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SAE Technical Paper Series

892371

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Aerospace Technology Conference and Exposition
Anaheim, California
September 25-28, 1989

ISSN 0148-7191

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Abstract

An efficient analytical methodology has been developed with the finite element method which may be used to predict the low frequency vibro-acoustic environment within an aerospace flight vehicle. This methodology includes general purpose capabilities for solving problems involving the effects of structure/acoustic interaction and random excitation pressure fields. Computational efficiency is enhanced by decoupling the structure from the acoustic volume, and taking advantage of reciprocity in the random vibration and vibro-acoustic formulations. The application of the analytical methodology to an example problem found good agreement with previous research, demonstrating the feasibility of the methodology described herein.

A VARIETY of random aero-acoustic loads act on the exterior of an aerospace flight vehicle during launch. These loads include propagating pressure fields which are representative of rocket noise produced during lift-off, and convecting pressure fields which are representative of the turbulent boundary layer created on the vehicle during operation at high flight dynamic pressures. These excitation pressure fields can induce random vibrations of the flight vehicle structure, which may be transmitted to the payload through structural attachment interfaces, or through the acoustic volume surrounding the payload (Figure 1). The combination of structure-borne vibration and acoustic impingement, known collectively as the *vibro-acoustic environment*, can impart significant dynamic loads to the payload and its components.

The analytical prediction of vibro-acoustic environments requires consideration of the complex

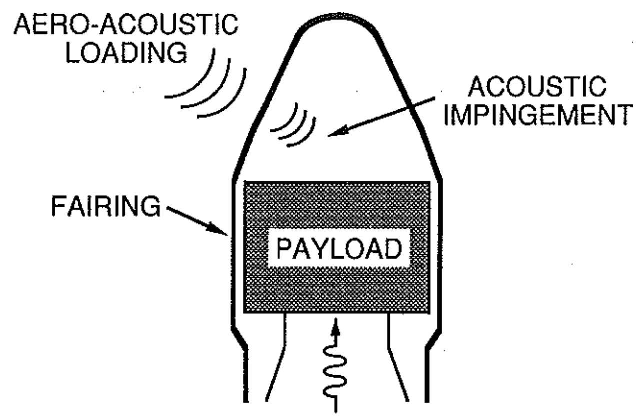

Figure 1. Vibro-Acoustic Environment

dynamic behavior and interactions among the flight vehicle structure, payload, and acoustic volume. Since these environments are random, it is necessary to describe the excitation, structural response, and acoustic response in terms of statistical averages, as opposed to explicit mathematical relationships. As a result, analytical prediction methods typically require highly complex models and specialized solution techniques that are generally difficult to implement and computationally intensive even for the simplest of problems. Efficient analytical methods are therefore needed in order to predict vibro-acoustic environments for the complex flight vehicle structural configurations and aero-acoustic loading conditions found in practice.

This paper describes an efficient analytical methodology based on the finite element method, which may be used to predict flight vehicle vibro-acoustic environments. This methodology is developed by addressing three important aspects of the prediction problem, including structure/acoustic interaction, random vibration analysis, and vibro-acoustic analysis. The effectiveness of the formulation is investigated through analysis of a sample problem consisting of a flexible panel backed by a closed rectangular cavity subjected to deterministic and random pressure field excitations. Following this evaluation are recommendations for further validating the methodology and applying it to typical aerospace flight vehicles.

Structure/Acoustic Interaction

The fundamental difficulty in using the finite element method for predicting vibro-acoustic environments is that it must address two inherently different formulations, involving structural vibrations and acoustic pressures. The complexity of the solution is further compounded when dynamic interactions between the structural and acoustic media are considered. These problems are typically solved by representing the acoustic volume by a three dimensional finite element model in which pressures are the unknown variables, and to represent the structure by another finite element model in which displacements are the unknown variables [1]*. In the absence of damping, this method involves the simultaneous solution of the following system of matrix equations†:

$$[M_s]\{\ddot{u}\} + [K_s]\{u\} = \{F_s\} + [A]^T\{p\} \qquad (1)$$

$$[M_a]\{\ddot{p}\} + [K_a]\{p\} = -[A]\{\ddot{u}\} \qquad (2)$$

The matrix product ($[A]^T\{p\}$) in Equation (1) can be regarded as the equivalent force acting on the structure due to acoustic back pressure, whereas the product ($-[A]\{\ddot{u}\}$) in Equation (2) can be regarded as the "pseudo-load" acting on the acoustic volume due to structural boundary accelerations. These terms are used to couple the acoustic pressures to structural accelerations, and generally cause the above system of matrix equations to become unsymmetric. Since it is generally difficult to obtain solutions of unsymmetric matrices with standard finite element solution algorithms, alternative solution methods have been developed. These include improved numerical integration schemes which can be used to solve the equations directly, or component mode synthesis techniques which can be used to symmetrically decouple the equations. A comprehensive review of these and other methods can be found in References [2] and [3].

