HOW TO HARNESS VOLATILITY TO UNLOCK ALPHA

The Excess Growth Rate: The Best-Kept Secret in Investing

June 2017

UNCORRELATED ANSWERS™

Richard Yasenchak, CFA
Senior Managing Director,
Head of Client Portfolio Management

Valerie Azuelos
Managing Director, Product Specialist

Vassilios Papathanakos, PhD
Executive Vice President
Deputy Chief Investment Officer

Executive Summary

- Volatility is traditionally viewed exclusively as a source of risk, but it can also be a source of alpha.
- A portfolio's compound return is the sum of its constituents' compound returns *plus* an additional component: the excess growth rate.
- The excess growth rate is solely a function of stocks' weights, volatilities and correlations. It measures the positive impact of diversification on a portfolio's compound return.
- Portfolios can be constructed to capture the excess growth rate with great consistency through diversification and systematic rebalancing.
- Rebalancing is the mechanism by which volatility is captured and diversification is replenished.

Intech®
—JANUS HENDERSON—

What is the excess growth rate?

A portfolio's compound return over multiple periods is not simply the weighted compound returns of the underlying stocks.

The underlying stocks' compound returns are indeed a component of the calculation. However, there is an additional contribution due to the diversification benefit a portfolio experiences based on the volatilities and correlations of the stocks within the portfolio. This additional component is known as a portfolio's *'Excess Growth Rate.'*[1]

To appreciate the excess growth rate and how it relates to the overall portfolio return, we first revisit how a portfolio return is calculated. It is well known that a portfolio's arithmetic return during a *single* period is the percentage change in portfolio value (including any dividend income) during that period. It is also widely accepted that the geometric, or compound, return is a preferable measure of a portfolio's average performance over multiple periods, rather than the average arithmetic return of these periods. For example, if a portfolio is down 20% in one period and up 20% in the following period, the portfolio performance over both periods is best represented by the compound return of about -4%,[2] and not by the average arithmetic return for the two periods of 0%. In general, for a given average arithmetic return, the more variability in return a portfolio experiences along the way, the more the compound return will differ from that average arithmetic return.[3] More importantly, a portfolio gains more by moderating large negative returns than it does by boosting positive returns by the same amount. As a result, a portfolio actually outperforms its constituents over the long term, because the diversification benefit of combining individual stocks into a portfolio and moderating any large negative returns allows the portfolio as a whole to compound more efficiently than the individual stocks over time.

Dr. E. Robert Fernholz introduced Stochastic Portfolio Theory in 1982[4] by showing mathematically that, when a portfolio's weights are kept constant through regular rebalancing, a portfolio's compound return exceeds that of its underlying stocks' compound returns. That is, a portfolio that continually rebalances to constant weights benefits from diversification by what he called the portfolio's *excess growth rate.*

Portfolio compound return =
weighted stock compound return + excess growth rate

[1] Other names for it include 'volatility capture,' 'volatility harvesting,' 'rebalancing premium,' and 'diversification premium.'

[2] The compound return is computed as (1+20%)*(1-20%)-1.

[3] The discrepancy is approximately equal to half the return variance.

[4] The original paper was (Fernholz and Shay; 1982); for mathematical introductions to the theory cf. (Fernholz; 2002) and (Fernholz and Karatzas; 2008). Compound returns are logarithmic.

The excess growth rate can be computed solely based on stocks' weights, volatilities and correlations – as opposed to the average returns of stocks relative to each other:[5]

$$\text{Excess growth} = \frac{1}{2}\,(\text{weighted stock variance} - \text{portfolio variance})$$

The excess growth rate quantifies directly the contribution to the portfolio's return due to diversification. Since the portfolio variance is less than the weighted stock variance (for long-only portfolios), the excess growth rate is a positive number: it always constitutes a positive contribution to the overall portfolio return. In practice it is also possible to adopt investment strategies which attempt to enhance this component of a portfolio's return in order to deliver a higher compound return over time.

For a portfolio, rebalancing is necessary for maintaining the excess growth rate, even though this is not obvious from looking at the formula. Without rebalancing, the portfolio becomes more concentrated in the stocks that performed well; this results in a decrease of the diversification gains, and a deterioration of the excess growth rate. Rebalancing is vital since it has the effect of replenishing diversification, and locking in previous gains from random price fluctuations, which otherwise would generally be lost.[6] Therefore, a rebalancing mechanism is critical to a strategy focused on exploiting diversification.

