

Day 1: Students take End-of-Unit Assessment to determine differentiation needs.

Day 4: Students take Performance Assessments.

Students	Assess	Differentiate		Assess
	Day 1	Day 2	Day 3	Day 4
All	End-of-Unit Assessment	Review Test		Performance Assessment Begin new unit
Scored 80% or Above		Extension	Extension	
Scored Below 80%		Reinforcement	Retest End-of-Unit Assessment	

Day 2: Students work on differentiation activities based on their performance on the assessment.

Day 3: Students continue with differentiation activities or retest.

Differentiation Informed by Data

TransMath offers multiple opportunities to assess, reinforce, and differentiate instruction:

VPORT Online Data-Management System

With one simple log-in, teachers can access student data to inform differentiation.

mBook Teacher Edition

The *mBook Teacher Edition*, accessed through VPORT, contains a multitude of online resources to access, reinforce, and differentiate instruction:

- **Teacher-Talk Tutorials** reinforce lesson concepts using narrated, animated visual models that make the concept concrete for the student
- **Interactive Click-Thru** slideshow presentations use visual models to concretely develop concepts
- **Whiteboard Activities** to motivate and engage students
- **On Track! Extension Activities**—multistep word problems designed for small groups to prepare students for high-stakes tests
- **Interactive Reinforcement Exercises**—online, interactive, multiple-choice activities that provide immediate feedback
- **Form B Retests** for Quizzes and End-of-Unit Assessments are available for downloading

mBook Study Guide for students provides online access to:

- The entire ***Student Text*** to review missed concepts
- **Teacher-Talk Tutorials** to reinforce difficult concepts
- **Interactive Reinforcement Exercises** to review, reinforce, and practice missed concepts

Overhead Meanipulative Set provides opportunities for multisensory modeling of missed concepts.

DAY 1

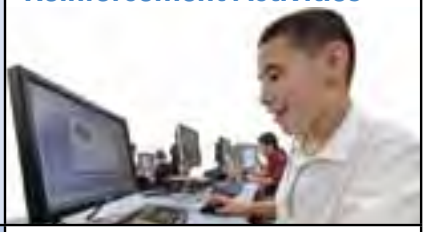
After administering End-of-Unit Assessment, determine differentiation by assessing student data.

- Administer assessment
- Enter scores
- Identify differentiation needs
- Establish small groups

DAY 2

Assign differentiation strategies depending on student needs.

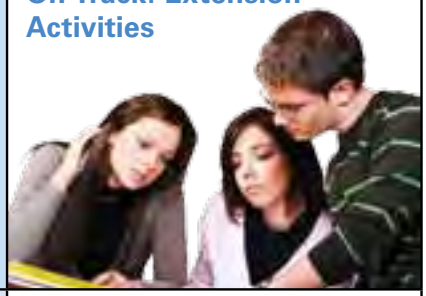
Student Score

80% or higher	On Track! Extension Activities 
Between 60–80%	Computer Station *mBook Study Guide* or Reinforcement Activities 
Below 60%	Teacher Reteach 

DAY 3

Students working on On Track! Activities continue while students who scored below 80% are retested.

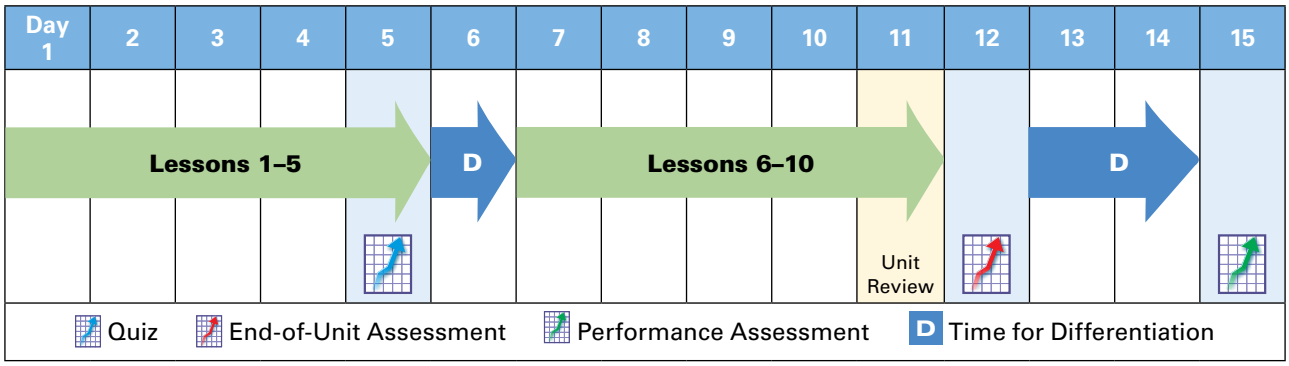
Student Score

80% or higher	On Track! Extension Activities 
Below 80%	Retest students that scored below 80% using End-of-Unit Assessment Form B 

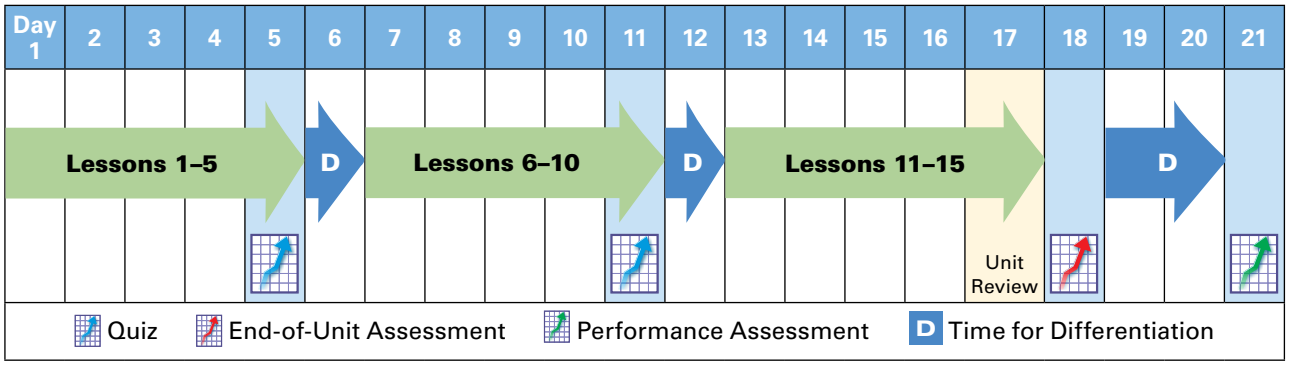
Pacing Guide at the Unit Level

Units are either 10 lessons or 15 lessons in length. *TransMath* lessons are designed for 50–60 minute lesson blocks per day.

10-Lesson Unit



15-Lesson Unit

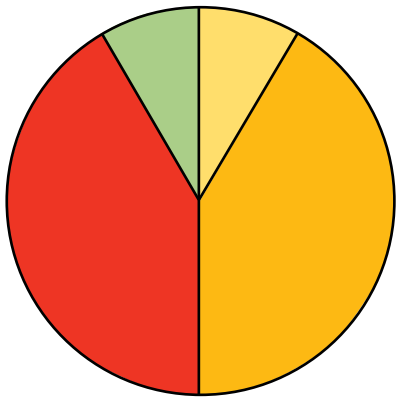


Pacing Guide at the Lesson Level

Every lesson has a predictable lesson structure. Although *TransMath* lessons are designed for 50–60 minute lesson blocks per day, adjustments can be made to fit multiple scheduling needs.

Lesson Structure	Approximate Time for a 50–60 Minute Lesson	
Skills Maintenance	Starts each lesson with distributed practice warm-ups	4–5 minutes
Building Number Concepts	Develops conceptual understanding of number, operation, and prealgebra topics through: • Teacher Modeling • Engagement Strategies • Extensive Use of Visual Models • Apply Skills Activities	20–25 minutes
Problem Solving	Develops conceptual understanding of geometry, measurement, data, and probability through: • Teacher Modeling • Engagement Strategies • Extensive Use of Visual Models • Rich, Grade-Level, Problem-Solving Activities	20–25 minutes
Homework	Provides daily, independent practice with lesson concepts and skills as well as earlier learned skills for continued distributed practice. Assignments take 15–20 minutes outside class.	5 minutes—Assign Homework

50–60 Minute Lesson



Daily Support for Teachers Through Technology ...

The *TransMath mBook Teacher Edition* provides powerful online resources that support teachers in the successful, daily implementation of this comprehensive curriculum.

Eliminates the Need for Multiple Books

- Provides access to the complete online *Teacher Guides*
- Online links to all student components

Supports Instruction Through Online Tools

- Allows for review of objectives and lesson plans
- Helps teachers prepare for lessons with Teacher-Talk Tutorials, which provide essential knowledge of math concepts to aid teacher modeling
- Provides Click-Thru slideshow presentations used to engage students when modeling initial lesson concepts
- Interactive Whiteboard presentations to promote student participation

Individualizes Instruction With Differentiation Tools

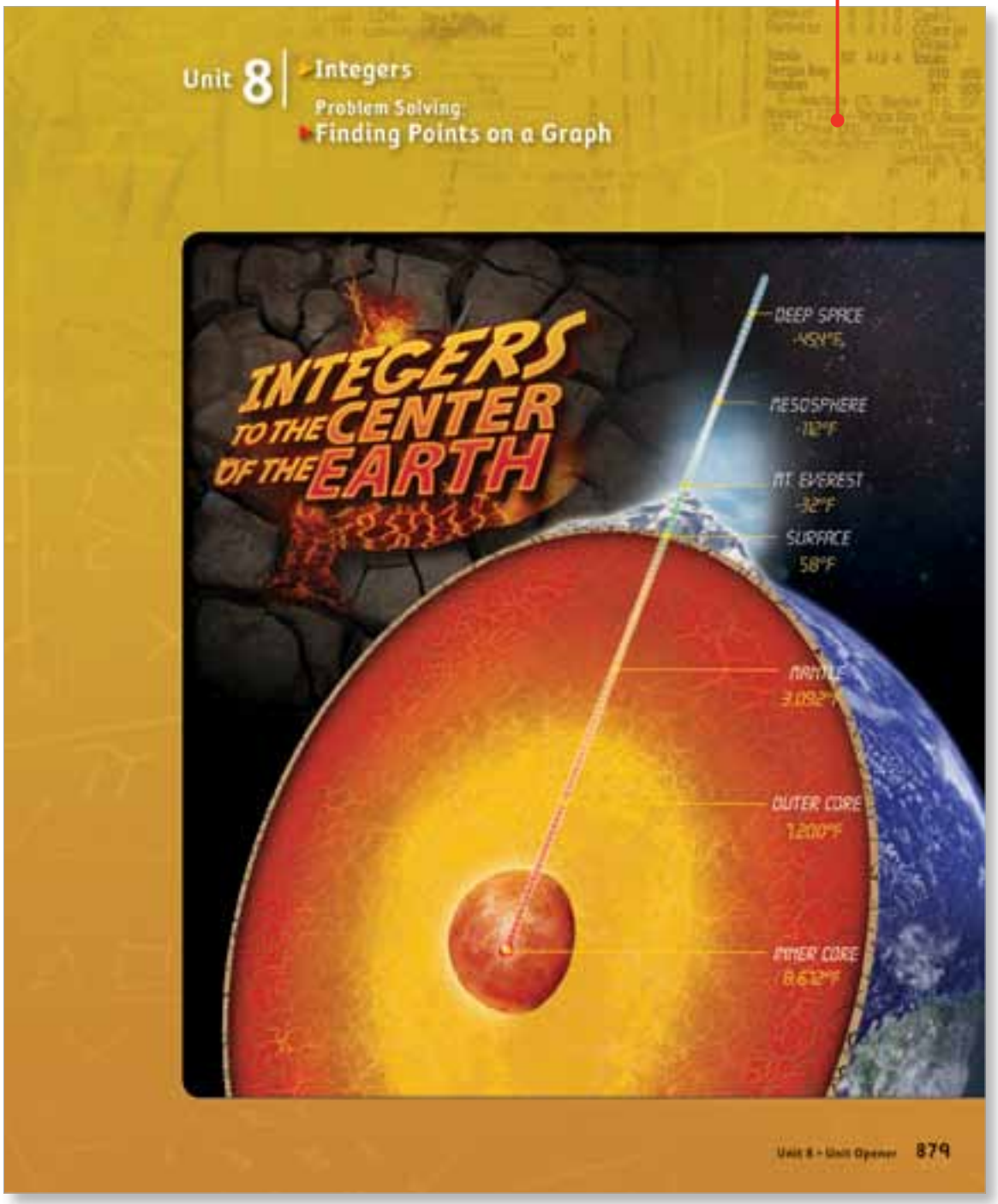
- Reinforces concepts and skills with Interactive Reinforcement Exercises
- Extends concepts and skills with On Track! Extension Activities
- Provides alternate form Quizzes and End-of-Unit Assessments for retesting
- Provides printable Interactive Text pages for reinforcement opportunities

Provides Access to State-Specific Tools

- Correlations to state standards
- Correlations to the NCTM Curriculum Focal Points

... and Through Print.

Thought-provoking, engaging unit openers



Making Sense of Rational Numbers, Teacher Guide, Unit 8

Teacher Support

***Student Text* pages** are showcased at point of use.

The unit opener **background information** provides real-world context to promote discussion.

Unit 8

Unit Opener: Background Information

Read through the Unit Opener of *Student Text*, pages 535–536, with students, then share some additional facts about the Earth's temperature.

- Heat stored in the Earth constantly rises to the surface, generally in hot springs. People have used this "magic water" throughout history. The Romans used it not only for relaxing soaks but also to heat buildings. The Maori of New Zealand used it for cooking, as did Native Americans. Since the 1960s, almost 200,000 homes in France have been heated using geothermal water.
- The coldest temperature ever recorded on Earth was at Vostok, Antarctica, on July 21, 1983. The thermometer read −129°F.
- When scientists calculate the average surface temperature of the Earth, they gather readings from many places—the hottest and the coldest, dry land and oceans. For the entire twentieth century, the average temperature of Earth was 58°F, but in recent years, it has risen to 59°F. Sub-Antarctic temperatures are rising faster than almost any others.

