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3D medial object computation using a domain Delaunay triangulation

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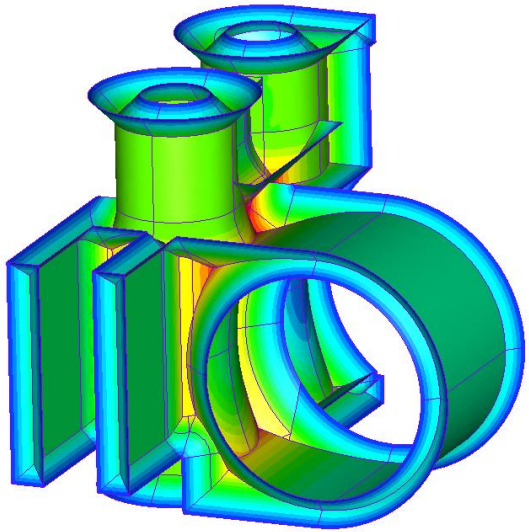
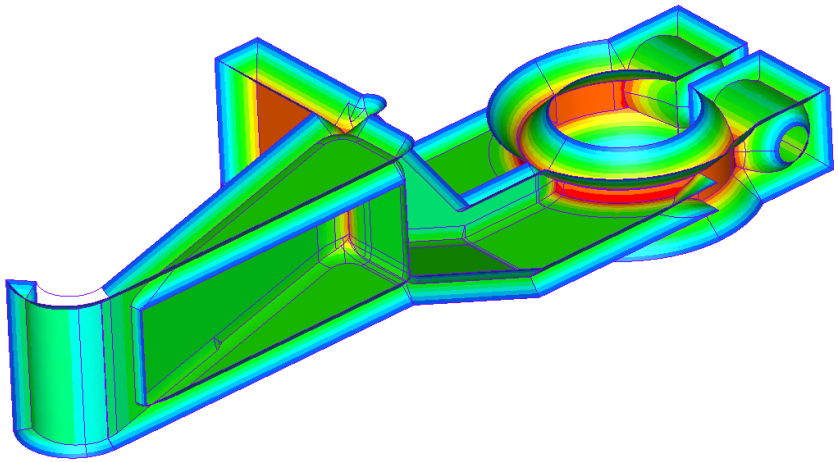
FOCUS INNOVATION CHALLENGE SOLUTION PARTNERSHIP INTEGRATION

Introduction

- TranscenData Europe Ltd.
  - Small company based near Cambridge
  - Suppliers of CADfix: a tool for translation, repair, and transformation of CAD models
  - 25 years of research into medial object
- Mature algorithm for computing 3D medial object
  - As an optional module for CADfix





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- Medial object: medial axis as a CAD model
  - Plus relationships with boundary geometry
- Why compute the medial object?
- History
- Basic principles
  - Using a Delaunay triangulation
  - Tolerant computation
  - Robustness
- Geometry preparation
- Our algorithm
- Medial object API
- Examples
- The future

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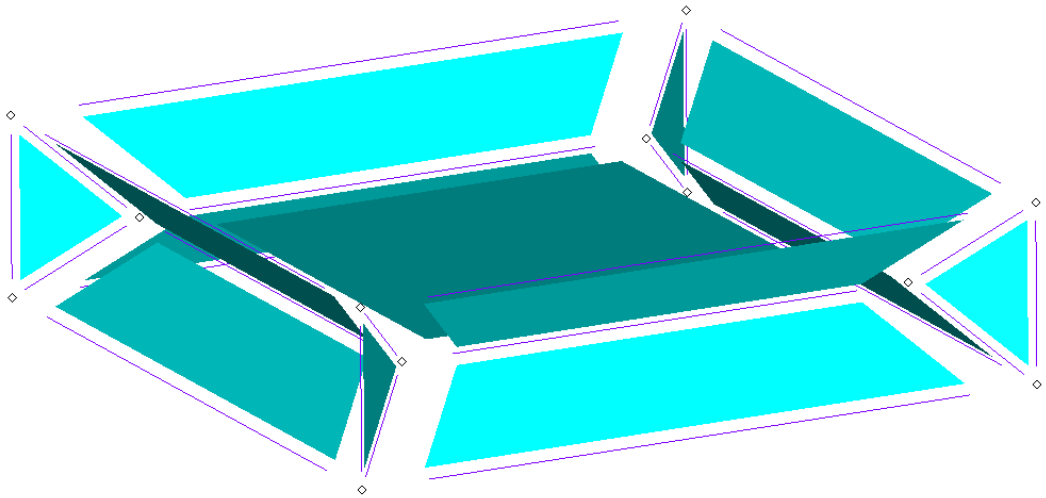
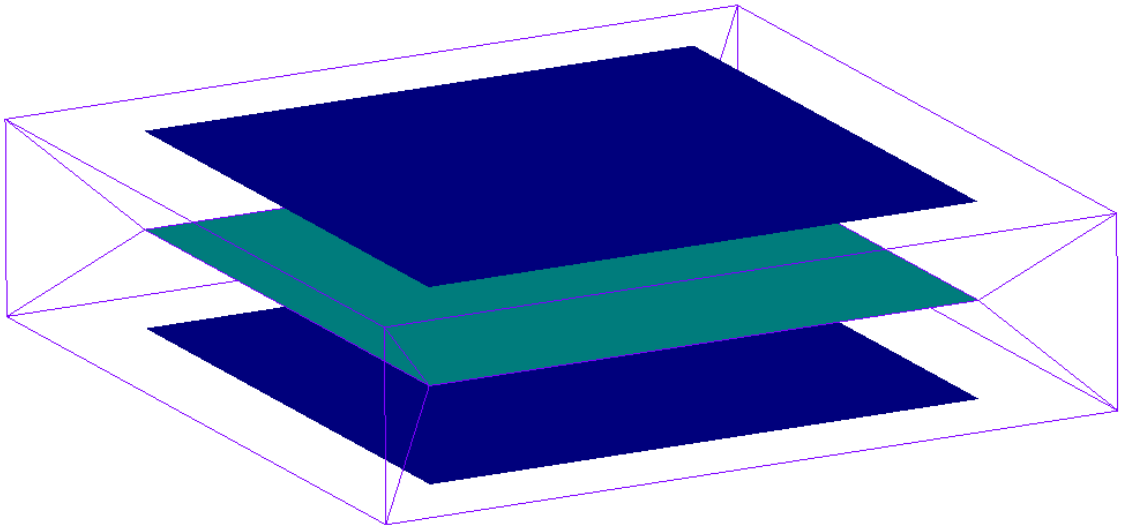
Medial object: medial axis transform as a CAD model

- CADfix computes **medial objects** from CAD solids
  - Medial axis represented as non-manifold CAD model
  - Medial edges are bounded by medial vertices
  - Medial faces are bounded by medial edges and embedded in medial surfaces
  - Result is a valid, if unusual, CAD object



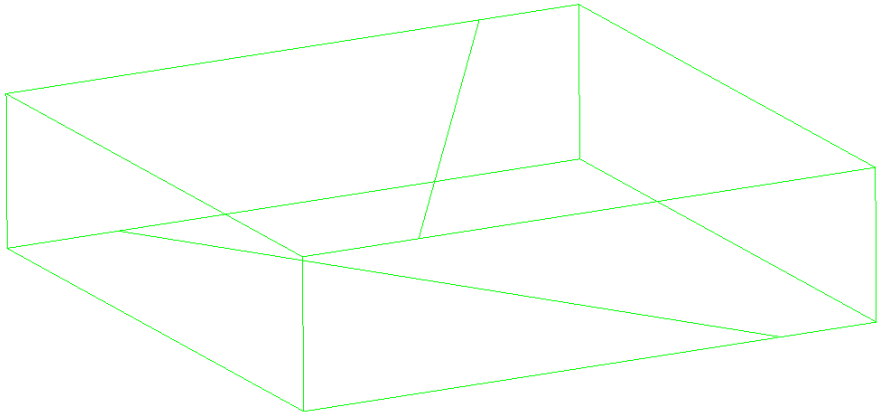
Medial object: touching sites

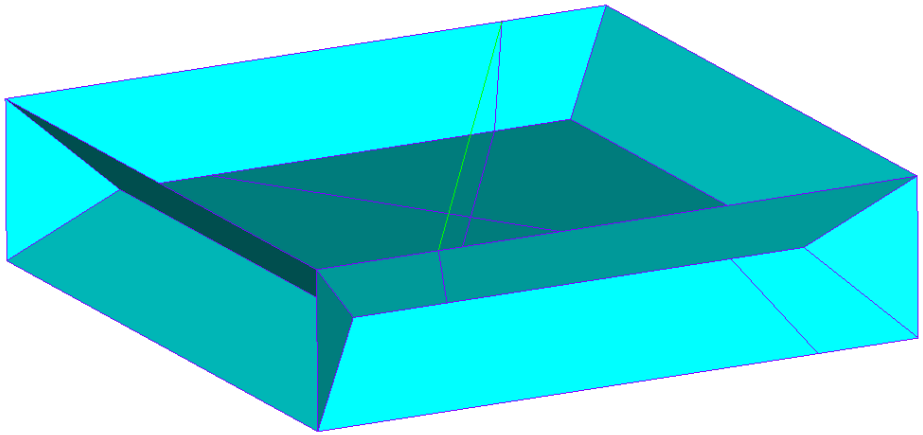
- Each medial entity has touching sites on the boundary
  - A touching site is a connected region touched by the spheres of a medial entity
    - For example, medial faces always have two touching sites
  - Touching sites have a set of **defining entities**
    - Boundary entities which are always touched by the medial spheres



Medial object: intermediate edges & vertices

- This means that the CAD topology is reflected in the medial object
  - Boundary edges with zero dihedral define “intermediate” medial edges connecting two medial faces





- The ability to reason about CAD models using these relationships is the real power of the medial object

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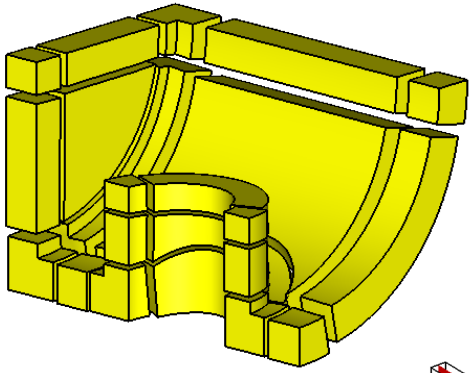
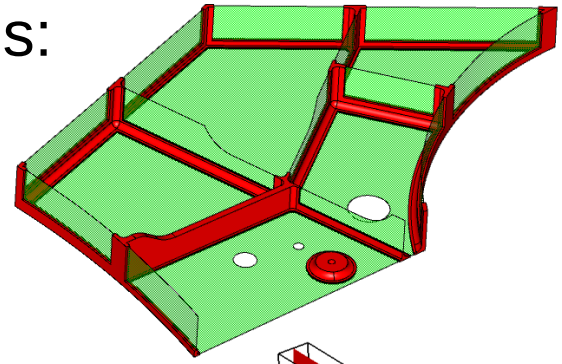
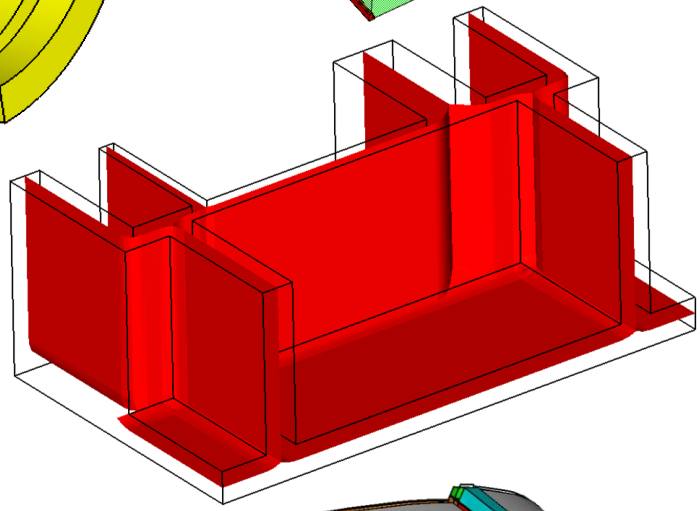
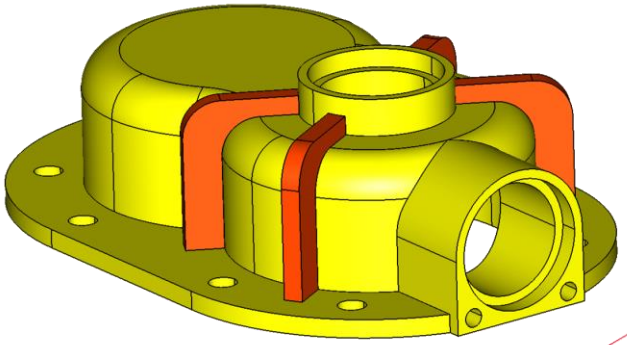
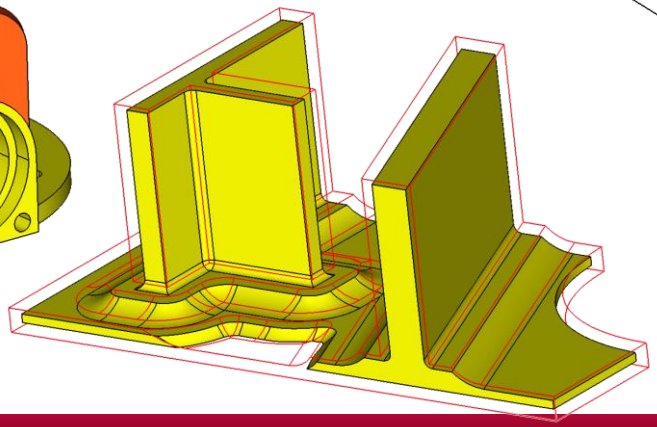
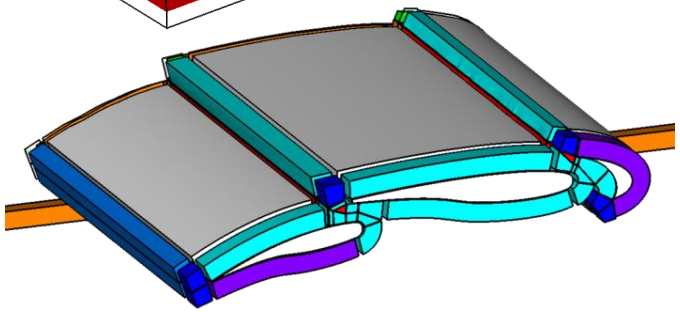
Why compute the medial object?

- Need for medial object driven by applications:
  - Hex meshing
  - Thin/thick subdivision
  - Feature recognition
  - Shelling
  - Midsurfacing
  - Blocking for CFD meshing
- ‘Object’ nature important for all apps

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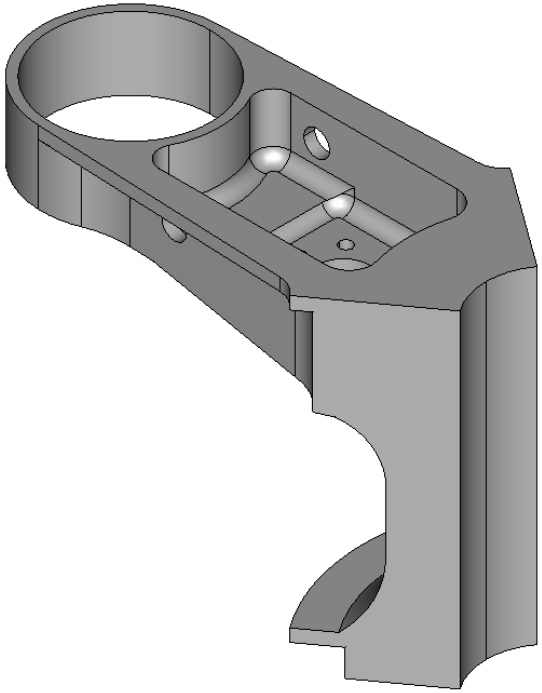
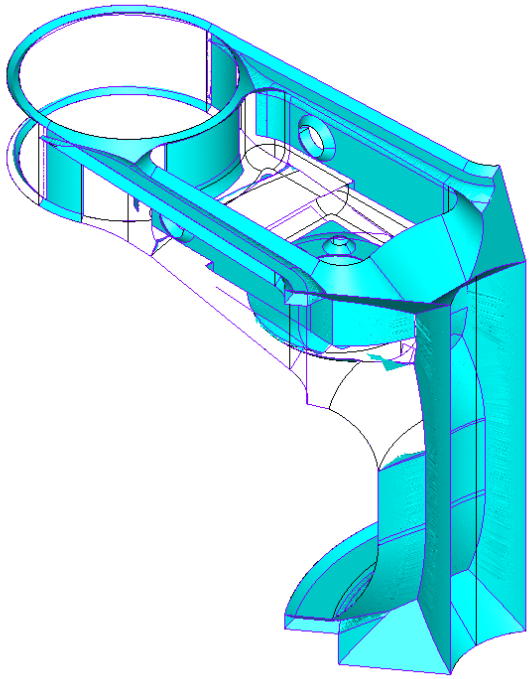
History

- Initially developed in 1990s in conjunction with Queen's University Belfast
  - Medial surface computation using a domain Delaunay triangulation, Damian Sheehy, 1994.
- Work at TranscenData restarted in 2005
  - Still in collaboration with Queen’s University
- Developed during several EU and TSB projects
  - GRIP, VIVACE, CRESCENDO, SILOET, ANSD, GHandI, SimOD
  - Always for a specific application

