Antileakage Fourier transform for seismic data regularization in higher dimensions

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ABSTRACT

Wide-azimuth seismic data sets are generally acquired more sparsely than narrow-azimuth seismic data sets. This brings new challenges to seismic data regularization algorithms, which aim to reconstruct seismic data for regularly sampled acquisition geometries from seismic data recorded from irregularly sampled acquisition geometries. The Fourier-based seismic data regularization algorithm first estimates the spatial frequency content on an irregularly sampled input grid. Then, it reconstructs the seismic data on any desired grid. Three main difficulties arise in this process: the “spectral leakage” problem, the accurate estimation of Fourier components, and the effective antialiasing scheme used inside the algorithm. The antileakage Fourier transform algorithm can overcome the spectral leakage problem and handles aliased data. To generalize it to higher dimensions, we propose an area weighting scheme to accurately estimate the Fourier components. However, the computational cost dramatically increases with the sampling dimensions. A windowed Fourier transform reduces the computational cost in high-dimension applications but causes undersampling in wavenumber domain and introduces some artifacts, known as Gibbs phenomena. As a solution, we propose a wavenumber domain oversampling inversion scheme. The robustness and effectiveness of the proposed algorithm are demonstrated with some applications to both synthetic and real data examples.

INTRODUCTION

Imaging subsalt structures under complex overburdens is a challenging task in seismic exploration due to the poor illumination of seismic waves. For some complex salt geometries, the conventional narrow-azimuth seismic survey gives poor images in subsalt areas. To solve the illumination problem, geophysicists designed wide-azimuth acquisition. The first attempt for marine acquisitions was called wide-azimuth towed streamer (WATS) acquisition. The WATS data sets allow us to greatly improve the signal-to-noise ratio in imaging by increasing the azimuthal fold and reducing the multiple interferences in the final images (Etgen, 2006). Geophysicists have also improved the processing and imaging technologies to take advantage of the new acquisition geometry. For example, 3D surface-related multiple elimination (SRME) algorithm (Verschuur et al., 1992) worked better with wide-azimuth data to predict and remove the multiple interference, tilted-transverse isotropy (TTI) velocity model building techniques were adapted to this type of data (Huang et al., 2008) and TTI reverse time migration has been developed to incorporate the azimuthal information into depth imaging (Baysal et al., 1983; Zhang and Zhang, 2009). These recently developed technologies demonstrated the significant improvements achieved for imaging the complex structures in the subsalt area. However, most of these new developments require regularly sampled seismic data as input, with a dense sampling rate (without aliasing) in all the spatial directions, i.e., inline, crossline, and vector offset. Due to the physical and economic constraints of the acquisition, the actual real seismic data are rarely acquired on such a required regular grid.

Seismic data regularization, which maps the seismic traces from an irregular grid to a regular one, provides the regularly sampled seismic data for 3D SRME and wave equation-based migration algorithms, including TTI reverse time migration. Different seismic data regularization algorithms have been developed during the last two decades. A series of algorithms is based on the integral of continuation operators (normally, Kirchhoff type). The integral is performed based on the traveltime calculation from a velocity model, thus the results depend on the given velocity model. In a complex geological structure case, when the velocity model has strong lateral variations, artifacts arise from this type of approach (Canning and Gardner, 1996; Bleistein and Jaramillo, 2000; Stolt, 2002). On the other hand, the algorithm is generally less effective at near offsets because of limited integration apertures (Fomel, 2003). When the in-
put data are coarsely sampled, the summation of aliased data creates strong artifacts. Additional treatments (such as local focus analysis, filtering, local inversion, etc.) are therefore required to remove these artifacts.

During the last two decades, the most popular data regularization algorithms were based on a convolution filter (Spitz, 1991; Abma and Claerbout, 1995), which can predict the data in such a way that the error is assumed to be white noise. Generally, the filter is calculated by specific inversion methods. The prediction error filter (PEF) is the most commonly used approach (Abma and Claerbout, 1995). It works efficiently on a regularly sampled input. The main advantage of PEF is that it handles aliased linear events (Spitz, 1991) with a proper local linear approximation.

