The Effect of Value Estimation Errors On Portfolio Growth Rates

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This article examines how value estimation errors effect the growth rates (i.e., expected continuous returns) and relative growth rates for the following four portfolio weighting methods: capitalization weights, estimation error independent weights (Treynor [2005]), Fundamental weights (Arnott et al. [2005] and Treynor [2005]), and Diversity weights (Fernholz [2002]).[1,2] It provides theoretical support, in the context of estimation error, for the empirical findings of Arnott et al. [2005] and Fernholz [2002] that many Fundamental-weighted and Diversity-weighted portfolios beat the market (i.e., capitalization weights) over time. The article gives empirical evidence to support its implication that equal-weight (which is one type of estimation error independent weight) portfolio returns should beat their corresponding capitalization weight portfolio returns over time. It also provides a theory for the size effect discussed in Fama and French [1992].[3]

In contrast to previous articles, this one addresses the effect of estimation errors on portfolio growth rates due to increased return volatility. It also examines the effect of mispricing corrections on stocks' and portfolios' returns.[4]

The main implication for investors is the following. To increase portfolio growth rates, investors should reduce the correlation between their portfolio weights and estimation errors. Given the positive cross-sectional correlation between estimation errors and capitalization (as discussed later), this can be done by underweighting high-capitalization stocks and overweighting low-capitalization stocks. The Fundamental weighting and Diversity weighting methods can be interpreted as ways to reduce the correlation between portfolio weights and estimation errors.

THE EFFECT OF MISPRICING CORRECTIONS ON STOCKS' AND PORTFOLIOS' RETURNS

Value estimation error is defined as the excess of a stock's capitalization over its value.[5] In this article, it is expressed as the continuous return necessary to go from stock value to stock price. Overvalued stocks have positive value estimation errors and undervalued stocks have negative value estimation errors.

Assuming that estimation errors are independent of value and are mean reverting to zero, i.e., capitalization tends to return to value over time, mispricing corrections reduce the return of overvalued stocks and increase the return of undervalued stocks.[6]

Since capitalization is the sum of value plus value estimation error, there tends to be a positive cross-sectional correlation between capitalization and value estimation error. High-capitalization stocks tend to be overvalued and low-capitalization stocks tend to be undervalued.[7] This does not mean that all high-capitalization stocks are overvalued or that all low-capitalization stocks are undervalued. Rather, the average estimation error for high-capitalization stocks as a group typically is positive and the average estimation error for low-capitalization stocks as a group typically is negative. This leaves room for many high-capitalization stocks to be undervalued and for many low-capitalization stocks to be overvalued.

Capitalization-weighted portfolios necessarily weight larger-capitalization stocks more heavily than smaller-capitalization stocks. Therefore, a positive cross-sectional correlation between capitalization and estimation error induces a reduction in return for capitalization-weighted portfolios, on average. This continuous return reduction is termed "mispricing correction drag."

Portfolios, with weights that are less cross-sectionally correlated with estimation error than capitalization weights, have less mispricing correction drag, and hence have an expected return advantage over capitalization-weighted portfolios. Many passive portfolios have weights that are less cross-sectionally correlated with estimation error than capitalization weights. Such passive portfolios ought to beat a broad capitalization-weighted benchmark over time. A well-known example of such a passive portfolio is an equal-weight portfolio, whose weights are cross-sectionally uncorrelated with everything.

THE EFFECT ON STOCKS' RETURN VOLATILITY AND PORTFOLIOS' EXCESS GROWTH RATES

Fernholz and Shay [1982] distinguish between a stock's or portfolio's expected arithmetic return and its growth rate (expected continuous return). Growth rates, not expected arithmetic returns, correspond to long-term returns. Growth rates are less than expected arithmetic returns because compounding introduces a drag due to return volatility. Estimation errors affect return volatility, and hence affect growth rates.

To see that growth rates correspond to long-term returns and are less than expected arithmetic returns, consider a volatile portfolio with a 50% chance of a one-period return of 20% + 100% = 120% and a 50% chance of a one-period return of 20% – 100% = –80%. This portfolio's one-period expected arithmetic return is (120 – 80)/2 = 20%. The portfolio's one-period expected arithmetic return is attractive.