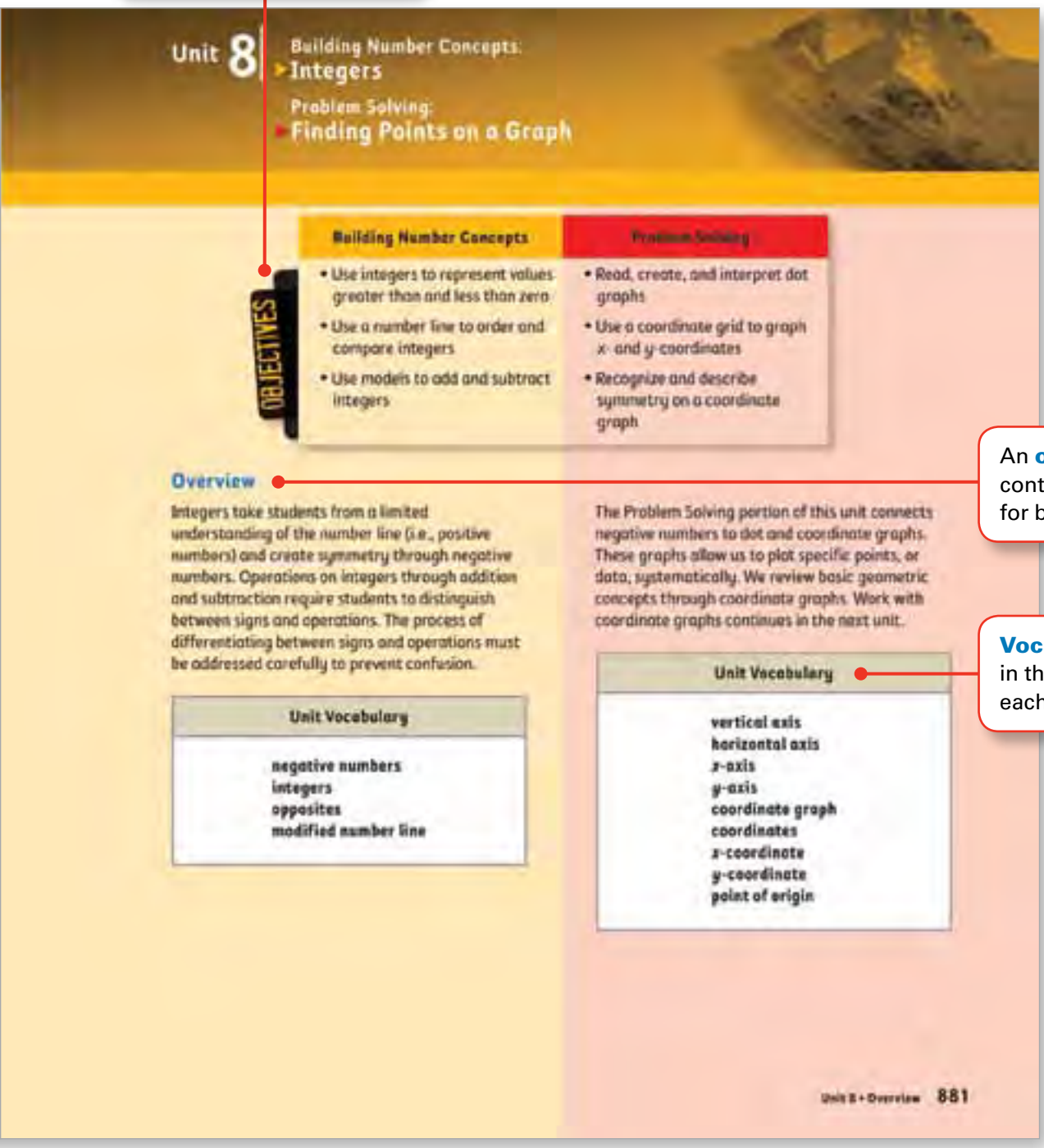
535

536

880 Unit 8 • Unit Opener

Making Sense of Rational Numbers, Teacher Guide, Unit 8

Unit objectives are identified for both strands.



Unit 8

Building Number Concepts:
Integers

Problem Solving:
Finding Points on a Graph

OBJECTIVES

Building Number Concepts	Problem Solving
• Use integers to represent values greater than and less than zero • Use a number line to order and compare integers • Use models to add and subtract integers	• Read, create, and interpret dot graphs • Use a coordinate grid to graph *x*- and *y*-coordinates • Recognize and describe symmetry on a coordinate graph

Overview

Integers take students from a limited understanding of the number line (i.e., positive numbers) and create symmetry through negative numbers. Operations on integers through addition and subtraction require students to distinguish between signs and operations. The process of differentiating between signs and operations must be addressed carefully to prevent confusion.

The Problem Solving portion of this unit connects negative numbers to dot and coordinate graphs. These graphs allow us to plot specific points, or data, systematically. We review basic geometric concepts through coordinate graphs. Work with coordinate graphs continues in the next unit.

An **overview** of unit content is provided for both strands.

Unit Vocabulary
negative numbers integers opposites modified number line

Unit Vocabulary
vertical axis horizontal axis *x*-axis *y*-axis coordinate graph coordinates *x*-coordinate *y*-coordinate point of origin

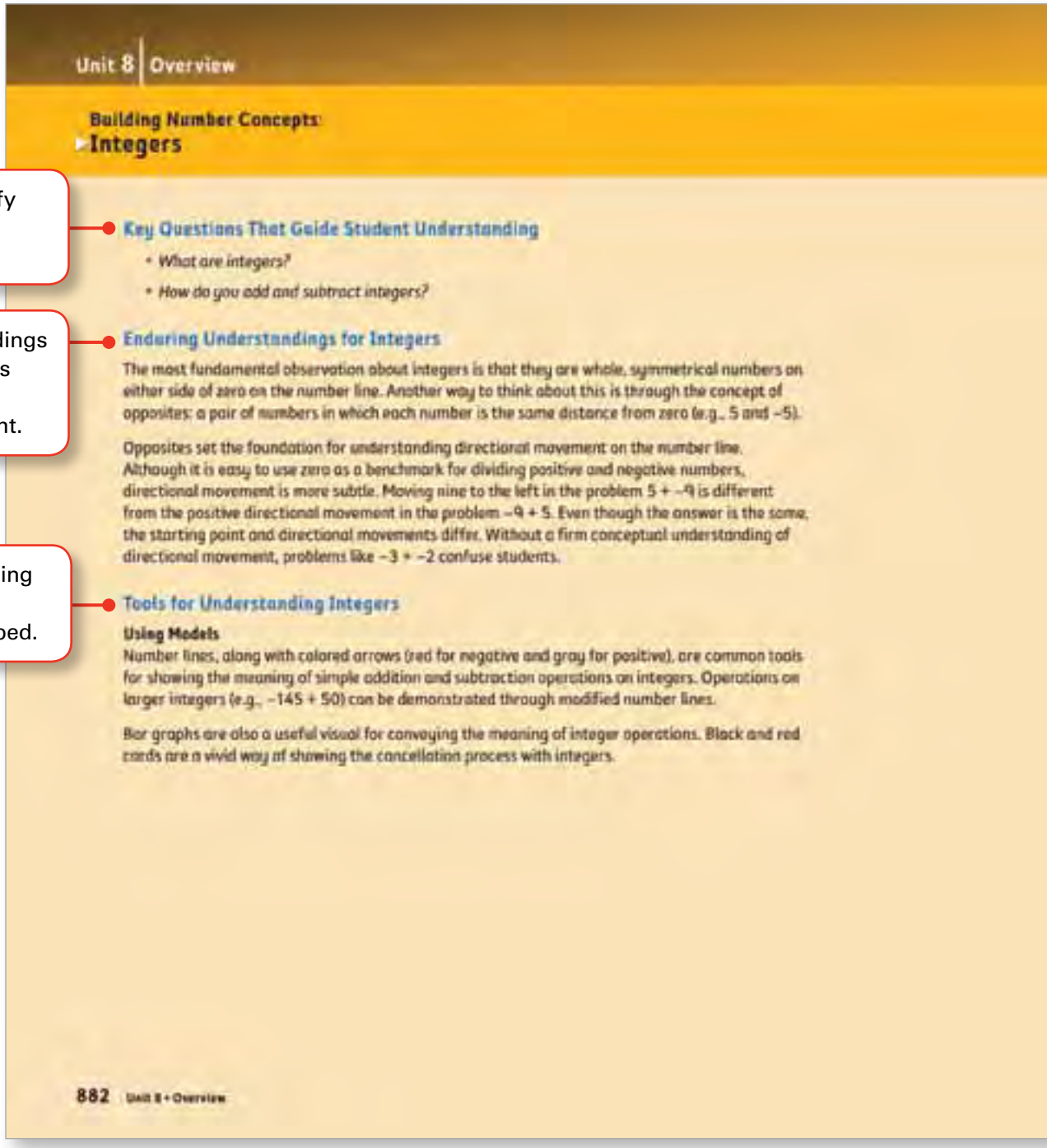
Vocabulary introduced in the unit is listed for each strand.

Unit 8 • Overview 881

Key questions identify **what** students need to know.

Enduring understandings explain **why** students need to know the concepts being taught.

Tools for understanding explain **how** the concepts are developed.



Unit 8 | Overview

Building Number Concepts:
Integers

Key Questions That Guide Student Understanding

- *What are integers?*
- *How do you add and subtract integers?*

Enduring Understandings for Integers

The most fundamental observation about integers is that they are whole, symmetrical numbers on either side of zero on the number line. Another way to think about this is through the concept of opposites: a pair of numbers in which each number is the same distance from zero (e.g., 5 and −5).

Opposites set the foundation for understanding directional movement on the number line. Although it is easy to use zero as a benchmark for dividing positive and negative numbers, directional movement is more subtle. Moving nine to the left in the problem 5 + −9 is different from the positive directional movement in the problem −9 + 5. Even though the answer is the same, the starting point and directional movements differ. Without a firm conceptual understanding of directional movement, problems like −3 + −2 confuse students.

Tools for Understanding Integers

Using Models

Number lines, along with colored arrows (red for negative and gray for positive), are common tools for showing the meaning of simple addition and subtraction operations on integers. Operations on larger integers (e.g., −145 + 50) can be demonstrated through modified number lines.

Bar graphs are also a useful visual for conveying the meaning of integer operations. Black and red cards are a vivid way of showing the cancellation process with integers.

882 Unit 8 • Overview

Making Sense of Rational Numbers, Teacher Guide, Unit 8

Problem Solving:
Finding Points on a Graph

Key Questions That Guide Student Understanding

- *Why do we need dot graphs when we have line graphs?*
- *What is so important about the structure of a coordinate graph?*

Enduring Understandings for Finding Points on a Graph

Dot graphs allow us to address two central concepts: 1) data on a dot graph are discrete data, and 2) dot graphs provide an informal introduction to coordinate graphs, which can be confusing because of the use of integer coordinates.

Dot graphs can be hard to interpret, which is why they are rarely used in graphing data. Nonetheless, dot graphs remind students that movement from one point to the next in line graphs implies continuous change. That is why it is so easy for students to misuse line graphs. The difference between discrete and continuous data is a major distinction in mathematics.

However, coordinate graphs are central to secondary mathematics. The location of *x*- and *y*-coordinates entails discrete data. The connection to negative numbers, the other focus of this unit, is timely because it enables students to see how integers are used in a visual display of data.

Tools for Understanding Finding Points on a Graph

Using Graphs
Dot graphs help students plot integers in an unusual way. More importantly, the graph helps students see the difference between discrete and continuous data. Dot graphs set the stage for coordinate graphs, which become another framework for using positive and negative integers.

Key questions identify **what** students need to know.

Enduring understandings explain **why** students need to know the concepts being taught.

Tools for understanding explain **how** the concepts are developed.

Lesson objectives are identified for each concept.

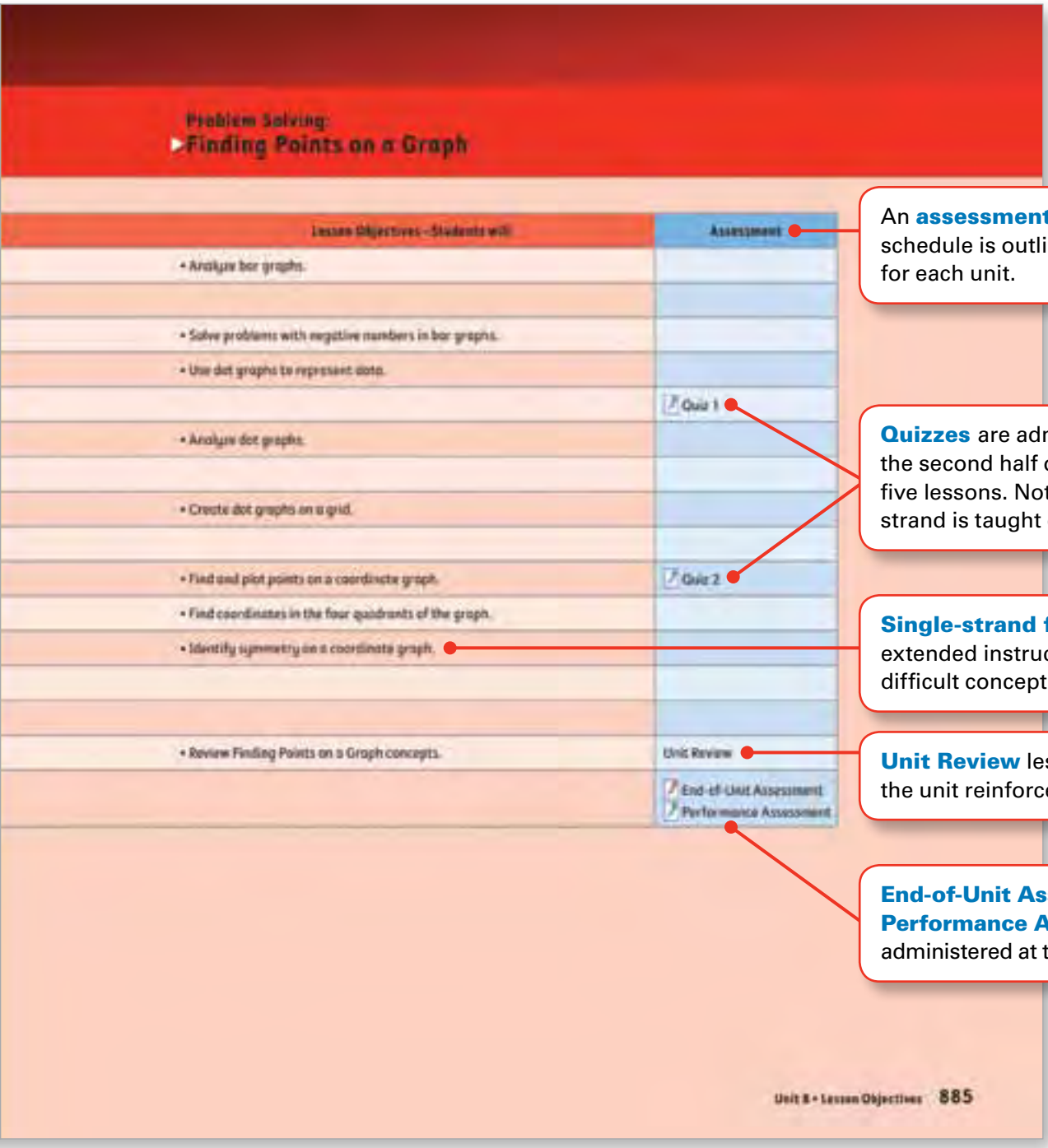
Unit 8 | Lesson Objectives

Building Number Concepts:
Integers

Lesson	Lesson Objectives–Students will:
1	• Analyze negative and opposite numbers.
2	• Represent change on the number line.
3	• Identify different contexts for using integers.
4	• Compare integers using number lines.
5	• Add integers.
6	• Perform addition with integers using black and red cards.
7	• Subtract positive integers.
8	• Subtract negative integers.
9	• Add and subtract positive and negative integers.
10	
11	
12	
13	• Solve problems with large integers.
14	• Study the difference between negative numbers and negative integers.
15	• Review Integers concepts.
Unit Assessments	End-of-Unit Assessment Performance Assessment

884 Unit 8 • Lesson Objectives

Making Sense of Rational Numbers, Teacher Guide, Unit 8



Problem Solving:
Finding Points on a Graph

Lesson Objectives—Students will	Assessment
• Analyze bar graphs.	
• Solve problems with negative numbers in bar graphs.	
• Use dot graphs to represent data.	
	Quiz 1
• Analyze dot graphs.	
• Create dot graphs on a grid.	
• Find and plot points on a coordinate graph.	Quiz 2
• Find coordinates in the four quadrants of the graph.	
• Identify symmetry on a coordinate graph.	
• Review Finding Points on a Graph concepts.	Unit Review
	End-of-Unit Assessment Performance Assessment

Unit 8 • Lesson Objectives 885

An **assessment** schedule is outlined for each unit.