All this implies that a portfolio's return can be increased by a combination of:

1. selecting stocks that have higher compound returns, and

2. combining stocks into a more favorable mix based on their variances and covariances.

The first component arises from stocks' cross-sectional returns, while the second component comes from stocks' volatility and correlations. The traditional approach to investments is to concentrate exclusively on maximizing the first component: selecting stocks by forecasting a price change before it happens based on fundamental characteristics and/or factor models.

In this paper, we focus on the second component of a portfolio's return, namely the excess growth rate: combining stocks more efficiently based on observations of price volatility and correlations. We estimate the excess growth rate for various portfolios and discuss how to increase its magnitude (via increased diversification) or its reliability (via optimization).

Rebalancing is vital since it has the effect of replenishing diversification, and locking in previous gains from random price fluctuations, which otherwise would generally be lost.

[5] If the portfolio weights are not constant (e.g., for a buy-and-hold strategy), the formula requires we use an appropriate average over time.

[6] For more information on rebalancing and its effect on portfolio performance, cf. (Banner, Schofield and Yasenchak; 2015).

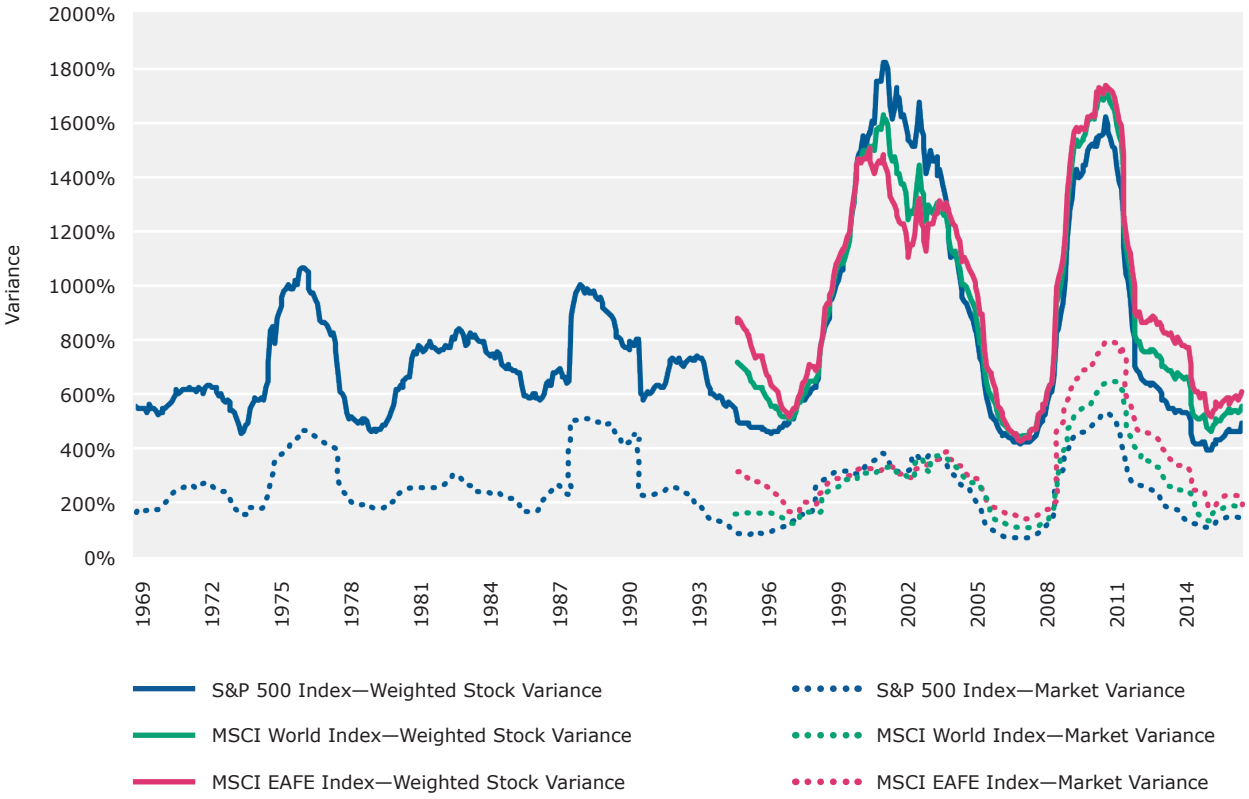
Capitalization-weighted portfolios

Actual market index portfolios are buy-and-hold, not regularly rebalanced, so they do not fully realize the potential diversification benefit in the form of a trading profit achievable through systematic rebalancing. Typically, the rebalancing activity for a market index portfolio is limited solely to the reconstitution trades necessary to maintain the index; these trades are infrequent by design, in order to reduce the turnover required to implement passive portfolios, and so a lot or most of the diversification benefit goes unrealized. The excess growth rates of the cap-weighted portfolios calculated in this section illustrate the diversification potential for a regularly rebalanced portfolio similar to the market, as driven by the covariance characteristics of stocks. We therefore estimate the maximum level of excess growth rate as opposed to the actual excess growth rate the corresponding index may have captured over time.

In Figure 1, we illustrate on the same chart the two components of the excess growth rate: the weighted stock variance and the overall market variance for a number of indexes.[7] As expected, for each index considered, the weighted stock variance exceeds the index's variance. That difference depends on the underlying stock correlations, as well as the market environment.

FIGURE 1

ROLLING 36-MONTH ANNUALIZED VARIANCE OF TOTAL ABSOLUTE LOGARITHMIC RETURNS*