Since most aerospace flight vehicle payload bay geometries involve relatively large acoustic volumes enclosed by relatively stiff structure, it is possible to take advantage of numerous analytical and experimental studies [3-12] which have determined that for such systems, the effects of the acoustic back pressure on the low frequency dynamic behavior of the structure are negligible. This is attributed to the fact that the impedance of the acoustic volume is very small in comparison to the impedance of the structure. Consequently, the dynamic behavior of the structure governs the overall response of the system, making it possible to use the *in vacuo* structural response to directly predict the sound pressure levels within the acoustic volume. Since dynamic interaction effects among the structure and acoustic volume are eliminated, this methodology is termed the "pseudo-coupled" formulation.

The pseudo-coupled formulation involves the development of individual finite element models of the structure and acoustic volume, utilizing standard structural and acoustic finite element modeling techniques [13]. Coincident gridpoints are defined at the structure/acoustic interface in order to facilitate the conversion of structural accelerations to acoustic pseudo-loads at these locations. Dealing first with the structural finite element model, Equation (1) is solved using standard finite element solution algorithms since the non-symmetric coupling terms representing the acoustic back pressure are assumed negligible (i.e., $[A]^T\{p\}=0$). Structural boundary accelerations ($\ddot{u}$) are recovered at gridpoints on the structure/acoustic interface, and these are converted to acoustic pseudo-loads by multiplying the gridpoint accelerations by their respective gridpoint areas (A). The acoustic pseudo-loads ($-[A]\{\ddot{u}\}$) are applied to the finite element model of the acoustic volume at the structure/acoustic interface gridpoints, and Equation (2) is solved with standard finite element acoustic analysis techniques. The combination of the structural and acoustic finite element analyses yields the sound pressure levels (p) within the acoustic volume due to external loads (F_s) applied to the structure. The pseudo-coupled formulation significantly reduces data processing requirements since it is not necessary to explicitly couple the structure to the acoustic volume, nor is it necessary to simultaneously solve Equations (1) and (2).

Random Vibration Formulation

The standard finite element formulation for random vibration analysis involving spatially-distributed excitation pressure fields usually requires the solution of matrix equations of the form [14]:

$$G_{xx}(\omega) = \sum_{j=1}^{q}\sum_{i=1}^{q} H_{ix}(\omega)\, G_{ij}(\omega)\, H_{jx}(\omega)^{*T} \qquad (3)$$

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* Numbers in brackets designate references at end of paper

† Nomenclature is defined at end of paper

where $G_{xx}(\omega)$ define elements of the spectral density matrix of the structural accelerations at the response locations (x), $G_{ij}(\omega)$ is the spectral density matrix of the excitation pressure field, and $H_{ix}(\omega)$ and $H_{jx}(\omega)^{*T}$ are frequency response function matrices relating accelerations at the response locations (x) to discrete input forces applied to arbitrary points (i and j) on the structure.

Each element of the excitation spectral density matrix, $G_{ij}(\omega)$, defines the auto and cross spectrum between pairs of discrete input locations on the structure. Assuming that the input excitation pressure field is homogeneous, elements of $G_{ij}(\omega)$ can be defined as functions of in-plane separation distances, Δx and Δy, between discrete pairs of input locations. For propagating and convecting pressure fields, these elements are commonly approximated in two dimensions by [15, 16]:

$$G_{ij}(\omega) = G_{pp}(\omega)\, \exp(-a_x|\Delta x|)\, \cos(k_x \Delta x) * \exp(-a_y|\Delta y|)\, \cos(k_y \Delta y) \qquad (4)$$

where $G_{pp}(\omega)$ is the auto or power spectral density (PSD) of the excitation pressure field, "a" is an exponential weight, and "k" is the wave number given by the radian excitation frequency divided by the trace velocity. For reverberant acoustic fields, elements of $G_{ij}(\omega)$ are often approximated by a diffuse noise model of the form [17]:

$$G_{ij}(\omega) = G_{pp}(\omega)\, \frac{\sin(k_x \Delta x)}{k_x \Delta x}\, \frac{\sin(k_y \Delta y)}{k_y \Delta y} \qquad (5)$$

where "k" is the radian excitation frequency divided by the speed of sound. For this case, the cross-spectra along the x and y axes are identical. It is often convenient to divide the excitation spectral density matrix by the excitation PSD, thereby converting $G_{ij}(\omega)$ to a correlation matrix. Use of the correlation matrix in place of the excitation spectral density matrix in Equation (3) results in the normalization of the response spectral density matrix to a unit PSD excitation.

