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Eureka Math™ Assessment Packet

Precalculus Module 4

Module 4

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End-of-Module Assessment	Qty: 30

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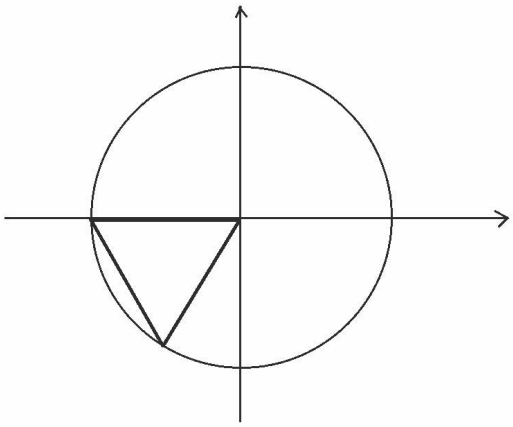
Name _____ Date _____

1. An equilateral triangle is drawn within the unit circle centered at the origin as shown.



Explain how one can use this diagram to determine the values of $\sin\left(\frac{4\pi}{3}\right)$, $\cos\left(\frac{4\pi}{3}\right)$, and $\tan\left(\frac{4\pi}{3}\right)$.

2. Suppose x is a real number with $0 < x < \frac{\pi}{4}$.

   a. Set $a = \sin(\pi - x)$, $b = \cos(\pi + x)$, $c = \sin(x - \pi)$, and $d = \cos(2\pi - x)$.
   Arrange the values a, b, c, and d in increasing order, and explain how you determined their order.

b. Use the unit circle to explain why $\tan(\pi - x) = -\tan(x)$.

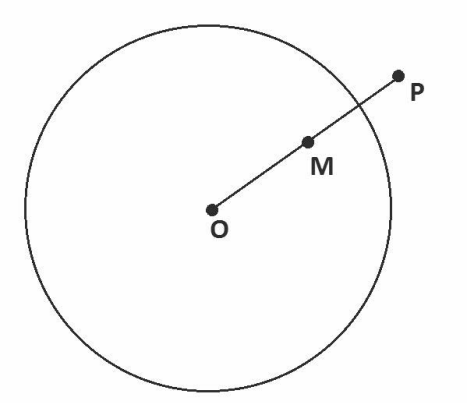
3.

a. Using a diagram of the unit circle centered at the origin, explain why $f(x) = \cos(x)$ is an even function.

b. Using a diagram of the unit circle centered at the origin, explain why $\sin(x - 2\pi) = \sin(x)$ for all real values x.

c. Explain why $\tan(x + \pi) = \tan(x)$ for all real values x.

4. The point P shown lies outside the circle with center O. Point M is the midpoint of $\overline{OP}$.



a. Use a ruler and compass to construct a line through P that is tangent to the circle.

b. Explain how you know that your construction does indeed produce a tangent line.

5. Each rectangular diagram below contains two pairs of right triangles, each having a hypotenuse of length 1. One pair of triangles has an acute angle measuring x radians. The other pair of triangles has an acute angle measuring y radians.

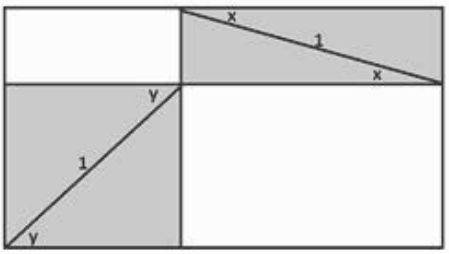


Figure 1

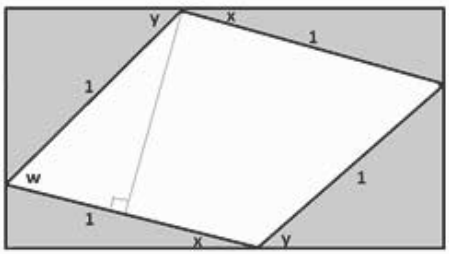


Figure 2

a. Using Figure 1, write an expression, in terms of x and y, for the area of the non-shaded region.

b. Figure 2 contains a quadrilateral which is not shaded and contains angle w. Write an expression, in terms of x and y, for the measure of angle w.

c. Using Figure 2, write an expression, in terms of w, for the non-shaded area. Explain your work.

