*THE OMEGA METHOD FOR DISCHARGE RATE EVALUATION**

*J. C. Leung***
Fauske & Associates, Inc.
16W070 West 83rd Street
Burr Ridge, Illinois 60521

INTRODUCTION

In 1986 the present author (Leung, 1986) published a generalized correlation for one-component homogeneous equilibrium flashing choked flow as reproduced in Fig. 1 (where the lines represent best-fit to the eleven fluids tested). Since then no fewer than ten publications have appeared, extending the methodology (or correlations) to cover non-flashing two-phase flow, flashing flow with noncondensables, two-phase flow in pipes with elevation change, subcooled inlet flashing flow, multicomponent systems, and safety/relief valve sizing (Leung, 1992). Such a methodology has come to be known as the "omega" method since an ω parameter comprising of dimensionless physical property groups was first introduced in the original publication. This paper serves to review its evolution as well as update its recent developments.

Relative to a single-phase system, two-phase flow has two additional degrees of freedom - one due to thermal non-equilibrium effect, and the other due to mechanical non-equilibrium resulting in slip between phases. It has been generally accepted that for high momentum discharges of a two-phase system, both thermal equilibrium and mechanical equilibrium can be assumed. This is the classical homogeneous (i.e. no slip) equilibrium flow model (HEM). Evaluation of this HEM usually involves a lengthy procedure and requires detailed thermodynamic property tabulation (Leung and Nazario, 1990). The ω method was hence

---

*Paper No. 46e, "Update on Relief System Design," AIChE Meeting, Denver, Colorado, August 16, 1994, unpublished.

**Leung, Inc., 1521 Evergreen Lane, Darien, Illinois 60561.
TEL: (708) 910-4316 FAX: (708) 910-3022

proposed as an alternative and has the attribute of bringing out key physical parameters influencing the compressible flow of a two-phase system.

TWO-PHASE EXPANSION LAW

A general expansion law or equation of state was proposed for a two-phase system undergoing decompression (pressure drop due to acceleration and friction loss),

$$\frac{v}{v_o} = \omega \left(\frac{P_o}{P} - 1 \right) + 1 \tag{1}$$

where the so-called "compressible flow parameter" ω is given by the following largely equivalent forms:

$$\omega = \frac{x_o v_{v\ell o}}{v_o} + \frac{C_p T_o P_o}{v_o} \left(\frac{v_{v\ell o}}{h_{v\ell o}} \right)^2 \tag{2a}$$

$$\omega = \frac{x_o v_{vo}}{v_o} + \frac{C_p T_o P_o}{v_o} \left(\frac{v_{v\ell o}}{h_{v\ell o}} \right)^2 \tag{2b}$$

$$\omega = \alpha_o + (1 - \alpha_o)\, \rho_{\ell o} T_o P_o \left(\frac{v_{v\ell o}}{h_{v\ell o}} \right)^2 \tag{2c}$$

Note that all properties are to be evaluated at the known inlet stagnation conditions. As will be shown later, a modified form for ω has been proposed recently which helps to remove the need for the empirical expressions used earlier. Referring to Eq. (2c), we see that ω is made up of two distinct terms (Leung, 1990b): the first reflects the compressibility of the two-phase mixture due to existing vapor volume, and the second reflects the compressibility due to phase change upon depressurization or flashing. For flashing two-phase flow systems, the second (phase change) term dominates until α_o approaches unity (all-vapor inlet). For non-flashing two-phase flow systems, the second term vanishes (no phase change) and ω becomes simply α_o. This result allows general solutions developed for flashing flow to be extended to non-flashing flow as well (Leung, 1990b).

NOZZLE FLOW - FLASHING AND NON-FLASHING

Based on the above-mentioned expansion law for two-phase systems, a generalized solution for flow through a perfect nozzle has been obtained (Leung, 1986; Leung and Epstein, 1990a). Sonic, choking or critical flow condition as defined by the maximum value in the mass flux G yields the following expression for the critical pressure ratio $\eta_c \equiv P_c/P_o$,

$$\eta_c^2 + (\omega^2 - 2\omega)(1 - \eta_c)^2 + 2\omega^2 \ln \eta_c + 2\omega^2(1 - \eta_c) = 0 \tag{3}$$

A graphical solution is illustrated in Fig. 2 (thick line k = 1). After determining η_c, its value is substituted into Eq. (4) (with $\eta = \eta_c$) to yield the critical mass flux G_c,

$$\frac{G}{\sqrt{P_o/v_o}} = \frac{\{-2[\omega \ln \eta + (\omega - 1)(1 - \eta)]\}^{1/2}}{\omega\left(\frac{1}{\eta} - 1\right) + 1} \tag{4}$$

Alternatively it can be used in the following normalized G^* formula governing sonic condition at the throat,

$$\frac{G_c}{\sqrt{P_o/v_o}} = \frac{\eta_c}{\sqrt{\omega}} \tag{5}$$

which would yield identical results. Note that Eq. (4) is a more general formula, valid under unchoked or subsonic flow condition. Thus if $P_c > P_a$ (or $\eta_c > \eta_a$), the downstream back pressure, flow is choked and Eqs. (3) and (5) apply. Otherwise $P_c < P_a$ (or $\eta_c < \eta_a$) flow is unchoked, then η is equated to η_a in Eq. (4) to obtain the unchoked mass flux.

The above choked flow solutions can be conveniently represented by Fig. 2 (Leung, 1992), covering the entire two-phase flow spectrum - flashing flow lies to the right of $\omega = 1$ and non-flashing flow lies to the left of $\omega = 1$. At $\omega = 1$ (or $\alpha_o = 1$), the present graph recovers the classical "isothermal" gas flow solution.

NOZZLE FLOW - INLET SUBCOOLED LIQUID

Flashing discharge of an inlet subcooled liquid can be treated in more or less the same manner (Leung and Grolmes, 1988). In this case the first term in ω parameter vanishes and we can define a "saturated" ω parameter as follows:

$$\omega_s = \rho_{\ell o} C_p T_o P_s \left(\frac{v_{v\ell o}}{h_{v\ell o}}\right)^2 \tag{6}$$

where P_s is the saturation (vapor) pressure corresponding to T_o. Now Eq. (1) takes the form

$$\frac{v}{v_o} = \omega_s \left(\frac{P_s}{P} - 1\right) + 1 \tag{1a}$$

The generalized solutions are divided into low and high subcooling regions, delineated by a transition saturation pressure ratio (P_{st}/P_o), i.e.,

$$\eta_{st} = \frac{2\,\omega_s}{1 + 2\,\omega_s} \tag{7}$$

In the low subcooling region where $\eta_s > \eta_{st}$ (or $P_s > \eta_{st} P_o$), the fluid attains flashing (two-phase) flow prior to reaching the exit (throat) location and the mass flux can be given by

$$\frac{G_c}{\sqrt{P_o \rho_{\ell o}}} = \frac{\left\{2(1 - \eta_s) + 2\left[\omega_s \eta_s \ln\left(\frac{\eta_s}{\eta}\right) - (\omega_s - 1)(\eta_s - \eta)\right]\right\}^{1/2}}{\omega_s \left(\frac{\eta_s}{\eta} - 1\right) + 1} \tag{8}$$

and for choking conditions to occur, the following equation yields the critical pressure ratio η_c

$$\frac{\left(\omega_s + \frac{1}{\omega_s} - 2\right)}{2\,\eta_s} \eta_c^2 - 2(\omega_s - 1)\,\eta_c + \omega_s \eta_s \ln\left(\frac{\eta_c}{\eta_s}\right) + \frac{3}{2}\,\omega_s \eta_s - 1 = 0 \tag{9}$$

Note that Eqs. (8) and (9) reduce exactly to Eqs. (4) and (3), respectively, for the saturated liquid inlet case with $\eta_s = 1$ (i.e. $P_s = P_o$).

In the high subcooling region where $\eta_s < \eta_{st}$ (or $P_s < \eta_{st} P_o$), no vapor is formed until the exit is reached, and Eq. (8) reduces to a Bernoulli-type formula,

$$\frac{G_c}{\sqrt{P_o \rho_{\ell o}}} = [2(1 - \eta_s)]^{0.5} \tag{10a}$$

or

$$G_c = [2\ \rho_{\ell o}\ (P_o - P_s)]^{0.5} \tag{10b}$$

and the critical pressure ratio is simply

$$\eta_c = P_s/P_o \tag{11}$$

or choking occurs at the saturation pressure corresponding to inlet temperature T_o. Figure 3 provides the graphical solutions for these equations.

NOZZLE FLOW - FLASHING WITH NONCONDENSABLE GAS

Flashing discharge in the presence of noncondensable gas is considered to be a hybrid two-phase flow system where the ω method would describe an expansion law separately in terms of the (condensable) vapor and (noncondensable) gas partial pressures (Leung and Epstein, 1991):

$$\frac{v}{v_o} = \omega\left(\frac{P_{vo}}{P_v} - 1\right) + 1 = \omega\left(\frac{1}{\eta_v} - 1\right) + 1 \tag{12}$$

and

$$\frac{v}{v_o} = \alpha_o\left(\frac{P_{go}}{P_g} - 1\right) + 1 = \alpha_o\left(\frac{1}{\eta_g} - 1\right) + 1 \tag{13}$$

Here ω is defined in terms of the vapor (flashing) component, similar to the subcooled inlet case,

$$\omega = \frac{x_o v_{v\ell o}}{v_o} + \frac{C_p T_o P_{vo}}{v_o}\left(\frac{v_{v\ell o}}{h_{v\ell o}}\right)^2 \tag{14a}$$

$$\omega = \alpha_o + (1 - \alpha_o)\, \rho_{\ell o} C_p T_o P_{vo}\left(\frac{v_{v\ell o}}{h_{v\ell o}}\right)^2 \tag{14b}$$

$$\omega = \alpha_o + (1 - \alpha_o)\, \omega_s \tag{14c}$$

Note that P_{vo} is the saturation pressure (P_s) corresponding to the inlet temperature T_o. Thus for a hybrid flow system, the key parameters are α_o and ω_s. For a given inlet condition, P_o, T_o, α_o, ω (or ω_s), y_{go} ($\equiv P_{go}/P_o$), the generalized mass flux expression can be written as

$$\frac{G}{\sqrt{P_o/v_o}} = \Big\{ 2 \Big[- \alpha_o y_{go} \ln \eta_g + (1 - \alpha_o)\, y_{go} (1 - \eta_g)$$

$$- \omega\, (1 - y_{go}) \ln \eta_v + (1 - \omega)(1 - y_{go})(1 - \eta_v)\Big]\Big\}^{1/2} \Big/ \left(\omega\left(\frac{1}{\eta_v} - 1\right) + 1\right) \tag{15}$$

An additional expression relating the two partial pressures, P_v and P_g, is found by combining Eqs. (12) and (13) to yield

$$\alpha_o\left(\frac{1}{\eta_g} - 1\right) = \omega\left(\frac{1}{\eta_v} - 1\right) \tag{16}$$

This result states that during the expansion process the two pressure ratios, η_g ($= P_g/P_{go}$) and η_v ($= P_v/P_{vo}$), are solely governed by the ratio α_o/ω, which is a measure of the relative "compressibility" of the non-flashing component to the flashing component. The condition governing exit choking is given by

$$\frac{G_c}{\sqrt{P_o/v_o}} = \left[\frac{y_{go}}{\alpha_o}\, \eta_g^2 + \frac{(1 - y_{go})}{\omega}\, \eta_v^2\right]^{1/2} \tag{17}$$

where the partial pressure ratios at choking will be given by the following

$$- \alpha_o\, y_{go} \ln \eta_g + (1 - \alpha_o)\, y_{go}\, (1 - \eta_g)$$

$$- \omega\, (1 - y_{go}) \ln \eta_v + (1 - \omega)(1 - y_{go})\, (1 - \eta_v)$$

$$= \frac{1}{2} \left[\frac{y_{go}}{\alpha_o} \eta_g^2 + \frac{(1 - y_{go})}{\omega} \eta_v^2 \right] \left[\omega \left(\frac{1}{\eta_v} - 1 \right) + 1 \right]^2 \tag{18}$$

This is a transcendental equation for either η_g or η_v as it is to be solved simultaneously with Eq. (16) for these "critical" pressure ratios. Once these ratios are found, the overall critical pressure ratio P_c/P_o is given by

$$\eta_c = y_{go}\, \eta_g + (1 - y_{go})\, \eta_v \tag{19}$$

In the case where the back pressure P_a is higher than the choking pressure P_c, then Eq. (16) is solved for P_g/P_{go} and P_v/P_{vo} subject to the obvious condition that P_g and P_v add up to P_a. Equation (15) is then used to yield the unchoked mass flux.