The standard random vibration formulation [13] requires that each row or column of the acceleration frequency response function matrices, $H_{ix}(\omega)$ and $H_{jx}(\omega)^{*T}$, be generated by applying discrete forces, $F_i(\omega)$, to each input location (i) of the finite element model, and calculating complex accelerations ($\ddot{u}$) at the response locations (x). By definition, the response accelerations, excitation forces, and frequency response functions are related by:

$$\ddot{u}_x(\omega) = F_i(\omega)\, H_{ix}(\omega) \qquad (6)$$

Assuming that the excitation forces are due to unit pressures, $p_i(\omega)$, applied to arbitrary areas, A_i, of the finite element model, Equation (6) can be expressed as follows:

$$\ddot{u}_x(\omega) = p_i(\omega)\, A_i\, H_{ix}(\omega) = A_i\, H_{ix}(\omega) \qquad (7)$$

The standard random vibration formulation is computationally intensive since a finite element model with 'q' input gridpoints, 'r' response gridpoints, and 's' discrete frequencies of interest requires (q x r x s) successive frequency response analyses to completely define the acceleration frequency response function matrices. For larger models involving complex geometries, high mode orders, wide frequency ranges, or numerous response locations, the amount of data which must be processed can quickly become unmanageable. These computational difficulties preclude the practical use of the standard random vibration formulation for the prediction of vibro-acoustic environments.

Following the method described in [18], it is possible to simplify the standard random vibration formulation by recognizing that reciprocity must exist among the input and response locations of the structure. Assuming that unit excitation forces, $F_x(\omega)$, are applied to the *response* locations (x), and accelerations, $\ddot{u}_i(\omega)$, are calculated at the *input* locations (i), the reciprocity relationship allows the acceleration frequency response function matrix to be defined by:

$$\ddot{u}_i(\omega) = F_x(\omega)\, H_{xi}(\omega) = H_{xi}(\omega) = H_{ix}(\omega) \qquad (8)$$

Substituting this definition of the acceleration frequency response function matrix into Equation (7) allows the complex accelerations at the response locations to be obtained by:

$$\ddot{u}_x(\omega) = A_i\, \ddot{u}_i(\omega) \qquad (9)$$

Use of Equation (9) to generate the acceleration frequency response function matrices significantly reduces data processing requirements in the random vibration formulation. Rather than requiring (q x r x s) successive frequency response analyses, these matrices can be completely defined in (r x s) analyses. Computational efficiency is further enhanced by recognizing that the acceleration frequency response function matrix is symmetric and the excitation cross-spectral density matrix is hermitian, allowing them to be defined by upper triangular terms only. Equation (3) can be modified for these simplifications as follows:

$$G_{xx}(\omega) = \sum_{j=1}^{q} \sum_{i=1}^{q} \ddot{u}_i(\omega)\, A_i\, G_{ij}(\omega)\, A_j\, \ddot{u}_j(\omega)^{*T} \qquad (10)$$

where the products $(\ddot{u}_i(\omega)A_i)$ and $(A_j\ddot{u}_j(\omega)^{*T})$ are used to generate the acceleration frequency response function matrices, $H_{ix}(\omega)$ and $H_{jx}(\omega)^{*T}$, respectively, and the elements of the excitation spectral density matrix, $G_{ij}(\omega)$, are defined by forms of Equation (4) or (5). This formulation has been validated on relatively simple structures and successfully applied to the prediction of the random vibration response of complex structures, including the Space Shuttle orbiter sidewall [18, 19].

Vibro-Acoustic Formulation

The random vibration formulation described above can be directly applied to the prediction of the vibro-acoustic response within the acoustic volume. This is accomplished by adapting the matrices in Equation (3) to the acoustic volume with the pseudo-coupled formulation as follows:

$$G_{yy}(\omega) = \sum_{j=1}^{q} \sum_{i=1}^{q} H'_{iy}(\omega)\, G'_{ij}(\omega)\, H'_{jy}(\omega)^{*T} \qquad (11)$$

where $G_{yy}(\omega)$ define elements of the spectral density matrix of the acoustic pressure at the response locations (y), $G'_{ij}(\omega)$ is the spectral density matrix of the acoustic pseudo-loads at the structure/acoustic interface, and $H'_{iy}(\omega)$ and $H'_{jy}(\omega)^{*T}$ are frequency response function matrices relating pressures at the response locations (y) within the acoustic volume to discrete input pseudo-loads applied to arbitrary points (i and j) on the structure/acoustic interface.