Fourier-based seismic data regularization algorithms (Sacchi and Ulrych, 1995; Duijndam et al., 1999; Hindriks and Duijndam, 2000; Xu et al., 2005b; Trad, 2009) provide another way of doing seismic data regularization. The philosophy of these algorithms is to estimate the Fourier coefficients from the input data as accurately as possible, then reconstruct the data to any desired grid. Different schemes have been proposed as, for example, irregular fast Fourier transform (FFT) (Hindriks and Duijndam, 2000), sparse inversion (Sacchi and Ulrych, 1995, Zwartjes and Sacchi, 2007), etc. These methods are efficient if the input data are on a regularly sampled grid, and the reconstruction is to fill in the missing traces. Its efficiency is due to the usage of the FFT. However, when these methods are applied to an irregularly sampled data set, the orthogonality among Fourier components no longer holds. The energy of one Fourier component can “leak” to the other components, and this crosstalk is called “spectral leakage.” In a Fourier-based regularization, direct estimation of Fourier coefficients from the discrete Fourier transform (DFT) or its alternatives (Beylkin, 1995) may lead to poor regularization results due to the ignoring of spectral leakage.

Xu and Pham (2004) proposed an alternative algorithm, called antileakage Fourier transform (ALFT), to resolve or at least reduce the spectral leakage problem. When applied to complex real data, the method proves efficient and accurate for seismic data regularization. Its implementation in both 2D and 3D for the assumption of common offset and common azimuth data demonstrated promising results (Xu and Pham, 2004; Xu et al., 2005b). The cascaded two pass of two 3D regularizations (common shot plus common receiver) allows us to recover irregularly sampled 5D seismic data using regularization schemes with lower dimensions (Xu et al., 2005a) and avoid the assumptions of common offset, or common azimuth. However, note that it may be less efficient in the case of filling irregular big acquisition holes. These holes generally have irregular shapes, with sizes strongly variable along the various dimensions of the acquisition geometry. The two-step regularization scheme is limited to filling the acquisition holes observed in the common shot and the common receiver subdata sets. In real data, the acquisition holes may be large in one direction but also reasonably small in other directions. Therefore, a high-dimensional interpolation algorithm is expected to have a better chance at efficiently filling in all the types of acquisition holes (Trad, 2009).

In this paper, after a brief review of the ALFT algorithm, we generalize the antileakage Fourier transform to high dimensions. We discuss two practical issues in this generalization. The first issue is how to accurately estimate the Fourier components using an irregular grid DFT integral. This is related to the choice of the integral weighting function (Beasley and Klotz, 1992). In the literature, the published algorithms to compute the integral weights are in either 1D or 2D (Canning and Gardner, 1996; Jousset et al., 2000; Zwartjes and Gislaf, 2002). For higher dimensions, we propose using a sampling density function to compute the weighting function on an irregularly sampled grid and adapt it into DFT summation, which is the kernel of the ALFT algorithm. The second issue is that the computational cost of ALFT depends on the number of wavenumbers and the number of input traces in all the dimensions. To reduce the cost, we can perform ALFT in a local window. However, small window size ALFT suffers from the frequency/wavenumber domain undersampling problem. In this paper, we discuss the sampling issue and demonstrate that a wavenumber domain oversampling improves the regularization. We propose to use a least-square inversion scheme to find the best-fit DFT coefficient in each ALFT iteration. Therefore, ALFT can be applied in small windows to reduce the computational cost. We have tested our algorithm on both the Mar-mousi synthetic data set and a real WATS data set in the Gulf of Mexico. Both tests give satisfactory regularization results.