* Source: Intech®. Data presented reflects past performance, which is no guarantee of future results.

[7] The time scale used for estimating these volatilities and correlations can be material. For example, one gets a different answer focusing on daily or annual periods. Each month, the weighted stock variance and portfolio variance are calculated on a rolling 36-month basis using the index's end-of-period weights.

In Figure 2, we approximately estimate the excess growth rate of these market portfolios by taking half the difference between the weighted stock variance and the index variance (half the difference between the pairs of similarly colored lines in Figure 1).[8]

We find that the excess growth rate of the market is cyclical and has been trending lower over the past few years. This is not a surprise given that cross-sectional stock volatility is also cyclical, and has similarly declined across the various markets. There have been other periods in which the excess growth rate of the market has been similarly low, and subsequently increased. We also observe that during periods of higher volatility (e.g., around the Tech Wreck, or the Global Financial Crisis), the potential benefit of diversification is greatest, but it is available to some degree in all periods.

The difference between the weighted average stock variance and the overall index variance illustrates the diversification potential driven by the covariance characteristics of stocks. The market is less diversified (more concentrated relative to an average stock) when the excess growth rate is lower, and more diversified when it is higher.

Alternative-beta portfolios

The market represents a broad equity portfolio, but it is typically not nearly as diversified as possible at comparable levels of absolute risk. As mentioned above, higher diversification should lead to a higher rate of capturing volatility through rebalancing, i.e., a higher excess growth rate. An easy[9] way to increase diversification relative to the market is through the construction of an equal-weighted portfolio. Figure 3 shows the excess growth rate for the equal-weighted portfolios corresponding to the same indexes as Figure 2.

From comparing Figures 2 and 3, it is clear that the increased diversification of the equal-weighted portfolio results directly in a higher excess growth rate (cf. Table 1). This table also shows the average relative return of the equal-weighted portfolio compared to the corresponding cap-weighted index. We find that the equal-weighted portfolio's relative return exceeds its estimated relative excess growth rate. Effectively, as the cap-weighted portfolio does not fully realize the potential trading profit, due to its buy-and-hold nature, the relative excess growth rate is an underestimate of the relative return of the

FIGURE 2

ANNUALIZED EXCESS GROWTH RATE FOR A PORTFOLIO REGULARLY REBALANCED TO THE MARKET WEIGHTS, ESTIMATED USING THE ROLLING 36-MONTH VARIANCE*

