

Teacher Edition

Eureka Math

Grade 5

Module 5

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Grade 5 • Module 5

Addition and Multiplication with Volume and Area

OVERVIEW

In this 25-day module, students work with two- and three-dimensional figures. Volume is introduced to students through concrete exploration of cubic units and culminates with the development of the volume formula for right rectangular prisms. The second half of the module turns to extending students' understanding of two-dimensional figures. Students combine prior knowledge of area with newly acquired knowledge of fraction multiplication to determine the area of rectangular figures with fractional side lengths. They then engage in hands-on construction of two-dimensional shapes, developing a foundation for classifying the shapes by reasoning about their attributes. This module fills a gap between Grade 4's work with two-dimensional figures and Grade 6's work with volume and area.

In Topic A, students extend their spatial structuring to three dimensions through an exploration of volume. Students come to see volume as an attribute of solid figures and understand that cubic units are used to measure it. Using improvised, customary, and metric units, they build three-dimensional shapes, including right rectangular prisms, and count units to find the volume. By developing a systematic approach to counting the unit cubes, students make connections between area and volume. They partition a rectangular prism into layers of unit cubes and reason that the number of unit cubes in a single layer corresponds to the number of unit squares on a face. They begin to conceptualize the layers themselves, oriented in any one of three directions, as iterated units. This understanding allows students to reason about containers formed by box templates and nets, reasonably predict the number of cubes required to fill them, and test their predictions by packing the containers.

Concrete understanding of volume and multiplicative reasoning come together in Topic B as the systematic counting from Topic A leads naturally to formulas for finding the volume of a right rectangular prism. Students solidify the connection between volume as *packing* and volume as *filling* by comparing the amount of liquid that fills a container to the number of cubes that can be packed into it. This connection is formalized as students see that 1 cubic centimeter is equal to 1 milliliter. Complexity increases as students use their knowledge that volume is additive to partition and calculate the total volume of solid figures composed of non-overlapping, rectangular prisms. Word problems involving the volume of rectangular prisms with whole number edge lengths solidify understanding and give students the opportunity to reason about scaling in the context of volume. Topic B concludes with a design project that gives students the opportunity to apply the concepts and formulas they have learned throughout Topics A and B to create a sculpture of a specified volume composed of varied rectangular prisms with parameters given in the project description.

In Topic C, students extend their understanding of area as they use rulers and set squares to construct and measure rectangles with fractional side lengths and find their areas. Students apply their extensive knowledge of fraction multiplication to interpret areas of rectangles with fractional side lengths and solve real-world problems involving these figures, including reasoning about scaling through contexts in which volumes are compared. Visual models and equations are used to represent the problems through the Read-Draw-Write (RDW) protocol.

In Topic D, students draw two-dimensional shapes to analyze their attributes and use those attributes to classify them. Familiar figures, such as parallelograms, rhombuses, squares, trapezoids, etc., have all been defined in earlier grades and, in Grade 4, students have gained an understanding of shapes beyond the intuitive level. Grade 5 extends this understanding through an in-depth analysis of the properties and defining attributes of quadrilaterals. Grade 4's work with the protractor is applied to construct various quadrilaterals. Using measurement tools illuminates the attributes used to define and recognize each quadrilateral. Students see, for example, that the same process they used to construct a parallelogram also produces a rectangle when all angles are constructed to measure 90° . Students then analyze defining attributes and create a hierarchical classification of quadrilaterals.

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions. Omit Lessons 8 and 9, in which students create sculptures out of multiple rectangular prisms. Instead, consider asking the art teacher to complete a similar project with students. Lessons 14 and 15 can be consolidated since they share the same objective. Use Problems 1 and 2 from Lesson 14 and Problems 1 and 2 from Lesson 15. Problem 3 from Lesson 15 can be an extension for early finishers. Omit Lesson 21, and instead, use it in a center or periodically as morning work.

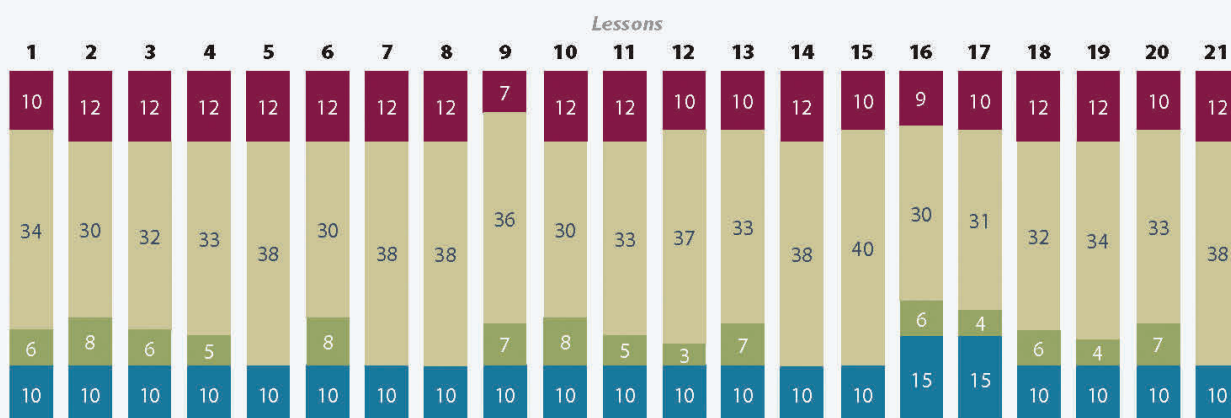
Note: If the drawing for Module 5, Topic D was done during Module 4, as suggested above, consolidate Lessons 16, 17, 18, and 19.



Distribution of Instructional Minutes

This diagram represents a suggested distribution of instructional minutes based on the emphasis of particular lesson components in different lessons throughout the module.

- **Fluency Practice**
- **Concept Development**
- **Application Problems**
- **Student Debrief**



Focus Grade Level Standards

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.¹

- Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
 - Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
- Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
 - A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
 - A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
 - Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
 - Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
 - Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

¹The balance of this cluster is addressed in Module 4.

Classify two-dimensional figures into categories based on their properties.

- Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
- Classify two-dimensional figures in a hierarchy based on properties.

Foundational Standards

- Recognize area as an attribute of plane figures and understand concepts of area measurement.
 - A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
 - A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
- Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
- Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
 - An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1/360$ of a circle is called a “one-degree angle,” and can be used to measure angles.
 - An angle that turns through n one-degree angles is said to have an angle measure of n degrees.
- Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
- Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
- Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
- Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

- Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
 - Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*

Focus Standards for Mathematical Practice

- **Make sense of problems and persevere in solving them.** Students work toward a solid understanding of volume through the design and construction of a three-dimensional sculpture within given parameters.
- **Reason abstractly and quantitatively.** Students make sense of quantities and their relationships when they analyze a geometric shape or real life scenario and identify, represent, and manipulate the relevant measurements. Students decontextualize when they represent geometric figures symbolically and apply formulas.
- **Construct viable arguments and critique the reasoning of others.** Students analyze shapes, draw conclusions, and recognize and use counterexamples as they classify two-dimensional figures in a hierarchy based on properties.
- **Model with mathematics.** Students model with mathematics as they make connections between addition and multiplication as applied to volume and area. They represent the area and volume of geometric figures with equations (and vice versa) and represent fraction products with rectangular areas. Students apply concepts of volume and area and their knowledge of fractions to design a sculpture based on given mathematical parameters. Through their work analyzing and classifying two-dimensional shapes, students draw conclusions about their relationships and continuously see how mathematical concepts can be modeled geometrically.
- **Attend to precision.** Mathematically proficient students try to communicate precisely with others. They endeavor to use clear definitions in discussion with others and their own reasoning. Students state the meaning of the symbols they choose, including using the equal sign (consistently and appropriately). They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, students have learned to examine claims and make explicit use of definitions.
- **Look for and make use of structure.** Students discern patterns and structures as they apply additive and multiplicative reasoning to determine volumes. They relate multiplying two of the dimensions of a rectangular prism to determining how many cubic units would be in each layer of the prism, as well as relate the third dimension to determining how many layers there are in the prism. This understanding supports students in seeing why volume can be computed as the product of three length measurements or as the product of one area by one length measurement. Additionally, recognizing that volume is additive allows students to find the total volume of solid figures composed of more than one non-overlapping right rectangular prism.

Overview of Module Topics and Lesson Objectives

Topics and Objectives		Days
A	Concepts of Volume	3
	Lesson 1: Explore volume by building with and counting unit cubes.	
	Lesson 2: Find the volume of a right rectangular prism by packing with cubic units and counting.	
	Lesson 3: Compose and decompose right rectangular prisms using layers.	
B	Volume and the Operations of Multiplication and Addition	6
	Lesson 4: Use multiplication to calculate volume.	
	Lesson 5: Use multiplication to connect volume as <i>packing</i> with volume as <i>filling</i> .	
	Lesson 6: Find the total volume of solid figures composed of two non-overlapping rectangular prisms.	
	Lesson 7: Solve word problems involving the volume of rectangular prisms with whole number edge lengths.	
	Lessons 8–9: Apply concepts and formulas of volume to design a sculpture using rectangular prisms within given parameters.	
Mid-Module Assessment: Topics A–B (assessment 1 day, return $\frac{1}{2}$ day, remediation or further applications $\frac{1}{2}$ day)		2
C	Area of Rectangular Figures with Fractional Side Lengths	6
	Lesson 10: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing, and relate to fraction multiplication.	
	Lesson 11: Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.	
	Lesson 12: Measure to find the area of rectangles with fractional side lengths.	
	Lesson 13: Multiply mixed number factors, and relate to the distributive property and the area model.	
	Lessons 14–15: Solve real-world problems involving area of figures with fractional side lengths using visual models and/or equations.	



Topics and Objectives		Days
D	Drawing, Analysis, and Classification of Two-Dimensional Shapes	6
	Lesson 16: Draw trapezoids to clarify their attributes, and define trapezoids based on those attributes.	
	Lesson 17: Draw parallelograms to clarify their attributes, and define parallelograms based on those attributes.	
	Lesson 18: Draw rectangles and rhombuses to clarify their attributes, and define rectangles and rhombuses based on those attributes.	
	Lesson 19: Draw kites and squares to clarify their attributes, and define kites and squares based on those attributes.	
	Lesson 20: Classify two-dimensional figures in a hierarchy based on properties.	
	Lesson 21: Draw and identify varied two-dimensional figures from given attributes.	
	End-of-Module Assessment: Topics A–D (assessment 1 day, return $\frac{1}{2}$ day, remediation or further applications $\frac{1}{2}$ day)	2
Total Number of Instructional Days		25

Terminology

New or Recently Introduced Terms

- Base (one face of a three-dimensional solid—often thought of as the surface on which the solid rests)
- Bisect (divide into two equal parts)
- Cubic units (cubes of the same size used for measuring volume)
- Height (adjacent layers of the base that form a rectangular prism)
- Hierarchy (series of ordered groupings of shapes)
- Unit cube (cube whose sides all measure 1 unit; cubes of the same size used for measuring volume)
- Volume of a solid (measurement of space or capacity)

Familiar Terms and Symbols²

- Angle (the union of two different rays sharing a common vertex)
- Area (the number of square units that covers a two-dimensional shape)
- Attribute (given quality or characteristic)

²These are terms and symbols students have seen previously.

- Cube (three-dimensional figure with six square sides)
- Degree measure of an angle (subdivide the length around a circle into 360 arcs of equal length; a central angle for any of these arcs is called a *one-degree angle* and is said to have an angle measure of 1 degree)
- Face (any flat surface of a three-dimensional figure)
- Kite (quadrilateral with two pairs of two equal sides that are also adjacent; a kite can be a rhombus if all sides are equal)
- Parallel lines (two lines in a plane that do not intersect)
- Parallelogram (four-sided closed figure with opposite sides that are parallel and equal)
- Perpendicular (two lines are *perpendicular* if they intersect, and any of the angles formed between the lines are 90° angles)
- Perpendicular bisector (line that cuts a line segment into two equal parts at 90°)
- Plane (flat surface that extends infinitely in all directions)
- Polygon (closed figure made up of line segments)
- Quadrilateral (closed figure with four sides)
- Rectangle (parallelogram with four 90° angles)
- Rectangular prism (three-dimensional figure with six rectangular sides)
- Rhombus (parallelogram with four equal sides)
- Right angle (angle formed by perpendicular lines; angle measuring 90°)
- Right rectangular prism (rectangular prism with only 90° angles)
- Solid figure (three-dimensional figure)
- Square units (squares of the same size—used for measuring)
- Three-dimensional figures (solid figures)
- Trapezoid (quadrilateral with at least one pair of parallel sides)
- Two-dimensional figures (figures on a plane)

Suggested Tools and Representations

- Area model
- Centimeter cubes
- Centimeter grid paper
- Isometric dot paper
- Patty paper (measuring 5.5 in \times 5.5 in)
- Protractor
- Ruler
- Set square or right angle template
- Tape diagram

Scaffolds³

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units*, please refer to “How to Implement *A Story of Units*.”

Assessment Summary

Type	Administered	Format
Mid-Module Assessment Task	After Topic B	Constructed response with rubric
End-of-Module Assessment Task	After Topic D	Constructed response with rubric

³Students with disabilities may require Braille, large print, audio, or special digital files. Please visit greatminds.org/contact to request information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.



Topic A

Concepts of Volume

Focus Standards:	■	Recognize volume as an attribute of solid figures and understand concepts of volume measurement.	
	□	A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.	
	□	A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	
	■	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	
Instructional Days: 3			
Coherence	-Links from:	G2–M8	Time, Shapes, and Fractions as Equal Parts of Shapes
		G3–M4	Multiplication and Area
		G3–M5	Fractions as Numbers on the Number Line
	-Links to:	G6–M5	Area, Surface Area, and Volume Problems

In Topic A, students extend their spatial structuring to three dimensions through an exploration of volume. They come to see volume as an attribute of solid figures and understand that cubic units are used to measure it. Using unit cubes, both customary and metric, students build three-dimensional shapes, including right rectangular prisms, and count to find the volume. By developing a systematic approach to counting the unit cubes, they make connections between area and volume.

Next, students pack rectangular prisms, folded from box templates, with centimeter cubes. This helps them visualize the layers of cubic units that compose volumes—an understanding that allows them to reasonably predict the number of cubes required to fill the containers and then test their predictions by packing the containers. Finally, students compose and decompose a rectangular prism from and into layers of unit cubes and reason that the number of unit cubes in a single layer corresponds to the number of unit squares on a face. They begin to conceptualize the layers themselves, oriented in any one of three directions, as iterated units.

A Teaching Sequence Toward Mastery of the Concepts of Volume

Objective 1: Explore volume by building with and counting unit cubes.
(Lesson 1)

Objective 2: Find the volume of a right rectangular prism by packing with cubic units and counting.
(Lesson 2)

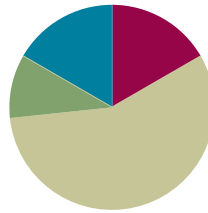
Objective 3: Compose and decompose right rectangular prisms using layers.
(Lesson 3)

Lesson 1

Objective: Explore volume by building with and counting unit cubes.

Suggested Lesson Structure

■ Fluency Practice	(10 minutes)
■ Concept Development	(34 minutes)
■ Application Problem	(6 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Multiply Whole Numbers Times Fractions Using Two Methods (5 minutes)
- Find the Area (5 minutes)

Multiply Whole Numbers Times Fractions Using Two Methods (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 4 content.

T: (Write $\frac{1}{2} \times 12 = \frac{\quad \times \quad}{2}$.) On your personal white board, write the complete number sentence.

S: (Write $\frac{1}{2} \times 12 = \frac{1 \times 12}{2}$.)

T: (Write $\frac{1}{2} \times 12 = \frac{1 \times 12}{2} = \frac{\quad}{2} = \underline{\quad}$.) Fill in the missing numbers.

S: (Write $\frac{1}{2} \times 12 = \frac{1 \times 12}{2} = \frac{12}{2} = 6$.)

T: (Write $\frac{1}{2} \times 12 = \frac{1 \times 12}{2}$.) Let's try another way. Divide by a common factor, and solve.

S: (Write $\frac{1}{2} \times 12 = \frac{1 \times \overset{6}{\cancel{12}}}{\underset{1}{\cancel{2}}} = 6$.)

T: Did you get the same answer using both methods?

S: Yes.

Continue with the following possible sequence: $16 \times \frac{1}{4}$, $16 \times \frac{3}{4}$, $\frac{2}{3} \times 9$, and $24 \times \frac{5}{6}$.

Find the Area (5 minutes)

Materials: (S) Personal white board

Note: Reviewing this Grade 4 concept prepares students to explore volume.

T: (Project a 4-inch by 2-inch rectangle.)
Name the shape.

S: Rectangle. → Parallelogram.
→ Quadrilateral.

T: (Write $__ \text{ in} \times __ \text{ in} = __ \text{ in}^2$.) This shape is a rectangle, though we could also call it a *quadrilateral* or *parallelogram*. On your personal white boards, write the area of the rectangle as a multiplication sentence starting with the length of the longest side.

S: (Write $4 \text{ in} \times 2 \text{ in} = 8 \text{ in}^2$.)

T: (Project a square with side lengths of 5 cm.) Name the shape.

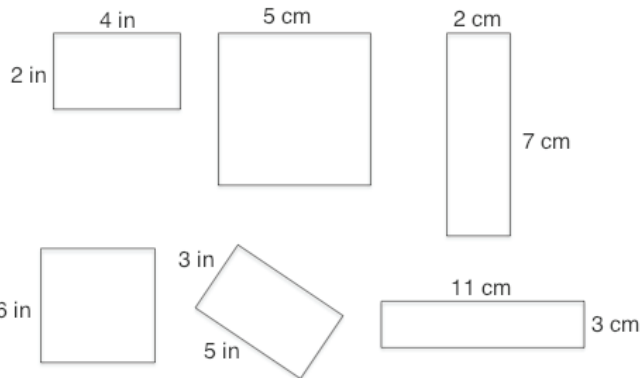
S: Square. → Rhombus. → Quadrilateral. → Parallelogram.

T: This shape is a square, but it is also correct to call it a *rhombus*, *quadrilateral*, or *parallelogram*. What is the measure of one of the square's sides?

S: 5 centimeters.

T: (Write $__ \text{ cm} \times __ \text{ cm} = __ \text{ cm}^2$.) On your boards, write the area of the square as a multiplication sentence using the measure of the square's sides.

S: (Write $5 \text{ cm} \times 5 \text{ cm} = 25 \text{ cm}^2$.)



Continue this process for the other squares and rectangles.

Concept Development (34 minutes)

Materials: (T) 20 centimeter cubes (S) Ruler, 20 centimeter cubes, centimeter grid paper (Template 1), isometric dot paper (Template 2)

Problem 1: Build a solid from cubes.

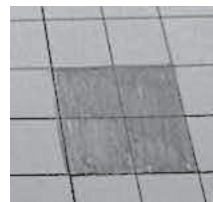
T: Shade a square on your centimeter grid paper with an area of 4 square units. (Pause to allow students to do this.)

T: This is going to be the foundation for our structure. Place 4 cubes directly on top of that square.

S: (Do so.)

T: Think of the first 4 cubes as the ground floor of a building. Make a second floor by putting another 4 cubes on top of them. (Pause.) How many cubes are there now?

S: 8 cubes.



- T: Did we change the ground floor? Why or why not? Turn and talk.
- S: No. We just built on top of it. → The second layer of cubes doesn't make it take up more space on the ground. → We built up, not out, so the structure got taller, not longer or wider.
- T: Put one more layer of 4. (Pause.) Explain to your partner how you know the total number of cubes.
- S: I just counted up from 8 as I put each cube. → Each floor had 4 blocks, so it's 3 fours. → I thought of 3 times 4, 12.
- T: What is the total number of cubes in your solid?
- S: 12 cubes.

Problem 2: Build solids with a given volume with cubic centimeters.

- T: Since this is a cube with each edge measuring 1 centimeter, we call this a **cubic centimeter**.
- T: (Hold up a centimeter cube.) These cubes can serve as a unit to measure the **volume** of your solid, the amount of space it takes up. What do we call this unit?
- S: A cubic centimeter.
- T: Just like we use squares to measure area in square units, we use cubes to measure volume in cubic units. (Write cubic unit, cubic centimeter, and cm^3 on the board.)
- T: (Hold up 2 cubes.) How many cubes?
- S: 2 cubes.
- T: How many cubic centimeters?
- S: 2 cubic centimeters.
- T: (Hold up 4 cubes in a square formation.)
- T: What is the volume of these 4 units together?
- S: 4 cubic centimeters.
- T: Work with a partner. On your grid paper, build three different solids with a volume of 4 cubic centimeters.

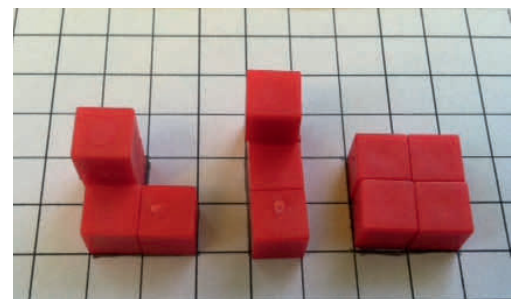
Give students time to build the structures. Move on to do likewise with five and then six cubes as time allows. While circulating, encourage students to use the words *volume* and *cubic centimeters* when answering questions.



**NOTES ON
MULTIPLE MEANS
OF REPRESENTATION:**

If only 1-inch cubes are available, adapt the lesson to work with 1-inch cubes. Try to obtain 1-inch grid paper from the Internet, or create it on the computer and print it for students to use.

Possible Solutions

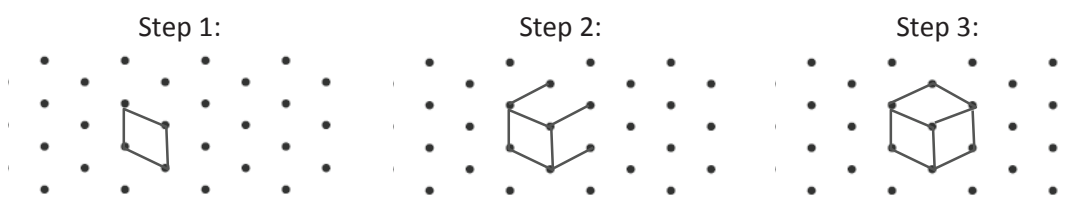


Problem 3: Represent solids on isometric dot paper.

- T: We are going to build some other structures, but we want to draw what we build. Let's learn how to use our isometric dot paper to draw our structures.
- T: We will start by drawing 1 cube. (Demonstrate while directing students in each step.)

Explain the process for drawing 1 cubic centimeter using the dot paper.

