A Story of Ratios®

Eureka Math helps students truly understand mathematics and connect it to the real world, preparing them to solve problems they have not encountered. Great Minds teachers and mathematicians believe that it is not enough for students to know the process for solving a problem; they need to understand why that process works.

Eureka Math presents mathematics as a story, one that develops from grades PK through 12. In *A Story of Ratios*, our middle school curriculum, this sequencing has been joined with methods of instruction that have been proven to work, in this nation and abroad.

Great Minds is here to make sure you succeed with an ever-growing library of resources, including free tip sheets, resource sheets, and full grade-level modules at eureka-math.org.

Sequence of Grade 8 Modules

Module 1: Integer Exponents and Scientific N tation
Module 2: The Concept of Congruence
Module 3: Similarity
Module 4: Linear Equations
Module 5: Examples of Functions from Geometry
Module 6: Linear Functions
Module 7: Introduction to Irrational Numbers Using Geometry

On the cover
An Elephant Fight (recto page; *Vasudeva Rescues Baby Krishna* on verso), India, Rajasthan, Kota, ca. 1800–1825.
Ink and opaque watercolor on paper, 14 3/8 × 22 3/16 in. (36.51 × 56.35 cm). Gift of Paul F. Walter (M.77.154.19a-b).
Location: Los Angeles County Museum of Art, Los Angeles, CA, USA

What does this painting have to do with math?
In an effo t to take advantage of every opportunity to build students' cultural literacy, Great Minds features an important work of art or architecture on the cover of each book we publish. We select images that we know students and teachers will love to look at again and again. These works also relate, in visual terms, to ideas taken up in the book. For hundreds of years, the drawing and painting of elephants in combat was a specialty in the north Indian kingdom of Kota. The elegantly simple style with which artists of this region captured the Herculean battle between two massive creatures reminds us of a major theme of *A Story of Ratios*—the relationship between quantities.

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EUREKA MATH®

GR8 MOD 1

EUREKA MATH

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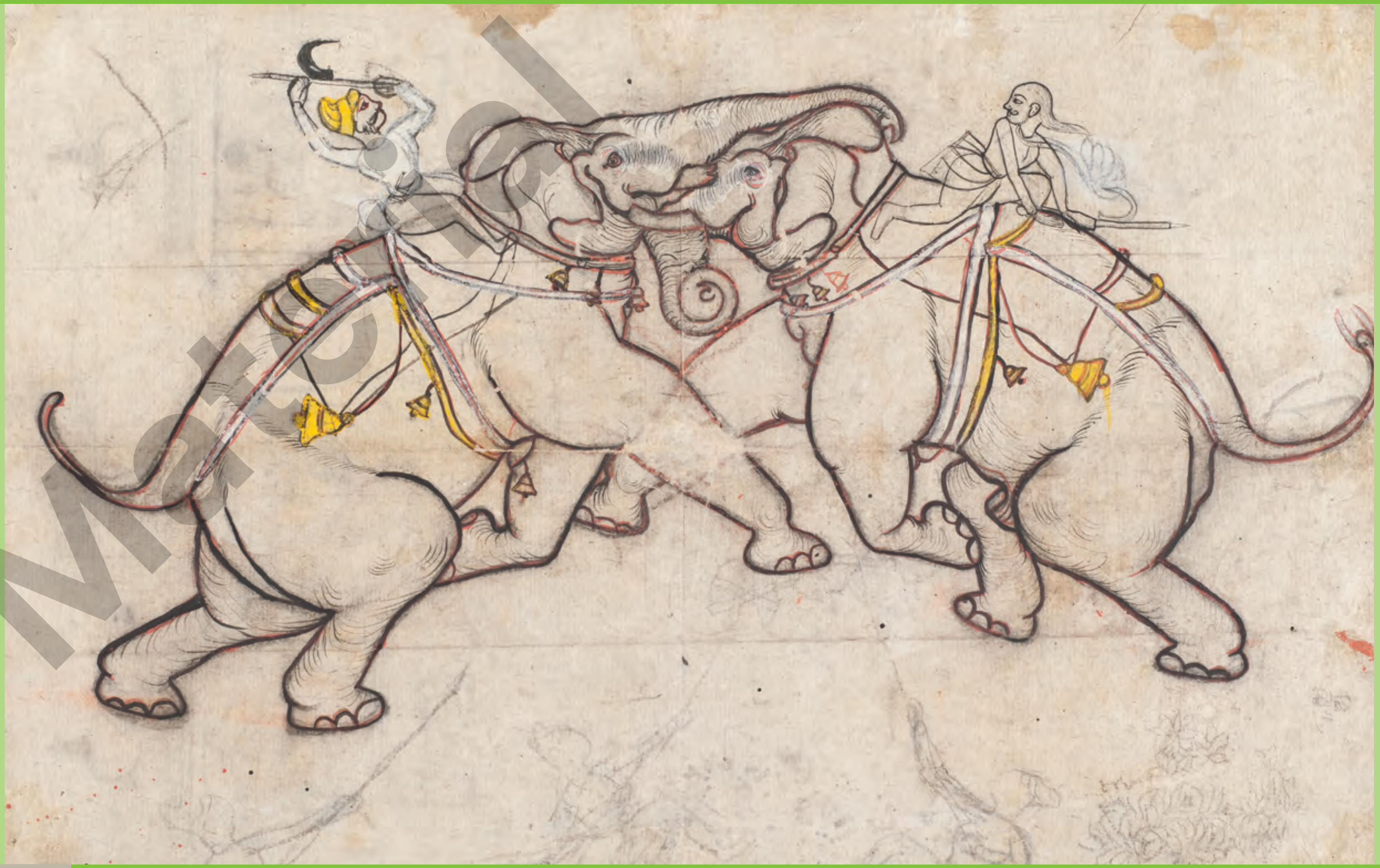

Eureka Math®
Grade 8
Module 1

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Student Name: ____________________

Learn, Practice, Succeed

Eureka Math™

Grade 8
Module 1

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Students, families, and educators:

Thank you for being part of the *Eureka Math*™ community, where we celebrate the joy, wonder, and thrill of mathematics.

In *Eureka Math* classrooms, learning is activated through rich experiences and dialogue. That new knowledge is best retained when it is reinforced with intentional practice. The *Learn, Practice, Succeed* book puts in students' hands the problem sets and fluency exercises they need to express and consolidate their classroom learning and master grade-level mathematics. Once students learn and practice, they know they can succeed.

What is in the Learn, Practice, Succeed *book?*

Fluency Practice: Our printed fluency activities utilize the format we call a Sprint. Instead of rote recall, Sprints use patterns across a sequence of problems to engage students in reasoning and to reinforce number sense while building speed and accuracy. Sprints are inherently differentiated, with problems building from simple to complex. The tempo of the Sprint provides a low-stakes adrenaline boost that increases memory and automaticity.

Classwork: A carefully sequenced set of examples, exercises, and reflection questions support students' in-class experiences and dialogue. Having classwork preprinted makes efficient use of class time and provides a written record that students can refer to later.

Exit Tickets: Students show teachers what they know through their work on the daily Exit Ticket. This check for understanding provides teachers with valuable real-time evidence of the efficacy of that day's instruction, giving critical insight into where to focus next.

Homework Helpers and Problem Sets: The daily Problem Set gives students additional and varied practice and can be used as differentiated practice or homework. A set of worked examples, Homework Helpers, support students' work on the Problem Set by illustrating the modeling and reasoning the curriculum uses to build understanding of the concepts the lesson addresses.

Homework Helpers and Problem Sets from prior grades or modules can be leveraged to build foundational skills. When coupled with *Affirm*™, *Eureka Math*'s digital assessment system, these Problem Sets enable educators to give targeted practice and to assess student progress. Alignment with the mathematical models and language used across *Eureka Math* ensures that students notice the connections and relevance to their daily instruction, whether they are working on foundational skills or getting extra practice on the current topic.

Where can I learn more about Eureka Math *resources?*

The Great Minds® team is committed to supporting students, families, and educators with an ever-growing library of resources, available at eureka-math.org. The website also offers inspiring stories of success in the *Eureka Math* community. Share your insights and accomplishments with fellow users by becoming a *Eureka Math* Champion.

Best wishes for a year filled with "aha" moments!

Jill Diniz

Jill Diniz
Chief Academic Officer, Mathematics
Great Minds

Contents

Module 1: Integer Exponents and Scientific Notation

5^6 means $5 \times 5 \times 5 \times 5 \times 5 \times 5$, and $\left(\frac{9}{7}\right)^4$ means $\frac{9}{7} \times \frac{9}{7} \times \frac{9}{7} \times \frac{9}{7}$.

You have seen this kind of notation before; it is called exponential notation. In general, for any number x and any positive integer n,

$$x^n = \underbrace{(x \cdot x \cdots x)}_{n \text{ times}}.$$

The number x^n is called x raised to the n^{th} power, where n is the exponent of x in x^n and x is the base of x^n.

Exercise 1

$\underbrace{4 \times \cdots \times 4}_{7 \text{ times}} =$

Exercise 2

$\underbrace{3.6 \times \cdots \times 3.6}_{\_\_\_\_\_ \text{ times}} = 3.6^{47}$

Exercise 3

$\underbrace{(-11.63) \times \cdots \times (-11.63)}_{34 \text{ times}} =$

Exercise 4

$\underbrace{12 \times \cdots \times 12}_{\_\_\_\_\_ \text{times}} = 12^{15}$

Exercise 5

$\underbrace{(-5) \times \cdots \times (-5)}_{10 \text{ times}} =$

Exercise 6

$\underbrace{\frac{7}{2} \times \cdots \times \frac{7}{2}}_{21 \text{ times}} =$

Exercise 7

$\underbrace{(-13) \times \cdots \times (-13)}_{6 \text{ times}} =$

Exercise 8

$\underbrace{\left(-\frac{1}{14}\right) \times \cdots \times \left(-\frac{1}{14}\right)}_{10 \text{ times}} =$

Exercise 9

$\underbrace{x \cdot x \cdots x}_{185 \text{ times}} =$

Exercise 10

$\underbrace{x \cdot x \cdots x}_{\_\_\_\_\_ \text{times}} = x^n$

Exercise 11

Will these products be positive or negative? How do you know?

$\underbrace{(-1) \times (-1) \times \cdots \times (-1)}_{\text{12 times}} = (-1)^{12}$

$\underbrace{(-1) \times (-1) \times \cdots \times (-1)}_{\text{13 times}} = (-1)^{13}$

Exercise 12

Is it necessary to do all of the calculations to determine the sign of the product? Why or why not?

$\underbrace{(-5) \times (-5) \times \cdots \times (-5)}_{\text{95 times}} = (-5)^{95}$

$\underbrace{(-1.8) \times (-1.8) \times \cdots \times (-1.8)}_{\text{122 times}} = (-1.8)^{122}$

Exercise 13

Fill in the blanks indicating whether the number is positive or negative.

If n is a positive even number, then $(-55)^n$ is ___________________________.

If n is a positive odd number, then $(-72.4)^n$ is ___________________________.

Exercise 14

Josie says that $\underbrace{(-15) \times \cdots \times (-15)}_{\text{6 times}} = -15^6$. Is she correct? How do you know?