It is impossible to present the entire solution in graphical form in terms of three independent parameters α_o, ω_s, and y_{go}. Figure 4 illustrates the dependence of the normalized critical mass flux G* and critical pressure ratio on α_o and y_{go} at a fixed ω_s value of 10. It is notable that at the limit of $\alpha_o = 0$ (absence of both vapor and gas at the inlet), both G* and P_c/P_o results are in perfect agreement with the choked flow solutions for subcooled liquid inlet case (i.e. Eqs. (8), (9), (10), and (11)); this observation is reassuring. Furthermore it can be shown that these hybrid equations reduce exactly to those for the pure flashing system when $y_{go} = 0$ and to that for the pure non-flashing system when $y_{go} = 1$.

PIPE FLOW - FLASHING AND NON-FLASHING

The treatment of pipe discharge is most conveniently described by an ideal inlet nozzle followed by a constant-diameter pipe flow section (Byrd et al., 1960). For the case of a sharp entrance inlet nozzle, an equivalent pipe length yielding 0.5 velocity head loss can be added to the pipe section. Figure 5 shows the flow system and the usage of subscript. As far as the ideal nozzle flow is concerned, the earlier expression, Eq. (4), is applicable here with $\eta = \eta_1$. For turbulent two-phase pipe flow with a constant friction factor f, the integral momentum equation is written as

$$4f\,\frac{L}{D} = \int_{P_1/P_o}^{P_2/P_o} \frac{-\dfrac{v}{v_o}\,d\left(\dfrac{P}{P_o}\right) - G^{*2}\,\dfrac{v}{v_o}\,d\left(\dfrac{v}{v_o}\right)}{\dfrac{1}{2}\,G^{*2}\left(\dfrac{v}{v_o}\right)^2 + Fi} \tag{20}$$

Here $Fi \equiv \rho_o\, g\, H/P_o/(4fL/D)$ is the so-called "flow inclination" number with H denoting the elevation change relative to the inlet. Horizontal flow yields a $Fi = 0$ and upflow would give a positive Fi number. Utilizing the expansion law, Eq. (1), the above momentum equation can be rewritten as (Leung and Epstein, 1990b)

$$4f\,\frac{L}{D} = -\int_{\eta_1}^{\eta_2} \frac{[(1-\omega)\eta^2 + \omega\eta]\left(1 - G^{*2}\,\dfrac{\omega}{\eta^2}\right)d\eta}{\dfrac{1}{2}\,G^{*2}\,[(1-\omega)\eta + \omega]^2 + \eta^2\,Fi} \tag{21}$$

which can be integrated in closed form, see Appendix A. For the special case of horizontal pipe flow ($Fi = 0$), Eq. (21) reduces to (Leung and Grolmes, 1987)

$$4f\,\frac{L}{D} = \frac{2}{G^{*2}}\left[\frac{\eta_1 - \eta_2}{1-\omega} + \frac{\omega}{(1-\omega)^2}\,\ln\frac{(1-\eta_2)\omega + \eta_2}{(1-\eta_1)\omega + \eta_1}\right]$$

$$- 2\ln\left[\frac{(1-\eta_2)\omega + \eta_2}{(1-\eta_1)\omega + \eta_1}\left(\frac{\eta_1}{\eta_2}\right)\right] \tag{22}$$

Note that the momentum equation yields the pressure drop in the pipe implicitly as a function of 4f L/D and mass flux. Finally a convenient formula governing choking at pipe exit is (similar to Eq. (5) for nozzle),

$$G_c^* = \frac{\eta_{2c}}{\sqrt{\omega}} \tag{23}$$

A subscript c is used here to denote choking. Thus if $\eta_{2c} > \eta_a$ (or $P_{2c} > P_a$), choking is implied and Eq. (23) is valid; Eqs. (4), (22), and (23) are used to solve for G^*, η_1 and η_2 for a given 4f L/D. If $\eta_{2c} < \eta_a$, no choking occurs; only Eqs. (4) and (22) are needed to solve for G^* and η_1 since $\eta_2 = \eta_a$ (unchoked exit).

The exit-choking solutions can be most conveniently presented in design-chart format, similar to Shapiro's (1953) chart for gas flow. Here the mass flux relative to that of a perfect nozzle as denoted by G_c/G_{oc} is plotted versus 4f L/D at ω values ranging from 0 to 100, i.e. covering both flashing and non-flashing flow regimes. Figures 6, 7, and 8 represent such solutions for Fi values of 0, 0.1 and 0.2, respectively. It is notable that the solution given by $\omega = 1$ in Fig. 6 is in perfect agreement with the traditional "isothermal" gas flow approximation. It is further noted that at $\omega = 0$, the solutions are in agreement with single-phase incompressible pipe flow results. Figure 9 presents the choking exit pressure ratio P_{2c}/P_o as function of G/G_o and ω.

PIPE FLOW - INLET SUBCOOLED LIQUID

Flashing flow discharge of initially subcooled liquid in pipes is a simple extension of the nozzle flow treatment (Leung and Ciolek, 1994). Again the inlet acceleration is treated as a perfect nozzle where for low subcooling with flashing within inlet nozzle, Eq. (8) applies (with $\eta = \eta_1$). For high subcooling with no flashing within inlet nozzle, Eq. (10a) becomes

$$G^* = [2\ (1 - \eta_1)]^{0.5} \tag{24}$$

The momentum equation, Eq. (20), can be easily integrated by partitioning into two distinct regions, a single-phase region and a two-phase region. Two situations can develop - Case I represents flashing occurring within the constant diameter pipe and Case II denotes flashing occurring within the inlet nozzle. Again for turbulent pipe flow with constant friction factor, we have, CASE I: Flashing Within Pipe ($P_1 > P_s$)

$$4f \frac{L}{D} = \frac{2}{G^{*2}} \left[(\eta_1 - \eta_s) + \frac{\eta_s - \eta_2}{1 - \omega_s} + \frac{\omega_s \eta_s}{(1 - \omega_s)^2} \ln \frac{(\eta_s - \eta_2)\, \omega_s + \eta_2}{\eta_s} \right]$$

$$- 2 \ln \left[\frac{(\eta_s - \eta_2)\, \omega_s + \eta_2}{\eta_s} \right] \tag{25a}$$

CASE II: Flashing Within Inlet Nozzle ($P_1 < P_s$)

$$4f \frac{L}{D} = \frac{2}{G^{*2}} \left[\frac{\eta_1 - \eta_2}{1 - \omega_s} + \frac{\omega_s \eta_s}{(1 - \omega_s)^2} \ln \frac{(\eta_s - \eta_2)\, \omega_s + \eta_2}{(\eta_s - \eta_1)\, \omega_s + \eta_1} \right]$$

$$- 2 \ln \left[\frac{(\eta_s - \eta_2)\, \omega_s + \eta_2}{(\eta_s - \eta_1)\, \omega_s + \eta_1} \left(\frac{\eta_1}{\eta_2} \right) \right] \tag{25b}$$

Finally, for the condition of exit choking the following expression provides the relationship between critical mass flux and pipe exit pressure (P_{2c}),

$$G_c^* = \frac{\eta_{2c}}{(\omega_s\, \eta_s)^{0.5}} \tag{26}$$

Thus for each case, we have three equations, Eqs. (8) or (24), (25) and (26) to solve for the three unknown - G_c^*, η_1 and η_2. As for the unchoked exit case, $\eta_2 = \eta_a$ (or $P_2 = P_a$) and only Eqs. (8) or (24) and (25) are required to solve for G^* and η_1 simultaneously. Note that Eq. (25b) reduces identically to Eq. (22) at zero subcooling (i.e. saturated inlet) when η_s = 1.

For sufficiently large inlet subcooling, a flow condition can be attained whereby only a single-phase liquid region prevails throughout the entire pipe, and flashing occurs just at the exit end of the pipe (i.e. $P_2 = P_s$). The transition saturation pressure ratio is given by

$$\eta_{st} = \frac{2\, \omega_s}{1 + 4f \dfrac{L}{D} + 2\, \omega_s} \tag{27}$$

which reduces exactly to Eq. (7) for the nozzle case when L/D approaches zero. If the inlet subcooling is high such that $\eta_s < \eta_{st}$, then the above methodology simply yields a critical mass flux given by

$$G_c^* = \left(\frac{2\,(1 - \eta_s)}{1 + 4f\,\dfrac{L}{D}} \right)^{0.5} \tag{28a}$$

or in dimensional form

$$G_c = \left(\frac{2\,\rho_{\ell o}\,(P_o - P_s)}{1 + 4f\,\dfrac{L}{D}} \right)^{0.5} \tag{28b}$$

Note that this expression differs from the classical incompressible pipe flow formula in that the pressure drop is not the total (P_o - P_a) but rather (P_o - P_s) due to choking condition at the exit.

PIPE FLOW - FLASHING WITH NONCONDENSABLE GAS

Treatment of pipe discharge of such a hybrid system is a natural extension of the nozzle development. Again the inlet is treated as an ideal nozzle and the previously derived inlet acceleration, Eq. (15), applies here. For turbulent pipe flow, the momentum equation, Eq. (20), can be expanded to yield

$$4f\,\frac{L}{D} = (1 - y_{go}) \int_{\eta_{v1}}^{\eta_{v2}} \frac{-\left(\dfrac{v}{v_o}\right) d\eta_v}{\dfrac{1}{2}\,G^{*2}\left(\dfrac{v}{v_o}\right)^2 + Fi} + y_{go} \int_{\eta_{g1}}^{\eta_{g2}} \frac{-\left(\dfrac{v}{v_o}\right) d\eta_g}{\dfrac{1}{2}\,G^{*2}\left(\dfrac{v}{v_o}\right)^2 + Fi}$$

$$+ \int_{v_1/v_o}^{v_2/v_o} \frac{-\,G^{*2}\left(\dfrac{v}{v_o}\right) d\left(\dfrac{v}{v_o}\right)}{\dfrac{1}{2}\,G^{*2}\left(\dfrac{v}{v_o}\right)^2 + Fi} \tag{29}$$

while noting Eqs. (12) and (13) provide the respective expansion law in terms of the vapor and gas partial pressures. Note that Eq. (29) after substitution of Eqs. (12) and (13) bears much resemblance to Eq. (21) and can be integrated in closed form as shown in Appendix B. For the special case of horizontal pipe flow, Eq. (29) yields

$$4f\,\frac{L}{D} = \frac{2}{G^{*2}}\left\{ y_{go}\left[\frac{\eta_{g1} - \eta_{g2}}{1 - \alpha_o} + \frac{\alpha_o}{(1 - \alpha_o)^2}\,\ln\frac{(1 - \eta_{g2})\,\alpha_o + \eta_{g2}}{(1 - \eta_{g1})\,\alpha_o + \eta_{g1}}\right]\right.$$

$$\left.(1 - y_{go})\left[\frac{\eta_{v1} - \eta_{v2}}{1 - \omega} + \frac{\omega}{(1 - \omega)^2}\,\ln\frac{(1 - \eta_{v2})\,\omega + \eta_{v2}}{(1 - \eta_{v1})\,\omega + \eta_{v1}}\right]\right\}$$

$$- 2\,\ln\left[\frac{(1 - \eta_{v2})\,\omega + \eta_{v2}}{(1 - \eta_{v1})\,\omega + \eta_{v1}}\left(\frac{\eta_{v1}}{\eta_{v2}}\right)\right] \tag{30}$$

where the partial pressures at locations 1 and 2 are related by (rewriting Eq. 16),

$$\eta_{g1} = \frac{1}{1 + \dfrac{\omega}{\alpha_o}\left(\dfrac{1}{\eta_{v1}} - 1\right)} \tag{31a}$$

$$\eta_{g2} = \frac{1}{1 + \dfrac{\omega}{\alpha_o}\left(\dfrac{1}{\eta_{v2}} - 1\right)} \tag{31b}$$

Finally the expression governing choking at pipe exit is similar to Eq. (17) where η_g and η_v becomes η_{g2} and η_{v2}, both evaluated at location 2. The exit pressure ratio η_2 is given by Eq. (19).