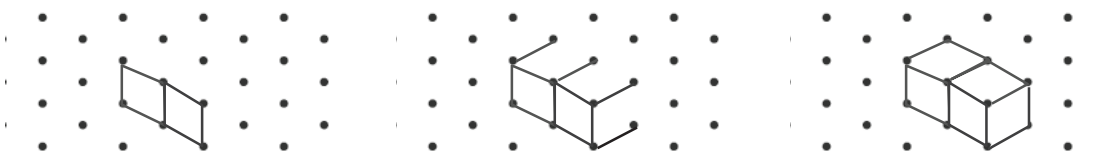
- Step 1: Connect four dots to make a parallelogram. This will represent one square face of the cube, viewed at an angle.
- Step 2: Draw three straight segments to the right from the two vertices on the top and the one on the bottom right.
- Step 3: Draw two segments to represent the missing edges.



T: Now we will put two cubes next to each other.

Explain the process for drawing 2 cubic centimeters.

- Step 1: Connect four dots to make a parallelogram.
- Step 2: Add another parallelogram that shares its right edge, just like your cubes.
- Step 3: Draw four straight segments to the right from the three vertices on the top and the one on the bottom right.
- Step 4: Draw three segments to represent the missing edges.



Allow students to practice several times. Then, choose examples of several students' work to show the class.

- T: With a partner, build a structure with no more than 10 cubes each. Then, draw your partner's structure on dot paper. Help each other figure out if it matches what you built.

Circulate and help students draw their figures. When they are comfortable with the process, move them to the Application Problem and Problem Set.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

The spatial reasoning required to draw centimeter cubes on isometric dot paper may be difficult for some students. Pattern block rhombuses may help students orient their drawings. Three rhombuses may be laid on paper (with or without dots) and traced to draw a cube.

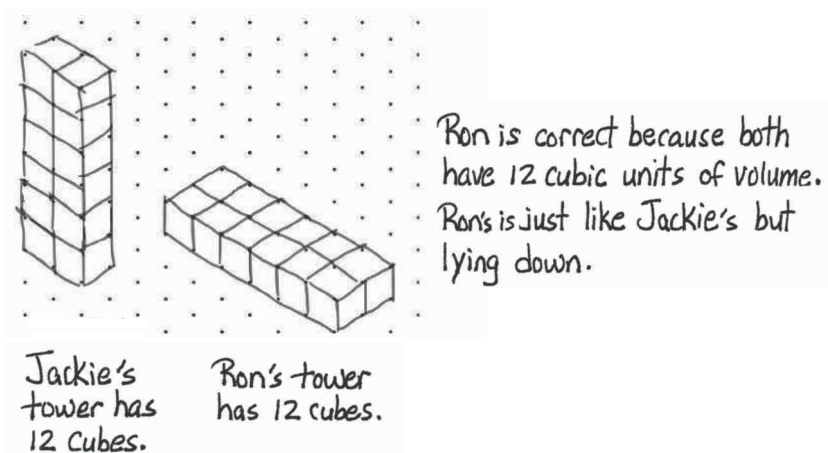
Students may also trace the yellow hexagon block and simply add three interior lines to create the cube.

Application Problem (6 minutes)

Jackie and Ron both have 12 centimeter cubes. Jackie builds a tower 6 cubes high and 2 cubes wide. Ron builds one 6 cubes long and 2 cubes wide.

Jackie says her structure has the greater volume because it is taller. Ron says that the structures have the same volume.

Who is correct? Draw a picture to explain how you know. Use grid paper if you wish.



Note: This Application Problem is intended to help students synthesize the parts they have learned in the lesson and build understanding toward subsequent lessons, where eventually they learn the formula for the volume of a rectangular prism.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding. Students should solve these problems using the RDW approach used for Application Problems.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Consider what students see written on the board or projected on a screen:

- At different times of the day, walk around the classroom to become aware of light patterns in the room and how sunlight affects the visibility of white boards.
- Students who are trying to process new or challenging information can benefit if the written or projected material is neat and precise. Therefore, be mindful to always practice writing clearly and legibly.

Name Kenny Date 5-18

1. Use your centimeter cubes to build the figures pictured below on centimeter grid paper. Find the total volume of each figure you built, and explain how you counted the cubic units. Be sure to include units.

A.

B.

C.

D.

E.

F.

Figure	Volume	Explanation
A	1 cm ³	I just counted one cube.
B	5 cm ³	I added 3 cubes and 2 cubes.
C	9 cm ³	I multiplied 3 layers x 3 cubes.
D	9 cm ³	I counted the front layer, then carefully counted the back.
E	12 cm ³	I counted the end layer (4 cubes), and then multiplied by 3.
F	20 cm ³	I counted the bottom layer, and then multiplied by 2. 10 x 2 = 20

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the purposeful sequencing of the Problem Set guide the selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Consider assigning incomplete problems for homework or at another time during the day.

Student Debrief (10 minutes)

Lesson Objective: Explore volume by building with and counting unit cubes.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

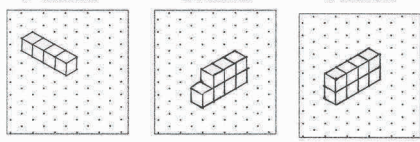
- In Problem 1, compare your answers for Figures C and D. What patterns do you notice?
- Compare your answers to Problem 2 with a partner. How were your drawings the same? Different?
- What was Joyce's mistake in Problem 3? What do you need to think about when counting cubic centimeters in drawings? How is it different from counting them in person? Is it possible for a drawing to fool you? Might some cubes be hidden, or might there be gaps that you cannot see?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

2. Build 2 different structures with the following volumes using your unit cubes. Then, draw one of the figures on the dot paper. One example has been drawn for you.


a. 4 cubic units b. 7 cubic units c. 8 cubic units



3. Joyce says that the figure below, made of 1 cm cubes, has a volume of 5 cubic centimeters.

a. Explain her mistake.

Joyce is not counting the one that is hidden. The cube that's on the second layer needs to be sitting on a hidden cube.



b. Imagine if Joyce adds to the second layer so the cubes completely cover the first layer in the figure above. What would be the volume of the new structure? Explain how you know.

The volume would be 10 cm^3 . I counted the first layer, and then multiplied by 2.

$$5 \text{ cm}^3 \times 2 = 10 \text{ cm}^3$$

Name _____

Date _____

1. Use your centimeter cubes to build the figures pictured below on centimeter grid paper. Find the total volume of each figure you built, and explain how you counted the cubic units. Be sure to include units.

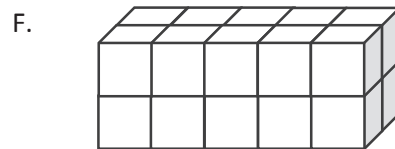
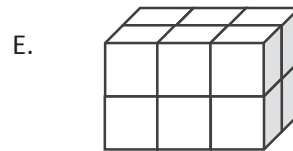
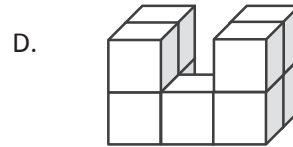
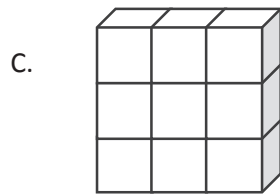
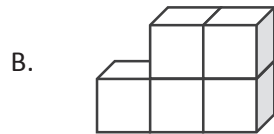
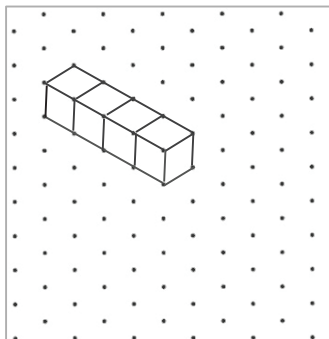


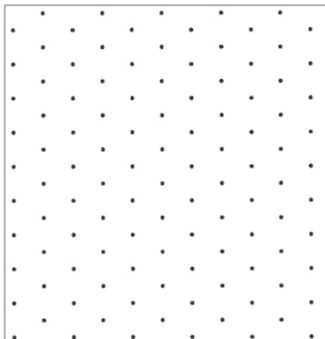
Figure	Volume	Explanation
A		
B		
C		
D		
E		
F		

2. Build 2 different structures with the following volumes using your unit cubes. Then, draw one of the figures on the dot paper. One example has been drawn for you.

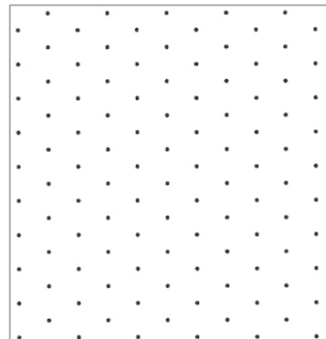
a. 4 cubic units



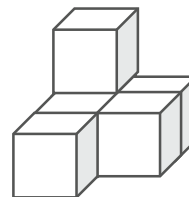
b. 7 cubic units



c. 8 cubic units



3. Joyce says that the figure below, made of 1 cm cubes, has a volume of 5 cubic centimeters.
- a. Explain her mistake.



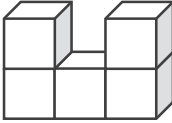
- b. Imagine if Joyce adds to the second layer so the cubes completely cover the first layer in the figure above. What would be the volume of the new structure? Explain how you know.

Name _____

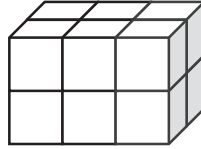
Date _____

1. What is the volume of the figures pictured below?

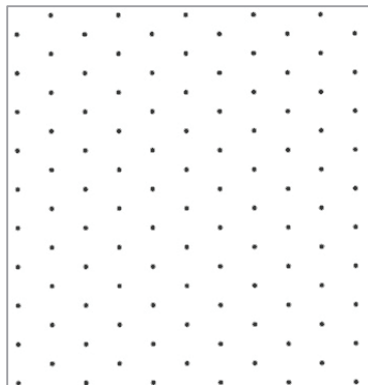
a.



b.



2. Draw a picture of a figure with a volume of 3 cubic units on the dot paper.



Name _____

Date _____

1. The following solids are made up of 1 cm cubes. Find the total volume of each figure, and write it in the chart below.

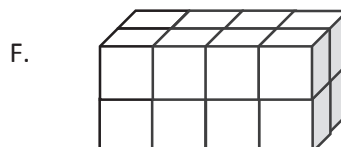
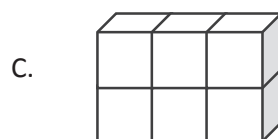
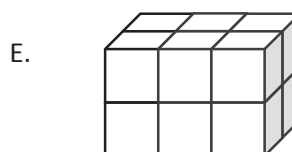
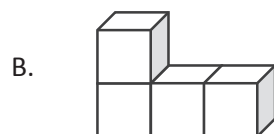
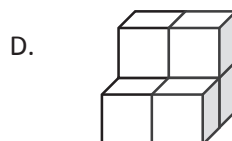
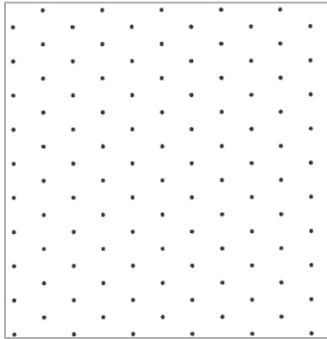


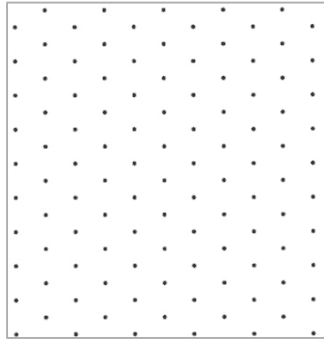
Figure	Volume	Explanation
A		
B		
C		
D		
E		
F		

2. Draw a figure with the given volume on the dot paper.

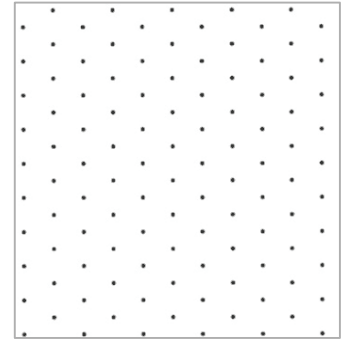
a. 3 cubic units



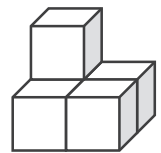
b. 6 cubic units



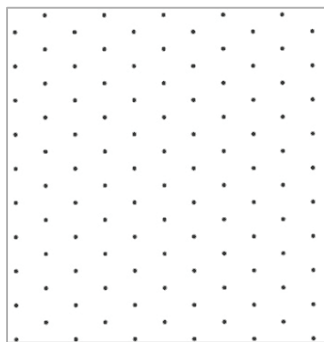
c. 12 cubic units

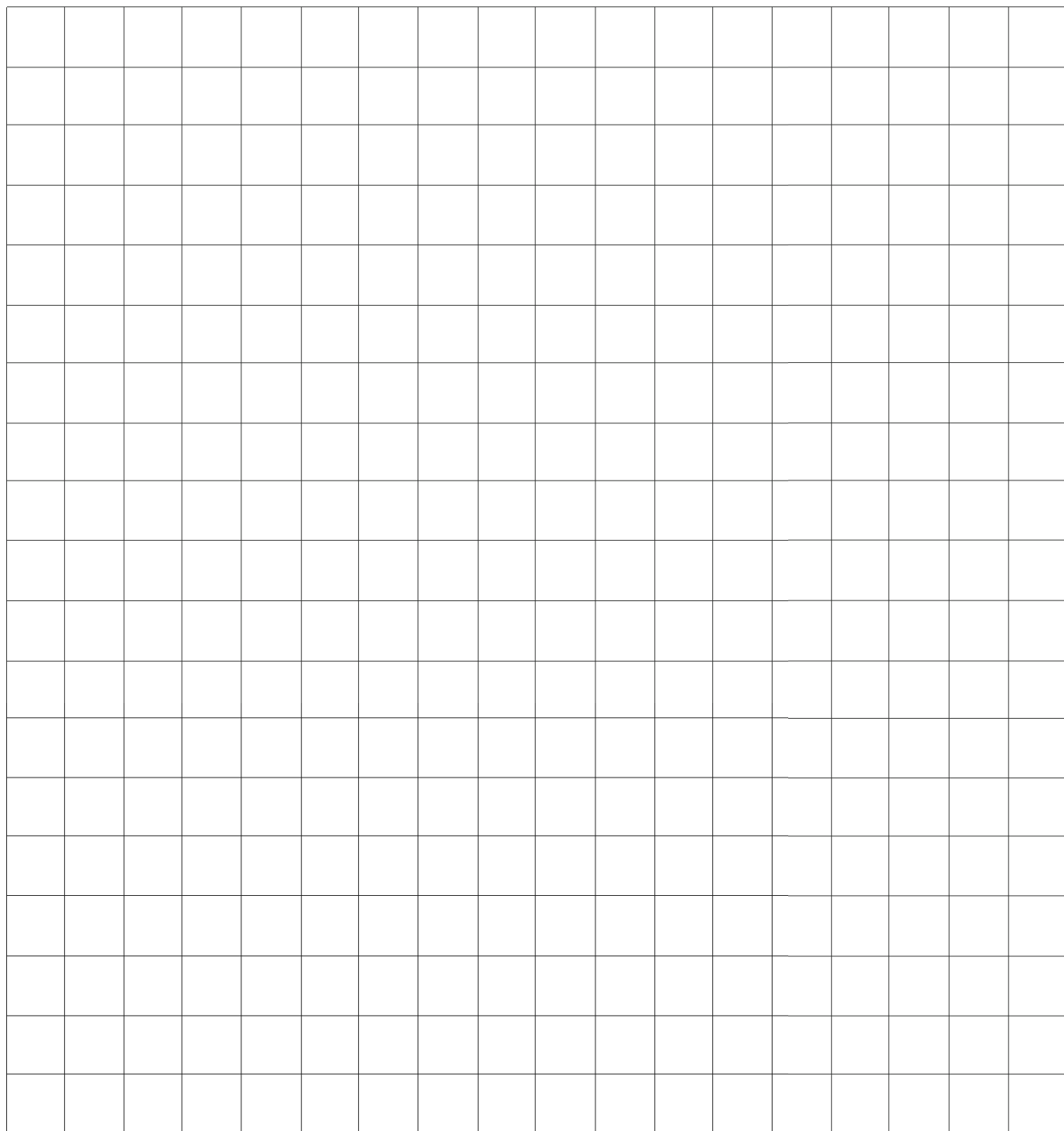


3. John built and drew a structure that has a volume of 5 cubic centimeters. His little brother tells him he made a mistake because he only drew 4 cubes. Help John explain to his brother why his drawing is accurate.

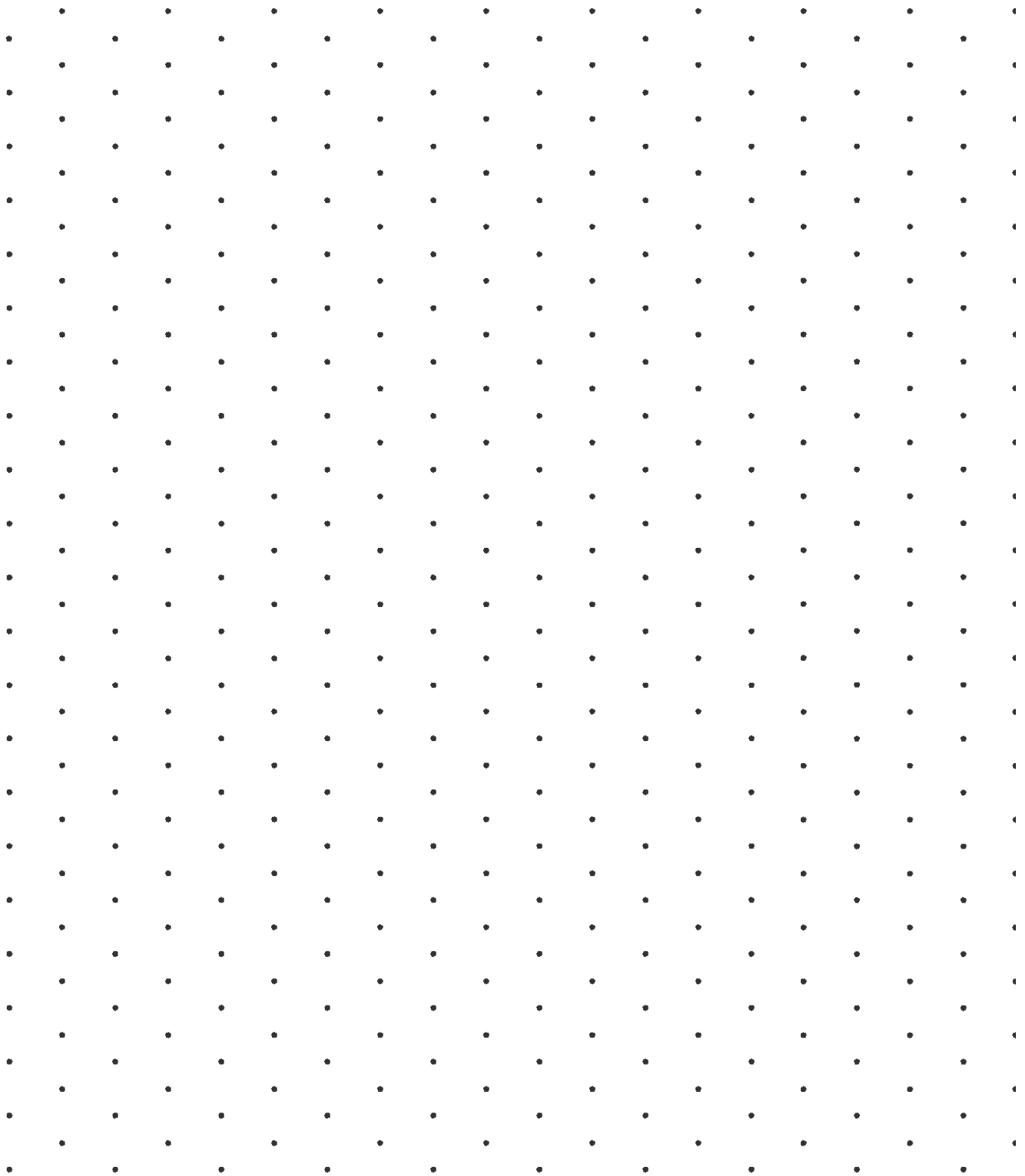


4. Draw another figure below that represents a structure with a volume of 5 cubic centimeters.





centimeter grid paper



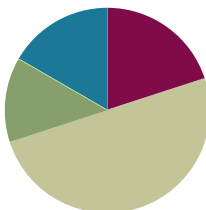
isometric dot paper

Lesson 2

Objective: Find the volume of a right rectangular prism by packing with cubic units and counting.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(8 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Multiply a Fraction and a Whole Number (4 minutes)
- Find the Area (4 minutes)
- Find the Volume (4 minutes)

Multiply a Fraction and a Whole Number (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 4 Lessons 6–8.

T: (Write $15 \div 3 = \underline{\quad}$.) Say the complete division sentence.

S: $15 \div 3 = 5$.

T: (Write $\frac{1}{3} \times 15 = \underline{\quad}$.) Say the complete multiplication sentence.

S: $\frac{1}{3} \times 15 = 5$.

T: (Write $\frac{2}{3} \times 15 = \underline{\quad}$.) On your personal white board, write the complete multiplication sentence.

S: (Write $\frac{2}{3} \times 15 = 10$.)

T: (Write $15 \times \frac{2}{3} = \underline{\quad}$.) Write the complete multiplication sentence.

S: (Write $15 \times \frac{2}{3} = 10$.)

Continue with the following possible sequence: $18 \div 6$, $\frac{1}{6} \times 18$, $\frac{5}{6} \times 18$, $18 \times \frac{5}{6}$, $\frac{1}{8} \times 32$, $32 \times \frac{1}{8}$, $32 \times \frac{5}{8}$, $\frac{2}{3} \times 18$, and $24 \times \frac{3}{4}$.

Find the Area (4 minutes)

Materials: (S) Personal white board

Note: Reviewing this Grade 4 concept prepares students to calculate volume.

T: (Project an 8 ft by 2 ft rectangle.)
Name the shape.

S: Rectangle. → Parallelogram.
→ Trapezoid. → Quadrilateral.

T: (Write $\underline{\hspace{1cm}}$ ft \times $\underline{\hspace{1cm}}$ ft = $\underline{\hspace{1cm}}$ ft².)
This shape is a rectangle, although we could also call it a *parallelogram*, *trapezoid*, or *quadrilateral*. On your personal white board, write the area of the rectangle as a multiplication sentence starting with the length of the longest side.

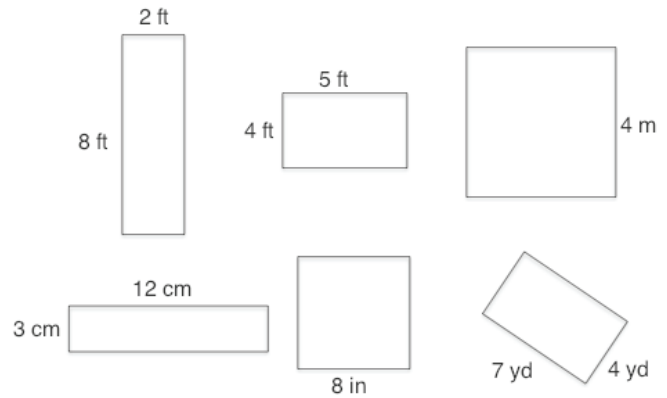
S: (Write $8 \text{ ft} \times 2 \text{ ft} = 16 \text{ ft}^2$.)