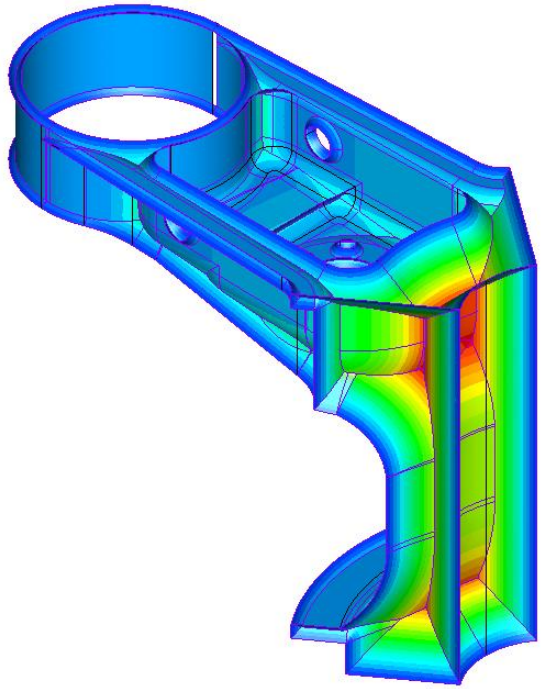
History



Test model



CADfix 7.0 SP3
(circa 2006)



CADfix 10 preview
(due Q4 2014)

Colours indicate medial radius

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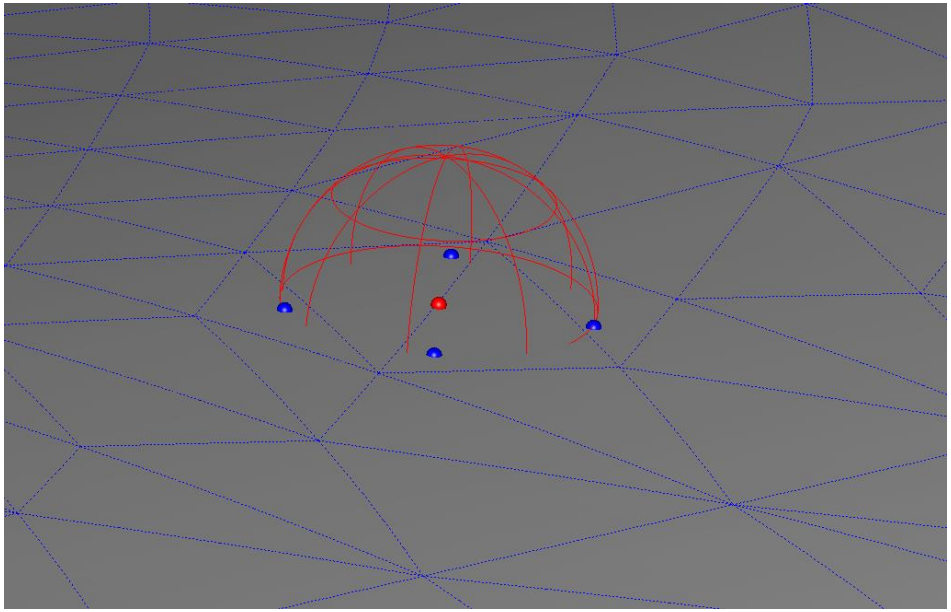
Basic principles: using a Delaunay triangulation

- The medial axis can be seen as the limit of a Voronoi diagram
  - Points sampled over the boundary of the object
  - As points become infinitely dense, Voronoi diagram approaches medial axis*
- A Delaunay mesh is the dual of the Voronoi diagram
  - In the limit, circumspheres of Delaunay tetrahedra become medial spheres
- Cannot use infinitely dense samples in real life
  - Choose sample points carefully
  - Care must be taken to ensure tetrahedra are isomorphic to medial spheres
    - Boundary segments of tetrahedra must be valid touching sites for medial sphere

Basic principles: using a Delaunay Triangulation

- *Except this only works completely in 2D
  - In 3D, Voronoi regions can exist centred on the boundary
  - Correspond to sliver tetrahedra
  - c.f. The Crust and the β-Skeleton: Combinatorial Curve Reconstruction, Nina Amenta, Marshall Bern, and David Eppstein.
- Sliver tetrahedra require special handling

Four sample points forming a sliver tetrahedron – see earlier talk on Introduction to medial axis transforms

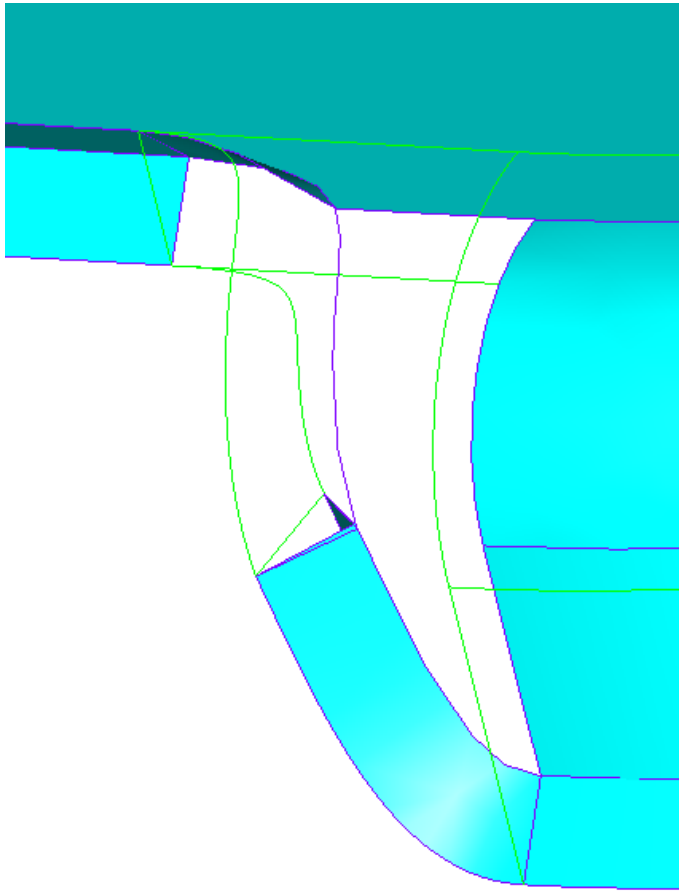


Basic principles: tolerant computation

- Medial axis is highly sensitive to small boundary features
  - Impractical to handle them exactly
  - Impossible when CAD model is ambiguous
  - Exact results are typically not what an application needs
- Compute the simplest medial object such that the inverse transform is within tolerance of the original object
  - Both distance and angular tangent/normal tolerance
  - c.f. Medial Meshes for Volume Approximation, Feng Sun, Yi-King Choi, Yizhou Yu, Wenping Wang
- Even more important as medial radius gets larger
  - For example, medial object of air volume around an aircraft

Basic principles: robustness

- Our algorithm may not work everywhere
  - Input geometry may contain errors
  - May be impossible to resolve medial topology without excessive boundary sampling
- Prefer incompleteness to incorrectness
  - Compute **partial** medial objects where necessary
  - Medial object is correct wherever it is present



Local failure to compute medial object does not affect the rest of the model

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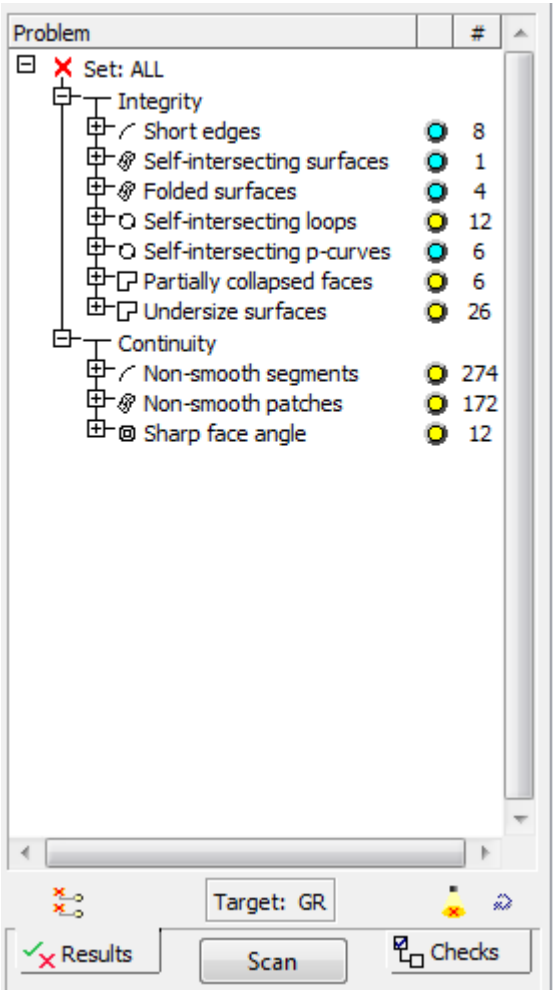
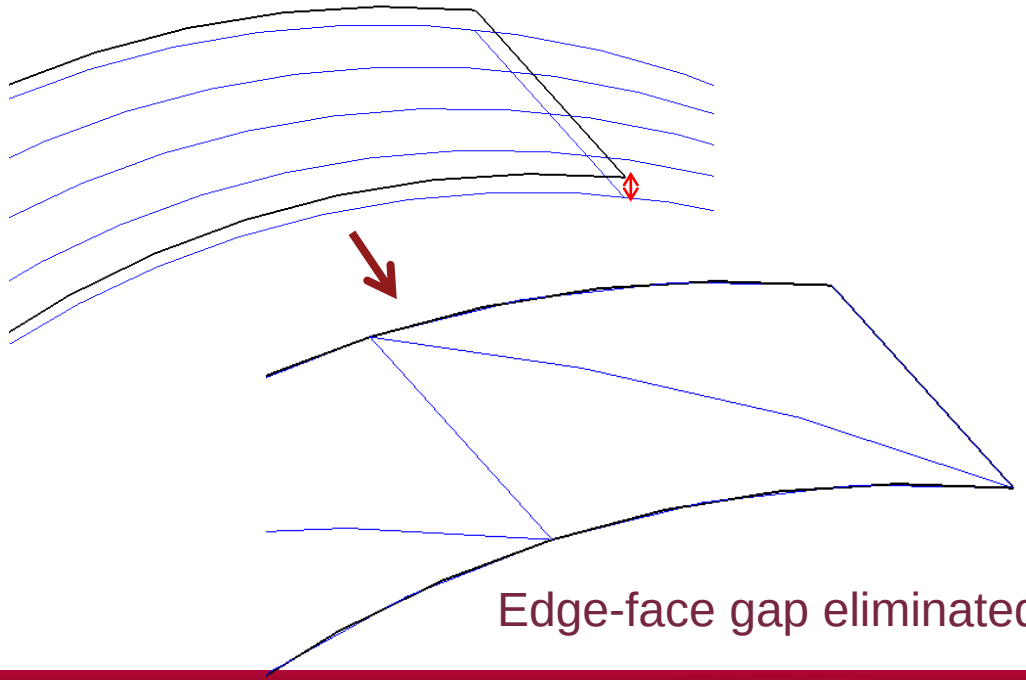
Geometry preparation

- CAD geometry often contains errors and artefacts unsuitable for medial object calculation
  - Edge-face and vertex-face gaps
  - Sharp edges
- Requirements similar to surface meshing
- CADfix has a “prepare” profile for medial object to detect and fix these issues



Geometry preparation

- Despite tolerances, the medial object can still be sensitive to poor quality geometry
  - Shapes with wobbly normal
  - Large edge-face and vertex-face gaps
- Can replace input geometry with a C1-continuous curved triangulation
  - Shapes can be effectively low-pass filtered
  - Edge-face and vertex-face gaps are blended into the interior
  - Deviation from original geometry can be controlled



Edge-face gap eliminated

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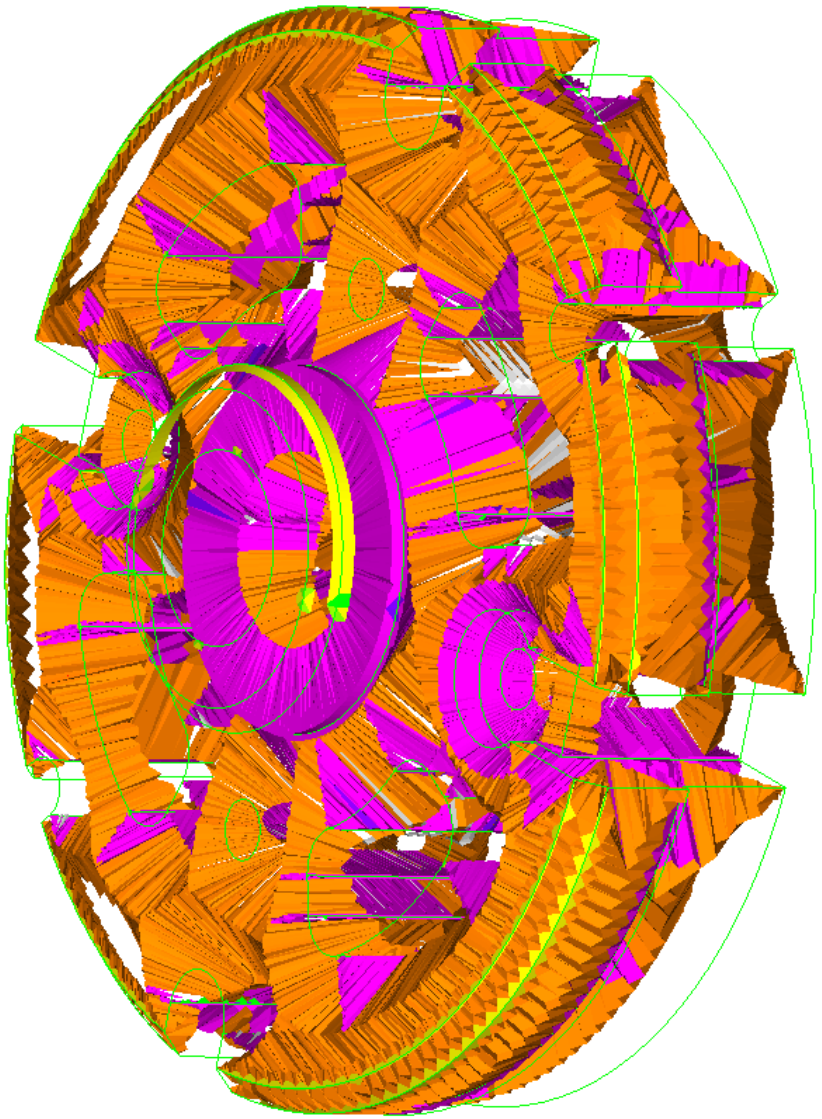
Algorithm overview

- Discover tetrahedra which represent medial vertices and edges only
  - Strict Delaunay mesh
    - Not constrained Delaunay
  - Localised boundary recovery
  - Conditioning
    - Check that tetrahedra are isomorphic to a medial sphere
    - Compute and check the medial sphere
- Create graph of medial topology
  - Walk tetrahedra to find medial vertices and edges
  - Reduce graph to minimal topology
  - Infer medial faces from edge loops
- Create medial geometry

Discovering medial tetrahedra

- Seeding phase
  - Find at least one tetrahedron on each medial edge network
- Walking phase
  - Take tetrahedron representing medial vertex or edge
  - Walk to neighbour
  - Recover boundary for neighbour
  - Condition to ensure isomorphic to medial sphere
  - Check if tetrahedron represents medial vertex or edge