For a given 4f L/D and inlet conditions, we have six equations - Eqs. (15), (17), (30), (31a), (31b) and (19) and six unknowns - G^*, η_{v1}, η_{g1}, η_{v2}, η_{g2}, and η_{2c}. In the case where the back pressure P_a is higher than the choking pressure P_{2c} (or $\eta_{2c}P_o$), then Eq. (31b) is solved for η_{g2} and η_{v2} subject to the condition that P_{g2} and P_{v2} add up to P_a. This eliminates the constraint governed by the exit choking expression, Eq. (17). Figure 10 illustrates how G_c/G_{oc} ratio varies as function of $4f\,\dfrac{L}{D}$ and y_{go} the mole fraction of noncondensable gas in the vapor phase.

It is noted that the hybrid flow solution presented herein reduces to all previously discussed flow configurations, i.e.

(1) Subcooled inlet pipe discharge solution when $\alpha_o = 0$, Fi = 0.
(2) Flashing pipe discharge solution when $y_{go} = 0$.
(3) Non-flashing pipe discharge solution when $y_{go} = 1$.
(4) Flashing nozzle discharge when $y_{go} = 0$, L/D = 0.
(5) Non-flashing nozzle discharge when $y_{go} = 1$, L/D = 0.
(6) Subcooled inlet nozzle discharge when $\alpha_o = 0$, L/D = 0.
(7) Hybrid flow nozzle discharge when $y_{go} > 0$, L/D = 0.

"MIXING RULE" IN HYBRID FLOW

For hybrid flow situations, it is often adequate to provide an approximation without solving the aforementioned algebraic set of equations. The following "mixing rule" is applicable to both nozzle flow and pipe flow of a flashing system in the presence of noncondensable gas. It is entirely an empirical observation as given by

$$G^* = \left[y_{go}\, G_g^{*2} + (1 - y_{go})\, G_v^{*2}\right]^{1/2} \tag{32a}$$

$$G = \left[y_{go}\, G_g^2 + (1 - y_{go})\, G_v^2\right]^{1/2} \tag{32b}$$

where G_g and G_g^* are the mass fluxes corresponding to a pure non-flashing flow system and G_v and G_v^* are the mass fluxes corresponding to a pure flashing flow system. Both G_g^* and G_v^* can be easily obtained from their corresponding expressions or design charts since they are dependent on values of α_o and ω, respectively.

MULTICOMPONENT AND NONIDEAL SYSTEMS

So far the ω method has been developed primarily for one- or two-component fluid. Limited success has been reported for multicomponent mixture exhibiting minor vapor-phase composition change during depressurization (Leung and Fisher, 1989; Leung and Nazario, 1990). In a more recent study (Nazario and Leung, 1992), a total of 15 different systems were evaluated at conditions varying from saturated liquid to almost all-vapor. Some of the systems were simple binaries while others were representative of actual multicomponent refinery streams. As shown in Fig. 11, a parity plot of the ω prediction results against the theoretical calculations (involving no fewer than ten flash calculations), a number of mixtures was found to fall outside the acceptable band ($\pm$ 15%). All the unacceptable deviations are on the conservative side though, resulting in oversize relief devices. Overall, the ω correlation appears adequate for reasonably narrow boiling mixtures (those with a difference in nominal atmospheric-pressure boiling temperature between the lightest and heaviest component of less than 80°C). But for wide boiling range mixtures and for non-ideal systems such as those containing hydrogen, in particular, the ω correlation underpredicts the flow significantly. In fact, for most of the hydrogen-containing mixtures, the flow can be closely approximated based on the purely non-flashing ($\omega = \alpha_o$) flow correlation. It was concluded that the simplifying assumptions used to arrive at the ω correlation do not accurately reflect the actual flashing behavior of these multicomponent mixtures. However, even for the most non-ideal systems examined, the normalized specific volume as a function of the pressure ratio, i.e. Eq. (1) can be described reasonably well with a straight line (up to the critical flow pressure). Hence it was led to suggest evaluating the ω parameter based on the actual flash behavior, say, to 90% of the inlet stagnation pressure. Thus, from Eq. (1), upon rearranging,

$$\omega = \frac{\dfrac{v_t}{v_o} - 1}{\dfrac{P_o}{P_t} - 1} \tag{33}$$

where $P_t/P_o = 0.9$, and v_t is calculated from the isenthalpic flash result. Figure 12 presents the results based on the newly defined ω value; the results are most encouraging in that all the previous deviations have disappeared. It should also be mentioned that a two-parameter expansion law has been used successfully to treat most non-ideal or complex mixtures of practical interest (Fauske & Associates, Inc., 1983, 1984; Morris, 1990). Here the equation of state takes the form

$$\frac{v}{v_o} = 1 + a\left(\frac{P_o}{P} - 1\right) + b\left(\frac{P_o}{P} - 1\right)^2 \qquad (34)$$

This reduces to Eq. (1) when a = ω and b = 0. By carrying out two flash calculations at, say, 0.9 P_o and 0.5 P_o, the a and b parameters can be found easily. The treatment of nozzle flow and pipe flow follows closely the development presented here although the actual numerical calculation is most expediently performed by the computer. Rather than seeking the analytical closed-form solution for the integral momentum equation based on Eq. (20), the integration can be efficiently carried out numerically.

SONIC VELOCITY AND CRITICAL FLOW

A critical flow condition is reached at a nozzle throat or pipe exit where a decompressive disturbance cannot travel upstream. No matter how low the downstream pressure drops to (even vacuum), the flow is unchanged. Physically the exit fluid (homogeneous) velocity has reached the local speed of sound or sonic velocity. For a two-phase homogeneous fluid, the sonic velocity c can be given by

$$c = v\left[-\left(\frac{\partial v}{\partial P}\right)_s\right]^{1/2} \qquad (35)$$

where the property derivative is to be evaluated along a constant entropy path. Furthermore for a decompressive disturbance, thermodynamic equilibrium between phases is closely approached. In a recent paper (Leung, 1993) the HEM sonic velocity according to Eq. (35) can be correlated by the expression

$$c^* \equiv \frac{c}{\sqrt{Pv}} = \frac{1}{\sqrt{\omega}} \qquad (36)$$

where ω is only a modified form of the previously discussed ω parameter, i.e.

$$\omega = \alpha\left(1 - 2\frac{Pv_{v\ell}}{h_{v\ell}}\right) + \frac{C_p TP}{v}\left(\frac{v_{v\ell}}{h_{v\ell}}\right)^2 \qquad (37)$$

The extra term is $(1 - 2\ Pv_{v\ell}/h_{v\ell})$ which is always slightly less than unity. It is of interest to note that for the case of all-vapor flow, ω takes on a value less than 1.0 (unlike previous expression where ω is 1.0 or slightly above 1.0). The success of such an ω as a scaling parameter is demonstrated in Fig. 13 where published theoretical two-phase sonic velocity data for four chemicals can be accurately correlated. In this comparison the data span the entire two-phase range with local quality ranging from 0 (saturated liquid) to 1.0 (saturated vapor).

Finally if the properties in Eq. (37) are evaluated at the inlet stagnation condition, the published theoretical HEM critical nozzle flow data for steam-water system can be closely represented by the ω solutions as shown in Fig. 14. Very little deviation is noted as the inlet quality x_o approaches unity (all steam). This therefore represents an improvement over the earlier empirical curve fit in the region when $\omega < 4.0$ (Leung, 1986).

CONCLUSION

The ω method represents a convenient way of estimating discharge flow of two-phase mixtures because it utilizes simply inlet physical properties. It further demonstrates that compressible two-phase flow can be correlated with only limited number of dimensionless physical property groups. Most situations of interest have been formulated and developed including flashing flow, non-flashing flow, flashing flow with subcooled liquid inlet, and flashing flow with noncondensable gas. Both nozzle and pipe discharge configurations are considered. It can be shown that compressible gas flow and incompressible liquid flow are special (limiting) solutions of the ω formulation. Recent development also demonstrates that two-phase sonic velocity data can be accurately correlated by the ω scaling parameter as well. The ω method tends to be more restrictive when applied to multicomponent systems. An alternate ω, determined with the help of an actual flash calculation, was suggested for those systems exhibiting wide boiling point range and containing light volatile gases such as hydrogen.

REFERENCES

Abramowitz and Stegun, Handbook of Mathematical Functions, Dover Publications, New York, Chapt. 3 (1972).

Byrd, R. B., Stewart, W. E., and Lightfoot, E. N., Transport Phenomena, Chapt. 15, Wiley, New York (1960).

Fauske & Associates, Inc., "SAFIRE Users Manual, Vol. V, Vent Flow Routines," AIChE/DIERS Report, FAI/84-5 (1984).

Fauske & Associates, Inc., "Emergency Relief Systems for Runaway Chemical Reactions and Storage Vessels: A Summary of Multiphase Flow Methods," AIChE/DIERS Report, FAI/83-27 (1983).

Leung, J. C., "A Generalized Correlation for One-Component Homogeneous Equilibrium Flashing Choked Flow," AIChE Journal, 32(10), 1743 (1986).

Leung, J. C. and Grolmes, M. A., "The Discharge of Two-Phase Flashing Flow in a Horizontal Duct," AIChE Journal, 33(3), 524 (1987); also errata, 34(6), 1030 (1988).

Leung, J. C. and Grolmes, M. A., "A Generalized Correlation for Flashing Choked Flow of Initially Subcooled Liquid," AIChE Journal, Vol. 34, 688-691 (1988).

Leung, J. C. and Fisher, H. G., "Two-Phase Flow Venting of Reactor Vessels," J. Loss Prevention Process Industries, 2(2), 78 (1989).

Leung, J. C. and Epstein, M., "A Generalized Correlation for Two-Phase Non-Flashing Homogeneous Choked Flow," ASME Trans. J. of Heat Transfer, 112(2), 528 (May, 1990a).

Leung, J. C. and Epstein, M., "The Discharge of Two-Phase Flashing Flow from an Inclined Duct," ASME Trans. J. of Heat Transfer, 112(2), 524 (May, 1990b).

Leung, J. C., "Two-Phase Flow Discharge in Nozzles and Pipes - A Unified Approach," J. Loss Prevention Process Industries, 3(27) (1990a).

Leung, J. C., "Similarity Between Flashing and Non-Flashing Two-Phase Flows," AIChE Journal, 36(5), 797 (1990b).

Leung, J. C. and Nazario, F. N., "Two-Phase Flashing Flow Methods and Comparisons," J. Loss Prevention Process Industries, 3(2), 253 (1990).

Leung, J. C. and Epstein, M., "Flashing Two-Phase Flow Including the Effects of Noncondensable Gases," ASME Trans. J. of Heat Transfer, 113(1), 269 (February, 1991).

Leung, J. C., "Size Safety Relief Valves for Flashing Liquids," Chem. Eng. Progress, 88(2), 70 (1992).

Leung, J. C., "A Theoretical Sonic Velocity Correlation for Homogeneous Two-Phase Mixtures," Paper submitted for publication (September, 1993).

Leung, J. C. and Ciolek, W. H., "Flashing Flow Discharge of Initially Subcooled Liquid in Pipes," ASME Trans. J. of Fluids Engineering, 116(3) (September, 1994).

Morris, S. D., "Flashing Flow Through Relief Lines, Pipe Breaks and Cracks," J. Loss Prevention Process Industries, 3(1), 17 (1990).

Nazario, F. N. and Leung, J. C., "Sizing Pressure Relief Valves in Flashing and Two-Phase Service: An Alternative Procedure," J. Loss Prevention Process Industries, 5(5), 263 (1992).

Shapiro, A., Compressible Fluid Flow, Vol. I, Chapt. 6, Ronald Press, New York (1953).