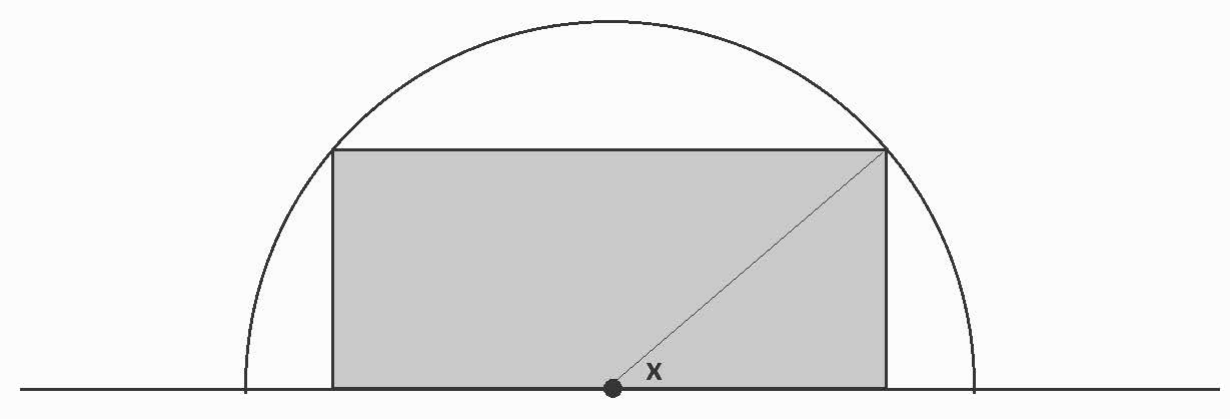
d. Use the results of parts (a), (b), and (c) to show why $\sin(x + y) = \sin(x)\cos(y) + \sin(y)\cos(x)$ is a valid formula.

e. Suppose α is a real number between $\frac{\pi}{2}$ and π and y is a real number between 0 and $\frac{\pi}{2}$. Use your result from part (d) to show the following:

$\cos(\alpha + y) = \cos(\alpha)\cos(y) - \sin(\alpha)\sin(y)$.

Explain your work.

6. A rectangle is drawn in a semicircle of radius 3 with its base along the base of the semicircle as shown.

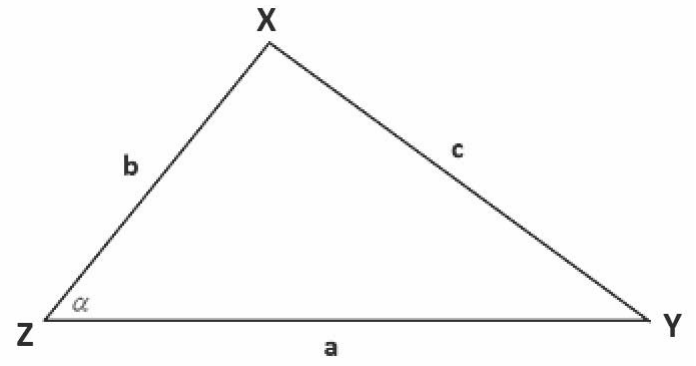


Find, to two decimal places, values for real numbers a and b so that $a\cos(x+b)$ represents the perimeter of the rectangle if the real number x is the measure of the angle shown.

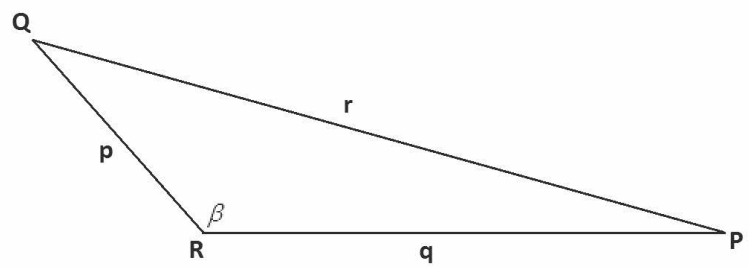
Name _____ Date _____

1.

a. In the following diagram, triangle XYZ has side lengths a, b, and c as shown. The angle α indicated is acute. Show that the area A of the triangle is given by $A = \frac{1}{2}ab\sin(\alpha)$.

