Converted-wave prestack time migration with a new approximate migration weight

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Summary

A new approximate migration weight is developed for Kirchhoff migration of converted-wave data. As with previous approximations it is based on the exact weight for a homogeneous medium. However, rather than assuming equality of travel path distances from source to image point and image point to receiver, it assumes that the total traveltime is partitioned in a way that is consistent with common-conversion point reflection. It is shown that, through solution of a cubic equation, this results in an efficient approach with no evaluations within the inner migration loop. Application of this new migration weight in prestack time migration of typical multicomponent land data shows that it yields migrated stacks and gathers very similar to those obtained using the exact homogeneous migration weight, and superior to those obtained using migration weights borrowed from P-wave migration theory, particularly in the near-surface region.

Introduction

Large volumes of multicomponent data have been and continue to be acquired. Extracting information from these data is desirable, but reality dictates that the geophysical community learns to use them one step at a time, as has been the pattern with compressional-wave data over many decades. Through consistent effort, the value of multicomponent data sets has gradually become more evident, and at present the converted-wave prestack time migration of reasonably flat geology is a standard deliverable from a number of vendors. Even so, improvements are still possible, as described in the present study.

Essential to the proper treatment of amplitudes in migration is incorporation of proper migration weights; however these can be expensive to apply, requiring evaluation inside the inner loops of Kirchhoff migration. For P-waves an approximate method which balances efficiency and accuracy was presented by Dellinger et al. (2000) and Zhang et al. (2000), and this is in common use throughout the industry. Miao et al. (2005) and Cary and Zhang (2010) presented extensions of the exact migration weight for PS data; Miao et al. (2005) also sought to develop an efficient and accurate approximation similar to that used in the P-wave case. Their analysis did not examine the issue in detail however, and a more satisfying and optimal approximation is desired.

In this work we review the strategy of migration weight approximation and present in detail a new approximation for converted-wave migration weights which achieves the same degree of accuracy as current approximations in use for P-wave migration. Single-trace responses as well as migrations to stacks and gathers are used to illustrate the value of this new method.

Theory of approximate PS migration weights

Review of PP case: Zhang et al. (2000) state that in a constant-velocity medium, the weight for a 3D common-offset prestack Kirchhoff migration of PP data is

$$\frac{z}{v^2}\left(\frac{t_s}{t_r}+\frac{t_r}{t_s}\right)\left(\frac{1}{t_r}+\frac{1}{t_s}\right), \qquad (1)$$

where t_s is the traveltime from source to image point, t_r is the time from image point to receiver, z the vertical distance to the image point, and v the velocity of the homogeneous medium. This expression must be evaluated in the inner migration loop in order to have access to the values of t_r and t_s, so it is time-consuming.

Following Dellinger (2000), Zhang et al. (2000) approximated $t_r = t_s = t / 2$ (where $t = t_s + t_r$ is the total traveltime**)**, which is exactly true for either zero-dip or for zero-offset configurations. This assumes that the weight can be approximated by the value it would have if it were a CMP reflection or if the source and receiver are at the equivalent offset position (Bancroft & Geiger, 1994). The weight above in equation 1 then simplifies to

$$\frac{8z}{v^2 t}=\frac{8}{v^2 t}\frac{t_0}{2v}=\frac{4t_0}{v^3 t}, \qquad (2)$$

where t_0 is the two-way vertical traveltime. The first quantity in equation 2 pertains to depth migration and the last to time migration. The $1/t$ factor of the approximation can be applied to the input before the migration loops and the z or t_0 factor can be applied to the image after the migration loops, so the additional time required for weighting is now negligible.

This is a good approximation for geologies which are largely horizontal as the CMP configuration makes the dominant contribution to the image, and in this approximation the CMP configuration is weighted correctly. Zheng et al. (2000) found it to give reasonable amplitude behavior in depth imaging applications.