ANTILEAKAGE FOURIER TRANSFORM ALGORITHM

The goal of the antileakage Fourier algorithm is to estimate the Fourier coefficients from irregularly sampled seismic traces with reduced wavenumber leakage. For a wide-azimuth survey, the seismic data are acquired at the surface. The acquisition geometry is described by the two dimensions of both shot location coordinates and receiver coordinates. The time is an additional dimension for the data set, but since the recorded time samples are on a regular grid, it is efficient to use a FFT to transform the data to the time frequency domain. Therefore, in the data regularization, we need to consider only the irregularities in the four spatial dimensions, either in the shot-receiver domain (x = (x,y,z,x,y)) or alternatively in the CDP-offset domain (x = (cdp,cdph,sh,sh)). The DFT on an irregular grid x can be expressed as seen in equation 1

$$\hat{f}(k) = \frac{1}{\Delta X}\sum_{x} w(x) f(x) e^{-2\pi i k x}$$

$$\hat{f}_{k}(x_{i}) = \hat{f}(k) e^{2\pi i k x_{i}}, \quad (1)$$

where w(x) is the integral weight that we will discuss later, $$\Delta X = \sum_{x} w(x)$$ is a normalization factor involving the summation of all the sampled integral weights, Np is the number of input points, $$\hat{f}(k)$$ is the Fourier coefficient for a wave number k, and $$\hat{f}_{k}(x_{i})$$ denotes the k component in the input data. The ALFT algorithm estimates the Fourier coefficients for all the wavenumbers k first. Then it selects the one with the maximum energy $$\hat{f}_{\max}(k)$$ and performs an inverse DFT back to the input grid. Finally, it updates the input by removing the contribution of the picked k component $$\hat{f}_{\max}(k)$$ from the input data as seen in equation 2

$$f_{k}(x_{i}) = f(x_{i}) - \hat{f}_{\max}(k) e^{2\pi i k x_{i}}. \quad (2)$$

Therefore, our regularization algorithm can be implemented in the following steps:

1) Initialize all the Fourier coefficients to zero.
2) Compute all the Fourier coefficients of the residual input data $$f_{k}(x_{i})$$ using equation 1.
3) Select the Fourier coefficient with the maximum energy, accumulate its contribution to the existing Fourier component.
4) Subtract the contribution of this coefficient from the input data (equation 2).
5) Iterate steps 2 to 4 until the updated residual input data are small enough.

The ALFT is an iterative process. The contributions to a given \( k \) component may be accumulated at different iterations because the base functions \( e^{-2\pi nk/k} \) in DFT are not orthogonal to each other. The whole ALFT process can be computationally intensive. On the estimation of \( N \) Fourier components with \( N_p \) input samples for each iteration, ALFT requires computation of all the \( N \) Fourier coefficients by a DFT, which has an order of \( N N_p \) floating operations, and it normally takes \( N \) iterations to converge. Therefore the total computational cost is in an order of \( N^2 N_p \). If \( N \) has same order of \( N_p \), the total computation cost is in an order of \( N_p^3 \).

**THE DFT WEIGHTING FUNCTION IN A HIGH-DIMENSIONAL ALFT**

When the data are sampled irregularly in 1D, the integral weight \( w(x_i) \) in equation 1 can be chosen as the distance between the two adjacent samples. In 2D, we can use the Voronoi Polygons algorithm to compute the weights (Canning and Gardner, 1998; Jousset et al., 2000), or a more sophisticated algorithm such as the Pipe-Menon algorithm (Zwartjes and Gisolf, 2002). In higher dimensions, geophysicists proposed a so-called hit-count algorithm, which counts the number of samples in a bin to evaluate the integral weight (Beasley and Klotz, 1992). The weighting function calculated from the hit-count algorithm is usually discontinuous when the sampling grid is irregular. The discontinuity affects the accuracy of the estimation of the Fourier components (Zwartjes and Gisolf, 2002).

