Summary

Mathematical morphological filtering (MMF) is a powerful tool for image processing based on the shape of the structure element (SE). It was introduced into seismic data processing to suppress noise and enhance signal quality. We explain the basic mathematic morphology concepts with set theory and define the basic and advanced morphological operations in seismic data processing. Unlike conventional seismic filtering techniques, MMF is a nonlinear operator so that it can more effectively isolate and attenuate seismic noise based on their shape differences from the signal. We apply different types of morphological filter on field (and various stages of processed) data to demonstrate their effectiveness for suppression of both coherent and incoherent noise and results in an improvement of signal to noise ratio.
Introduction

Seismic data are always contaminated with different types of noise, including coherent and incoherent noise. These noise types will negatively affect the result of seismic data processing, such as deghosting, demultiple and migration, thus finally degrade the quality of the seismic imaging, inversion and interpretation. It is very important to attenuate the noise and improve the seismic signal to noise ratio. Conventional seismic noise attenuation methods are usually implemented by using the difference of frequency, wavenumber, dip and coherency etc. between signal and noise.

In the past few decades, many approaches from various fields have been introduced to seismic processing to attenuate noise in seismic data. Mathematical morphological filtering (MMF) was first introduced from image processing into seismic data processing for the removal of abnormal amplitudes (Wang 2005). The basic idea of this method is to use a so-called structuring element (SE) (a small section of signal with a specific structure) sliding on the seismic data by performing a set of logical calculus, thus smooth the input signal, and remove the abnormal high and low points. More researchers developed MMF to remove different types of noise since then. Li et al. (2016) proposed a compound morphological top-hat filter to attenuate low-frequency noise in microseismic monitoring. Huang et al. (2018) extended these methods to a planar MMF from the time direction to time-spatial direction to suppress coherent noise.

We here present in this paper a variant of basic and advanced mathematical morphological filters, and their applications for attenuation of random noise and coherent noise in premigration data, and in improvement of the signal-to-noise ratio of postmigration gathers.

Methodology

In image processing, mathematical morphology is used to investigate the interaction between an image and a certain chosen structuring element using the basic operation of erosion and dilation. Mathematical morphology stands somewhat apart from traditional linear image processing, since the basic operations of morphology are nonlinear in nature, and thus make use of a totally different type of algebra than linear algebra. Therefore, different types of signals are easier to be isolated by morphological operations than linear operations, just as sparse-inversion denoise generally achieves better results than normal least-square inversion methods.

In theory, morphological calculation is a set operation. Assume we have two sets A and B, the dilation A by B, denoted by $A \oplus B$, is defined as

$$A \oplus B = \{x | x = a + b \ for \ a \in A, b \in B\}. \quad (1)$$

Generally, A is the object of interest and B is the SE. It is apparent the dilation operation can “grow” or “thicken” the object. On the other hand, the erosion of A by B, denoted by $A \ominus B$, is defined as

$$A \ominus B = \{x | x + b \in A, for \ every \ b \in B\}. \quad (2)$$

Erosion is an operation that can “shrink” or “thin” the object. Morphological opening and closing operations can be derived with the combination of dilation and erosion. The opening operation $A \circ B$ and closing operation $A \cdot B$ can be defined as

$$A \circ B = (A \ominus B) \oplus B, \quad (3)$$

and

$$A \cdot B = (A \oplus B) \ominus B. \quad (4)$$

Furthermore, the combination of the opening and closing operation forms the MMF as

$$MMF_B(A) = \frac{1}{2}((A \circ B) \cdot B + (A \cdot B) \circ B), \quad (5)$$

where $MMF_B$ denotes the MMF with structure element B. Many interesting morphological filters can be formed using residues, i.e., the differences of two or more common operations. For an example, we have noticed the erosion and dilation operations act only at the edges of objects, so we can detect edges by examining the difference between an original image and its erosion and dilation. The morphological gradient operation is defined by

$$Grad_B(A) = A \oplus B - A \ominus B. \quad (6)$$
This gradient is the two sides of the actual edges, which can be decomposed into two “half” gradients—inner gradient $Grad^-$ and outer gradient $Grad^+$:

$$Grad^- (A) = A - (A \oplus B),$$  \hspace{1cm} (7)

and

$$Grad^+ (A) = (A \ominus B) - A.$$  \hspace{1cm} (8)

As morphological gradient equivalent of the mathematical gradient, there is also an equivalent of Laplacian: the morphological Laplacian is defined at

$$Lap_B (A) = Grad^+ (A) - Grad^- (A)$$  \hspace{1cm} (9)