T: (Project a square with side lengths 4 m.) Name the shape.

S: Square. → Rhombus. → Rectangle. → Parallelogram. → Trapezoid. → Quadrilateral.

T: (Write $\underline{\hspace{1cm}}$ m \times $\underline{\hspace{1cm}}$ m = $\underline{\hspace{1cm}}$ m².) On your personal white board, write the area of the square as a multiplication sentence using the measure of the square's sides.

S: (Write $4 \text{ m} \times 4 \text{ m} = 16 \text{ m}^2$.)



Continue the process for the other squares and rectangles.

Find the Volume (4 minutes)

Materials: (S) Personal white board

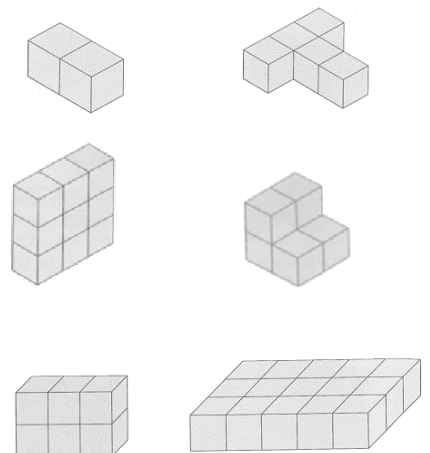
Note: This fluency activity reviews Lesson 1.

T: (Project the first of the images to the right.)
Each cube is 1 cubic centimeter.

T: (Write Volume = $\underline{\hspace{1cm}}$ cubic cm.) On your personal white board, complete the equation.

S: (Write Volume = 2 cubic cm.)

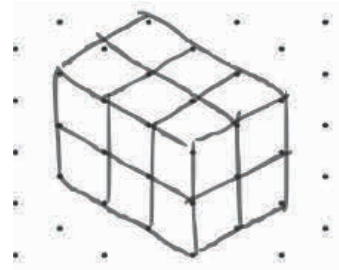
Continue the process for the remaining images.



Application Problem (8 minutes)

Mike uses 12 centimeter cubes to build structures. Use centimeter cubes to build at least 3 different structures with the same volume as Mike's. Record one of your structures on dot paper.

Note: This Application Problem is designed to bridge from the previous lesson. Students work with different factors of 12 to discover different ways of arranging the cubes. This leads to a better understanding of how rectangular prisms are constructed from identical cubes and toward developing an understanding of volume. Students may work with a partner if desired.



Concept Development (30 minutes)

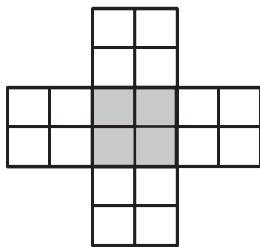
Materials: (S) Pencil, centimeter grid paper (Lesson 1 Template 1, also needed for Homework), scissors, tape, 50 centimeter cubes, net (Template), Problem Set

Note: This lesson uses the Problem Set as part of the lesson.

Problem 1(a)

Project the image from Problem 1(a) from the Problem Set.

- T: To make a box, copy this image on grid paper by first shading the bottom of the box and then outlining the figure. (If necessary, model how to draw onto grid paper.)
- T: Now, cut around the outside. The bottom is shaded, so fold up the flaps to make the sides of the box. Crease well, and tape to make the edges of the box. (Model cutting and folding as necessary.)



- T: Fill the box with cubic centimeters to find how many cubes fill the box up.
- S: Eight cubes.
- T: What is the volume of the box?
- S: 8 cubic centimeters. → 8 centimeters cubed.
- T: Talk to your partner about different ways to pack and count the cubic centimeters.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Breaking down a tissue or cereal box to show how the sides form a flat shape and then building it back into a box may be helpful for students to understand the figures used in the lesson to make the boxes.

Be aware that spatial skills and fine motor skills vary widely among fifth graders. Some may require more time to cut, fold, and tape the boxes. Proximity to the teacher and the demonstration can support students whose spatial skills are developing.

- S: You can just count one by one. → You can put in a row of two on the bottom and then another row on top of that to have four. It looks like a slice. Another slice makes four more. → You can put four on the bottom and another four on top of that. 2×4 is 8. → You can count by two or by four.

Problem 1(b)

- T: Let's fold to make another box with rectangular sides, or a **rectangular prism**.

Follow the same procedure as with Problem 1(a) to have students make the prism and pack the cubes into the box.

- T: What is the volume of this box?

S: 16 cubic centimeters.

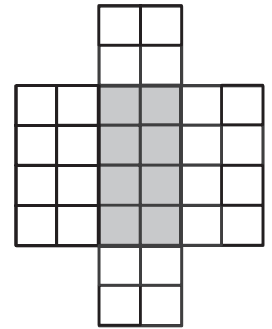
- T: How does its volume compare with the volume of the first rectangular prism?

S: It doubled.

- T: Interesting. Why do you think that might be? Turn and talk.

S: It was like two of the first box laid side by side. → The bottom was twice as long, but it had the same number of layers, so it was 16.
→ The sides of the box were the same height. Just the cubic centimeters on the bottom doubled.

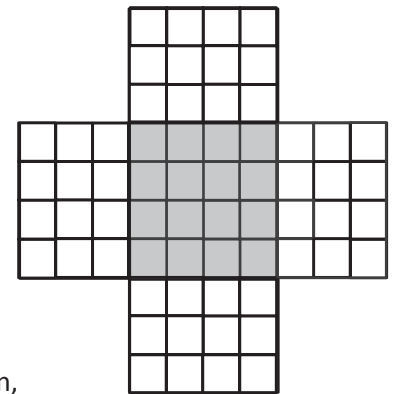
- T: Look at the image on the board. Talk to your partner about how you might find the volume without packing it.



Problem 1(c)

- T: (Project or show the next figure.) This time, put in the first layer of cubes on the bottom without cutting to actually make the box. (Pause.) What do you think the volume of this box will be?

S: The flaps show the number of layers. → The bottom has 16 cubes, and there are 3 layers. → The bottom is 4 by 4, and it looks like it will be 3 layers, so ... 4×4 is 16, and double is 32, and another 16 is 48, so 48. → I think it will be 48 cubic centimeters because 16×3 is 48.



Allow students to answer Problems 2(a) and 2(b) independently. Check the answers and students' thinking together following the sequence above. Then, distribute the box patterns on the Template for students to cut out, and have them work independently on Problem 3.

Student Debrief (10 minutes)

Lesson Objective: Find the volume of a right rectangular prism by packing with cubic units and counting.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How did you pack your boxes in Problem 1 (a), (b), and (c)? Cube by cube, row by row, or layer by layer? Did the way you packed your boxes change from problem to problem? If so, how and why did your thinking change?
- In Problem 2, how did you verify your prediction? Did your prediction change between 2 (a) and (c)? Why or why not?
- What did you discover in Problem 3? Did your discovery match your prediction? Could you have used fewer cubes to make your prediction? Why or why not?
- How has your understanding of the term *volume* changed from yesterday to today?



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Encourage students who easily grasp this concept and move quickly through the Problem Set to think about the results of the same problems if the units were 2 cm cubes, 3 cm cubes, and so on.

Name: Ada Date: _____

1. Shade the following figures on centimeter grid paper. Cut and fold each to make 3 open boxes, taping them so they hold their shapes. Pack each box with cubes. Write how many cubes fill the box.

a. Number of cubes: 8

b. Number of cubes: 16


c. Number of cubes: 48

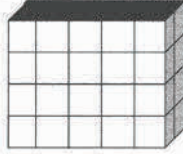
2. Predict how many centimeter cubes will fit in each box and briefly explain your prediction. Use cubes to find the actual volume. (The figures are not drawn to scale.)

a. Prediction: 8 cm³
Actual: 8 cm³
It's 4 cubes across and 2 deep, so 8 cubes altogether.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

b.  Prediction: 16 cm³
Actual: 16 cm³
There are 2 layers, top and bottom. Each layer has 8 cubes.
 $8 \text{ cubes} \times 2 = 16 \text{ cubes}$.

c.  Prediction: 40 cm³
Actual: 40 cm³
There are 4 layers. Each layer has 10 cubes.
 $10 \text{ cubes} \times 4 = 40 \text{ cubes}$.

3. Cut out the net in the template, and fold it into a cube. Predict the number of 1-centimeter cubes that would be required to fill it.

a. Prediction: 24 cubes

b. Explain your thought process as you made your prediction.
I saw that the net had six faces, so I knew it would not be an open box. When I folded the net, I discovered it had the shape of a cube.

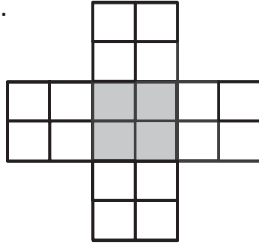
c. How many 1-centimeter cubes are used to fill the figure? Was your prediction accurate?
It took 27-centimeter cubes to fill the figure. My Prediction of 24 cubes wasn't 100% accurate but it was close.

Name _____

Date _____

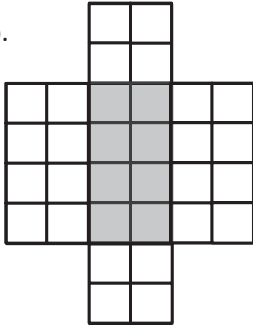
1. Shade the following figures on centimeter grid paper. Cut and fold each to make 3 open boxes, taping them so they hold their shapes. Pack each box with cubes. Write how many cubes fill each box.

a.



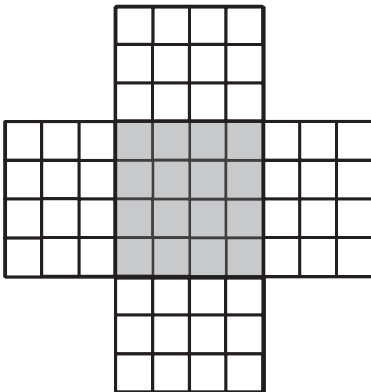
Number of cubes: _____

b.



Number of cubes: _____

c.



Number of cubes: _____

2. Predict how many centimeter cubes will fit in each box, and briefly explain your predictions. Use cubes to find the actual volume. (The figures are not drawn to scale.)

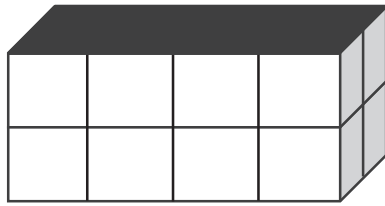
a.



Prediction: _____

Actual: _____

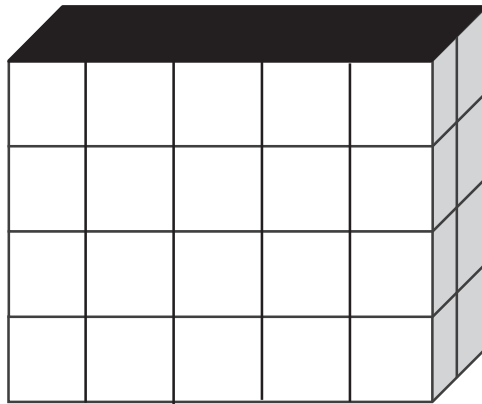
b.



Prediction: _____

Actual: _____

c.



Prediction: _____

Actual: _____

3. Cut out the net in the template, and fold it into a cube. Predict the number of 1-centimeter cubes that would be required to fill it.

a. Prediction: _____

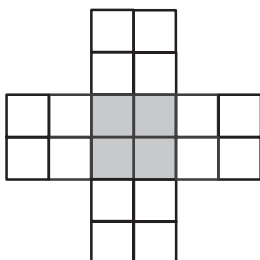
b. Explain your thought process as you made your prediction.

c. How many 1-centimeter cubes are used to fill the figure? Was your prediction accurate?

Name _____

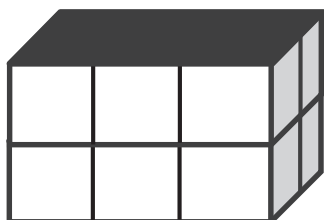
Date _____

1. If this figure were to be folded into a box, how many cubes would fill it?



Number of cubes: _____

2. Predict how many centimeter cubes will fit in the box, and briefly explain your prediction. Use cubes to find the actual volume. (The figure is not drawn to scale.)



Prediction: _____

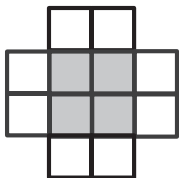
Actual: _____

Name _____

Date _____

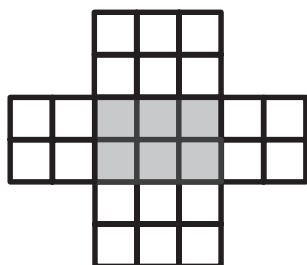
1. Make the following boxes on centimeter grid paper. Cut and fold each to make 3 open boxes, taping them so they hold their shapes. How many cubes would fill each box? Explain how you found the number.

a.



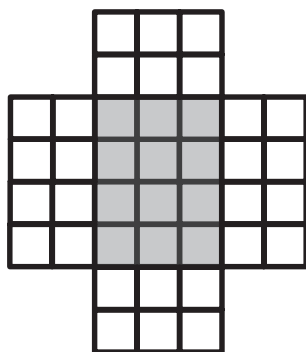
Number of cubes: _____

b.



Number of cubes: _____

c.



Number of cubes: _____

2. How many centimeter cubes would fit inside each box? Explain your answer using words and diagrams on each box. (The figures are not drawn to scale.)

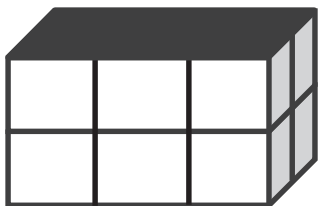
a.



Number of cubes: _____

Explanation:

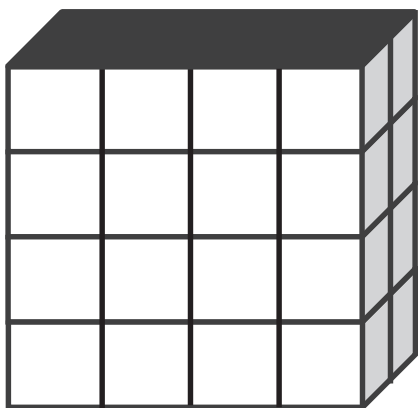
b.



Number of cubes: _____

Explanation:

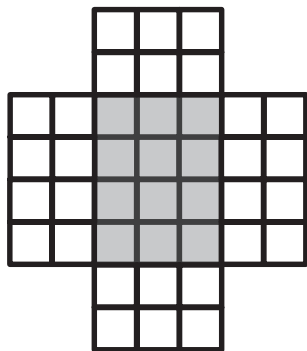
c.

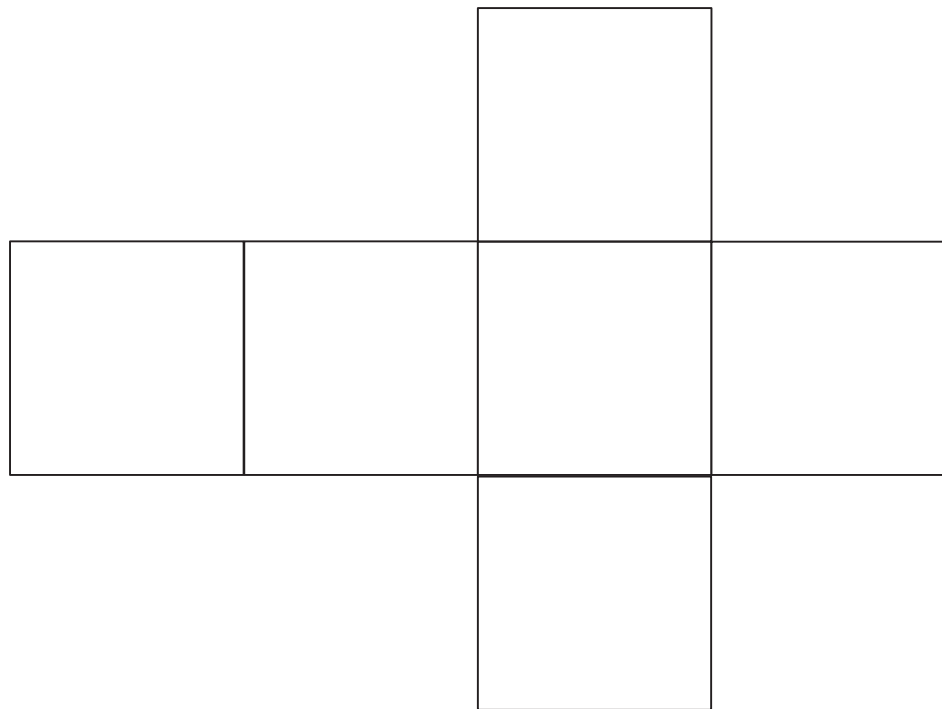
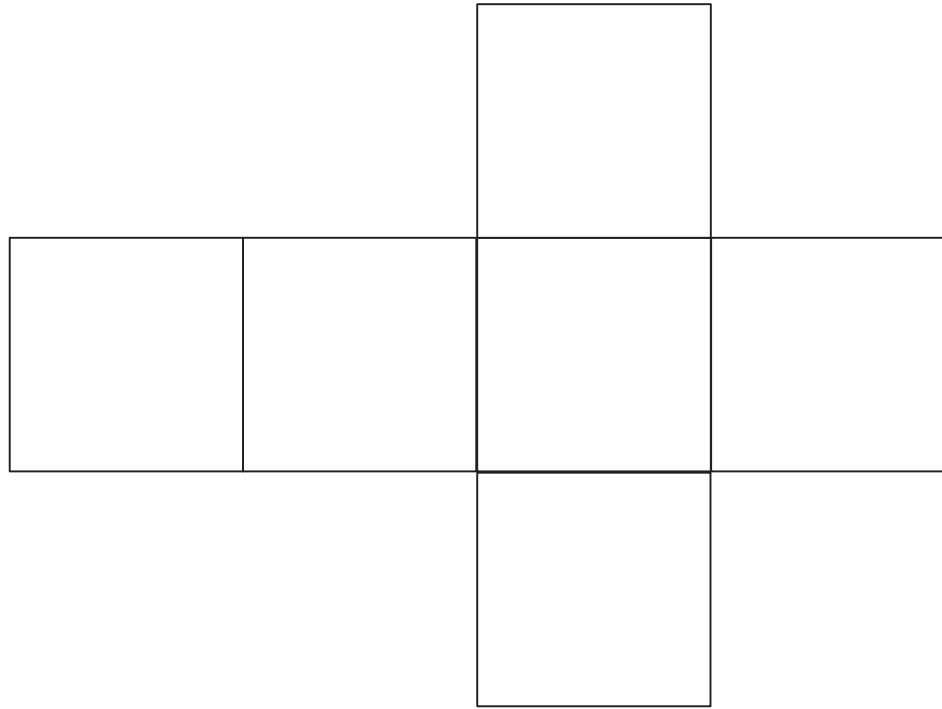


Number of cubes: _____

Explanation:

3. The box pattern below holds 24 1-centimeter cubes. Draw two different box patterns that would hold the same number of cubes.





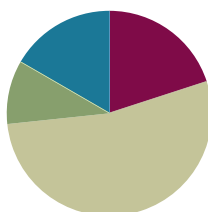
net

Lesson 3

Objective: Compose and decompose right rectangular prisms using layers.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(6 minutes)
■ Concept Development	(32 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Sprint: Multiply a Fraction and Whole Number (8 minutes)
- Find the Volume (4 minutes)

Sprint: Multiply a Fraction and Whole Number (8 minutes)

Materials: (S) Multiply a Fraction and Whole Number Sprint

Note: This Sprint reviews content from Module 4 Lessons 6–8.

Find the Volume (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 1–2.

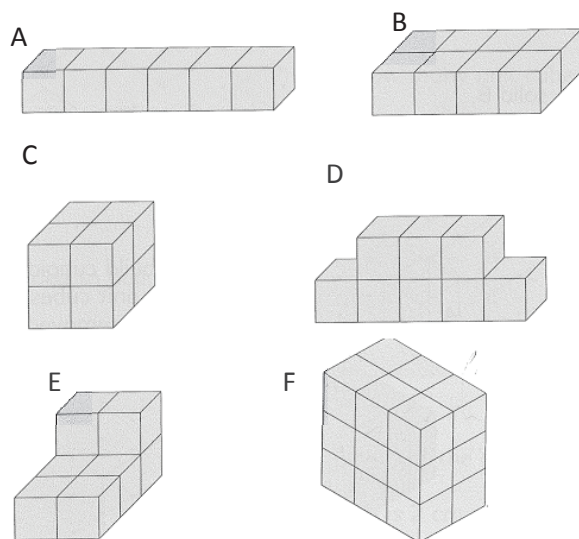
T: (Project Image A, pictured to the right.) Each cube is 1 cubic centimeter. How many cubes are there? Respond on your personal white board.

S: 6.

T: Write the volume on your personal white board with the correct units.

S: 6 cubic centimeters.

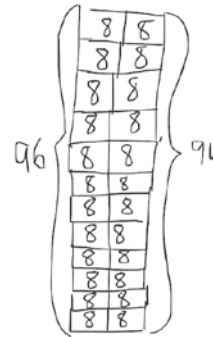
Follow this sequence for the other images pictured to the right.



Application Problem (6 minutes)

An ice cube tray has two rows of 8 ice cubes. How many ice cubes are in a stack of 12 ice cube trays? Draw a picture to explain your reasoning.

Note: This Application Problem encourages students to visualize layers in the stack that will be helpful as students refine their understanding of volume in today's lesson.



$$2 \times 96 = 192$$

There are 192 ice cubes.

Concept Development (32 minutes)

Materials: (T) 27 centimeter cubes (S) 27 centimeter cubes, rectangular prism recording sheet (Template)

- T: Build this with your own cubes. (Show 4 cubes in a square formation stacked vertically—2 layers with 2 cubes in each layer.)
- T: What's the volume of this rectangular prism?
- S: 4 cubic centimeters.
- T: Let's add layers horizontally. Add another layer *next* to the first one.
- S: (Work.)
- T: What is the volume?
- S: 8 cubic centimeters.
- T: Add 3 more layers next to the first two. (Pause for students to do this.)
- T: What is the volume now?
- S: 20 cubic centimeters.
- T: How did you figure that out? Turn and talk.
- S: I added the first 8 to the 12 more that I added. → I saw 5 along the bottom, and there were 2 layers going back, so that makes 10, and 2 layers going up makes 20. → I knew that I had 27 cubes to start, and I only have 7 left.
- T: (Project a blank rectangular prism from the recording sheet, or draw one on the board.) Let's record how we built the layers. Use the first rectangular prism in the row of your recording sheet.
- T: How many layers did we build in all?
- S: 5.
- T: Let's show that by partitioning the prism, vertically like bread slices, into 5 layers. (Partition the prism vertically into 5 equal sections.) Make your prism look like mine. How many cubes were in each layer?
- S: 4 cubes.