Name ____ Date____________________

1.

a. Express the following in exponential notation:

$$\underbrace{(-13) \times \cdots \times (-13)}_{35 \text{ times}}.$$

b. Will the product be positive or negative? Explain.

$$\underbrace{(-13) \times \cdots \times (-13)}_{35 \text{ times}}.$$

2. Fill in the blank:

$$\underbrace{\frac{2}{3} \times \cdots \times \frac{2}{3}}_{\_\_\_\_\text{times}} = \left(\frac{2}{3}\right)^4$$

3. Arnie wrote:

$$\underbrace{(-3.1) \times \cdots \times (-3.1)}_{4 \text{ times}} = -3.1^4$$

Is Arnie correct in his notation? Why or why not?

Use what you know about exponential notation to complete the expressions below.

1. $\underbrace{(-2) \times \cdots \times (-2)}_{35 \text{ times}} = (-\mathbf{2})^{\mathbf{35}}$

2. $\underbrace{\left(\frac{9}{2}\right) \times \cdots \times \left(\frac{9}{2}\right)}_{12 \text{ times}} = \left(\frac{\mathbf{9}}{\mathbf{2}}\right)^{\mathbf{12}}$

When the base (the number being repeatedly multiplied) is negative or fractional, I need to use parentheses. If I don't, the number being multiplied will not be clear. Some may think that the 2 or only the numerator of the fraction gets multiplied.

3. $\underbrace{8 \times \cdots \times 8}_{\_\_\_\_ \text{ times}} = 8^{56}$

The exponent states how many times the 8 is multiplied. It is multiplied 56 times, so that is what is written in the blank.

4. Rewrite each number in exponential notation using 3 as the base.
   a. $9 = \mathbf{3 \times 3 = 3^2}$
   b. $27 = \mathbf{3 \times 3 \times 3 = 3^3}$
   c. $81 = \mathbf{3 \times 3 \times 3 \times 3 = 3^4}$
   d. $243 = \mathbf{3 \times 3 \times 3 \times 3 \times 3 = 3^5}$

All I need to do is figure out how many times to multiply 3 in order to get the number I'm looking for in parts (a)–(d).

5. Write an expression with (-2) as its base that will produce a negative product.

One possible solution is shown below.

$(-\mathbf{2})^{\mathbf{3}} = (-\mathbf{2}) \times (-\mathbf{2}) \times (-\mathbf{2}) = -\mathbf{8}$

To produce a negative product, I need to make sure the negative number is multiplied an odd number of times. Since the product of two negative numbers results in a positive product, multiplying one more time will result in a negative product.

1. Use what you know about exponential notation to complete the expressions below.

$\underbrace{(-5) \times \cdots \times (-5)}_{17 \text{ times}} =$

$\underbrace{3.7 \times \cdots \times 3.7}_{\_\_\_\_ \text{ times}} = 3.7^{19}$

$\underbrace{7 \times \cdots \times 7}_{\_\_\_\_ \text{ times}} = 7^{45}$

$\underbrace{6 \times \cdots \times 6}_{4 \text{ times}} =$

$\underbrace{4.3 \times \cdots \times 4.3}_{13 \text{ times}} =$

$\underbrace{(-1.1) \times \cdots \times (-1.1)}_{9 \text{ times}} =$

$\underbrace{\left(\frac{2}{3}\right) \times \cdots \times \left(\frac{2}{3}\right)}_{19 \text{ times}} =$

$\underbrace{\left(-\frac{11}{5}\right) \times \cdots \times \left(-\frac{11}{5}\right)}_{\_\_\_\_ \text{ times}} = \left(-\frac{11}{5}\right)^{x}$

$\underbrace{(-12) \times \cdots \times (-12)}_{\_\_\_\_ \text{ times}} = (-12)^{15}$

$\underbrace{a \times \cdots \times a}_{m \text{ times}} =$

2. Write an expression with (-1) as its base that will produce a positive product, and explain why your answer is valid.

3. Write an expression with (-1) as its base that will produce a negative product, and explain why your answer is valid.

4. Rewrite each number in exponential notation using 2 as the base.

$8 =$ $\quad$ $16 =$ $\quad$ $32 =$

$64 =$ $\quad$ $128 =$ $\quad$ $256 =$

5. Tim wrote 16 as $(-2)^4$. Is he correct? Explain.

6. Could -2 be used as a base to rewrite 32? 64? Why or why not?

In general, if x is any number and m, n are positive integers, then

$$x^m \cdot x^n = x^{m+n}$$

because

$$x^m \times x^n = \underbrace{(x \cdots x)}_{m \text{ times}} \times \underbrace{(x \cdots x)}_{n \text{ times}} = \underbrace{(x \cdots x)}_{m+n \text{ times}} = x^{m+n}.$$

Exercise 1

$14^{23} \times 14^8 =$

Exercise 2

$(-72)^{10} \times (-72)^{13} =$

Exercise 3

$5^{94} \times 5^{78} =$

Exercise 4

$(-3)^9 \times (-3)^5 =$

Exercise 5

Let a be a number.

$a^{23} \cdot a^8 =$

Exercise 6

Let f be a number.

$f^{10} \cdot f^{13} =$

Exercise 7

Let b be a number.

$b^{94} \cdot b^{78} =$

Exercise 8

Let x be a positive integer. If $(-3)^9 \times (-3)^x = (-3)^{14}$, what is x?

What would happen if there were more terms with the same base? Write an equivalent expression for each problem.

Exercise 9

$9^4 \times 9^6 \times 9^{13} =$

Exercise 10

$2^3 \times 2^5 \times 2^7 \times 2^9 =$

Can the following expressions be written in simpler form? If so, write an equivalent expression. If not, explain why not.

Exercise 11

$6^5 \times 4^9 \times 4^3 \times 6^{14} =$

Exercise 12

$(-4)^2 \cdot 17^5 \cdot (-4)^3 \cdot 17^7 =$

Exercise 13

$15^2 \cdot 7^2 \cdot 15 \cdot 7^4 =$

Exercise 14

$2^4 \times 8^2 = 2^4 \times 2^6 =$

Exercise 15

$3^7 \times 9 = 3^7 \times 3^2 =$

Exercise 16

$5^4 \times 2^{11} =$

Exercise 17

Let x be a number. Rewrite the expression in a simpler form.

$(2x^3)(17x^7) =$

Exercise 18

Let a and b be numbers. Use the distributive law to rewrite the expression in a simpler form.

$a(a + b) =$

Exercise 19

Let a and b be numbers. Use the distributive law to rewrite the expression in a simpler form.

$b(a+b) =$

Exercise 20

Let a and b be numbers. Use the distributive law to rewrite the expression in a simpler form.

$(a+b)(a+b) =$

In general, if x is nonzero and m, n are positive integers, then

$$\frac{x^m}{x^n} = x^{m-n}.$$

Exercise 21

$\frac{7^9}{7^6} =$

Exercise 22

$\frac{(-5)^{16}}{(-5)^7} =$

Exercise 23

$\frac{\left(\frac{8}{5}\right)^9}{\left(\frac{8}{5}\right)^2} =$

Exercise 24

$\frac{13^5}{13^4} =$

Exercise 25

Let a, b be nonzero numbers. What is the following number?

$$\frac{\left(\frac{a}{b}\right)^9}{\left(\frac{a}{b}\right)^2} =$$

Exercise 26

Let x be a nonzero number. What is the following number?

$$\frac{x^5}{x^4} =$$

Can the following expressions be written in simpler forms? If yes, write an equivalent expression for each problem. If not, explain why not.

Exercise 27

$$\frac{2^7}{4^2} = \frac{2^7}{2^4} =$$

Exercise 29

$$\frac{3^5 \cdot 2^8}{3^2 \cdot 2^3} =$$

Exercise 28

$$\frac{3^{23}}{27} = \frac{3^{23}}{3^3} =$$

Exercise 30

$$\frac{(-2)^7 \cdot 95^5}{(-2)^5 \cdot 95^4} =$$

Exercise 31

Let x be a number. Write each expression in a simpler form.

a. $\frac{5}{x^3}(3x^8) =$

b. $\frac{5}{x^3}(-4x^6) =$

c. $\frac{5}{x^3}(11x^4) =$

Exercise 32

Anne used an online calculator to multiply $2\ 000\ 000\ 000\ \times 2\ 000\ 000\ 000\ 000$. The answer showed up on the calculator as $4\mathrm{e} + 21$, as shown below. Is the answer on the calculator correct? How do you know?

Name ____ Date____________________

Write each expression using the fewest number of bases possible.

1. Let a and b be positive integers. $23^a \times 23^b =$

2. $5^3 \times 25 =$

3. Let x and y be positive integers and $x > y$. $\frac{11^x}{11^y} =$

4. $\frac{2^{13}}{2^3} =$

Let x, a, and b be numbers and $b \neq 0$. Write each expression using the fewest number of bases possible.

1. $(-7)^3 \cdot (-7)^4 =$

   $\boldsymbol{(-7)^{3+4}}$

I have to be sure that the base of each term is the same if I intend to use the identity $x^m \cdot x^n = x^{m+n}$ for Problems 1–4.

2. $x^5 \cdot x^6 =$

   $\boldsymbol{x^{5+6}}$

3. $\left(\frac{2}{3}\right)^7 \cdot \left(\frac{2}{3}\right)^5 =$

   $\boldsymbol{\left(\frac{2}{3}\right)^{7+5}}$

4. $2^4 \cdot 8^2 =$

   $\boldsymbol{2^4 \cdot (2^3)^2 =}$

   $\boldsymbol{2^4 \cdot 2^3 \cdot 2^3 =}$

   $\boldsymbol{2^{4+3+3}}$

For this problem, I know that $8 = 2^3$, so I can transform the 8 to have a base of 2.

5. $ab^3 \cdot a^4b^5 =$

   $\boldsymbol{a \cdot a^4 \cdot b^3 \cdot b^5 =}$

   $\boldsymbol{a^{1+4} \cdot b^{3+5}}$

In this problem, I see two bases, a and b. I can use the commutative property to reorder the a's so that they are together and reorder the b's so they are together.

6. $\frac{(-3)^5}{(-3)^2} =$

   $\boldsymbol{(-3)^{5-2}}$

When the bases are the same for division problems, I can use the identity $\frac{x^m}{x^n} = x^{m-n}$.

7. $\frac{a^2b^5}{b^2} =$

   $\boldsymbol{a^2 \cdot \frac{b^5}{b^2} =}$

   $\boldsymbol{a^2b^{5-2}}$

8. $\frac{27}{3^2} =$

   $\boldsymbol{\frac{3^3}{3^2} =}$

   $\boldsymbol{3^{3-2}}$

The number 27 is the same as $3 \times 3 \times 3$, which is equal to 3^3.

1. A certain ball is dropped from a height of x feet. It always bounces up to $\frac{2}{3}x$ feet. Suppose the ball is dropped from 10 feet and is stopped exactly when it touches the ground after the 30^{th} bounce. What is the total distance traveled by the ball? Express your answer in exponential notation.

Bounce	Computation of Distance Traveled in Previous Bounce	Total Distance Traveled (in feet)
1		
2		
3		
4		
30		
n		

2. If the same ball is dropped from 10 feet and is caught exactly at the highest point after the 25^{th} bounce, what is the total distance traveled by the ball? Use what you learned from the last problem.