Consider what happens over time. Each period corresponds to the flip of a coin. For the coin, there will be about 50% heads and 50% tails over the long term. Consequently, the portfolio's return will be 120% in about half the periods and –80% in about half the periods. The typical two-period result will be one 120% return and one –80% return. A dollar invested in the portfolio will have a typical two-period result of up 120% to 2.20 and then down 80% to 0.44, or down 80% to 0.20 and then up by 120% to 0.44. In each case, the dollar typically becomes 0.44 after two periods, a return of –56% every two periods, or –33.7% per period. The portfolio's long-term return is –33.7% per period versus its expected arithmetic return of 20% per period. The portfolio is unattractive.

Suppose the portfolio's volatility is reduced but its expected return remains the same. Make the two possible portfolio arithmetic returns 20% + 5% = 25% and 20% – 5% = 15%. Now, the portfolio's expected arithmetic return and long-term return are 20% and 19.9%, respectively. Evidently, the difference between long-term returns and expected arithmetic returns is due to return volatility.

The gap between a stock's (or portfolio's) long-term return and its expected arithmetic return is eliminated if continuous returns are used. Return, R, is the ratio of change in value to beginning value. Continuous return, r, equals $\log(1 + R)$.

In the first example above, the two possible continuous returns are $\log(1 + 1.2) = \log(2.2) = 0.7885$, or 78.85%, and $\log(1 + (-0.8)) = \log(0.2) = -1.609$, or –160.9%. This implies an expected continuous return of (78.85% – 160.9%)/2 = –41.05%. The continuous return corresponding to the example's long-term return of –33.7% is $\log(1 - 0.337) = -0.4105$, or –41.05%.

Since this article focuses on growth rates and they correspond to expected continuous returns, the remainder of the article uses the continuous return format to simplify the analysis.

Fernholz and Shay [1982] show that a stock's (or portfolio's) expected continuous return is approximately its expected arithmetic return less one-half of its continuous return variance. Thus, they quantify the growth rate drag due to volatility as half the continuous return variance. They also note that a portfolio's variance is less than its weighted average stock variance (due to diversification) so that the expected growth rate drag due to return volatility is less for a portfolio than for its stocks. They define a portfolio's Excess Growth Rate as the portfolio's growth rate less the weighted average growth rate of its stocks and show that it is one-half the excess of the weighted average variance of the portfolio's stocks over the portfolio's variance. Thus, the portfolio's Excess Growth Rate is the weighted average growth rate drag of the portfolio's stocks due to return volatility less the portfolio's growth rate drag due to return volatility. It depends only on the portfolio's weights and the stocks' variance-covariance matrix. It has nothing to do with the stocks' expected returns or growth rates.[8]

For an example of how a portfolio's Excess Growth Rate is calculated, suppose you have a two-stock portfolio where stock 1 has weight 1/3 and return variance = 0.16, stock 2 has weight 2/3 and return variance = 0.09, and the correlation between stock 1 and 2 returns is 0.7. Assume the weights are capitalization weights and let this be known as the "capitalization-weighted baseline case." The weighted average variance of the portfolio's stocks is $[(1/3)(0.16) + (2/3)(0.09)] = 0.1133$. The portfolio's variance is $[(1/3)^2(0.16) + (2/3)^2(0.09) + 2(1/3)(2/3)(0.7)(\sqrt{0.16})(\sqrt{0.09})] = 0.0951$. Thus, the capitalization-weighted baseline case portfolio's Excess Growth Rate is $(0.1133 - 0.0951)/2 = 0.0091$.

Estimation error volatility increases stocks' return variances and increases a portfolio's Excess Growth Rate.