In practical seismic data processing, if we set $f = f(n)$ to represent time series of seismic data, $g = g(n)$ to represent the structure element, the morphological operation dilation and erosion can be defined as

$$(f \oplus g)(n) = \max \left( f(n - n') + g(n') \right) \quad n' \in [-N', N']$$  \hspace{1cm} (10)

and

$$(f \ominus g)(n) = \min \left( f(n + n') - g(n') \right) \quad n' \in [-N', N']$$  \hspace{1cm} (11)

where $n$ and $n'$ represent samples, and $N'$ is half size of the SE. We can define other morphological operations and MMF for seismic data processing by substituting equations 10 and 11 into the equations from 3 to 9. The SE is the only parameter for MMF or any morphological operation, i.e., a specific morphological operation is determined if the SE is given. The most common 1D SE is a semicircle type function which can be defined as

$$g(n') = a \sqrt{1 - \left( \frac{n'}{N'} \right)^2}$$  \hspace{1cm} (13)

where $N'$ is the size, and $a \geq 0$ is the height of the SE. All morphological operations can be easily expanded to 2D case by selecting 2D SE and applying to 2D seismic data.

Next, we will present the application of morphological operations on real seismic data for attenuation of different type of noise.

**Random noise attenuation**

![Figure 1](image-url)  
*Figure 1* Common-offset gather of NW Africa Atlantic Margin data. a) blended source, b) after deblending with enhanced adaptive subtraction method, c) MMF on deblended data, and d) difference between b and c.
Figure 1 shows how MMF removes the residual noise after simultaneous source deblending. Figure 1a is a common offset gather of NW Africa Atlantic Margin survey which is continuously recorded simultaneous-source data. After normal enhanced adaptive subtraction (EAS) (Liu et al. 2015) deblending processing (Figure 1b), there are still obvious residuals of the secondary source energy overlaid on the top of primary source. Since the geology is very complex, and the arrival time of the secondary source happens to be coincident with the arrival time of the multiples of the primary source, further deblending or traditional denoising may hurt the continuity of the multiples of the primary source, and hence affect the demultiple results. MMF with properly selected SE can effectively attenuate the residual blended noise (Figure 1c) without visible damage to the primary events (Figure 1d).

Coherent noise attenuation

Figure 2 shows the 2D stack data where the Laplacian mode of MMF with a flat-top SE is being used to model residual multiples that were left over by a conventional model-based demultiple approach. Figure 2a is a stack section of a 2D line which has already undergone model based 2D SRME and 2D shallow water multiple elimination (SWME). In dipping geological areas and regions of complex geology, 2D model-based multiple approaches fail to model the acquired multiples due to the 3D nature of the data and multiples, leaving coherent residual multiples behind as seen in Figure 2a. Figure 2b shows the raw MMF Laplacian multiple model. We can see the MMF in Laplacian mode has modelled most of the coherent multiples and does not contain much underlying primary data. A muted version of this MMF Laplacian model is then adaptively subtracted from the input to produce the section shown in Figure 2c. Figure 2d is the difference of the subtraction result.

![Figure 2 Stack of NW Africa Atlantic Margin 2D Data. a) stack with model-based demultiple already applied, showing residual dipping multiple energy, b) raw MMF Laplacian model, c) stack after adaptive subtraction of MMF model d) subtraction difference between a and c.](image)

Postmigration denoising and signal enhancement

Figure 3 shows how we apply MMF for signal enhancement and noise attenuation on raw migrated 2D PSTM CDP gathers from a Red Sea data set which presents processing challenges from both a structural and from a residual noise perspective. By performing the MMF filtering process in time-slice transform (Figure 3c and 3f), we aim to preserve events that are locally or continuously flat with respect to offset.
As such, multiples, apex-shifted multiples, linear noise, spurious noise and migration stretch are all suppressed. We observe a signal enhancement effect on the stack where coherent events become more coherent (Figure 3a and Figure 3d), and flat events in CDP gathers are enhanced (Figure 3b and 3e). We expect that subsequent AVO analysis and angle-limited stacks will benefit significantly.

![Figure 3](image)

**Figure 3**: 2D Red Sea data prestack Kirchhoff migration before MMF filtering. a) Stack image, b) CDP gathers, c) time-slice and after MMF filtering. d) Stack image, e) CDP gathers and f) time-slice in time-slice domain.

**Conclusions**

MMF is a very powerful nonlinear image processing tool and was introduced into seismic data processing for denoising to improvement of signal to noise ratio. Unlike conventional seismic denoising tools that separate noise and signals by their different frequency, wavenumber, amplitude and coherency etc., MMF takes the advantage of the shape differences between noises and signal. Our field data examples have demonstrated that MMF can effectively attenuate different types of seismic noise, including coherent and incoherent noise in both premigration and postmigration data.

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**References**