3. Let a and b be numbers and $b \neq 0$, and let m and n be positive integers. Write each expression using the fewest number of bases possible:

$(-19)^5 \cdot (-19)^{11} =$	$2.7^5 \times 2.7^3 =$
$\frac{7^{10}}{7^3} =$	$\left(\frac{1}{5}\right)^2 \cdot \left(\frac{1}{5}\right)^{15} =$
$\left(-\frac{9}{7}\right)^m \cdot \left(-\frac{9}{7}\right)^n =$	$\frac{ab^3}{b^2} =$

4. Let the dimensions of a rectangle be $(4 \times (871209)^5 + 3 \times 49762105)$ ft. by $(7 \times (871209)^3 - (49762105)^4)$ ft. Determine the area of the rectangle. (Hint: You do not need to expand all the powers.)

5. A rectangular area of land is being sold off in smaller pieces. The total area of the land is 2^{15} square miles. The pieces being sold are 8^3 square miles in size. How many smaller pieces of land can be sold at the stated size? Compute the actual number of pieces.

For any number x and any positive integers m and n,

$$(x^m)^n = x^{nm}$$

because

$$\begin{aligned}(x^m)^n &= (\underbrace{x \cdot x \cdots x}_{m \text{ times}})^n \\ &= \underbrace{(\underbrace{x \cdot x \cdots x}_{m \text{ times}}) \times \cdots \times (\underbrace{x \cdot x \cdots x}_{m \text{ times}})}_{n \text{ times}} \\ &= x^{nm}.\end{aligned}$$

Exercise 1

$(15^3)^9 =$

Exercise 2

$\left((-2)^5\right)^8 =$

Exercise 3

$(3.4^{17})^4 =$

Exercise 4

Let s be a number.

$(s^{17})^4 =$

Exercise 5

Sarah wrote $(3^5)^7 = 3^{12}$. Correct her mistake. Write an exponential equation using a base of 3 and exponents of 5, 7, and 12 that would make her answer correct.

Exercise 6

A number y satisfies $y^{24} - 256 = 0$. What equation does the number $x = y^4$ satisfy?

For any numbers x and y, and positive integer n,

$$(xy)^n = x^n y^n$$

because

$$\begin{aligned}(xy)^n &= \underbrace{(xy)\cdots(xy)}_{n \text{ times}}\\ &= \underbrace{(x \cdot x \cdots x)}_{n \text{ times}} \cdot \underbrace{(y \cdot y \cdots y)}_{n \text{ times}}\\ &= x^n y^n.\end{aligned}$$

Exercise 7

$(11 \times 4)^9 =$

Exercise 8

$(3^2 \times 7^4)^5 =$

Exercise 9

Let a, b, and c be numbers.

$(3^2 a^4)^5 =$

Exercise 10

Let x be a number.

$(5x)^7 =$

Exercise 11

Let x and y be numbers.

$(5xy^2)^7 =$

Exercise 12

Let a, b, and c be numbers.

$(a^2bc^3)^4 =$

Exercise 13

Let x and y be numbers, $y \neq 0$, and let n be a positive integer. How is $\left(\frac{x}{y}\right)^n$ related to x^n and y^n?

Name ____ Date____________________

Write each expression as a base raised to a power or as the product of bases raised to powers that is equivalent to the given expression.

1. $(9^3)^6 =$

2. $(113^2 \times 37 \times 51^4)^3 =$

3. Let x, y, z be numbers. $(x^2yz^4)^3 =$

4. Let x, y, z be numbers and let m, n, p, q be positive integers. $(x^m y^n z^p)^q =$

5. $\frac{4^8}{5^8} =$

Lesson Notes

Students will be able to rewrite expressions involving powers to powers and products to powers. The following two identities will be used:

For any number x and any positive integers m and n, $(x^m)^n = x^{mn}$.

For any numbers x and y and positive integer n, $(xy)^n = x^n y^n$.

In the lesson today, we learned to use the identity to simplify these expressions. If the directions say "show in detail", or "prove," I know I need to use the identities and properties I knew before this lesson to show that the identity I learned today actually holds true.

Show (prove) in detail why $(3 \cdot x \cdot y)^5 = 3^5 \cdot x^5 \cdot y^5$.

$\boldsymbol{(3 \cdot x \cdot y)^5 = (3 \cdot x \cdot y) \cdot (3 \cdot x \cdot y) \cdot (3 \cdot x \cdot y) \cdot (3 \cdot x \cdot y) \cdot (3 \cdot x \cdot y)}$ ***By the definition of exponential notation***

$\boldsymbol{= (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \cdot (x \cdot x \cdot x \cdot x \cdot x) \cdot (y \cdot y \cdot y \cdot y \cdot y)}$ ***By the commutative and associative properties***

$\boldsymbol{= 3^5 \cdot x^5 \cdot y^5}$ ***By the definition of exponential notation or by the first law of exponents***

$\boldsymbol{= 3^5 x^5 y^5}$

If I am going to use the first law of exponents to explain this part of my proof, I might want to show another line in my work that looks like this: $3^{1+1+1+1+1} \cdot x^{1+1+1+1+1} \cdot y^{1+1+1+1+1}$.

1. Show (prove) in detail why $(2 \cdot 3 \cdot 7)^4 = 2^4 3^4 7^4$.

2. Show (prove) in detail why $(xyz)^4 = x^4 y^4 z^4$ for any numbers x, y, z.

3. Show (prove) in detail why $(xyz)^n = x^n y^n z^n$ for any numbers x, y, and z and for any positive integer n.

Number Correct: ______

Applying Properties of Exponents to Generate Equivalent Expressions—Round 1

Directions: Simplify each expression using the laws of exponents. Use the least number of bases possible and only positive exponents. All letters denote numbers.

1.	$2^2 \cdot 2^3$		23.	$6^3 \cdot 6^2$	
2.	$2^2 \cdot 2^4$		24.	$6^2 \cdot 6^3$	
3.	$2^2 \cdot 2^5$		25.	$(-8)^3 \cdot (-8)^7$	
4.	$3^7 \cdot 3^1$		26.	$(-8)^7 \cdot (-8)^3$	
5.	$3^8 \cdot 3^1$		27.	$(0.2)^3 \cdot (0.2)^7$	
6.	$3^9 \cdot 3^1$		28.	$(0.2)^7 \cdot (0.2)^3$	
7.	$7^6 \cdot 7^2$		29.	$(-2)^{12} \cdot (-2)^1$	
8.	$7^6 \cdot 7^3$		30.	$(-2.7)^{12} \cdot (-2.7)^1$	
9.	$7^6 \cdot 7^4$		31.	$1.1^6 \cdot 1.1^9$	
10.	$11^{15} \cdot 11$		32.	$57^6 \cdot 57^9$	
11.	$11^{16} \cdot 11$		33.	$x^6 \cdot x^9$	
12.	$2^{12} \cdot 2^2$		34.	$2^7 \cdot 4$	
13.	$2^{12} \cdot 2^4$		35.	$2^7 \cdot 4^2$	
14.	$2^{12} \cdot 2^6$		36.	$2^7 \cdot 16$	
15.	$99^5 \cdot 99^2$		37.	$16 \cdot 4^3$	
16.	$99^6 \cdot 99^3$		38.	$3^2 \cdot 9$	
17.	$99^7 \cdot 99^4$		39.	$3^2 \cdot 27$	
18.	$5^8 \cdot 5^2$		40.	$3^2 \cdot 81$	
19.	$6^8 \cdot 6^2$		41.	$5^4 \cdot 25$	
20.	$7^8 \cdot 7^2$		42.	$5^4 \cdot 125$	
21.	$r^8 \cdot r^2$		43.	$8 \cdot 2^9$	
22.	$s^8 \cdot s^2$		44.	$16 \cdot 2^9$	

Number Correct: ______
Improvement: ______

Applying Properties of Exponents to Generate Equivalent Expressions—Round 2

Directions: Simplify each expression using the laws of exponents. Use the least number of bases possible and only positive exponents. All letters denote numbers.

1.	$5^2 \cdot 5^3$		23.	$7^3 \cdot 7^2$	
2.	$5^2 \cdot 5^4$		24.	$7^2 \cdot 7^3$	
3.	$5^2 \cdot 5^5$		25.	$(-4)^3 \cdot (-4)^{11}$	
4.	$2^7 \cdot 2^1$		26.	$(-4)^{11} \cdot (-4)^3$	
5.	$2^8 \cdot 2^1$		27.	$(0.2)^3 \cdot (0.2)^{11}$	
6.	$2^9 \cdot 2^1$		28.	$(0.2)^{11} \cdot (0.2)^3$	
7.	$3^6 \cdot 3^2$		29.	$(-2)^9 \cdot (-2)^5$	
8.	$3^6 \cdot 3^3$		30.	$(-2.7)^5 \cdot (-2.7)^9$	
9.	$3^6 \cdot 3^4$		31.	$3.1^6 \cdot 3.1^6$	
10.	$7^{15} \cdot 7$		32.	$57^6 \cdot 57^6$	
11.	$7^{16} \cdot 7$		33.	$z^6 \cdot z^6$	
12.	$11^{12} \cdot 11^2$		34.	$4 \cdot 2^9$	
13.	$11^{12} \cdot 11^4$		35.	$4^2 \cdot 2^9$	
14.	$11^{12} \cdot 11^6$		36.	$16 \cdot 2^9$	
15.	$23^5 \cdot 23^2$		37.	$16 \cdot 4^3$	
16.	$23^6 \cdot 23^3$		38.	$9 \cdot 3^5$	
17.	$23^7 \cdot 23^4$		39.	$3^5 \cdot 9$	
18.	$13^7 \cdot 13^3$		40.	$3^5 \cdot 27$	
19.	$15^7 \cdot 15^3$		41.	$5^7 \cdot 25$	
20.	$17^7 \cdot 17^3$		42.	$5^7 \cdot 125$	
21.	$x^7 \cdot x^3$		43.	$2^{11} \cdot 4$	
22.	$y^7 \cdot y^3$		44.	$2^{11} \cdot 16$	

We have shown that for any numbers x, y, and any positive integers m, n, the following holds

$$x^m \cdot x^n = x^{m+n} \quad (1)$$

$$(x^m)^n = x^{mn} \quad (2)$$

$$(xy)^n = x^n y^n. \quad (3)$$

Definition: ____

Exercise 1

List all possible cases of whole numbers m and n for identity (1). More precisely, when $m > 0$ and $n > 0$, we already know that (1) is correct. What are the other possible cases of m and n for which (1) is yet to be verified?

Exercise 2

Check that equation (1) is correct for each of the cases listed in Exercise 1.

Exercise 3

Do the same with equation (2) by checking it case-by-case.

Exercise 4

Do the same with equation (3) by checking it case-by-case.

Exercise 5

Write the expanded form of $8{,}374$ using exponential notation.

Exercise 6

Write the expanded form of $6{,}985{,}062$ using exponential notation.

Name ____ Date____________________

1. Simplify the following expression as much as possible.

$$\frac{4^{10}}{4^{10}} \cdot 7^0 =$$

2. Let a and b be two numbers. Use the distributive law and then the definition of zeroth power to show that the numbers $(a^0 + b^0)a^0$ and $(a^0 + b^0)b^0$ are equal.

