# A hybrid crossgather curvelet domain multiple elimination method and its application

Y. Zhai<sup>\*</sup>, Z. Liu, J. Sheng, B. Wang, and J. Heim, TGS Geophysical Company

## Summary

Recent research shows that the curvelet domain multiple elimination is more effective than the conventional timespace domain approach. There are two categories of curvelet domain methods: direct matching and inversion-based. We propose a hybrid method that combines the advantages of the two categories. First, multiple-contaminated data and the predicted model are transformed to the curvelet domain. We then break the multiple adaptation into two stages: a global adaptation that corrects significant errors in the multiple model among the curvelets with similar dips, temporal and spatial coordinates across neighboring gathers, followed by a step that improves the coefficients by solving an optimization problem with sparsity constraints. The crossline data coherency is considered and a robust primarymultiple separation is achieved without performing 3D curvelet transforms. Finally, both multiple-free data and adaptive multiple are transformed back to the time-space domain. More than one iteration might be run on the residuals until only the incoherent noise is left. We tested the proposed method on a 2D line from the Gigante survey in the southern Gulf of Mexico. The result shows improved multiple attenuation in comparison with an inversion-based method in curvelet domain.

### Introduction

For decades, multiple attenuation has been an important step in seismic processing, though recent efforts try to include the full waveform in the migration engine (Berkhout, 2012, Verschuur et al., 2016). Multiple attenuation usually consists of two major steps: multiple prediction and adaptive subtraction. Despite great success, none of the multiple prediction methods provide a "perfect" multiple model. In fact, predicted multiples can be severely mismatched. Therefore, the adaptive subtraction is crucial to match amplitudes, phase, and kinematics to the input data.

Least-squares (LS) matching in the time-space (TX) domain has been the most widely used adaptive subtraction tool (Berkhout and Verschuur, 1997). It adapts the multiple model to the input (primaries and multiples) to minimize the residual energy in L2 norm. This method is often applied over several iterations (in various sort domains) and is robust and efficient in regions where primary and multiple events are well separated, but can cause significant damage to primary events in other regions. L1 norm matching (Guitton and Verschuur, 2004) can reduce the primary damage when the multiples are relatively weak, but the improvement is not significant, while the computational cost increases significantly (Abma et al., 2005).

Processing the data in the curvelet domain has been an attractive alternative to the TX domain ever since it was introduced by Candes and Donoho (2000), as seismic events are naturally decomposed into a linearly weighted sum of curvelets with signature scales, dips, and time and space coordinates (Figure 1). Adaptive subtraction methods in curvelet domain can be implemented through either direct matching or solving an optimization problem. Neelamani et al. (2008a) proposed to match the complex coefficients of the multiples to ones of the data elementwise. Saab et al. (2007) proposed a primary-multiple separation scheme by solving an optimization problem with extra sparsity constraints on both primaries and multiples.

These methods generally provide an uplift to LS adaptive subtraction in TX domain, but have some disadvantages as well. Direct matching is relatively easy to implement, but it is difficult to set the bounds of the adaptation, leading to "over-adapting", thus primary damage is common (Nguyen and Dyer, 2016). Inversion-based methods allow for the



Figure 1: Synthetic data with water bottom multiples (a), multiple model (b), and multiple's curvelet amplitudes (c). S1-S6 represents coefficients at six scales in the curvelet domain.

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trade-off between overall data fitting and sparseness of the models. However, it requires a relatively accurate initial multiple model before it can find a reasonable solution (Wu and Hung, 2015) in several iterations, which add to the already significant compute-time. Our proposed method combines advantages of the two approaches outlined above. First, we obtain an initial solution by applying a global matching operator to the predicted multiple in the curvelet domain. Then, we obtain the multiple-free data and the adaptive multiple by solving the optimization. The crossline data coherence is considered by grouping curvelets with similar dip, temporal and spatial locations between neighboring gathers.

#### Method

We divide our method into four steps. First, we apply a fast discrete curvelet transform (FDCT) with wrapping (Candes et al., 2006) to the input data and predicted multiple model. Second, we apply a global matching operator to the curvelet coefficients of the multiple model. Curvelets are orthogonal, therefore it is possible to apply this operator to individual scales independently to make the process more efficient. Nguyen and Dyer (2016) proposed a global matching scheme in a subband (wedge) that contains curvelets with common scales and dips, before applying a sample-bysample matching. For real data, the errors of predicted multiples might vary significantly within a wedge. For example, amplitude and phase of high order multiples (usually in the deep section) in the SRME model can be very different from true multiples, whereas low-order multiples (usually in the shallow section) are much closer. On the other hand, noise with unbalanced amplitude contaminates the multiple model throughout the whole gather (Kustowski et al., 2013). Therefore, the wedge-based global matching scheme might have difficulties handling real data.

To overcome this issue, we group the curvelet coefficients of the multiple model by similar dips, temporal and spatial locations, rather than by wedges (Figure 2a). We first find the most significant curvelets within a selective group of the multiple model, as they are more likely multiples than erroneously predicted primaries or noise. We then calculate the mean amplitude and phase of these coefficients, as well as the mean values of the corresponding coefficients of the input data. Next, we apply the amplitude ratio and phase differences of the mean values to the multiple coefficients. We repeat this procedure until all dips, temporal and spatial locations are covered.