Quizzes are administered during the second half of the lesson—every five lessons. Note that only one strand is taught on these days.

Single-strand focus provides extended instructional time for difficult concepts.

Unit Review lessons at the end of the unit reinforce student learning.

End-of-Unit Assessments and **Performance Assessments** are administered at the end of every unit.

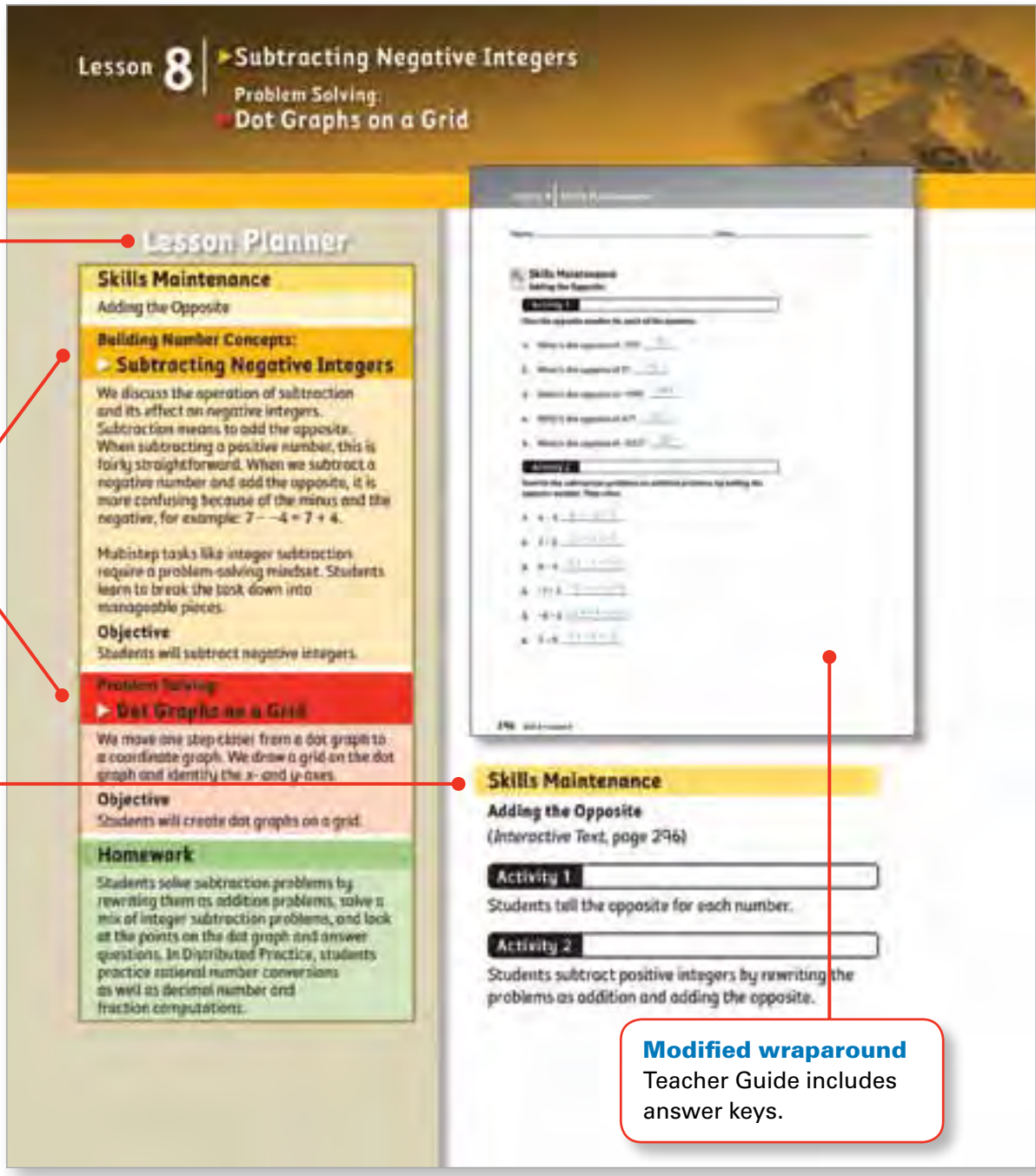
Lesson Plans That Drive Instruction

Provide Everything Needed for Successful Implementation

Every lesson begins with an at-a-glance **Lesson Planner.**

Dual topics provide a balance of conceptual understanding and problem-solving applications.

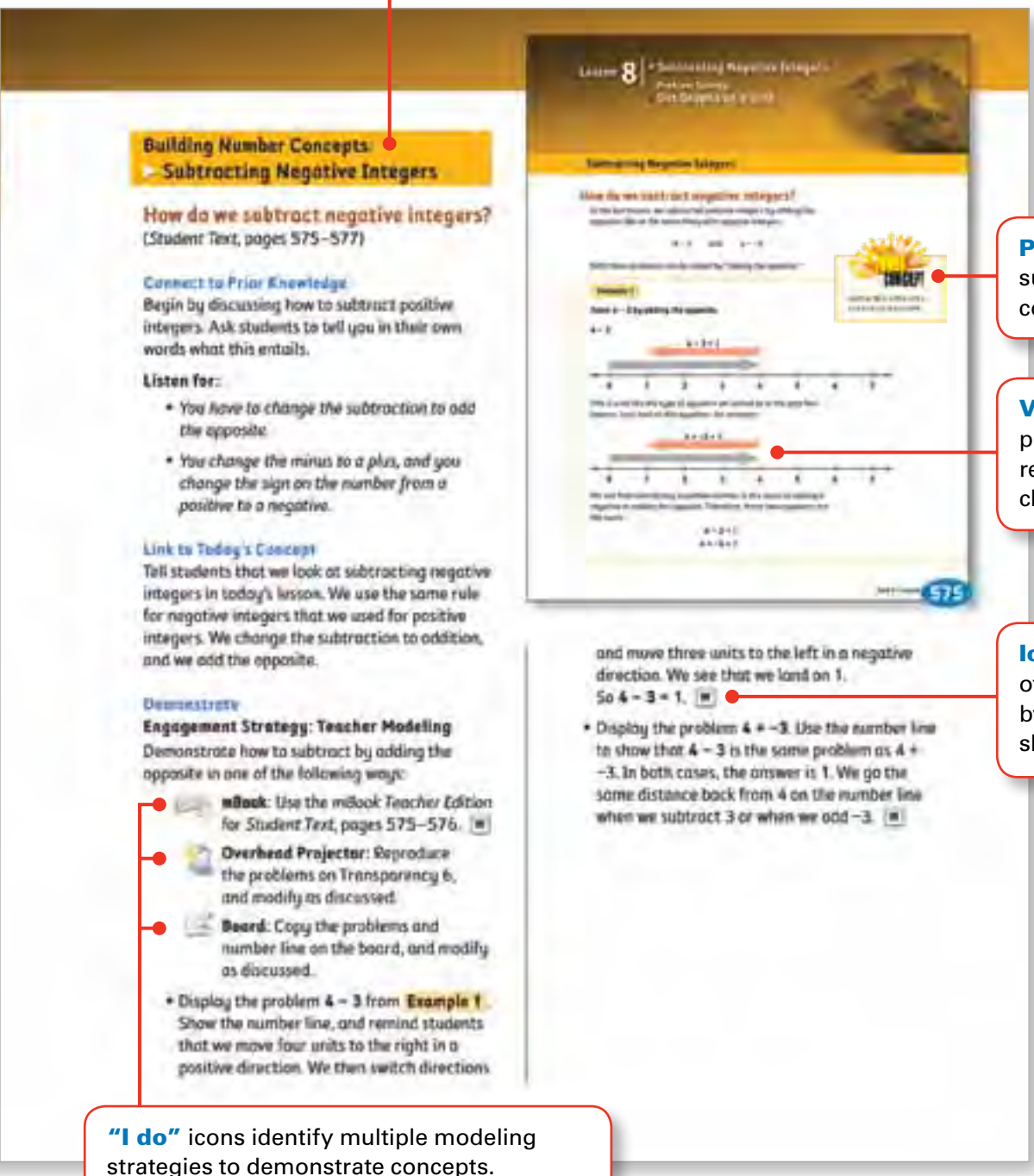
Skills Maintenance distributes practice across lessons and takes four to five minutes at the beginning of class.



Modified wraparound Teacher Guide includes answer keys.

Making Sense of Rational Numbers, Teacher Guide, Unit 8, Lesson 8

The **Building Number Concepts** strand takes 20 to 25 minutes of the lesson, focusing on development of the concept with rich practice opportunities.



Building Number Concepts
Subtracting Negative Integers

How do we subtract negative integers?
(Student Text, pages 575–577)

Connect to Prior Knowledge
Begin by discussing how to subtract positive integers. Ask students to tell you in their own words what this entails.

Listen for:

- *You have to change the subtraction to add the opposite.*
- *You change the minus to a plus, and you change the sign on the number from a positive to a negative.*

Link to Today's Concept
Tell students that we look at subtracting negative integers in today's lesson. We use the same rule for negative integers that we used for positive integers. We change the subtraction to addition, and we add the opposite.

Demonstrate
Engagement Strategy: Teacher Modeling
Demonstrate how to subtract by adding the opposite in one of the following ways:

- **mBook**: Use the *mBook Teacher Edition* for *Student Text*, pages 575–576.
- **Overhead Projector**: Reproduce the problems on Transparency 6, and modify as discussed.
- **Board**: Copy the problems and number line on the board, and modify as discussed.

- Display the problem **4 – 3** from **Example 1**. Show the number line, and remind students that we move four units to the right in a positive direction. We then switch directions and move three units to the left in a negative direction. We see that we land on 1. So **4 – 3 = 1**.
- Display the problem **4 + –3**. Use the number line to show that **4 – 3** is the same problem as **4 + –3**. In both cases, the answer is 1. We go the same distance back from 4 on the number line when we subtract 3 or when we add –3.

Power Concepts summarize critical concepts for students.

Visual models provide a concrete representation of challenging concepts.

Icons identify the parts of instruction supported by the Click-Thru slideshow presentation.

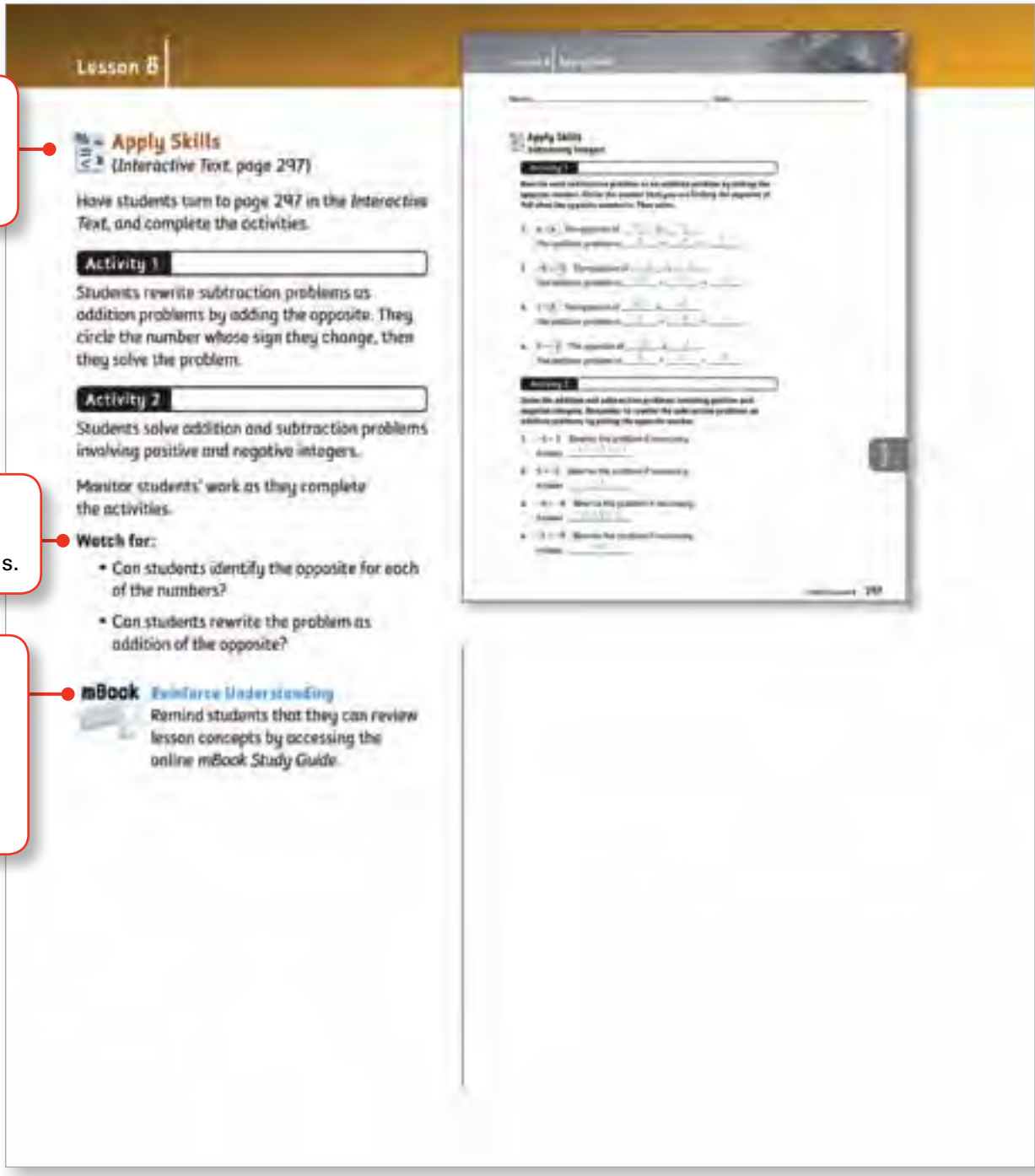
"I do" icons identify multiple modeling strategies to demonstrate concepts.