Let x, y, f, and g be numbers ($x, y, f, g \neq 0$). Simplify each of the following expressions.

1. $\frac{x^6}{x^6}$

$= \boldsymbol{x^{6-6}}$
$= \boldsymbol{x^0}$
$= \mathbf{1}$

In class, I know we defined a number raised to the zeroth power as 1.

2. $\frac{x^3y^4}{x^3y^4}$

$= \boldsymbol{x^{3-3}y^{4-4}}$

$= \boldsymbol{x^0y^0}$

$= \mathbf{1 \cdot 1}$

$= \mathbf{1}$

3. $4^5 \cdot \frac{1}{4^5}$

$= \dfrac{\mathbf{4^5}}{\mathbf{4^5}}$

$= \mathbf{4^{5-5}}$

$= \mathbf{4^0}$
$= \mathbf{1}$

I have to use the rule for multiplying fractions.

I multiply the numerator times the numerator and the denominator times the denominator.

4. $3^7 \cdot \frac{1}{3^5} \cdot 3^5 \cdot \frac{1}{3^7} \cdot 3^2 \cdot \frac{1}{3^2}$

$= \dfrac{\mathbf{3^7 \cdot 1 \cdot 3^5 \cdot 1 \cdot 3^2 \cdot 1}}{\mathbf{3^5 \cdot 3^7 \cdot 3^2}}$

$= \dfrac{\mathbf{3^{7+5+2}}}{\mathbf{3^{5+7+2}}}$

$= \dfrac{\mathbf{3^{14}}}{\mathbf{3^{14}}}$

$= \mathbf{3^{14-14}}$

$= \mathbf{3^0}$

$= \mathbf{1}$

5. $\frac{f^4 \cdot g^3}{g^3 \cdot f^4}$

$= \dfrac{\boldsymbol{f^4 \cdot g^3}}{\boldsymbol{f^4 \cdot g^3}}$

$= \boldsymbol{f^{4-4} \cdot g^{3-3}}$

$= \boldsymbol{f^0 g^0}$

$= \mathbf{1 \cdot 1}$

$= \mathbf{1}$

6. $\left(8^2(2^6)\right)^0$

$= \mathbf{(8^2)^0 \cdot (2^6)^0}$

$= \mathbf{8^{2 \cdot 0} \cdot 2^{6 \cdot 0}}$

$= \mathbf{8^0 \cdot 2^0}$

$= \mathbf{1 \cdot 1}$

$= \mathbf{1}$

There is a power outside of the grouping symbol. That means I must use the third and second laws of exponents to simplify this expression.

Let x, y be numbers $(x, y \neq 0)$. Simplify each of the following expressions.

1. $\dfrac{y^{12}}{y^{12}} =$	2. $9^{15} \cdot \dfrac{1}{9^{15}} =$
3. $(7(123456.789)^4)^0 =$	4. $2^2 \cdot \dfrac{1}{2^5} \cdot 2^5 \cdot \dfrac{1}{2^2} =$
5. $\dfrac{x^{41}}{y^{15}} \cdot \dfrac{y^{15}}{x^{41}} =$	

Definition: For any nonzero number x, and for any positive integer n, we define x^{-n} as $\frac{1}{x^n}$.

Note that this definition of negative exponents says x^{-1} is just the reciprocal, $\frac{1}{x}$, of x.

As a consequence of the definition, for a nonnegative x and all *integers* b, we get

$$x^{-b} = \frac{1}{x^b}$$

Exercise 1

Verify the general statement $x^{-b} = \frac{1}{x^b}$ for $x = 3$ and $b = -5$.

Exercise 2

What is the value of (3×10^{-2})?

Exercise 3

What is the value of (3×10^{-5})?

Exercise 4

Write the complete expanded form of the decimal 4.728 in exponential notation.

For Exercises 5–10, write an equivalent expression, in exponential notation, to the one given, and simplify as much as possible.

Exercise 5

$5^{-3} =$

Exercise 6

$\frac{1}{8^9} =$

Exercise 7

$3 \cdot 2^{-4} =$

Exercise 8

Let x be a nonzero number.

$x^{-3} =$

Exercise 9

Let x be a nonzero number.

$\frac{1}{x^9} =$

Exercise 10

Let x, y be two nonzero numbers.

$xy^{-4} =$

We accept that for nonzero numbers x and y and all integers a and b,

$$x^a \cdot x^b = x^{a+b}$$
$$(x^b)^a = x^{ab}$$
$$(xy)^a = x^a y^a.$$

We claim

$$\frac{x^a}{x^b} = x^{a-b} \qquad \text{for all integers } a, b.$$

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \qquad \text{for any integer } a.$$

Exercise 11

$\frac{19^2}{19^5} =$

Exercise 12

$\frac{17^{16}}{17^{-3}} =$

Exercise 13

If we let $b = -1$ in (11), a be any integer, and y be any nonzero number, what do we get?

Exercise 14

Show directly that $\left(\frac{7}{5}\right)^{-4} = \frac{7^{-4}}{5^{-4}}$.

Equation Reference Sheet

For any numbers x, y [$x \neq 0$ in (4) and $y \neq 0$ in (5)] and any positive integers m, n, the following holds:
$x^m \cdot x^n = x^{m+n}$ (1)
$(x^m)^n = x^{mn}$ (2)
$(xy)^n = x^n y^n$ (3)
$\frac{x^m}{x^n} = x^{m-n}$ (4)
$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ (5)
For any numbers x, y and for all whole numbers m, n, the following holds:
$x^m \cdot x^n = x^{m+n}$ (6)
$(x^m)^n = x^{mn}$ (7)
$(xy)^n = x^n y^n$ (8)
For any nonzero number x and all integers b, the following holds:
$x^{-b} = \frac{1}{x^b}$ (9)
For any numbers x, y and all integers a, b, the following holds:
$x^a \cdot x^b = x^{a+b}$ (10)
$(x^b)^a = x^{ab}$ (11)
$(xy)^a = x^a y^a$ (12)
$\frac{x^a}{x^b} = x^{a-b} \quad x \neq 0$ (13)
$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \quad x, y \neq 0$ (14)

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Write each expression in a simpler form that is equivalent to the given expression.

1. $76543^{-4} =$

2. Let f be a nonzero number. $f^{-4} =$

3. $671 \times 28796^{-1} =$

4. Let a, b be numbers $(b \neq 0)$. $ab^{-1} =$

5. Let g be a nonzero number. $\frac{1}{g^{-1}} =$

Lesson Notes

You will need your Equation Reference Sheet. The numbers in parentheses in the solutions below correlate to the reference sheet.

Examples

1. Compute: $(-2)^4 \cdot (-2)^3 \cdot (-2)^{-2} \cdot (-2)^0 \cdot (-2)^{-2}$

$$= (-\mathbf{2})^{\mathbf{4+3+(-2)+0+(-2)}}$$
$$= (-\mathbf{2})^{\mathbf{3}}$$
$$= -\mathbf{8}$$

Negative exponents follow the same identities as positive exponents and exponents of zero.

2. Without using (10), show directly that $(y^{-1})^6 = y^{-6}$.

$(\boldsymbol{y^{-1}})^{\mathbf{6}} = \left(\frac{\mathbf{1}}{\boldsymbol{y^1}}\right)^{\mathbf{6}}$ ***By definition of negative exponents*** **(9)**

$= \frac{\mathbf{1^6}}{\boldsymbol{y^6}}$ ***By*** $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ **(14)**

$= \frac{\mathbf{1}}{\boldsymbol{y^6}}$

$= \boldsymbol{y^{-6}}$ ***By definition of negative exponents*** **(9)**

3. Without using (13), show directly that $\frac{6^{-9}}{6^3} = 6^{-12}$.

$\frac{\mathbf{6^{-9}}}{\mathbf{6^3}} = \mathbf{6^{-9}} \cdot \frac{\mathbf{1}}{\mathbf{6^3}}$ ***By product formula for complex fractions***

$= \frac{\mathbf{1}}{\mathbf{6^9}} \cdot \frac{\mathbf{1}}{\mathbf{6^3}}$ ***By definition of negative exponents*** **(9)**

$= \frac{\mathbf{1}}{\mathbf{6^9 \cdot 6^3}}$ ***By product formula for complex fractions***

$= \frac{\mathbf{1}}{\mathbf{6^{9+3}}}$ ***By*** $x^m \cdot x^n = x^{m+n}$ **(10)**

$= \frac{\mathbf{1}}{\mathbf{6^{12}}}$

$= \mathbf{6^{-12}}$ ***By definition of negative exponents*** **(9)**

1. Compute: $3^3 \times 3^2 \times 3^1 \times 3^0 \times 3^{-1} \times 3^{-2} =$

   Compute: $5^2 \times 5^{10} \times 5^8 \times 5^0 \times 5^{-10} \times 5^{-8} =$

   Compute for a nonzero number, a: $a^m \times a^n \times a^l \times a^{-n} \times a^{-m} \times a^{-l} \times a^0 =$

2. Without using (10), show directly that $(17.6^{-1})^8 = 17.6^{-8}$.

3. Without using (10), show (prove) that for any whole number n and any positive number y, $(y^{-1})^n = y^{-n}$.

4. Without using (13), show directly without using (13) that $\frac{2.8^{-5}}{2.8^7} = 2.8^{-12}$.

The Laws of Exponents

For $x, y \neq 0$, and all integers a, b, the following holds:

$$x^a \cdot x^b = x^{a+b}$$
$$(x^b)^a = x^{ab}$$
$$(xy)^a = x^a y^a.$$

Facts we will use to prove (11):

(A) (11) is already known to be true when the integers a and b satisfy $a \geq 0, b \geq 0$.

(B) $x^{-m} = \frac{1}{x^m}$ for any whole number m.

(C) $\left(\frac{1}{x}\right)^m = \frac{1}{x^m}$ for any whole number m.

Exercise 1

Show that **(C)** is implied by equation (5) of Lesson 4 when $m > 0$, and explain why **(C)** continues to hold even when $m = 0$.

Exercise 2

Show that **(B)** is in fact a special case of (11) by rewriting it as $(x^m)^{-1} = x^{(-1)m}$ for any whole number m, so that if $b = m$ (where m is a whole number) and $a = -1$, (11) becomes **(B)**.

Exercise 3

Show that **(C)** is a special case of (11) by rewriting **(C)** as $(x^{-1})^m = x^{m(-1)}$ for any whole number m. Thus, **(C)** is the special case of (11) when $b = -1$ and $a = m$, where m is a whole number.

Exercise 4

Proof of Case (iii): Show that when $a < 0$ and $b \geq 0$, $(x^b)^a = x^{ab}$ is still valid. Let $a = -c$ for some positive integer c. Show that the left and right sides of $(x^b)^a = x^{ab}$ are equal.

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1. Show directly that for any nonzero integer x, $x^{-5} \cdot x^{-7} = x^{-12}$.

2. Show directly that for any nonzero integer x, $(x^{-2})^{-3} = x^{6}$.

Lesson Notes

You will need your Equation Reference Sheet. The numbers in parentheses in the solutions below correlate to the reference sheet.

Examples

1. A very contagious strain of bacteria was contracted by two people who recently travelled overseas. When the couple returned, they then infected three people. The next week, each of those three people infected three more people. This infection rate continues each week. By the end of 5 weeks, how many people would be infected?

