Mullins effect on anti-vibration products with residual strain

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Abstract: This paper presents an engineering approach to simulating the loading-unloading and reloading-unloading histories of rubber products with the Mullins effect in industry, including the stress softening and residual deformation. The work is a further study extended from Luo et al. using rebound energy (resilience) approach. This rebound resilience is a key engineering constant in the functions, a physically meaningful concept. It should be noted that the proposed model does not need a negative stress state as a necessary pre-requisite for including the residual strain. The simulation was performed using well-know non-linear software Abaqus and validated by a well defined experiment. It is shown that this method can be easily incorporated with FE software, e.g. Abaqus. The proposed approach provides the reliable prediction and can be used for engineering design and industrial applications.

Keywords: Mullins effect, Rubber, design, resilience

1. Introduction

Rubber-like materials exhibit an appreciable change in their mechanical properties during the loading-unloading process, especially in first few loading-unloading cycles from a virgin state. This stress-softening phenomenon under this condition is referred to as the Mullins effect: Mullins [1-2] and Harwood et al., [3]. Abaqus provides a method to predict this effect by specifying three parameters in a FEA model. However, it is not clear for these parameters to be related to physical concepts. Some efforts have been made to attempt for modeling actual applications of the Mullins effect by manually correcting these three parameters [4]. The authors concluded that progressive stress-softening damage was not supported by the Abaqus Mullins effect model and that a more simplified approach would most likely provide a better cost-benefit ratio in many practical applications. In this study, two more functions are included in the strain energy model so that it can capture the observed softening and residual strain response simultaneously in addition to a single function used previously by Luo et al. [5-7]. Also, the proposed approach can be used for calculation on reloading-unloading response that is different from first loading and unloading cycle. A rubber component, manufactured in Trelleborg IAVS and not a simple test specimen, was selected for experiment and verification. In industry, rubber products are normally designed for loadings applied along compression and shear directions.

2. Resilience approach model

There are several hyperelastic material models that are commonly used to describe rubber and other elastomeric materials based on strain energy potential or strain energy density. The hyperelastic models for rubber material without the Mullins effect can be expressed in the following general form:

$$W = W_1(\bar{I}) + W_2(J_{el}) \quad (1)$$

In order to obtain the response of rubber material under an unloading condition, a function is added to the first part W_I in equation (1). To include residual deformation, both a second function and a residual deformation function are incorporated into equation (1) as a damage function. Hence, a constitutive model based on resilience approach is formed as follows:

$$W = \theta(\beta)W_I(\bar{I}) - \theta_r(\beta)R(\bar{I}_1, \xi, \beta) + W_J(J_{el}) \quad (2)$$

where

β is a nominal scale variable with a loading-unloading process

β = 0 when a loading (including reloading) or an unloading starts

β = 1 when a loading (including reloading) or an unloading finishes

ξ is a residual deformation variable

During first loading from a virgin state:

$$\theta(\beta) = \theta_r(\beta) = 1 \quad (3)$$

$$R\left(\bar{I}_1, \xi, \beta\right) = 0 \quad (4)$$

During unloading:

$$\theta(\beta) = 1 - (1 - \theta_0)\beta^{\alpha_u} \quad (5)$$

$$\theta_r(\beta) = 1 - (1 - \theta_0)\beta^{\alpha_u} \quad (6)$$

$$R_u\left(\bar{I}_1, \xi, \beta\right) = \xi\beta^5 \bar{I}_1^{\,2} \quad (7)$$

θ_0 is the rebound resilience of a rubber material, a constant value for a given compound

α_u is an index variable that can be considered an unloading energy release rate

During reloading:

$$\theta(\beta) = \theta_0 + m\beta^{\alpha_r} \tag{8}$$

$$\theta_r(\beta) = \theta_0 \tag{9}$$

$$R_r\left(\bar{I}_1, \xi, \beta\right) = \xi(1-\beta)^{10}\,\bar{I}_1^{\,2} \tag{10}$$

m is the reloading decay function and varies with reloading cycles

$$m = \frac{1}{P_0}(P_n - \theta_0 P_0) \tag{11}$$

P_0 is the maximum initial load.