To see an example where estimation error volatility increases a portfolio's Excess Growth Rate, suppose that estimation error increases each stock's variance by factor k. In this case, the excess growth rate increases by the factor k. To illustrate this, consider the capitalization-weighted baseline case and suppose that estimation error caused each stocks' variance to rise 10% (i.e., $k = 1.1$). Now stock 1 has return variance = 0.176 and stock 2 has return variance = 0.099. The weighted average variance of the portfolio's stocks is $[(1/3)(0.176) + (2/3)(0.099)] = 0.12467$. The portfolio's variance is $[(1/3)^2(0.176) + (2/3)^2(0.099) + 2(1/3)(2/3)(0.7)(\sqrt{0.176})(\sqrt{0.099})] = 0.10462$. Thus, the portfolio's Excess Growth Rate is $(0.12467 - 0.10462)/2 = 0.01002$ (which is 1.1 times the capitalization-weighted baseline case Excess Growth Rate of 0.00911).

THE ESTIMATION ERRORS' EFFECT ON PORTFOLIO GROWTH RATES INTERPRETED IN THE CONTEXT OF VARIOUS PORTFOLIO WEIGHTING METHODS AND A SIZE EFFECT

Capitalization-Weighted Portfolios

Estimation errors impact a capitalization-weighted portfolio's growth rate in two ways. A capitalization-weighted portfolio tends to experience drag from mispricing corrections and gain due to a higher portfolio Excess Growth Rate. If the mispricing correction rate is large enough, the mispricing correction drag will more than offset any gain from the higher portfolio Excess Growth Rate and the capitalization-weighted portfolio's growth rate will be less than it would be without estimation errors.

A rough approximation is that value-weighted and capitalization-weighted portfolios of the same stocks have similar Excess Growth Rates. If so, value-weighted portfolios, which have no mispricing correction drag, tend to have higher growth rates than capitalization-weighted portfolios, which do have mispricing correction drag.

Estimation Error Independent Portfolio Weights

There exist portfolio weights that are independent of estimation error, in the sense of a zero cross-sectional (point in time) correlation between portfolio weights and estimation errors. Such weights are termed "estimation error independent weights" and are related to the "market value indifferent" weights analyzed in Treynor [2005]. One example of estimation error independent weights is an equal-weighted portfolio, whose weights have a zero correlation with everything. Other examples are value-weighted portfolios and, in an expectations sense, randomly chosen portfolio weights, and all portfolio weights that are functions of quantities that are uncorrelated with estimation error.[9]

Relative to capitalization weights, estimation error independent weights do not suffer from mispricing correction drag, which is an advantage.

Capitalization-weighted and estimation error independent weighted portfolios often have similar Excess Growth Rates. To see this, reconsider the baseline case. For an equal-weighted portfolio, stocks 1 and 2 each have weight 1/2. The weighted average variance of the portfolio's stocks is $[(1/2)(0.16) + (1/2)(0.09)] = 0.125$. The portfolio's variance is $[(1/2)^2(0.16) + (1/2)^2(0.09) + 2(1/2)(1/2)(0.7)(\sqrt{0.16})(\sqrt{0.09})] = 0.1087$. Thus, the equal-weighted portfolio's Excess Growth Rate is $(0.125 - 0.1087)/2 = 0.0082$. This is reasonably close to the capitalization-weighted portfolio's baseline case Excess Growth Rate of 0.0091.

Because estimation error independent weight portfolios do not suffer from mispricing correction drag, yet often have similar Excess Growth Rates, they can be expected to have a higher growth rate than their corresponding capitalization-weight portfolios. An interesting implication of the foregoing is that all estimation error independent weight portfolios have the same expected mispricing correction drag advantage over their corresponding capitalization weight portfolios.

Empirically, equal-weight portfolio returns beat capitalization-weight portfolio returns. As shown in the top half of the exhibit, using CRSP monthly returns for the NYSE index over the 1928–2007 period, the equal-weighted portfolio continuous return exceeds that for the capitalization-weighted portfolio by 0.13% monthly (which is statistically significant at the 1% level). Also in each of the four independent 20-year subperiods, the NYSE index equal-weighted portfolio continuous return exceeds that for its capitalization-weighted portfolio (though the difference is statistically significant at the 10% level only for the first and third subperiods). Similar results hold using CRSP monthly continuous returns for the AMEX index (over the August 1962–2007 period) and for the NASDAQ index (over the 1973–2007 period) and each of their approximately 20-year half periods. Similarly, as shown in the bottom half of the exhibit, for the CRSP monthly continuous returns, positive and typically statistically significant alphas result when the equal-weighted index return—risk-free return is regressed on the value-weighted index return—risk-free return for each of these three indices over their whole periods and their subperiods.