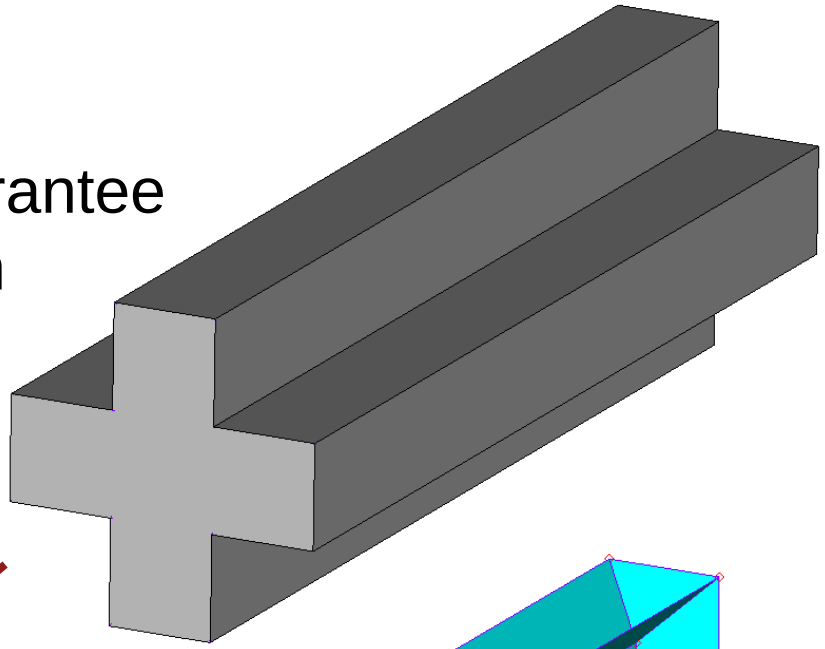


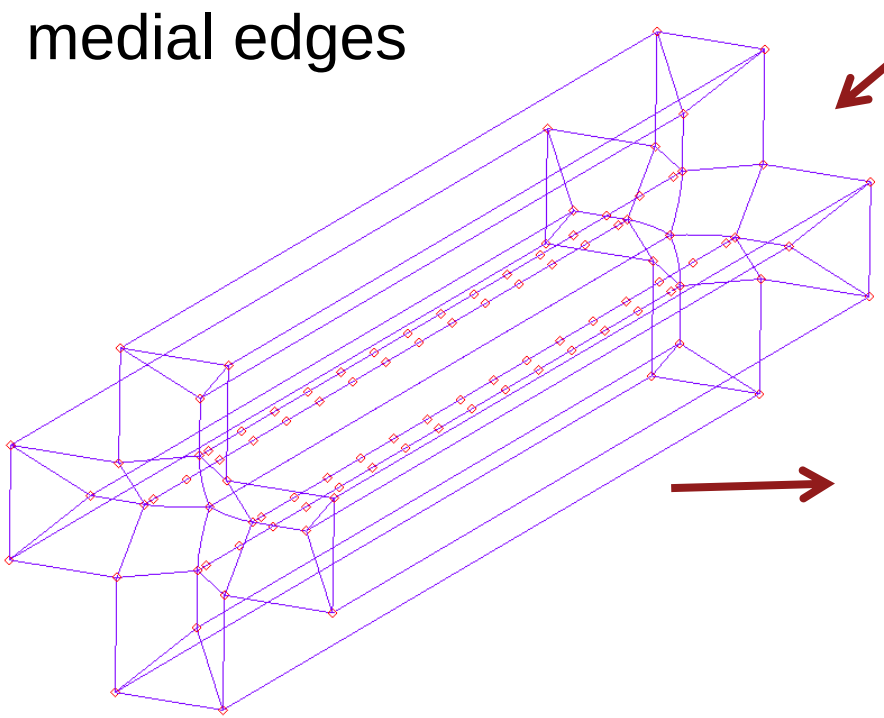
Creating medial topology

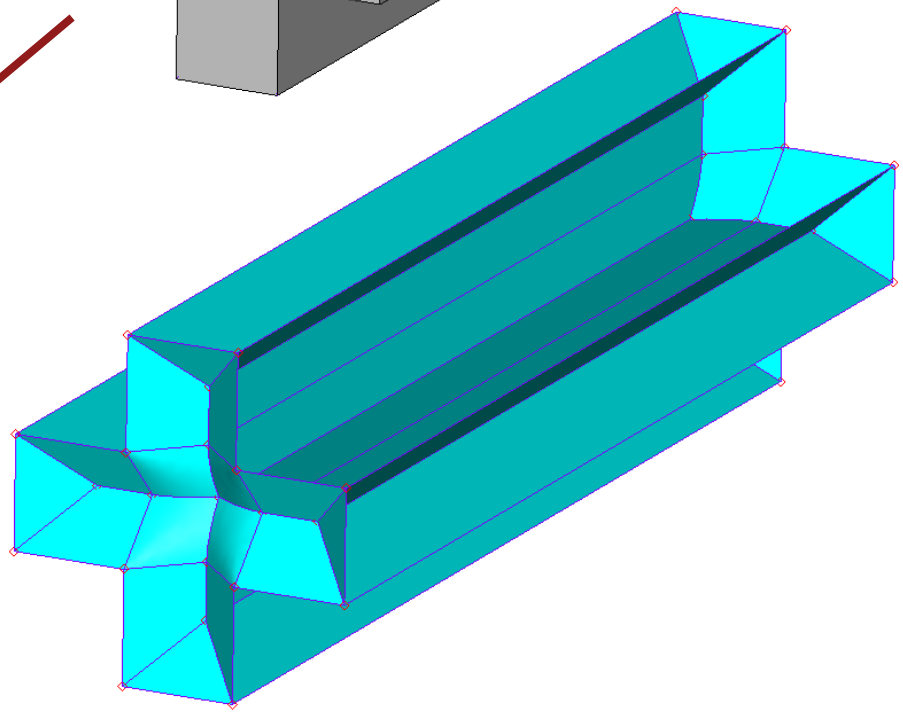
- Basic topology can be found by walking tetrahedra
- Tetrahedra can only touch four boundary entities
  - Delaunay property does not give unique topology when medial sphere touches too many entities
    - > 4 entities for vertices
    - > 3 entities for edges
  - Delaunay property does not give unique topology when medial sphere touches a finite region of the boundary
    - Finite contact
  - Produces fragmented topology which must be merged out
- Tetrahedra may be defined by features smaller than our tolerance
  - Collapse medial topology to reach desired fidelity
- Medial faces are implied by loops of medial edges

Creating medial topology

- Delaunay criterion does not guarantee unique topology along the length of this model
- Topology reduction needed to correct fragmented medial edges

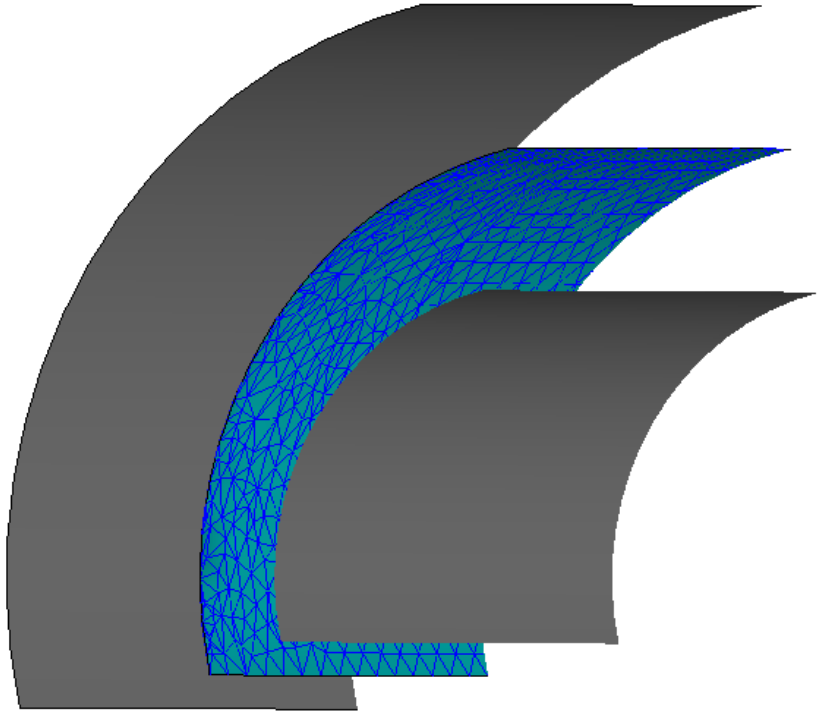






Geometry creation

- Analytic geometry used in special cases
- Generate curvature sensitive meshes of medial edges and faces elsewhere
  - Including radius data
  - Based on Adaptive Curvature-Sensitive Meshing of the Medial Axis, Ang, Pin Yang and Cecil G. Armstrong, *Proceedings, 10th International Meshing Roundtable*, Sandia National Laboratories, pp.155-165, October 7-10 2001.



Medial geometry between two CAD faces

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Medial object API

- CADfix has a geometry API
  - Procedural APIs in C and Fortran
  - Object-oriented APIs in C++, Python, Java, and TCL
- Stable 2D medial object API
- Alpha 3D medial object API
- Access to complete medial object structure
  - Medial topology and geometry
  - Relationships with boundary geometry
- Used by several partners
  - Queen's University Belfast
  - Sandia
  - Carnegie Mellon

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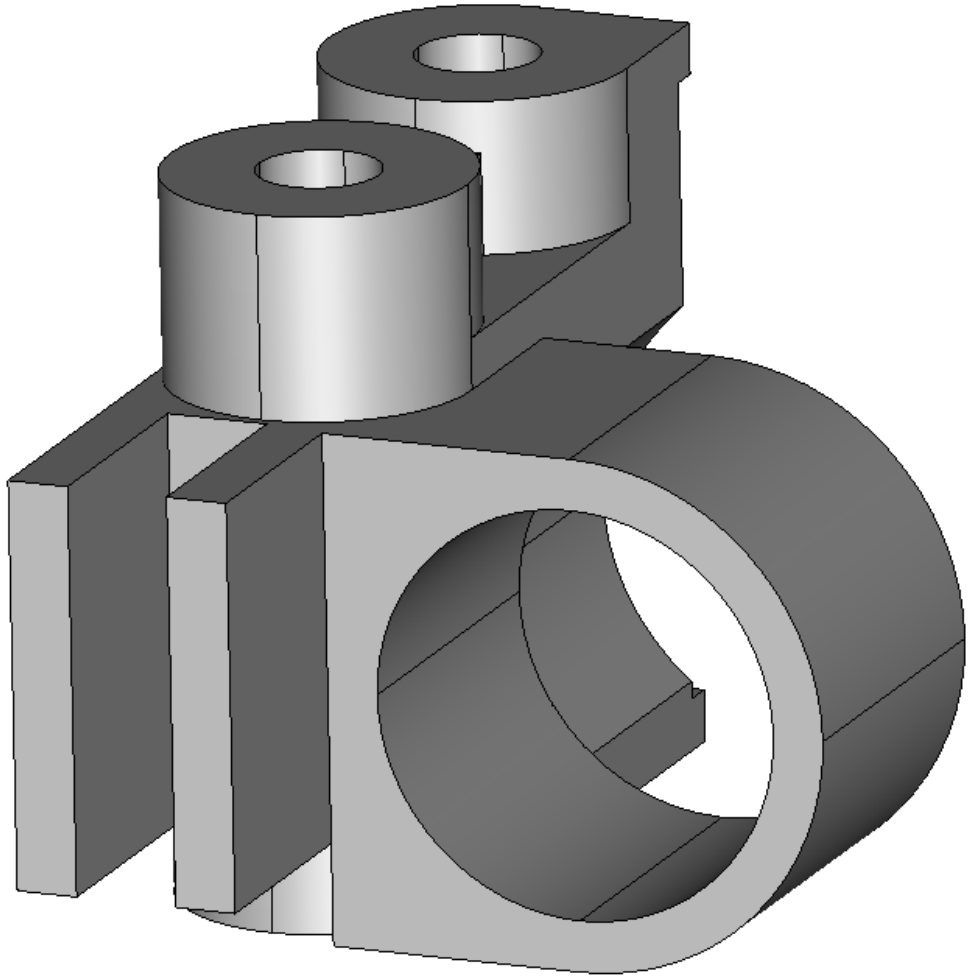
Example: suspension part

- Runtime: 21 seconds – 57 boundary faces, 289 medial faces



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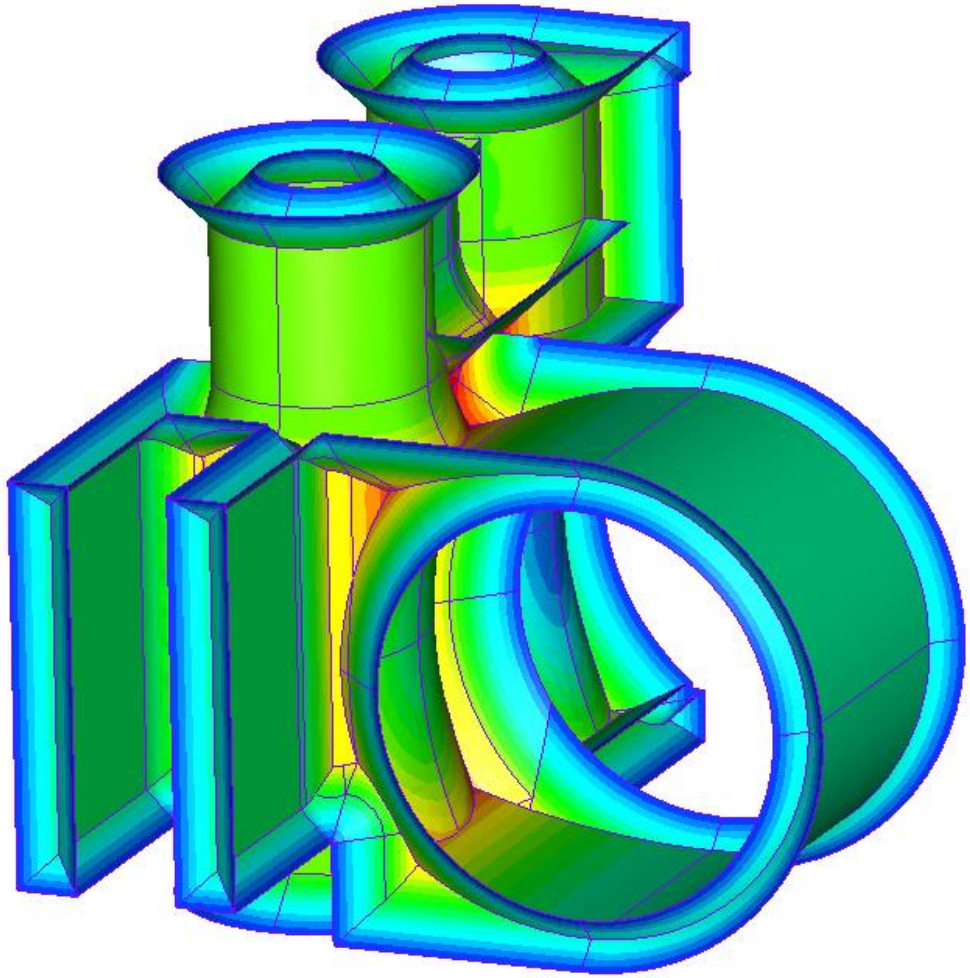
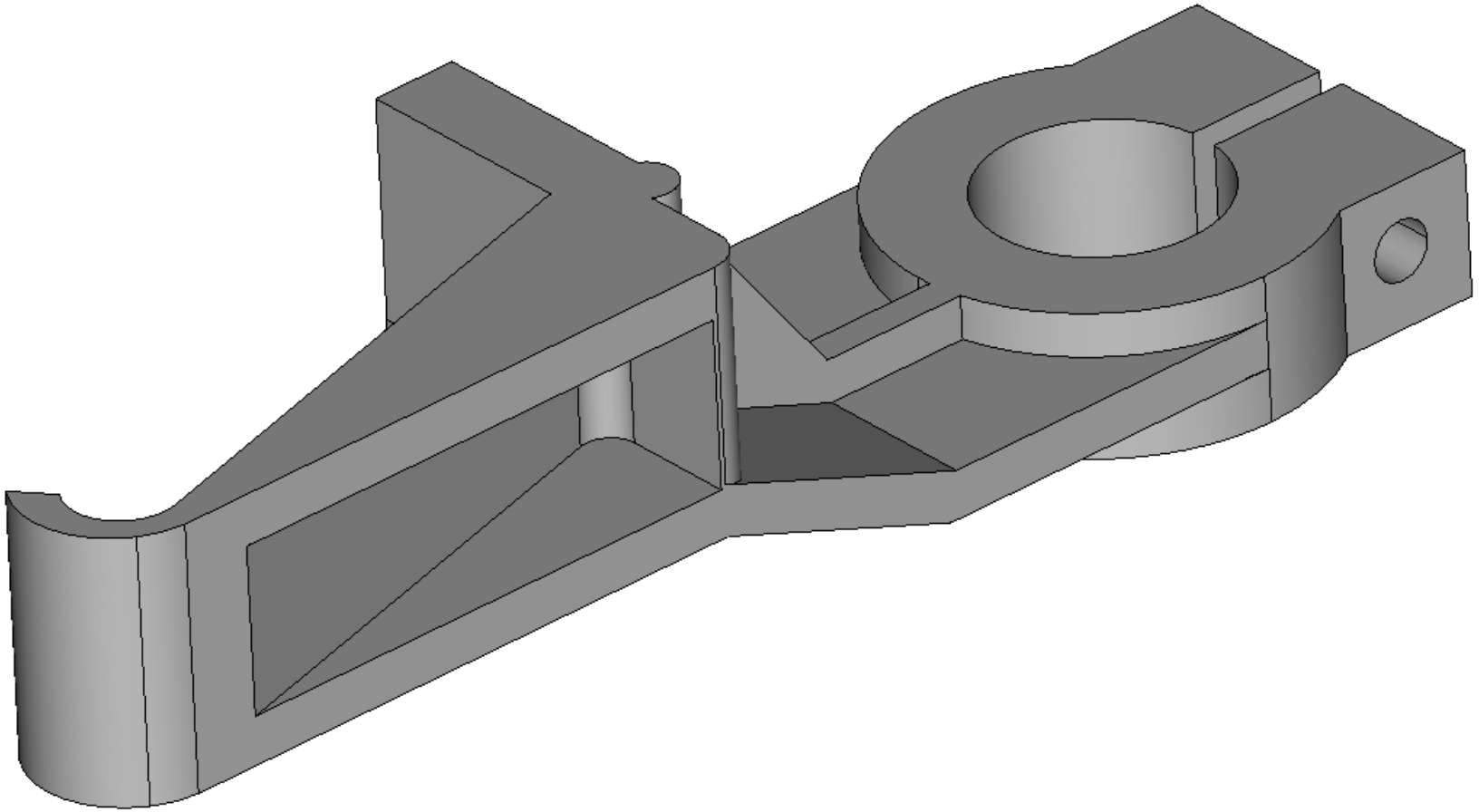