T: Record that on each layer that we drew. (Write a 4 on each of the vertical layers.) Write a number sentence that expresses the volume of this prism using these layers. Turn and talk.

S: We could write $4\text{ cm}^3 + 4\text{ cm}^3 + 4\text{ cm}^3 + 4\text{ cm}^3 + 4\text{ cm}^3 = 20\text{ cm}^3$. → Since all the layers are the same, we could write $5 \times 4\text{ cubic cm} = 20\text{ cubic cm}$.

T: (Draw the table on the board.) I'll record that in a table. Now, imagine that we could partition this prism horizontally into layers like a cake, like our ice cube trays. What might that look like? Work with your partner to show the layers on the next prism in the row (point to the next prism on the recording sheet), and tell how many cubes would be in each. Use your cubes to help you.

S: The prism is 2 units high, so we could cut the prism in half horizontally from left to right. That would be 10 cubes in each one. → We could make a top layer of 10 cubes and a bottom layer of 10 cubes.

T: Let's record your thinking. (Draw the figure to the right.) Write a number sentence that expresses the volume of the prism using these layers.

S: $10\text{ cm}^3 + 10\text{ cm}^3 = 20\text{ cm}^3$.
→ $2 \times 10\text{ cubic cm} = 20\text{ cubic cm}$.

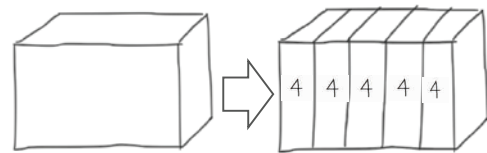
T: Let's record that information in our table. (Record.) Work with your partner to find one last way that we can partition this prism into layers. Use the third prism on your recording sheet to label the layers, and write the number of cubes in each layer. Then, write a number sentence to explain your thinking.

S: (Work to draw the third figure and write the number sentences.)

T: I'll record this last bit of information in our table. (Record.)

T: Now, let's draw the different layers together. Use the last prism in the row of your recording sheet.

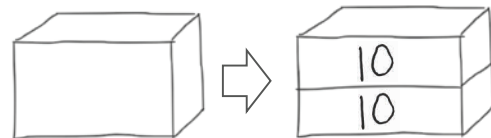
- Step 1: Draw vertical lines to show the 5 layers of 4 cubes each that remind us of bread slices. (Point to the table's first line.)
- Step 2: Draw a horizontal line to show the two layers of 10 cubes each that remind us of layers of cake. (Point to the table's second line.)
- Step 3: Draw both a horizontal and a vertical line to show the front and back layers of 10 each. (Point to the table's last line.)



$$4\text{ cm}^3 + 4\text{ cm}^3 + 4\text{ cm}^3 + 4\text{ cm}^3 + 4\text{ cm}^3 = 20\text{ cm}^3$$

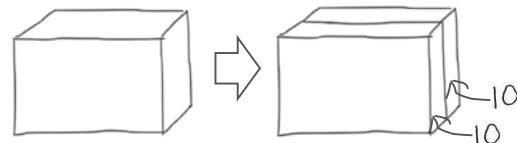
$$5 \times 4\text{ cubic cm} = 20\text{ cubic cm}$$

Number of Layers	Cubes in Each Layer	Volume
5	4	20 cm^3
2	10	20 cm^3
2	10	20 cm^3



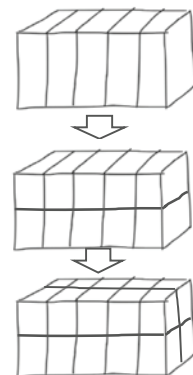
$$10\text{ cm}^3 + 10\text{ cm}^3 = 20\text{ cm}^3$$

$$2 \times 10\text{ cubic cm} = 20\text{ cubic cm}$$



$$10\text{ cm}^3 + 10\text{ cm}^3 = 20\text{ cm}^3$$

$$2 \times 10\text{ cubic cm} = 20\text{ cubic cm}$$

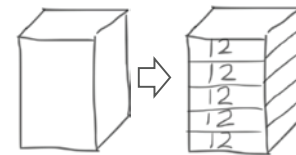
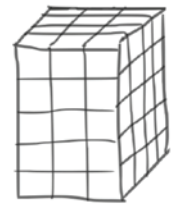


- T: What is the volume of the prism?
 S: 20 cubic centimeters.
 T: Build a prism with a partner that has one 3 cube by 3 cube layer. (Demonstrate building this with the cubes.)
 T: What is the volume?
 S: 9 cubic centimeters.
 T: Add another layer of cubes on top that completely cover the first layer.
 T: What is the volume now? How do you know?
 S: It's 18 cubic centimeters because now, we have 2 groups of 9 cubic centimeters. → Two layers with 9 cubes each is 18 cubic centimeters.
 T: Now, add another layer. What is the volume?
 S: 27 cubic centimeters.
 T: What is the overall shape of your rectangular prism?
 S: A cube!
 T: Use the set of cubes on your recording sheet to show the three ways of layering using the same system we just did with our 2 by 2 by 5 rectangular prism.
 S: (Work.)
 T: (Project or draw an image of a $3 \times 4 \times 5$ rectangular prism. Direct students to the set of vertical prisms on the rectangular prism recording sheet.) Imagine what the bottom layer of this prism would look like. Describe it to your partner, and then build it.
 S: There would be 3 rows with 4 cubes in each row.
 → There would be 12 cubes in all. It would be 3 cubes wide and 4 cubes long and 1 cube high. → This would be like a 4 by 3 rectangle, but it is 1 centimeter tall. (Build.)
 T: Here's the same prism but without the unit cubes drawn. How might we represent the bottom layer on this picture? Use your recording sheet, and talk to your partner.
 S: We could draw a horizontal slice toward the bottom and label it with 12. → I can see in the drawing that there are 5 layers in all, so I'll need to make the bottom about 1 fifth of the prism and put 12 on it.
 T: What is the volume of the single layer?
 S: 12 cubic centimeters.
 T: What is the volume of the prism with 5 of these layers?
 S: I know there are 5 layers that are the same, so $12 \text{ cm}^3 + 12 \text{ cm}^3 + 12 \text{ cm}^3 + 12 \text{ cm}^3 + 12 \text{ cm}^3$, so 60 cm^3 . → It's 5×12 cubic cm, so 60 cubic cm.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge students who quickly grasp the decompositions by asking them to determine a "rule" for finding the volume and test it for different rectangular prisms. They might also be asked to calculate the volume of the prisms as if they were built from 2 cm cubes. Ask them to explain what would happen to the volume if the dimensions of the cubes were doubled or tripled.



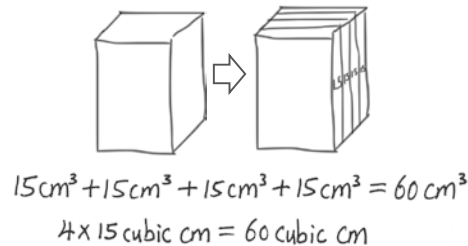
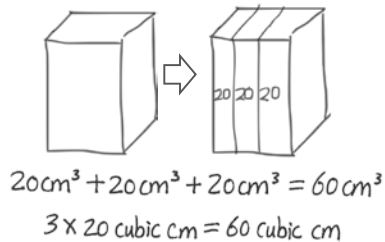
$$12 \text{ cm}^3 + 12 \text{ cm}^3 + 12 \text{ cm}^3 + 12 \text{ cm}^3 + 12 \text{ cm}^3 = 60 \text{ cm}^3$$

$$5 \times 12 \text{ cubic cm} = 60 \text{ cubic cm}$$

T: What other ways could we partition this prism into layers? Turn and talk, and then draw a picture of your thinking on the recording sheet.

S: (Draw.)

Possible Solutions



Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Compose and decompose right rectangular prisms using layers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Name Georgie Date _____

1. Use the prisms to find the volume.

- Build the rectangular prism pictured to the left with your cubes, if necessary.
- Decompose it into layers in 3 different ways and show your thinking on the blank prisms.
- Complete the missing information in the table.

a.

Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
2	12	24 cubic cm
6	4	24 cubic cm
2	12	24 cubic cm

b.

Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
3	8	24 cubic cm
4	6	24 cubic cm
2	12	24 cubic cm

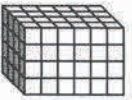
Any combination of the questions below may be used to lead the discussion.

- In Problem 1, how did you decide how to go about decomposing the prisms? Is there a different way or order in which you could have done it?
- Problem 4 uses meters instead of centimeters. What, if anything, did that change in how you drew your picture? How about in how you figured out the volume?
- What was Josh having a hard time visualizing in Problem 2? Which layers are easier for you to visualize? Which are the hardest? How can you make the hardest layers easier to see?
- At what point did you not need to model with the physical cubes anymore?
- How did the Application Problem connect to today's lesson? How are stacks of ice trays different from the prisms in the lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

2. Josh and Jonah were finding the volume of the prism to the right. The boys agree that 4 layers can be added together to find the volume. Josh says that he can see on the end of the prism that each layer will have 16 cubes in it. Jonah says that each layer has 24 cubes in it. Who is right? Explain how you know using words, numbers, and/or pictures.

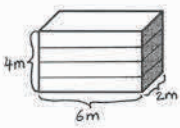


Jonah is right. Each layer has 6 across and 4 deep, so 24 cubes in each layer. 24×4 is 96. Josh sees 16 cubes on the end layer, but he'd have to multiply by 6 going across, not 4. He would get $16 \times 4 = 64$, not 96, so on Jonah is right.

3. Marcos makes a prism 1 inch by 5 inches by 5 inches. He then decides to create layers equal to his first one. Fill in the chart below, and explain how you know the volume of each new prism.

Number of layers	Volume	Explanation
2	50 in^3	Each layer is 25 in^2 . 2 layers is $2 \times 25 \text{ in}^2 = 50 \text{ in}^3$
4	100 in^3	4 layers is double 2 layers, so $2 \times 50 \text{ in}^3 = 100 \text{ in}^3$
7	175 in^3	I multiplied 1 layer (25 in^2) by 7. $7 \times 25 \text{ in}^2 = 175 \text{ in}^3$

4. Imagine the rectangular prism below is 6 meters long, 4 meters tall, and 2 meters wide. Draw horizontal lines to show how the prism could be decomposed into layers that are 1 meter in height.



It has 4 layers from bottom to top.
Each horizontal layer contains 12 cubic meters.
The volume of this prism is 48 m^3

A

Number Correct: _____

Multiply a Fraction and a Whole Number

1.	$\frac{1}{5} \times 2 =$	
2.	$\frac{1}{5} \times 3 =$	
3.	$\frac{1}{5} \times 4 =$	
4.	$4 \times \frac{1}{5} =$	
5.	$\frac{1}{8} \times 3 =$	
6.	$\frac{1}{8} \times 5 =$	
7.	$\frac{1}{8} \times 7 =$	
8.	$7 \times \frac{1}{8} =$	
9.	$3 \times \frac{1}{10} =$	
10.	$7 \times \frac{1}{10} =$	
11.	$\frac{1}{10} \times 7 =$	
12.	$4 \div 2 =$	
13.	$4 \times \frac{1}{2} =$	
14.	$6 \div 3 =$	
15.	$\frac{1}{3} \times 6 =$	
16.	$10 \div 5 =$	
17.	$10 \times \frac{1}{5} =$	
18.	$\frac{1}{3} \times 9 =$	
19.	$\frac{2}{3} \times 9 =$	
20.	$\frac{1}{4} \times 8 =$	
21.	$\frac{3}{4} \times 8 =$	
22.	$\frac{1}{6} \times 12 =$	

23.	$\frac{5}{6} \times 12 =$	
24.	$\frac{1}{3} \times 15 =$	
25.	$\frac{2}{3} \times 15 =$	
26.	$15 \times \frac{2}{3} =$	
27.	$\frac{1}{5} \times 15 =$	
28.	$\frac{2}{5} \times 15 =$	
29.	$\frac{4}{5} \times 15 =$	
30.	$\frac{3}{5} \times 15 =$	
31.	$15 \times \frac{3}{5} =$	
32.	$18 \times \frac{1}{6} =$	
33.	$18 \times \frac{5}{6} =$	
34.	$\frac{5}{6} \times 18 =$	
35.	$24 \times \frac{1}{4} =$	
36.	$\frac{3}{4} \times 24 =$	
37.	$32 \times \frac{1}{8} =$	
38.	$32 \times \frac{3}{8} =$	
39.	$\frac{5}{8} \times 32 =$	
40.	$32 \times \frac{7}{8} =$	
41.	$\frac{5}{9} \times 54 =$	
42.	$63 \times \frac{7}{9} =$	
43.	$56 \times \frac{3}{7} =$	
44.	$\frac{6}{7} \times 49 =$	

B

Number Correct: _____

Improvement: _____

Multiply a Fraction and a Whole Number

1.	$\frac{1}{7} \times 2 =$	
2.	$\frac{1}{7} \times 3 =$	
3.	$\frac{1}{7} \times 4 =$	
4.	$4 \times \frac{1}{7} =$	
5.	$\frac{1}{10} \times 3 =$	
6.	$\frac{1}{10} \times 7 =$	
7.	$\frac{1}{10} \times 9 =$	
8.	$9 \times \frac{1}{10} =$	
9.	$3 \times \frac{1}{8} =$	
10.	$5 \times \frac{1}{8} =$	
11.	$\frac{1}{8} \times 5 =$	
12.	$10 \div 5 =$	
13.	$10 \times \frac{1}{5} =$	
14.	$9 \div 3 =$	
15.	$\frac{1}{3} \times 9 =$	
16.	$10 \div 2 =$	
17.	$10 \times \frac{1}{2} =$	
18.	$\frac{1}{3} \times 6 =$	
19.	$\frac{2}{3} \times 6 =$	
20.	$\frac{1}{6} \times 12 =$	
21.	$\frac{5}{6} \times 12 =$	
22.	$\frac{1}{4} \times 8 =$	

23.	$\frac{3}{4} \times 8 =$	
24.	$\frac{1}{5} \times 15 =$	
25.	$\frac{2}{5} \times 15 =$	
26.	$\frac{4}{5} \times 15 =$	
27.	$\frac{3}{5} \times 15 =$	
28.	$15 \times \frac{3}{5} =$	
29.	$\frac{1}{3} \times 15 =$	
30.	$\frac{2}{3} \times 15 =$	
31.	$15 \times \frac{2}{3} =$	
32.	$24 \times \frac{1}{6} =$	
33.	$24 \times \frac{5}{6} =$	
34.	$\frac{5}{6} \times 24 =$	
35.	$20 \times \frac{1}{4} =$	
36.	$\frac{3}{4} \times 20 =$	
37.	$24 \times \frac{1}{8} =$	
38.	$24 \times \frac{3}{8} =$	
39.	$\frac{5}{8} \times 24 =$	
40.	$24 \times \frac{7}{8} =$	
41.	$\frac{5}{9} \times 63 =$	
42.	$54 \times \frac{7}{9} =$	
43.	$49 \times \frac{3}{7} =$	
44.	$\frac{6}{7} \times 56 =$	

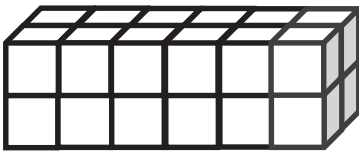
Name _____

Date _____

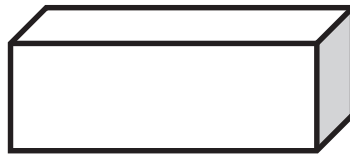
1. Use the prisms to find the volume.

- Build the rectangular prism pictured below to the left with your cubes, if necessary.
- Decompose it into layers in three different ways, and show your thinking on the blank prisms.
- Complete the missing information in the table.

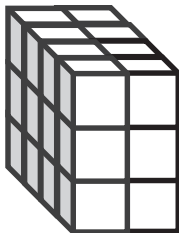
a.



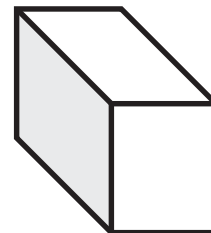
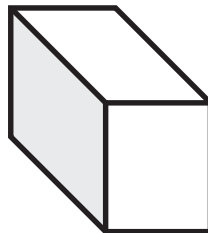
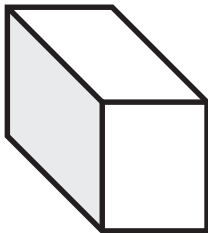
Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
		cubic cm
		cubic cm
		cubic cm



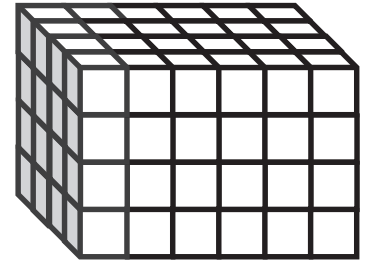
b.



Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
		cubic cm
		cubic cm
		cubic cm



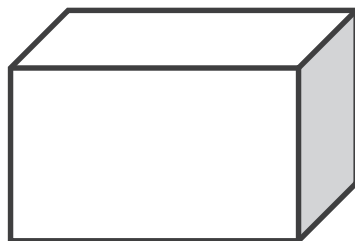
2. Josh and Jonah were finding the volume of the prism to the right. The boys agree that 4 layers can be added together to find the volume. Josh says that he can see on the end of the prism that each layer will have 16 cubes in it. Jonah says that each layer has 24 cubes in it. Who is right? Explain how you know using words, numbers, and/or pictures.



3. Marcos makes a prism 1 inch by 5 inches by 5 inches. He then decides to create layers equal to his first one. Fill in the chart below, and explain how you know the volume of each new prism.

Number of Layers	Volume	Explanation
2		
4		
7		

4. Imagine the rectangular prism below is 6 meters long, 4 meters tall, and 2 meters wide. Draw horizontal lines to show how the prism could be decomposed into layers that are 1 meter in height.



It has _____ layers from bottom to top.

Each horizontal layer contains _____ cubic meters.

The volume of this prism is _____.

Name _____

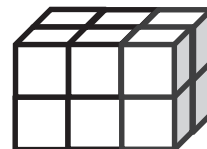
Date _____

1. Use unit cubes to build the figure to the right, and fill in the missing information.

Number of layers: _____

Number of cubes in each layer: _____

Volume: _____ cubic centimeters

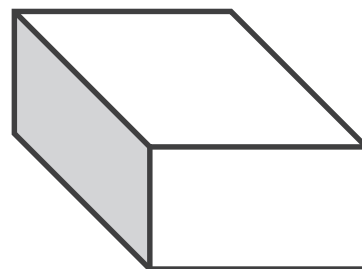


2. This prism measures 3 units by 4 units by 2 units. Draw the layers as indicated.

Number of layers: 4

Number of cubic units in each layer: 6

Volume: _____ cubic centimeters



Name _____

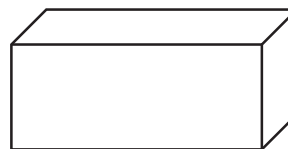
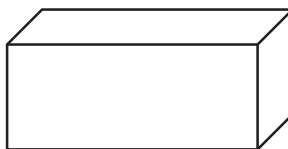
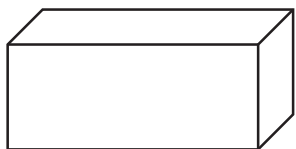
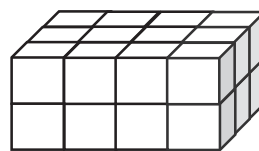
Date _____

1. Use the prisms to find the volume.

- The rectangular prisms pictured below were constructed with 1 cm cubes.
- Decompose each prism into layers in three different ways, and show your thinking on the blank prisms.
- Complete each table.

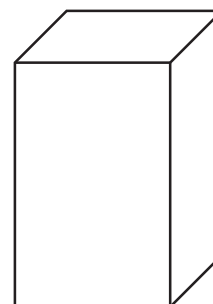
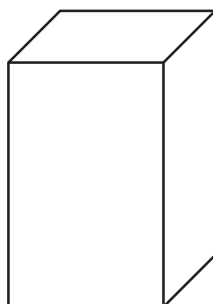
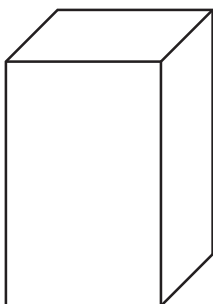
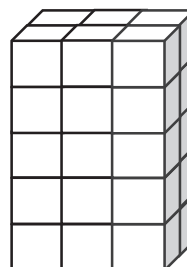
a.

Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
		cubic cm
		cubic cm
		cubic cm

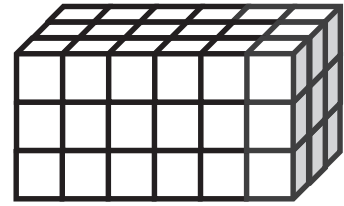


b.

Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
		cubic cm
		cubic cm
		cubic cm



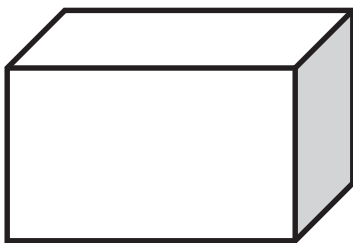
2. Stephen and Chelsea want to increase the volume of this prism by 72 cubic centimeters. Chelsea wants to add eight layers, and Stephen says they only need to add four layers. Their teacher tells them they are both correct. Explain how this is possible.



3. Juliana makes a prism 4 inches across and 4 inches wide but only 1 inch tall. She then decides to create layers equal to her first one. Fill in the chart below, and explain how you know the volume of each new prism.

Number of Layers	Volume	Explanation
3		
5		
7		

4. Imagine the rectangular prism below is 4 meters long, 3 meters tall, and 2 meters wide. Draw horizontal lines to show how the prism could be decomposed into layers that are 1 meter in height.



It has _____ layers from top to bottom.