The ALFT algorithm is less sensitive to the choice of the weighting functions, because it is an iterative DFT process and, at the end, the contributions from all the iterations sum up to match the original data. However, a proper integral weighting function gives a better estimate of each Fourier component, speeds up the iterative process, and improves the regularization result. In our work, we propose a simple solution to compute the weighting function based on a sampling density function \( \sigma(x) \) that will be defined later.

Let us demonstrate the basic concept in 1D. The sampling location function is defined as the collection of all the locations where the measurements are made. It could be represented as the sum of normalized \( \delta \) functions with the supports at the sampled locations. In Figure 1a, the irregularly sampled locations \( x_i \) are generated by a random function. We can build a sampling location function \( \mathcal{L}(x) \) on this type of irregular grid (Figure 1b).

To use Fourier reconstruction methods, usually we assume that the function \( f(x) \) is continuous and band-limited. When it is measured at discrete samples, each measurement contributes to the reconstruction as a weighted local base function. For example, on a regularly sampled grid, according to the Shannon sampling theorem, the function can be well reconstructed by \( \text{sinc} \) functions centered at each sample. This implies that one can treat each measurement as a continuous density, rather than a single spike. This concept can be generalized to irregularly sampled measurements in higher dimensions to build the weighting function \( w(x_i) \). The construction of \( w(x_i) \) is performed by convolving a distribution function with the sampling location function. The distribution function can be a boxcar function, a B-spline function, or a Gaussian function. We choose to use the Gaussian function as our distribution function due to its smoothness and computational convenience.

The convolution of the sampling location function with a Gaussian function \( G(x) \) gives the sampling density function \( \sigma(x) \)

\[
\sigma(x) = G(x) \ast \mathcal{L}(x), \quad G(x) = \left( \frac{1}{\pi b} \right)^{\frac{m}{2}} e^{-\frac{x^2}{b}},
\]

where \( m \) is the number of dimensions of the vector \( x \). After the convolution, the blurred sampling location is a positive smooth function, covering the whole input area (Figure 1c). We define the weighting function in equation 4 as

\[
w(x) = \frac{1}{\sigma(x)} \quad \Delta x = \sum_{\ell = N_p} w(x_i).
\]

For seismic data regularization applications, the input seismic trace coordinates are irregularly sampled, but we can choose the sampling rate on the survey design for common middle point (CMP) and offsets and rescale those coordinates to a unit sampling rate, thus we have \( b = 1 \) in \( G(x) \) (equation 3).

**SAMPLING IN THE WAVE-NUMBER DOMAIN**

As previously mentioned, the ALFT is computationally intensive because the algorithm requires iterative DFTs. In each dimension, DFT has an order of \( N_p^2 \) floating point operations, compared to the FFT algorithm that requires an order of \( N_p \ln N_p \) floating point opera-

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Integral weight for an irregularly sampled grid. (a) Synthetic irregularly sampled measurements location \( x_i \); (b) sampling location function \( \mathcal{L}(x) \); (c) sampling density function \( \sigma(x) \) obtained applying a Gaussian function blurring.
tions. In higher dimensions, the total number of the input samples is equal to the product of the sample numbers in all the dimensions that dramatically increases the computational cost and can easily exceed the capacity of today’s super computers. One solution is to compute the DFTs in smaller windows in all the spatial dimensions. For example, if we divide the $N_p$ samples into M subwindows, the number of floating point operations will be reduced from $N_p^3$ to $(N_p/M)^3 M$, which greatly reduces the computational cost.

However, for a conventional DFT, a smaller window size leads to fewer wavenumber components with a bigger wavenumber sampling interval. For seismic data, each wavenumber component physically represents a spatial direction of propagation of seismic waves. To properly represent the reflection information embedded in the seismic data, we need adequately sampled wavenumbers. On the other hand, the conventional DFT assumes a periodic boundary condition, and windowing the seismic data apparently breaks this assumption. As a result, a spatial wraparound appears near the edges of the windows and produces serious Gibbs phenomena when the window size becomes smaller. In the following, we propose using an accurate oversampling scheme to overcome the problems.