Example: bracket

- Runtime: 35 seconds – 51 boundary faces, 283 medial faces



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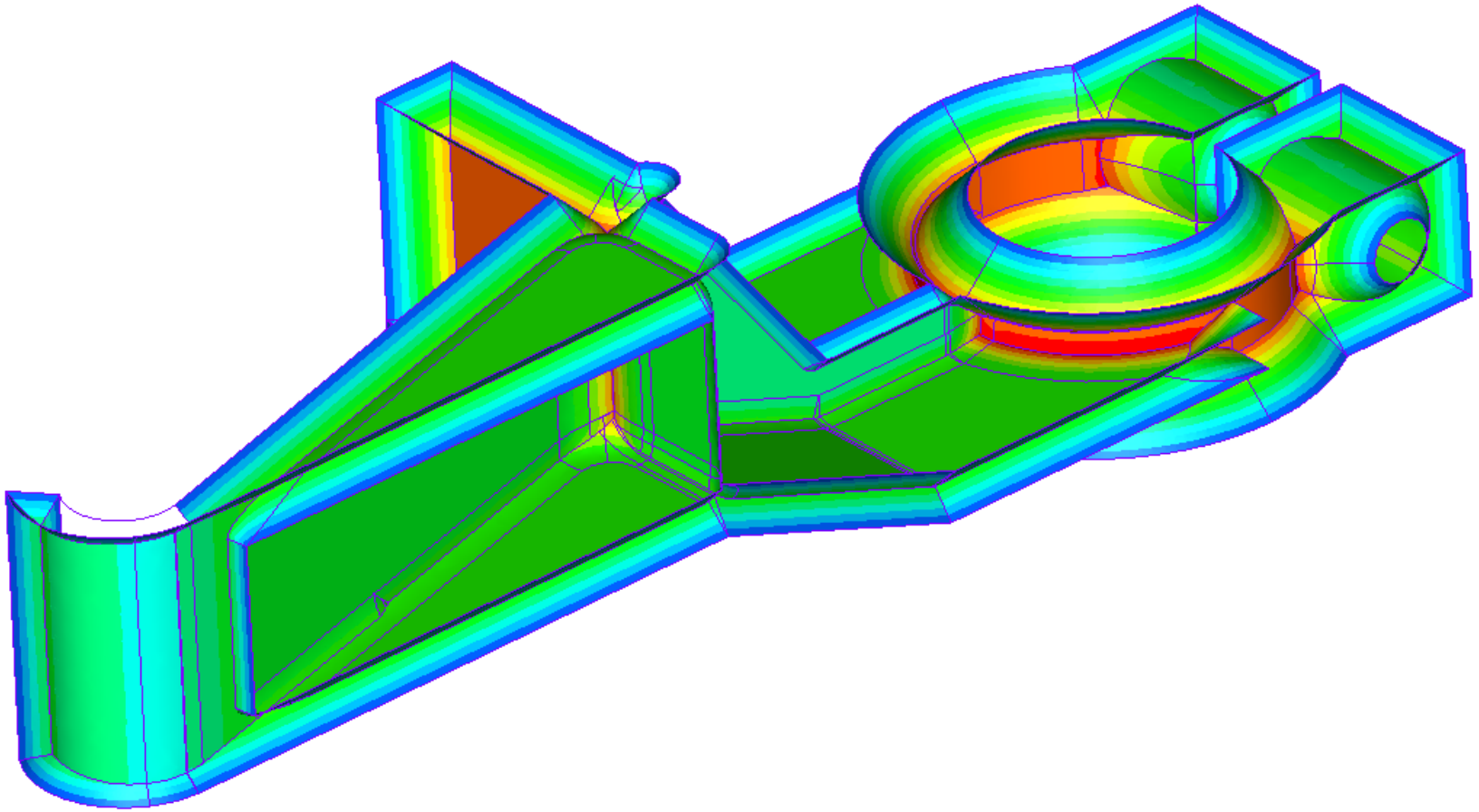
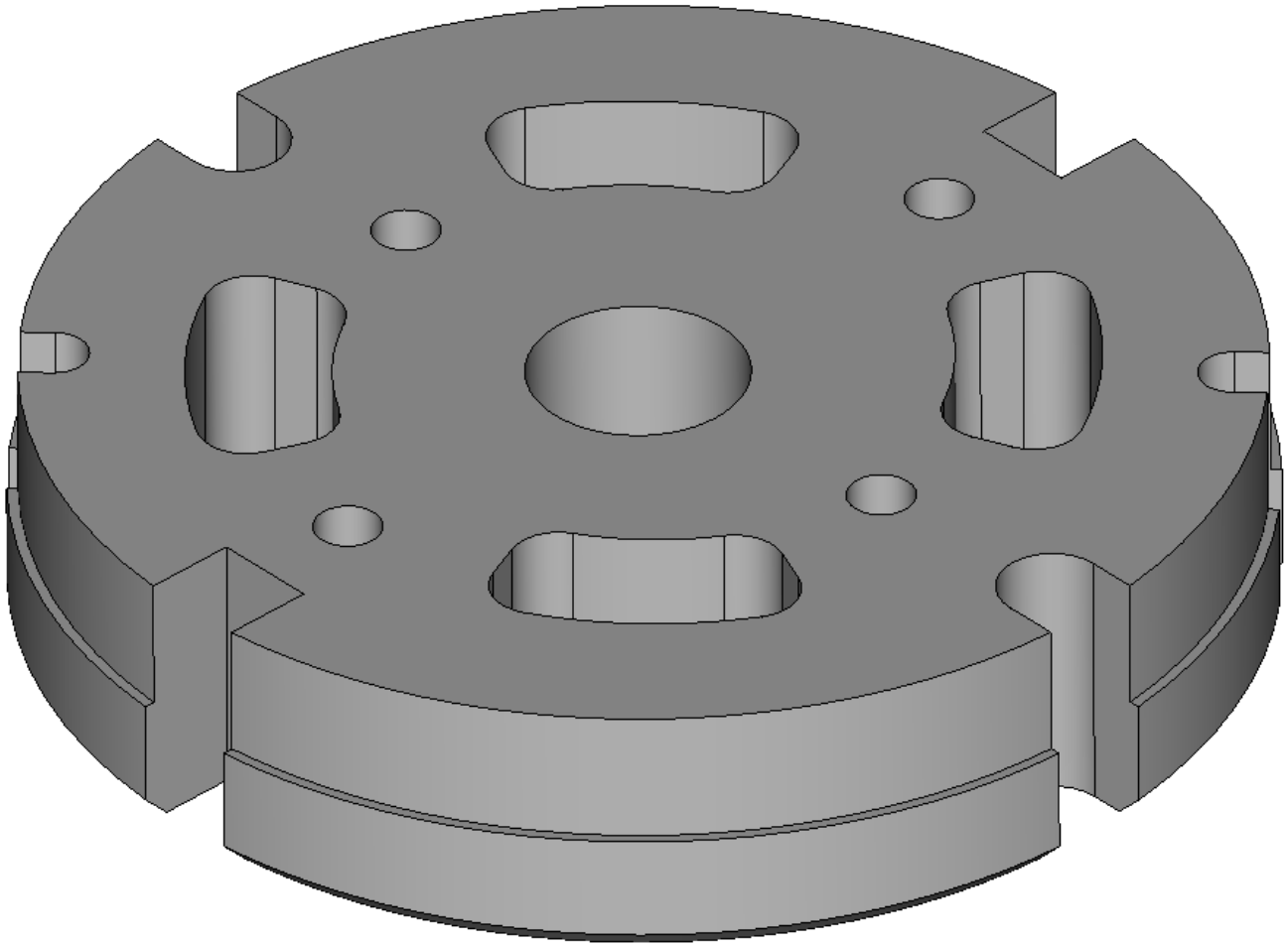


Example: motor base

- Runtime: 1 minute – 88 boundary faces, 418 medial faces



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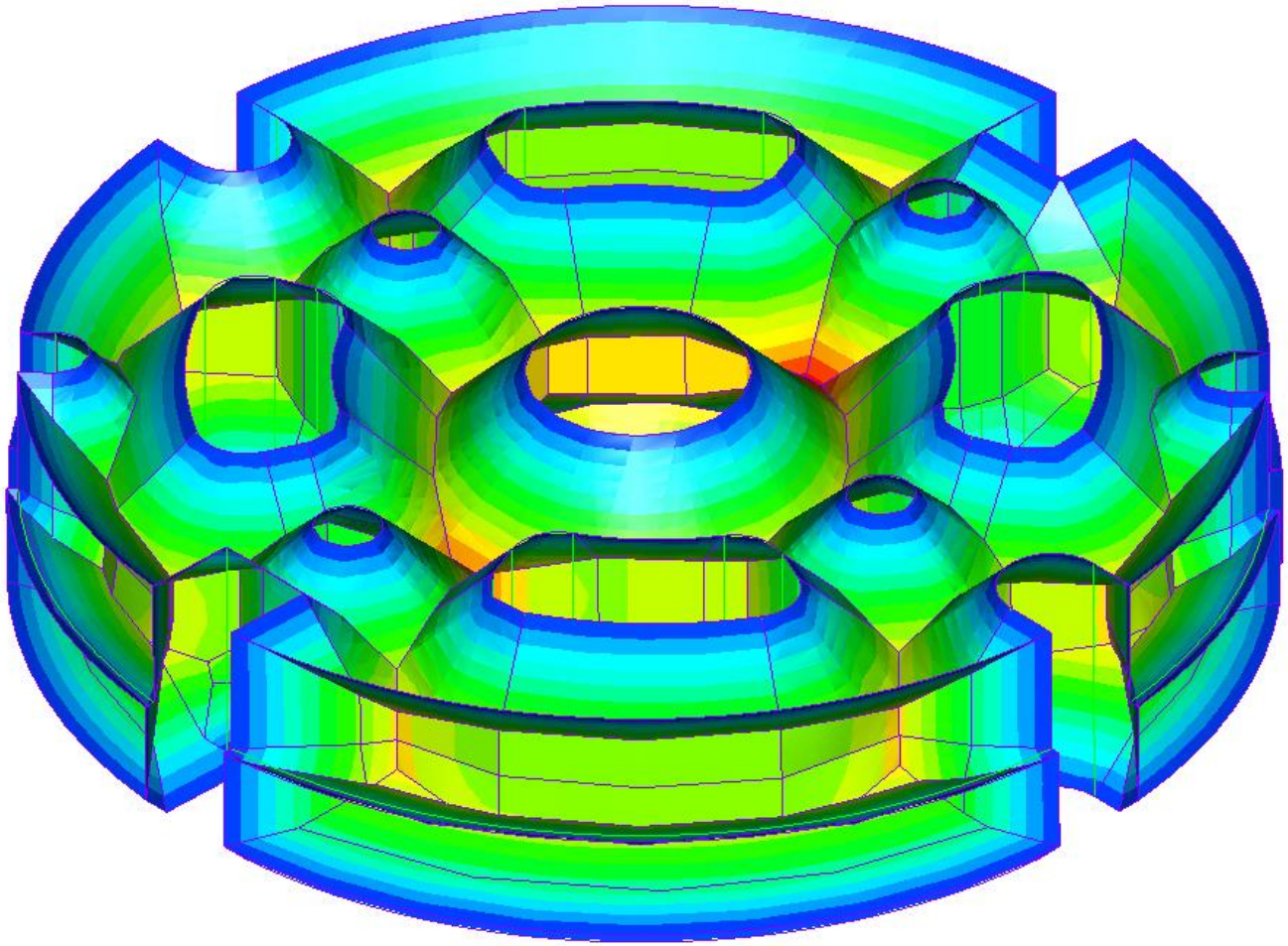
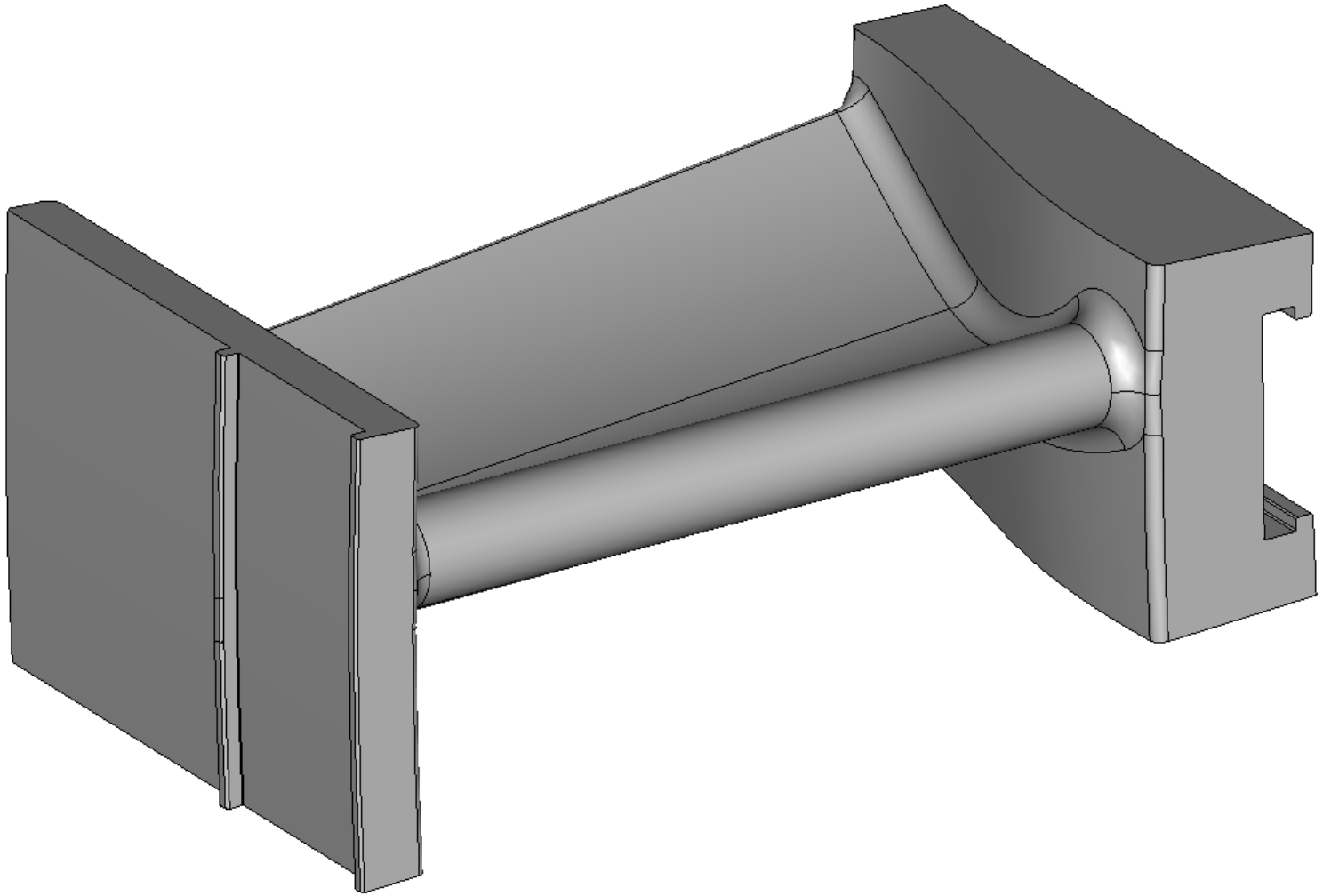


Example: turbine blade

- Runtime: 5 minutes 30 seconds – 107 boundary faces, 321 medial faces



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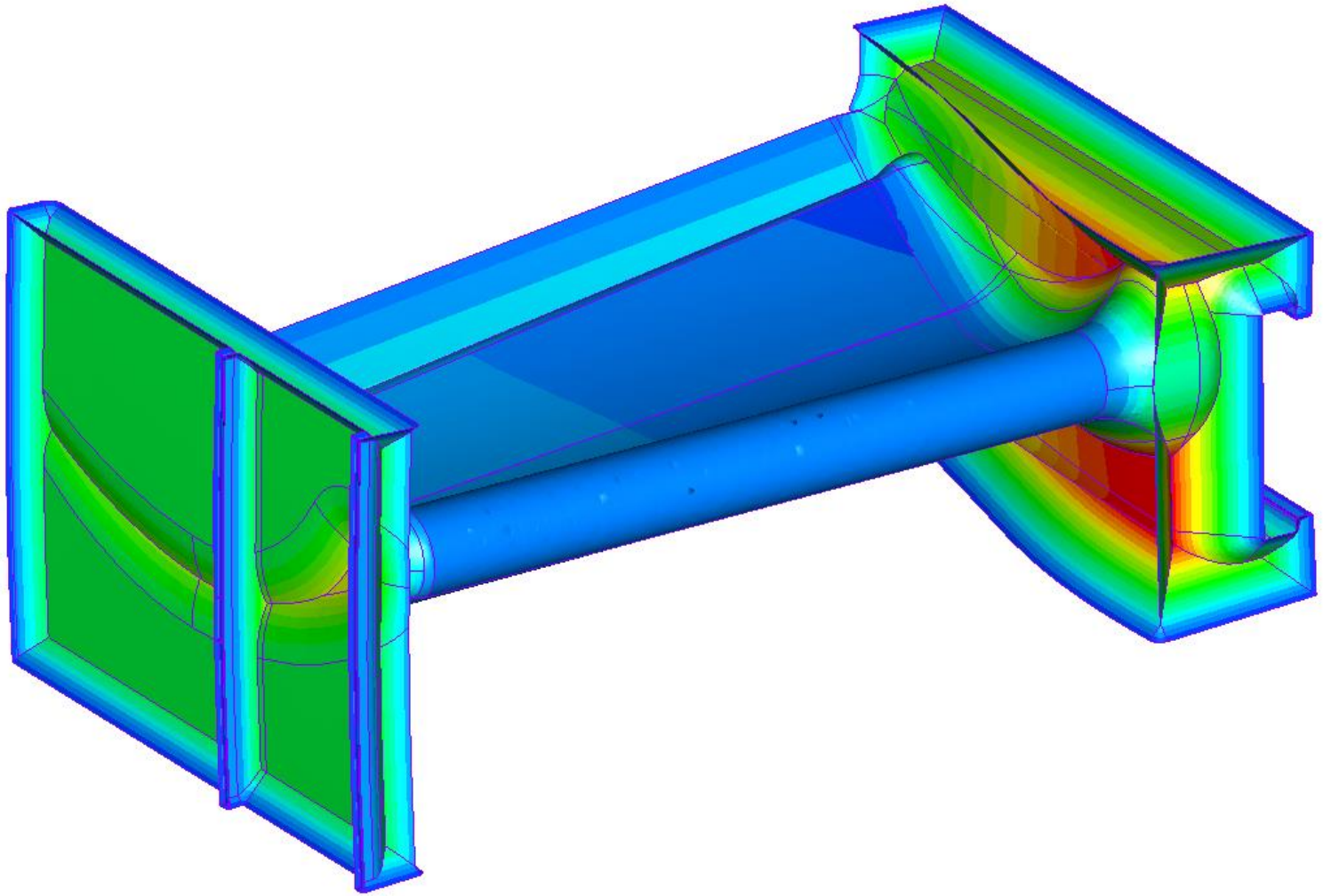
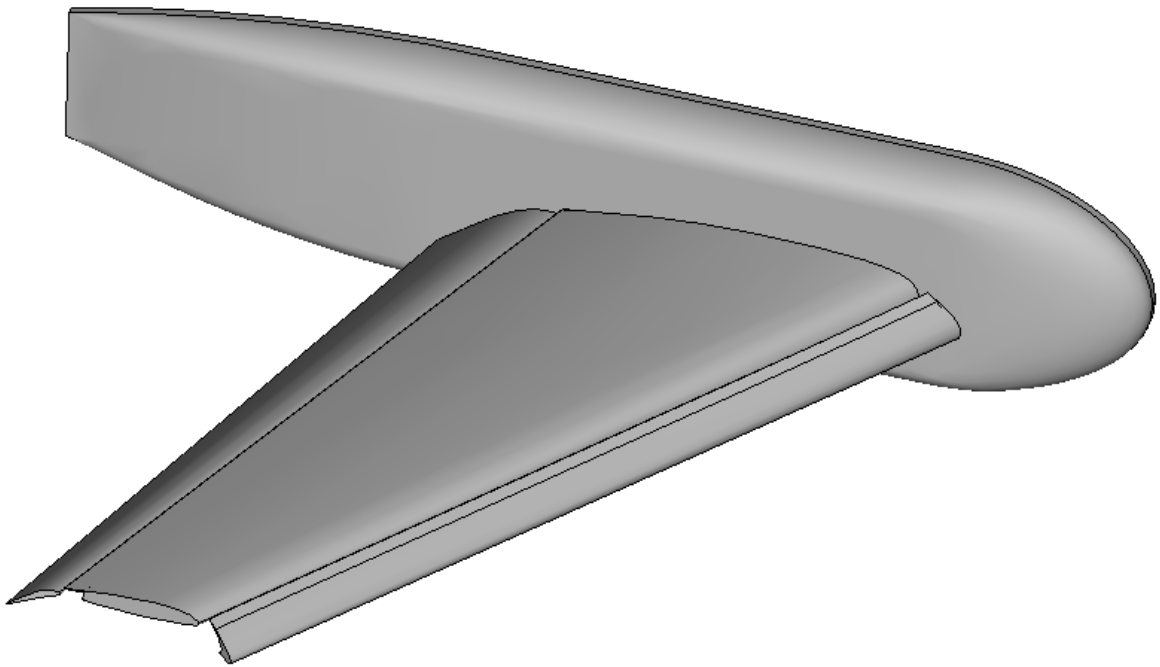