NOTATION

c	=	sonic velocity
c^*	=	normalized sonic velocity
C_p	=	liquid specific heat at constant pressure
D	=	pipe diameter
f	=	Fanning friction factor
Fi	=	flow inclination number
G	=	mass velocity or flux
G^*	=	normalized mass flux
$h_{v\ell}$	=	latent heat of vaporization
H	=	elevation change
L	=	pipe length
k	=	gas or vapor specific heat ratio
P	=	absolute pressure

P_i	=	partial pressure of phase i, i = v or g
T	=	absolute temperature
v	=	specific volume
x	=	vapor mass fraction or quality
y_{go}	=	inlet gas mole fraction in vapor phase
α_o	=	inlet void fraction
ρ	=	fluid density
η	=	P/P_o pressure ratio
η_g	=	P_g/P_{go} partial pressure ratio
η_v	=	P_v/P_{vo} partial pressure ratio
ω	=	parameter as defined by Eq. (2)

Subscripts

a	=	refers to ambient pressure
c	=	critical or choked
g	=	gas
ℓ	=	liquid
o	=	stagnation inlet condition
s	=	saturated condition
v	=	vapor
$v\ell$	=	difference between vapor and liquid properties
1	=	pipe inlet at constant diameter section
2	=	pipe exit

APPENDIX A

The integrand in Eq. (21) is first expanded and rearranged to yield

$$4f\,\frac{L}{D} = -\int_{\eta_1}^{\eta_2} \frac{(1-\omega)\eta^2 + \omega\eta - \omega(1-\omega)G^{*2} - \omega^2\,\dfrac{G^{*2}}{\eta}}{\left[\dfrac{G^{*2}}{2}(1-\omega)^2 + \mathrm{Fi}\right]\eta^2 + G^{*2}\,\omega(1-\omega)\eta + \dfrac{G^{*2}\omega^2}{2}}\,d\eta \tag{A-1}$$

Letting $X(\eta)$ denote the denominator of the integrand, we can write out the four pertinent integrals as

$$4f\,\frac{L}{D} = \underbrace{-\int_{\eta_1}^{\eta_2} \frac{(1-\omega)\eta^2 d\eta}{X(\eta)}}_{I_1} \underbrace{-\int_{\eta_1}^{\eta_2} \frac{\omega\eta d\eta}{X(\eta)}}_{I_2}$$

$$\underbrace{+\int_{\eta_1}^{\eta_2} \frac{\omega(1-\omega)G^{*2} d\eta}{X(\eta)}}_{I_3} \underbrace{+\int_{\eta_1}^{\eta_2} \frac{\omega^2 G^{*2} d\eta}{\eta X(\eta)}}_{I_4} \tag{A-2}$$

Now making use of the integral table (Abramowitz and Stegun, 1972), and by denoting

$$a \equiv \frac{G^{*2}}{2}\,\omega^2 \tag{A-3}$$

$$b \equiv G^{*2}\omega(1-\omega) \tag{A-4}$$

$$c \equiv \frac{G^{*2}}{2}(1-\omega)^2 + \mathrm{Fi} \tag{A-5}$$

$$q \equiv 4ac - b^2 = 2G^{*2}\,\omega^2\,\mathrm{Fi} \tag{A-6}$$

the solution for the fundamental integral, $\int d\eta/X(\eta)$, is given in the following form:

$$I_0(\eta) \equiv \int \frac{d\eta}{X(\eta)}$$

$$= \frac{2}{\sqrt{q}} \tan^{-1} \frac{2c\eta + b}{\sqrt{q}} \quad (q > 0 \quad \text{or} \quad Fi > 0, \text{ upflow}) \tag{A-7a}$$

$$= \frac{1}{\sqrt{-q}} \ln \left| \frac{2c\eta + b - \sqrt{-q}}{2c\eta + b + \sqrt{-q}} \right| \quad (q < 0 \quad \text{or} \quad Fi < 0, \text{ downflow}) \tag{A-7b}$$

$$= - \frac{2}{2c\eta + b} \quad (q = 0 \quad \text{or} \quad Fi = 0, \text{ horizontal flow}) \tag{A-7c}$$

Via integration by parts, the four pertinent integrals can be evaluated as follows:

$$I_1 = (1 - \omega) \left[\frac{\eta_2 - \eta_1}{c} - \frac{b}{2c^2} \ln \frac{X(\eta_2)}{X(\eta_1)} + \frac{b^2 - 2ac}{2c^2} (I_0(\eta_2) - I_0(\eta_1)) \right] \tag{A-8}$$

$$I_2 = \omega \left[\frac{1}{2c} \ln \frac{X(\eta_2)}{X(\eta_1)} - \frac{b}{2c} (I_0(\eta_2) - I_0(\eta_1)) \right] \tag{A-9}$$

$$I_3 = G^{*2} \omega (1 - \omega) [I_0(\eta_2) - I_0(\eta_1)] \tag{A-10}$$

$$I_4 = G^{*2} \omega^2 \left[\frac{1}{2a} \ln \frac{\eta_2^2 X(\eta_1)}{\eta_1^2 X(\eta_2)} - \frac{b}{2a} (I_0(\eta_2) - I_0(\eta_1)) \right] \tag{A-11}$$

Finally the desired result is

$$4f \frac{L}{D} = - I_1 - I_2 + I_3 + I_4 \tag{A-12}$$

APPENDIX B

The first integrand in Eq. (29) is expanded and rearranged to yield

$$J_v = - \int_{\eta_{v1}}^{\eta_{v2}} \frac{(1 - \omega)\,\eta_v^2 + \omega\,\eta_v}{\left[\frac{1}{2}\,G^{*2}(1 - \omega)^2 + Fi\right]\eta_v^2 + G^{*2}\omega(1 - \omega)\,\eta_v + \frac{1}{2}\,G^{*2}\omega^2}\,d\eta_v \qquad \text{(B-1)}$$

Note that this integral is similar to ($I_1 + I_2$) of Eq. (A-2) with η_v replacing η. Thus the solution can be given by Eqs. (A-8) and (A-9), i.e.

$$J_v = - I_1(\eta_v) - I_2(\eta_v) \qquad \text{(B-2)}$$

where I_1 and I_2 are to be evaluated in terms of η_v.

The second integrand in Eq. (29) can be shown to yield a similar form

$$J_g = - \int_{\eta_{g1}}^{\eta_{g2}} \frac{(1 - \alpha_o)\,\eta_g^2 + \alpha_o\,\eta_g}{\left[\frac{1}{2}\,G^{*2}(1 - \alpha_o)^2 + Fi\right]\eta_g^2 + G^{*2}\alpha_o(1 - \alpha_o)\,\eta_g + \frac{1}{2}\,G^{*2}\alpha_o^2}\,d\eta_g \qquad \text{(B-3)}$$

Note the similarity between Eqs. (B-3) and (B-1); hence Eqs. (A-8) and (A-9) can provide the solution if the following substitutions are made: ω replaced by α_o in Eqs. (A-3), (A-4), (A-5), (A-6), (A-8), and (A-9), and I_1 and I_2 of Eqs. (A-8) and (A-9) are in terms of η_g instead of η. Hence

$$J_g = - I_1(\eta_g) - I_2(\eta_g) \qquad \text{(B-4)}$$

The third integrand in Eq. (29) after expansion resembles I_3 and I_4 terms in Eq. (A-2) with η_v replacing η. Hence the final closed-form solution for Eq. (29) can be represented by

$$4f\,\frac{L}{D} = -\,(1 - y_{go})[I_1(\eta_v) + I_2(\eta_v)]$$

$$-\,y_{go}[I_1(\eta_g) + I_2(\eta_g)]$$

$$+\,I_3(\eta_v) + I_4(\eta_v) \qquad \text{(B-5)}$$

For the case when Fi = 0, the above equation reduces exactly to the solution for the horizontal case of Eq. (30).

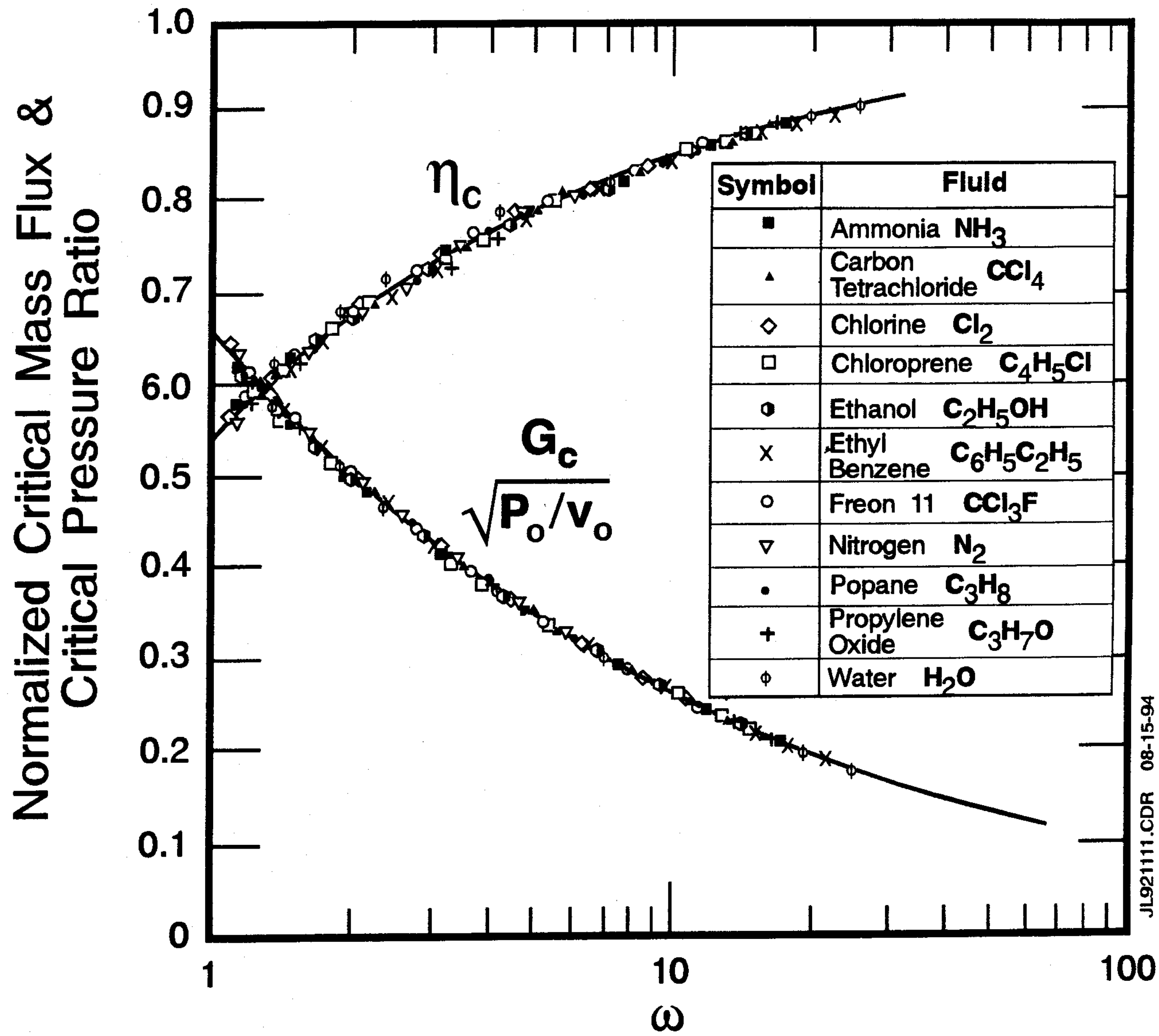


Figure 1 Correlation for calculated HEM nozzle choked flow data based on ω parameter.

