Hex Meshing - what has the medial object to offer ?

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What's the Problem ?

There is a very simple algorithm for dividing an object into hexahedra: first divide it into tetrahedra, and then divide each tetrahedron into four hexahedra by midpoint subdivision.

If you don't want such a dense mesh, use bigger tets. In fact you can use extremely large tets and then split the resulting hexahedral blocks regularly into finer elements.

Most people who have addressed the hex-meshing problem over the last 40 years are well aware of this trite construction. When you look at such a mesh you see that it isn't what you wanted: it is only what you asked for.

There is some small print and we need to look at the real requirements. We want a mesh which gives accurate results for a small number of degrees of freedom. The irregularities in the mesh from the above algorithm mean that the cost-accuracy tradeoff is almost as bad as for a pure tet mesh. A smoothly flowing hex mesh has been found to be significantly better.

The need for smooth flow will become more pronounced as the Iso Geometric Analysis message that spline elements are really cost-effective is listened to and found to be good value. Spline elements are even more demanding of smooth flow.

Requirements

We need

Smoothly flowing meshes
with Basis Functions matching
in orientation and density
the spatial frequencies of
the solution looked for.

This is, of course, not always completely possible. There are at least two situations where the ideal hex mesh does not exist.

(i) the four-valent vertex at the top of a square pyramid cannot be filled by the corner of a hex element.

This is a lovely illustration that although a hex mesh can be proved to exist for any object with an even number of element edges on every edge, the hex mesh that exists may be far from what you want. The number of elements in it can be far greater than what you would estimate from the volume of the object and the lengths of the element edges on the edges of the object.

(ii) tangentially tapering steps. If we have a circular boss on a square base and the diameter of the boss is the same as the edge length of the base it will not be possible to put a well-shaped element into the base under the step.

Don't expect a hex-meshing algorithm to outperform a skilled human. You can and should expect it to tell you when you have asked the impossible.

Less dramatic is the fact that totally smooth flow is possible only when the object is, topologically, a pile of bricks. A face with five sides needs a point somewhere in the middle where four quad element faces meet. If you take the top layer of elements off, you see the same pattern underneath, telling you that there is actually an element edge with other than four elements around it. It also tells you that such an irregularity has to be part of a chain. Such chains can form closed loops, they can terminate at the surface of the object, and they can collide with other chains (in a fairly small number of possible configurations). In my view identifying the necessary pattern of such irregularities and the way they meet is the essence of hex meshing.

What does the medial object offer ?

(i) Evidence about what is opposite to what. A Plastering algorithm must surely find information valuable about what the mesh on a face is going to have to be compatible with when it collides.

(ii) Evidence about distances.

(iii) Evidence about orientation.