Teacher Support

"We do" practice provides a safe environment to monitor student understanding.

Watch for: questions help teachers recognize common misconceptions.

Students can review lesson concepts by accessing the online ***mBook Study Guide,*** which includes Teacher-Talk Tutorials.



Lesson 8

Apply Skills

Interactive Text, page 297)

Have students turn to page 297 in the *Interactive Text*, and complete the activities.

Activity 1

Students rewrite subtraction problems as addition problems by adding the opposite. They circle the number whose sign they change, then they solve the problem.

Activity 2

Students solve addition and subtraction problems involving positive and negative integers.

Monitor students' work as they complete the activities.

Watch for:

- Can students identify the opposite for each of the numbers?
- Can students rewrite the problem as addition of the opposite?

mBook Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Making Sense of Rational Numbers, Teacher Guide, Unit 8, Lesson 8

The **Problem Solving** strand takes 20 to 25 minutes of the lesson for instruction and application of topics in a problem-solving environment.

Problem Solving

Dot Graphs on a Grid

How is elevation presented using negative integers?

(*Student Text*, pages 578–579)

Demonstrate

- Have students look at page 578 of the *Student Text*. Have students read the description of elevation and various locations, such as Mt. Whitney and Death Valley, and their relationship to sea level.
- Remind students that we discussed elevation and sea level in Lesson 3 when we looked at common uses of integers. This is a good real-world context for seeing positive and negative numbers.
- Next have students read through the scenario that sets the stage for today's activity. It is about two people hiking on a trail in Death Valley. The elevation of the trail goes above and below sea level at various points.
- Have students look at the map and the table. The map shows the different points of the trail. The table shows the elevation.
- Point out to students the *x*-axis on the graph on the following page represents trail points A–J. The elevation is labeled *y*, which is the vertical axis on the dot graph. We use these *x*- and *y*-values to find the location of each trail point on the dot graph.

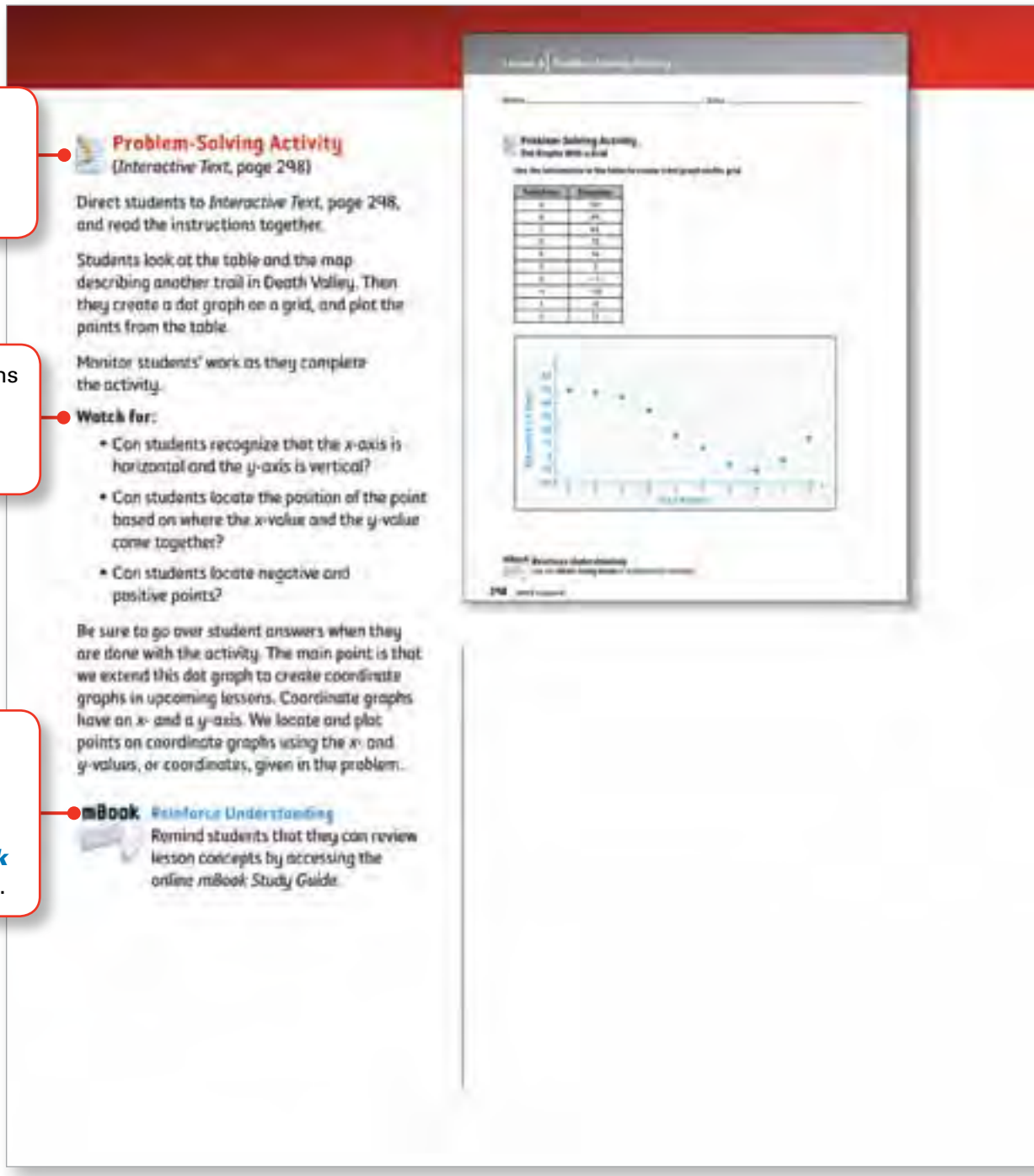
Embedded engagement strategies provide varied opportunities for student discussion.

Teacher Support

"We do" in-class practice provides a safe environment to monitor student understanding.

Watch for: questions help teachers recognize common misconceptions.

Students can review lesson concepts by accessing the online ***mBook Study Guide***.



Problem-Solving Activity
(Interactive Text, page 298)

Direct students to *Interactive Text*, page 298, and read the instructions together.

Students look at the table and the map describing another trail in Death Valley. Then they create a dot graph on a grid, and plot the points from the table.

Monitor students' work as they complete the activity.

Watch for:

- Can students recognize that the *x*-axis is horizontal and the *y*-axis is vertical?
- Can students locate the position of the point based on where the *x*-value and the *y*-value come together?
- Can students locate negative and positive points?

Be sure to go over student answers when they are done with the activity. The main point is that we extend this dot graph to create coordinate graphs in upcoming lessons. Coordinate graphs have an *x*- and a *y*-axis. We locate and plot points on coordinate graphs using the *x*- and *y*-values, or coordinates, given in the problem.

mBook Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Making Sense of Rational Numbers, Teacher Guide, Unit 8, Lesson 8

"You do" homework is designed to provide 15–20 minutes of out-of-class practice of lesson concepts and skills each day.

Lesson 8

Homework

Go over the instructions on pages 580–581 of the *Student Text* for each part of the homework.

Activity 1

Students rewrite subtraction problems as addition problems by adding the opposite. Then they solve the problems.

Activity 2

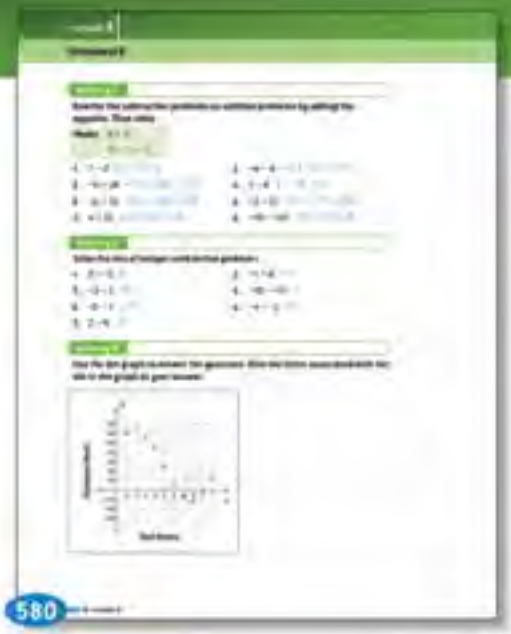
Students solve a mix of integer subtraction problems where they are subtracting both positive and negative integers.

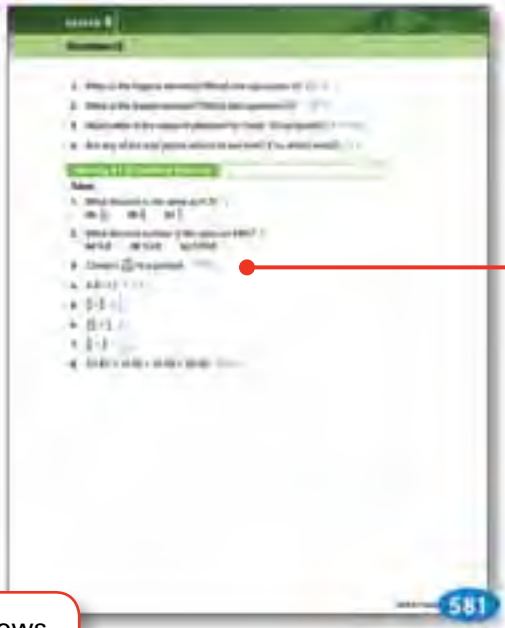
Activity 3

Students look at the points on the dot graph and answer questions.

Activity 4 – Distributed Practice

Students practice rational number conversions, as well as decimal number and fraction computations, to become more proficient in these skills.





Homework answers are provided at point of use.

Distributed Practice allows for continuous practice of previously learned concepts and skills.

Daily Support for Students and Parents Through Technology ...

TransMath is accessible to both students and parents—anytime, anywhere—through the *TransMath mBook Study Guide*. This online system provides students and parents with:

- **The complete *Student Text***
  - Access to all lesson pages
  - Homework pages online—no need to take books home
- **Online Reinforcement**
  - Teacher-Talk Tutorials narrate and animate initial lesson concepts using concrete visual models to aid conceptual understanding
  - Interactive Reinforcement Exercises provide immediate corrective feedback and track student progress
  - Anytime, anywhere access to lesson pages to review missed content

***VmathLive* is the interactive, online format that supports *TransMath* lessons and:**

- Provides computational practice and tutorials of key math concepts
- Provides self-paced learning practice in a computer-based environment
- Reinforces essential math skills and strategies
- Provides real-time math skills competitions with learners around the world

Teacher-Talk Tutorial from *TransMath mBook Study Guide*, Unit 1, Lesson 1

... and Through Print.

Engaging, **thought-provoking** openers start each unit.

Provides Multiple Models and Strategies for Understanding Lesson Concepts.

Daily questioning tecÚiques establish prior knowledge and build foundational understanding.

Lesson 11 ▸Division With Decimal Numbers

Problem Solving: ▸Understanding the Area of a Circle

▸Division With Decimal Numbers

How do we show decimal number division using an area model?

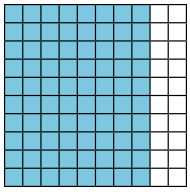
We use the 100-square grid model to help us understand what is going on when we divide with decimal numbers. The 100-square grid is an area model, and it is what we used to show multiplication. Here is a simple division problem.

0.8 ÷ 0.2

We begin by converting 0.8 and 0.2 to their equivalents 0.80 and 0.20, because we are using a 100-square grid.

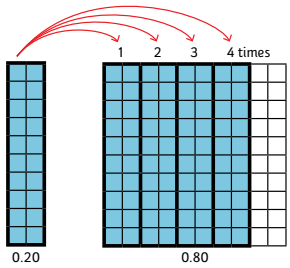
0.80 ÷ 0.20

Next, we shade the portion of the grid equivalent to 0.80.



0.80 is shaded

Then we show what 0.20 looks like. It will divide, or break up, the 0.80. As we see in the example, it breaks up 0.80 exactly four times.



The decimal number 0.20 divides or breaks up 0.80 four times.

That means that 0.80 ÷ 0.20 = 4.

What happens when we divide a small decimal number like 0.05 into a number like 0.30? We see in Example 1 that we can use the same process. This time we have to think of 0.05 as 5 out of 100. That is the number dividing, or breaking up, 0.30.

Visual models take the place of extensive text explanations and provide engaging opportunities for students to build conceptual understanding.

Making Sense of Rational Numbers, Student Text, Unit 5, Lesson 11

> Studies that included visual representations along with the other components of explicit instruction tended to produce significant positive effects.
>
> —*National Mathematics Advisory Panel's Final Report*, page 48

Daily **homework** provides independent practice to solidify key skills.

Lessons provide meaningful applications of **problem-solving strategies** to prepare for higher-level mathematical thinking.

Lesson 11

▶Problem Solving: **Understanding the Area of a Circle**

How can we make sense of the area formula for a circle?

We've learned the formula for the area of a circle. It looks like this:

Area of a circle = $\pi \cdot r^2$

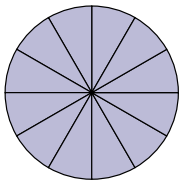
What does each symbol mean?

Symbol	Meaning
π	This is the symbol for pi.
r	This is the symbol for radius.
r^2	This is the same thing as radius • radius or $r \cdot r$.

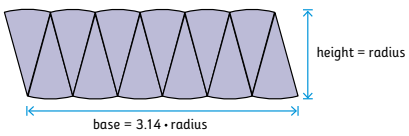
The formula for the area of a circle is 3.14 • radius • radius. Why do we use this formula? We'll use triangles to figure it out.

Example 1

Show the area formula of a circle with pictures.
We start with a circle divided into 12 parts.