P_n is the n^{th} reloading decay force and can be obtained from experimental data as a function of reloading cycle n.

α_r is an index variable that can be considered a reloading energy rate

3. Case verification

A typical rubber anti-vibration component is selected for the verification, as shown in figure 1 (a). The finite element model is shown in Figure 1 (b). The finite element model consists of 1,400 elements and 4400 degrees of freedom.

Abaqus has very good user subroutine facilities, where a UHYPER is suitable for this simulation. The loading condition was a compressive load applied along the vertical direction. The simulation results together with the test curve, are shown in Figure 2. The comparison has indicated very good agreement and the main characteristics are properly simulated.

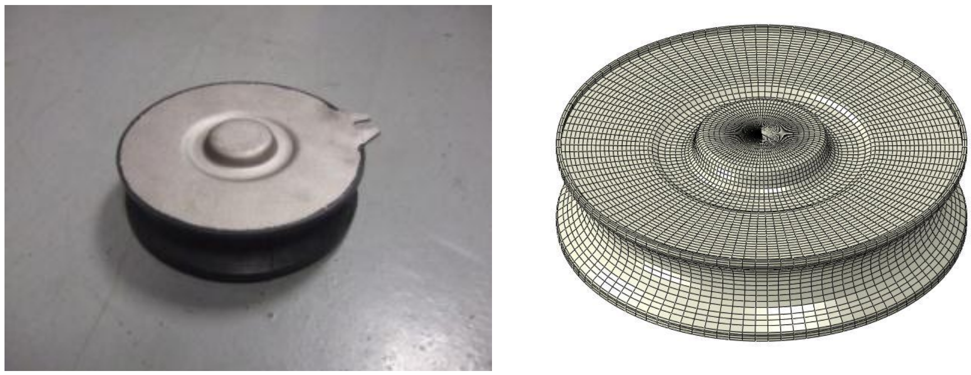

Figure 1. The rubber component and its finite element model

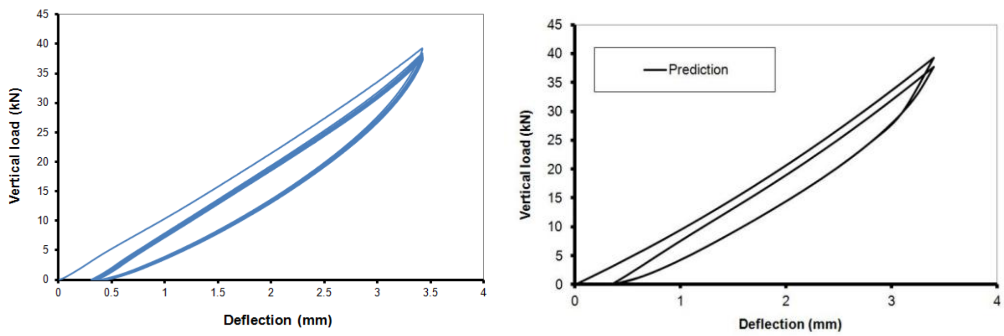

Figure 2. The comparison between the experiment and the simulation

4. Conclusion and suggestion

This study presents an engineering approach to simulating the loading-unloading and reloading-unloading histories of rubber products with the Mullins effect in industry, including the stress softening and residual deformation. The work is a further study extended from Luo et al. using rebound energy (resilience) approach. This rebound resilience is a key engineering constant in the functions, a physically meaningful concept. It should be noted that the proposed model does not need a negative stress state as a necessary pre-requisite for including the residual strain. The following parameters are defined in the functions: rebound resilience, unloading energy release rate, reloading energy rate, residual deformation variable and reloading decay function. From simulation side, only three parameters with physical meanings are needed: unloading energy release rate, reloading energy rate, residual deformation variable. The proposed approach provides the reliable prediction and can be used for engineering design and industrial applications. Nevertheless, the proposed approach should be further verified using more engineering cases.

5. References

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