Week of Return	$\mathbf{2+3}$
Week 1	$\mathbf{(3\times 3)+(2+3)}$
Week 2	$\mathbf{(3^2\times 3)+(3\times 3)+(2+3)}$
Week 3	$\mathbf{(3^3\times 3)+(3^2\times 3)+(3\times 3)+(2+3)}$
Week 4	$\mathbf{(3^4\times 3)+(3^3\times 3)+(3^2\times 3)+(3\times 3)+(2+3)}$
Week 5	$\mathbf{(3^5\times 3)+(3^4\times 3)+(3^3\times 3)+(3^2\times 3)+(3\times 3)+(2+3)}$

The 3 people infected upon return each infect 3 people.

Therefore, in week 1, there are 9 new infected people, or $(3\times 3)=3^2$.

Those 9 people infect 3 people each, or 27 new people. $(3^2\times 3)=3^3$

2. Show directly that $r^{-10}\cdot r^{-12}=r^{-22}$.

$\boldsymbol{r^{-10}\cdot r^{-12}=\frac{1}{r^{10}}\cdot\frac{1}{r^{12}}}$ ***By definition of negative exponents*** **(9)**

$\boldsymbol{=\frac{1}{r^{10}\cdot r^{12}}}$ ***By product formula for complex fractions***

$\boldsymbol{=\frac{1}{r^{10+12}}}$ ***By $x^m\cdot x^n=x^{m+n}$ for whole numbers m and n*** **(6)**

$\boldsymbol{=\frac{1}{r^{22}}}$

$\boldsymbol{=r^{-22}}$ ***By definition of negative exponents*** **(9)**

"Show directly" and "prove" mean the same thing: I should use the identities and definitions I know are true for whole numbers to prove the identities are also true for integer exponents.

1. You sent a photo of you and your family on vacation to seven Facebook friends. If each of them sends it to five of their friends, and each of those friends sends it to five of their friends, and those friends send it to five more, how many people (not counting yourself) will see your photo? No friend received the photo twice. Express your answer in exponential notation.

# of New People to View Your Photo	*Total # of People to View Your Photo*

2. Show directly, without using (11), that $(1.27^{-36})^{85} = 1.27^{-36 \cdot 85}$.

3. Show directly that $\left(\frac{2}{13}\right)^{-127} \cdot \left(\frac{2}{13}\right)^{-56} = \left(\frac{2}{13}\right)^{-183}$.

4. Prove for any nonzero number x, $x^{-127} \cdot x^{-56} = x^{-183}$.

5. Prove for any nonzero number x, $x^{-m} \cdot x^{-n} = x^{-m-n}$ for positive integers m and n.

6. Which of the preceding four problems did you find easiest to do? Explain.

7. Use the properties of exponents to write an equivalent expression that is a product of distinct primes, each raised to an integer power.

$$\frac{10^5 \cdot 9^2}{6^4} =$$

Fact 1: *The number* 10^n*, for arbitrarily large positive integers* n*, is a* big number in the sense that given a number M (no matter how big it is) there is a power of 10 that exceeds M.

Fact 2: *The number* 10^{-n}*, for arbitrarily large positive integers* n*, is a* small number in the sense that given a positive number S (no matter how small it is), there is a (negative) power of 10 that is smaller than S.

Exercise 1

Let $M = 993{,}456{,}789{,}098{,}765$. Find the smallest power of 10 that will exceed M.

Exercise 2

Let $M = 78{,}491\frac{899}{987}$. Find the smallest power of 10 that will exceed M.

Exercise 3

Let M be a positive integer. Explain how to find the smallest power of 10 that exceeds it.

Exercise 4

The chance of you having the same DNA as another person (other than an identical twin) is approximately 1 in 10 trillion (one trillion is a 1 followed by 12 zeros). Given the fraction, express this very small number using a negative power of 10.

$$\frac{1}{10\,000\,000\,000\,000}$$

Exercise 5

The chance of winning a big lottery prize is about 10^{-8}, and the chance of being struck by lightning in the U.S. in any given year is about $0.000\,001$. Which do you have a greater chance of experiencing? Explain.

Exercise 6

There are about 100 million smartphones in the U.S. Your teacher has one smartphone. What share of U.S. smartphones does your teacher have? Express your answer using a negative power of 10.

Name ____ Date____________________

1. Let $M = 118{,}526.65902$. Find the smallest power of 10 that will exceed M.

2. Scott said that 0.09 was a bigger number than 0.1. Use powers of 10 to show that he is wrong.

1. What is the smallest power of 10 that would exceed 6,234,579?

   M has 7 digits, so a number with 8 digits will exceed it.

   $$\boldsymbol{M = 6,234,579 < 9,999,999 < 10,000,000 = 10^7}$$

   The smallest power of 10 that would exceed $6,234,579$ is 10^7.

   If I create a number with the same number of digits as M but with all nines, I know that number will exceed M. If I then add 1, I will have a number that can be written as a power of 10.

2. Which number is equivalent to 0.001: 10^3 or 10^{-3}? How do you know?

   10^{-3} is equivalent to 0.001. Positive powers of 10 create large numbers, and negative powers of 10 create numbers smaller than one. The number 10^{-3} is equal to the fraction $\frac{1}{10^3}$, which is the same as $\frac{1}{1000}$ and 0.001. Since 0.001 is a small number, its power of 10 should be negative.

3. Jessica said that 0.0001 is bigger than 0.1 because the first number has more digits to the right of the decimal point. Is Jessica correct? Explain your thinking using negative powers of 10 and the number line.

   $\boldsymbol{0.0001 = \frac{1}{10000} = 10^{-4}}$ ***and*** $\boldsymbol{0.1 = \frac{1}{10} = 10^{-1}}$

   On a number line 10^{-1} is farther from 0 than 10^{-4}, meaning that 10^{-1} is larger than 10^{-4}. Therefore, Jessica is incorrect because $0.0001 < 0.1$.

   I have to remember that negative exponents behave differently than positive exponents. I have to think about the number line and that the further right a number is, the larger the number is.

4. Order the following numbers from least to greatest:

   $10^2 \quad 10^{-4} \quad 10^0 \quad 10^{-3}$

   $$\boldsymbol{10^{-4} < 10^{-3} < 10^0 < 10^2}$$

   Since all of the bases are the same, I just need to make sure I have the exponents in order from least to greatest.

1. What is the smallest power of 10 that would exceed $987{,}654{,}321{,}098{,}765{,}432$?

2. What is the smallest power of 10 that would exceed $999{,}999{,}999{,}991$?

3. Which number is equivalent to $0.000\,000\,1$: 10^7 or 10^{-7}? How do you know?

4. Sarah said that $0.000\,01$ is bigger than 0.001 because the first number has more digits to the right of the decimal point. Is Sarah correct? Explain your thinking using negative powers of 10 and the number line.

5. Order the following numbers from least to greatest:

$$10^5 \qquad 10^{-99} \qquad 10^{-17} \qquad 10^{14} \qquad 10^{-5} \qquad 10^{30}$$

Number Correct: ______

Applying Properties of Exponents to Generate Equivalent Expressions—Round 1

Directions: Simplify each expression using the laws of exponents. Use the least number of bases possible and only positive exponents. When appropriate, express answers without parentheses or as equal to 1. All letters denote numbers.

1.	$4^5 \cdot 4^{-4}$		23.	$\left(\frac{1}{2}\right)^6$	
2.	$4^5 \cdot 4^{-3}$		24.	$(3x)^5$	
3.	$4^5 \cdot 4^{-2}$		25.	$(3x)^7$	
4.	$7^{-4} \cdot 7^{11}$		26.	$(3x)^9$	
5.	$7^{-4} \cdot 7^{10}$		27.	$(8^{-2})^3$	
6.	$7^{-4} \cdot 7^9$		28.	$(8^{-3})^3$	
7.	$9^{-4} \cdot 9^{-3}$		29.	$(8^{-4})^3$	
8.	$9^{-4} \cdot 9^{-2}$		30.	$(22^0)^{50}$	
9.	$9^{-4} \cdot 9^{-1}$		31.	$(22^0)^{55}$	
10.	$9^{-4} \cdot 9^0$		32.	$(22^0)^{60}$	
11.	$5^0 \cdot 5^1$		33.	$\left(\frac{1}{11}\right)^{-5}$	
12.	$5^0 \cdot 5^2$		34.	$\left(\frac{1}{11}\right)^{-6}$	
13.	$5^0 \cdot 5^3$		35.	$\left(\frac{1}{11}\right)^{-7}$	
14.	$(12^3)^9$		36.	$\frac{56^{-23}}{56^{-34}}$	
15.	$(12^3)^{10}$		37.	$\frac{87^{-12}}{87^{-34}}$	
16.	$(12^3)^{11}$		38.	$\frac{23^{-15}}{23^{-17}}$	
17.	$(7^{-3})^{-8}$		39.	$(-2)^{-12} \cdot (-2)^1$	
18.	$(7^{-3})^{-9}$		40.	$\frac{2y}{y^3}$	
19.	$(7^{-3})^{-10}$		41.	$\frac{5xy^7}{15x^7y}$	
20.	$\left(\frac{1}{2}\right)^9$		42.	$\frac{16x^6y^9}{8x^{-5}y^{-11}}$	
21.	$\left(\frac{1}{2}\right)^8$		43.	$(2^3 \cdot 4)^{-5}$	
22.	$\left(\frac{1}{2}\right)^7$		44.	$(9^{-8})(27^{-2})$	

Number Correct: ______

Improvement: ______

Applying Properties of Exponents to Generate Equivalent Expressions—Round 2

Directions: Simplify each expression using the laws of exponents. Use the least number of bases possible and only positive exponents. When appropriate, express answers without parentheses or as equal to 1. All letters denote numbers.

1.	$11^5 \cdot 11^{-4}$		23.	$\left(\frac{3}{7}\right)^5$	
2.	$11^5 \cdot 11^{-3}$		24.	$(18xy)^5$	
3.	$11^5 \cdot 11^{-2}$		25.	$(18xy)^7$	
4.	$7^{-7} \cdot 7^9$		26.	$(18xy)^9$	
5.	$7^{-8} \cdot 7^9$		27.	$(5.2^{-2})^3$	
6.	$7^{-9} \cdot 7^9$		28.	$(5.2^{-3})^3$	
7.	$(-6)^{-4} \cdot (-6)^{-3}$		29.	$(5.2^{-4})^3$	
8.	$(-6)^{-4} \cdot (-6)^{-2}$		30.	$(22^6)^0$	
9.	$(-6)^{-4} \cdot (-6)^{-1}$		31.	$(22^{12})^0$	
10.	$(-6)^{-4} \cdot (-6)^0$		32.	$(22^{18})^0$	
11.	$x^0 \cdot x^1$		33.	$\left(\frac{4}{5}\right)^{-5}$	
12.	$x^0 \cdot x^2$		34.	$\left(\frac{4}{5}\right)^{-6}$	
13.	$x^0 \cdot x^3$		35.	$\left(\frac{4}{5}\right)^{-7}$	
14.	$(12^5)^9$		36.	$\left(\frac{6^{-2}}{7^5}\right)^{-11}$	
15.	$(12^6)^9$		37.	$\left(\frac{6^{-2}}{7^5}\right)^{-12}$	
16.	$(12^7)^9$		38.	$\left(\frac{6^{-2}}{7^5}\right)^{-13}$	
17.	$(7^{-3})^{-4}$		39.	$\left(\frac{6^{-2}}{7^5}\right)^{-15}$	
18.	$(7^{-4})^{-4}$		40.	$\frac{42ab^{10}}{14a^{-9}b}$	
19.	$(7^{-5})^{-4}$		41.	$\frac{5xy^7}{25x^7y}$	
20.	$\left(\frac{3}{7}\right)^8$		42.	$\frac{22a^{15}b^{32}}{121ab^{-5}}$	
21.	$\left(\frac{3}{7}\right)^7$		43.	$(7^{-8} \cdot 49)^{-5}$	
22.	$\left(\frac{3}{7}\right)^6$		44.	$(36^9)(216^{-2})$	

Exercise 1

The Federal Reserve states that the average household in January of 2013 had $\$7,122$ in credit card debt. About how many times greater is the U.S. national debt, which is $\$16,755,133,009,522$? Rewrite each number to the nearest power of 10 that exceeds it, and then compare.