Then we put the 12 triangles together so that they look like this:



We multiply by p because the base is part of the circumference.

Lesson 11

Homework

Activity 1

Solve each decimal number problem.

1. 332.7 − 123.88
2. 23.85 + 78.93 + 56.1 + 19.79
3. 2.64 • 0.5
4. 30.01 • 0.4
5. 332.9 + 128.33 + 786.29
6. 889.4 − 567.99

Activity 2

Divide.

1. 27/3
2. 810/9
3. 32/8
4. 42/7
5. 640/80
6. 300/6
7. 28/4
8. 350/50

Activity 3

Answer each question about circles.

1. If the radius is 3.5, what is the diameter?
2. If the diameter is 12, what is the radius?
3. If the radius is 2.3, what is the circumference?
   Remember the formula for circumference is $2 \cdot \pi \cdot r$. ($\pi = 3.14$)
4. If the diameter is 5.5, what is the area?
   Remember the formula for area is $\pi \cdot r^2$. ($\pi = 3.14$)

Activity 4 • Distributed Practice

Solve.

1. $\frac{4}{8} + \frac{1}{3}$
2. $\frac{7}{9} - \frac{1}{6}$
3. $\frac{3}{4} - \frac{1}{2}$
4. $1\frac{2}{3} \cdot 5\frac{1}{2}$
5. $\frac{8}{9} \div \frac{1}{2}$
6. $\frac{4}{8} \cdot \frac{3}{7}$

Unit 5 • Lesson 11 397

Key Concepts Consolidated in Unit Review Lessons Reinforce Student Learning.

Lesson 15 | Unit Review
▶Operations on Decimal Numbers
Problem Solving:
▶Area of Two-Dimensional Shapes

▶Operations on Decimal Numbers

What are the key ideas for adding, subtracting, multiplying, and dividing decimal numbers?

Lining up decimal numbers based on place value and the decimal point is essential. If we don't line up decimal numbers correctly, we will make a lot of mistakes. The decimal portion of a number is always less than one. It doesn't matter how many places there are to the right of the decimal point.

Review 1

How do we line up decimal numbers when we add or subtract?

12.105 + 1.7

$$\begin{array}{r} 12.105 \\ +\ 1.700 \\ \hline \end{array}$$

We line up the numbers based on the decimal point. We use zeros to make sure that we are adding the correct place values.

Review 2

What do the numbers after the decimal point mean?

Look at the number 23.999999999999999999999.

This looks like a large number, but all of the 9s to the right of the decimal point are still less than 1. If we round this number, it is 24. That is only 1 more than 23.

The decimal number 23.999999999999999999999 rounds to 24.

412 Unit 5 • Lesson 15

Making Sense of Rational Numbers, Student Text, Unit 5, Lesson 15

Lesson 15

What happens when we multiply decimal numbers?

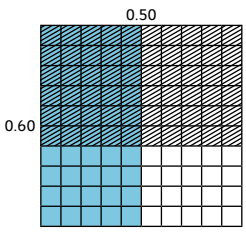
Multiplying decimal numbers is similar to multiplying fractions. When we multiply two decimal numbers, the product is usually less than the two other numbers. We can use a 100-square grid to help understand this. Knowing that the product is smaller helps us understand why we move the decimal points to the left. It makes sense because the product should be smaller than the other two numbers.

Review 1

What happens when we multiply decimal numbers?

Let's multiply 0.60 • 0.50.

100-Square Grid



0.60 • 0.50 = 0.30

The answer 0.30 is smaller than 0.60 or 0.50.

Traditional Algorithm

$$\begin{array}{r} 0.60 \\ \times\ 0.50 \\ \hline 00 \\ 3000 \\ \hline 0.3000 \end{array}$$

Since there are four decimal places in the numbers we are multiplying, we move the decimal point four places to the right.

Unit 5 • Lesson 15 413

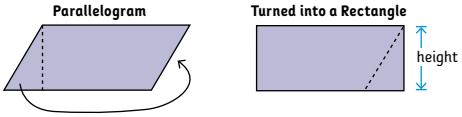
Lesson 15

To find the area formula for parallelograms, we can move one part of the shape and turn the parallelogram into a rectangle.

Review 2

What is the area of a parallelogram?

We can use what we know about the area of rectangles to find the area of a parallelogram. When we move part of the parallelogram, we make a rectangle. The base of the rectangle is the same as the base of the parallelogram.



To find the area of the parallelogram, use the formula for area of a rectangle. It is important to remember that the height of the parallelogram is the vertical distance between the bases, not the slanted end of the shape.

Area of a parallelogram = base • height

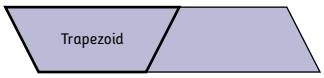
Lesson 15

The area formulas for other polygons have to do with triangles and quadrilaterals. The area formula for a trapezoid doesn't make sense until we translate and reflect a copy of the trapezoid.

Review 3

How do we find the area of a trapezoid?

We can use what we know about the area of parallelograms to find the area of a trapezoid. When we translate and reflect a copy of the trapezoid, we create a parallelogram. The base of the parallelogram is the sum of the bases of the trapezoid.



To find the area of the trapezoid, we take one half of the parallelogram. This leads us to the area formula for a trapezoid.

Area of a trapezoid = $\frac{1}{2}$ • (base 1 + base 2) • *h*

Making Sense of Rational Numbers, Student Text, Unit 5, Lesson 15

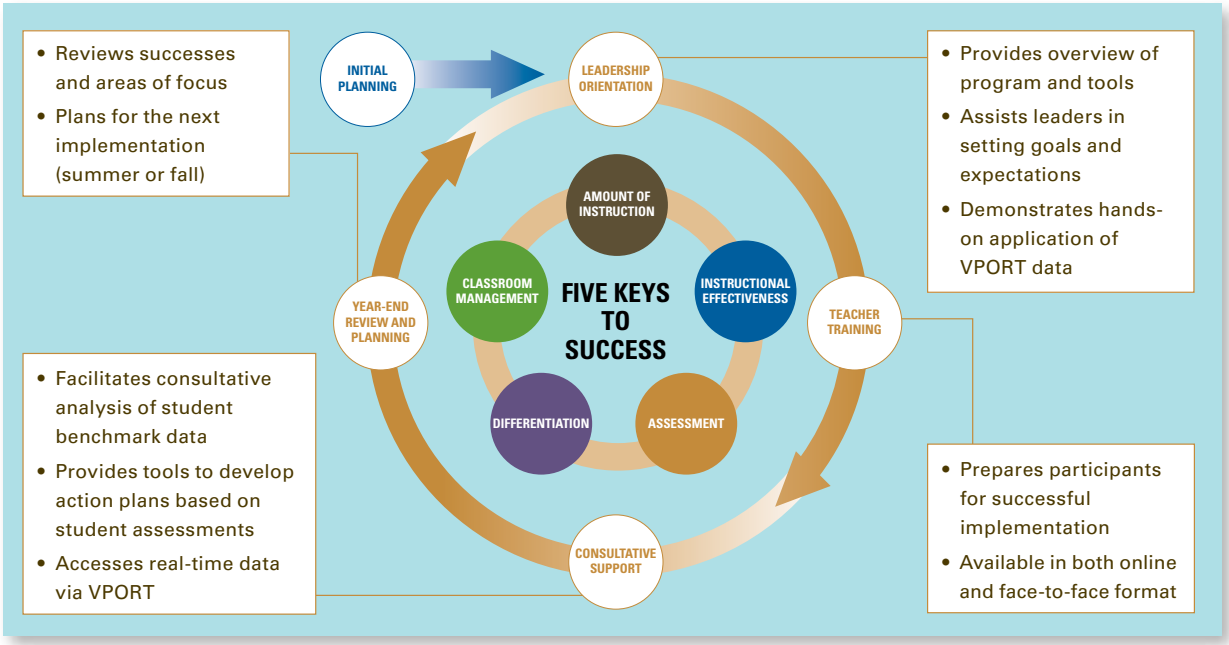
At Cambium Learning Group, we understand that intervention solutions don't come from programs alone. Voyager's professional development partnership provides ongoing training and implementation support to maximize the effectiveness of instruction.

Focus on Fidelity

Voyager provides award-winning professional development to support effective teaching practices. The hands-on, interactive design can be used in structured environments or in self-paced individual settings to help teachers be successful from the start. Participants learn to:

- Use the VPORT data-management system to assess students and differentiate instruction
- Apply new research and best practices
- Implement the program with ease and fidelity

The Voyager professional development partnership extends throughout the school year and integrates continuous training and support services with detailed reporting on student achievement for teachers and administrators. Our services embody the **five keys to success.**



Initial Planning, Leadership Orientation, and Teacher Training

The professional development partnership begins with collaborative planning between district leadership and Voyager's support staff. This **initial planning** involves customizing program training and support to align with district expectations and goals.

Voyager's **leadership orientation** provides an opportunity for school leaders to review program components and VPORT, Voyager's online data management system. Leaders establish implementation goals and expectations as well as an implementation plan and timeline for their school.

Student success depends on the strength of the teacher, and Voyager's training focuses on improving the quality of instruction by increasing teacher knowledge. Voyager's professional development is unmatched in the industry, offering **teacher training** through face-to-face sessions and an online course.

Consultative Support and Year-end Review/Planning

VPORT provides educators with immediate and transparent real-time data to track student progress throughout the year. With Voyager's **consultative support**, educators learn to:

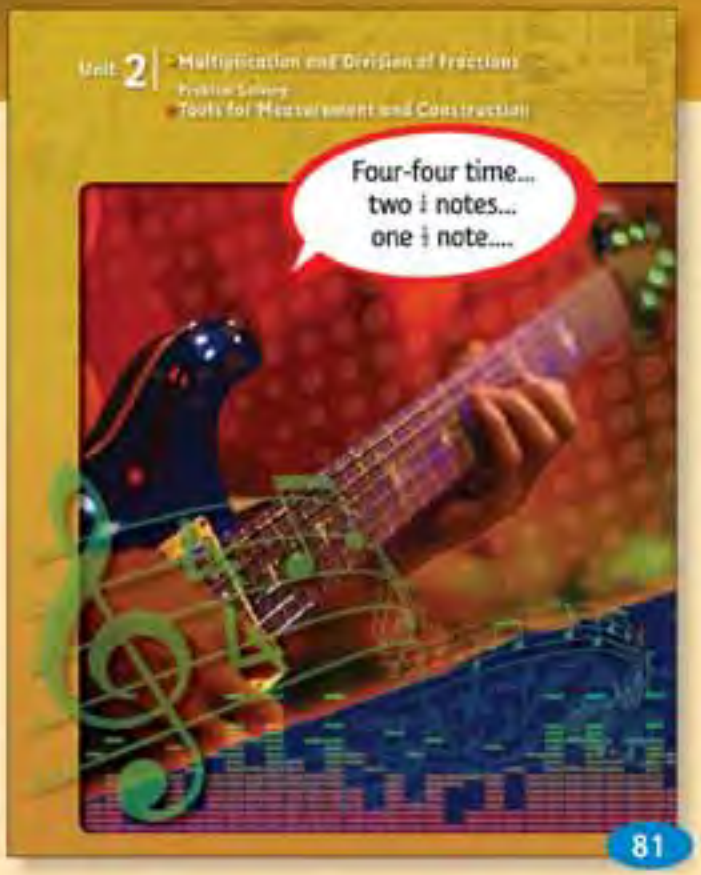
- Identify student needs
- Monitor student progress against goals
- Evaluate student learning
- Adjust instruction based on skills and needs

One of the most important benefits of the Voyager partnership occurs during the **year-end review and planning** stage. Administrators and Voyager support personnel review student progress made during the year and examine areas of focus for the following year. Working collaboratively, they analyze benchmark data and set goals for summer and fall implementations.

Unit Opener: Background Information

With students, read through the Unit Opener of *Student Text*, pages 81–82. Then share some of these musical notes.

- Classical musician Wolfgang Mozart wrote his first piece at age five or six.
- J. S. Bach composed more than 1,100 works in his lifetime.
- The longest song ever performed is "Marathon 2" by Mark Mallman, which lasts 52 hours.
- Math and music have important connections. For example, the rhythm of music is measured in beats, usually four beats to a measure. Whatever combination of notes used, the total beats must add up to the measure. A measure of four beats could be two half notes (two beats each) or one half note plus two quarter notes.
- Each note on the scale emits a certain sound frequency measured in hertz, or cycles per second. The note A has a frequency of 440 hertz. Doubling the frequency produces an A one octave higher. Halving it yields an A one octave lower.
- Music has patterns, just as numbers have patterns. Musicians can transpose a song from one key to another by following the logical pattern that keeps the spacing between the notes the same.
- Gottfried Wilhem von Leibniz, a mathematician, once said this about music: "Music is the pleasure the human mind experiences from counting without being aware that it is counting."



Unit 2

Building Number Concepts:
Multiplication and Division of Fractions

Problem Solving:
Tools for Measurement and Construction

OBJECTIVES

Building Number Concepts	Problem Solving
• Use models to show multiplication and division of fractions • Understand how multiplication and division of fractions is different from whole numbers • Use the traditional methods to multiply and divide fractions	• Develop an understanding of basic geometric terms • Measure lengths and angles using a variety of tools and units • Use a compass to complete basic geometric constructions

Overview

Learning how to multiply and divide fractions is often a turning point for many students because common denominators are unnecessary. This change in operations with fractions can lead students to believe that math has arbitrary rules and procedures. To combat such misunderstandings, this unit focuses on visual representations and a logical comprehension of multiplication and division of fractions.

Unit Vocabulary

- area model
- area
- factors
- common factors
- greatest common factor
- commute
- traditional method
- multiplicand

The Problem Solving strand of this unit focuses on rulers, protractors, and compasses. These tools play a critical role in measurement and geometry and are fundamental to many occupations. We build on the use of these tools throughout the next three units.

Unit Vocabulary

- point
- line segment
- line
- ray
- metric ruler
- millimeters
- centimeters
- angle
- vertex
- right angle
- acute angle
- obtuse angle
- protractor
- bisect
- equilateral triangle

Building Number Concepts:
Multiplication and Division of Fractions

Key Questions That Guide Student Understanding

- *Why do we generally get answers that are smaller when we multiply fractions?*
- *Why do we generally get answers that are larger when we divide fractions?*

Enduring Understandings for Multiplication and Division of Fractions

When we multiply two fractions, the product is generally smaller than the multiplicands. This is the opposite of what happens when we multiply two whole numbers. To show this concept, choose your numbers carefully, and talk students through the multiplication process. When we multiply $\frac{1}{2}$ by $\frac{6}{8}$, we take $\frac{1}{2}$ of $\frac{6}{8}$. The product is a fractional portion of the quantity $\frac{6}{8}$.