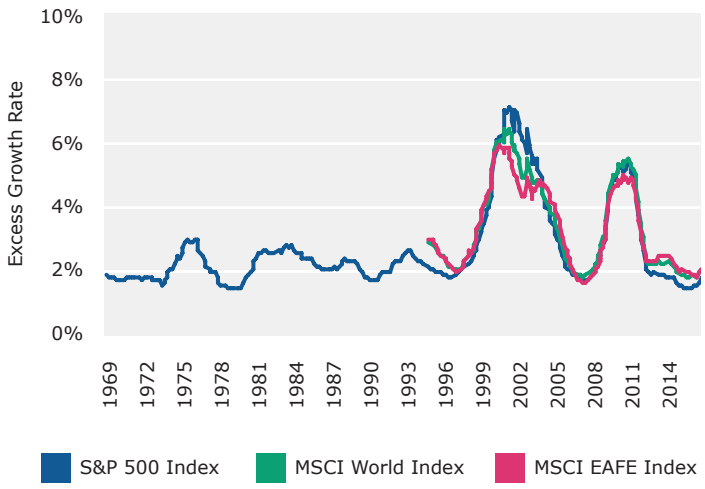
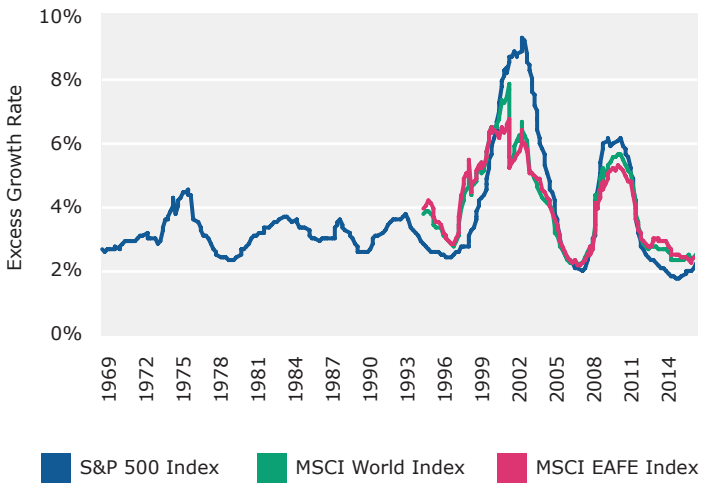


FIGURE 3

ANNUALIZED EXCESS GROWTH RATE FOR AN EQUAL-WEIGHTED PORTFOLIO, ESTIMATED USING THE ROLLING 36-MONTH VARIANCE*



* Source: Intech®. Data presented reflects past performance, which is no guarantee of future results.

[8] This is an approximation primarily because the market portfolios are buy-and-hold; in order to arrive at a better estimate, we need to establish an average holding period, and evaluate the formula for the market weights *averaged* over this period. Furthermore, we would need to estimate the volatilities and correlations at the time scale of the average holding period.

[9] In the sense that no covariance estimates or optimization is necessary. This is not typically an easy portfolio to implement at scale, or at tolerable risk.

TABLE 1

AVERAGE ANNUALIZED EXCESS GROWTH RATE (EGR) FOR THE MARKET AND THE EQUAL-WEIGHTED PORTFOLIO FOR VARIOUS INDEXES

	Market EGR potential (maximum)	EQW Estimated EGR	EQW Relative EGR (minimum)	EQW Estimated Relative Return
S&P 500 Index	2.64%	3.65%	1.01%	1.86%
MSCI World Index	3.37%	4.08%	0.71%	1.04%
MSCI EAFE Index	3.29%	4.05%	0.76%	1.63%

equal-weighted portfolio.[10] In Table 1, we describe the market's excess growth rate as the *potential maximum* if we regularly rebalanced the cap-weighted index,[11] but the actual excess growth rate is lower because it's limited to the reconstitution of trades that occur less frequently by design. Conversely, we describe the equal-weighted portfolio's relative excess growth rate as a *minimum,* which increases as the market's excess growth rate decreases.

The case of the equal-weighted portfolio is representative of a large class of so-called smart-beta, or alternative-beta, strategies. These are diversified portfolios that are regularly rebalanced in order to stay close to target weights that are constructed according to specific rules. Besides the equal-weighted portfolio, other popular examples include fundamental indexation, quantitative value strategies, and various types of minimum and low-volatility portfolios.

Independently of other premia captured by these alternative-beta strategies, such diversified portfolios result in a higher excess growth rate than the market. This potential for improved long-term performance is realized through the systematic rebalancing, but not in a targeted or optimized way.