Exercise 2

There are about $3,000,000$ students attending school, kindergarten through Grade 12, in New York. Express the number of students as a single-digit integer times a power of 10.

The average number of students attending a middle school in New York is 8×10^2. How many times greater is the overall number of K–12 students compared to the average number of middle school students?

Exercise 3

A conservative estimate of the number of stars in the universe is 6×10^{22}. The average human can see about 3,000 stars at night with his naked eye. About how many times more stars are there in the universe compared to the stars a human can actually see?

Exercise 4

The estimated world population in 2011 was 7×10^{9}. Of the total population, 682 million of those people were left-handed. Approximately what percentage of the world population is left-handed according to the 2011 estimation?

Exercise 5

The average person takes about $30{,}000$ breaths per day. Express this number as a single-digit integer times a power of 10.

If the average American lives about 80 years (or about $30{,}000$ days), how many total breaths will a person take in her lifetime?

Name ____ Date____________________

Most English-speaking countries use the short-scale naming system, in which a trillion is expressed as $1{,}000{,}000{,}000{,}000$. Some other countries use the long-scale naming system, in which a trillion is expressed as $1{,}000{,}000{,}000{,}000{,}000{,}000{,}000$. Express each number as a single-digit integer times a power of ten. How many times greater is the long-scale naming system than the short-scale?

1. A 250 gigabyte hard drive has a total of 250,000,000,000 bytes of available storage space. A 3.5 inch double-sided floppy disk widely used in the 1980's could hold about 8×10^5 bytes. How many double-sided floppy disks would it take to fill the 250 gigabyte hard drive?

$$\mathbf{250,000,000,000 \approx 3 \times 10^{11}}$$

$$\begin{aligned}\frac{3 \times 10^{11}}{8 \times 10^5} &= \frac{3}{8} \times \frac{10^{11}}{10^5} \\ &= 0.375 \times 10^{11-5} \\ &= 0.375 \times 10^6 \\ &= 375,000\end{aligned}$$

I know that when the question says, "How many will it take to fill…," it means to divide.

It would take $375,000$ floppy disks to fill the 250 gigabyte hard drive.

2. A calculation of the operation $2,000,000 \times 3,000,000,000$ gives an answer of 6e+15. What does the answer of 6e+15 on the screen of the calculator mean? Explain how you know.

The answer means 6×10^{15}. This is known because

$$\begin{aligned}(2 \times 10^6) \times (3 \times 10^9) &= (2 \times 3) \times (10^6 \times 10^9) \\ &= 6 \times 10^{6+9} \\ &= 6 \times 10^{15}\end{aligned}$$

I know that multiplication follows the commutative and associative properties. I can then use the first law of exponents to simplify the expression.

3. An estimate of the number of neurons in the brain of an average rat is 2×10^8. A cat has approximately 8×10^8 neurons. Which animal has a greater number of neurons? By how much?

$$\mathbf{8 \times 10^8 > 2 \times 10^8}$$

$$\begin{aligned}\frac{8 \times 10^8}{2 \times 10^8} &= \frac{8}{2} \times \frac{10^8}{10^8} \\ &= 4 \times 10^{8-8} \\ &= 4 \times 10^0 \\ &= 4 \times 1 \\ &= 4\end{aligned}$$

I need to divide to figure out how many times larger a cat's number of neurons is than a rat's number of neurons.

A cat has 4 times as many neurons as a rat.

1. The Atlantic Ocean region contains approximately 2×10^{16} gallons of water. Lake Ontario has approximately $8{,}000{,}000{,}000{,}000$ gallons of water. How many Lake Ontarios would it take to fill the Atlantic Ocean region in terms of gallons of water?

2. U.S. national forests cover approximately $300{,}000$ square miles. Conservationists want the total square footage of forests to be $300{,}000^2$ square miles. When Ivanna used her phone to do the calculation, her screen showed the following:

   a. What does the answer on her screen mean? Explain how you know.
   b. Given that the U.S. has approximately 4 million square miles of land, is this a reasonable goal for conservationists? Explain.

3. The United States is responsible for about $20{,}000$ kilograms of carbon emission pollution each year. Express this number as a single-digit integer times a power of 10.

4. The United Kingdom is responsible for about 1×10^4 kilograms of carbon emission pollution each year. Which country is responsible for greater carbon emission pollution each year? By how much?

A positive, finite decimal s is said to be written in scientific notation if it is expressed as a product $d \times 10^n$, where d is a finite decimal so that $1 \leq d < 10$, and n is an integer.

The integer n is called the order of magnitude of the decimal $d \times 10^n$.

Are the following numbers written in scientific notation? If not, state the reason.

Exercise 1

1.908×10^{17}

Exercise 2

0.325×10^{-2}

Exercise 3

7.99×10^{32}

Exercise 4

$4.0701 + 10^7$

Exercise 5

18.432×5^8

Exercise 6

8×10^{-11}

Use the table below to complete Exercises 7 and 8.

The table below shows the debt of the three most populous states and the three least populous states.

State	Debt (in dollars)	Population (2012)
California	$407,000,000,000$	$38,000,000$
New York	$337,000,000,000$	$19,000,000$
Texas	$276,000,000,000$	$26,000,000$
North Dakota	$4,000,000,000$	$690,000$
Vermont	$4,000,000,000$	$626,000$
Wyoming	$2,000,000,000$	$576,000$

Exercise 7

a. What is the sum of the debts for the three most populous states? Express your answer in scientific notation.

b. What is the sum of the debt for the three least populous states? Express your answer in scientific notation.

c. How much larger is the combined debt of the three most populous states than that of the three least populous states? Express your answer in scientific notation.

Exercise 8

a. What is the sum of the population of the three most populous states? Express your answer in scientific notation.

b. What is the sum of the population of the three least populous states? Express your answer in scientific notation.

c. Approximately how many times greater is the total population of California, New York, and Texas compared to the total population of North Dakota, Vermont, and Wyoming?

Exercise 9

All planets revolve around the sun in elliptical orbits. Uranus's furthest distance from the sun is approximately 3.004×10^9 km, and its closest distance is approximately 2.749×10^9 km. Using this information, what is the average distance of Uranus from the sun?

Name ____ Date____________________

1. The approximate total surface area of Earth is $5.1 \times 10^8\ \text{km}^2$. All the salt water on Earth has an approximate surface area of $352{,}000{,}000\ \text{km}^2$, and all the freshwater on Earth has an approximate surface area of $9 \times 10^6\ \text{km}^2$. How much of Earth's surface is covered by water, including both salt and fresh water? Write your answer in scientific notation.

2. How much of Earth's surface is covered by land? Write your answer in scientific notation.

3. Approximately how many times greater is the amount of Earth's surface that is covered by water compared to the amount of Earth's surface that is covered by land?

Definitions

A positive decimal is said to be written in scientific notation if it is expressed as a product $d \times 10^n$, where d is a decimal greater than or equal to 1 and less than 10 and n is an integer.

The integer n is called the order of magnitude of the decimal $d \times 10^n$.

Examples

1. Write the number 32,000,000,000 in scientific notation.

$\mathbf{32,000,000,000 = 3.2 \times 10^{10}}$

I will place the decimal between the 3 and 2 to achieve a value that is greater than 1 and smaller than 10.

I will need to multiply 3.2 by 10^{10} because I need to write an equivalent form of 32,000,000,000.

2. What is the sum of 5.4×10^7 and 8.24×10^9?

To add terms, they need to be like terms. I know that means that the magnitudes, or the powers, need to be equal.

$\mathbf{(5.4 \times 10^7) + (8.24 \times 10^9)}$

$\mathbf{= (5.4 \times 10^7) + (8.24 \times (10^2 \times 10^7))}$ ***By first law of exponents***

$\mathbf{= (5.4 \times 10^7) + ((8.24 \times 10^2) \times 10^7)}$ ***By associative property of multiplication***

$\mathbf{= (5.4 \times 10^7) + (824 \times 10^7)}$

I know that "$\times 10^2$" multiplies 8.24 by 100.

$\mathbf{= (5.4 + 824) \times 10^7}$ ***By distributive property***

$\mathbf{= 829.4 \times 10^7}$

$\mathbf{= (8.294 \times 10^2) \times 10^7}$

$\mathbf{= 8.294 \times 10^9}$

The last step is to write this in scientific notation.

3. The Lextor Company recently posted its quarterly earnings for 2014.
   Quarter 1: 2.65×10^6 dollars
   Quarter 2: 1.6×10^8 dollars
   Quarter 3: 6.1×10^6 dollars
   Quarter 4: 2.25×10^8 dollars

   What is the average earnings for all four quarters? Write your answer in scientific notation.

$$\begin{aligned}
\textbf{Average Earnings} &= \frac{(2.65 \times 10^6) + (1.6 \times 10^8) + (6.1 \times 10^6) + (2.25 \times 10^8)}{4} \\
&= \frac{(2.65 \times 10^6) + (1.6 \times 10^2 \times 10^6) + (6.1 \times 10^6) + (2.25 \times 10^2 \times 10^6)}{4} \\
&= \frac{(2.65 \times 10^6) + (160 \times 10^6) + (6.1 \times 10^6) + (225 \times 10^6)}{4} \\
&= \frac{(2.65 + 160 + 6.1 + 225) \times 10^6}{4} \\
&= \frac{393.75 \times 10^6}{4} \\
&= \frac{393.75}{4} \times 10^6 \\
&= 98.4375 \times 10^6 \\
&= 9.84375 \times 10^7
\end{aligned}$$

The average earnings in 2014 for the Lextor Company is 9.84375×10^7 dollars.

1. Write the number $68{,}127{,}000{,}000{,}000{,}000$ in scientific notation. Which of the two representations of this number do you prefer? Explain.

2. Here are the masses of the so-called inner planets of the solar system.

   Mercury: 3.3022×10^{23} kg
   Earth: 5.9722×10^{24} kg
   Venus: 4.8685×10^{24} kg
   Mars: 6.4185×10^{23} kg

   What is the average mass of all four inner planets? Write your answer in scientific notation.

Exercise 1

The speed of light is $300{,}000{,}000$ meters per second. The sun is approximately 1.5×10^{11} meters from Earth. How many seconds does it take for sunlight to reach Earth?