Dividing fractions is based on the observation that the quotient is generally bigger than the quantity being divided (or dividend). When we divide $\frac{3}{4}$ by $\frac{1}{2}$, we use $\frac{1}{2}$ to partition $\frac{3}{4}$ one and one-half times. Again, this is the opposite of what happens with whole-number division.

Tools for Understanding Multiplication and Division of Fractions

Using Models

Rectangular area models and fraction bars are essential tools for helping students think about multiplying and dividing fractions. Area models help students see how we take a portion of another fraction. Fraction bars help students see how in the division process, one fraction partitions another.

Using Strategies for Fraction Operations

Students easily confuse what method to use for operations with fractions. We help students understand the rationale behind multiplication of fractions by stressing the idea that we take a fraction of a fraction (e.g., we take $\frac{1}{3}$ of $\frac{3}{4}$). When we divide fractions, we break up, or partition, one fraction by another.

Problem Solving:

Tools for Measurement and Construction

Key Questions That Guide Student Understanding

- *What are important tools and benchmarks for measuring angles?*
- *How can we construct angles and shapes using a variety of tools?*

Enduring Understandings for Measurement and Construction

Understanding angles and how to measure them is important for geometry and secondary subjects like trigonometry. Knowing basic types of angles provides a foundation for understanding properties of triangles and other polygons. Students need to understand angles in relationship to benchmarks (e.g., 45 degrees, 90 degrees). Protractors, scales, and rulers provide a high level of precision in measurement. Compasses are not only precise in creating arcs, semicircles, and circles, but they allow students to create and investigate angles without the use of protractors. Finally, it is important for students to use benchmarks and approximations as ways of putting exact measurements in context. For example, an 87-degree angle is almost a right angle.

Tools for Understanding Measurement and Construction

Using a Variety of Tools

To students, protractors are impractical tools. They present degrees in both directions, making the tool hard to read. Creating protractors out of paper helps students see the logic of the tool and to focus on important benchmarks, such as 45 and 90 degrees. Protractors are also helpful in showing how to construct benchmark measurements. Such practice makes the transition to actual protractors more sensible to students.

We use the same logic for metric and U.S. Customary rulers. We have used the metric ruler as a way of reinforcing the base 10 system and because it is easier to read. In this unit, we contrast the metric ruler with the US customary ruler and then measure objects using benchmarks such as $\frac{1}{4}$ and $\frac{1}{2}$ inch.

Lesson 1 ▸The Concept of Multiplication

Problem Solving: ▸Points, Line Segments, Lines, and Rays

Lesson Planner

Vocabulary Development

point
line segment
line
ray

Skills Maintenance

Multiplication Facts

Building Number Concepts:
▸The Concept of Multiplication

We review the concept of multiplication using a number line. We also multiply fractions by whole numbers using fraction bars.

Objective
Students will multiply whole numbers by fractions using number lines and fraction bars.

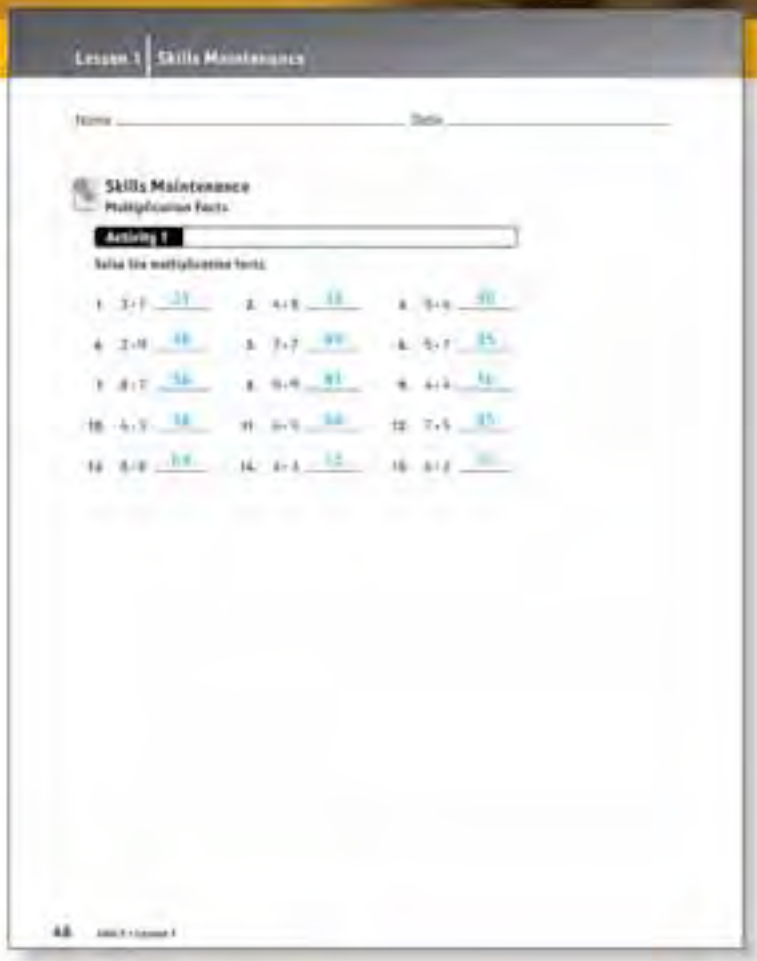
Problem Solving:
▸Points, Line Segments, Lines, and Rays

We introduce important geometry terms: points, line segments, lines, and rays. To explain these terms, we use the context of a surveyor's job.

Objective
Students will identify lines, line segments, points, and rays using a real-world context.

Homework

Students multiply whole numbers by fractions using number lines and fraction bars. In Distributed Practice, students practice operations with whole numbers so they can continue to improve their skills.


Lesson 1 | Skills Maintenance

Name _____ Date _____

Skills Maintenance
Multiplication Facts

Activity 1

Solve the multiplication facts.

1. 3 × 7 = 21
2. 4 × 8 = 32
3. 5 × 9 = 45
4. 2 × 10 = 20
5. 7 × 7 = 49
6. 5 × 7 = 35
7. 8 × 7 = 56
8. 9 × 9 = 81
9. 4 × 3 = 12
10. 6 × 5 = 30
11. 6 × 6 = 36
12. 7 × 5 = 35
13. 8 × 8 = 64
14. 6 × 3 = 18
15. 6 × 2 = 12

48 Unit 2 • Lesson 1

Skills Maintenance

Multiplication Facts

(*Interactive Text*, page 46)

Activity 1

Students solve multiplication facts.

Building Number Concepts:
▶ The Concept of Multiplication

What is multiplication?
(*Student Text*, pages 83–86)

Connect to Prior Knowledge
Begin by reminding students about multiplication with whole numbers. Ask students to think of different situations that might require us to use multiplication.

Link to Today's Concept
Tell students that in today's lesson, we look at multiplication as a number of sets of another number.

Demonstrate
Engagement Strategy: Teacher Modeling
Demonstrate the concept of multiplication in one of the following ways:

mBook: Use the *mBook Teacher Edition* for *Student Text*, pages 83–84. m

Overhead Projector: Reproduce *Student Text*, pages 83–84, on a transparency.

- Display **5 • 20**. Ask students to think about situations in everyday life that require multiplication. Ask them where we would use a problem like 5 • 20 in the real world.
- Guide students to the illustration of the rows of chairs. In this example, we set up five rows of chairs for a concert with 20 chairs in each row. If we multiply 5 by 20, the product will be the total number of chairs for the concert. m
- Now ask for volunteers to give more examples of when we might need to use multiplication. Prompt students to explain why multiplication was needed in the example.

Listen for:

- Examples that represent multiplication.
- Key words such as of, sets of, and groups of. For example, the chairs are five sets of 20.

- Remind students that they already know how to multiply whole numbers. Display the problem **3 • 2 = 6**. Point out that 3 • 2 = 6 is the same as 3 × 2 = 6. Tell students that in higher level math, we often use *x* as a variable, so we need to use a different symbol for multiplication to avoid confusion. m

What is multiplication? (continued)

Demonstrate

Multiply Using a Number Line

- Discuss multiplication using a number line. Show students how to represent whole-number multiplication on the number line using the problem **3 • 2 = 6**.
- Display the number line, and count out three sets of two. Point out to students that when we take a unit of 2 three times, you get 6. Another way to say this is 3 sets of 2. [M]

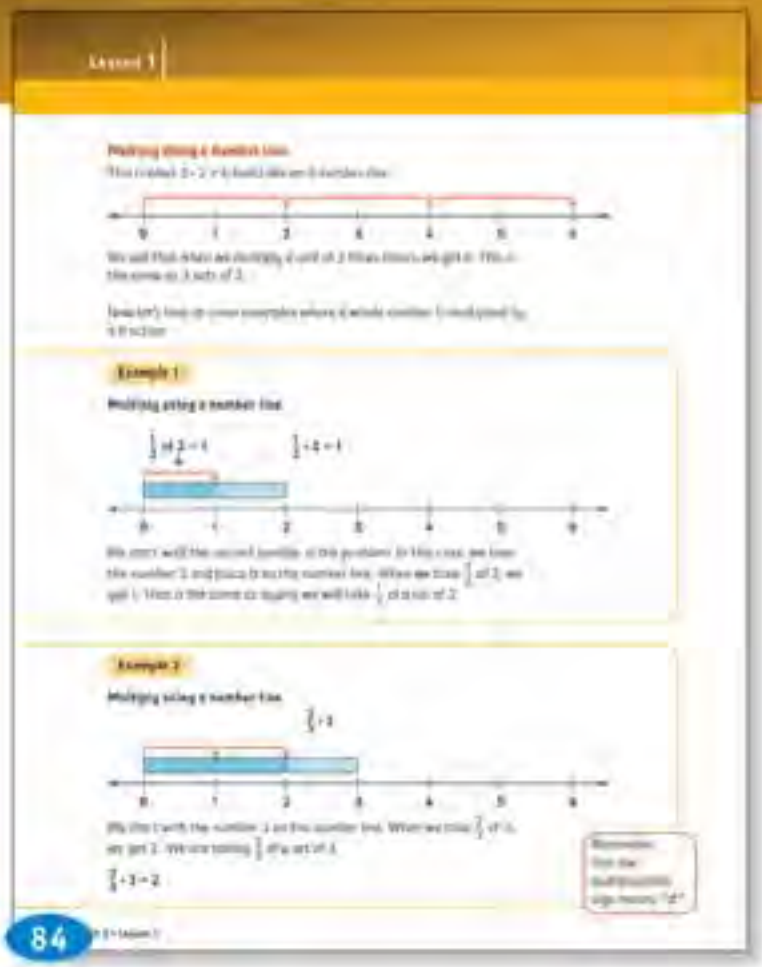
Check for Understanding

Engagement Strategy: Think Tank

Distribute pieces of paper to students, and have them write their names on the papers. Then write the problem **2 • 5 = 10** on the board. Have students draw a number line to represent the multiplication problem. When students finish, collect their papers, and put them into a container. Draw a paper out, and share the answer with the class. If it is correct, congratulate the student. Invite the student to explain the answer with the class.

Demonstrate

- Next have students look at **Example 1**. We demonstrate a fraction multiplied by a whole number using a number line and fraction bars. The problem is $\frac{1}{2} \cdot 2 = 1$.
- Tell students that we start by representing the second number in the problem. We start with the whole number 2 on the number line and take half of it to get an answer of 1.
- Look at **Example 2**. This example illustrates a fraction times a whole number using a number line and fraction bars. The problem is $\frac{2}{3} \cdot 3 = 2$.

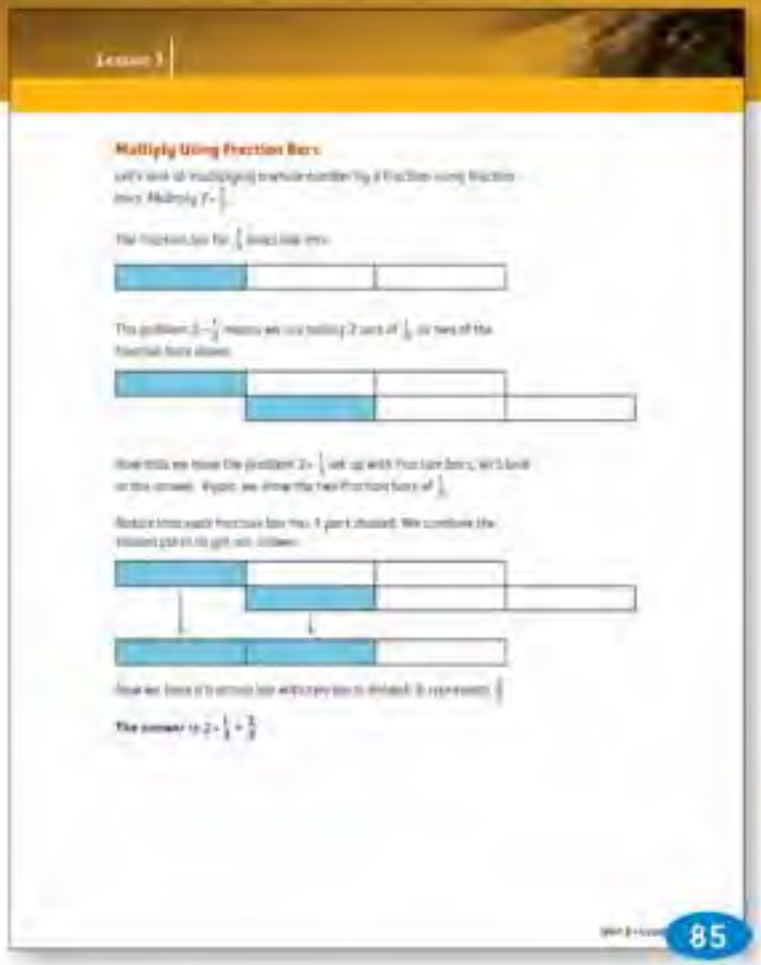


- Refer to the fraction bars for each step. We start with the number 3 on the number line. Then we take $\frac{2}{3}$ of it. Count out two of the rectangles.
- Remind students that the multiplication sign means "of." For example, $\frac{2}{3} \cdot 3$ is the same as saying "$\frac{2}{3}$ of a set of 3s."

Demonstrate

Multiplication Using Fraction Bars

- Next explain how we use fraction bars to represent the multiplication of a whole number by a fraction. The problem is $2 \cdot \frac{1}{3}$. This means we need two sets of $\frac{1}{3}$. We shade one fraction bar to represent $\frac{1}{3}$. We need two sets of these fraction bars.
- Explain that we shade another fraction bar to represent $\frac{1}{3}$. We have two fraction bars that are $\frac{1}{3}$. We combine them and get $\frac{2}{3}$.