Example: NASA trap wing [external & truncated]

- Runtime: 7 minutes – 27 boundary faces, 108 medial faces



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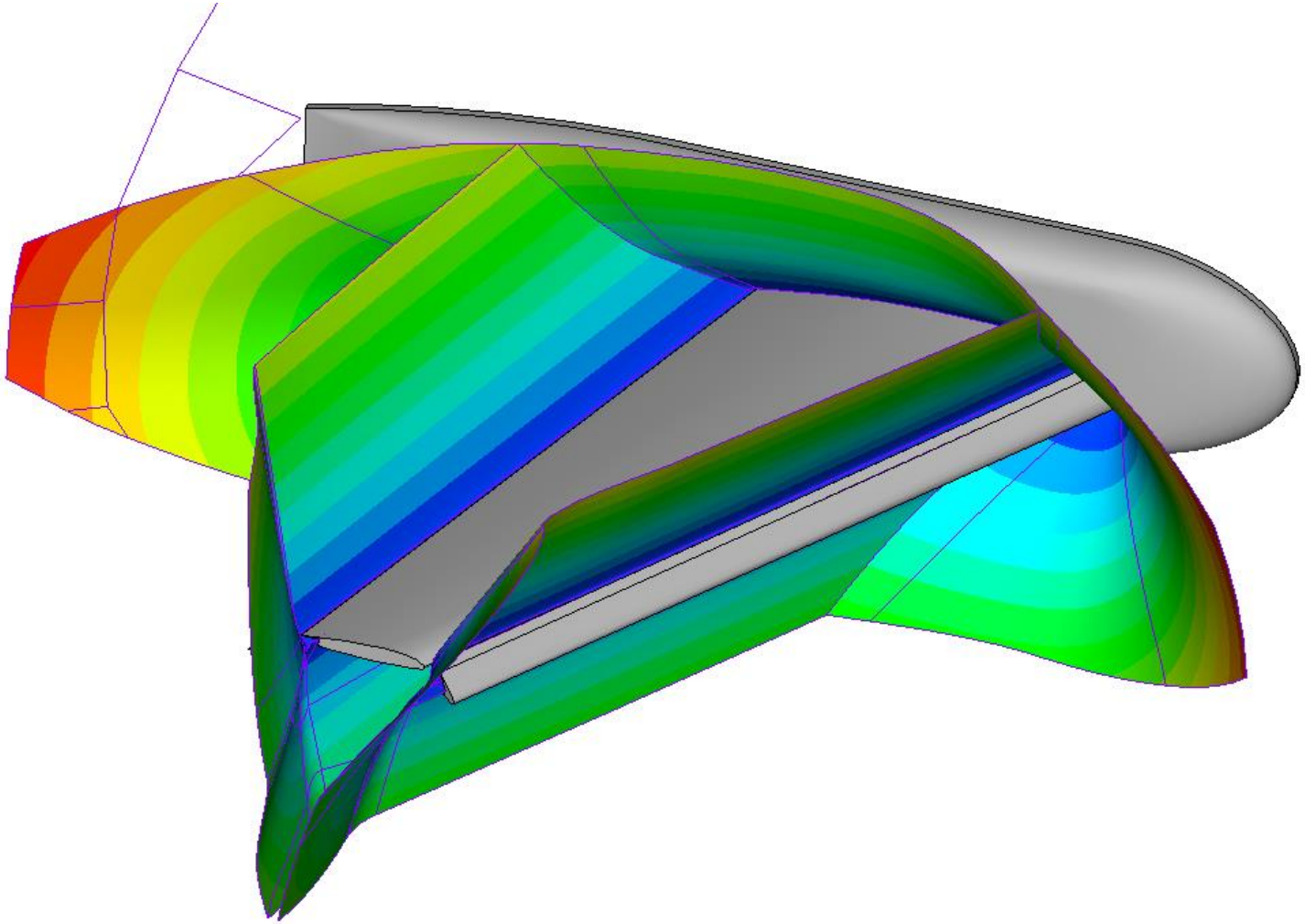

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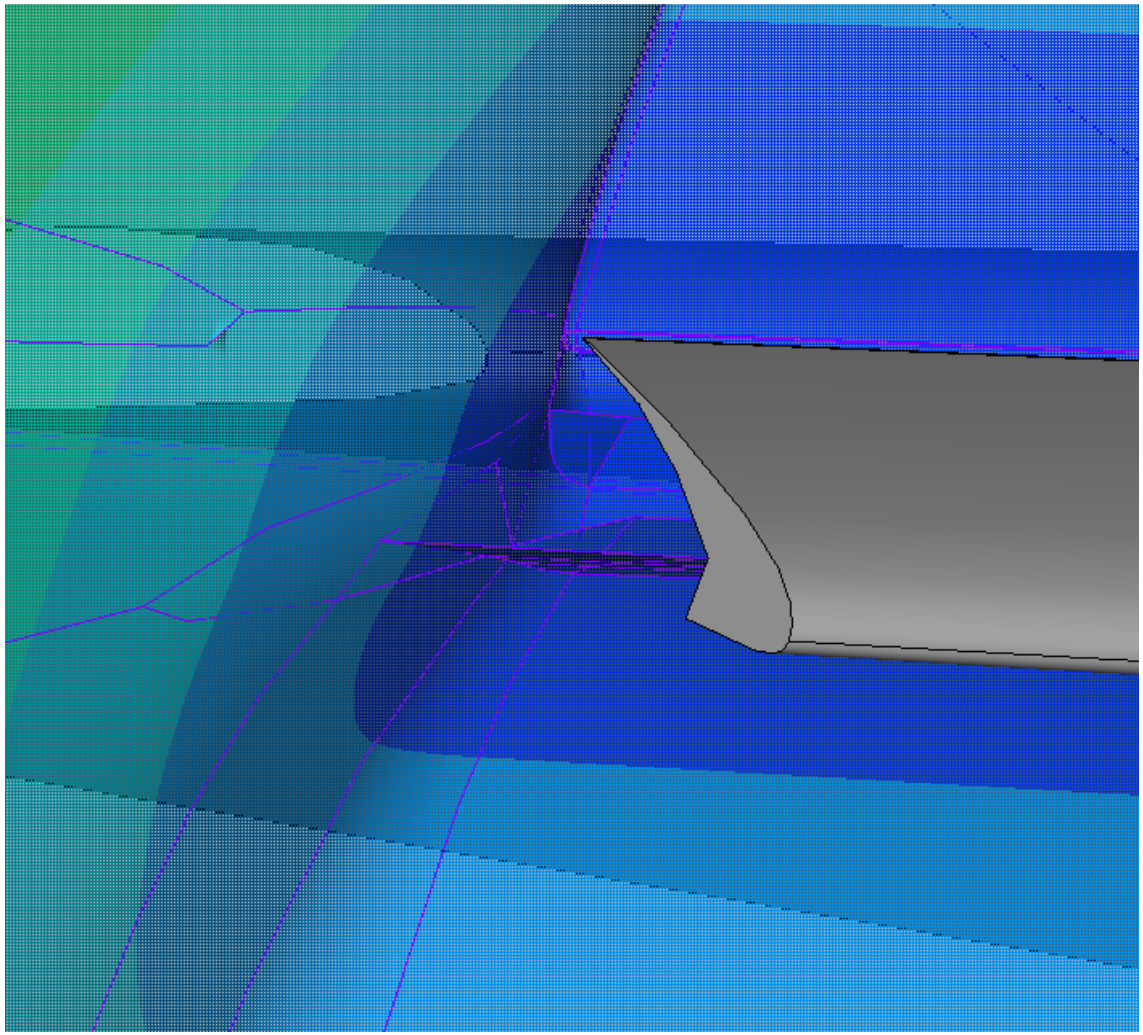

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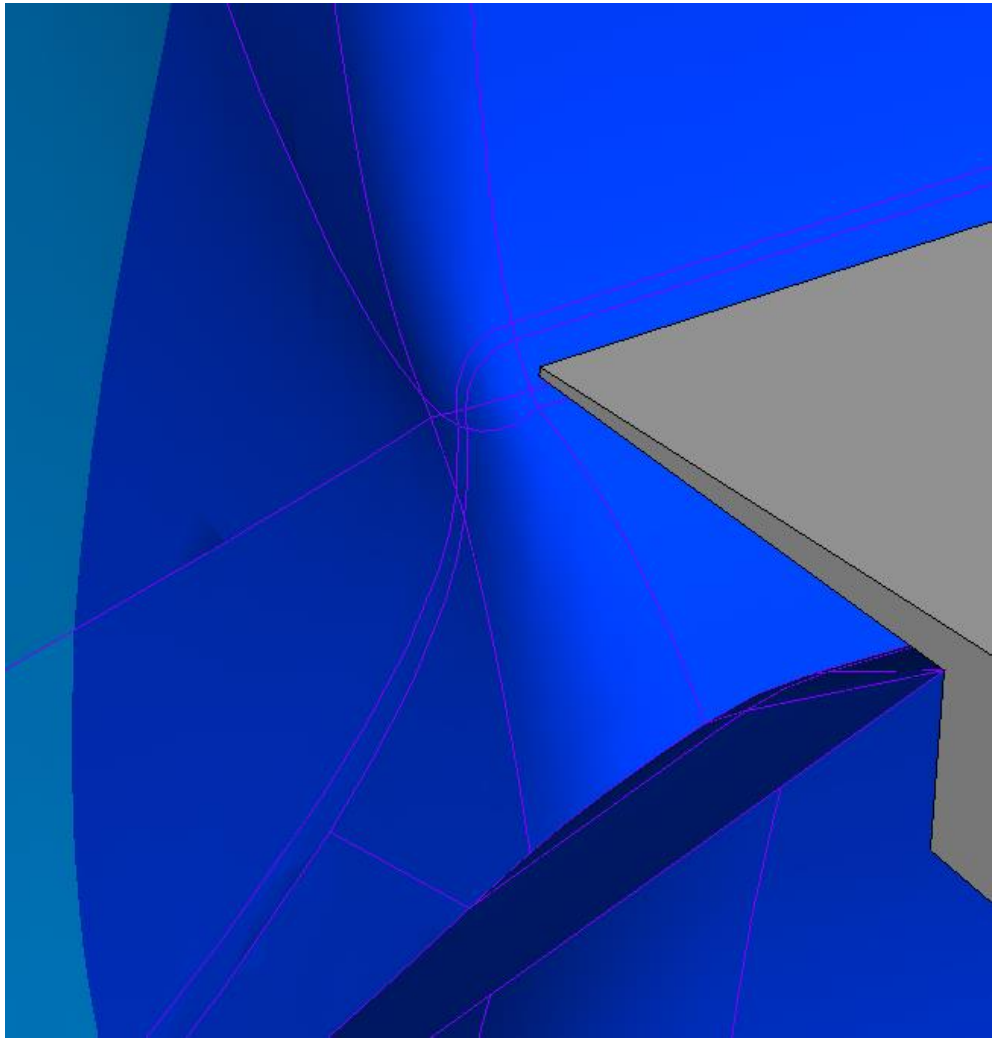
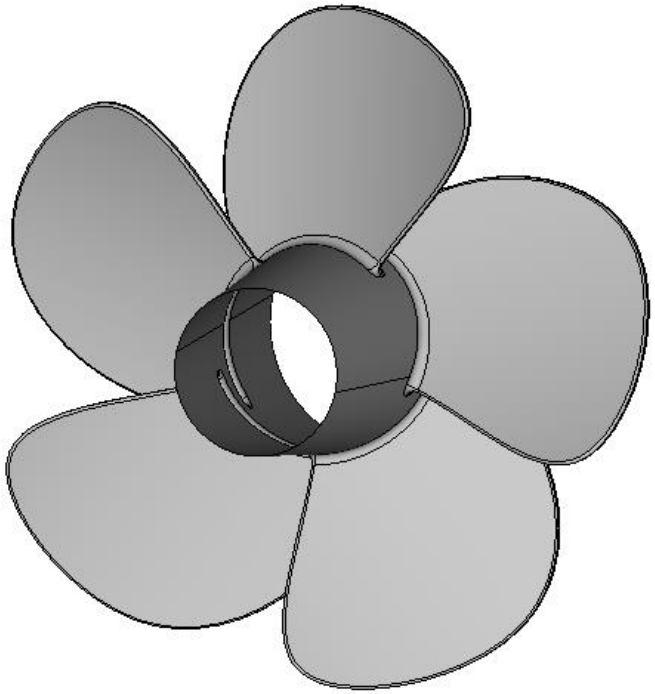


Example: five blade propeller [external & truncated]

- Runtime: 18 minutes – 73 boundary faces, 103 medial faces



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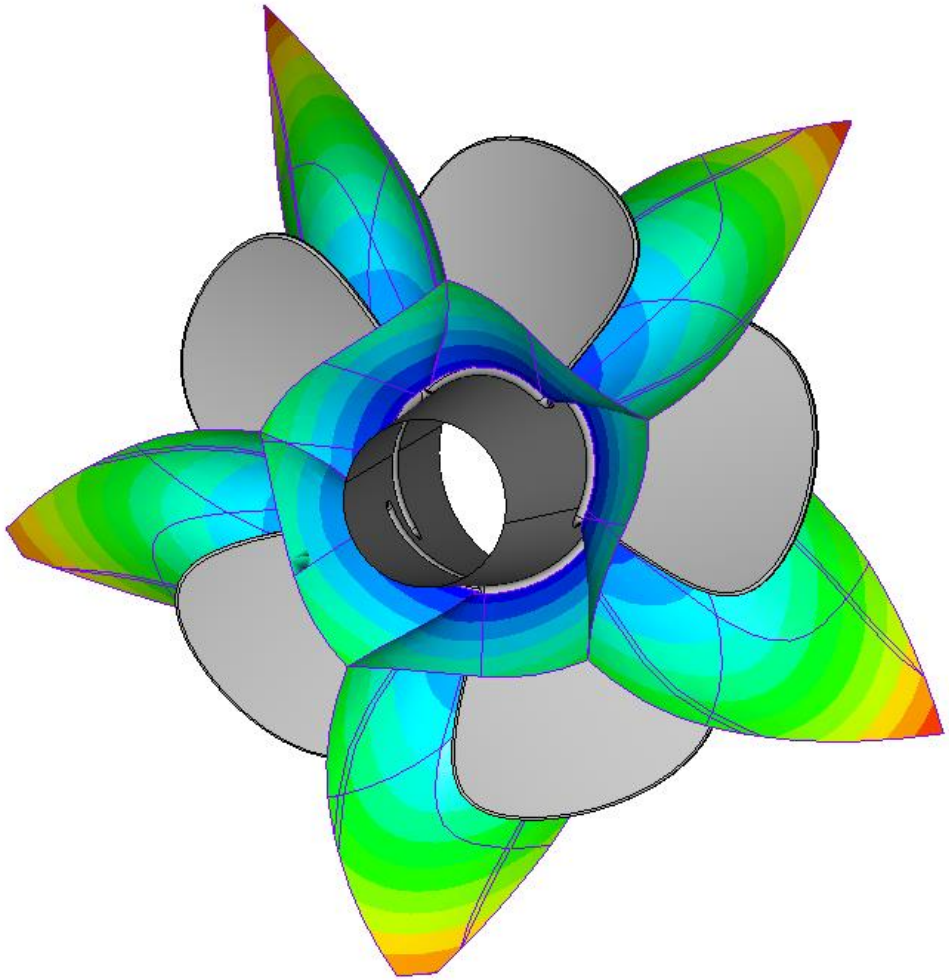