Exercise 2

The mass of the moon is about 7.3×10^{22} kg. It would take approximately $26{,}000{,}000$ moons to equal the mass of the sun. Determine the mass of the sun.

Exercise 3

The mass of Earth is 5.9×10^{24} kg. The mass of Pluto is $13{,}000{,}000{,}000{,}000{,}000{,}000{,}000$ kg. Compared to Pluto, how much greater is Earth's mass than Pluto's mass?

Exercise 4

Using the information in Exercises 2 and 3, find the combined mass of the moon, Earth, and Pluto.

Exercise 5

How many combined moon, Earth, and Pluto masses (i.e., the answer to Exercise 4) are needed to equal the mass of the sun (i.e., the answer to Exercise 2)?

Name ______________________________ Date ____________

1. The speed of light is 3×10^8 meters per second. The sun is approximately $230{,}000{,}000{,}000$ meters from Mars. How many seconds does it take for sunlight to reach Mars?

2. If the sun is approximately 1.5×10^{11} meters from Earth, what is the approximate distance from Earth to Mars?

1. A lightning bolt produces 1.1×10^{10} watts of energy in about 1 second. How much energy would that bolt of lightning produce if it lasted for 24 hours? (Note: 24 hours is 86,400 seconds.)

$\left(\mathbf{1.1 \times 10^{10}}\right) \times \mathbf{86,400}$

$= \left(\mathbf{1.1 \times 10^{10}}\right) \times \left(\mathbf{8.64 \times 10^{4}}\right)$

$= (\mathbf{1.1 \times 8.64}) \times \left(\mathbf{10^{10} \times 10^{4}}\right)$

$= \mathbf{9.504 \times 10^{10+4}}$

$= \mathbf{9.504 \times 10^{14}}$

I need to take the amount of energy produced in one second and multiply it by 86,400.

A lightning bolt would produce 9.504×10^{14} watts of energy if it lasted 24 hours.

2. There are about 7,000,000,000 people in the world. In Australia, there is a population of about 2.306×10^{7} people. What is the difference between the world's and Australia's populations?

$\mathbf{7,000,000,000 - 2.306 \times 10^{7}}$

$= \left(\mathbf{7 \times 10^{9}}\right) - \left(\mathbf{2.306 \times 10^{7}}\right)$

$= \left(\mathbf{7 \times 10^{2} \times 10^{7}}\right) - \left(\mathbf{2.306 \times 10^{7}}\right)$

$= \left(\mathbf{700 \times 10^{7}}\right) - \left(\mathbf{2.306 \times 10^{7}}\right)$

$= (\mathbf{700 - 2.306}) \times \mathbf{10^{7}}$

$= \mathbf{697.694 \times 10^{7}}$

$= \mathbf{6.97694 \times 10^{9}}$

Just like in the last lesson, I need to make sure the numbers have the same order of magnitude (exponent) before I actually subtract.

The difference between the world's and Australia's populations is about 6.97694×10^{9} people.

3. The average human adult body has about 5×10^{13} cells. A newborn baby's body contains approximately 2.5×10^{12} cells.

   a. Find the combined number of cells.

$$\begin{aligned}\textbf{Combined Number of Cells} &= (\mathbf{5 \times 10^{13}}) + (\mathbf{2.5 \times 10^{12}}) \\ &= (\mathbf{5 \times 10^{1} \times 10^{12}}) + (\mathbf{2.5 \times 10^{12}}) \\ &= (\mathbf{50 \times 10^{12}}) + (\mathbf{2.5 \times 10^{12}}) \\ &= (\mathbf{50 + 2.5}) \times \mathbf{10^{12}} \\ &= \mathbf{52.5 \times 10^{12}} \\ &= \mathbf{5.25 \times 10^{13}}\end{aligned}$$

   The number of cells in a human adult and baby combined is 5.25×10^{13} cells.

   b. Given that the number of cells in the average elephant is approximately 1.5×10^{27}, how many times larger is the number of cells in an elephant than the number of cells in a human adult and baby combined?

$$\begin{aligned}\frac{\mathbf{1.5 \times 10^{27}}}{\mathbf{5.25 \times 10^{13}}} &= \frac{\mathbf{1.5}}{\mathbf{5.25}} \times \frac{\mathbf{10^{27}}}{\mathbf{10^{13}}} \\ &\approx \mathbf{0.286 \times 10^{27-13}} \\ &= \mathbf{0.286 \times 10^{14}} \\ &= \mathbf{2.86 \times 10^{13}}\end{aligned}$$

   The number of cells in an elephant is about 2.86×10^{13} times larger than the number of cells in a human adult and baby combined.

1. The sun produces 3.8×10^{27} joules of energy per second. How much energy is produced in a year? (Note: a year is approximately $31{,}000{,}000$ seconds).

2. On average, Mercury is about $57{,}000{,}000$ km from the sun, whereas Neptune is about 4.5×10^{9} km from the sun. What is the difference between Mercury's and Neptune's distances from the sun?

3. The mass of Earth is approximately 5.9×10^{24} kg, and the mass of Venus is approximately 4.9×10^{24} kg.
   a. Find their combined mass.
   b. Given that the mass of the sun is approximately 1.9×10^{30} kg, how many Venuses and Earths would it take to equal the mass of the sun?

Exercise 1

The mass of a proton is

$$0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 001\ 672\ 622 \text{ kg.}$$

In scientific notation it is

Exercise 2

The mass of an electron is

$$0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 910\ 938\ 291 \text{ kg.}$$

In scientific notation it is

Exercise 3

Write the ratio that compares the mass of a proton to the mass of an electron.

Exercise 4

Compute how many times heavier a proton is than an electron (i.e., find the value of the ratio). Round your final answer to the nearest one.

Example 2

The U.S. national debt as of March 23, 2013, rounded to the nearest dollar, is \$16,755,133,009,522. According to the 2012 U.S. census, there are about 313,914,040 U.S. citizens. What is each citizen's approximate share of the debt?

$$\begin{aligned}\frac{1.6755 \times 10^{13}}{3.14 \times 10^8} &= \frac{1.6755}{3.14} \times \frac{10^{13}}{10^8} \\ &= \frac{1.6755}{3.14} \times 10^5 \\ &= 0.533598\ldots \times 10^5 \\ &\approx 0.5336 \times 10^5 \\ &= 53360\end{aligned}$$

Each U.S. citizen's share of the national debt is about \$53,360.

Exercise 5

The geographic area of California is $163{,}696$ sq. mi., and the geographic area of the U.S. is $3{,}794{,}101$ sq. mi. Let's round off these figures to 1.637×10^5 and 3.794×10^6. In terms of area, roughly estimate how many Californias would make up one U.S. Then compute the answer to the nearest ones.

Exercise 6

The average distance from Earth to the moon is about 3.84×10^5 km, and the distance from Earth to Mars is approximately 9.24×10^7 km in year 2014. On this simplistic level, how much farther is traveling from Earth to Mars than from Earth to the moon?

Name ____ Date ____________________

1. Two of the largest mammals on earth are the blue whale and the African elephant. An adult male blue whale weighs about 170 tonnes or long tons. (1 tonne $= 1000$ kg)

   Show that the weight of an adult blue whale is 1.7×10^5 kg.

2. An adult male African elephant weighs about 9.07×10^3 kg.

   Compute how many times heavier an adult male blue whale is than an adult male African elephant (i.e., find the value of the ratio). Round your final answer to the nearest one.

1. Which of the two numbers below is greater? Explain how you know.

   8.25×10^{15} and 8.2×10^{20}

   The number $\mathbf{8.2 \times 10^{20}}$ is greater. When comparing each numbers order of magnitude, it is obvious that $\mathbf{20 > 15}$; therefore, $\mathbf{8.2 \times 10^{20} > 8.25 \times 10^{15}}$.

   To figure out which number is greater, I need to look at the order of magnitude (exponent) of each number.

2. About how many times greater is 8.2×10^{20} compared to 8.25×10^{15}?

$$\begin{aligned} \frac{8.2 \times 10^{20}}{8.25 \times 10^{15}} &= \frac{8.2}{8.25} \times \frac{10^{20}}{10^{15}} \\ &= 0.993939\ldots \times 10^{20-15} \\ &\approx 0.99 \times 10^{5} \\ &= 99,000 \end{aligned}$$

   $\mathbf{8.2 \times 10^{20}}$ is about $\mathbf{99,000}$ times greater than $\mathbf{8.25 \times 10^{15}}$.

3. Suppose the geographic area of Los Angeles County is $4,751$ square miles. If the state of California has an area of 1.637×10^{5} square miles, that means that it would take approximately 35 Los Angeles Counties to make up the state of California. As of 2013, the population of Los Angeles County was 1×10^{7} people. If the population were proportional to area, what would be the population of the state of California? Write your answer in scientific notation.

$$\begin{aligned} 1 \times 10^{7} \times 35 &= 35 \times 10^{7} \\ &= (3.5 \times 10) \times 10^{7} \\ &= 3.5 \times (10 \times 10^{7}) \\ &= 3.5 \times 10^{8} \end{aligned}$$

   The population of California would be $\mathbf{3.5 \times 10^{8}}$ people.

   Since it takes about 35 Los Angeles Counties to make up the state of California, then what I need to do is multiply the population of Los Angeles County by 35.

   The expression 35×10^{7} is not in scientific notation because 35 is too large (it has to be less than 10). I can rewrite 35 as 3.5×10 because $35 = 3.5 \times 10$.

1. There are approximately 7.5×10^{18} grains of sand on Earth. There are approximately 7×10^{27} atoms in an average human body. Are there more grains of sand on Earth or atoms in an average human body? How do you know?

2. About how many times more atoms are in a human body compared to grains of sand on Earth?

3. Suppose the geographic areas of California and the U.S. are 1.637×10^5 and 3.794×10^6 sq. mi., respectively. California's population (as of 2012) is approximately 3.804×10^7 people. If population were proportional to area, what would be the U.S. population?

4. The actual population of the U.S. (as of 2012) is approximately 3.14×10^8. How does the population density of California (i.e., the number of people per square mile) compare with the population density of the U.S.?

Exercise 1

A certain brand of MP3 player will display how long it will take to play through its entire music library. If the maximum number of songs the MP3 player can hold is $1{,}000$ (and the average song length is 4 minutes), would you want the time displayed in terms of seconds-, days-, or years-worth of music? Explain.

Exercise 2

You have been asked to make frosted cupcakes to sell at a school fundraiser. Each frosted cupcake contains about 20 grams of sugar. Bake sale coordinators expect 500 people will attend the event. Assume everyone who attends will buy a cupcake; does it make sense to buy sugar in grams, pounds, or tons? Explain.

Exercise 3

The seafloor spreads at a rate of approximately 10 cm per year. If you were to collect data on the spread of the seafloor each week, which unit should you use to record your data? Explain.

The gigaelectronvolt, $\frac{\text{GeV}}{c^2}$, is what particle physicists use as the unit of mass.

1 gigaelectronvolt = 1.783×10^{-27} kg

Mass of 1 proton = $1.672\ 622 \times 10^{-27}$ kg

Exercise 4

Show that the mass of a proton is $0.938 \frac{\text{GeV}}{c^2}$.