We use the simple function $y = 0.1 x$ (Figure 2a) to illustrate the problems and our solution. First, let us sample the function from 0 to 63 with a regular sampling step of 1. Because the input data are sampled on a regular grid, the Fourier components are orthogonal to each other and there is no frequency leakage. Therefore, a straightforward ALFT cannot improve the reconstruction in this ideal case. Figure 2b shows the continuous reconstruction of Fourier components computed by a conventional FFT. The ringing artefacts appearing at the two edges correspond to the famous Gibbs phenomenon. This example tells us that truncating the sampling range introduces noise near the boundary, especially for longer wavelength events. We have to overcome these boundary truncation Gibbs artefacts when using smaller window sizes to speed up an ALFT regularization.

The solution we adopt is to oversample the data in the wavenumber domain. In fact, the wavenumber $k$ in the DFT equation 1 can be an arbitrary value. Instead of using equation 1 to estimate $\hat{f}(k)$, we propose solving for $\hat{f}(k)$ by minimizing the following objective function in equation 5:

$$\text{Error} = \sum_{\ell \in N_p} \| f(x_\ell) - \hat{f}(k)e^{2\pi ikx_\ell} \|.$$  (5)

Minimizing this objective function is an inversion process with a single unknown. It does not add much computational cost, but the sampling step in the wavenumber domain can be arbitrarily smaller than in a conventional DFT (equation 1). The number of wavenumbers may be much bigger than when using equation 1 for conventional small window size applications. It may even reach the same number of wavenumbers as when using equation 1 for large window size applications. In a small window size application, the number $N_p$ in equation 1 is much smaller than that obtained for large window size applications, which significantly saves computing time especially in the case of numerous dimensions. This is the point that dramatically reduces the computational cost. With smaller window sizes, the seismic events are close to linear events, and this provides us with an easy antialiasing scheme inside ALFT algorithm (Schonewille et al., 2009). One remark is that if $f(x_\ell)$ is a real function, we have to enforce that $\hat{f}(-k) = \hat{f}(k)^*$, where $\hat{f}(k)^*$ is the conjugated complex of $\hat{f}$. Therefore, we need to modify the objective function (equation 5) as follows

$$\text{Error} = \sum_{\ell \in N_p} \| f(x_\ell) - \hat{f}(k)e^{2\pi ikx_\ell} - \hat{f}^*(k)e^{-2\pi ikx_\ell} \|.$$  (6)

Solving for equation 6 gives different Fourier coefficients than a conventional DFT does. It can provide as many Fourier coefficients as we need to recover the locally sampled function. We call the Fourier transform based on equation 6 the “best-fit DFT,” and use it to replace the DFT in our ALFT kernel. We first test an oversampling of 16 times using the objective function given in equation 6, which means that the new wavenumber sampling step $\Delta k$ is only $\frac{1}{16}$ of a conventional FFT’s. Figure 3 shows the spectra comparison: the red line is the spectrum computed by DFT (1); the blue line is computed by the best-fit DFT (6); and the green line is computed by ALFT using the best-fit DFT as the kernel. We notice that the first nonzero wavenumber components of both the best-fit DFT and the new ALFT are the same and have much stronger energy than other components (detail shows in Figure 3b). Especially, the spectrum of ALFT only has one big nonzero value that means that the ALFT with the best-fit DFT converges after one iteration. This demonstrates that the best-fit DFT provides much better convergence than DFT as the “engine” of ALFT. Figure 4 shows the reconstructed continuous function. Compared to the exact solution, it has very small errors and the biggest mismatches happen near the two edges (see Figure 4b).