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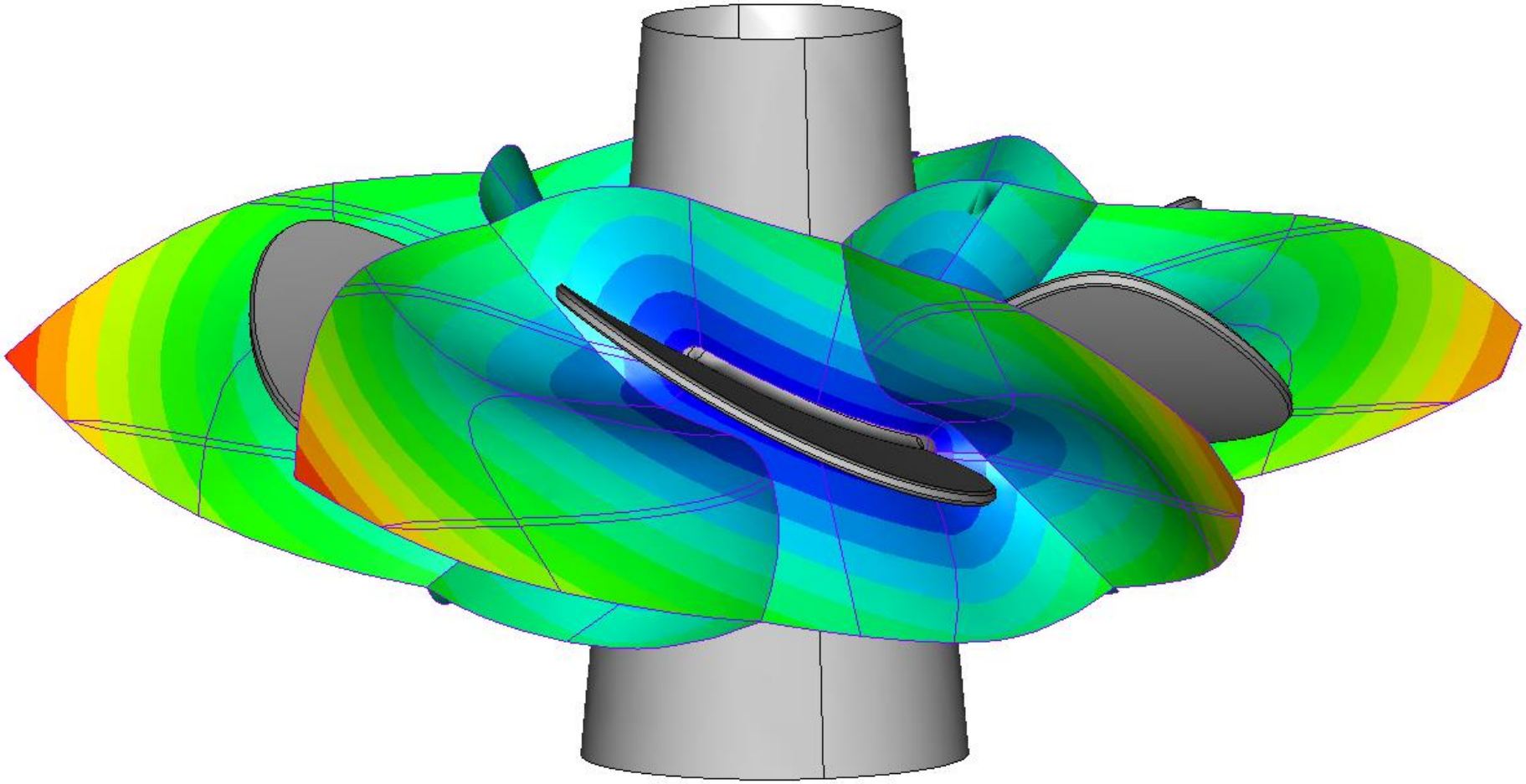

Example: five blade propeller [external & truncated]

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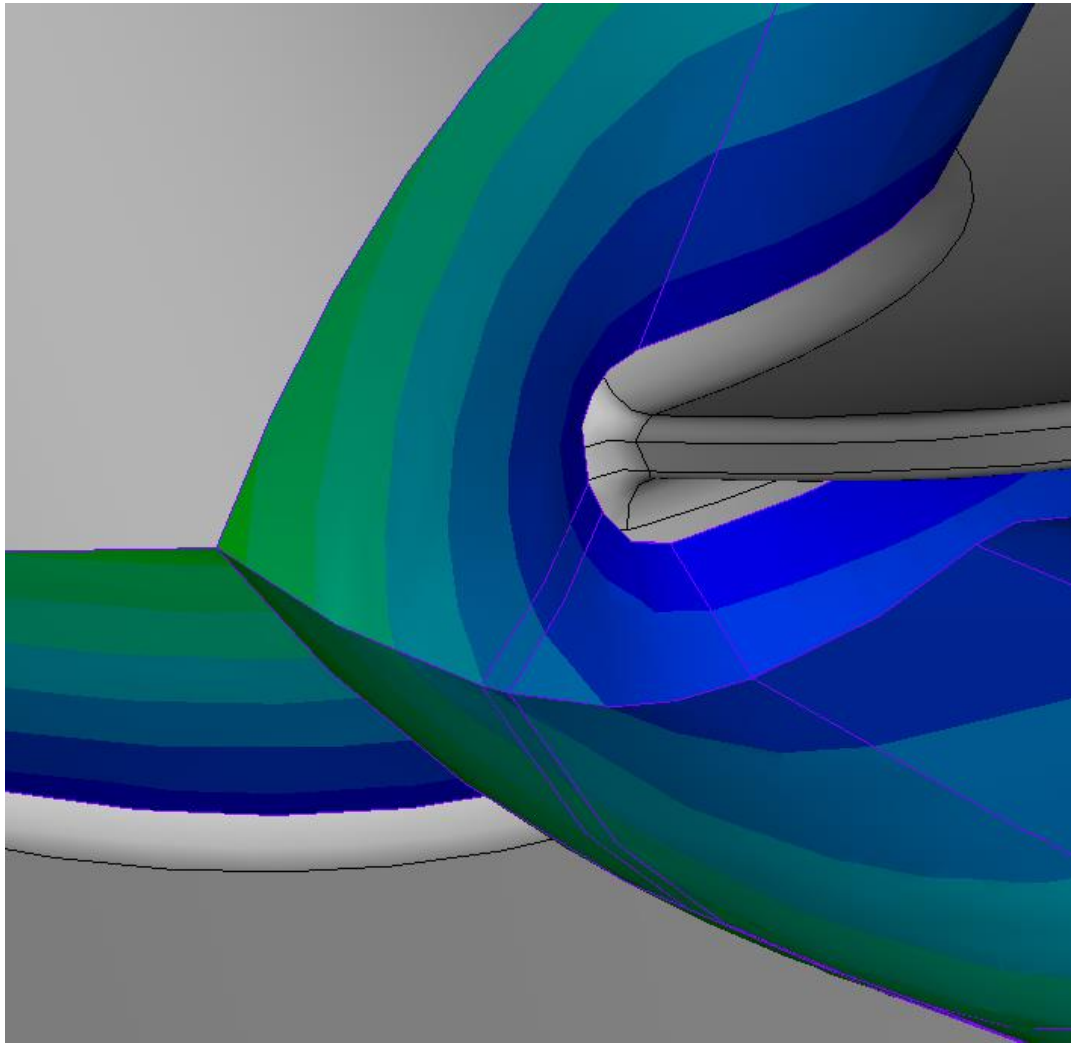
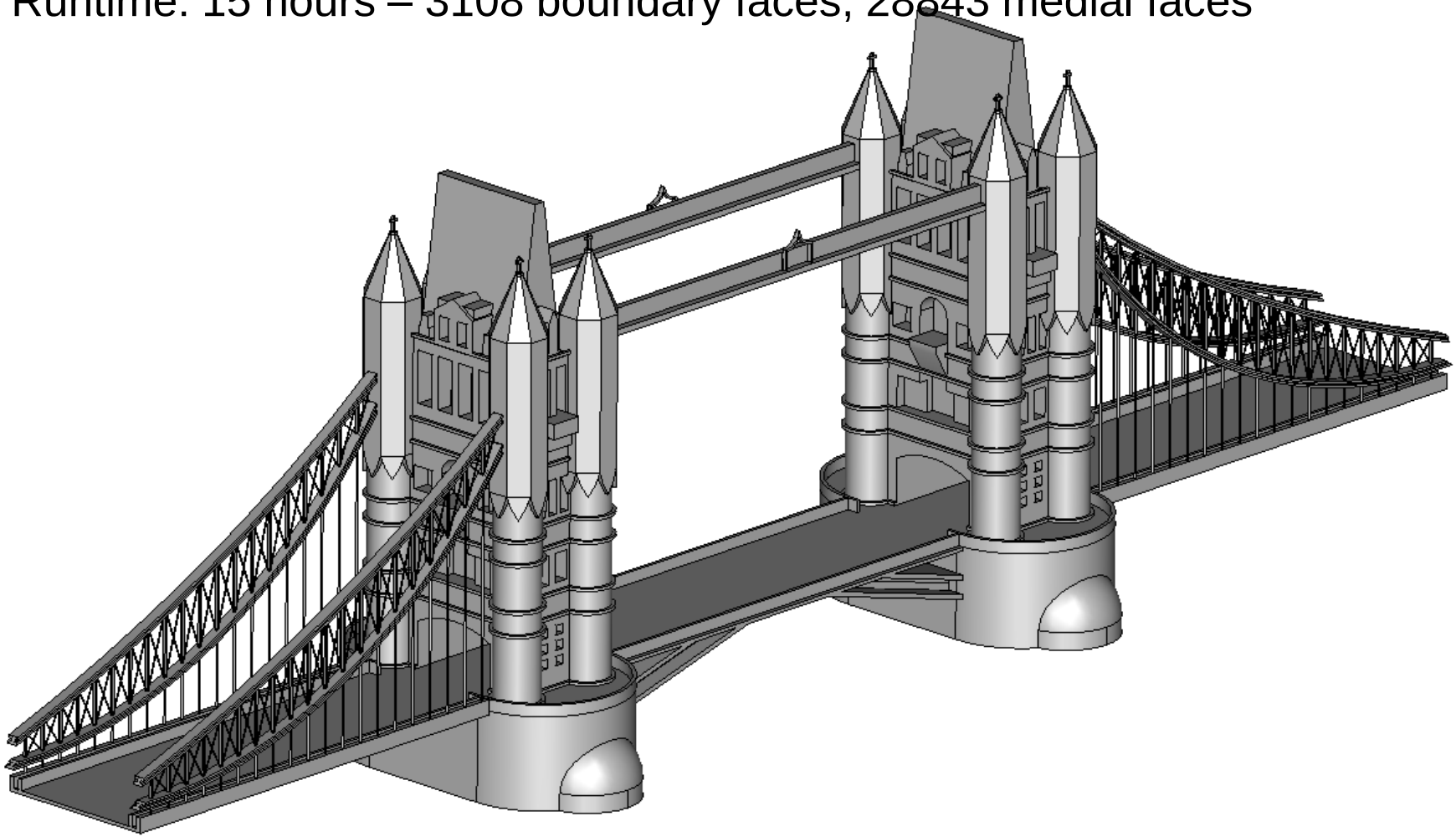


Example: Tower Bridge

- Runtime: 15 hours – 3108 boundary faces, 28843 medial faces



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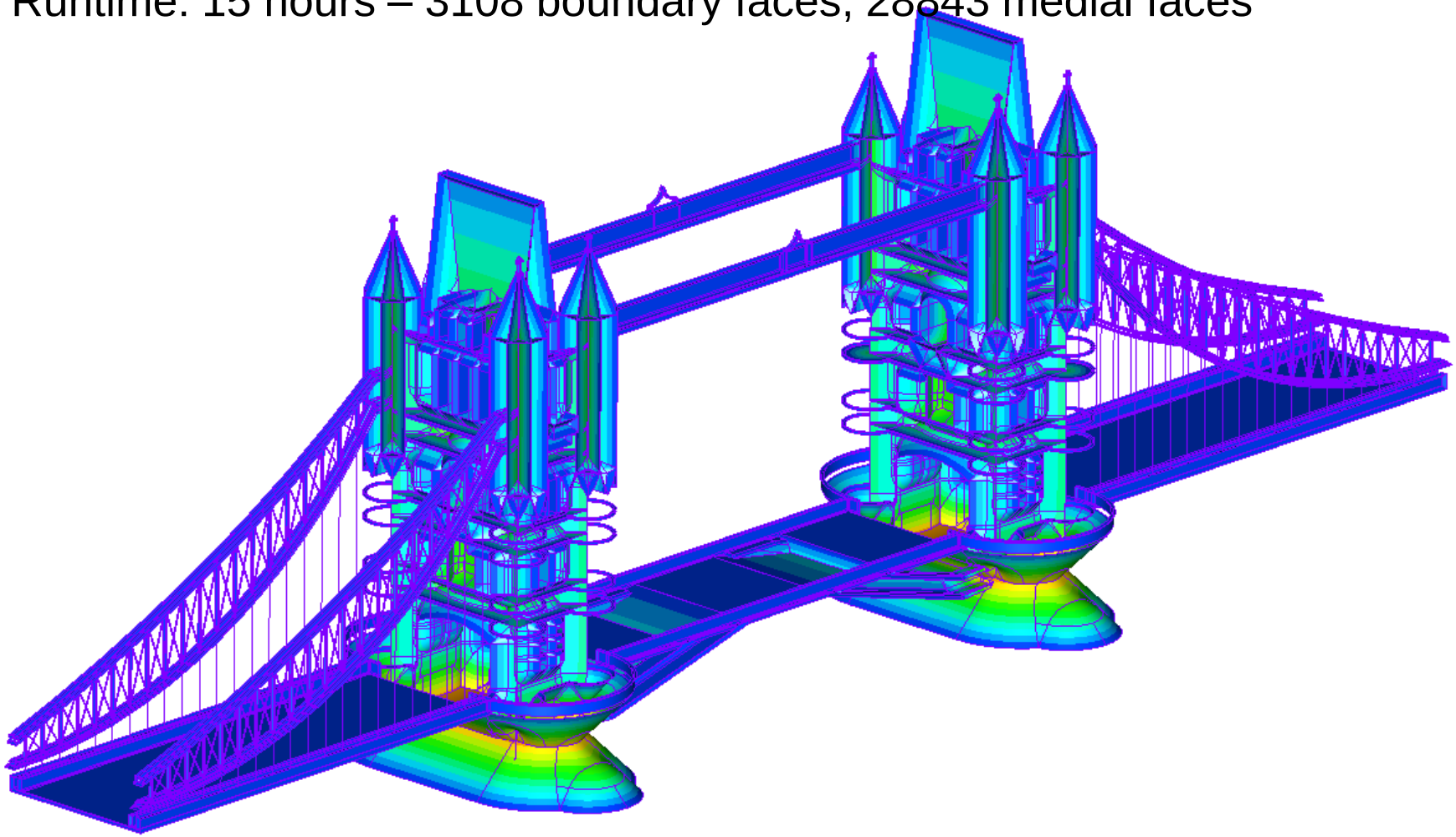

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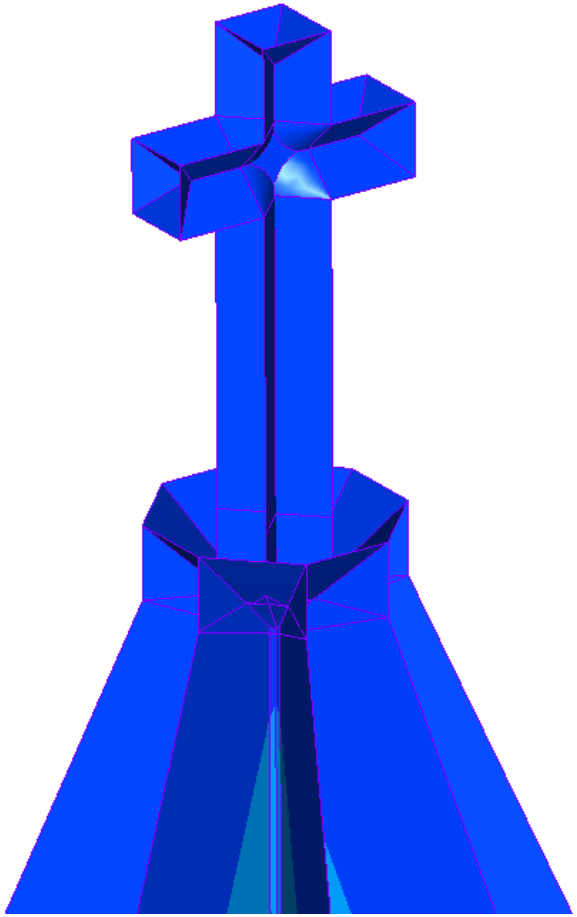