Fundamentally-Weighted Portfolios

Fundamental portfolio weights, introduced in Arnott et al. [2005], are weights that are based on fundamental company quantities such as book value or sales. Arnott et al. [2005] empirically shows that several Fundamentally-weighted portfolios beat the market (i.e., capitalization weights). Fundamental weights may be estimation error neutral (discussed in the appendix), estimation error independent, or neither. Whether a particular Fundamental weight scheme meets either criterion may vary over time.

To the extent Fundamental weights have a lower correlation with estimation error than capitalization weights, it is reasonable to expect that Fundamental weights have a higher growth rate than capitalization weights, which is an advantage.

Suppose it is possible to identify Fundamental weights that, in effect, constitute a valid valuation model. These Fundamental weights should approximate estimation error independent weights and should have a growth rate advantage over capitalization weights. Such Fundamental weights are related to Fundamental analysis, as practiced by financial analysts. In this sense, Fundamental weights are nothing new.

Such fundamental weights can achieve an efficiency advantage because they need not systematically overweight (underweight) a capitalization-weighted benchmark's smaller (larger) stocks. They can approximate estimation error independence by overweighting (underweighting) undervalued (overvalued) stocks, relative to capitalization weights, at all capitalization levels.

A shortcoming of Fundamental weights that are functions of historical Fundamental variables is that they are not likely to adequately reflect future business prospects. Since future business prospects are the major determinant of value, such Fundamental weights are not likely to reflect value accurately, hence they probably have substantial estimation errors. Nothing precludes a cross-sectional correlation between such Fundamental weights and estimation errors. Consequently, nothing assures that Fundamental weights are estimation error independent or that Fundamental weights necessarily have lower mispricing correction drag than capitalization weights.

The Excess Growth Rates for Fundamentally-weighted and capitalization-weighted portfolios are not easily compared because the Fundamental weights could be calculated in so many different ways. However, Fundamentally-weighted portfolios do not necessarily have higher Excess Growth Rates than capitalization-weighted portfolios.

Because Fundamentally-weighted portfolios need not have lower mispricing correction drag or higher Excess Growth Rates, they need not have higher growth rates than capitalization weights.

Fundamental weights are no panacea. For example, if all investors adopted the same Fundamental weights, the Fundamental weights would become the market's weights and the Fundamental portfolio would be a capitalization-weighted portfolio. Any argument against capitalization weights would then apply to Fundamental weights.

EXHIBIT

Monthly Continuous Return (Equal Weighted—Capitalization Weighted)

Index	Start	End	mean	t	
NYSE	19280131	20071231	0.0013	2.8138	c
NYSE	19280131	19471231	0.0031	2.1725	b
NYSE	19480131	19671229	0.0005	1.1944	
NYSE	19680131	19871231	0.0011	1.4827	a
NYSE	19880129	20071231	0.0003	0.4512	
AMEX	19620831	20071231	0.0017	2.7679	c
AMEX	19620831	19841231	0.0022	2.3252	b
AMEX	19850131	20071231	0.0012	1.5291	a
NASD	19730131	20071231	0.0014	1.8432	b
NASD	19730131	19901231	0.0011	1.4059	a
NASD	19910131	20071231	0.0017	1.2785	

a = significant at 10% confidence level
b = significant at 5% confidence level
c = significant at 1% confidence level

Regression Equation using Monthly Continuous Returns (Equal Weight – R_f) = $\alpha + \beta$ (Capitalization Weight – R_f) + ε

Index	Start	End	alpha	
NYSE	19280131	20071231	0.0005	
NYSE	19280131	19471231	0.0020	b
NYSE	19480131	19671229	0.0002	
NYSE	19680131	19871231	0.0006	
NYSE	19880129	20071231	0.0005	
AMEX	19620831	20071231	0.0013	b
AMEX	19620831	19841231	0.0015	a
AMEX	19850131	20071231	0.0012	a
NASD	19730131	20071231	0.0015	b
NASD	19730131	19901231	0.0011	
NASD	19910131	20071231	0.0021	b

a = significant at 10% confidence level
b = significant at 5% confidence level
c = significant at 1% confidence level