On the irregular grid, Figure 5a shows how the function is sampled on a randomly generated irregular grid within the same sampling range as in Figure 2a. Figure 5b shows the spectrums of the irregularly sampled input. Compared with Figure 3b, we see that the spectrums of the new ALFT (green line) are almost identical for both

![Figure 2. Regularly sampled function $y = 0.1 x$ and its Fourier reconstruction. (a) The function $y = 0.1 x$ on a regular grid with sampling rate of 1 on the support $x \in [0, 63]$; (b) Reconstructed continuous function using Fourier components computed by FFT, the Gibbs effect is very clear.](image-url)
regularly and randomly sampled grid. Figure 6a shows the reconstructed continuous function by ALFT. The greatest error shows up in the area with a large sampling hole. Further improvement can be obtained by reducing the wavenumber interval \( \Delta k \). Figure 7 shows the experiment with a wavenumber resampling ratio of 32. The ALFT spectrum (Figure 7a) shows that the magnitude of the first nonzero wavenumber is almost doubled (compare to Figure 5b), and the reconstructed continuous function exhibits a much reduced error.

In fact, the input function in our test \( y = 0.1x \) could be approximated by \( y = 0.1/\alpha \sin(\alpha x) \), with the coefficient \( \alpha \) sufficiently small. This is an ultra-low frequency function, and truncating the function implicitly defined in FFT is usually too large to reasonably recover such a function. The wavenumber domain oversampling we propose provides the possibility to estimate the spectrum on much lower wavenumber samples and to produce a better reconstruction.

ALFT is a data-driven algorithm. In practice, the wavenumber domain sampling rate \( \Delta k \) can never be small enough to exactly reconstruct the input function \( y = 0.1x \). However, a better reconstruction is obtained by increasing the sampling ratio. Similarly, the seismic events may have arbitrary dips, considering that seismic waves can propagate in any direction. When working on a small window size, with a limited spatial sampling rate, the wavenumber sampling rate implicitly defined in FFT is usually too large to accurately describe the wave propagation. The oversampling in the wavenumber domain represents the seismic events more accurately and allows ALFT to produce better regularization results.

**NUMERICAL EXAMPLES**

We first apply the algorithm on a synthetic data set. To demonstrate the capabilities to handle complex seismic events, we chose the Marmousi data set though it is a 2D synthetic data set that contains only two spatial dimensions (one for the shots and one for the receivers). We randomly dropped 10\% of the input traces. The missing traces create a 2D spatial irregularity and require the ALFT algorithm to reconstruct them. The nearest offset, \( h = 200 \text{ m} \), contains a high number of seismic complexities, such as crossing events, amplitude changes along the events, and diffractions. Since this first offset is on the edge of acquisition, the largest reconstruction errors usually show up on it. Figure 8a shows the original Marmousi data set. Figure 8b shows the input data for ALFT. Figure 8c is the output of ALFT. Compared with the perfect solution (Figure 8a), we observe that the major seismic events are well reconstructed by our ALFT algorithm.

We then demonstrate the high-dimensional ALFT reconstruction on a wide-azimuth marine seismic data set, acquired in the Garden Banks area in the Gulf of Mexico. The data set is indexed by the midpoint (2D vector), the vector offset (2D), and the recording time. After a natural binning, the data are sparsely sampled by 50 m in the inline direction (along midpoint X coordinate), 60 m in the midpoint Y direction, 300 m in the offset X, and 1200 m in offset Y directions. The time sampling rate is 12 ms. Dipping events are poorly sampled in the spatial directions. As the time sampling is regular, the antileakage schemes were applied on vector midpoint and vector offset simultaneously. Once the five-dimensional Fourier coefficients are obtained, the back transform to regular grid is a conventional FFT.

In Figure 9a, three inline sections of input seismic data at near, middle, and far offsets are shown. For the display purpose, we ap-

![Figure 3](image3.png)  
**Figure 3.** The spectrums on a grid of wavenumber resampling ratio of 16. The red line is the spectrum computed by the DFT; blue line is the spectrum computed by the best-fit DFT; green line is the spectrum computed by ALFT. (a) The full spectrum; (b) the zoom-in over an area of big difference.