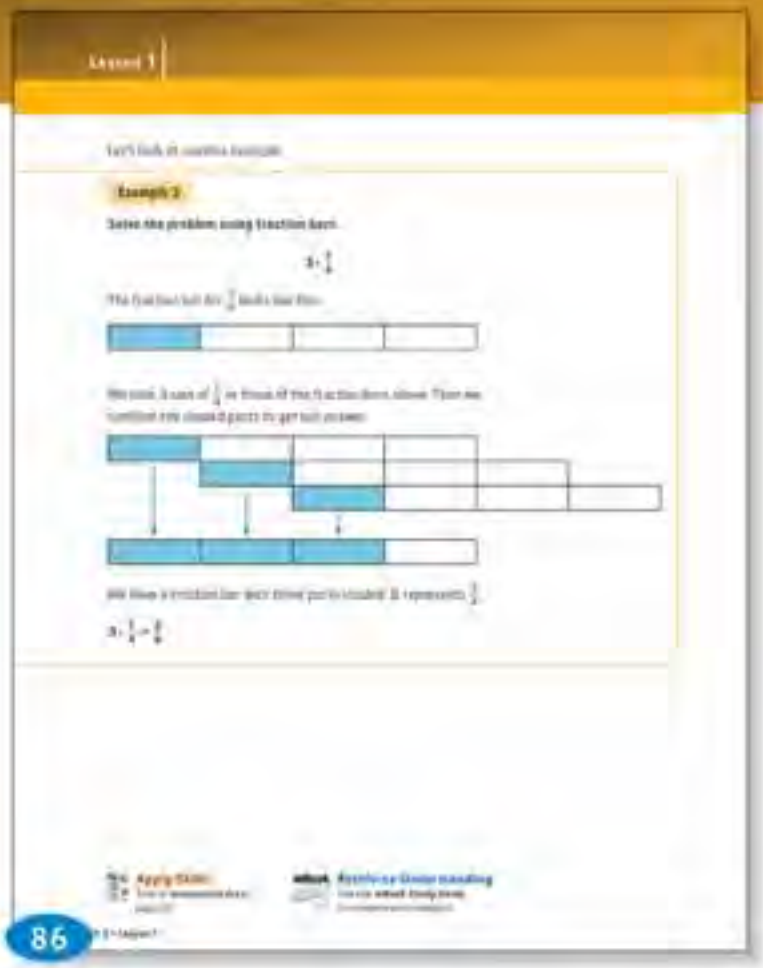
What is multiplication? (continued)

Demonstrate

- Next have students look at page 86 of the *Student Text*. **Example 3** is another problem that uses fraction bars. In the problem $3 \cdot \frac{1}{4}$, we shade a fraction bar to represent $\frac{1}{4}$. We need three of these.
- Point out the three fraction bars to represent the three sets of $\frac{1}{4}$. We combine them and get $\frac{3}{4}$: $3 \cdot \frac{1}{4} = \frac{3}{4}$.
- Ask students to summarize multiplication of a fraction by a whole number.

Listen for:

- *You think of it as a fraction of a number. For example, $\frac{1}{2} \cdot 4$ means half of four.*
- *You can use a number line to represent the multiplication. You show the whole number and then show how to take a fraction of it.*
- *You can use fraction bars to represent the multiplication. You shade a fraction bar to represent the fraction and then you see how many you need.*



Check for Understanding

Engagement Strategy: Look About

Write the problem $2 \cdot \frac{2}{5}$ on the board. Tell students that they will use fraction bars to solve the problem with the help of the whole class ($2 \cdot \frac{2}{5} = \frac{4}{5}$). Students draw fraction bars and write their solutions in large writing on a piece of paper or a dry erase board. When students finish their work, they should hold up their answers for everyone to see.

If students are not sure about the answer, prompt them to look about at other students' solutions to help with their thinking. Review the answers after all students have held up their solutions.

Reinforce Understanding

If students need further practice, have them draw fraction bars to solve the following problems:

$5 \cdot \frac{1}{6}$ ($5 \cdot \frac{1}{6} = \frac{5}{6}$)

$3 \cdot \frac{1}{3}$ ($3 \cdot \frac{1}{3} = \frac{3}{3}$)

Apply Skills

(*Interactive Text*, page 47)

Have students turn to page 47 in the *Interactive Text* and complete the activities.

Activity 1

Students rewrite whole numbers as fractions. Ask students to think about what we learned about the meaning of numerator and denominator. Be sure students see that we are able to rewrite any whole number as a fraction with 1 in the denominator. They use this idea extensively in upcoming lessons.

Activity 2

Students look at shaded fraction bars that represent a multiplication problem involving a fraction and a whole number and tell what problem is represented.

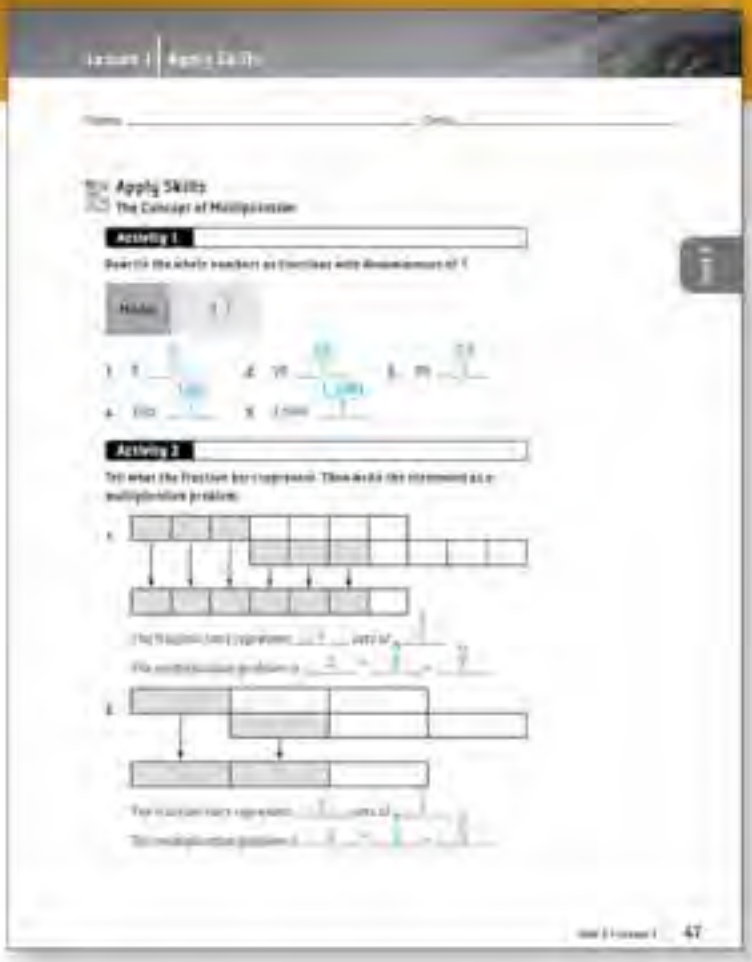
Monitor students' work as they complete the activities.

Watch for:

- Can students rewrite a whole number as a fraction?
- Can students recognize the problem represented by the fraction bars?

mBook **Reinforce Understanding**

Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.


Apply Skills

Activity 1

Activity 2

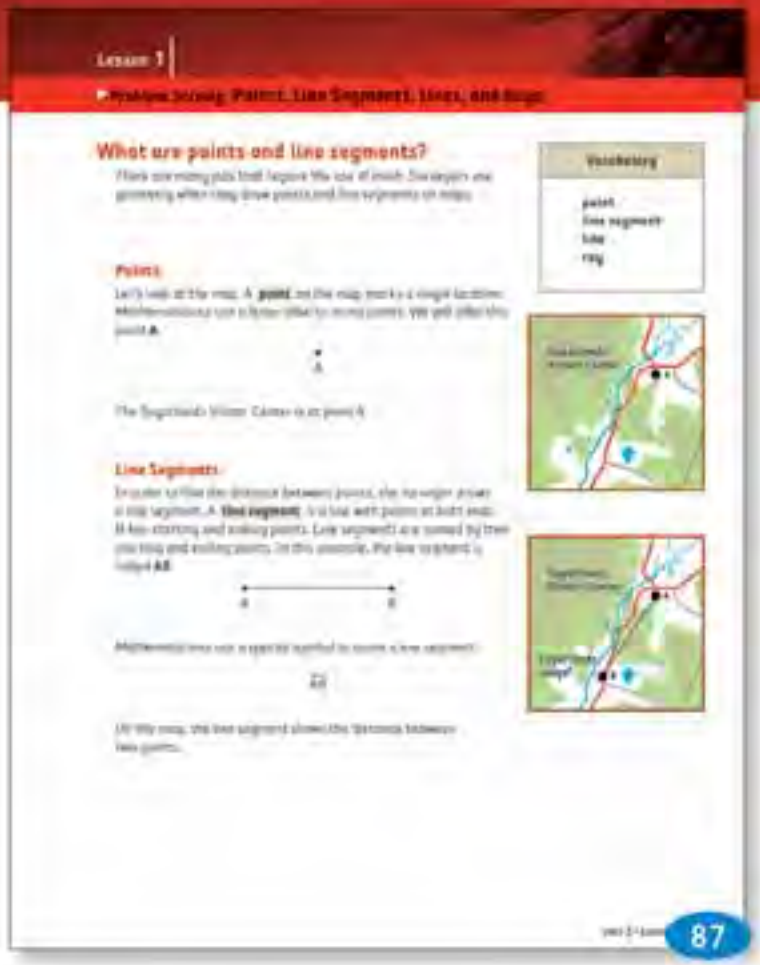
47

Problem Solving:
▶ Points, Line Segments, Lines, and Rays

What are points and line segments?
(*Student Text*, page 87)

Build Vocabulary
Start by asking students if they know what surveyors do. Explain that surveyors use math in their jobs. Explain that a **point** marks a single location, and a **line segment** is a line with points at both ends. Surveyors draw points and line segments on maps.

Demonstrate
- Show students the map on page 87 of the *Student Text*. Explain that single locations are labeled by points on a map. We use a letter to label the point. Refer to the map, and explain that the Sugarlands Visitor Center is at Point *A* on the map.
- Next explain that a line segment is a line with two points at the end. Walk across the room and explain to students that you started at Point *A* and walked in a line and stopped at Point *B*. The line in between the two points is the line segment.
- Point out that the same is true on the map. The line segment from Point *A* to Point *B* on the map is the distance between the Sugarlands Visitors Center and the Sugarlands Lodge.
- Explain that we use a symbol to name a line segment: $\overline{AB}$.



What are lines and rays?

(*Student Text*, page 88)

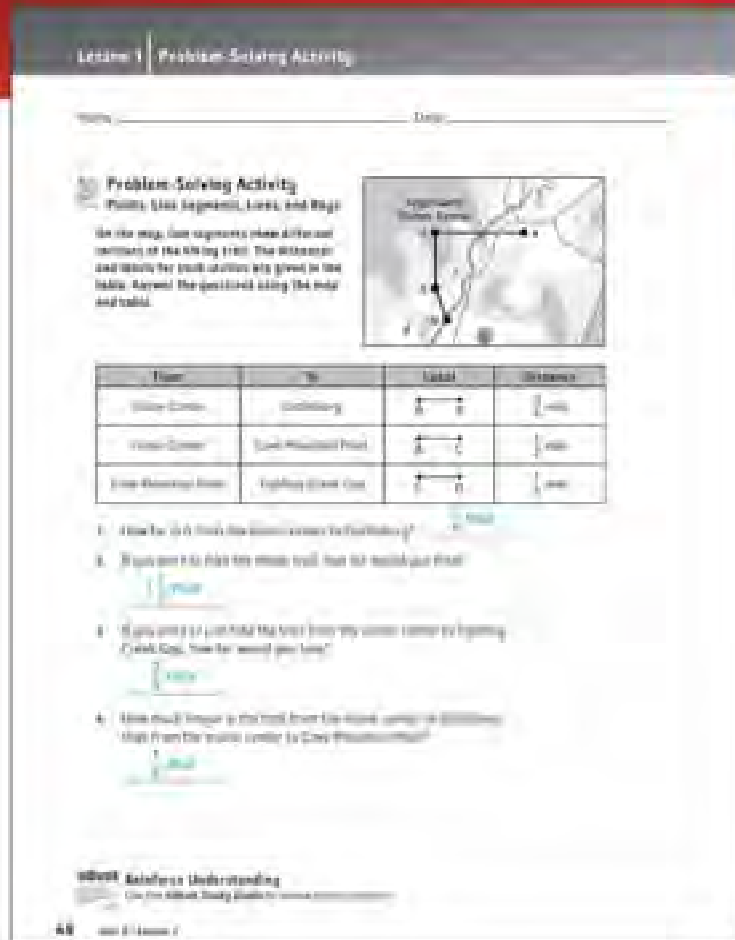
Demonstrate

- Discuss the terms **lines** and **ray**. Explain that lines and rays are both named by points, but they are not line segments.
- Read page 88 of the *Student Text* with students. Explain that lines are different from line segments. Line segments have a starting point and an ending point. Lines go on forever, and are represented by an arrow at each end.
- Show students the line CD and point out the symbol: $\overleftrightarrow{CD}$.
- Explain the definition of a ray. A ray is like a line segment because it has a starting point, but it does not have an ending point. A ray is similar to a line because it goes on forever, but only in one direction.
- Show students the ray EF, and point out the symbol used to write it: $\overrightarrow{EF}$.
- Summarize by explaining that mathematicians use this standard notation for naming and writing about lines and rays. Finally, tell students that these terms and notations are used in many different professions in addition to surveying.
- Ask students to summarize the vocabulary they learned.

Listen for:

- *A point is a dot that represents a location.*
- *A line segment has a beginning and an end. You call the line segment by the name of the beginning and ending points.*
- *A line goes on forever. It has arrows at each end. You name the line segment by two points on the line.*

- *A ray goes on forever in one direction but has a point at the other end. You call the ray by the end point and another point on the ray.*
- Be sure students have a basic understanding of these terms.


Lesson 1 | Problem-Solving Activity

Problem-Solving Activity

Reinforce Understanding

Problem-Solving Activity
(*Interactive Text*, page 48)

Have students turn to *Interactive Text*, page 48, and complete the activity.

Have students take out a blank sheet of lined paper and refer to the map and table as they answer the questions. Once students complete the questions, discuss them in class. Be sure students use the new vocabulary—points and line segments—as they discuss their work.

Monitor students' work as they complete the activity.