In the same manner that structural boundary accelerations can be used to define the acoustic pseudo-loads, the spectral density matrix of the structural boundary accelerations can be used to define the spectral density matrix of the acoustic pseudo-loads at the structure/acoustic interface. This is accomplished by using Equation (10) to obtain $G_{xx}(\omega)$, and directly substituting $G_{xx}(\omega)$ for $G'_{ij}(\omega)$ in Equation (11), as follows:

$$G_{yy}(\omega) = \sum_{j=1}^{q} \sum_{i=1}^{q} H'_{iy}(\omega)\, G_{xx}(\omega)\, H'_{jy}(\omega)^{*T} \qquad (12)$$

The vibro-acoustic formulation requires the application of acoustic pseudo-loads, $(\{\ddot{u}_i(\omega)\}\,[A_i])$, at discrete input locations on the structure/acoustic interface, and the calculation of complex pressures, $p_y(\omega)$, at the response locations (y) within the acoustic volume. The collection of pressures for all discrete input points (q) on the structure/fluid interface completely define the pressure frequency response function matrices, $H'_{iy}(\omega)$ and $H'_{jy}(\omega)^{*T}$. Assuming that unit accelerations are used to define the acoustic pseudo-loads at the structure/acoustic interface, the pressure at the response location is related to the pressure frequency response function by:

$$p_y(\omega) = \ddot{u}_i(\omega)\, A_i\, H'_{iy}(\omega) = A_i\, H'_{iy}(\omega) \qquad (13)$$

which is similar to the acceleration/pressure relationship given for the structure in Equation (7). For a finite element model with 'q' input gridpoints on the structure/acoustic interface, 'r' response gridpoints within the acoustic volume, and 's' discrete frequencies of interest, it is necessary to conduct (q x r x s) successive frequency response analyses in order to completely define the pressure frequency response function matrices.

By a method similar to that described above for the random vibration formulation, it is possible to simplify the standard finite element vibro-acoustic formulation by applying unit acoustic pseudo-loads at the *response* locations within the acoustic volume and calculating complex pressures at the *input* locations on the structure/acoustic interface. The complex pressures at the response locations can therefore be generated by scaling the complex pressures at the input locations by the gridpoint areas at the input locations as follows:

$$p_x(\omega) = A_i\, p_i(\omega) \qquad (14)$$

These pressures are collected to form the pressure frequency response function matrices for the acoustic volume. Rather than requiring (q x r x s) successive frequency response analyses, these matrices can be completely defined in (r x s) analyses. Computational efficiency is further enhanced by recognizing that these matrices are symmetric, allowing them to be defined by upper triangular terms only. Using the above definitions, the vibro-acoustic formulation reduces to:

$$G_{yy}(\omega) = \sum_{j=1}^{q} \sum_{i=1}^{q} p_i(\omega)\, A_i\, G_{xx}(\omega)\, A_j\, p_j(\omega)^{*T} \qquad (15)$$