PS case: An exact, constant-velocity weight for PS migration, analogous to equation 1, is

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$$\frac{\dfrac{z}{v_c^2}\left(\gamma\left[\gamma+\cos(\theta_s+\theta_r)\right]\dfrac{t_s}{t_r}+\dfrac{1}{\gamma}\left[\dfrac{1}{\gamma}+\cos(\theta_s+\theta_r)\right]\dfrac{t_r}{t_s}\right)}{\left(\gamma+\dfrac{1}{\gamma}+2\cos(\theta_s+\theta_r)\right)\left(\gamma\dfrac{1}{t_r}+\dfrac{1}{\gamma}\dfrac{1}{t_s}\right)^{-1}}. \qquad (3)$$

Here $v_c^2 = v_p v_s$, where v_p and v_s are the P-wave and S-wave velocities of the medium, $\gamma = v_p / v_s$, and θ_s and θ_r are the angles of the P-wave and S-wave travelpaths with respect to vertical. We found the expression of Miao et al. (2005) to be more accurate than that of Cary & Zhang (2010) and obtained equation 3 from the expression for the horizontal layering case in Miao et al. (2005), simplifying it for the case of a homogeneous medium. [Note that setting $\gamma = 1$ in equation 3 yields equation 1, except for a factor of 2 missing from Zhang et al. (2000).] Like equation 1, the expression in equation 3 must be evaluated in the inner migration loop; however it is more complicated than the PP expression, so that it would be even more time-consuming to apply.

Miao et al. (2005) suggest that equation 3 could be approximated following the same practice as in the PP case. Presumably they refer to the fact that, for instance, in the CMP geometry ($\theta_s = \theta_r$) we have $t_r / t_s = \gamma$ which simplifies equation 3 to equation 2 with $8/v^2$ replaced by the constant factor $\left(\sqrt{\gamma}+1/\sqrt{\gamma}\right)^2/v_c^2$. We refer to this as the midpoint weight approximation (MPWA). However the CMP geometry does not make the principal contribution to the image in PS migration, so the MPWA is not an ideal approximation for the PS case. For a largely horizontal geology the principal contribution to the image comes from CCP (common conversion point) reflections. The purpose of this abstract is to present details of a migration weight approximation which is as accurate for PS data as equation 2 is for PP data.

The general strategy of the approximation, which we refer to as the CPWA (conversion point weight approximation), is very simple; it is to set t_r and t_s to values which sum to the original total traveltime t, but which correspond to a CCP reflection. This requires solving a cubic equation, as described next, but this can be done outside the inner loop and is thus not a burden in terms of computing time.

Derivation of the approximate converted-wave migration weight: Beginning from the exact weight for a homogeneous medium (equation 3) let us express it in terms of raypath distances, r_s and r_r, rather than traveltimes, using the relations $t_s = r_s / v_p$ and $t_r = r_r / v_s$. This yields

$$\frac{\dfrac{z}{v_c^2}\left(\left[\gamma+\cos(\theta_s+\theta_r)\right]\dfrac{r_s}{r_r}+\left[\dfrac{1}{\gamma}+\cos(\theta_s+\theta_r)\right]\dfrac{r_r}{r_s}\right)}{\left(\gamma+\dfrac{1}{\gamma}+2\cos(\theta_s+\theta_r)\right)\left(\gamma\dfrac{v_c/\sqrt{\gamma}}{r_r}+\dfrac{1}{\gamma}\dfrac{v_c\sqrt{\gamma}}{r_s}\right)^{-1}}. \qquad (4)$$

We can further represent this in terms of the ratio $r \equiv r_r / r_s$ and the product $p \equiv r_s r_r$ giving us

$$\frac{\dfrac{z}{\sqrt{p}v_c}\left(\left[\gamma+\cos(\theta_s+\theta_r)\right]\dfrac{1}{r}+\left[\dfrac{1}{\gamma}+\cos(\theta_s+\theta_r)\right]r\right)}{\left(\gamma+\dfrac{1}{\gamma}+2\cos(\theta_s+\theta_r)\right)\left(\dfrac{\sqrt{\gamma}}{\sqrt{r}}+\dfrac{\sqrt{r}}{\sqrt{\gamma}}\right)^{-1}}. \qquad (5)$$

The MPWA can be obtained from this exact PS weight by setting $r = 1$ and $p = t^2 v_c^2 \gamma / (1+\gamma)^2$, consistent with a CMP reflection geometry. To obtain the more accurate CPWA we must find r and p for a CCP configuration. For given values of t, γ, v_c, and offset h, we require three relations:

1) $r_s^2 - h_s^2 = r_r^2 - h_r^2 \left(= z^2\right)$ from which we obtain

$$1/r - r = (2h_s - h)h / p, \qquad (6)$$

where h_s, h_r are lateral distances from source and receiver to CCP location, so that $h = h_s + h_r$.

