Medial Axis - Introduction

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These notes are intended to provide some common vocabulary for subsequent presentations and discussion. They are deliberately somewhat superficial.

Definitions

If an object is viewed as a point set in the classic CSG tradition, then the medial axis is

(i) the set of sphere making multiple tangential contact with the boundary.

This is sometimes abbreviated to

(ii) the set of the centres of those circles.

This naturally suggests that it can be computed by a Delaunay triangulation approach.

It can also be defined as

(iii) the set of singularities of the field of distance from the boundary.

This naturally suggests that it can be computed using a grass-fire algorithm.

If an object is viewed as a Brep, which is somewhat richer than CSG, because the pieces of boundary have identity, the medial axis can be seen as

(iv) a partitioning of the object into equivalence classes, each class containing those points which are closer to a particular boundary entity than to any other.

In this context the medial object can have not just radius information, but also boundary support entity information associated with each piece of it.

Other Similar Objects

There are some other objects which are not quite the same thing.

(i) The unflapped medial object. Often the parts of the medial axis which meet the actual boundary at corners need to be handled differently in applications. They can be removed.

(ii) The midsurface. This is a property of a thin object, which is some surface-like entity which lies somewhere in the middle of the object. It is seldom defined precisely, but often does what is usually required in some application or another.

(iii) The metric medial axis. This is defined in terms of a distance field as above, but where distance is not Euclidean, but depends on some local application-specific measure of distance. This might be useful for meshing where heavy grading is required in the mesh, and the measure of distance might be number of elements required along some interval.

Mathematical Properties

The medial object is invariant under similarity transforms (translation, rotation and scaling) but not under affine transforms (shearing and non-isotropic scaling).

It is not manifold, and so it could not be held in the original Brep solid modellers which insisted that every edge was the boundary of exactly two faces. You need a non-manifold modeller to hold it.

Its structure is not continuous with respect to small changes in the shape of the object. Technically the geometry is continuous if the boundary of the object is totally smooth ($C\infty$), but since a change in the data structures may be needed to hold a new piece, even if it is of infinitesimal length, this is a bit pedantic. This lack of continuity makes the medial object seriously sensitive to noise in the representation. It is important that the medial object which is computed is of what the CAD model was intended to represent, rather than of what it actually is.

When what is stored under the heading of 'medial object' contains radius information as well as sphere centres, the original geometry can, in principle, be recovered from it. If it also contains support entity information, the boundary structure can also be recovered.

Applicable Properties

This is not an exhaustive list. Please discover new ones.

(i) Offsets. An offset object can be recovered by increasing the radius function uniformly.

(ii) Oppositeness. Each piece of medial axis contains information about its support entities, which have to be opposite each other in a useful sense that there is a sphere making tangential contact with the interiors of both.

(iii) Angle between 'opposite' entities. The gradient of the medial radius indicates the angle which a pair of opposite faces make.