Diversity-Weighted Portfolios

Fernholz [2002] defines a passive portfolio termed a Diversity-weighted portfolio. A Diversity-weighted portfolio's weights are functions of an associated capitalization-weighted benchmark portfolio's weights.[10] The Diversity-weighted portfolio's weights, as a percentage of its benchmark portfolio's weights, monotonically decrease with increases in its benchmark portfolio's weights. The benchmark's larger capitalization stocks have less weight under Diversity weighting, and the benchmark's smaller capitalization stocks have more weight under Diversity weighting. For example, using the formula given in the previous endnote and assuming that parameter $p = 0.8$, stock 1 of the Diversity-weighted portfolio's version of the baseline case has weight $(1/3)^{0.8}/[(1/3)^{0.8} + (2/3)^{0.8}] = 0.3648$, which exceeds the capitalization weight (i.e., 1/3) for this small-capitalization stock. Stock 2 has Diversity weight 0.6352, which is less than the capitalization weight (i.e., 2/3) for this high-capitalization stock.

Fernholz [2002] empirically shows that a Diversity-weighted portfolio outperforms its benchmark over time.[11] In the context of this article, a possible partial explanation of the Fernholz [2002] empirical result is that a Diversity-weighted portfolio has less expected mispricing correction drag than its capitalization-weighted benchmark and similar Excess Growth Rates. To see that Diversity-weighted and capitalization-weighted portfolios may have similar Excess Growth Rates, consider the baseline case. The diversity-weighted portfolio's weighted average variance of the stocks is $[(0.3648)(0.16) + (0.6352)(0.09)] = 0.1345$. The portfolio's variance is $[(0.3648)^2(0.16) + (0.6352)^2(0.09) + 2(0.3648)(0.6352)(0.7)(\sqrt{0.16})(\sqrt{0.09})] = 0.1194$. Thus, the diversity-weighted portfolio's Excess Growth Rate is approximately $(0.1345 - 0.1194)/2 = 0.0076$. This is close to the capitalization-weighted portfolio's Excess Growth Rate, which was calculated earlier as 0.0091.

A Theoretical Size Effect

In the context of this article's analysis, capitalization weights typically are positively cross-sectionally correlated with estimation errors. The positive correlation induces a return drag due to mispricing corrections. Portfolio weights that are less positively correlated with estimation errors tend to suffer less mispricing correction drag. One way of reducing the correlation is to overweight smaller stocks relative to capitalization weights. Portfolios that overweight smaller stocks relative to capitalization weights tend to suffer less mispricing drag, and hence tend to have higher growth rates. This theoretical size effect partially explains the traditional size effect noted by others, e.g., Fama and French [1992].

Apparently, on average, Arnott et al.'s Fundamental indexes overweight their capitalization-weighted benchmark's smaller-capitalization stocks and underweight their capitalization-weighted benchmark's larger-capitalization stocks. This suggests that there is a positive impact of estimation errors on the difference between the returns of Arnott et al.'s Fundamental indexes and capitalization-weighted portfolios due to differential mispricing correction drag or, equivalently, a benefit from a size effect.

CONCLUSIONS

This article analyzes the impact of value estimation errors on portfolio growth rates and how the effect differs depending on the portfolio weighting method. In contrast to previous articles, it addresses the effect of estimation errors on portfolio growth rates due to increased return volatility. It also examines the effect of mispricing corrections on stocks' and portfolios' returns. Growth rates for several portfolio weighting methods are compared to their corresponding capitalization-weighted portfolio's growth rate. Examples include estimation error independent weights (which include equal weights), Fundamental weights, and Diversity weights.

Capitalization-weighted portfolios experience drag from mispricing corrections but gain from estimation error volatility's contribution to their Excess Growth Rates. The net effect depends on the balance between estimation error volatility and the speed of mispricing corrections. A relatively high rate of mispricing corrections leads to a net negative impact.