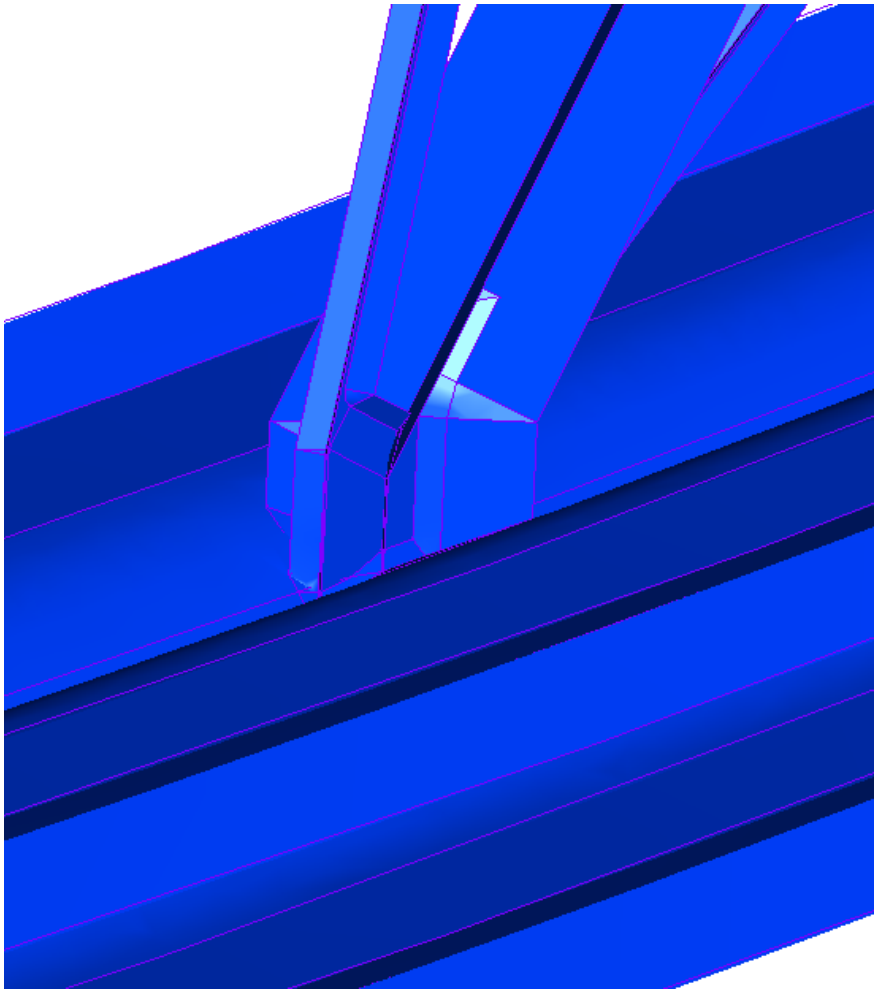

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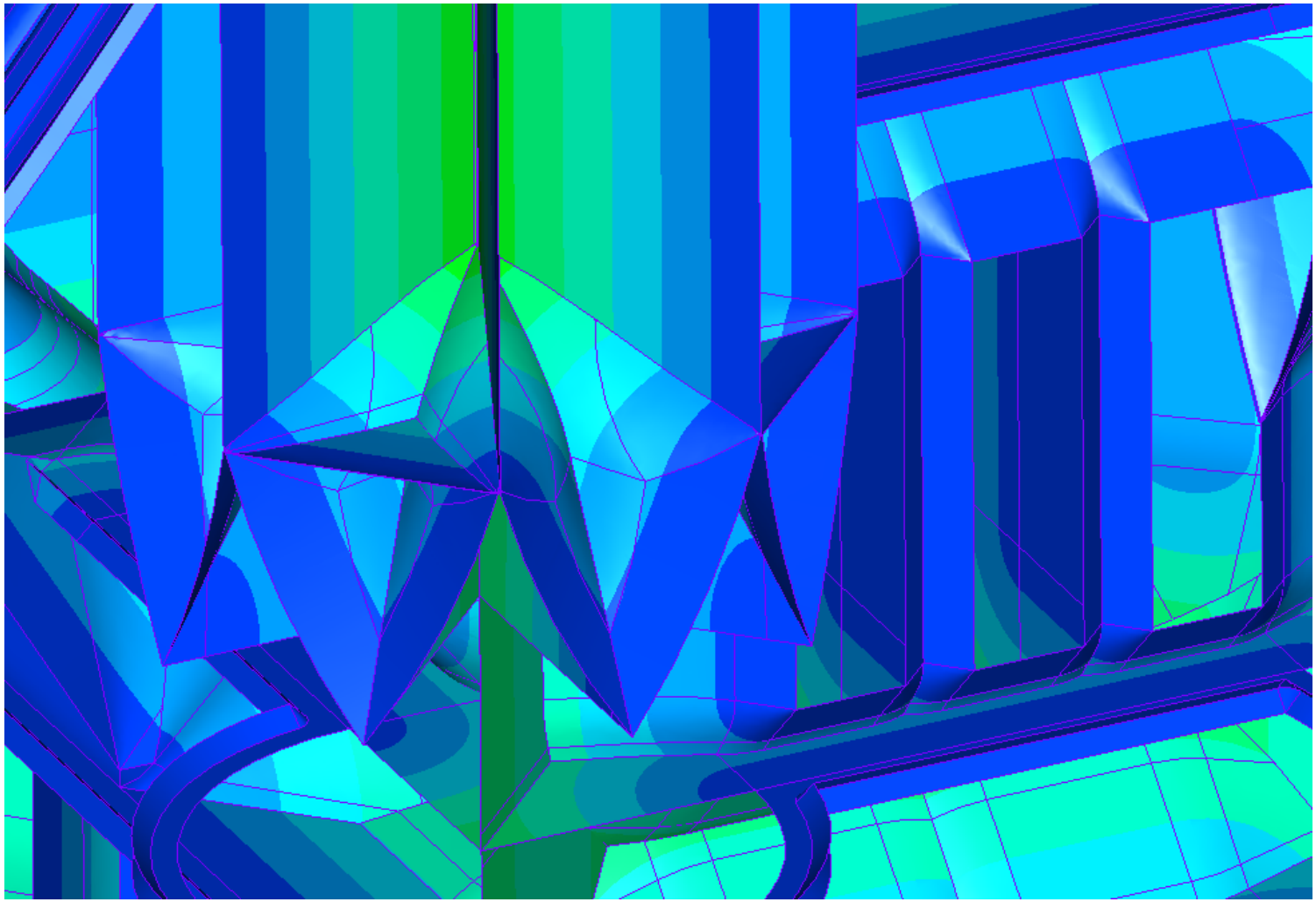
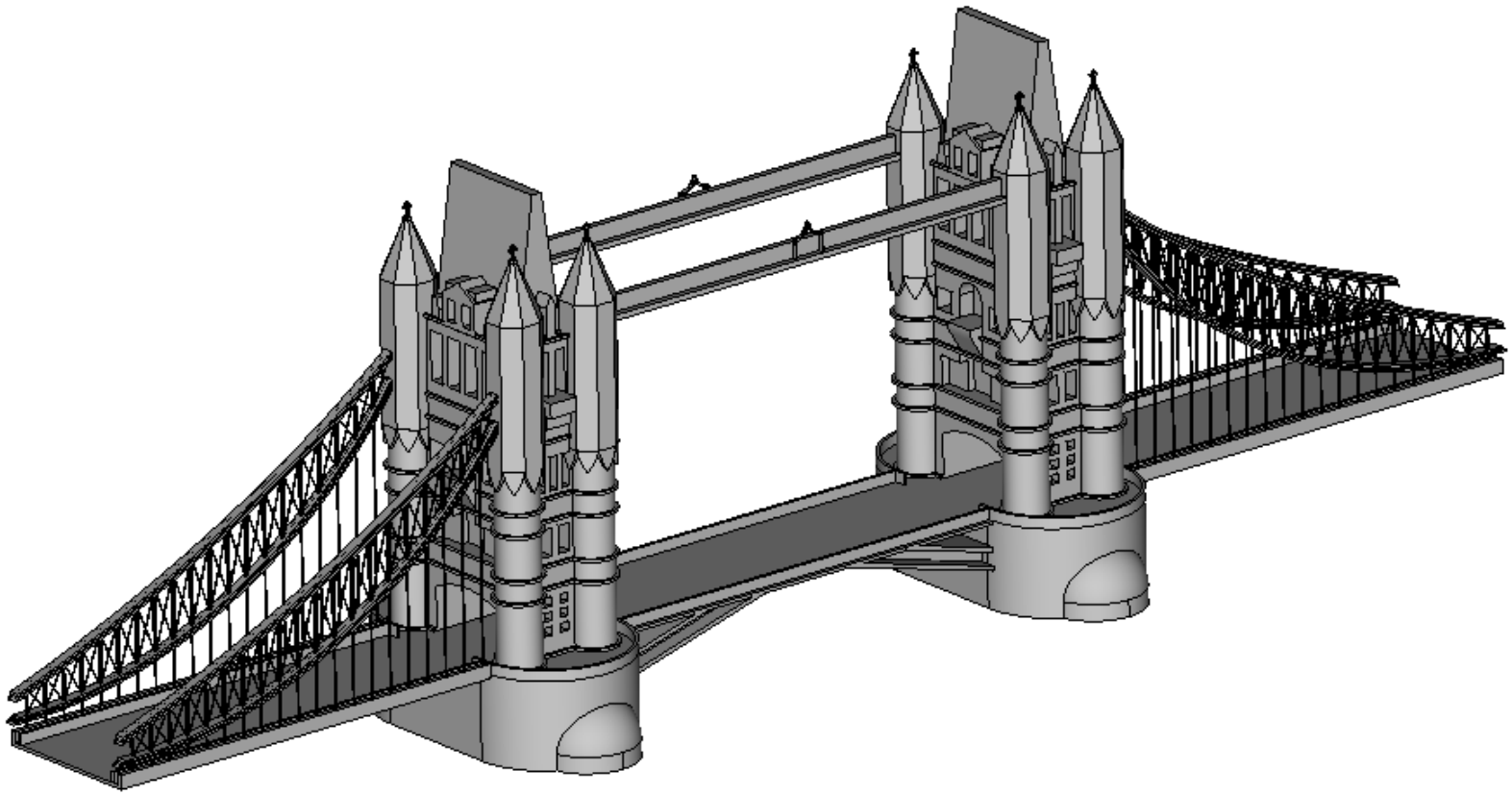


Example: Tower Bridge [external & truncated]

- Runtime: 9 hours 40 minutes. 3114 boundary faces, 22133 medial faces



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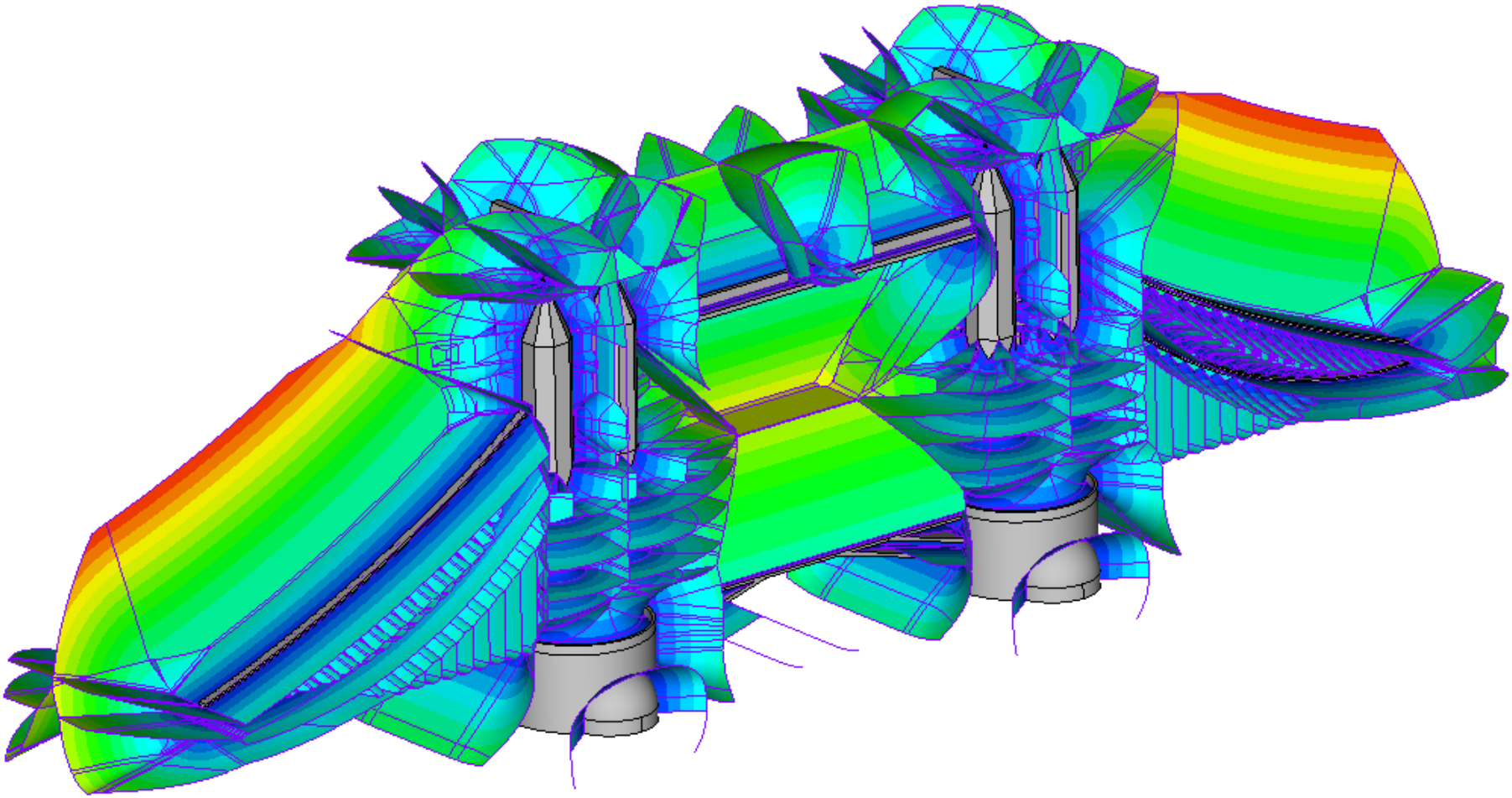

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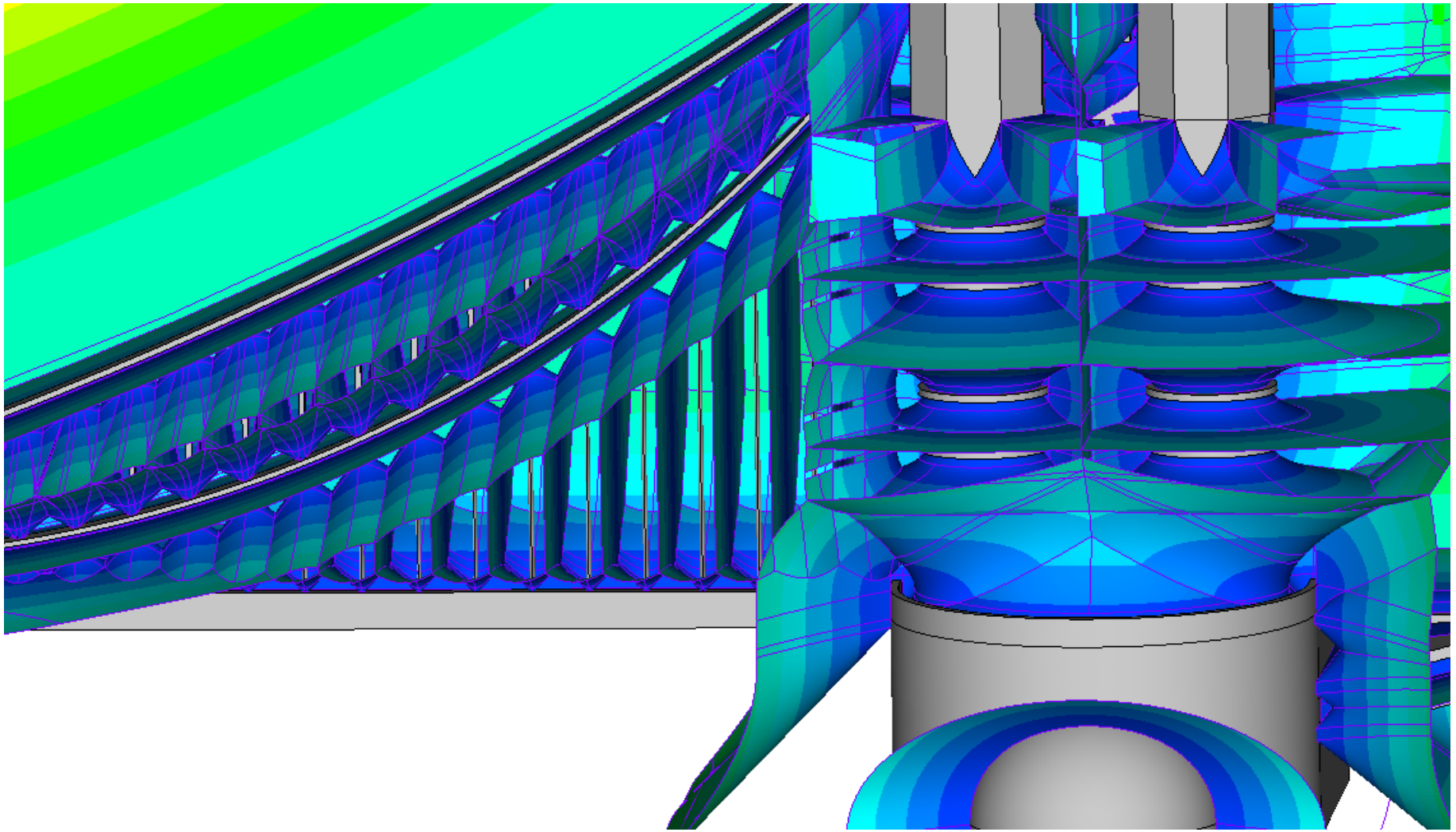