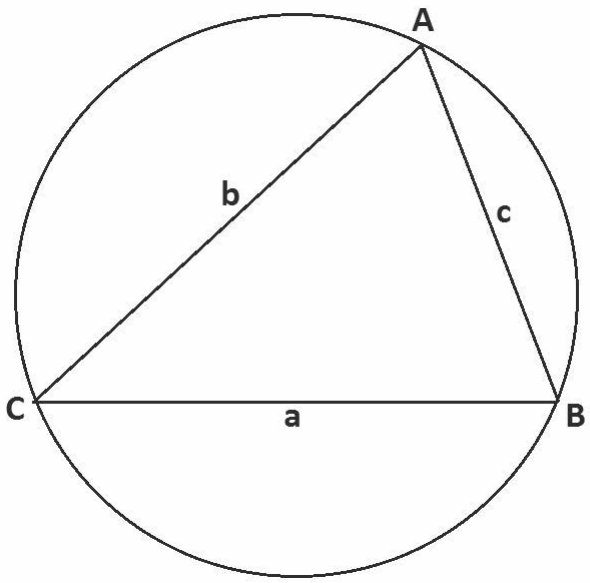


b. In the following diagram, triangle PQR has side lengths p, q, and r as shown. The angle β indicated is obtuse. Show that the area A of the triangle is given by $A = \frac{1}{2}pq\sin(\beta)$.



c. To one decimal place, what is the area of the triangle with sides of lengths 10 cm, 17 cm, and 21 cm? Explain how you obtain your answer.

2. Triangle ABC with side lengths a, b, and c as shown is circumscribed by a circle with diameter d.



a. Show that $\frac{a}{\sin(A)} = d$.

b. The law of sines states that $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ for any triangle ABC with side lengths a, b, and c (with the side of length a opposite vertex A, the side of length b opposite vertex B, and the side of length c opposite vertex C). Explain why the law of sines holds for all triangles.

c. Prove that $c^2 = a^2 + b^2 - 2ab\cos(C)$ for the triangle shown in the original diagram.

3. Beatrice is standing 20 meters directly east of Ari, and Cece is standing 15 meters directly northeast of Beatrice.

   a. To one decimal place, what is the distance between Ari and Cece?

   b. To one decimal place, what is the measure of the smallest angle in the triangle formed by Ari, Beatrice, and Cece?

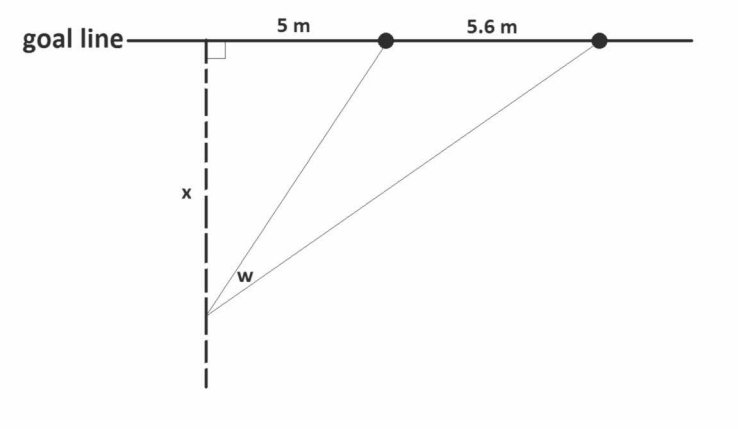
4.

a. Is it possible to construct an inverse to the sine function if the domain of the sine function is restricted to the set of real values between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$? If so, what is the value of $\sin^{-1}\left(\frac{1}{2}\right)$ for this inverse function? Explain how you reach your conclusions.

b. Is it possible to construct an inverse to the cosine function if the domain of the cosine function is restricted to the set of real values between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$? If so, what is the value of $\cos^{-1}\left(-\frac{1}{2}\right)$ for this inverse function? Explain how you reach your conclusions.

c. Is it possible to construct an inverse to the tangent function if the domain of the tangent function is restricted to the set of real values between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$? If so, what is the value of $\tan^{-1}(-1)$ for this inverse function? Explain how you reach your conclusions.

5. The diagram shows part of a rugby union football field. The goal line (marked) passes through two goal posts (marked as black circles) set 5.6 meters apart.