In popular science writing, a commonly used unit is the light-year, or the distance light travels in one year (note: one year is defined as 365.25 days).

$$1 \text{ light-year} = 9{,}460{,}730{,}472{,}580.800 \text{ km} \approx 9.46073 \times 10^{12} \text{ km}$$

Exercise 5

The distance of the nearest star (Proxima Centauri) to the sun is approximately $4.013\ 336\ 473 \times 10^{13}$ km. Show that Proxima Centauri is 4.2421 light-years from the sun.

Exploratory Challenge 2

Suppose you are researching atomic diameters and find that credible sources provided the diameters of five different atoms as shown in the table below. All measurements are in centimeters.

1×10^{-8}	1×10^{-12}	5×10^{-8}	5×10^{-10}	5.29×10^{-11}

Exercise 6

What new unit might you introduce in order to discuss the differences in diameter measurements?

Exercise 7

Name your unit, and explain why you chose it.

Exercise 8

Using the unit you have defined, rewrite the five diameter measurements.

Name ____ Date ____________________

1. The table below shows an approximation of the national debt at the beginning of each decade over the last century. Choose a unit that would make a discussion about the growth of the national debt easier. Name your unit, and explain your choice.

Year	Debt in Dollars
1900	2.1×10^{9}
1910	2.7×10^{9}
1920	2.6×10^{10}
1930	1.6×10^{10}
1940	4.3×10^{10}
1950	2.6×10^{11}
1960	2.9×10^{11}
1970	3.7×10^{11}
1980	9.1×10^{11}
1990	3.2×10^{12}
2000	5.7×10^{12}

2. Using the new unit you have defined, rewrite the debt for years 1900, 1930, 1960, and 2000.

1. What is the average of the following two numbers?

3.257×10^3 and 3.1×10^3

$$\frac{\mathbf{3.257 \times 10^3 + 3.1 \times 10^3}}{\mathbf{2}} = \frac{\mathbf{(3.257 + 3.1) \times 10^3}}{\mathbf{2}}$$
$$= \frac{\mathbf{6.357 \times 10^3}}{\mathbf{2}}$$
$$= \frac{\mathbf{6.357}}{\mathbf{2}} \times \mathbf{10^3}$$
$$= \mathbf{3.1785 \times 10^3}$$

To find the average, I need to add the two numbers and then divide by 2. Since the numbers are raised to the same power of 10, I really only need to add 3.257 and 3.1.

2. Assume you are given the data below and asked to decide on a new unit in order to make comparisons and discussions of the data easier.

1.9×10^{15}	3.75×10^{19}
9.26×10^{16}	7.02×10^{19}
4.56×10^{17}	2.4×10^{3}

I need to examine the exponents to see which is most common or which exponent most numbers would be close to. Since I'm deciding the unit, I just need to make sure my choice is reasonable.

a. What new unit would you select? Name it and express it using a power of 10.

I would choose to use $\mathbf{10^{18}}$ as my unit. I'm ignoring the number with $\mathbf{10^3}$ because it is so much smaller than the other numbers. Most of the other numbers are close to $\mathbf{10^{18}}$. I will name my unit $\boldsymbol{Q}$.

b. Rewrite at least two pieces of data using the new unit.

$$\frac{\mathbf{1.9 \times 10^{15}}}{\mathbf{10^{18}}} = \mathbf{1.9 \times 10^{15-18}} = \mathbf{1.9 \times 10^{-3}} = \mathbf{0.0019}$$

$\mathbf{1.9 \times 10^{15}}$ ***rewritten in the new unit is*** $\mathbf{0.0019}\boldsymbol{Q}$.

$$\frac{\mathbf{7.02 \times 10^{19}}}{\mathbf{10^{18}}} = \mathbf{7.02 \times 10^{19-18}} = \mathbf{7.02 \times 10^{1}} = \mathbf{70.2}$$

$\mathbf{7.02 \times 10^{19}}$ ***rewritten in the new unit is*** $\mathbf{70.2}\boldsymbol{Q}$.

To rewrite the data, I will take the original number and divide it by the value of my unit, Q, which is 10^{18}.

1. Verify the claim that, in terms of gigaelectronvolts, the mass of an electron is 0.000511.

2. The maximum distance between Earth and the sun is 1.52098232×10^8 km, and the minimum distance is 1.47098290×10^8 km.[1] What is the average distance between Earth and the sun in scientific notation?

3. Suppose you measure the following masses in terms of kilograms:

2.6×10^{21}	9.04×10^{23}
8.82×10^{23}	2.3×10^{18}
1.8×10^{12}	2.103×10^{22}
8.1×10^{20}	6.23×10^{18}
6.723×10^{19}	1.15×10^{20}
7.07×10^{21}	7.210×10^{29}
5.11×10^{25}	7.35×10^{24}
7.8×10^{19}	5.82×10^{26}

What new unit might you introduce in order to aid discussion of the masses in this problem? Name your unit, and express it using some power of 10. Rewrite each number using your newly defined unit.

[1]Note: Earth's orbit is elliptical, not circular.

There is a general principle that underlies the comparison of two numbers in scientific notation: *Reduce everything to whole numbers if possible.* To this end, we recall two basic facts.

1. Inequality (A): Let x and y be numbers and let $z > 0$. Then $x < y$ if and only if $xz < yz$.
2. Comparison of whole numbers:
    a. If two whole numbers have different numbers of digits, then the one with more digits is greater.
    b. Suppose two whole numbers p and q have the same number of digits and, moreover, they agree digit-by-digit (starting from the left) until the n^{th} place. If the digit of p in the $(n+1)^{\text{th}}$ place is greater than the corresponding digit in q, then $p > q$.

Exercise 1

The Fornax Dwarf galaxy is 4.6×10^5 light-years away from Earth, while Andromeda I is 2.430×10^6 light-years away from Earth. Which is closer to Earth?

Exercise 2

The average lifetime of the tau lepton is 2.906×10^{-13} seconds, and the average lifetime of the neutral pion is 8.4×10^{-17} seconds. Explain which subatomic particle has a longer average lifetime.

Exploratory Challenge 1/Exercise 3

THEOREM: *Given two positive numbers in scientific notation, $a \times 10^m$ and $b \times 10^n$, if $m < n$, then $a \times 10^m < b \times 10^n$.*

Prove the theorem.

Exercise 4

Compare 9.3×10^{28} and 9.2879×10^{28}.

Exercise 5

Chris said that $5.3 \times 10^{41} < 5.301 \times 10^{41}$ because 5.3 has fewer digits than 5.301. Show that even though his answer is correct, his reasoning is flawed. Show him an example to illustrate that his reasoning would result in an incorrect answer. Explain.

Exploratory Challenge 2/Exercise 6

You have been asked to determine the exact number of Google searches that are made each year. The only information you are provided is that there are 35,939,938,877 searches performed each week. Assuming the exact same number of searches are performed each week for the 52 weeks in a year, how many total searches will have been performed in one year? Your calculator does not display enough digits to get the exact answer. Therefore, you must break down the problem into smaller parts. Remember, you cannot approximate an answer because you need to find an exact answer. Use the screen shots below to help you reach your answer.

Yahoo! is another popular search engine. Yahoo! receives requests for $1{,}792{,}671{,}355$ searches each month. Assuming the same number of searches are performed each month, how many searches are performed on Yahoo! each year? Use the screen shots below to help determine the answer.

Name ____ Date ____________________

1. Compare 2.01×10^{15} and 2.8×10^{13}. Which number is larger?

2. The wavelength of the color red is about 6.5×10^{-9} m. The wavelength of the color blue is about 4.75×10^{-9} m. Show that the wavelength of red is longer than the wavelength of blue.

1. If $a \times 10^n < b \times 10^n$, what are some possible values for a and b? Explain how you know.

   When two numbers are each raised to the same power of $\mathbf{10}$, in this case the power of $\boldsymbol{n}$, then you only need to look at the numbers $\boldsymbol{a}$ and $\boldsymbol{b}$ when comparing the values (Inequality (A) guarantees this). Since we know that $\boldsymbol{a \times 10^n < b \times 10^n}$, then we also know that $\boldsymbol{a < b}$. Then a possible value for $\boldsymbol{a}$ is $\mathbf{5}$, and a possible value for $\boldsymbol{b}$ is $\mathbf{6}$ because $\mathbf{5 < 6}$.

   I recall that Inequality (A) says: Let x and y be numbers and let $z > 0$. Then $x < y$ if and only if $xz < yz$.

2. Assume that $A \times 10^{-5}$ is not written in scientific notation and A is positive. That means that A is greater than zero but not necessarily less than 10. Is it possible to find a number A so that $A \times 10^{-5} < 1.1 \times 10^5$ is not true? If so, what number could A be?

   Since $\mathbf{10^{-5} = 0.00001}$ and $\mathbf{1.1 \times 10^5 = 110000}$, then a number for $\boldsymbol{A}$ bigger than $\mathbf{1.1 \times 10^{10}}$ would show that $\boldsymbol{A \times 10^{-5} < 1.1 \times 10^5}$ is not true.

   If $\boldsymbol{A = 1.1 \times 10^{10}}$, then by substitution

$$\begin{aligned} \boldsymbol{A \times 10^{-5}} &= \mathbf{(1.1 \times 10^{10}) \times 10^{-5}} \\ &= \mathbf{1.1 \times 10^{10+(-5)}} \\ &= \mathbf{1.1 \times 10^{5}}. \end{aligned}$$

   If $\boldsymbol{A = 1.1 \times 10^{10}}$, then $\boldsymbol{A \times 10^{-5} = 1.1 \times 10^5}$.
   Therefore, $\boldsymbol{A}$ can be any number as long as $\boldsymbol{A > 1.1 \times 10^{10}}$.

   If $A \times 10^{-5} < 1.1 \times 10^5$ is not written in scientific notation, it means that A can be a really large number. I am being asked if there is a number I can think of that, when multiplied by 10^{-5}, or its equivalent, $\frac{1}{100000}$, would be larger than 1.1×10^5.

3. Which of the following two numbers is greater?
   2.68941×10^{27} or 2.68295×10^{27}

   Since $\mathbf{2.68941 > 2.68295}$, then
   $\mathbf{2.68941 \times 10^{27} > 2.68295 \times 10^{27}}$.

   Since both numbers are raised to the same power (Inequality (A) again), all I need to compare is 2.68941 and 2.68295.

1. Write out a detailed proof of the fact that, given two numbers in scientific notation, $a \times 10^n$ and $b \times 10^n$, $a < b$, if and only if $a \times 10^n < b \times 10^n$.
   a. Let A and B be two positive numbers, with no restrictions on their size. Is it true that $A \times 10^{-5} < B \times 10^5$?
   b. Now, if $A \times 10^{-5}$ and $B \times 10^5$ are written in scientific notation, is it true that $A \times 10^{-5} < B \times 10^5$? Explain.

2. The mass of a neutron is approximately 1.674927×10^{-27} kg. Recall that the mass of a proton is 1.672622×10^{-27} kg. Explain which is heavier.

3. The average lifetime of the Z boson is approximately 3×10^{-25} seconds, and the average lifetime of a neutral rho meson is approximately 4.5×10^{-24} seconds.
   a. Without using the theorem from today's lesson, explain why the neutral rho meson has a longer average lifetime.
   b. Approximately how much longer is the lifetime of a neutral rho meson than a Z boson?

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