The growth rates for the estimation error independent weighting methods exceed the capitalization-weighted growth rate because they have a lower mispricing correction drag than the capitalization-weighting method and have portfolio Excess Growth Rates that are typically similar to that of the capitalization-weighting method. Evidence was provided in this article that equally-weighted portfolio (i.e., one of several estimation error independent weighting methods) returns exceed those for capitalization-weighted portfolios.

Fundamental weighting and Diversity weighting methods may have portfolio growth rates that exceed their corresponding capitalization-weighted portfolio growth rates because they may have a lower mispricing correction drag than the capitalization-weighting method does and portfolio Excess Growth Rates that are typically similar to that for the capitalization weighting method. Thus, the article provides theoretical support, in the context of estimation error, for the empirical findings of Arnott et al. [2005] and Fernholz [2002] that many Fundamental-weighted and Diversity-weighted portfolio returns beat the market's (i.e., capitalization-weighted) returns. The article also notes weaknesses of the Fundamental weighting approach. The empirical results quoted in the literature that many Fundamental-weighted portfolio returns beat the market's (i.e., capitalization-weighted) returns should not be taken to hold generally.

The main implication for investors is the following. To increase portfolio growth rates, investors should reduce the cross-sectional correlation between their portfolio weights and estimation errors. Given the positive correlation between estimation errors and capitalization, this can be done by underweighting high-capitalization stocks and overweighting low-capitalization stocks. The Fundamental weighting and Diversity weighting methods can be interpreted as ways to reduce the correlation between portfolio weights and estimation errors.

Large-capitalization stocks tend to suffer mispricing correction drag and small-capitalization stocks tend to enjoy mispricing correction gain. This is a size effect that partially explains the traditional size effect noted by others, e.g., Fama and French [1992].

An appendix derives the estimation errors' effect on portfolios' growth rates. It is available upon request.

ENDNOTES

[1]This article emphasizes readability. A more rigorous and comprehensive version is available from the authors.

[2]"Fundamental" and "Diversity" are capitalized because Fundamental weights and Diversity weights are the names of specific products marketed to investors, hence are proper nouns.

[3]An alternative theory is that small stocks have greater risk-adjusted returns because the model for measuring risk is deficient (see Fama and French).

[4]While this article refers to "stocks," the idea it addresses (i.e., the effect of value estimation errors on portfolio growth rates) is not unique to stocks; it is applicable to "assets" generally. Credit for this point belongs to an anonymous reviewer.

[5]"Value" and the "value-weighted" portfolios discussed in this article are theoretical constructs. They are unobservable.

[6]The assumption that estimation error is independent of value is reasonable because any correlation implies that assets are systematically mispriced over time according to their value and that estimates of true value are systematically biased.

[7]For intuition, consider the special case where all values are equal and there are estimation errors. In this case, there is perfect positive correlation between price and estimation error.

[8]To focus on portfolio growth rates, we assume that stocks' growth rates are not directly affected by estimation error. In this case, the effect of estimation error on a portfolio's growth rate is equivalent to the estimation error effect on the portfolio's Excess Growth Rate.

[9]Fundamental weights, discussed later in the article, may or may not approximate estimation error independence at various times.

[10]Denote stock *i*'s weight in a capitalization-weighted benchmark by μ_i and its weight in the Diversity-weighted portfolio by π_i. Choose a parameter, *p*, between 0 (which corresponds to equal weights) and 1 (which corresponds to capitalization weights). Then the Diversity-weighted portfolio with parameter *p*, has the following weights.

$$\pi_i = \frac{\mu_i^p}{\Sigma_j \mu_j^p}$$

[11]Fernholz [2002] also shows that a Diversity-weighted portfolio's continuous relative return can be perfectly attributed to the change in the benchmark's log Diversity (a specific function that measures the extent to which capital is spread across the benchmark's stocks) and a positive drift termed the Kinetic Differential. Finally, Fernholz [2002] shows that log Diversity is bounded, hence a Diversity-weighted portfolio outperforms its benchmark over time by the amount of its Kinetic Differential.

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