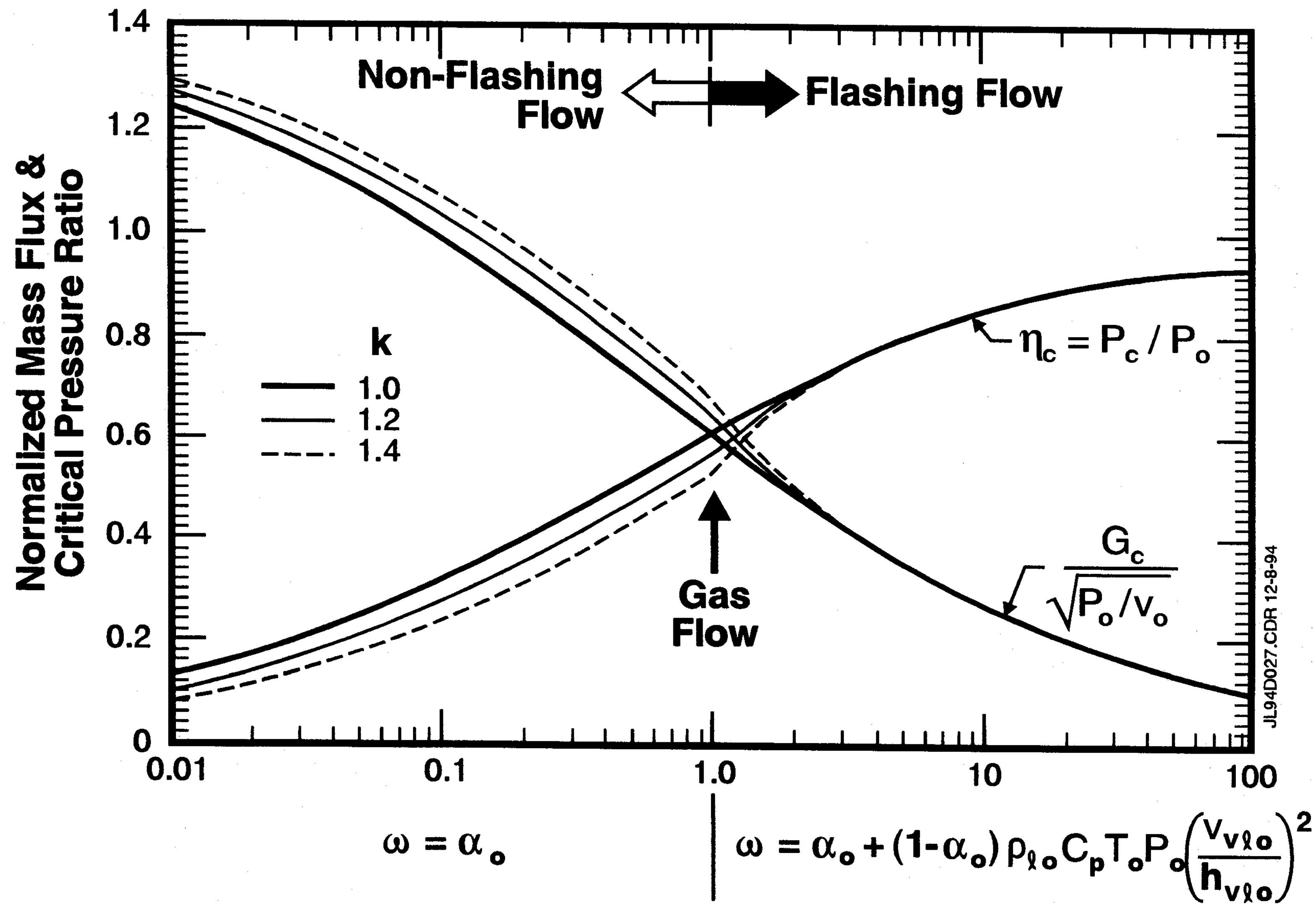


Figure 2 Correlation for nozzle critical flow of flashing and non-flashing systems.

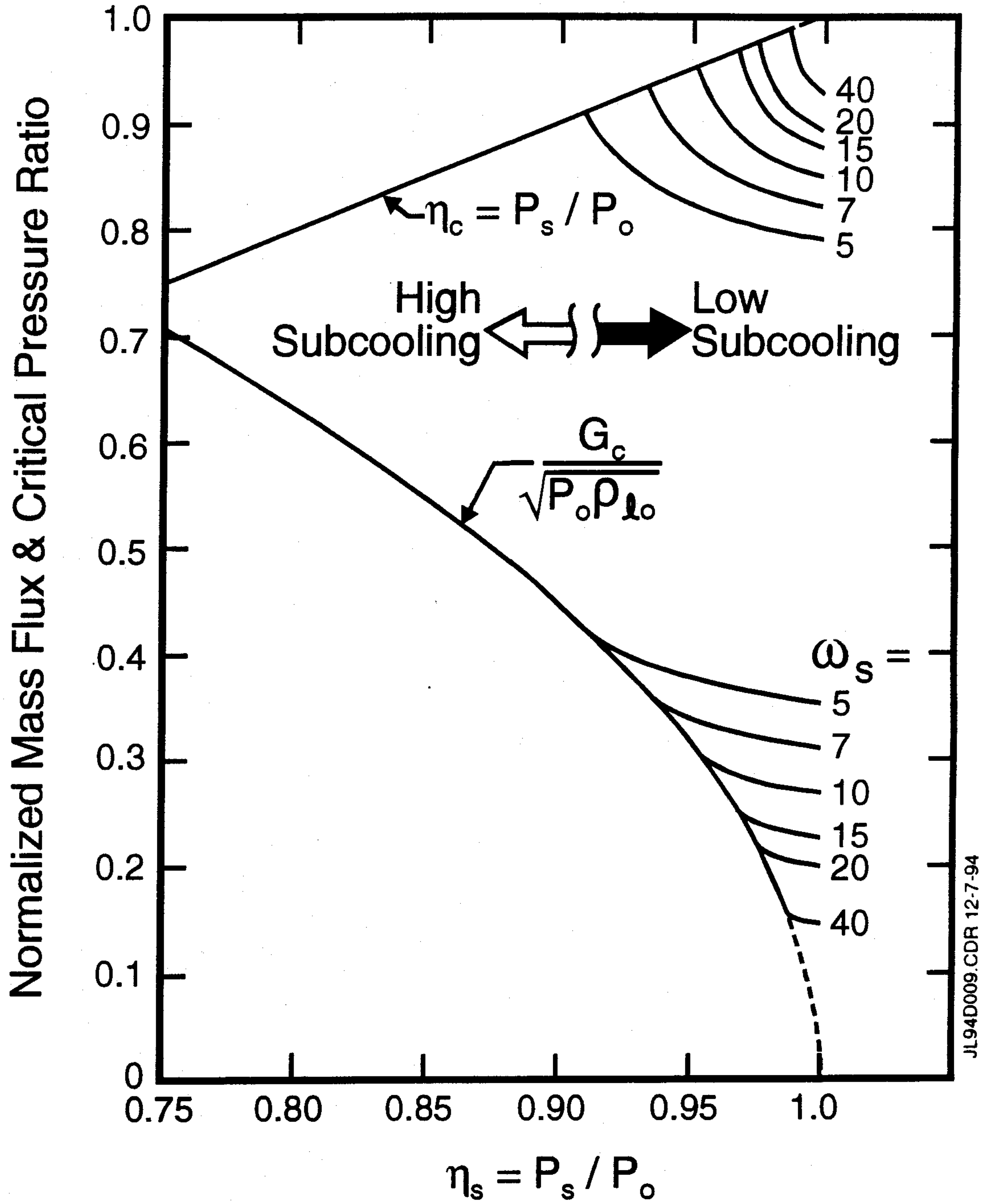


Figure 3 Correlation for nozzle critical flow of inlet subcooled liquid.

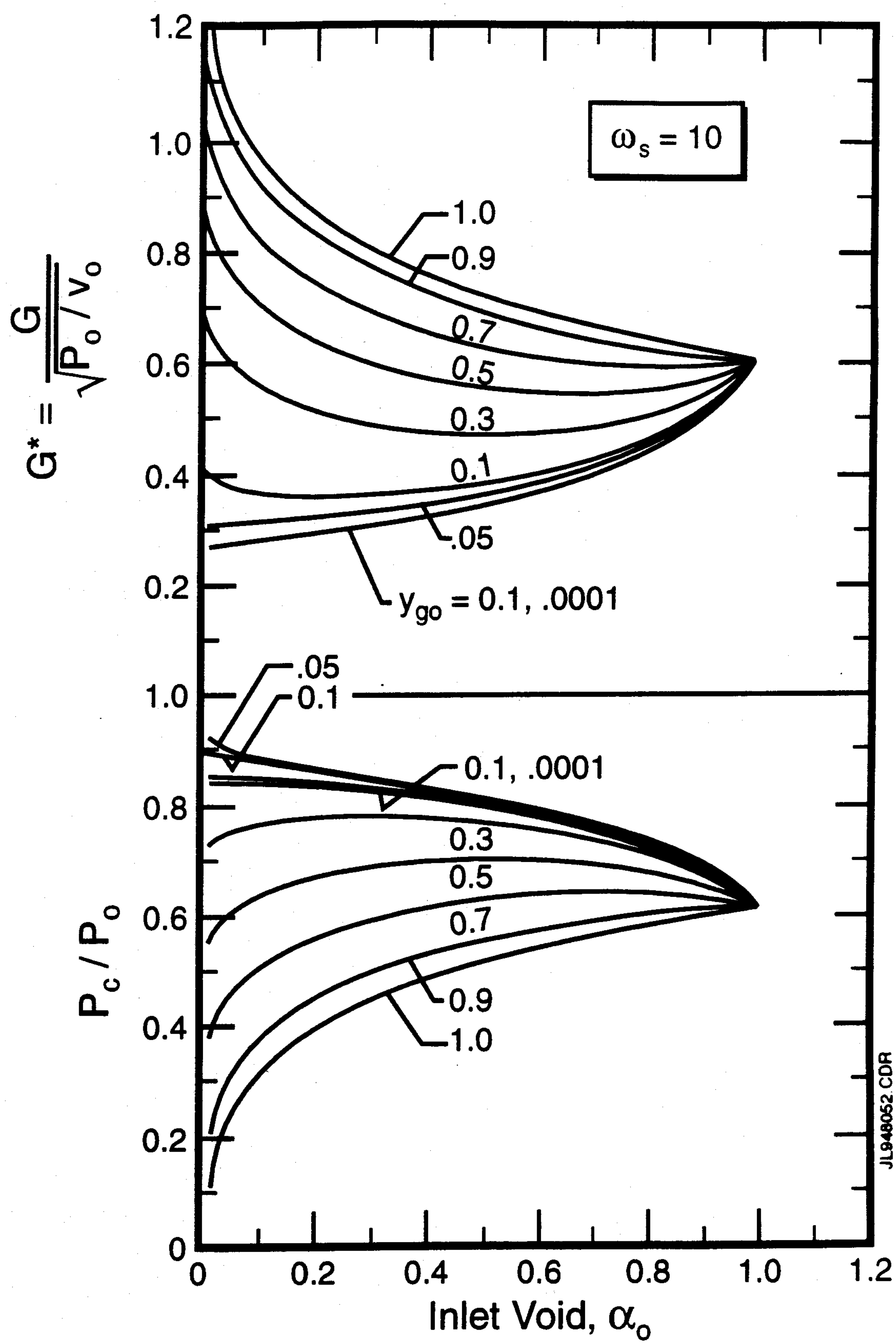


Figure 4 Typical solutions for hybrid flow system for an ω_s value of 10.