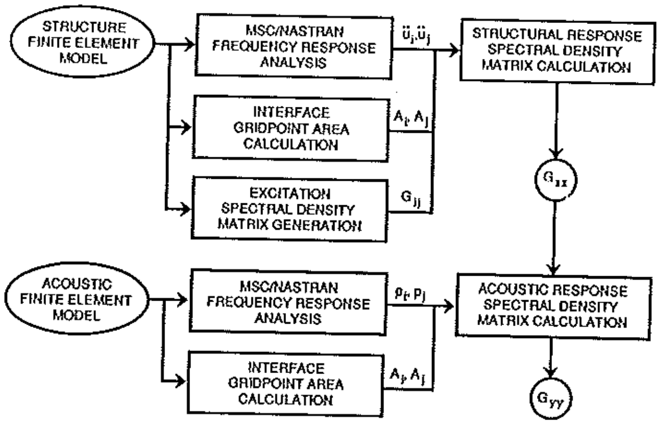

Figure 2. Schematic Diagram of Vibro-Acoustic Environment Prediction Methodology

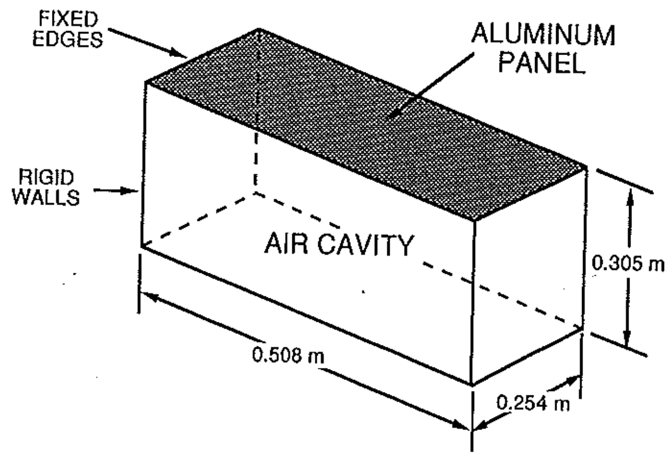

Figure 3. Panel/Cavity System

where the products ($p_i(\omega)A_i$) and ($A_j p_j(\omega)^{*T}$) are used to generate the pressure frequency response function matrices, and the spectral density matrix of the structural boundary accelerations, $G_{xx}(\omega)$, are obtained from Equation (10). The vibro-acoustic environment prediction methodology defined by Equations (10) and (15) was implemented with the MSC/NASTRAN finite element code [13] and a stand-alone Fortran program. Key features and important functions of this methodology are shown schematically in Figure 2.

Table 1.
Panel/Cavity Physical and Material Properties

Aluminum Panel:	
Young's Modulus (E)	68.9E+3 MPa
Poisson's Ratio (ν)	0.30
Density (ρ)	2767.0 kg/m^3
Thickness (t)	0.00127 m
Damping (% critical)	1.0 @ 100 Hz
Air Cavity:	
Bulk Modulus (κ)	0.143 MPa
Density (ρ)	1.227 kg/m^3
Damping (% critical)	1.0 @ 500 Hz

Example Application

An example application was developed consisting of a rectangular cavity containing air, with one flexible aluminum wall (Figure 3 and Table 1). This problem was chosen in order to compare the analytical vibro-acoustic response predictions to experimental results of a similar configuration in References [7] and [8]. Although attempts were made to analytically duplicate the experiment, the lack of design details in the References required several approximations in the analytical models. Among these approximations are the use of rigid boundary constraints to model the plexiglass cavity walls and the application of fixed boundary constraints to model the panel attachment to the cavity. Other approximations involved the direct adaptation of the experimentally-determined structural damping, which varied with the inverse of the frequency, and acoustic damping, which varied with the inverse of the frequency squared. These properties were approximated by modal viscous damping in the finite element models. In light of these approximations, the verification of the analytical methodology was conducted in somewhat of a qualitative manner. Future work will involve the development and detailed modeling of a controlled experiment in order to fully validate the vibro-acoustic response predictions.

Initial investigations involved the prediction of the panel and acoustic response to a deterministic excitation. The finite element model of the *in vacuo* panel was subjected to a loudspeaker excitation which resulted in a completely correlated sinusoidal pressure field acting on the panel surface at a sound pressure level of 100 dB. Accelerations for gridpoints on the structure/acoustic interface were computed and converted to acoustic pseudo-loads. These pseudo-loads were then used as input to the finite element model of the acoustic volume, and the pressure response was calculated at a single location within the acoustic volume at a depth of 76 mm below the panel center. Analysis results for the displacement response of the panel center (Figure 4), taken together with analysis results for the pressure response within the acoustic cavity (Figure 5), indicate that symmetric volume displacing panel modes dominate the overall response of the system. This should be expected since the loudspeaker excitation was distributed symmetrically about the panel center.

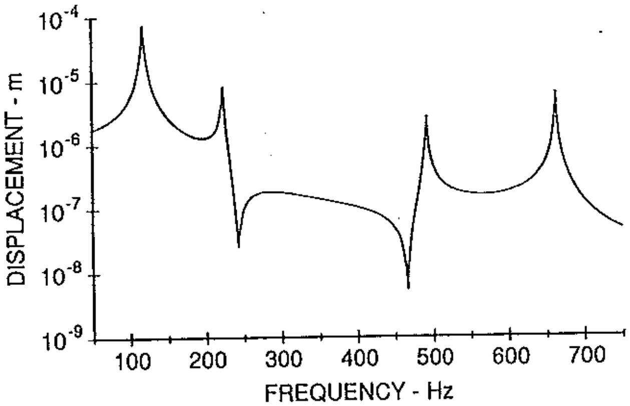

Figure 4. Displacement Response of Panel Due to 100 dB Sinusoidal Loudspeaker Excitation