This step will yield an adapted multiple model, which should be close to the "correct" model, and can serve as the initial solution of the inversion. In practice, we adopt the method proposed by Saab et al. (2007).

$$\begin{split} f(C_{p}, C_{m}) &= \left\| C_{p} \right\|_{1, w_{p}} + \left\| C_{m} \right\|_{1, w_{m}} + \left\| AC_{m} - AC_{m*} \right\|_{2}^{2} + \\ \eta \left\| A(C_{p} + C_{m}) - d \right\|_{2}^{2} \end{split} \tag{1}$$

where  $C_*$  are curvelet coefficients of the primary (p) and multiple (m), respectively;  $||C_*||_{1,W_*}$  are weighted L1 norms of the coefficients; *A* denotes the inverse curvelet transform (CT) with wrapping; *d* is the input data with multiple contamination. If controls the trade-off between the model sparsity, multiple fitting, and overall data fitting. Note that  $C_{m*}$  represents the adapted multiple coefficients after the global matching. This optimization problem can be solved by applying a soft-thresholding operator (Daubechies, et al., 2003). Once  $C_p$  and  $C_m$  are satisfactory, we apply the inverse CT to obtain the multiple-free data and adapted multiple model.

The CT is computer resource demanding, thus most successful curvelet domain applications are implemented in 2D, though multidimensional CT has been introduced for more than a decade (Ying et al., 2005). Neelamani et al. (2008b) propose a "curvelets + wavelets" method by applying a 1D wavelet transform in the crossline direction and a 2D CT in the inline direction. To avoid the 3D CT's, we group the curvelet coefficients not only across "wedges" but also between the neighboring gathers in the global matching step (Figure 2b). A better estimate of the mean values can be achieved by taking into account the data coherency in the crossline direction. After we obtain the globally-matched model, we use the average thresholds when solving the optimization problem. These will result in robust primary-multiple separation without performing expensive 3D CT's.

The 4<sup>th</sup> term of equation 1 includes all residual energy, i.e. incoherence noise. However, due to the compromise between model sparsity and overall data fitting, some weak events (primaries or multiples) might still exist in the residual. Several iterations of residual adaptive matching that separate the residual primaries and multiples are necessary to guarantee minimum energy loss and primary damage, through the whole process (Figure 3).

### Data Example

We applied the proposed method to a 2D line of the Gigante project. Gigante is an 186,000 km 2D survey acquired in the Gulf of Mexico (Figure 4). We selected line 238 because of its complex multiple content that leads to difficult multiple elimination. The predicted SRME model has strong amplitude and phase distortions, as well as kinematic errors. In fact, 2D multiple models usually contain more kinematic errors than 3D models due to out of plane contributions, leading to more challenges for the adaptive subtraction. For comparison, we also show the multiple elimination result of an inversion-based curvelet domain method (referred as CINV) that takes a TX domain matched model as the initial input.



Figure 2: Curvelet coefficients of the multiple model at scale 5. Each block in a) represents a "wedge" that includes coefficients with common dips. Boxes with different colors represent selective groups with similar dips, temporal and spatial locations. b) schematically shows how coefficients from neighboring gathers are grouped.

Figure 5a and 5b are the input data and SRME model, respectively. The model contains significant amplitude and phase distortions that are common for SRME (Dragoset et al., 2010). Some kinematic errors also exist. Figure 5c shows the final primary section after 15 iterations of inversion by Saab's approach. The multiples are significantly attenuated, but residual multiples are still visible across the section (black arrows). The result of the proposed hybrid crossgather method (referred as HCG) is shown in Figure 5d. It produces a better multiple elimination result, and as expected, does not seem to attenuate primary events. The crosswedge and crossgather global matching provides a better initial model with minimum primary leakage for the inversion, and the extra iterations on the residuals ensure minimal primary damage.

#### Discussion

The proposed method can easily be extended to allow the input of several multiple models. For example, in shallow water environments, a water bottom-related multiple model and SRME model are used in combination to attenuate different types of multiples (Zhai et al., 2015). Neelamani et al. (2008) propose a method to adapt all multiple models simultaneously. Following this approach, in the global matching step the adapted curvelet coefficients of multiple model 1 are obtained, and multiple model 2 is adapted by matching the curvelet coefficients to the residual. All

multiple models can be adapted sequentially, and a combined, globally matched model, is output by linear summation of all adapted models (Figure 3).



Figure 3: Flow chart of the proposed method.



Figure 4: Gigante map. Blue lines are 2D lines that covers > a 600,000 km<sup>2</sup> area. Line 238 is marked by black.

## Conclusions

We propose a hybrid crossgather curvelet domain method that efficiently adapts multiple models to input data containing primaries and multiples. We first apply a global matching operator to the predicted multiple model in curvelet domain by adapting the coefficients with similar dips, temporal and spatial coordinates across neighboring gathers. The crossline data coherency is taken into account this way without calculating 3D curvelet transforms. Then, we apply an inversion-based approach to further improve the result. Several iterations might be necessary to ensure minimum primary damage. We have shown that the proposed method provides a superior adaptive subtraction of multiple models on real data from the Gulf of Mexico in comparison to the conventional adaptive subtraction method in TX domain followed by an inversion-based method in curvelet domain that relies on the TX domain result. The

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method can easily be extended to handle several input models created by different modeling algorithms.

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Figure 5: Channel gathers of line 238. a) input data; b) SRME model; c) CINV result; d) HCG result with 3 iterations

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