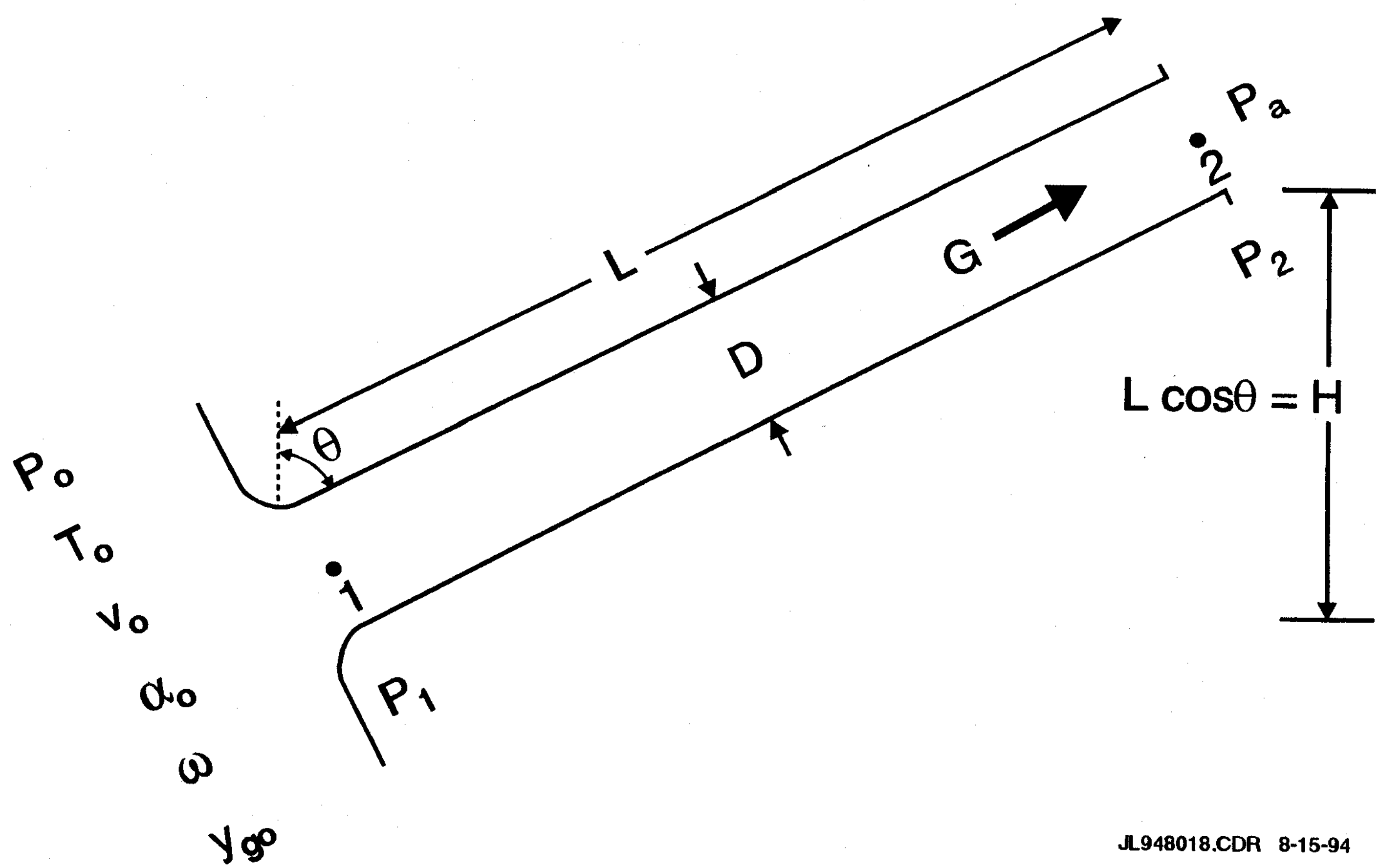
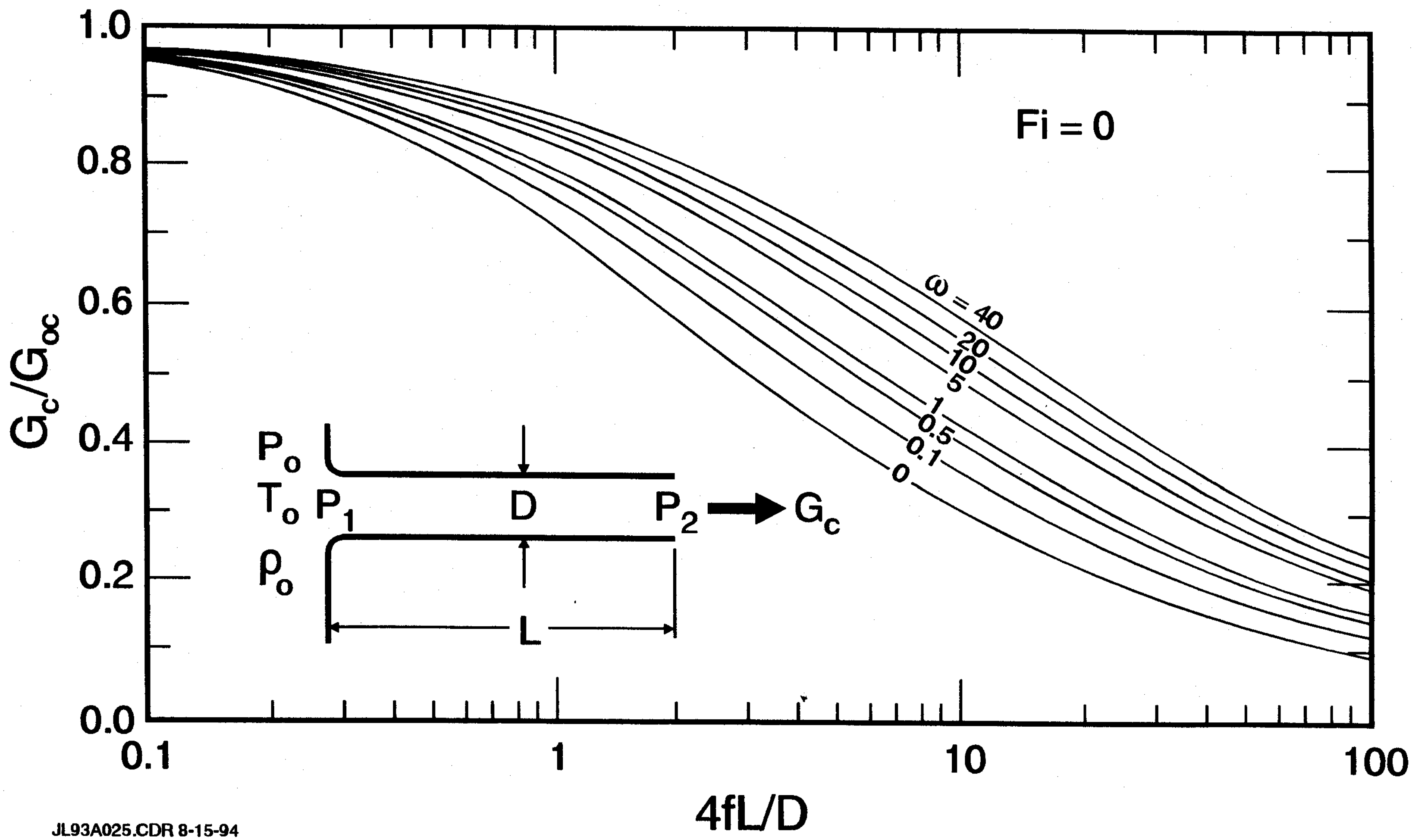


Figure 5 System model and notation for general pipe discharge configuration.



JL93A025.CDR 8-15-94

Figure 6 Choked flow discharge from pipe with Fi = 0 (horizontal).

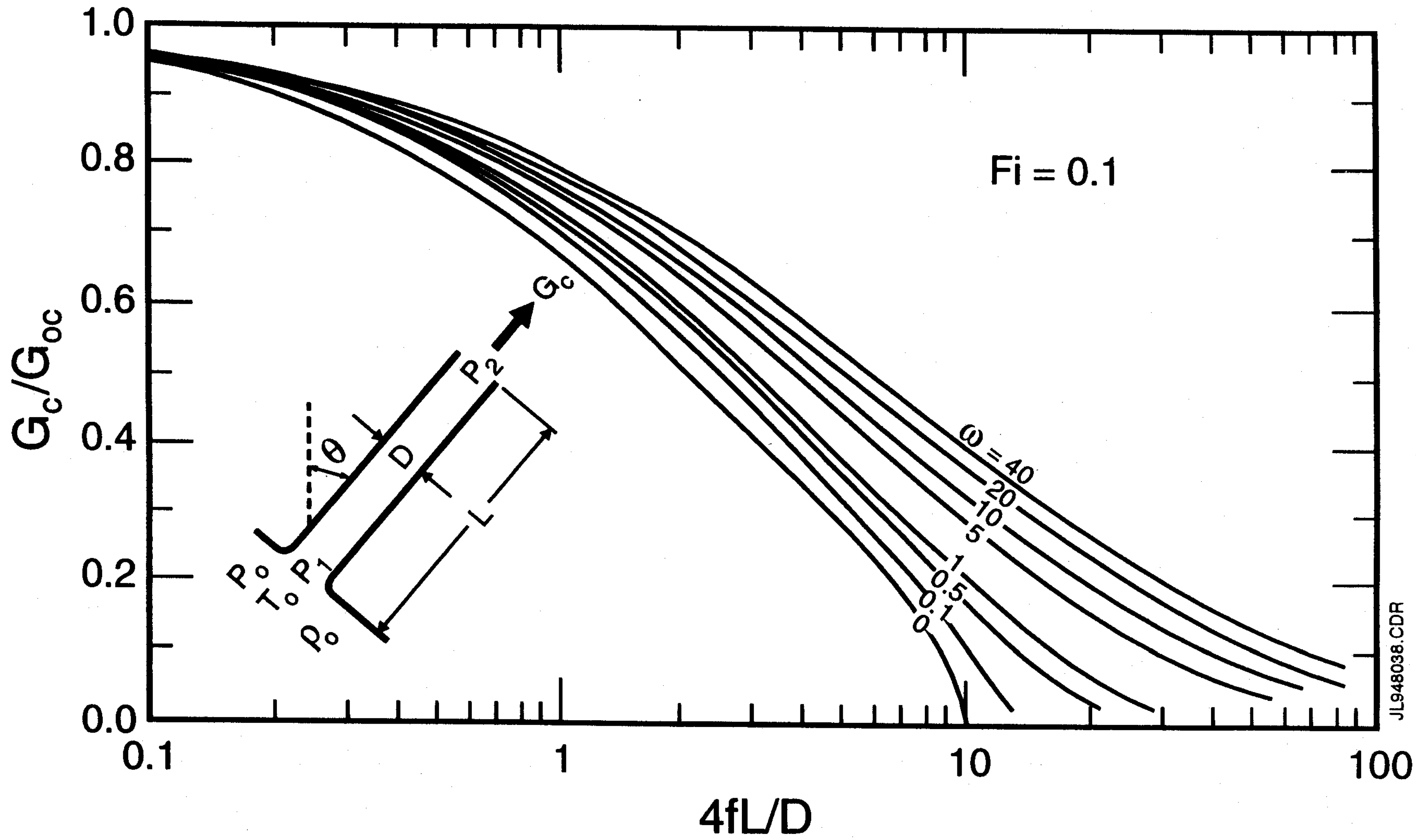


Figure 7 Choked flow discharge from pipe with Fi = 0.1.

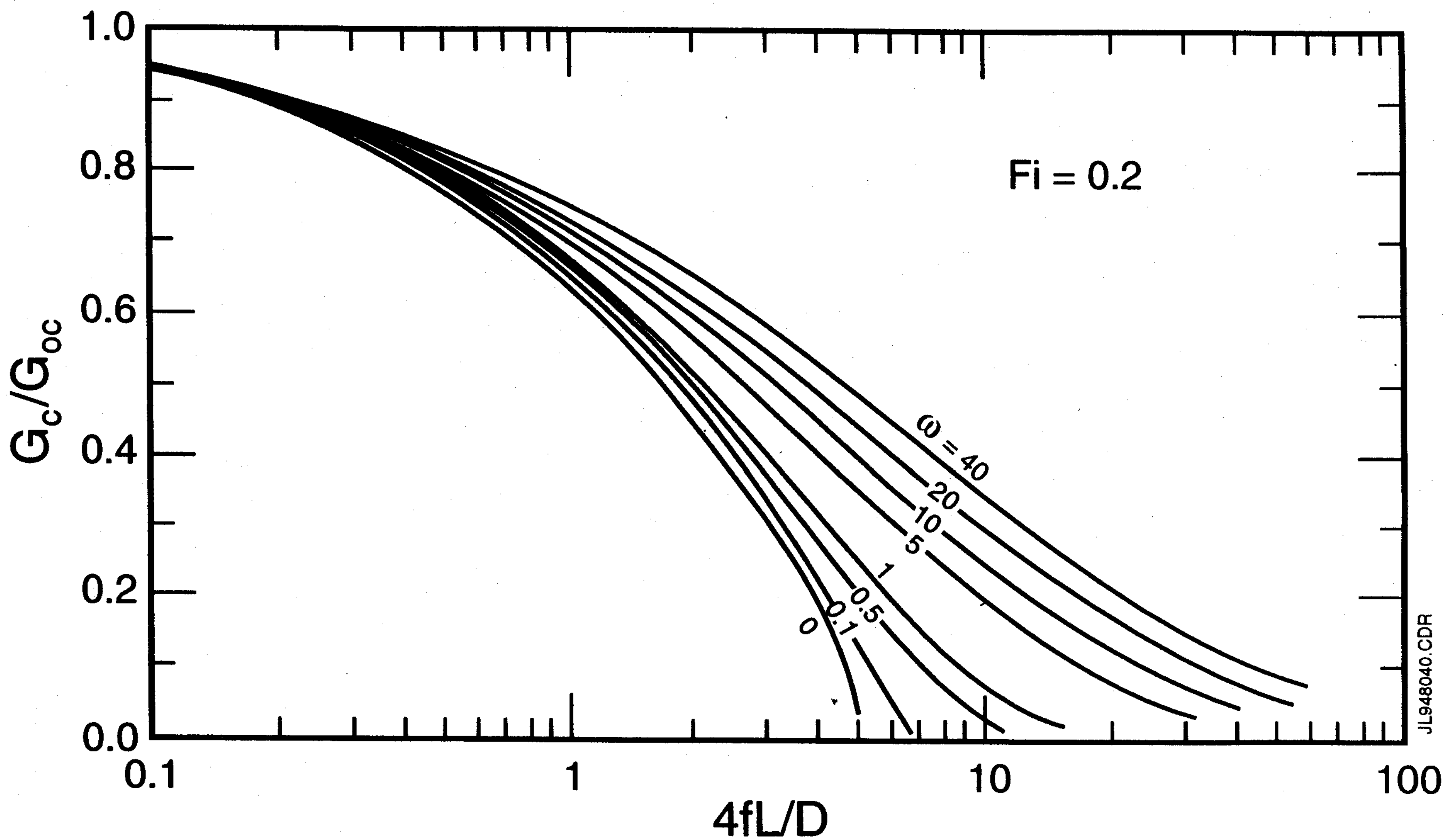


Figure 8 Choked flow discharge from pipe with Fi = 0.2.