![Figure 4](image4.png)  
**Figure 4.** The reconstructed continuous function by ALFT. (a) The full function; (b) the zoom-in over a large difference area (red line is the exact solution).
plied a partial NMO to the data to move the traces to the regular bin centers. In this data set, the offset sampling is 300 m in X and 1200 m in Y. As a result, a severe spatial aliasing occurs in offset directions, especially in the offset Y direction. Therefore, the data regularization algorithm with strong anti-aliasing capability is required to generate all the traces to the bin centers of regularized vector offsets.

Figure 9a shows complicated seismic signals due to complex geology; for example, the reflections and diffractions from the top of salt, crossing seismic events with multiples, and dramatic amplitude changes along the seismic events. From our experience, some conventional 3D regularization methods are applied on the data after partial NMO and it works well on near offsets but poorly on middle and far offset sections. This is because the partial NMO is an oversimplified assumption. It is difficult to align all the seismic events effectively by a NMO when the geological structures are complex, and the biggest moveout errors usually show up on far offsets. As shown in Figure 9a (middle and right), the multiple reflections on the middle and far offsets are blurred. This is because that the velocity in NMO correction is chosen to flatten the primary events but is not suitable for the multiples. Therefore, such highly aliased multiples are difficult to be correctly interpolated by a common offset/azimuth regularization scheme (Xu et al., 2005b).

Figure 9b shows the data regularization results produced by our 5D regularization algorithm. The same inline offset and crossline offset sections are plotted as shown in Figure 9a. Because the output seismic traces after regularization are located on the expected regular grid, here we directly plot them without a NMO correction. In Figure 9b, all the traces are recalculated with a 5D inverse FFT using the ALFT Fourier components (including the traces in the acquisi-

Figure 5. Function \( y = 0.1x \) on an irregular grid and its spectra. (a) Function \( y = 0.1x \) irregularly sampled on the support \( x \in (0 \rightarrow 64) \); (b) red line is the spectrum computed by the DFT; blue line is the spectrum computed by the best-fit DFT; green line is the spectrum computed by ALFT.

Figure 6. The reconstructed continuous function by ALFT. (a) The full function; (b) the zoom-in over a large difference area (red line is the exact solution).

Figure 7. The experiment with a wavenumber resampling ratio 32. (a) A zoom-in over the spectrum; (b) a zoom-in over an error in the reconstructed continuous function.
tion holes). When compared with the seismic sections before regularization (Figure 9a), it is clear that the method we proposed greatly reduces the blurring and aliasing effects and enhances the coherence of the seismic events.

**Figure 9.** Data regularization on a Gulf of Mexico wide-azimuth real data set. Left: near offset h = 450 m; middle: middle offset h = 2250 m; right: far offset h = 5250 m. (a) The input seismic data; (b) the ALFT output.

**Figure 8.** Data regularization on the Marmousi synthetic data set (offset = 200 m). (a) Original data set; (b) The input data set, where we randomly dropped 10% traces; (c) the reconstructed data set.

**Figure 7.** Time (s) measurements for different offsets in the Gulf of Mexico. (a) 2250 m; right: far offset h

**CONCLUSION**

We have generalized ALFT to high dimensions and applied it to wide-azimuth marine seismic data regularization. The efficiency of the method depends on the size of the local window used for each irregular DFT. We propose a frequency/wavenumber domain oversampling scheme inside of ALFT to overcome the Gibbs artifacts at the boundaries. We applied the algorithm on a wide-azimuth data set from the Gulf of Mexico. This numerical test shows promising results. With abundant information from wide-azimuth data, such a regularization method will improve prestack seismic processing, especially SRME and wave equation-based imaging techniques.

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