Optimized portfolios

We saw that equity strategies regularly rebalanced to market indexes, as well as equal-weighted portfolios, have a positive excess growth rate that tends to vary over time. Can an actively managed portfolio target a more stable excess growth rate? What about a higher risk-adjusted return? The answer to both questions is absolutely 'Yes!'

An actively managed portfolio has three main opportunities to generate value relative to the broad market, or relative to simple alternative-beta strategies:[12]

- Identify which specific stocks have a higher potential to contribute to diversification through directly estimating the volatilities and correlations of all of the securities in the investable universe.
- Achieve an optimal (not necessarily maximum) amount of diversification by using a portfolio optimization.
- Implement the trading necessary to realize as much as possible of this potential diversification benefit, net of transaction costs.

Can an actively managed portfolio target a more stable excess growth rate? What about a higher risk-adjusted return? The answer to both questions is absolutely 'Yes!'

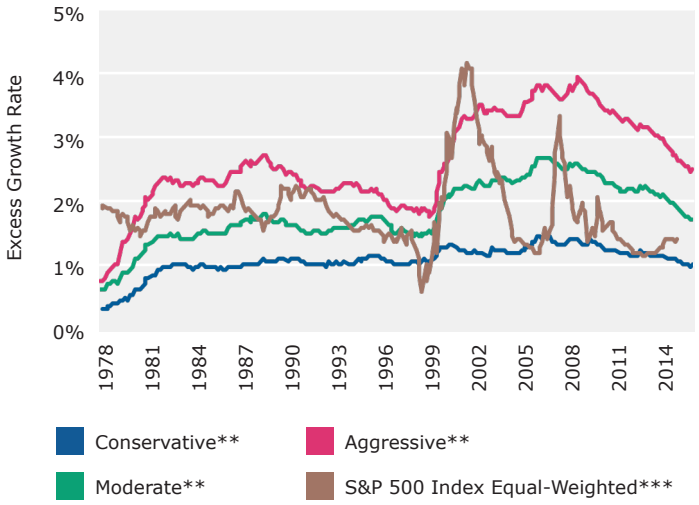
[10] The difference between the relative return and the relative EGR provides an estimate of how much diversification benefit the market portfolio gives up through lack of regular rebalancing. We see that the broader the universe of constituents, the lower the discrepancy: 90bps for S&P 500, 80 bps for MSCI EAFE, 30 bps for MSCI World.

[11] In this context, rebalancing the index refers to frequently trading to beginning-of-period weights. For example, rebalance every day back to the beginning-of-month index weights.

[12] Due to their construction methodology, 'passive' alternative-beta strategies cannot effectively take advantage of more than one of these opportunities; for example, if they incorporate a portfolio optimization, they will also typically incorporate a constraint that directly limits the opportunity set accessible to the optimization, for the sake of maintaining low turnover.

FIGURE 4

ANNUALIZED EXCESS GROWTH RATE FOR S&P 500-BASED PORTFOLIOS*



The result of simulating actively managed strategies is shown in Figure 4 and Table 2. In the simulations, we make realistic, if conservative assumptions about actually implementing the portfolio; for example, we assume transaction costs of 40 bps per unit trading.

For the sake of demonstration, we present the effect of targeting three different levels of long-term return. As can be seen from the excess growth rate formula, an increase in diversification (properly measured) is linked directly to an increase in a portfolio's compound return. This implies that the long-term return of a portfolio relative to a cap-weighted index can be boosted by selecting weights that improve diversification, and/or with increasing the efficiency of the rebalancing methodology.

If the goal is to maximize the consistency of outperformance, the objective of the optimization should not be to maximize the excess growth rate. Instead, a more risk-controlled approach is to target a level of outperformance and then minimize the tracking error. Similarly, if the objective is to maximize the consistency of absolute performance, the same approach works, except that the absolute volatility should be minimized instead of the tracking error. In Figure 4, we compare the S&P 500 equal-weighted portfolio's relative excess growth rate to the estimated trading profit of three optimized portfolios.