Watch for:

- Can students find distances in the table?
- Can students compute the length of the whole trail based on the map and the table of data?
- Can students determine the length of part of the trail?
- Can students identify the need for subtraction?

mBook Reinforce Understanding
Remind students that they can review lesson concepts by accessing the online *mBook Study Guide*.

Homework

Go over the instructions for each activity on page 89 of the *Student Text*.

Activity 1

Students solve simple multiplication problems with a fraction and a whole number using number lines.

Activity 2

Students show each multiplication problem with a fraction bar.

Activity 3 • Distributed Practice

Students practice operations with whole numbers so they can continue to improve their skills.


Lesson 1

Homework

Activity 1

Draw a number line to help you multiply by a fraction.

Model: $\frac{1}{3} \times 3$

0 1 2 3

1. $\frac{1}{3} \times 4$
2. $\frac{1}{3} \times 9$
3. $\frac{1}{3} \times 6$
4. $\frac{1}{3} \times 12$
5. $\frac{1}{3} \times 15$
6. $\frac{1}{3} \times 12$

See Additional Answers below.

Activity 2

Show each problem using fraction bars.

1. $\frac{1}{2} \times 4$
2. $\frac{1}{3} \times 6$
3. $\frac{1}{4} \times 8$
4. $\frac{1}{5} \times 5$
5. $\frac{1}{6} \times 6$
6. $\frac{1}{2} \times 4$

See Additional Answers below.

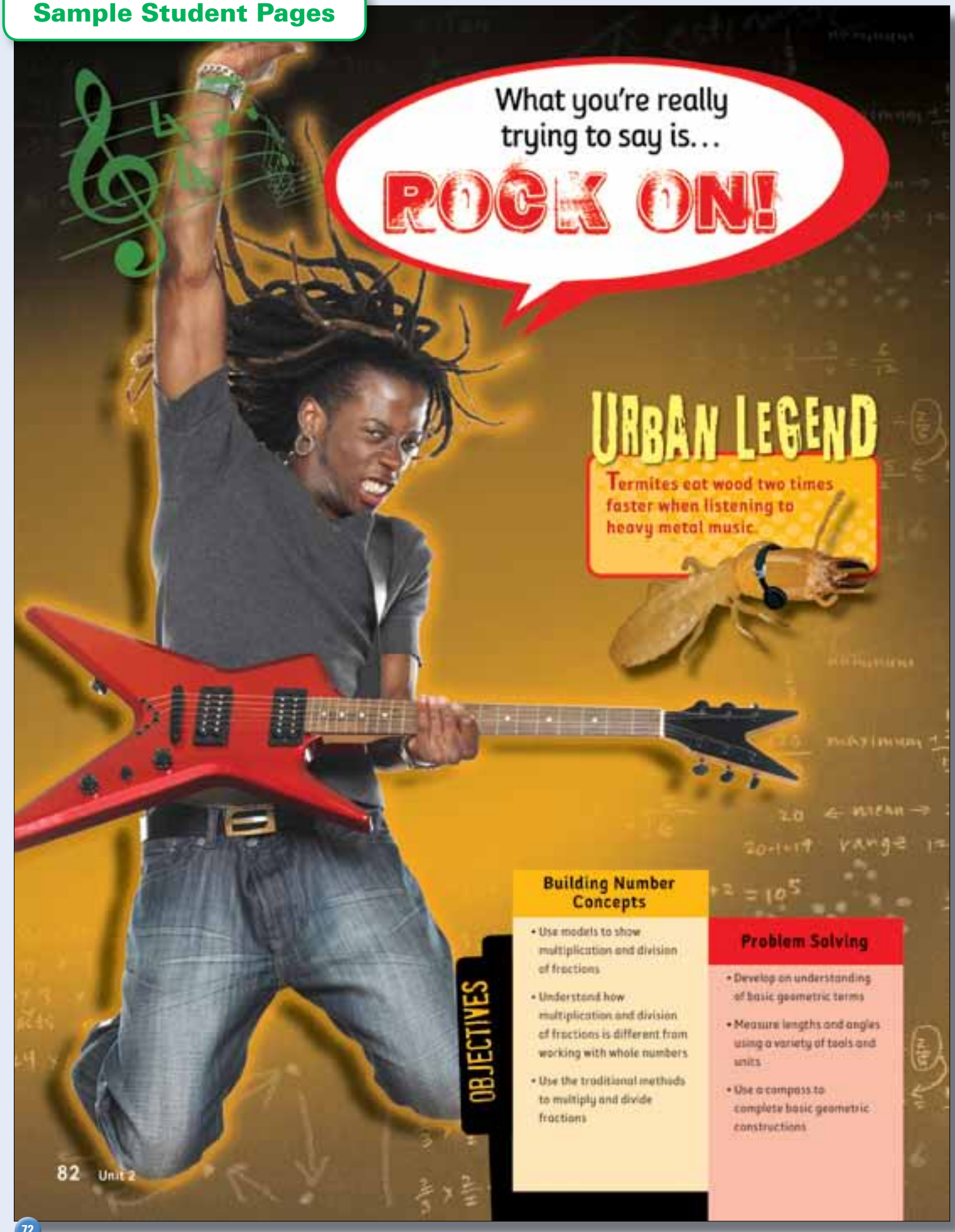
Activity 3 • Distributed Practice

Solve.

1. $167 + 14 = 671$
2. $512 - 107 = 412$
3. $611 - 238 = 173$
4. $21 + 7 = 217$

89

(*Additional Answers continue on Appendix, page A2.*)


What you're really trying to say is...
ROCK ON!
URBAN LEGEND
Termites eat wood two times faster when listening to heavy metal music.
OBJECTIVES
Building Number Concepts
• Use models to show multiplication and division of fractions
• Understand how multiplication and division of fractions is different from working with whole numbers
• Use the traditional methods to multiply and divide fractions
Problem Solving
• Develop an understanding of basic geometric terms
• Measure lengths and angles using a variety of tools and units
• Use a compass to complete basic geometric constructions
82 Unit 2

Lesson 1

The Concept of Multiplication

Problem Solving:

Points, Line Segments, Lines, and Rays

The Concept of Multiplication

What is multiplication?

Let's review the concept of multiplication. We can model the following problem in different ways.

Model 1:

5 · 20

Model 2:

The model shows five sets of twenty.

Until now, the numbers we have multiplied in this program have been whole numbers. Let's look at a multiplication problem with whole numbers:

$$3 \cdot 2 = 6$$

Notice the symbol we are using for multiplication. Remember: 3 · 2 is the same as 3 × 2.

Multiply Using a Number Line

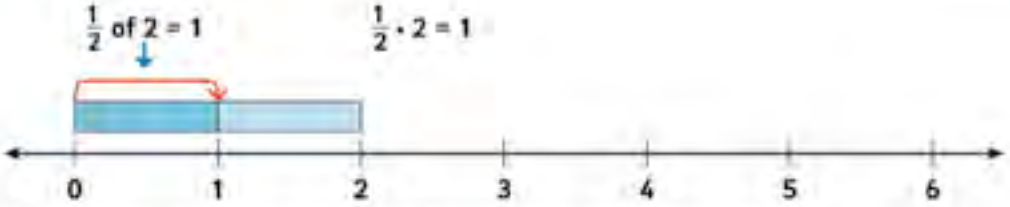
This is what $3 \cdot 2 = 6$ looks like on a number line.

We see that when we multiply a unit of 2 three times, we get 6. This is the same as 3 sets of 2.

Now let's look at some examples where a whole number is multiplied by a fraction.

Example 1

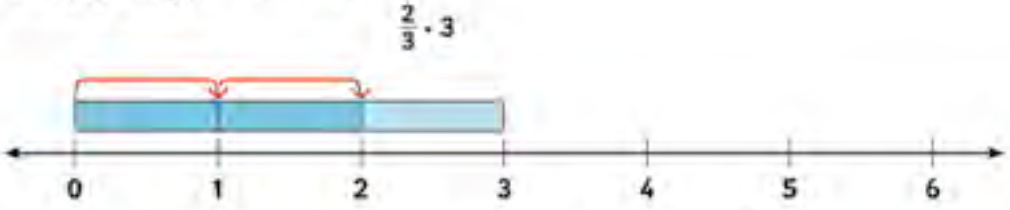
Multiply using a number line.



We start with the second number in the problem. In this case, we take the number 2 and place it on the number line. When we take $\frac{1}{2}$ of 2, we get 1. That is the same as saying we will take $\frac{1}{2}$ of a set of 2.

Example 2

Multiply using a number line.



We start with the number 3 on the number line. When we take $\frac{2}{3}$ of 3, we get 2. We are taking $\frac{2}{3}$ of a set of 3.

$\frac{2}{3} \cdot 3 = 2$

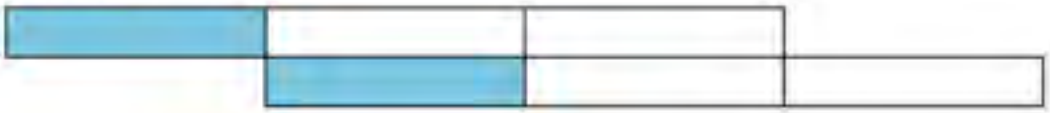
Remember that the multiplication sign means "of."

Multiply Using Fraction Bars

Let's look at multiplying a whole number by a fraction using fraction bars. Multiply $2 \cdot \frac{1}{3}$.

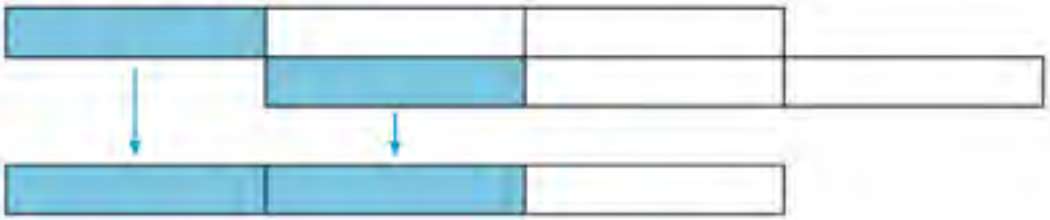
The fraction bar for $\frac{1}{3}$ looks like this:

The problem $2 \cdot \frac{1}{3}$ means we are taking 2 sets of $\frac{1}{3}$, or two of the fraction bars above.



Now that we have the problem $2 \cdot \frac{1}{3}$ set up with fraction bars, let's look at the answer. Again, we show the two fraction bars of $\frac{1}{3}$.

Notice that each fraction bar has 1 part shaded. We combine the shaded parts to get our answer.



Now we have a fraction bar with two parts shaded. It represents $\frac{2}{3}$.

The answer is $2 \cdot \frac{1}{3} = \frac{2}{3}$.

Let's look at another example.

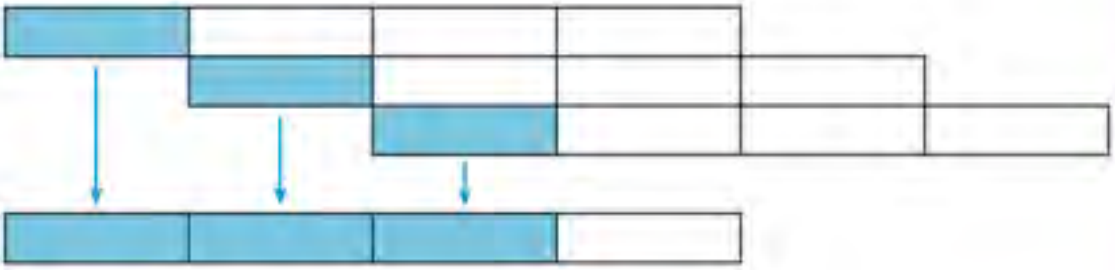
Example 3

Solve the problem using fraction bars.

$$3 \cdot \frac{1}{4}$$

The fraction bar for $\frac{1}{4}$ looks like this:

We take 3 sets of $\frac{1}{4}$ or three of the fraction bars above. Then we combine the shaded parts to get our answer.



We have a fraction bar with three parts shaded. It represents $\frac{3}{4}$.

$3 \cdot \frac{1}{4} = \frac{3}{4}$

Apply Skills
Turn to *Interactive Text*, page 47.

mBook Reinforce Understanding
Use the *mBook Study Guide* to review lesson concepts.

▶Problem Solving: Points, Line Segments, Lines, and Rays

What are points and line segments?

There are many jobs that require the use of math. Surveyors use geometry when they draw points and line segments on maps.

Vocabulary

point
line segment
line
ray

Points

Let's look at the map. A **point** on the map marks a single location. Mathematicians use a letter label to name points. We will label this point **A**.

The Sugarlands Visitor Center is at point A.

Line Segments

In order to find the distance between points, the surveyor draws a line segment. A **line segment** is a line with points at both ends. It has starting and ending points. Line segments are named by their starting and ending points. In this example, the line segment is called **AB**.

Mathematicians use a special symbol to name a line segment:

$\overline{AB}$

On the map, the line segment shows the distance between two points.

What are lines and rays?

Here are two more terms we need to know:

Lines

Lines are different from line segments. Lines have arrows at each end. This means that lines go on forever. They are named by two points on the line. In this example, it's CD.

Mathematicians use a special symbol to name a line:

$\overleftrightarrow{CD}$

Rays

A **ray** has a starting point and then goes on forever in one direction. Rays are named by the endpoint and some other point on the ray. In this example, the ray is named EF.

Mathematicians use a special symbol for naming a ray:

$\overrightarrow{EF}$

These terms are not just used by surveyors, but they are also used by people in other lines of work. There are many professions that require people to work closely with maps. Some people build highways and roads. Others design parks, hiking trails, and bike routes. There are people who plan cities. Pilots use lines to plan their routes. As we can see, many people use geometry in their everyday lives.

Problem-Solving Activity
Turn to *Interactive Text*, page 48.

mBook Reinforce Understanding
Use the *mBook Study Guide* to review lesson concepts.

Homework

Activity 1

Draw a number line to help you multiply by a fraction.

Model $\frac{1}{3} \cdot 3$

1. $\frac{1}{2} \cdot 4$
2. $\frac{1}{3} \cdot 6$
3. $\frac{1}{4} \cdot 8$
4. $\frac{1}{2} \cdot 12$
5. $\frac{2}{3} \cdot 10$
6. $\frac{3}{4} \cdot 12$

Activity 2

Show each problem using fraction bars.

1. $2 \cdot \frac{1}{3}$
2. $3 \cdot \frac{1}{5}$
3. $2 \cdot \frac{1}{4}$
4. $4 \cdot \frac{1}{8}$
5. $4 \cdot \frac{1}{6}$
6. $3 \cdot \frac{1}{8}$

Activity 3 • Distributed Practice

Solve.

1. $352 + 19$
2. $512 - 101$
3. $417 - 238$
4. 31×7

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