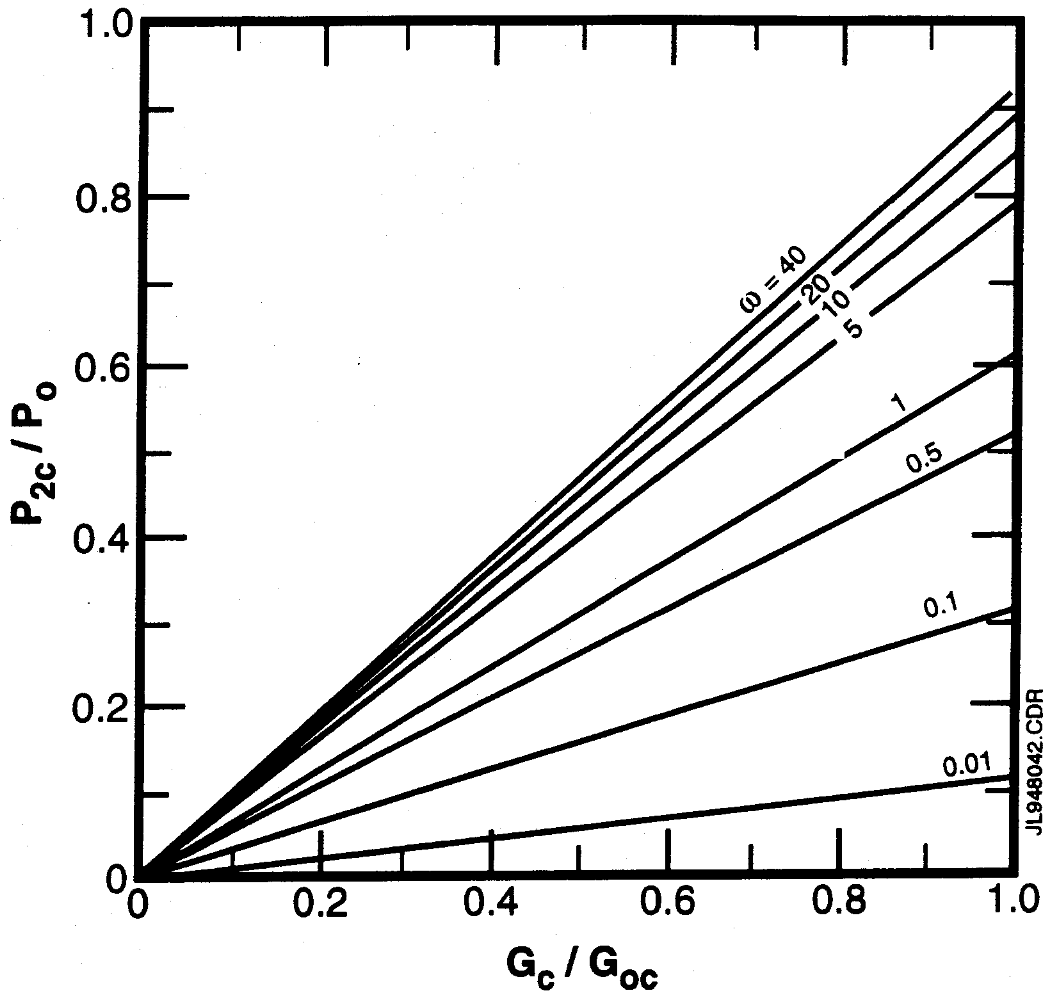


Figure 9 Correlation for exit choking pressure ratio in pipe discharge.

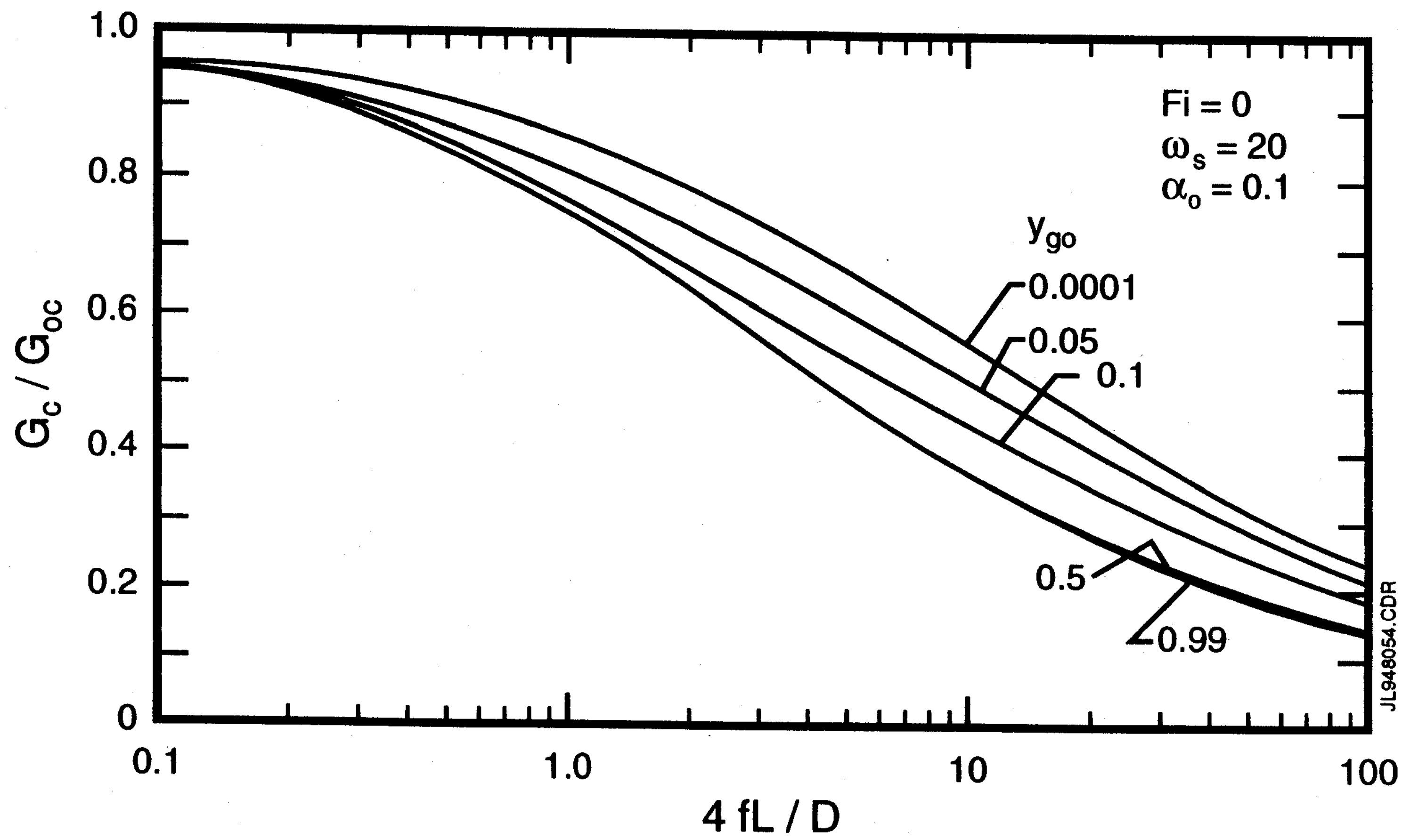


Figure 10 Influence of gas mole fraction on critical mass flux for Fi = 0 (horizontal pipe), $\omega_s = 20$ and $\alpha_o = 0.1$.

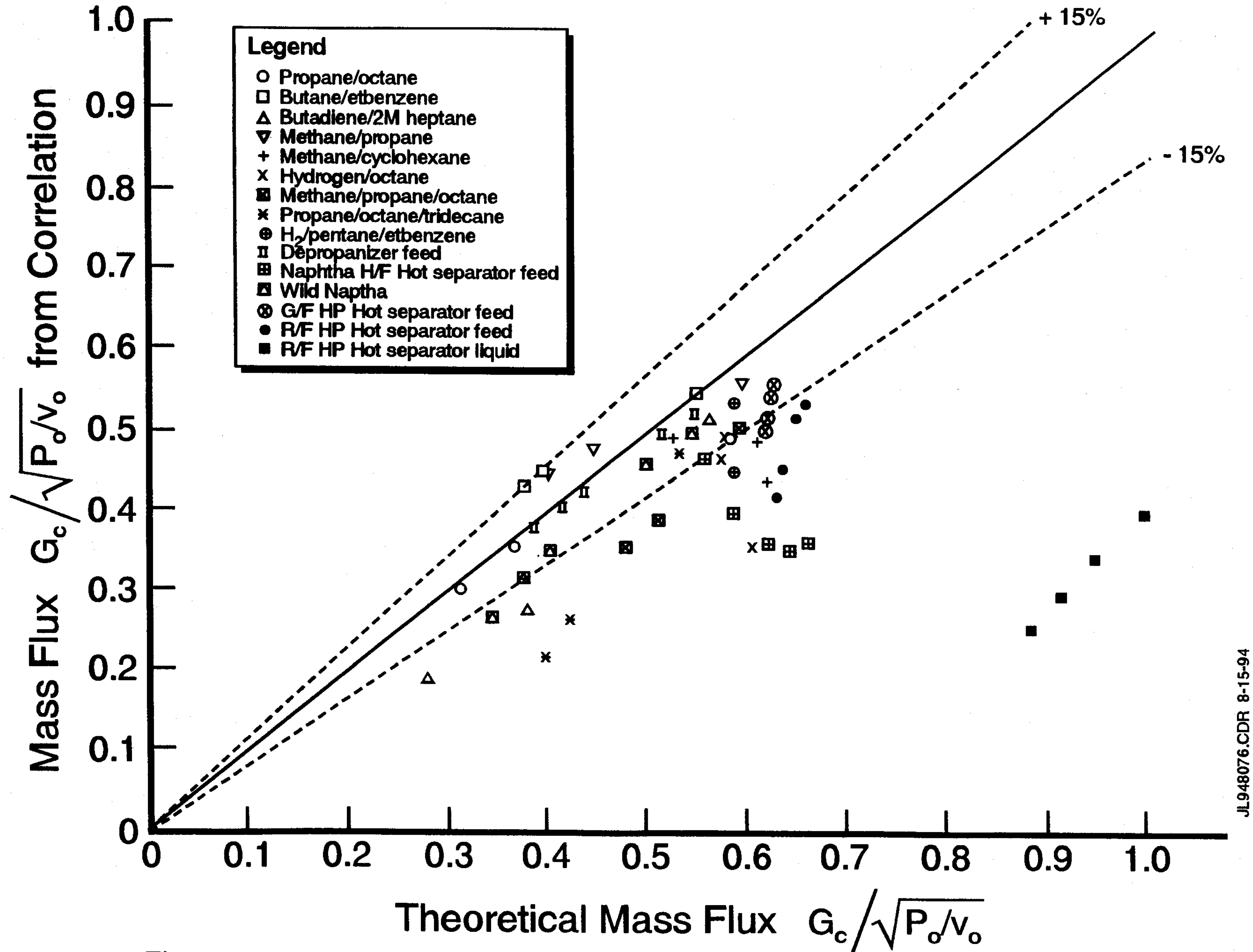


Figure 11 Prediction of choked flow of multicomponent mixtures based on ω method.