TABLE 2

RISK-ADJUSTED PERFORMANCE FOR PORTFOLIOS RELATIVE TO THE S&P 500 INDEX*

(all quantities are expressed in logarithmic terms)

PORTFOLIO	Relative Return	Tracking Error	Information Ratio
Equal-weighted	2.40%	5.46%	0.44
Conservative	1.17%	1.40%	0.84
Moderate	2.09%	2.37%	0.89
Aggressive	3.50%	4.11%	0.85

Figures 2 and 3 showed that the excess growth rate for non-optimized diversified portfolios can be highly variable. This is not necessary when harnessing the excess growth rate as seen in Figure 4: the diversification benefit relative to the market can be kept within a relatively narrow range by modulating the active share and other portfolio characteristics, based on the underlying market risk structure. Such techniques can be used to construct robust investment portfolios in equities.

We can summarize some of the main observations above in the following table:

PORTFOLIO	Cap-Weighted	Equal-Weighted	Optimized
Style	Passive	Smart Beta	Active
Diversification	Low	Moderate	Optimal
Rebalancing	Infrequent	Regular	Optimal
Risk-adjusted active return	Zero	Low	High

* See Simulations Disclaimer at the end of the paper for additional information.

** Conservative, Moderate and Aggressive optimized portfolios, optimally rebalanced. Estimated using a trading-profit attribution methodology. Note that the optimized portfolio can target different levels of EGR, and at a higher consistency than an unmanaged portfolio. For an introduction to the trading-profit attribution, cf. (Yasenchak and Papathanakos; 2015).

*** Estimated using 36-month rolling variances gross of transaction costs; same as in Figures 2 and 3 respectively.

Conclusion

The excess growth rate is an important source of long-term returns, and is always positive across all-long portfolios.

When many stocks are combined in a portfolio that is regularly rebalanced, their interactions cause the portfolio to have a higher compound return than the weighted compound return of the stocks in the portfolio.

Infrequently traded portfolios, such as market indexes, do not benefit from this potential alpha opportunity nearly as much as would be possible given their constituents. This is very meaningful for a long-term investor because the difference in the realized excess growth rate between a market index and a regularly traded, risk-controlled, portfolio can amount to hundreds of basis points per year, even net of trading costs.

Simulations disclaimer

The simulations shown are hypothetical, do not reflect the results or risks associated with actual trading, have been prepared with the benefit of hindsight, and are for illustrative purposes only. The simulations have been prepared by Intech® and have not been independently verified. Simulations potentially allow investors to understand and evaluate how a strategy or investment process would have performed hypothetically during certain time periods. Simulations may be theoretically changed from time to time and results can differ. Simulations do not reflect material, economic, and market factors, assume that markets were sufficiently liquid to permit the types of trading used to produce the results, and assume that all securities were available during the time period(s) presented.

The simulations include the reinvestment of all dividends, interest, and capital gains, and trading costs, but do not reflect the deduction of investment advisory and other fees. Thus, any simulated results will be reduced by advisory fees and any other expenses, which will materially lower results over time. Past performance of simulated data is no guarantee of future results. Investing involves risk, including fluctuation in value, the possible loss of principal and total loss of investment. There are numerous other factors related to the markets in general or to the implementation of any specific trading strategy, which cannot be fully accounted for in the preparation of simulations, all of which can adversely affect actual trading and final results.

This material is has been prepared for designated institutional and professionals investors. The views expressed herein are subject to change based on market and other conditions. This material is for general information purposes only and is not intended as investment advice, as an offer or solicitation of an offer to sell or buy, or as an endorsement, recommendation, or sponsorship of any company, security, advisory service, or fund. The contents are current at the time of writing. Although the information contained herein has been obtained from sources believed to be reliable, its accuracy and completeness cannot be guaranteed. This information should not be used as the sole basis for investment decisions.

Intech® is a specialized global asset management firm that harnesses stock price volatility as a source of excess return and a key to risk control. Founded in 1987 in Princeton, NJ by pioneering mathematician Dr. E. Robert Fernholz, Intech® serves institutional investors across five continents, delivering relative return, low volatility, adaptive volatility and absolute return investment solutions.

525 Okeechobee Boulevard, Suite 1800
West Palm Beach, FL 33401
(561) 775-1100
intechinvestments.com

C-0318-1933 03-31-19

FOR INSTITUTIONAL INVESTOR USE ONLY

Intech®
—JANUS HENDERSON—