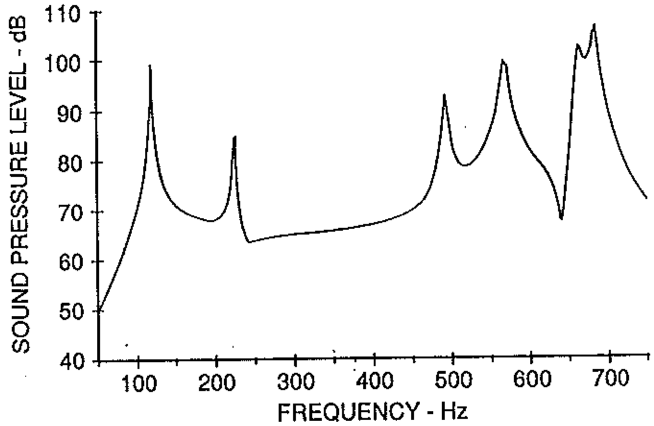

Figure 5. Acoustic Pressure Within Cavity Due to 100 dB Sinusoidal Loudspeaker Excitation

Comparison of the analytical response predictions with the experimental results [7] found good agreement for the prediction of the panel and acoustic volume natural frequencies, however, only fair agreement for the maximum amplitudes of the panel displacement and cavity internal pressure at the panel fundamental frequency. Specifically, the analysis predicted that the maximum displacement at the panel center and the maximum pressure within the acoustic volume would each be about 2.5 times lower than the experimentally-determined values. A parametric evaluation of the effects of panel and acoustic damping on the accuracy of the analytical solution determined that the panel damping was by far the critical parameter which governed the overall response of the system. This should be expected since the impedance of the panel is much greater than that of the acoustic volume. In order to obtain closer agreement with the experimental results for the panel displacement and cavity internal pressure, the panel damping was reduced by a factor of 2.5, resulting in a panel damping of roughly four tenths of one percent of critical damping at the panel fundamental frequency. This damping value is considered reasonable for fully-fixed aluminum panels subjected to acoustic excitation [20]. Other improvements which could be incorporated into the analytical models to obtain even closer agreement with the experimental results include the use of improved methods for representing the panel edge constraints and the flexibility of the cavity walls. These effects were not investigated for the present study, however.

Subsequent analyses were conducted to predict the panel and acoustic response to random, spatially-distributed excitation pressure fields. For these analyses, turbulent boundary layer and reverberant field excitations were applied to the exterior of the panel [16, 17], and the vibro-acoustic response of the panel and acoustic volume were calculated. A single response location within the acoustic volume was defined at a location 76 mm below the center of the panel. For simplicity, the response locations on the structure/acoustic interface were chosen at five locations defined by the panel center and each of the quarter points. For practical applications, however, the maximum discrete frequency of interest must be considered when calculating the locations of the response points on the structure/acoustic interface. This is necessary since the minimization of Equations (4) and (5) with respect to frequency will define the maximum separation distances (Δx and Δy) for which a correlated pressure field may exist.

Dealing first with the panel, unit forces were applied to the response locations and frequency response analyses were performed to obtain complex accelerations for each gridpoint on the model. These accelerations were multiplied by their respective gridpoint areas with Equation (9) in order to generate the acceleration frequency response function matrices. Correlation functions between the gridpoints on the panel were developed by forms of Equations (4) and (5), assuming that the excitation PSD, $G_{pp}(\omega)$, was normalized to unity. These correlation functions were assembled to form the correlation matrices, $G_{ij}(\omega)$, for the turbulent boundary layer and reverberant field excitations. Equation (10) was then used with each correlation matrix, and the panel acceleration response spectral density matrices were computed. Auto spectra of the acceleration response at the panel center normalized to unit PSD turbulent boundary layer and reverberant field excitations can be seen in Figures 6 and 7, respectively.