2) Snell's law, $(\sin\theta_s)/v_p = (\sin\theta_r)/v_s$, from which we obtain

$$h_s / h = \gamma / (r+\gamma). \qquad (7)$$

3) $t = t_s + t_r$ from which we obtain

$$(t v_c)^2 / p = r\gamma + 2 + 1/(r\gamma). \qquad (8)$$

We use equations 7 and 8 to eliminate h_s and p from equation 6 which yields a cubic equation in r:

$$r^3\left(\gamma-\tau^2\right)+r^2\left(2-\gamma\tau^2-\gamma^2\right)+r\left(\tau^2-2\gamma+1/\gamma\right)+\left(\gamma\tau^2-1\right)=0, \qquad (9)$$

where $\tau \equiv t\, v_c / h$ is a dimensionless time. We can solve equation 9 for r, then substitute into equation 8 to obtain p. These values (r and p) can then be used in equation 5 to calculate the CPWA. Note from the cosine law for a CCP configuration we find $\cos(\theta_s + \theta_r) = (r + 1/r - h^2 / p) / 2$.

Because p and r for a CCP configuration depend on t but not on z or t_0, only the z or t_0 factor in equation 5 needs to be applied to the image after migration, and everything else can be applied to the input prior to migration. Nothing needs to be applied inside the migration loops.

Equation 9 is reminiscent of the cubic equation obtained by Schneider (2002). His equation yields the conversion point, which also could be used as a starting point to calculate r and p. This would be less direct than using equations 8 and 9, but would still yield r and p values for use in equation 5.

The above is for the 3D case. The analogue of equation 5 for the 2.5D case is

$$\frac{\dfrac{z}{p^{1/4}v_c}\left(\left[\gamma+\cos(\theta_s+\theta_r)\right]\dfrac{1}{r}+\left[\dfrac{1}{\gamma}+\cos(\theta_s+\theta_r)\right]r\right)}{\left(\gamma+\dfrac{1}{\gamma}+2\cos(\theta_s+\theta_r)\right)\Big/\sqrt{\dfrac{\sqrt{\gamma}}{\sqrt{r}}+\dfrac{\sqrt{r}}{\sqrt{\gamma}}}}. \qquad (10)$$

r and p for this expression can be obtained following the same method as for the 3D case.

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Examples of prestack time migration with approximate and exact weights

In this section we demonstrate the effect of choosing either the MPWA or CPWA in converted-wave prestack time migration. We illustrate this with i) single-trace responses, ii) migrated stacks, and iii) migrated gathers.

All examples are from the Firestone 3C/2D dataset. These data were collected in February 2013 from both Vibroseis and dynamite sources with 110 ft (35.5 m) source interval and 55 ft (16.76 m) receiver interval.

Single-trace responses. Figure 1a illustrates migration of a single trace using the exact homogeneous weight of equation 3. Figure 1b shows the difference between Figure 1a and the analogous result obtained using the MPWA weight (which is of a similar form to the PP weight in equation 2). As expected, for all times they are equal at the midpoint of the trace. However, in Figure 1c, the difference between Figure 1a and the CPWA result shows that the position at which they are equal varies with time, tracing out the locus of CCPs. These results suggest that for the final migration products the CPWA will have its greatest advantage over the MPWA in the near surface.

Migrated stacks. Figure 2 shows near-surface detail of the migrated stacks. As anticipated from the single-trace responses, the CPWA result is superior to the MPWA result in this region. At later times the results are more similar.

Migrated gathers. Figure 3 shows near-surface detail of a single migrated gather. Both the MPWA and CPWA are most accurate at short offsets, but the CPWA would provide better input than the MPWA for a postmigration AVO analysis. At later times the three results become more similar.

Conclusions

A new approximate migration weight has been obtained for prestack Kirchhoff migration of converted wave data. It is expected that this weight provides an optimal balance of accuracy and efficiency for current processing standards. It provides results very similar to the exact homogeneous weight for reasonably horizontal geologies, but requires no additional evaluation within the inner migration loop.