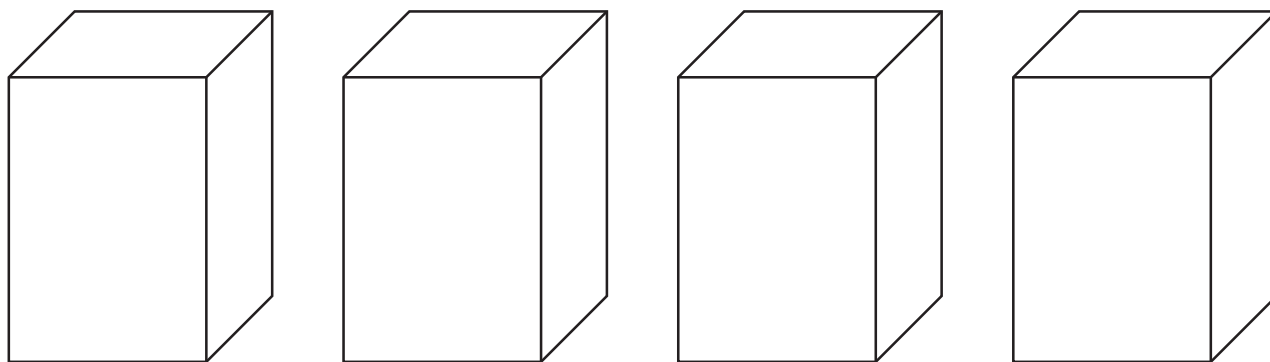
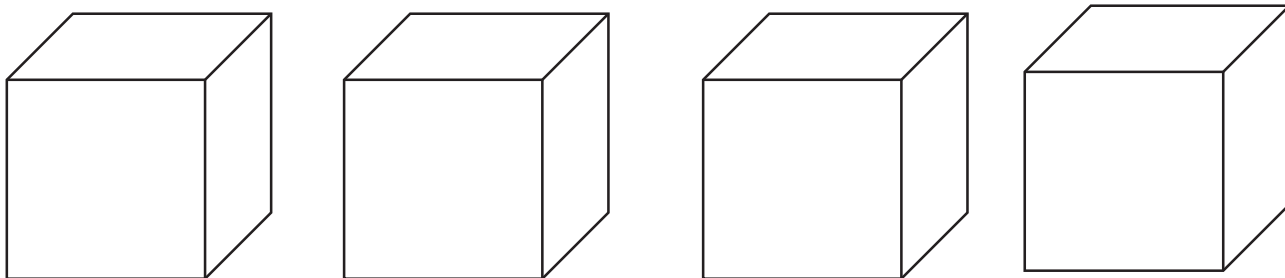
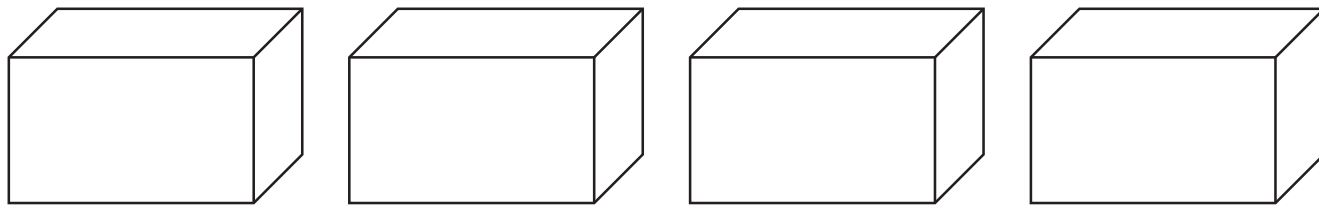
Each horizontal layer contains _____ cubic meters.

The volume of this prism is _____

Name _____

Date _____

Use these rectangular prisms to record the layers that you count.

_____
rectangular prism recording sheet



Topic B

Volume and the Operations of Multiplication and Addition

- Focus Standards:**
- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
 - A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
 - A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
 - Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
 - Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
 - Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
 - Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Instructional Days: 6

Coherence

-Links from:	G3–M4	Multiplication and Area
-Links to:	G6–M5	Area, Surface Area, and Volume Problems

Concrete understanding of volume and multiplicative reasoning come together in Topic B as the systematic counting from Topic A leads naturally to formulas for finding the volume of a right rectangular prism. Students come to see that multiplying the edge lengths or multiplying the height by the area of the base yields an equivalent volume to that found by packing and counting unit cubes.

Next, students solidify the connection of volume as *packing* with volume as *filling* by comparing the amount of liquid that fills a container to the number of cubes that can be packed into it. This connection is formalized

as students see that 1 cubic centimeter is equal to 1 milliliter. Complexity increases as students use their knowledge that volume is additive to partition and calculate the total volume of solid figures composed of non-overlapping rectangular prisms.

Word problems involving the volume of rectangular prisms with whole number edge lengths solidify understanding and give students the opportunity to reason about scaling in the context of volume. This topic concludes with a design project that allows students to apply the concepts and formulas they have learned throughout Topics A and B to create a sculpture of a specified volume composed of varied rectangular prisms with parameters stipulated in the project description.

A Teaching Sequence Toward Mastery of Volume and the Operations of Multiplication and Addition

Objective 1: Use multiplication to calculate volume.
(Lesson 4)

Objective 2: Use multiplication to connect volume as *packing* with volume as *filling*.
(Lesson 5)

Objective 3: Find the total volume of solid figures composed of two non-overlapping rectangular prisms.
(Lesson 6)

Objective 4: Solve word problems involving the volume of rectangular prisms with whole number edge lengths.
(Lesson 7)

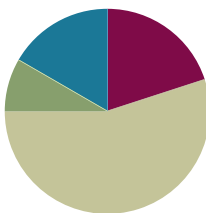
Objective 5: Apply concepts and formulas of volume to design a sculpture using rectangular prisms within given parameters.
(Lessons 8–9)

Lesson 4

Objective: Use multiplication to calculate volume.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Multiply Fractions (4 minutes)
- Find the Area (4 minutes)
- Find the Volume (4 minutes)

Multiply Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews Module 4 content.

T: (Write $\frac{1}{3}$ of $\frac{4}{5}$ is ____.) Write the fraction of a set as a multiplication expression.

S: (Write $\frac{1}{3} \times \frac{4}{5}$.)

T: Draw a rectangle, and shade in $\frac{4}{5}$.

S: (Draw a rectangle, partition it into 5 equal units, and shade 4 of the units.)

T: To show $\frac{1}{3}$ of $\frac{4}{5}$, how many equal parts do we need?

S: 3.

T: Show 1 third of 4 fifths.

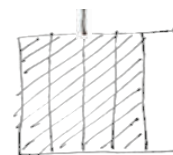
S: (Partition the 4 fifths into thirds, and shade 1 third.)

T: Make the other units the same size as the double-shaded ones.

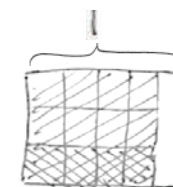
S: (Extend the horizontal thirds across the remaining units using dotted lines.)

T: What unit do we have now?

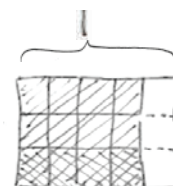
S: Fifteenths.



$$\frac{4}{5}$$



$$\frac{1}{3} \times \frac{4}{5}$$



$$\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$$

T: How many fifteenths are double shaded?

S: Four.

T: Write the product, and say the sentence.

S: (Write $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$.) $\frac{1}{3}$ of $\frac{4}{5}$ is 4 fifteenths.

Continue with the following possible sequence: $\frac{2}{3} \times \frac{4}{5}$, $\frac{3}{5} \times \frac{2}{3}$, and $\frac{3}{5} \times \frac{5}{8}$.

Find the Area (4 minutes)

Materials: (S) Personal white board

Note: Reviewing this Grade 4 concept prepares students to calculate volume.

T: (Project the square with side lengths 10 cm.)

T: How long are the square's sides?

S: 10 cm.

T: (Write ____ cm \times ____ cm = ____ cm².) On your personal white board, write the area of the square as a multiplication sentence, including the units.

S: (Write 10 cm \times 10 cm = 100 cm².)

T: (Project a rectangle labeled 3 ft by 13 ft.)

T: What is the measure of the rectangle's longest side?

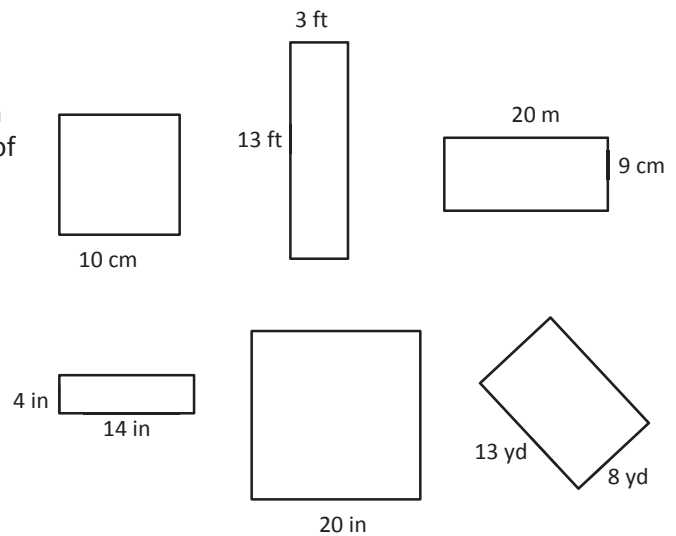
S: 13 ft.

T: What is the measure of the rectangle's shortest side?

S: 3 ft.

T: (Write ____ ft \times ____ ft = ____ ft².) Write the area of the rectangle as a multiplication sentence starting with the length of the longest side.

S: (Write 13 ft \times 3 ft = 39 ft².)



Continue this process with the other rectangles and square.

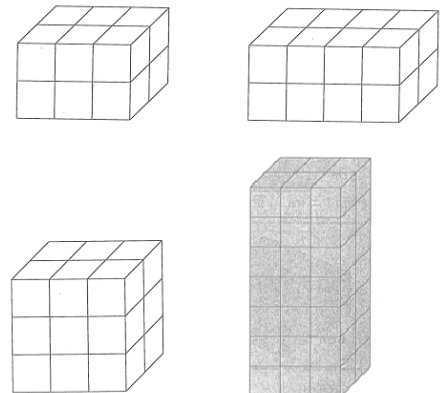
Find the Volume (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 1 and 2.

T: (Project the first image to the right. Number of horizontal layers: 2.) Each cube is 1 cubic centimeter.

T: (Underneath, write Number of cubes in each horizontal layer: ____.) Fill in the blank.



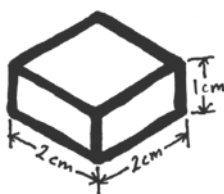
- S: (Write Number of cubes in each horizontal layer: 6.)
- T: (Write Number of cubes in each horizontal layer: 6. Beneath it, write Volume = ____ cubic centimeters + ____ cubic centimeters.) Fill in the blanks.
- S: (Write Volume = 6 cubic centimeters + 6 cubic centimeters.)
- T: (Write Volume = 6 cubic centimeters + 6 cubic centimeters. Beneath it, write Volume = ____ cubic centimeters.)
- S: (Write Volume = 12 cubic centimeters.)

Continue this process for the remaining prisms.

Application Problem (5 minutes)

Draw a $2\text{ cm} \times 2\text{ cm} \times 1\text{ cm}$ rectangular prism on the board, or project an image of one on the board.

Karen says that the volume of this prism is 5 cm^3 and that she calculated it by adding the sides together. Give the correct volume of this prism, and explain Karen's error.



The Prism is 2 cubes long, 2 cubes wide and 1 cube high, which is the same thing as 2 rows with 2 cubes in each row. The correct Volume is 4 cm^3 . Karen should not be adding all the sides together. She should count the total cubic units in the Prism.

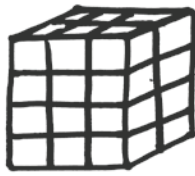
Note: To find the volume of this figure, Karen could add 2 and 2 (the number of centimeter cubes in each row) but not by adding all three dimensions.

Concept Development (33 minutes)

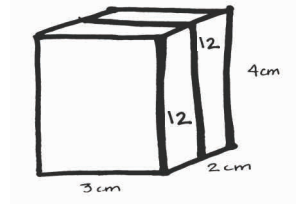
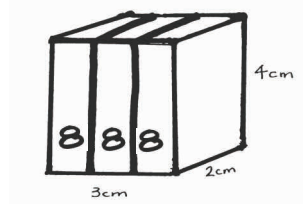
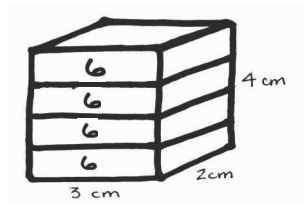
Materials: (T) Images of rectangular prisms to project (S) Personal white board, rectangular prism recording sheet (Lesson 3 Template)

Part 1: Find the volume of multilayer prisms using multiplication.

- T: (Project the leftmost image on the next page.) Record the length, width, and height of this rectangular prism on your recording sheet. Then, decompose the prism into layers three different ways to find the volume like we did together yesterday.
- S: (Work on the recording sheet to show the three different decompositions pictured.)



$$\begin{aligned} l &= 3 \text{ cm} \\ w &= 2 \text{ cm} \\ h &= 4 \text{ cm} \end{aligned}$$



T: Let's record some information about our prism in this table. Look at this layer on the top. How many cubes are in each layer? How do you know?

S: There are 6 cubes. It is 3 cubes by 2 cubes.
→ I counted them. → It's like an array,
 $3 \times 2 = 6$.

T: (Record in the table as 3×2 .)

Cubes in Each Layer	Number of Layers	Volume
(3×2)	4	24 cm^3
(2×4)	3	24 cm^3
(3×4)	2	24 cm^3

Follow a similar sequence to record the other decompositions.

T: How do we use this information to find the volume of the prism? Turn and talk.

S: With 4 layers, that's 4 copies of the same array of cubes, 4 times 6. That's 24 cubic centimeters.
→ I see 3 layers that each have 8 cubes in them. Eight cubes 3 times is 24 cubes. That's 24 cubic centimeters. → Three times 4 shows the cubes in the first layer on the front, but I need 2 of those, so 2 twelves make 24 cubic centimeters. → Count the layers. Four layers, and each layer is a 3 cm by 2 cm by 1 cm prism; 6 fours is 24. The volume is 24 cubic centimeters.

T: (Record the number of layers and volumes in the table.)

T: (Hold up a cube.) We know that this is 1 cubic centimeter. Look at one face of this cube (point to one face); what is the area of this face?

S: 1 square centimeter.

T: (Point to the face on the top of the first prism.) If 1 square unit is the area of one cube's face, and there are 6 cubes that make up this face, what is the area of this face? Write a number sentence to show the area. Be sure to include the units.

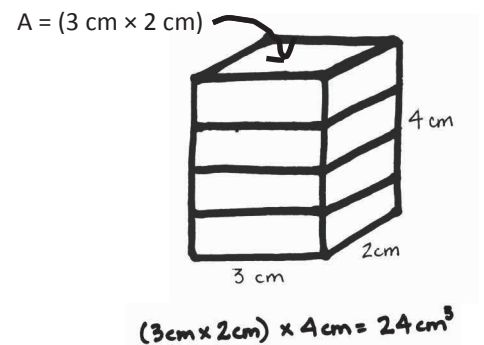
S: $3 \text{ cm} \times 2 \text{ cm} = 6 \text{ cm}^2$.

T: What do you notice about the area of this face and the number of cubes in this layer?

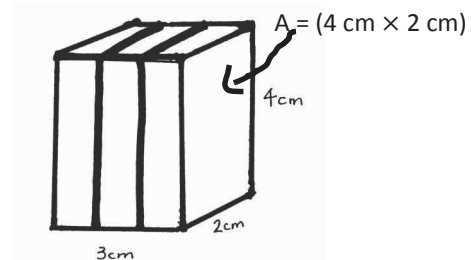
S: They are the same.

T: A moment ago, we said that to find the volume, we had to account for the number of layers in the prism. How many layers are under this face?

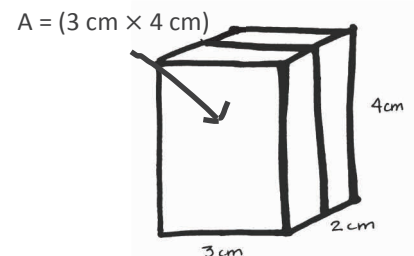
S: 4.



- T: Which dimension of the prism gives us that number?
- S: The height.
- T: How many centimeters is the height? Give me the unit, too.
- S: 4 centimeters.
- T: So, we can find the volume by multiplying the area of this face by the height. (Write $(3 \text{ cm} \times 2 \text{ cm})$.) The height, 4 cm, happens to tell us the number of the layers. Show me the multiplication sentence you can use to find the volume of this prism that matches this way of seeing the layers.
- S: $V = (3 \text{ cm} \times 2 \text{ cm}) \times 4 \text{ cm} = 24 \text{ cubic cm}$. $\rightarrow V = 6 \text{ cm}^2 \times 4 \text{ cm} = 24 \text{ cm}^3$.
- T: (Write $V = (3 \text{ cm} \times 2 \text{ cm}) \times 4 \text{ cm} = 24 \text{ cm}^3$ and $6 \text{ cm}^2 \times 4 \text{ cm} = 24 \text{ cm}^3$ on the board.) I notice some of you wrote $6 \text{ cm}^2 \times 4 \text{ cm}$, and others multiplied centimeters by centimeters by centimeters. What happens to the square units when you multiply them by the third factor? Why? Talk with a partner.
- S: When multiplying a square unit times one more unit, it becomes a cubic unit. \rightarrow You start out with length units, the second factor makes them square units, and the third factor makes them cubic units. \rightarrow To measure area, we use squares. To measure volume, we use cubes. The third factor means we don't just have flat squares, but cubes.
- T: Is this the same volume we found when we counted by the number of cubes in each layer?
- S: Yes.
- T: Let's use this method again, but I'd like to use the area of this face. (Point to the layer on the end.) Write a multiplication expression that shows how to find the area of this face.
- S: $4 \text{ cm} \times 2 \text{ cm}$.
- T: (Write $(4 \text{ cm} \times 2 \text{ cm})$.) To find the volume, we need to know how many layers are to the left of this face. What dimension of this prism tells us how many layers this time? How many centimeters is that? Turn and talk.
- S: This time there are 3 layers. \rightarrow The length is the one that shows how many layers this time. It's 3 centimeters. \rightarrow The prism is 3 centimeters long. This shows the layers beside this face.
- T: (Write $(4 \text{ cm} \times 2 \text{ cm}) \times 3 \text{ cm}$.) Multiply to find the volume.
- S: (Work to find 24 cm^3 .)
- T: (Project the image of the prism shown to the right.) Now, let's look at this last decomposition. Find the area of the front face. Tell which dimension shows the layers, and work with your partner to write an expression to find the volume. Turn and talk.
- S: The area of this face is 3 cm times 4 cm. That's 12 square centimeters. There are 2 layers that are each 1 cm. $3 \times 4 \times 2 = 24$. The volume is 24 cubic centimeters. \rightarrow The area is 12 square centimeters, and the width is 2 cm. Twelve square centimeters times 2 centimeters is 24 cubic centimeters.

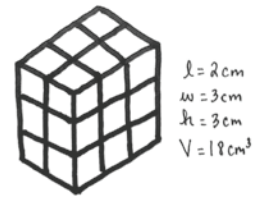


$$(4 \text{ cm} \times 2 \text{ cm}) \times 3 \text{ cm} = 24 \text{ cm}^3$$



$$(3 \text{ cm} \times 4 \text{ cm}) \times 2 \text{ cm} = 24 \text{ cm}^3$$

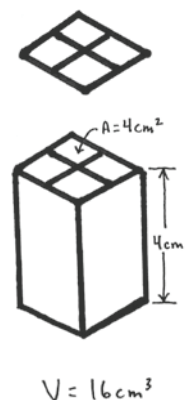
- T: This is the same volume as before. Look at all three multiplication sentences. What patterns do you notice? Turn and talk.
- S: The volume is the same every time. → We are multiplying all the sides together, but they are in a different order. → When we multiply the length of the sides together, we get the same volume as when we counted the layers.
- T: So, does centimeters times centimeters times centimeters give us centimeters cubed? Why or why not? Turn and talk.
- S: Yes. There are three measurements that are centimeters, and then the answer is in cubic units. → True. There are three factors that have centimeter units. So, the product has to be cubic units because cubes measure space in three dimensions!
- T: Let's see if this pattern holds. (Display the image of the prism shown below.) Record the dimensions of this prism. What's different about it?
- S: It's the same width and length, but now, the height is 1 cm shorter. → There are 6 fewer cubic centimeters in this one. → There are still some 2×3 layers in this one.
- T: How would you find its volume? Turn and talk.
- S: I can subtract 6 cubic units from the 24 cubic units in the 4-layer prism. That makes the volume 18 cubic centimeters. → I can multiply the 6 cubes in the top layer by 3 layers. That's 18 cubic centimeters. → I can multiply 2 cm times 3 cm times 3 cm, which is 18 cubic centimeters. → The end has a 6 cm^2 area and 3 layers, so $6 \text{ cm}^2 \times 3 \text{ cm} = 18 \text{ cm}^3$. → The front face is different now. It is 3 cm by 3 cm. There is 1 layer behind the $3 \text{ cm} \times 3 \text{ cm}$ face for a total of 2 layers. $3 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm} = 18 \text{ cm}^3$.
- T: Let's record this. (Record.)
- T: What can we conclude about finding volume from these examples?
- S: We can multiply the sides to find the volume. → If we know the area of one face and multiply by the number of those layers, we can find volume. → Yes, but the number of layers is just the length of the remaining side.



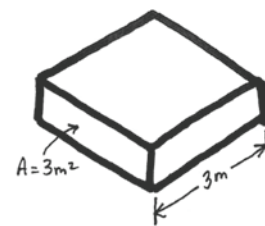
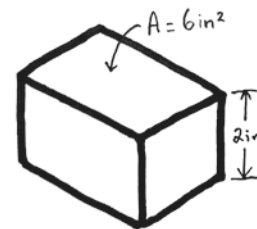
$$\begin{aligned}(3 \text{ cm} \times 2 \text{ cm}) \times 3 \text{ cm} &= 18 \text{ cm}^3 \\ (2 \text{ cm} \times 3 \text{ cm}) \times 3 \text{ cm} &= 18 \text{ cm}^3 \\ (3 \text{ cm} \times 3 \text{ cm}) \times 2 \text{ cm} &= 18 \text{ cm}^3\end{aligned}$$

Part 2: Calculate the volume when the area of one side is given.

- T: (Post image illustrated to the right.) This image represents the top face of a rectangular prism. If the prism is made of 1 cm cubes, what is the area of this face?
- S: 4 square centimeters.
- T: (Write $A = 4 \text{ cm}^2$. Then, post the image of the prism with a height of 4 cm.) If the rectangular prism that sits below this face is built of centimeter cubes and has a height of 4 cm, how many layers of centimeter cubes are in the prism?
- S: 4 layers.