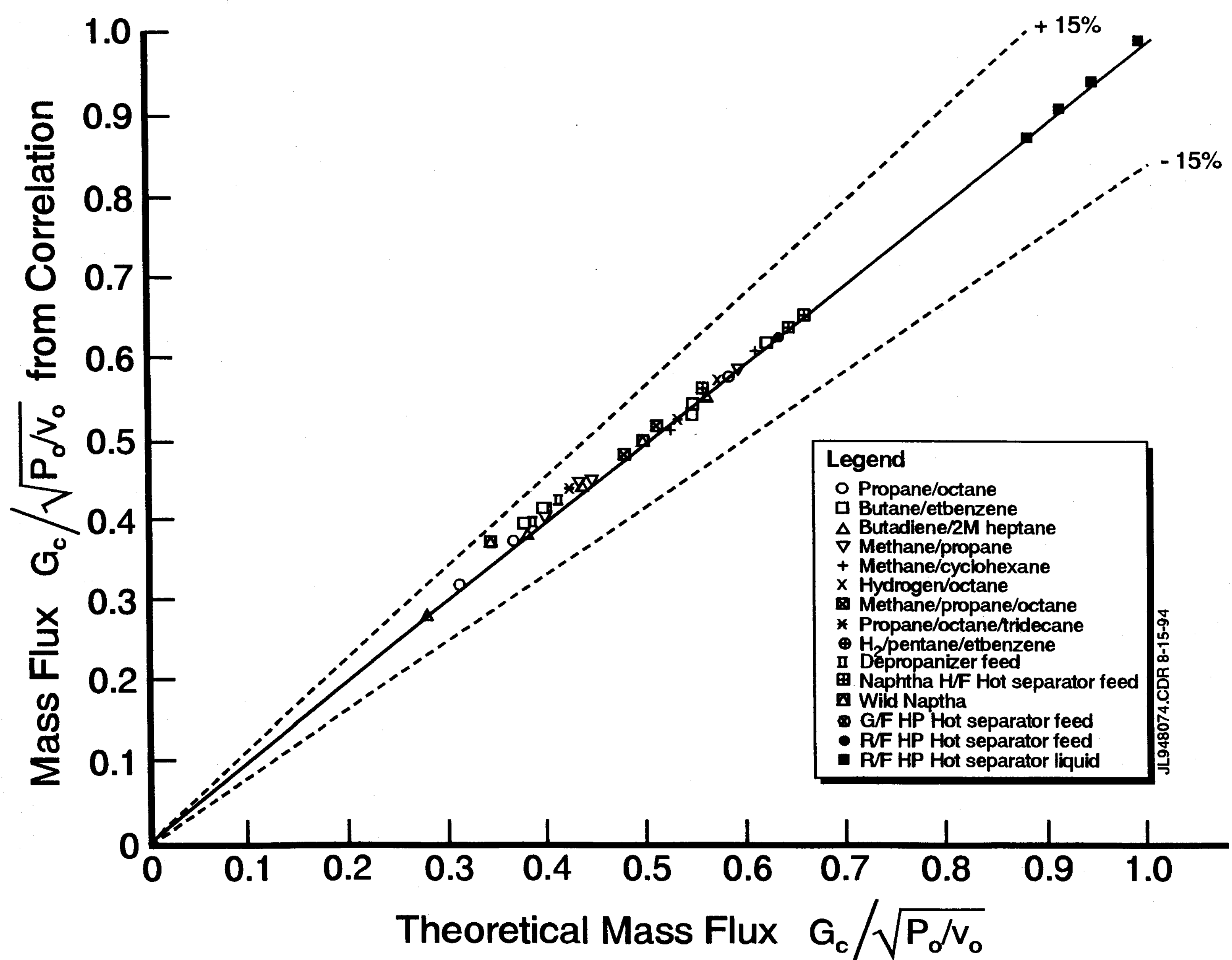


Figure 12 Prediction of choked flow of multicomponent mixtures based on modified ω method.

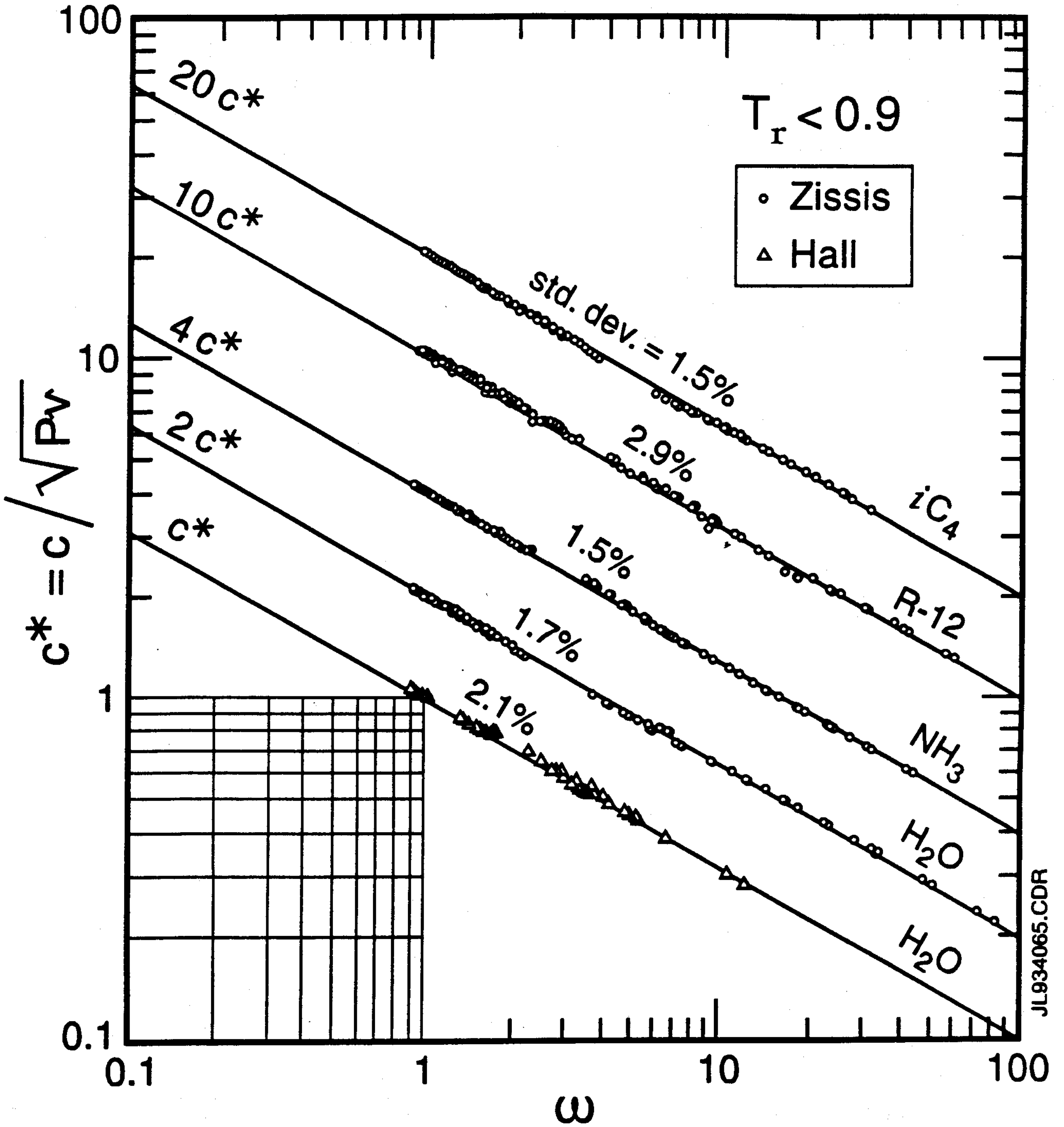


Figure 13 **Prediction of calculated two-phase sonic velocity data based on ω method.**

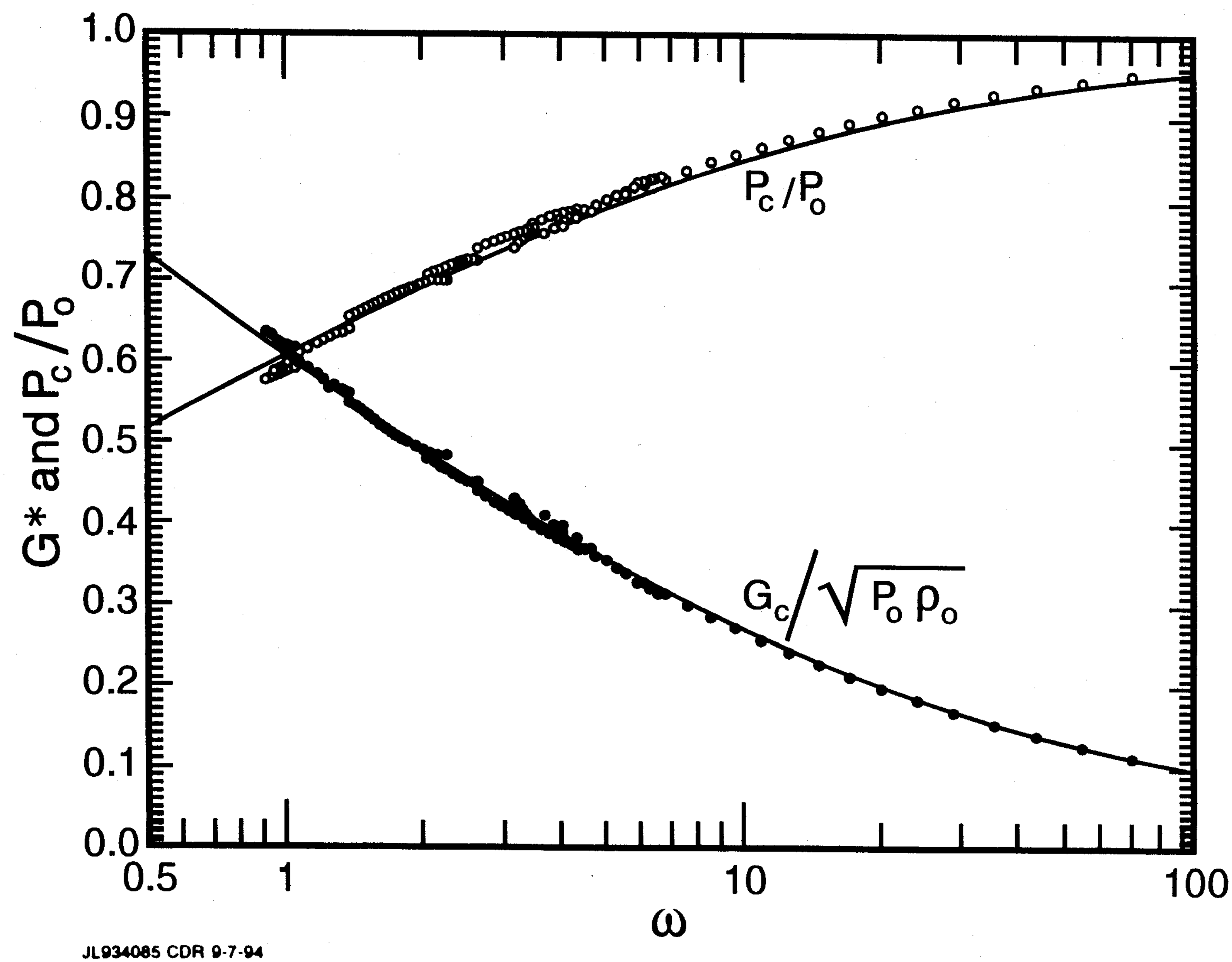


Figure 14 Prediction of theoretical HEM water data based on ω parameter of Eq. (37).