Acknowledgements

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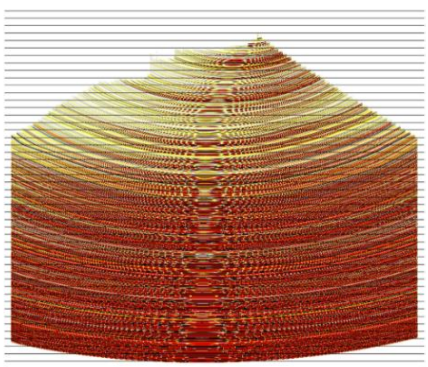
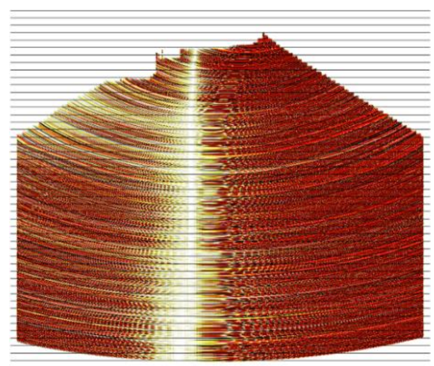
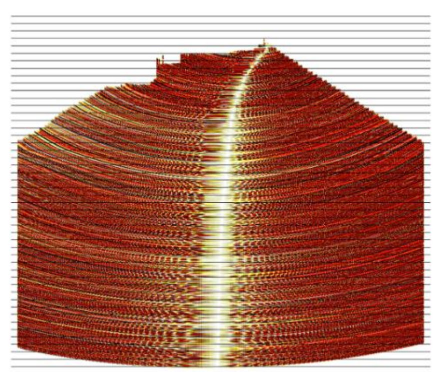


a)



b)



c)

Figure 1. Single-trace responses. a) The "exact" response, using the exact homogeneous weight in equation 3. b) The difference of the exact response and the MPWA response. c) The difference of the exact response and the CPWA response. The amplitudes in b) and c) have been significantly increased in order to emphasize the location of zero differences.

Between b) and c) we have indicated the approximate lateral location of the source (S), midpoint (M), asymptotic conversion point bin (A), and receiver (R), as given from the geometry of the input trace. From this we see that the exact and MPWA results are identical (difference=0) at the midpoint, while the exact and CPWA results are identical at CCP locations.

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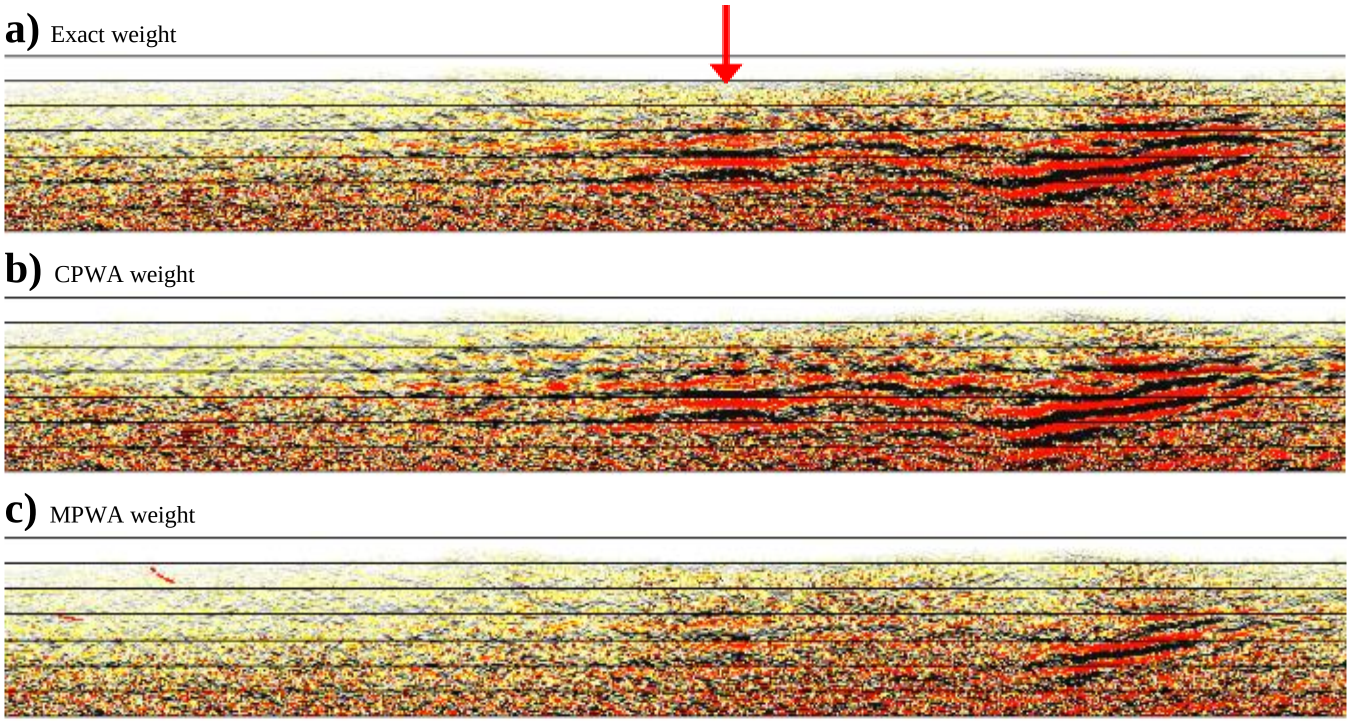