- T: How can we use the layers to find the volume? Turn and talk.
- S: I can see that the length is 2 cm and the width is also 2 cm, so if the height is 4 cm, I can multiply 2 by 2 by the number of layers, which is 4, to get the volume. → Since the area of the top is 2 times 2 cm, which is 4 cm^2 , we can just multiply the area of the top by the height to find the volume.
- T: Show me the multiplication sentence you can use to find the volume of this prism.
- S: $V = (2\text{ cm} \times 2\text{ cm}) \times 4\text{ cm} = 16\text{ cubic cm}$. → $V = 4\text{ cm}^2 \times 4\text{ cm} = 16\text{ cm}^3$.
- T: (Write $V = 16\text{ cm}^3$ on the board.)
- T: (Post the image to the right on the board.) What's different about this prism?
- S: We can't see the individual cubes in the face with the area.
→ We don't know the dimensions of the top face, just the area.
- T: Do we need the dimensions of that top face to find the volume? Why or why not?
- S: No. We can use the area. → We don't need to know how many cubes are in each layer. We just want the total volume. The area of the top and the height are enough to find volume.
- T: Work with a neighbor to find the volume of this prism.
- S: (Work to show $V = 6\text{ in}^2 \times 2\text{ in} = 12\text{ in}^3$.)
- T: (Post the final image, to the right, on the board.) Compare this prism to the last one. Turn and talk.
- S: This one shows just the area again. → This one shows the area of the front and the width. We can still just multiply them. → This time, we have the area of a different face. We have the area of the front face and the width of the prism, which tells how many layers are behind the front face.
- T: Find the volume of this prism.
- S: (Work to show $V = 3\text{ m}^2 \times 3\text{ m} = 9\text{ m}^3$.)



Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use multiplication to calculate volume.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Name: Kathy Date: _____

1. Each rectangular prism is built from centimeter cubes. State the dimensions and find the volume.

a. Length: 5 cm Width: 2 cm Height: 2 cm Volume: 20 cm^3
 $5 \times 2 \times 2 = 20$

b. Length: 3 cm Width: 2 cm Height: 4 cm Volume: 24 cm^3
 $3 \times 2 \times 4 = 24$

c. Length: 4 cm Width: 2 cm Height: 4 cm Volume: 32 cm^3
 $4 \times 2 \times 4 = 32$

d. Length: 4 cm Width: 3 cm Height: 3 cm Volume: 36 cm^3
 $4 \times 3 \times 3 = 36$

2. Write a multiplication sentence that you could use to calculate the volume for each rectangular prism in Problem 1. Include the units in your sentences.

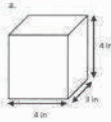
a. $5\text{ cm} \times 2\text{ cm} \times 2\text{ cm} = 20\text{ cm}^3$ b. $3\text{ cm} \times 2\text{ cm} \times 4\text{ cm} = 24\text{ cm}^3$
c. $4\text{ cm} \times 2\text{ cm} \times 4\text{ cm} = 32\text{ cm}^3$ d. $4\text{ cm} \times 3\text{ cm} \times 3\text{ cm} = 36\text{ cm}^3$


Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

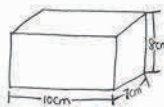
- Explain how the prisms in Problems 1(d) and 3(b) are similar and different. Identify the dimensions of another prism that would have an equivalent volume to Problem 1(d).
- Explain how we get cubic units when we multiply to find volume.
- (Connect the term *face* with the term *base*. Discuss with students that these two terms may be used interchangeably when dealing with right rectangular prisms.) Why could we think of any face as the base of our prism? (Discuss the fact that if we imagine rotating the prism so that the chosen face lies at the bottom, or what we typically think of as the base, the remaining dimension can be thought of as the height of the prism.)
- In Problem 4, what would happen to the volume of Tyron's box if he doubled the height to 16 cm? If he halved the length? If he doubled the height while halving the length?
- Explain your thought process on Problem 5 as you found the error in Aaron's thinking.
- Compare your earlier strategies for finding volume to the method we learned today. How is the formula for finding the volume of rectangular prisms helpful?

3. Calculate the volume of each rectangular prism. Include the units in your number sentences.

a.  $V = 4 \text{ in} \times 3 \text{ in} \times 4 \text{ in} = 48 \text{ in}^3$


b.  $V = 3 \text{ m} \times 2 \text{ m} \times 6 \text{ m} = 36 \text{ m}^3$

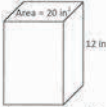
4. Tyron is constructing a box in the shape of a rectangular prism to store his baseball cards. It has a length of 10 centimeters, a width of 7 centimeters, and a height of 8 centimeters. What is the volume of the box?

 $V = l \times w \times h$
 $= 10 \text{ cm} \times 7 \text{ cm} \times 8 \text{ cm}$
 $= 70 \text{ cm}^2 \times 8 \text{ cm}$
 $= 560 \text{ cm}^3$

The volume of the box is 560 cubic centimeters.

5. Aaron says more information is needed to find the volume of the prisms. Explain why Aaron is mistaken, and calculate the volume of the prisms.

a.  Aaron can multiply the area of the face by the width to find the volume.
 $V = 60 \text{ cm}^2 \times 5 \text{ cm} = 300 \text{ cm}^3$

b.  Aaron can multiply the area of the face by the height to find the volume.
 $V = 20 \text{ in}^2 \times 12 \text{ in} = 240 \text{ in}^3$

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



NOTES ON VOCABULARY:

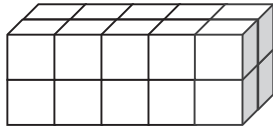
While it is true that any face of a rectangular prism may serve as the base, it is not true for other prisms or cylinders. For example, a right triangular prism has two triangular bases, but the remaining rectangular faces are not bases.

Name _____

Date _____

1. Each rectangular prism is built from centimeter cubes. State the dimensions, and find the volume.

a.



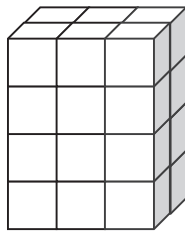
Length: _____ cm

Width: _____ cm

Height: _____ cm

Volume: _____ cm^3

b.



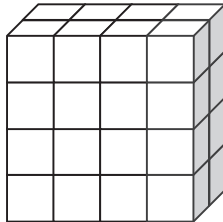
Length: _____ cm

Width: _____ cm

Height: _____ cm

Volume: _____ cm^3

c.



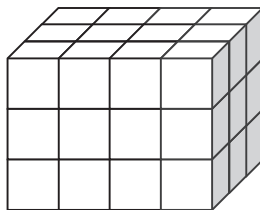
Length: _____ cm

Width: _____ cm

Height: _____ cm

Volume: _____ cm^3

d.



Length: _____ cm

Width: _____ cm

Height: _____ cm

Volume: _____ cm^3

2. Write a multiplication sentence that you could use to calculate the volume for each rectangular prism in Problem 1. Include the units in your sentences.

a. _____

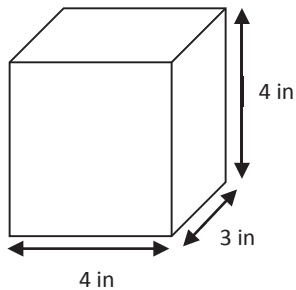
b. _____

c. _____

d. _____

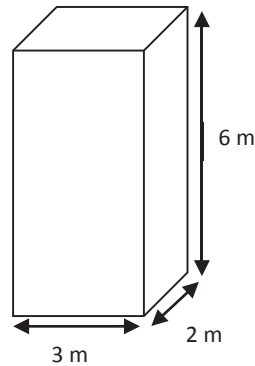
3. Calculate the volume of each rectangular prism. Include the units in your number sentences.

a.



V = _____

b.

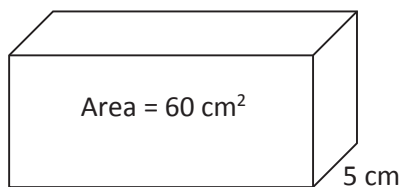


V = _____

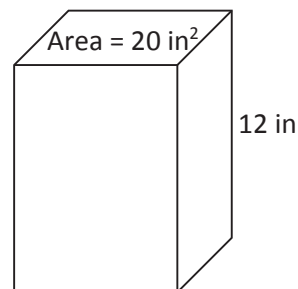
4. Tyron is constructing a box in the shape of a rectangular prism to store his baseball cards. It has a length of 10 centimeters, a width of 7 centimeters, and a height of 8 centimeters. What is the volume of the box?

5. Aaron says more information is needed to find the volume of the prisms. Explain why Aaron is mistaken, and calculate the volume of the prisms.

a.



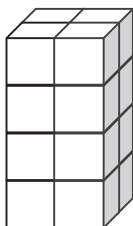
b.



Name _____

Date _____

1. Calculate the volume of prism.



Length: _____ mm

Width: _____ mm

Height: _____ mm

Volume: _____ mm^3

Write the multiplication sentence that shows how you calculated the volume. Be sure to include the units.

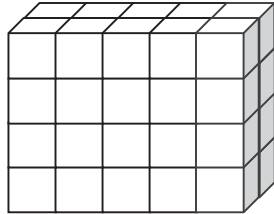
2. A rectangular prism has a top face with an area of 20 ft^2 and a height of 5 ft. What is the volume of this rectangular prism?

Name _____

Date _____

1. Each rectangular prism is built from centimeter cubes. State the dimensions, and find the volume.

a.



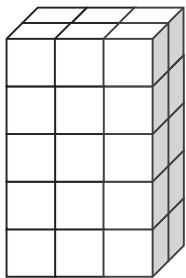
Length: _____ cm

Width: _____ cm

Height: _____ cm

Volume: _____ cm^3

b.



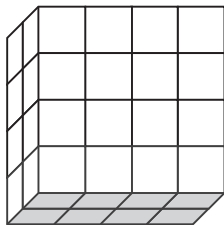
Length: _____ cm

Width: _____ cm

Height: _____ cm

Volume: _____ cm^3

c.



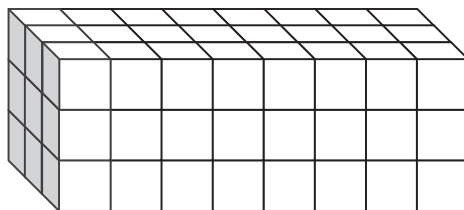
Length: _____ cm

Width: _____ cm

Height: _____ cm

Volume: _____ cm^3

d.



Length: _____ cm

Width: _____ cm

Height: _____ cm

Volume: _____ cm^3

2. Write a multiplication sentence that you could use to calculate the volume for each rectangular prism in Problem 1. Include the units in your sentences.

a. _____

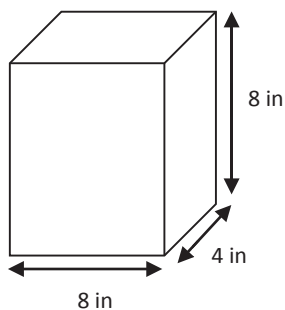
b. _____

c. _____

d. _____

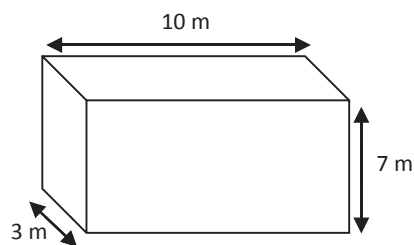
3. Calculate the volume of each rectangular prism. Include the units in your number sentences.

a.



Volume: _____

b.



Volume: _____

4. Mrs. Johnson is constructing a box in the shape of a rectangular prism to store clothes for the summer. It has a length of 28 inches, a width of 24 inches, and a height of 30 inches. What is the volume of the box?

5. Calculate the volume of each rectangular prism using the information that is provided.

a. Face area: 56 square meters

Height: 4 meters

b. Face area: 169 square inches

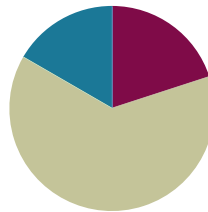
Height: 14 inches

Lesson 5

Objective: Use multiplication to connect volume as *packing* with volume as *filling*.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Concept Development	(38 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Count by Cubic Centimeters (2 minutes)
- Find the Area (4 minutes)
- Find the Volume (6 minutes)

Count by Cubic Centimeters (2 minutes)

Note: This fluency activity prepares students for today's lesson.

T: Count by 100 cubic centimeters to 1,000 cubic centimeters. (Write as students count.)

100 cm ³	200 cm ³	300 cm ³	400 cm ³	500 cm ³	600 cm ³	700 cm ³	800 cm ³	900 cm ³	1,000 cm ³
100 mL	200 mL	300 mL	400 mL	500 mL	600 mL	700 mL	800 mL	900 mL	1 liter

S: 100 cm³, 200 cm³, 300 cm³, 400 cm³, 500 cm³, 600 cm³, 700 cm³, 800 cm³, 900 cm³, 1,000 cm³.

T: Count by 100 mL. (Write as students count.)

S: 100 mL, 200 mL, 300 mL, 400 mL, 500 mL, 600 mL, 700 mL, 800 mL, 900 mL, 1,000 mL.

T: 1,000 mL = 1 liter. Count by 100 mL again. This time, when you come to 1,000 mL, say 1 liter. (Write as students count.)

S: 100 mL, 200 mL, 300 mL, 400 mL, 500 mL, 600 mL, 700 mL, 800 mL, 900 mL, 1 liter.

Find the Area (4 minutes)

Materials: (S) Personal white board

Note: Reviewing this Grade 4 concept prepares students to calculate volume. Images not drawn to scale.

T: (Project a rectangle with side lengths of 6 cm and 4 cm.) What is the length of the rectangle's longest side?

S: 6 cm.

T: What is the length of the rectangle's shortest side?

S: 4 cm.

T: (Write $__ \text{ cm} \times __ \text{ cm} = __ \text{ cm}^2$.) On your personal white board, write the area of the rectangle as a multiplication sentence including the units.

S: (Write $6 \text{ cm} \times 4 \text{ cm} = 24 \text{ cm}^2$.)

T: (Project a square with a given length of 9 cm.) Name the shape.

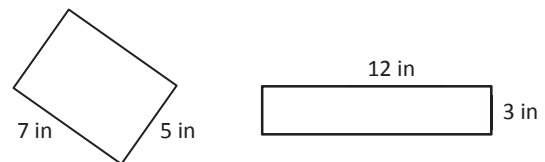
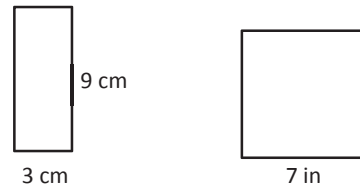
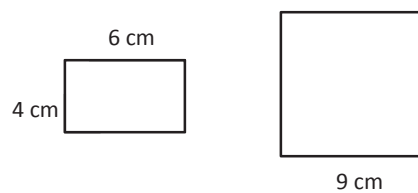
S: Square.

T: What is the length of the square's sides?

S: 9 cm.

T: (Write $__ \text{ cm} \times __ \text{ cm} = __ \text{ cm}^2$.) Write the area of the square as a multiplication sentence including the units.

S: (Write $9 \text{ cm} \times 9 \text{ cm} = 81 \text{ cm}^2$.)



Continue this process for the other rectangles and squares.

Find the Volume (6 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews Lesson 4. Images not drawn to scale.

T: (Project the 4 cm by 5 cm by 2 cm rectangular prism illustrated to the right.) What's the length of the rectangular prism?

S: 4 cm.

T: What's the width?

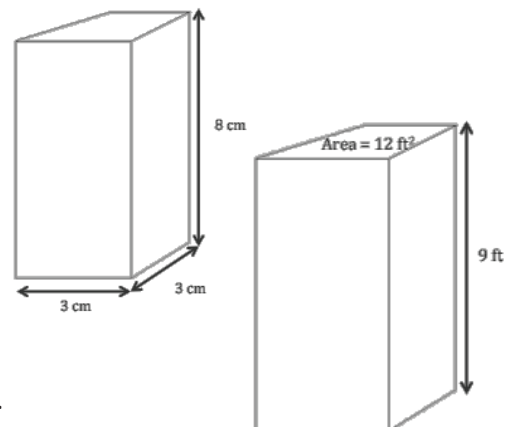
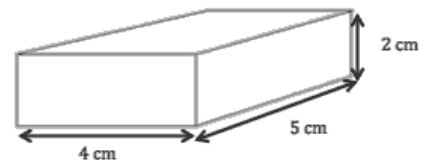
S: 5 cm.

T: What's the height?

S: 2 cm.

T: (Write $__ \text{ cm} \times __ \text{ cm} \times __ \text{ cm} = __ \text{ cm}^3$.) On your personal white board, calculate the volume.

S: (Write $4 \text{ cm} \times 5 \text{ cm} \times 2 \text{ cm} = 40 \text{ cm}^3$.)



Repeat this process for the 3 cm by 3 cm by 8 cm rectangular prism.

T: (Project the rectangular prism to the right.) Say the given area of the rectangular prism's front face.

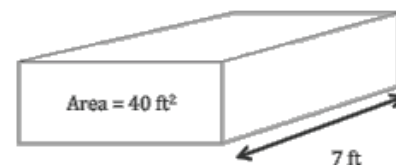
S: 40 ft^2 .

T: Say the given width.

S: 7 ft.

T: (Write $V = \underline{\hspace{1cm}} \text{ft}^3$.) On your personal white board, calculate the volume.

S: (Write $V = 280 \text{ ft}^3$.)



Repeat this process for the rectangular prism with a face of 12 ft^2 and a height of 9 ft.

Concept Development (38 minutes)

Materials: (S) Per group: centimeter cubes, several small watertight containers (preferably right rectangular prisms) marked with a horizontal line for measuring, small pitcher of water, graduated cylinder labeled with mL, class data recording sheet poster, ruler or tape measure, Problem Set (Problems 1-3)

Note: Because today's lesson is a hands-on exploration, time for the Application Problem has been given to the Concept Development.

Before class, prepare a large poster or sheet for groups to record their findings. Be sure to use cubes that are denser than water for the displacement exploration.

Problem 1

Investigate $1 \text{ cm}^3 = 1 \text{ mL}$.

T: What are some ways that we can determine the volume of the box you've been given using the materials on your table?

S: We can pack it with cubes and count. → We can pack the bottom layer and then use the cubes to find how many layers. → We could find the area of any base and then count the layers. → We can measure the sides and then multiply the three dimensions.

T: Measure the inside dimensions of your box using the line that's drawn as the height, and multiply to find the volume. Then, confirm the measurement by packing the box to the line that's drawn. Record the volume in cubic centimeters on Problem #1 of your Problem Set.

S: (Work.)



NOTES ON MULTIPLE MEANS OF REPRESENTATION AND MATERIALS:

Fancy toothpicks, straight pins, and some office supplies come in clear rectangular boxes suitable for this activity. The horizontal fill line for the water can be drawn at a height that matches the number of cubes that can be packed into the box. If these are not readily available, small rectangular breath mint boxes or metal spice boxes can be used. However, students may have to estimate if the corners are rounded. It is best to test the boxes and volumes before implementing this lesson.

An alternative approach is to gather a collection of small gift boxes and use salt rather than water to fill them. While salt is not a liquid, it does behave like one for the purposes of the first activity.

The second activity must be done with water and a centimeter cube that sinks. If a dense cube is not available, students should use a drinking straw or coffee stirrer to submerge the cube completely.

If resources are limited, this may be done as a demonstration and then as a center over the course of a few days in small groups.

- T: Now I would like you to find the amount of liquid your container will hold. Any ideas how you might do this using the materials on your table? Turn and talk.
- S: We could pour in some water and then measure the water with the graduated cylinder. → We could fill the container with water and then use a measuring cup to measure the water. That would tell us the amount it will hold.
- T: What units are used on the graduated cylinder?
- S: Milliliters.
- T: Pour the water to the fill line. Then, measure the amount of water by carefully pouring it into the graduated cylinder. Record the liquid volume on Problem #2 of your Problem Set. Once your group is done, have a member of your group record your data onto the class poster.
- S: (Work and record.)
- T: (Circulate, asking students to describe what they're doing. Encourage use of the terms *volume*, *capacity*, and other unit language.)
- T: Now that we've recorded our findings, let's look at the volume data. What do you notice about the volume as measured by the cubes and the liquid volume?
- S: They are the same. → Our box packed 36 cubes, and it held 36 mL of water. → Although our prism was a different size than the first group's, our packing and filling amounts were the same. → Ours was really close—just one cubic centimeter more than the milliliters.
- T: What can you say about the relationship of 1 milliliter to 1 cubic centimeter?
- S: They seem to be the same. → I think they are equal.
- T: There's a way we can show that these two measurements are equal. Put water into your graduated cylinder to any measuring point other than the fill line. Be careful to fill it exactly to the line you choose. For example, you might fill your graduated cylinder to 15 mL.
- S: (Pour.)
- T: Now, pour in 1 more milliliter of water, and describe what happens to the water level.
- S: The water went up one more line. → The water rose because we put more in.
- T: Record the new amount of water on Problem #3 of your Problem Set. What will happen to the water level if we place 1 cube in the graduated cylinder? Tell your partner.
- S: It will go up again. → The water will rise because the cube pushes some of the water out of the way.
- T: Let's find out how far the water will rise. Place 1 centimeter cube into the water. Describe what happens to your partner.
- S: (Work and discuss.)
- T: How did the water level change?
- S: The water rose. → It looks like there's more water in the graduated cylinder. → The water went up 1 mL.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

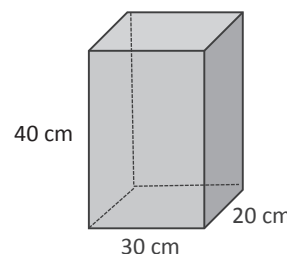
When dividing students into groups, be sure to choose members with different strengths. Try to assign tasks such as recorder, builder, pourer, and measurer.

- T: We didn't actually put more water in, and yet the cube caused a rise in the water level equal to when we put 1 mL of water in the graduated cylinder. From this investigation and from our work with the boxes, what can we say about the relationship between 1 mL of water and 1 cubic cm?
- S: They are equal. → I know they are equal because I measured my box and got the same number of cubes as milliliters. → I know they are equal because one cube made the water go up 1 milliliter.
- T: Yes. We have seen that $1 \text{ cm}^3 = 1 \text{ mL}$. (Write $1 \text{ cm}^3 = 1 \text{ mL}$ on the board.) This is an important relationship that will help us solve problems.

Problem 2

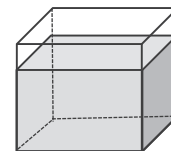
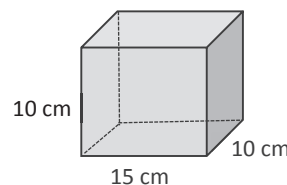
A rectangular tank measures 30 cm by 20 cm by 40 cm. How many milliliters of water are in the tank when it is full? How many liters is that?

- T: Let's use what we've learned about volume as filling to solve this problem. We need to find the volume of the water in the tank. What do we know about the tank that can help us?
- S: We know the size of the tank. → Since the water is filled to the top, the volume of the tank will be the same as the volume of the water.
- T: Find the volume of the tank.
- S: (Work to find 24,000 cubic centimeters.)
- T: We discovered today that 1 cubic centimeter is equal to 1 mL. Since this is true, how many milliliters of water are in the tank when it is full?
- S: 24,000 mL.
- T: How many liters is that?
- S: 24 liters.