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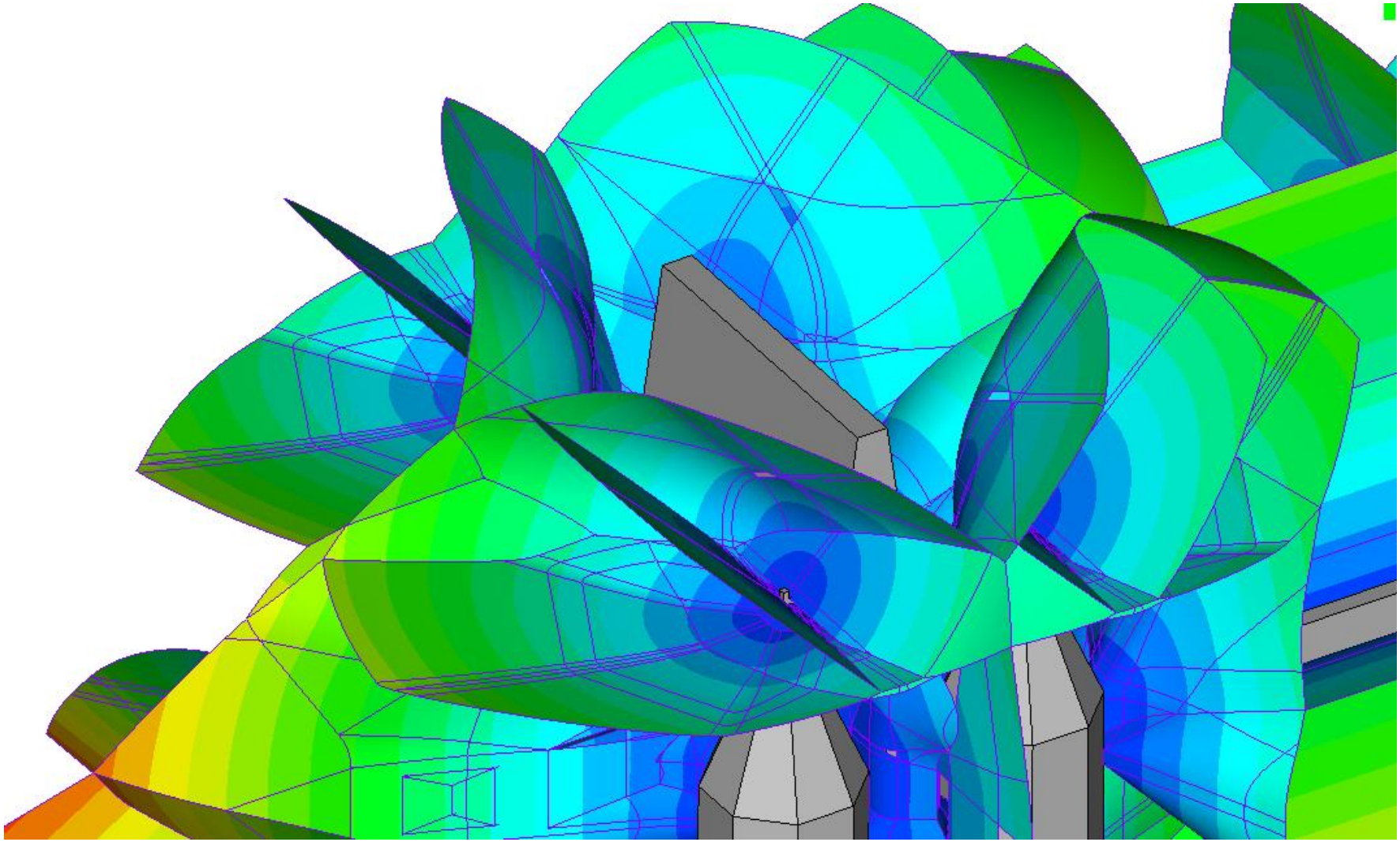
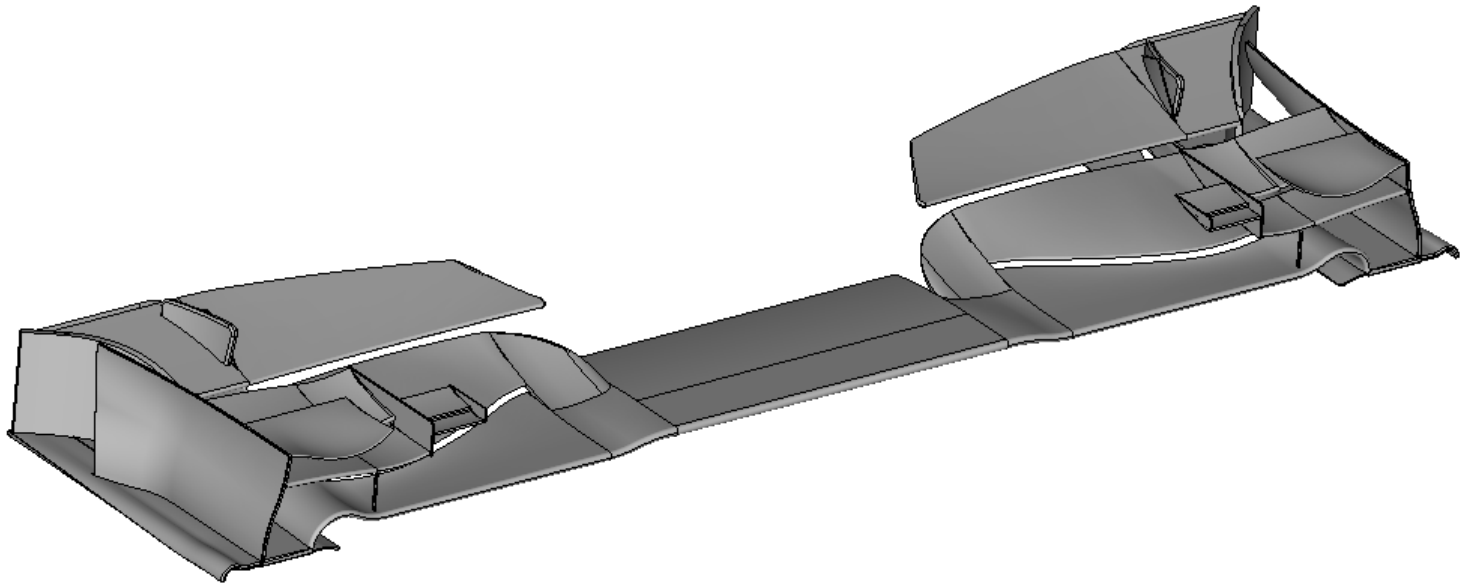


Example: Formula 1 front wing [external & truncated]

- Runtime: 5 hours 30 minutes – 441 boundary faces, 1555 medial faces



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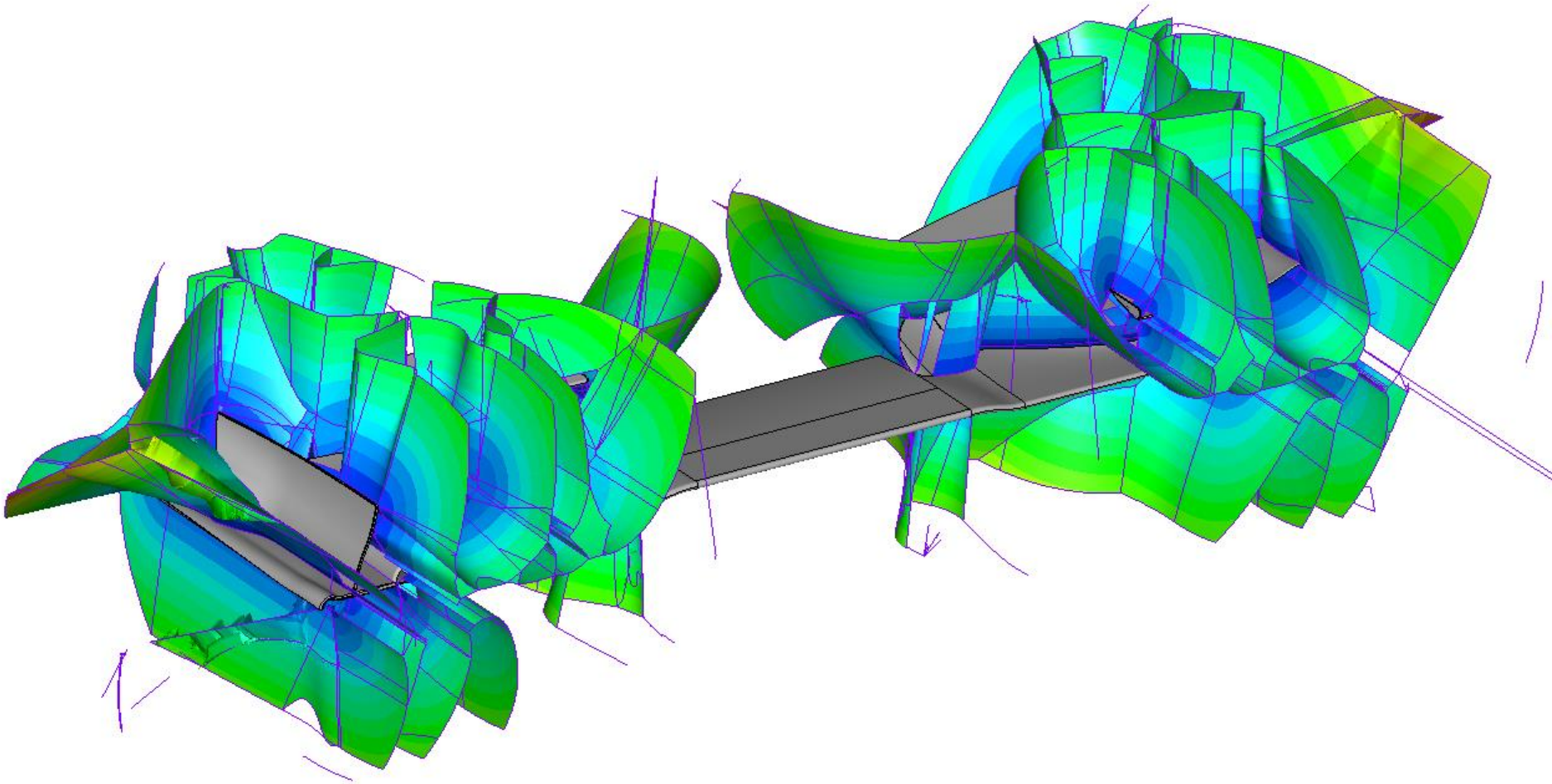

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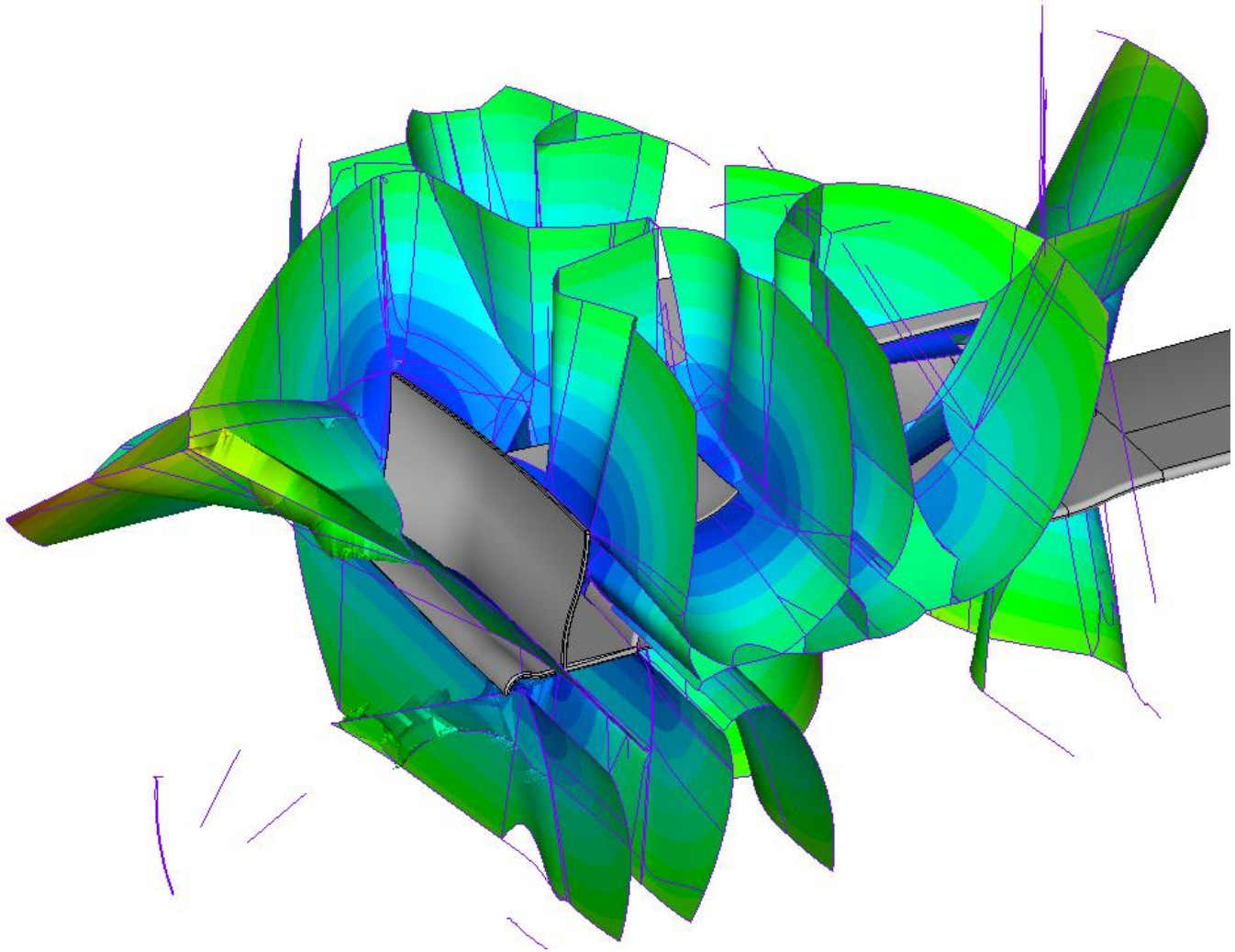

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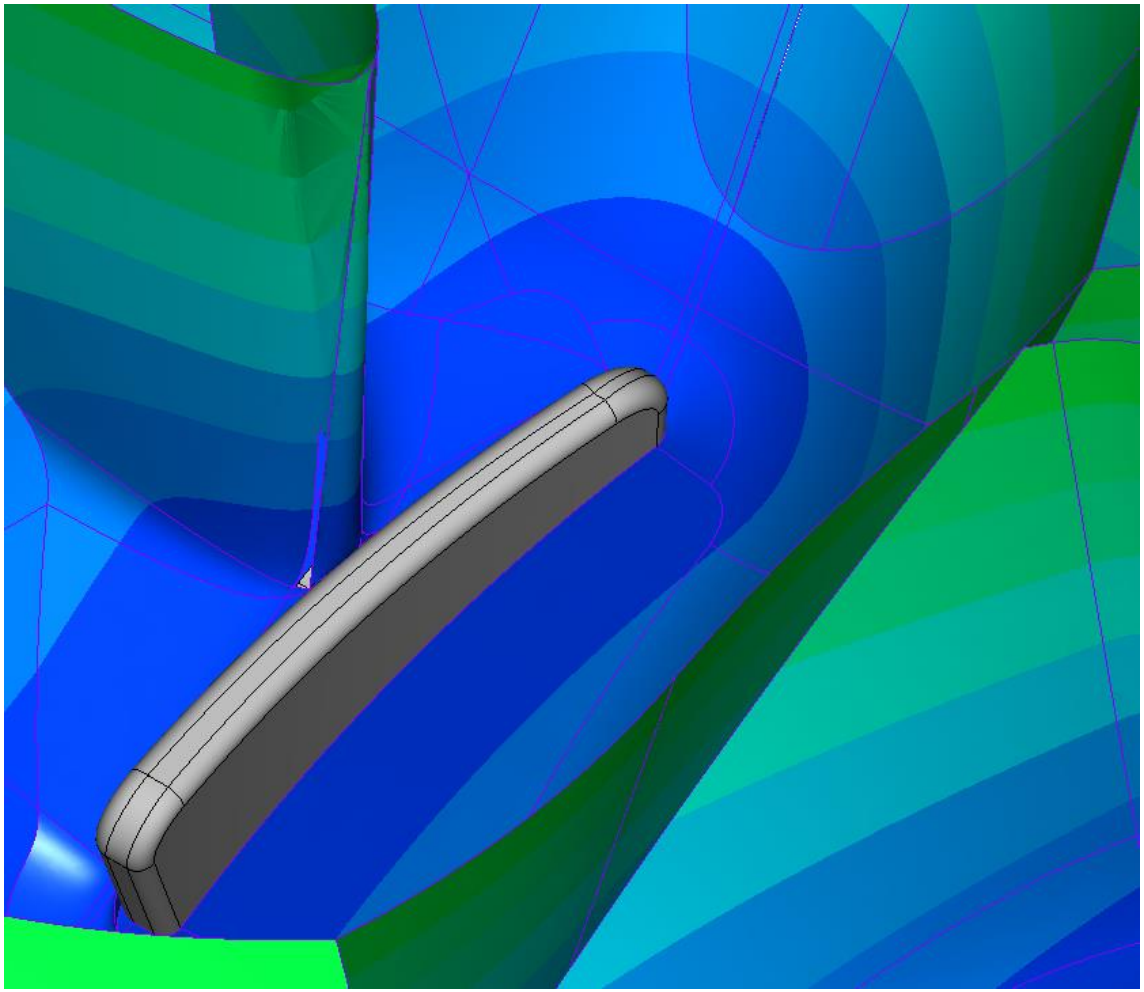

Contents

- Medial object: medial axis as a CAD model
  - Plus relationships with boundary geometry
- Why compute the medial object?
- History
- Basic principles
  - Using a Delaunay triangulation
  - Tolerant computation
  - Robustness
- Geometry preparation
- Our algorithm
- Medial object API
- Examples
- The future

The future

- Loose ends
  - Remaining areas where topology is not discovered correctly
  - Medial geometry may not reach required accuracy
  - Possible to miss disconnected medial edge networks
- Incremental generation
  - Would like to update medial object when CAD geometry is altered
- Performance
  - Has not been a focus for us
  - Currently okay for batch processes, not good interactively
  - Finite contact handling is expensive
- Parallelisation
  - Current algorithm is serial
  - We have ideas for parallelising several areas

Conclusions

- The 3D medial object is the medial axis transform as a CAD model
  - Computed from a CAD model
  - With relationships back to boundary CAD entities
- CADfix contains a mature algorithm for computing 3D medial objects
  - Based on a Delaunay triangulation
- Tolerant computation and geometry preparation is key
  - Combat medial axis instability to get expected results
  - Compute the simplest medial object such that the inverse transform is within tolerance of the original object
- Successful medial objects can be calculated for real, complex, industrial models