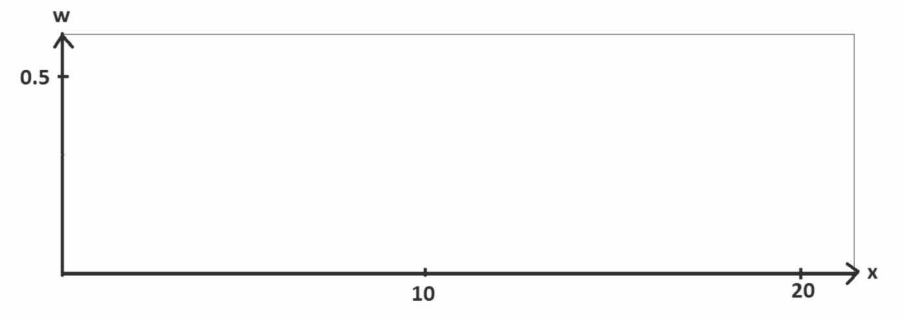


According to the rules of the game, an attempt at a conversion must be taken at a point on a line through the point of touchdown and perpendicular to the goal line. If a touchdown occurred 5 meters to one side of a goal post on the goal line, for example, the dashed line in the diagram indicates the line on which the conversion must be attempted.

Suppose the conversion is attempted at a distance of x meters from the goal line. Let w be the angle (measured in radians) indicated subtended by the goal posts.

a. Using inverse trigonometric functions, write an expression for w in terms of the distance x.

b. Using a graphing calculator or mathematics software, sketch a copy of the graph of the angle measure w as a function of x on the axes below. Indicate on your sketch the value of x that maximizes w. What is that maximal angle measure? (Give all your answers to two decimal places.)



c. In the original diagram, we see that the angle of measure w is one of three angles in an obtuse triangle. To two decimal places, what is the measure of the obtuse angle in that triangle when w has its maximal possible measure?

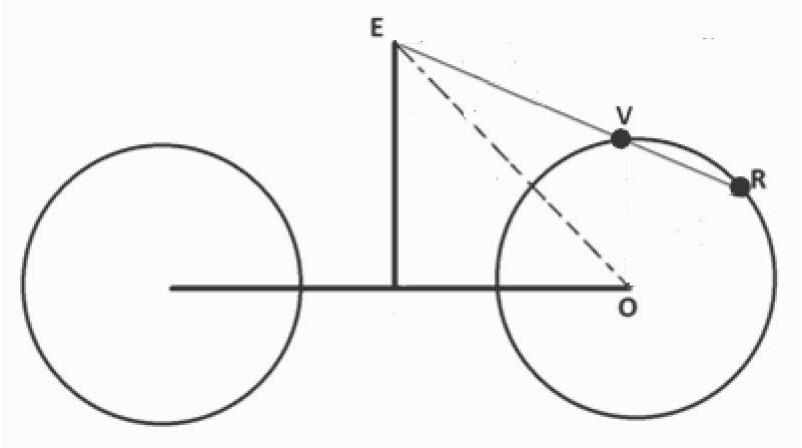
6. While riding her bicycle, Anu looks down for an instant to notice a reflector attached to the front wheel near its rim. As the bicycle moves, the wheel rotates and the position of the reflector relative to the frame of the bicycle changes. Consequently, the angle down from the horizontal that Anu needs to look in order to see the reflector changes with time.

   Anu also notices the air valve on the rim of the front wheel tire and observes that the valve and the reflector mark off about one-sixth of the perimeter of the front wheel.

   As Anu rides along a straight path, she knows that there will be a moment in time when the reflector, the valve, and her eye will be in line. She wonders what the angle between the horizontal from her eye and the line from her eye to the reflector passing through the valve is at this special moment.

   She estimates that the reflector and the valve are each 1.5 feet from the center of the front wheel, that her eye is 6 feet away from the center of that wheel, and that the line between her eye and the wheel center is $45°$ down from the horizontal.

   According to these estimates, what is the measure, to one decimal place in radians, of the angle Anu seeks?



In this diagram, O represents the center of the front wheel, E the location of Anu's eye, and R and V the positions of the reflector and valve, respectively, at the instant R, V, and E are collinear.