Problem 3

- a. A small fish tank is filled to the top with water. If the tank measures 15 cm by 10 cm by 10 cm, what is the volume of water in the tank? Express your answer in mL.
- b. After a week, water evaporates out of the tank, so the water is 9 cm high. What is the volume of the water in the tank?
- T: (Project Problem 3(a) and the accompanying image onto the board.) Using what we've talked about today, turn and talk to your partner, and find the volume of water in the tank in cubic centimeters and in milliliters.
- S: All we need to do is multiply the sides because the water is all the way to the top. → Since the water fills the whole tank, we can just multiply $15 \times 10 \times 10$. That's 1,500 cubic centimeters. → It's easy to find the volume. It's 1,500 cubic centimeters. We have to say it in milliliters. That's exactly the same number, so it's 1,500 mL.
- T: (Project Problem 3(b).) Let's imagine that some of the water evaporated out of the tank. Now the water is only 9 cm deep. Does this change the height of the tank? Why or why not?
- S: No, because the tank doesn't change size.
- T: Does this change the area of the bottom of the tank?



- S: No. The tank is still the same size.
- T: Will the volume of the water change? Why or why not?
- S: The volume in the tank will be less because some of the water is gone. → The water won't be as high.
→ The water level will go down by 1 cm, so that's like pouring out a layer of 15 by 10 centimeters.
- T: Find the volume of the water in the tank now. Turn and talk.
- S: The bottom of the tank is the same, so the water is spread out on the bottom the same way as before. The only thing that is different is the height of the water. I'll multiply 15 and 10 and then multiply by 9. That's 1,350 cubic centimeters of water. → The part of the water that is gone is $15 \times 10 \times 1$. That's 150 cubic centimeters. I can subtract that from 1,500. That will be 1,350 cubic centimeters still in the tank.
- T: What is the volume of the water in the tank now?
- S: 1,350 cubic centimeters.
- T: Express that in milliliters.
- S: 1,350 mL.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

While the relationship of $1 \text{ cm}^3 = 1 \text{ mL}$ seems a simple one numerically, the concept behind this relationship—that of volume as *filling* as well as *packing* (especially when comparing a rectangular container to a cylindrical one)—is more complex. Be sure to offer many opportunities for students to encounter this concept beyond today's lesson. Ask often if an amount of liquid will fit into rectangular containers and how it might be confirmed without pouring.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use multiplication to connect volume as *packing* with volume as *filling*.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be

Name: Cheney Date: _____

1. Determine the volume of two boxes on the table using cubes and then confirm by measuring and multiplying.

Box Number:	Number of cubes packed:	Measurements:	Volume:		
		Length	Width	Height	
1	32	4 cm	4 cm	2 cm	32 cm ³
2	20	2 cm	5 cm	2 cm	20 cm ³

2. Using the same boxes from #1, record the amount of liquid that your box can hold.

Box Number:	Liquid the box can hold:
1	32 mL
2	20 mL

3. Shade to show the water in the graduated cylinder.

At first:
4 mL

After 1 mL water added:
5 mL

After 1 cm cube added:
6 mL

addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

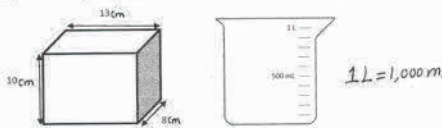
Any combination of the questions below may be used to lead the discussion.

- Have you ever heard the term *cc's* on TV or in real life? In what context have you heard the term? What do you think *cc's* means? (Students might say that medicines, intravenous fluids, and injections are often ordered in *cc's*. Explain that a *cc* is a cubic centimeter, equivalent to 1 milliliter.)
- Problem 6 describes the height of the water using the word *depth*. Discuss the connection between these two terms. How is height like depth? When might you use the word *height* to describe a figure, and when might *depth* be more appropriate? Can the words be used interchangeably? (Have students rephrase problems from the Problem Set to test. English language learners especially will benefit from such a discussion.)
- (Problem 7 asks students to extend their knowledge of cubic centimeters and milliliters to liters. Allow students to work together to think through this task if necessary and then explain their thinking to another partner group. Discuss other connections as well.) If 1 cubic centimeter is equal to 1 milliliter, to what liquid measure is 1 cubic meter equivalent? How could you find out? (Students can draw to investigate using $100\text{ cm} = 1\text{ m}$. Building a cubic container from meter sticks in the classroom helps students visualize the actual volume of 1 kiloliter. They might also imagine pouring 1,000 liter bottles of water or 500 2-liter soft drinks into that single container.)
- (Ask students to generate as many rectangular prisms with whole number sides as they can that would hold 1 liter of liquid. Although the dimensions are all factors of 1,000, the shapes of the containers may be drastically different. Students might even be encouraged to draw the containers on isometric dot paper for comparison.) What do the sides of all these containers have in common? (All are factors of 1,000.) Because they all have the same factors, are they all the same shape? Why or why not?

4. What conclusion can you draw about 1 cubic centimeter and 1 mL?

When 1 cubic centimeter is added, the water level rises 1 mL. Therefore, 1 cubic cm is equal to 1 mL. $1\text{ cm}^3 = 1\text{ mL}$

5. The tank, shaped like a rectangular prism, is filled to the top with water.



Will the beaker hold all the water in the tank? If yes, how much more will the beaker hold? If no, how much more will the tank hold than the beaker? Explain how you know.

$V_{\text{tank}} = 13\text{ cm} \times 8\text{ cm} \times 10\text{ cm} = 1,040\text{ cm}^3$. No, the beaker holds 40 mL less than the tank. $1\text{ L} = 1,000\text{ mL}$, and $1,040\text{ cm}^3 = 1,040\text{ mL}$. $1,040\text{ mL}$ is 40 mL more than 1,000 mL.

6. A rectangular fish tank measures 26 cm by 20 cm by 18 cm. The tank is filled with water to a depth of 15 cm.

a. What is the volume of the water in mL?

$26\text{ cm} \times 20\text{ cm} \times 15\text{ cm} = 7,800\text{ cm}^3 = 7,800\text{ mL}$

b. How many liters is that?

$7,800\text{ mL} \div 1,000 = 7.8\text{ L}$

c. How many more mL of water will be needed to fill the tank to the top? Explain how you know.

$26\text{ cm} \times 20\text{ cm} \times 3\text{ cm} = 1,560\text{ cm}^3 = 1,560\text{ mL}$. The remaining part is $26\text{ cm} \times 20\text{ cm} \times 3\text{ cm}$. I multiplied to find the volume there is left to fill.

7. A rectangular container is 25 cm long and 20 cm wide. If it holds 1 liter of water when full, what is its height?

$25\text{ cm} \times 20\text{ cm} = 500\text{ cm}^2$. $1\text{ L} = 1,000\text{ cm}^3$. $1,000\text{ cm}^3 \div 500\text{ cm}^2 = 2\text{ cm}$. It is 2 cm high.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Date _____

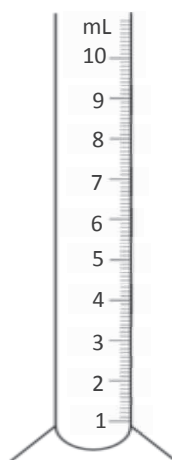
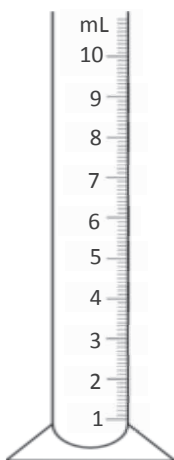
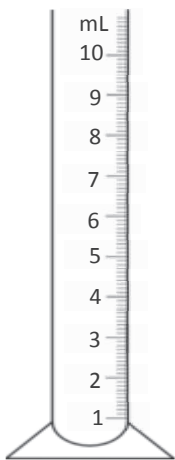
1. Determine the volume of two boxes on the table using cubes, and then confirm by measuring and multiplying.

Box Number	Number of Cubes Packed	Measurements			Volume
		Length	Width	Height	

2. Using the same boxes from Problem 1, record the amount of liquid that your box can hold.

Box Number	Liquid the Box Can Hold
	mL
	mL

3. Shade to show the water in the graduated cylinder.



At first:

_____ mL

After 1 mL water added:

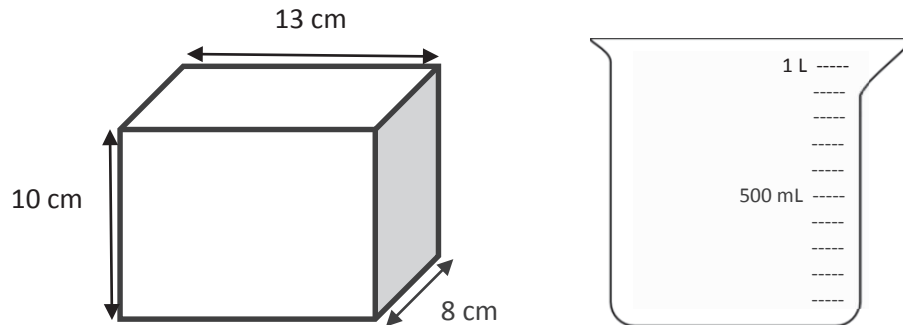
_____ mL

After 1 cm cube added:

_____ mL

4. What conclusion can you draw about 1 cubic centimeter and 1 mL?

5. The tank, shaped like a rectangular prism, is filled to the top with water.

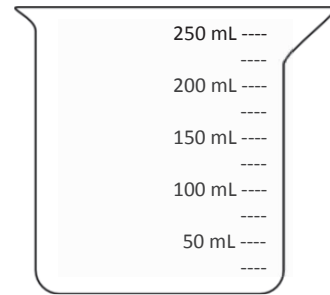
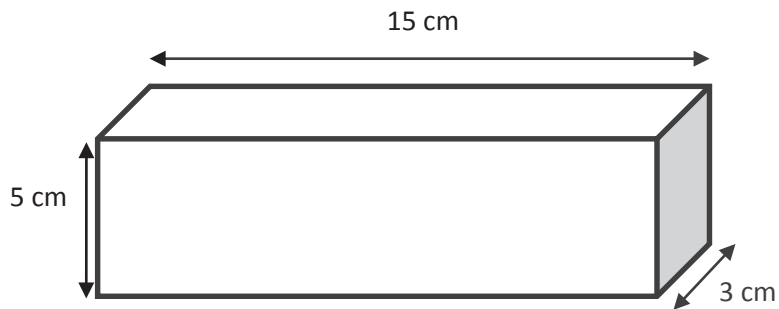


Will the beaker hold all the water in the tank? If yes, how much more will the beaker hold? If no, how much more will the tank hold than the beaker? Explain how you know.

6. A rectangular fish tank measures 26 cm by 20 cm by 18 cm. The tank is filled with water to a depth of 15 cm.
- What is the volume of the water in mL?
 - How many liters is that?
 - How many more mL of water will be needed to fill the tank to the top? Explain how you know.
7. A rectangular container is 25 cm long and 20 cm wide. If it holds 1 liter of water when full, what is its height?

Name _____

Date _____

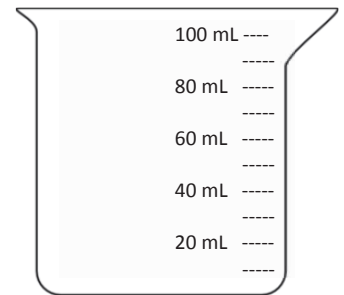


- a. Find the volume of the prism.
- b. Shade the beaker to show how much liquid would fill the box.

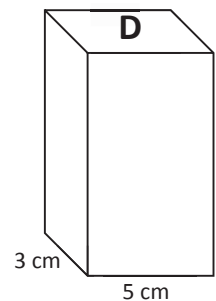
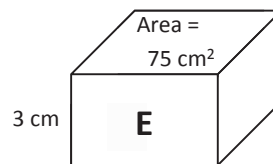
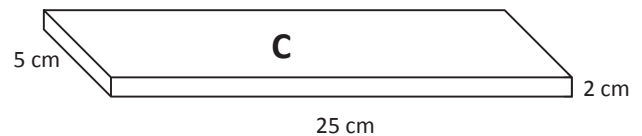
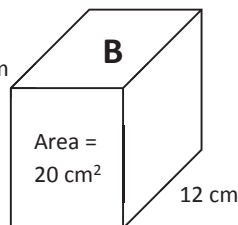
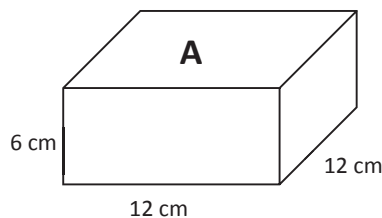
Name _____

Date _____

1. Johnny filled a container with 30 centimeter cubes. Shade the beaker to show how much water the container will hold. Explain how you know.



2. A beaker contains 250 mL of water. Jack wants to pour the water into a container that will hold the water. Which of the containers pictured below could he use? Explain your choices.



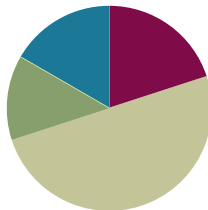
3. On the back of this paper, describe the details of the activities you did in class today. Include what you learned about cubic centimeters and milliliters. Give an example of a problem you solved with an illustration.

Lesson 6

Objective: Find the total volume of solid figures composed of two non-overlapping rectangular prisms.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(8 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Multiply Fractions (3 minutes)
- Count by Cubic Centimeters (3 minutes)
- Find the Volume (6 minutes)

Multiply Fractions (3 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews Module 4.

T: (Write $\frac{1}{2} \times \frac{1}{2} = \underline{\hspace{1cm}}$.) Complete the number sentence.

S: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Continue this process with $\frac{1}{2} \times \frac{1}{3}$ and $\frac{1}{2} \times \frac{1}{8}$.

T: (Write $\frac{2}{3} \times \frac{1}{5} = \underline{\hspace{1cm}}$.) On your personal white board, complete the number sentence.

S: (Write $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$.)

T: (Write $\frac{2}{3} \times \frac{1}{3} = \underline{\hspace{1cm}}$.) Say the completed number sentence.

S: $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$.

Repeat this process with $\frac{1}{3} \times \frac{3}{3}$, $\frac{1}{4} \times \frac{3}{5}$, and $\frac{3}{4} \times \frac{3}{5}$.

T: (Write $\frac{2}{3} \times \frac{2}{5} = \underline{\hspace{1cm}}$.) Complete the number sentence.

S: $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$.

Continue this process for $\frac{3}{4} \times \frac{3}{5}$.

T: (Write $\frac{1}{5} \times \frac{3}{4} = \underline{\hspace{1cm}}$.) On your personal white board, write the equation with the product.

S: (Write $\frac{1}{5} \times \frac{3}{4} = \frac{3}{20}$.)

T: (Write $\frac{3}{4} \times \frac{4}{3} = \underline{\hspace{1cm}}$.) Write the equation with the product.

S: (Write $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$.)

Continue with the following possible sequence: $\frac{2}{3} \times \frac{3}{4}$, $\frac{5}{8} \times \frac{2}{3}$, and $\frac{2}{5} \times \frac{3}{8}$.

Count by Cubic Centimeters (3 minutes)

Note: This fluency activity prepares students for today's lesson.

T: Count by twos to 10. (Write as students count.)

S: 2, 4, 6, 8, 10.

T: Count by two-hundreds to 1,000. (Write as students count.)

S: 200, 400, 600, 800, 1,000.

T: Count by 200 cm³ to 1,000 cm³. (Write as students count.)

S: 200 cm³, 400 cm³, 600 cm³, 800 cm³, 1,000 cm³.

T: Count by 200 cm³. This time, when you come to 1,000 cm³, say 1 liter. (Write as students count.)

S: 200 cm³, 400 cm³, 600 cm³, 800 cm³, 1 liter.

2	4	6	8	10
200	400	600	800	1,000
200 cm ³	400 cm ³	600 cm ³	800 cm ³	1,000 cm ³
200 cm ³	400 cm ³	600 cm ³	800 cm ³	1 liter



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Have students skip-count as a group as they did in Grades K and 1 when they were counting by twos and threes. This time, when they get to 1,000 cm³, they should say 1 liter.

Find the Volume (6 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 4. Images not drawn to scale.

T: (Project a 3 cm by 4 cm by 2 cm rectangular prism.)
What's the length of the rectangular prism?

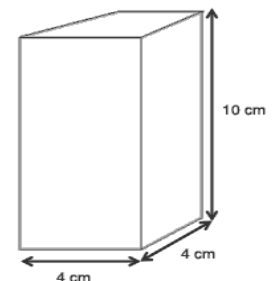
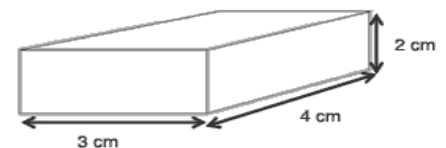
S: 3 cm.

T: What's the width?

S: 4 cm.

T: What's the height?

S: 2 cm.



T: (Write $__ \text{ cm} \times __ \text{ cm} \times __ \text{ cm} = __ \text{ cm}^3$.) On your personal white board, calculate the volume.

S: (Write $3 \text{ cm} \times 4 \text{ cm} \times 2 \text{ cm} = 24 \text{ cm}^3$.)

Repeat the process for the 4 cm by 4 cm by 10 cm rectangular prism.

T: (Project the rectangular prism that has a given volume of 40 in^3 , length of 4 in, and width of 5 in.) What's the length of the rectangular prism?

S: 4 in.

T: What's the width of the rectangular prism?

S: 5 in.

T: What's the volume of the rectangular prism?

S: 40 in^3 .

T: (Write $40 \text{ in}^3 = 4 \text{ in} \times 5 \text{ in} \times __ \text{ in}$.) On your personal white board, fill in the missing side length. If you need to, write a division sentence to calculate your answer.

S: (Write $40 \text{ in}^3 = 4 \text{ in} \times 5 \text{ in} \times 2 \text{ in}$.)

Repeat the process for the rectangular prism with a given volume of 120 in^3 , length of 3 in, and width of 4 in.

T: (Project the rectangular prism with a face having a given area of 30 ft^2 and a given width of 6 ft.) Say the given area of the prism's front face.

S: 30 ft^2 .

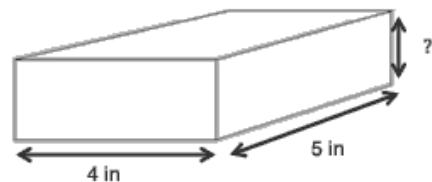
T: Say the given width.

S: 6 ft.

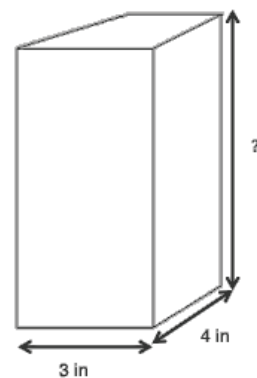
T: (Write $V = __ \text{ ft}^3$.) On your personal white board, calculate the volume.

S: (Write $V = 180 \text{ ft}^3$.)

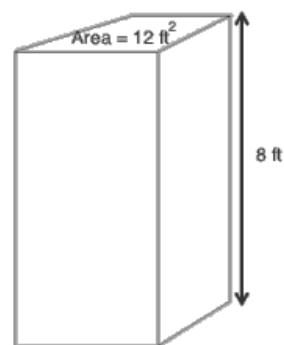
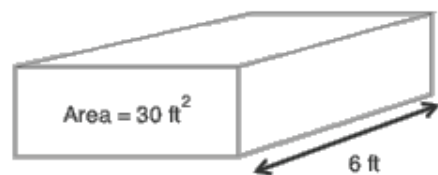
Repeat the process for the rectangular prism with a face having a given area of 12 ft^2 and a height of 8 ft.



$$\text{Volume} = 40 \text{ in}^3$$



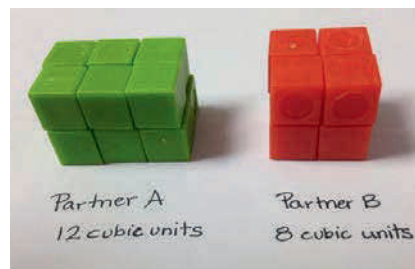
$$\text{Volume} = 120 \text{ in}^3$$



Concept Development (30 minutes)

Materials: (T) Drawing of rectangular prism figures

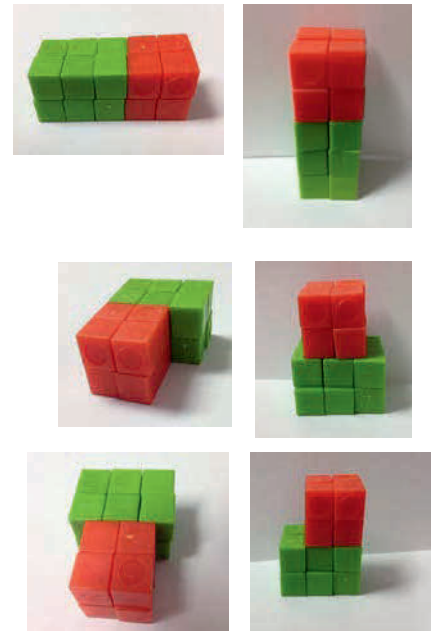
(S) 15 centimeter cubes, dot paper



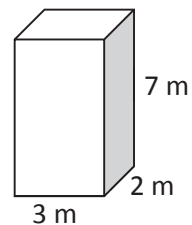
Problem 1

Build and combine structures, and then find the total volume.

- T: Partner A, use one color cube to build a structure that is 3 cm by 2 cm by 2 cm. Partner B, use a different color to build a cube that is 2 cm long on every side. Record the volume of your structures.
- S: (Work.)
- T: Keeping their original dimensions, how could you combine the two structures you've built? Turn and talk. Then, find the volume of your new structures.
- S: We could put the cube on top of the rectangular prism.
 → We could put them beside each other on the end.
 → We could make an L. → The volume is 20 cubic units.
- T: Now, build a different structure using the two prisms, and find the volume.
- S: (Work.)
- T: How did you find the volume of your new structures?
- S: We counted all the blocks. → We knew that one was 12 cubic units and the other one was 8 cubic units. We just added that together to get 20 cubic units.
- T: When you built the second structure, did the volume change? Why or why not?
- S: It did not change the volume. There were still 20 cubic units. → It doesn't matter how we stacked the two prisms together. The volume of each one is the same every time, and the volume of the whole thing is still 20 cubic units. → The total volume is always going to be the volume of the red structure plus the volume of the green structure, no matter how we stack them.

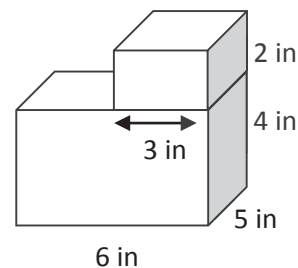
Possible Combinations**Problem 2**

- T: (Project or draw on the board the 3 m × 2 m × 7 m prism on the right.) What is the volume of this prism?
- S: 42 m³.
- T: Imagine another prism identical to this one. If we glued them together to make a bigger prism, how could we find the volume? Turn and talk. Then, find the volume.
- S: We already know that the volume of the first one is 42 m³. We could just add another 42 m³ to it. That would be 84 m³. → We could multiply 42 by 2 since they are just alike. That's 84 m³.