For the acoustic volume, a unit acoustic pseudo-load was applied at the response location within the acoustic volume and a frequency response analysis was conducted to obtain complex pressures

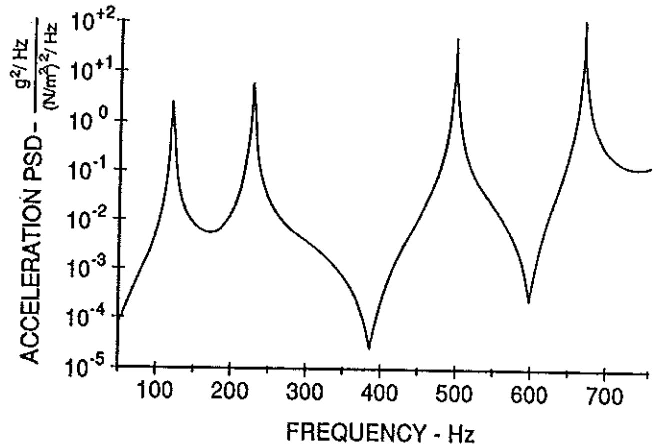

Figure 6. Acceleration Response of Panel Due to Unit PSD Turbulent Boundary Layer Excitation

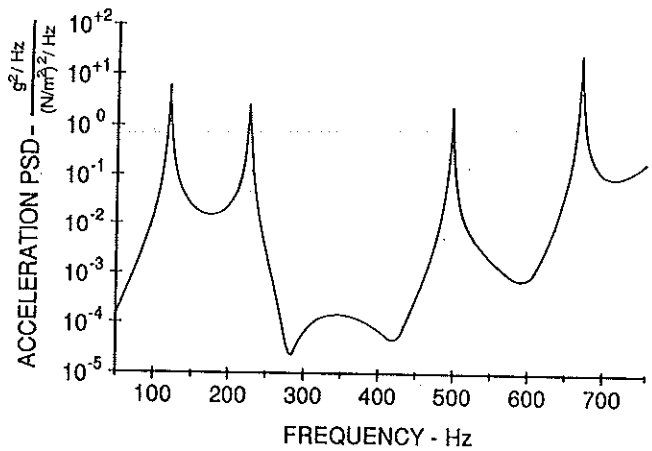

Figure 7. Acceleration Response of Panel Due to Unit PSD Reverberant Field Excitation

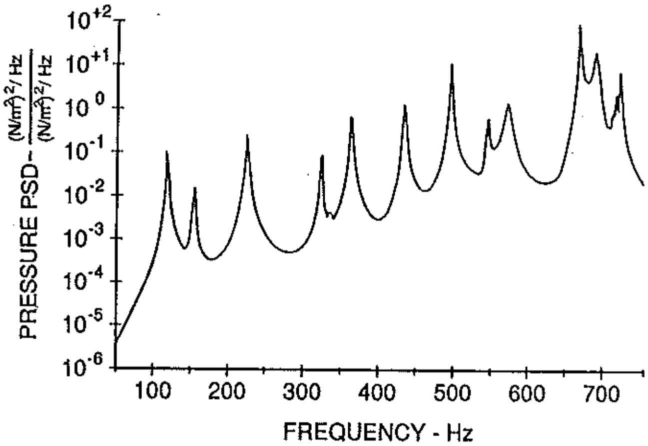

Figure 8. Acoustic Pressure Within Cavity Due to Unit PSD Turbulent Boundary Layer Excitation

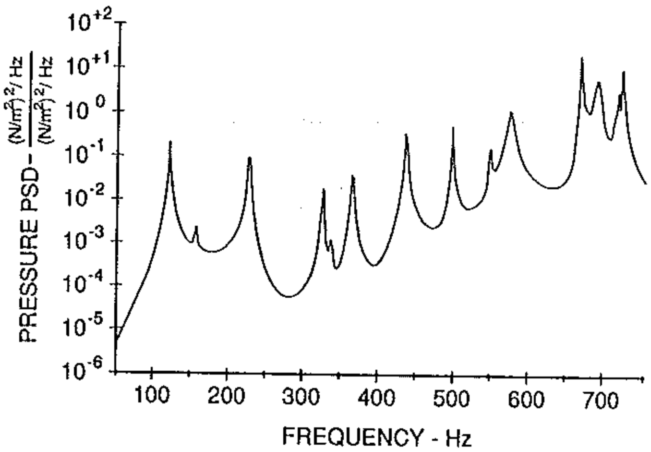

Figure 9. Acoustic Pressure Within Cavity Due to Unit PSD Reverberant Field Excitation

at the five input locations on the structure/acoustic interface. The pressure frequency response function matrices were generated in Equation (14), and used in Equation (15) with the appropriate spectral density matrix of the structural response, $G_{XX}(\omega)$, obtained above for the turbulent boundary layer and reverberant field excitations. These analyses yielded the spectral density matrices of the vibro-acoustic response within the cavity normalized to unit PSD turbulent boundary layer and reverberant field excitations applied to the panel (Figures 8 and 9). Comparison of the random vibro-acoustic response with the deterministic response obtained above (Figure 5) indicates numerous peaks in the vibro-acoustic response at non-symmetric, as well as symmetric panel and acoustic volume modes. These peaks are attributed to the effects of correlation among the discrete input points on the structure/acoustic interface, and can become more pronounced if additional input locations are prescribed. These analyses demonstrated that it is feasible to obtain highly detailed vibro-acoustic response predictions in an efficient manner with the pseudo-coupled formulation.