Figure 2: Near-surface detail of migrated stacks obtained using a) the exact homogeneous weight, b) the conversion-point weight approximation (CPWA), and c) the midpoint weight approximation (MPWA). The vertical time scale is from 0.0 – 0.7 s. The red arrow at top indicates the location of the migrated gather displayed in Figure 3. The CPWA presented here yields a result superior to the MPWA result.

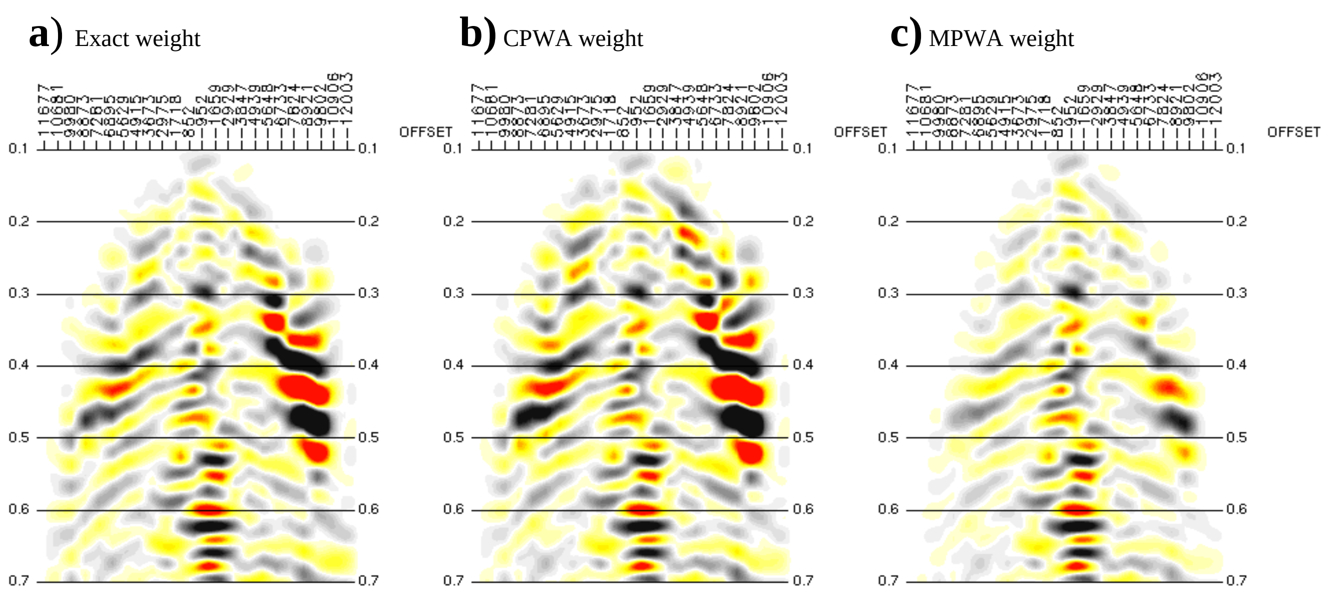

Figure 3: Near-surface detail of migrated gathers obtained using a) the exact homogeneous weight, b) the conversion-point weight approximation (CPWA), and c) the midpoint weight approximation (MPWA). The location of the gather in the stack is indicated by the red arrow in Figure 2. The CPWA result is much more similar to the exact result than is the MPWA result.

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EDITED REFERENCES

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