The Measurement of Diversity

Diversity is a measure of the spread of capital across an equity index or market. Diversity is lower when capital is concentrated mostly into a few large stocks, and is higher when capital is spread more uniformly. Changes in diversity are caused by the ebb and flow of capital between larger and smaller stocks.

Diversity is measured by a function $\mathbf{D}_p$ of the cap weights of the index. Suppose we have an index of n stocks with cap weights $w_1, \ldots, w_n$, so $w_1 + \cdots + w_n = 1$. The diversity of this index is

$$\mathbf{D}_p(w_1, \ldots, w_n) = \left(n^{p-1} \sum_{i=1}^{n} w_i^p \right)^{1/p},$$

where p is a constant $0 < p < 1$. $\mathbf{D}_p$ reaches its maximum value of 1 if all the weights are equal, and it reaches its minimum value when all the capital is concentrated into a single stock. The value of p used for the S&P 500 Index is $p = .76$, and we shall use this value in the illustrations that follow.

In Figure 1 we see the cap weight distribution of a hypothetical index of 20 stocks. (The distribution in Figure 1 actually corresponds to the percentiles of the cap-weighted S&P 500 on December 31, 1997.) The diversity of this 20-stock index, as measured by $\mathbf{D}_{.76}$, is .8396.
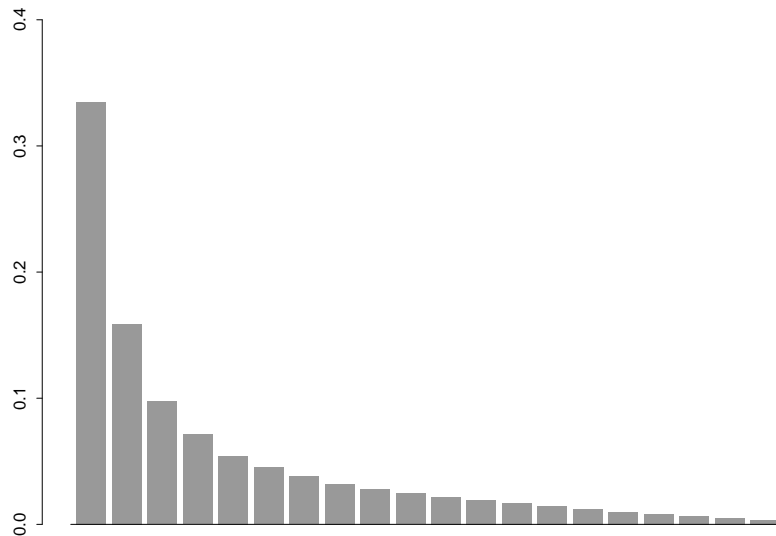

Figure 1: 20-Stock index, $\mathbf{D}_{.76} = .8396$.

The next two figures show modifications of this index with higher and lower diversity.
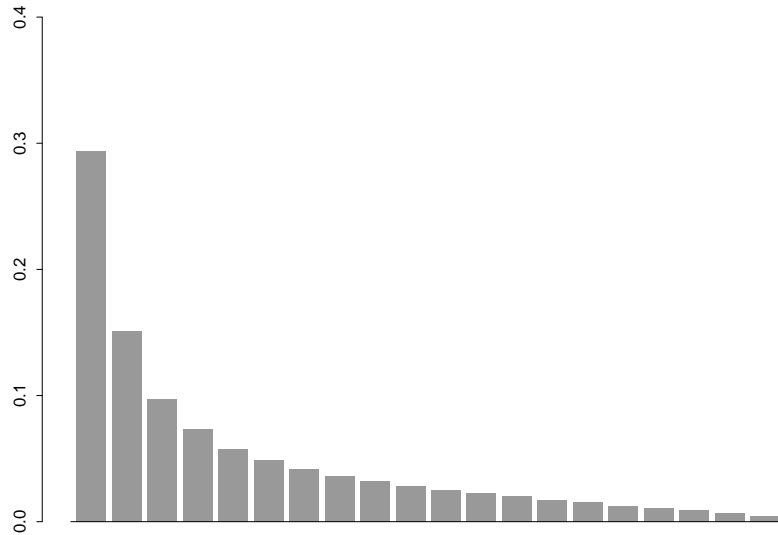

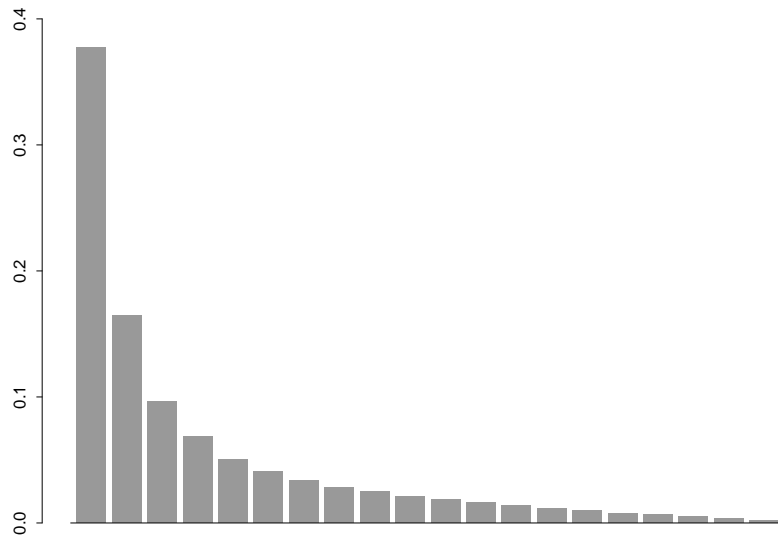Figure 2: More diverse distribution, $\mathbf{D}_{.76} = .8676$.

Figure 3: Less diverse distribution, $\mathbf{D}_{.76} = .8105$.

In Figure 2, some of the capital has been removed from the larger stocks and redistributed among the smaller stocks: for example, the weight of the largest stock has declined from about 34% to about 29%. The logarithmic increase in diversity when going from the distribution in Figure 1 to that of Figure 2 is 3.28%. Figure 3 has more capital in the larger

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stocks and less in the smaller stocks, so it has lower diversity: the change in diversity in going from Figure 1 to Figure 3 is -3.53%. This means that the diversity of the distribution in Figure 2 is 6.81% higher than the diversity of the distribution in Figure 3.

As can be seen from these charts, changes in diversity represent the movement of capital between larger stocks and smaller stocks. Historically, the diversity of the market has been variable over the short term, but stable and mean reverting over the long term. Figure 4 shows the cumulative changes in diversity for the Center for Research in Security Prices (CRSP) U.S. equity market database over the period from 1927 to 1997. The chart shows only changes caused by capital gains or losses among the stocks. Changes in diversity caused by changes in the composition of the market or by corporate actions such as takeovers or spinoffs are not included.

Figure 4: Cumulative changes in $\mathbf{D}_{.76}$, 1927–1997.

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