Comparison of the vibro-acoustic response predictions with the experimental results [8] was not possible since the experiment involved the measurement of sound transmission through the panel due to white noise applied by a single loudspeaker. The resulting excitation pressure field, although possessing random frequency content, was not appropriate for generating a completely random environment. Consequently, the vibro-acoustic analysis for this excitation reduces to the deterministic formulation which has been analyzed above. Random excitation in the present context implies random incidence, frequency content, phase, amplitude, etc., for which meaningful results for convecting, propagating, and diffuse excitations can be obtained only through tests in wind tunnels, siren facilities, or acoustic reverberation chambers. In this regard, work is currently underway to develop a controlled experiment to validate the vibro-acoustic formulation. This work will involve the prediction of the vibro-acoustic response of a simple panel/cavity system subjected to multi-point random excitation. Future work may involve the application of the analytical methodology to the prediction of sound pressure levels within a flight vehicle fairing during testing in an acoustic reverberation chamber, or the use of an instrumented fairing during launch to obtain data for comparison with analytical predictions for the vibro-acoustic response to convecting and propagating excitation pressure fields.

Summary and Conclusions

An efficient analytical methodology has been developed with the finite element method which may be used to predict the low frequency vibro-acoustic environment within an aerospace flight vehicle. This methodology greatly simplifies the vibro-acoustic environment prediction problem by use of a pseudo-coupled structure/acoustic interaction formulation, whereby structural accelerations are used to develop acoustic pseudo-loads. Computational efficiency is further enhanced by taking advantage of reciprocity to develop frequency response functions for the structure and acoustic volume in the random vibration and vibro-acoustic formulations. The application of the analytical methodology to an example problem found good agreement with previous research, demonstrating the feasibility of the methodology for the prediction of vibro-acoustic environments.

Further developments are required before this analytical methodology can be applied in a general manner to typical aerospace flight vehicles. These include (a) the development and modeling of more closely controlled experiments to further validate the analytical methodology, (b) the experimental determination of spatial correlation functions for typical flight vehicle fairing geometries subjected to reverberant, convecting, and propagating pressure fields, and (c) the development of analytical methods to represent non-linear structural and acoustic damping, including visco-elastic constrained layers and acoustic blankets. These developments will allow for the wide application of the analytical methodology to a variety of aerospace flight vehicles.

Acknowledgements

The author wishes to acknowledge the contributions of Dr. Sheldon Rubin, Ms. Carole S. Tanner, and Mr. Jeffrey J. Muhle of The Aerospace Corporation, El Segundo, CA, Mr. Allan G. Piersol of Piersol Engineering, Woodland Hills, CA, and Dr. John F. Wilby of Atlantic Applied Research Corporation, Van Nuys, CA. This work was sponsored by the United States Air Force Space Systems Division under contract F04701-88-C-0089.

Nomenclature

A	structure/acoustic interface area matrix
a	exponential weight
F	excitation forces
$G_{ij}(\omega)$	structural excitation spectral density matrix
$G'_{ij}(\omega)$	acoustic excitation spectral density matrix
$G_{pp}(\omega)$	power spectral density of excitation pressure
$G_{xx}(\omega)$	structural response spectral density matrix
$G_{yy}(\omega)$	acoustic response spectral density matrix
$H_{ix}(\omega)$	acceleration frequency response function
$H'_{iy}(\omega)$	pressure frequency response function
K	structural stiffness matrix
k	wave number
M	mass matrix
p	acoustic pressures
q	total number of discrete input locations
r	total number of discrete response locations
s	total number of discrete frequencies
u	structural displacements
Δx, Δy	separation distances in x- and y-directions
Σ	summation
ω	radian frequency

Subscripts:

a	property of acoustic volume
i, j	arbitrary locations
p	property of excitation pressure
s	property of structure
x	response location on structure
y	response location in acoustic volume

Superscripts:

*	complex conjugate
¨	second derivative with respect to time
T	transpose

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Printed in U.S.A.