Problem 3

- T: (Project or draw on the board the composite structure.) How is this drawing different from the last one?
- S: There are two different-size boxes this time. → The little box on top only has measurements on the length and the height.
- T: There are a lot of markings on this figure. We'll need to be careful that we use the right ones when we find the volume. Find the volume of the bottom box.
- S: (Work to find 120 cubic inches.)
- T: What about the one on the top? I heard someone say that there isn't a width measurement on the drawing. How will we find the volume? Turn and talk.
- S: The boxes match up exactly in the drawing on the width. That means the width of the top box is the same as the bottom one, so it's still 5 inches wide. → You can tell the top and bottom box are the same width, so just multiply $3 \times 5 \times 2$.
- T: What is the volume of the top box?
- S: 30 cubic inches.
- T: How could we find the total volume?
- S: Add the two together.
- T: Say the number sentence with the units.
- S: $120 \text{ cubic inches} + 30 \text{ cubic inches} = 150 \text{ cubic inches}$.

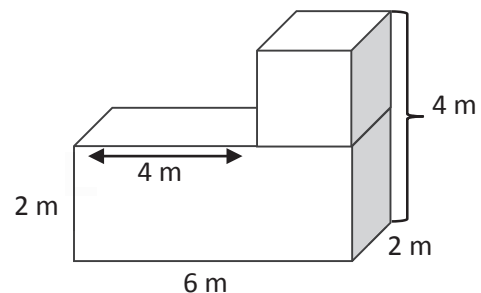


**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

Continuing to use the cubes to construct the prisms can help to keep kinesthetic learners on task. The calculation of the volume can still be made with the formula.

Problem 4

- T: (Project or draw the figure given on the right.) Compare this figure to the last one.
- S: There are two different boxes again. → There's a little one and a big one, like last time. → This time, there's a bracket on the height of both boxes. → There's no length or width or height measurement on the top box this time.
- T: If there are no measurements on the top box alone, how might we still calculate the volume? Turn and talk.
- S: We can tell the length of the top box by looking at the 6 meters along the bottom. The other box has 4 meters sticking out on the top of the box. That means the box must be 2 meters long. → The length is 6 minus 4. That's 2. → The width is easy. It's the same as the bottom box, so that's 2 meters. → The height of both boxes is 4 meters. If the bottom box is 2 meters, then the top box must also be 2 meters.
- T: What is the volume of the top prism? Say the number sentence.
- S: $2 \text{ m} \times 2 \text{ m} \times 2 \text{ m} = 8 \text{ cubic meters}$.



- T: What is the volume of the bottom prism? Say the number sentence.
- S: $6 \text{ m} \times 2 \text{ m} \times 2 \text{ m} = 24 \text{ cubic meters}$.
- T: What's the total volume of both? Say the number sentence.
- S: $8 \text{ cubic meters} + 24 \text{ cubic meters} = 32 \text{ cubic meters}$.

Problem 5

Two rectangular prisms have a combined volume of 135 cubic meters. Prism A has double the volume of Prism B.

- What is the volume of each prism?
 - If one face of Prism A has an area of 10 square meters, what is its height?
- T: Let's use a tape diagram to help us with this problem. Read it with me.

T/S: (Read.)

T: What can we draw from the first sentence?

S: A tape diagram for each prism. → Two tape diagrams labeled *Prism A's volume* and *Prism B's volume*. → A bracket on both to show they are 135 cubic meters total. → Their total volume is 135 cubic meters.

T: What does the next sentence tell us, and how can we represent it?

S: Prism A is double the volume of Prism B. → We need 2 units for Prism A. → Prism A's tape should be twice as long as Prism B's.

T: Show that in your diagram. Then, use this information to solve for the volumes of both prisms.

S: (Work.)

T: What is the volume of each prism?

S: Prism A is 90 cubic meters, and Prism B is 45 cubic meters.

T: To find the height of Prism A, what do we need to think about? Turn and talk, and then solve.

S: We know the area of one face. If we multiply the area by something, we should get the volume of 90 m^3 . The area is 10 m^2 , and 10 times 9 is 90. It is 9 meters tall. → We can divide 90 by 10 and get 9 meters tall.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge students whose spatial skills allow them to see these figures easily by having them draw a figure consisting of three different prisms on dot paper with just enough information given to calculate the volume of the figure. They should calculate the volume of their own figures and then exchange figures with a partner.

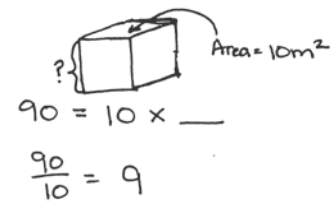
Students can write about the minimum information necessary to calculate the volume of a composite figure.



$$3 \text{ units} = 135$$

$$1 \text{ unit} = \frac{135}{3} = 45$$

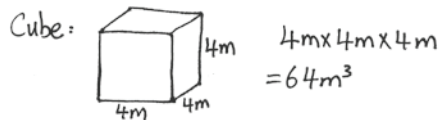
Prism A's volume is $2 \times 45 \text{ cm}^3 = 90 \text{ cm}^3$.
Prism B's volume is 45 cm^3 .



Prism A is 9 meters high.

Application Problem (8 minutes)

A storage company advertises three different choices for all your storage needs: “The Cube,” a true cube with a volume of 64 m^3 ; “The Double” (double the volume of “The Cube”); and “The Half” (half the volume of “The Cube”). What could be the dimensions of the three storage units? How might they be oriented to cover the most floor space? The most height?



Double:
 Could be $8\text{m} \times 4\text{m} \times 4\text{m} = 128\text{m}^3$
 most floor space: $8\text{m} \times 16\text{m}$
 (or 128m^2 base and 1m high.)
 most height: $1\text{m} \times 1\text{m} \times 128\text{m}$

Half:
 Could be $4\text{m} \times 4\text{m} \times 2\text{m} = 32\text{m}^3$
 most floor space: $8\text{m} \times 4\text{m}$
 (or 32m^2 base and 1m high.)
 most height: $1\text{m} \times 1\text{m} \times 32\text{m}$

Note: Students use the knowledge that a cube's sides are all equal to find the side as 4 meters. (Side \times side \times side = 64 m^3 , so each side must be 4 m.) While students may approach halving or doubling the other storage units using different approaches (for example, doubling any of the dimensions singly), this problem affords an opportunity to discuss the many options of orienting the storage units to give the most practical increase or decrease in square footage or height for storage of various items. The problem also reinforces the part to whole relationships of volume.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

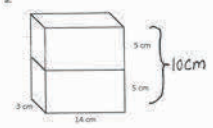
Student Debrief (10 minutes)

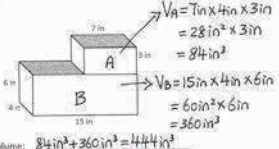
Lesson Objective: Find the total volume of solid figures composed of two non-overlapping rectangular prisms.

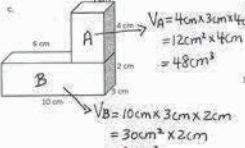
The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

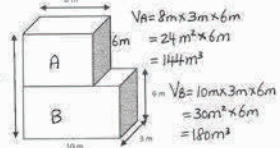
Name: Michelle Date: _____

1. Find the total volume of the figures, and record your solution strategy.

a. 
 Volume: $V = 14\text{cm} \times 3\text{cm} \times 10\text{cm} = 420\text{cm}^3$
 Solution Strategy: I combined the 2 heights to get 10cm. Then I just used the formula for Volume.

b. 
 $V_A = 7\text{in} \times 4\text{in} \times 3\text{in} = 28\text{in}^2 \times 3\text{in} = 84\text{in}^3$
 $V_B = 15\text{in} \times 4\text{in} \times 6\text{in} = 60\text{in}^2 \times 6\text{in} = 360\text{in}^3$
 Volume: $84\text{in}^3 + 360\text{in}^3 = 444\text{in}^3$
 Solution Strategy: Prism A & B have the same width, so I used the Volume formula & then added the 2 volumes to find the total.

c. 
 $V_A = 4\text{cm} \times 3\text{cm} \times 4\text{cm} = 12\text{cm}^2 \times 4\text{cm} = 48\text{cm}^3$
 $V_B = 10\text{cm} \times 3\text{cm} \times 2\text{cm} = 30\text{cm}^2 \times 2\text{cm} = 60\text{cm}^3$
 Volume: $48\text{cm}^3 + 60\text{cm}^3 = 108\text{cm}^3$
 Solution Strategy: 10cm - 6cm shows that the length of A is 4cm. Then I found the volume of A & B and added them together.

d. 
 $V_A = 8\text{m} \times 3\text{m} \times 6\text{m} = 24\text{m}^2 \times 6\text{m} = 144\text{m}^3$
 $V_B = 10\text{m} \times 3\text{m} \times 6\text{m} = 30\text{m}^2 \times 6\text{m} = 180\text{m}^3$
 Volume: $144\text{m}^3 + 180\text{m}^3 = 324\text{m}^3$
 Solution Strategy: 12m - 6m shows that the height of A is 6m. Again, I found the volume of each Prism then added them together.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What advice would you give to a friend who was having trouble picturing the dimensions on a composite figure? What helps you to figure out missing dimensions?
- (If all students use an addition strategy to find the total volume of the figures, suggest to them the alternate strategy of subtracting the missing part. For example, Problem 1(d) offers such a strategy.) Imagine that the top prism was once identical to the bottom prism (much like the figure in Problem 1(a). If these two prisms were identical, what would the total volume be? (360 cubic meters.) Now, imagine that the end was removed from the top prism. What would the volume of that removed part be? ($2\text{ m} \times 3\text{ m} \times 6\text{ m} = 36\text{ cubic meters.}$) If we wanted to know the total volume of the figure with this part removed, how could we do that? (Subtract. $360\text{ m}^3 - 36\text{ m}^3 = 324\text{ m}^3$.) Is this the same volume we found when we added the two prisms? (Yes!)

2. A sculpture (pictured below) is made of two sizes of rectangular prisms. One size measures 13 in by 8 in by 2 in. The other size measures 9 in by 8 in by 18 in. What is the total volume of the sculpture?

Volume A = $13\text{ in} \times 8\text{ in} \times 2\text{ in}$
 $= 208\text{ in}^3$

Volume B = $9\text{ in} \times 8\text{ in} \times 18\text{ in}$
 $= 1,296\text{ in}^3$

The total volume of the sculpture is $3,840\text{ in}^3$

3. The combined volume of two identical cubes is 128 cubic centimeters. What is the side length of each cube?

total volume: 128 cm^3
 64 cm^3 64 cm^3

Each side length on each cube is 4 cm.

4. A rectangular tank with a base area of 24 cm^2 is filled with water and oil to a depth of 9 cm. The oil and water separate into two layers when the oil rises to the top. If the thickness of the oil layer is 4 cm, what is the volume of the water?

$24\text{ cm}^2 \times 5\text{ cm} = 120\text{ cm}^3$

The volume of the water is 120 cm^3 .

5. Two rectangular prisms have a combined volume of 432 cubic feet. Prism A has half the volume of Prism B.

a. What is the volume of Prism A? Prism B?
 The volume of Prism A is 144 ft^3 .
 The volume of Prism B is 288 ft^3 .

b. If Prism A has a base area of 24 ft^2 , what is the height of Prism A?
 $144 \div 24 = 6$. The height of Prism A is 6 ft.

c. If Prism B's base is $\frac{1}{3}$ the area of Prism A's base, what is the height of Prism B?
 $288 \div 16 = 18$. The height of Prism B is 18 ft.

- How did the Application Problem help you solve Problem 3?
- Compare your approach to solving Problem 4 with that of the person sitting next to you. How is your thinking alike? How is it different?
- (Allow students to share the tape diagrams that they used to solve Problem 5. If students need more scaffolding, use the suggested sequence in the Concept Development to guide their drawing.) How is the problem like the one we did together? How is it different? (It uses the word *half* instead of *double*. This time, Prism B is the larger.) What was the effect on the height of the two prisms by making the base of Prism B smaller than the one on Prism A?
- Is a shorter container always a smaller volume? Give some examples of prisms to support your answer.

Exit Ticket (3 minutes)

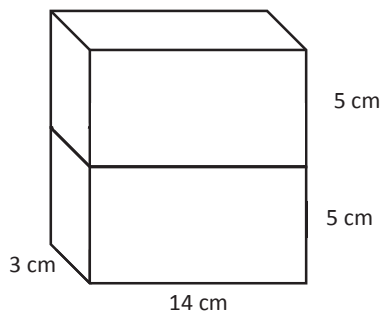
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____

Date _____

1. Find the total volume of the figures, and record your solution strategy.

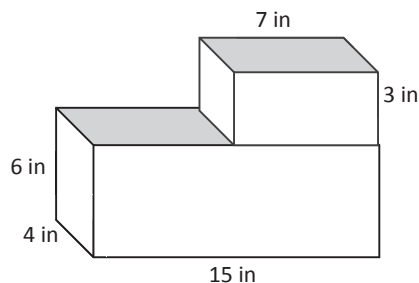
a.



Volume: _____

Solution Strategy:

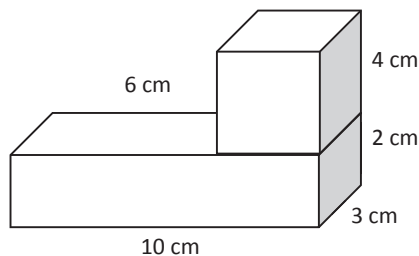
b.



Volume: _____

Solution Strategy:

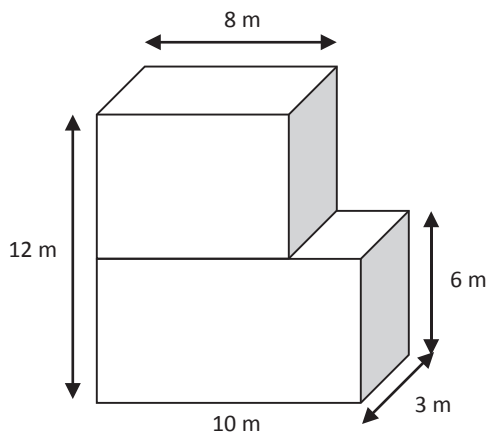
c.



Volume: _____

Solution Strategy:

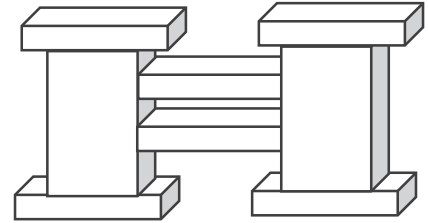
d.



Volume: _____

Solution Strategy:

2. A sculpture (pictured below) is made of two sizes of rectangular prisms. One size measures 13 in by 8 in by 2 in. The other size measures 9 in by 8 in by 18 in. What is the total volume of the sculpture?



3. The combined volume of two identical cubes is 128 cubic centimeters. What is the side length of each cube?
4. A rectangular tank with a base area of 24 cm^2 is filled with water and oil to a depth of 9 cm. The oil and water separate into two layers when the oil rises to the top. If the thickness of the oil layer is 4 cm, what is the volume of the water?

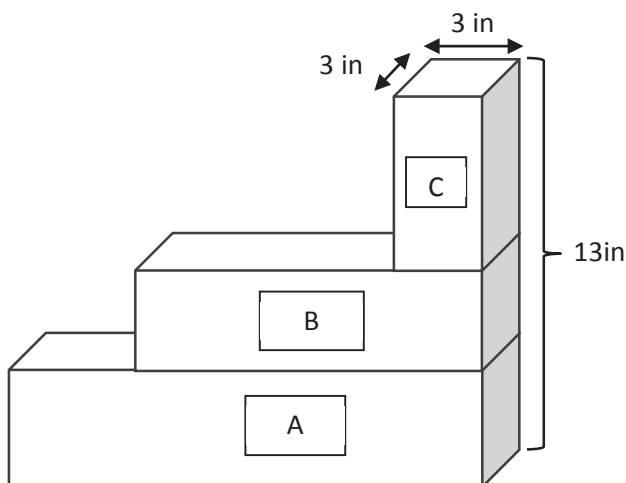


5. Two rectangular prisms have a combined volume of 432 cubic feet. Prism A has half the volume of Prism B.
- What is the volume of Prism A? Prism B?
 - If Prism A has a base area of 24 ft^2 , what is the height of Prism A?
 - If Prism B's base is $\frac{2}{3}$ the area of Prism A's base, what is the height of Prism B?

Name _____

Date _____

The image below represents three planters that are filled with soil. Find the total volume of soil in the three planters. Planter A is 14 inches by 3 inches by 4 inches. Planter B is 9 inches by 3 inches by 3 inches.

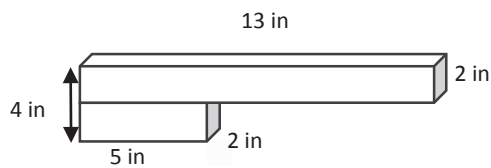


Name _____

Date _____

1. Find the total volume of the figures, and record your solution strategy.

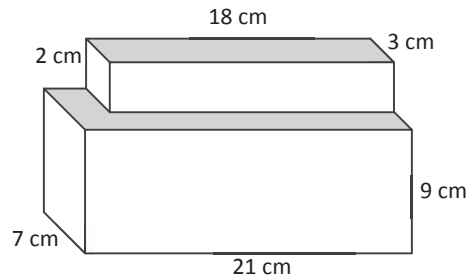
a.



Volume: _____

Solution Strategy:

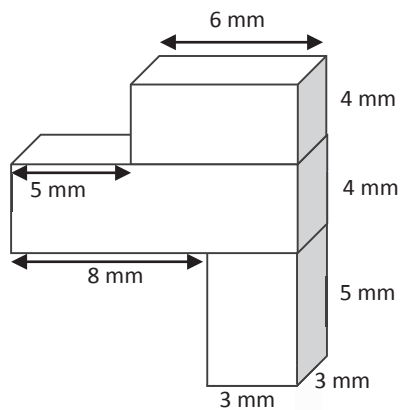
b.



Volume: _____

Solution Strategy:

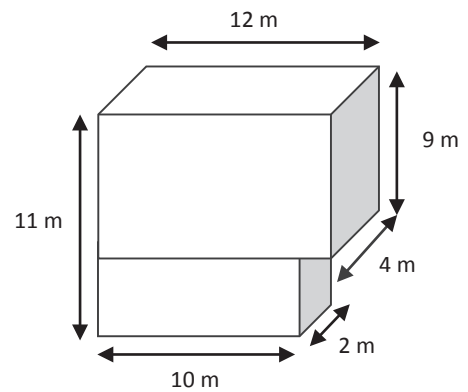
c.



Volume: _____

Solution Strategy:

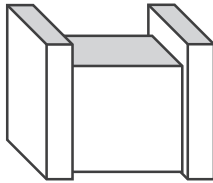
d.



Volume: _____

Solution Strategy:

2. The figure below is made of two sizes of rectangular prisms. One type of prism measures 3 inches by 6 inches by 14 inches. The other type measures 15 inches by 5 inches by 10 inches. What is the total volume of this figure?



3. The combined volume of two identical cubes is 250 cubic centimeters. What is the measure of one cube's edge?
4. A fish tank has a base area of 45 cm^2 and is filled with water to a depth of 12 cm. If the height of the tank is 25 cm, how much more water will be needed to fill the tank to the brim?



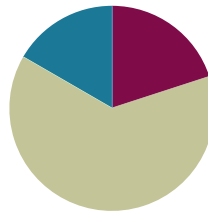
5. Three rectangular prisms have a combined volume of 518 cubic feet. Prism A has one-third the volume of Prism B, and Prisms B and C have equal volume. What is the volume of each prism?

Lesson 7

Objective: Solve word problems involving the volume of rectangular prisms with whole number edge lengths.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Concept Development	(38 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Sprint: Multiply Fractions (8 minutes)
- Find the Volume (4 minutes)

Sprint: Multiply Fractions (8 minutes)

Materials: (S) Multiply Fractions Sprint

Note: This fluency activity reviews Module 4.

Find the Volume (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews Lesson 5. Images not drawn to scale.

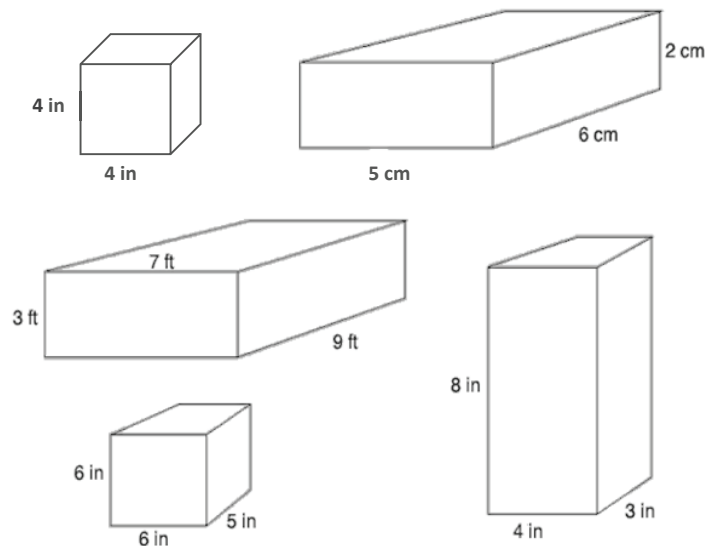
T: On your personal white board, write the formula for finding the volume of a rectangular prism.

S: (Write $V = l \times w \times h$.)

T: (Write $V = l \times w \times h$.) Project a rectangular prism with a length of 5 cm, width of 6 cm, and height of 2 cm.

T: Write a multiplication sentence to express the volume of the rectangular prism.

S: (Beneath $V = l \times w \times h$, write $V = 5 \text{ cm} \times 6 \text{ cm} \times 2 \text{ cm}$. Beneath it, write $V = 60 \text{ cm}^3$.)



Continue this process with the other rectangular prisms.

T: (Project a cube with side lengths equal to 4 inches.) Name the prism.

S: Cube.

T: Write a multiplication sentence to show the volume of the cube.

S: (Write $V = 4 \text{ in} \times 4 \text{ in} \times 4 \text{ in}$. Beneath it, write $V = 64 \text{ in}^3$.)

Concept Development (38 minutes)

Materials: (S) Problem Set

Note: The time normally allotted for the Application Problem has been included in the Concept Development portion of today's lesson.

Suggested Delivery of Instruction for Solving Lesson 7's Word Problems

1. Model the problem.

Have two pairs of students who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something?
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give everyone two minutes to finish work on that question, sharing their work and thinking with a peer. All students should write their equations and statements of the answer.

3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solutions.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

The problems in today's lesson are focused on planters and gardening. The lesson could serve as a springboard for planning a school garden. Students could plan planters as Geoffrey does in the lesson given parameters of height or base area. Students might also research optimal soil depths (and thus volume) for particular plants and vegetables to incorporate into their designs.

Bringing their designs into reality is the ultimate real-world problem connection. If it is possible, let them try. Writing a proposal to present the designs (along with estimated costs) to the principal or school board encompasses many skills, but even cardstock planters with paper flowers can be a rewarding experience.

Note: Geoffrey builds rectangular planters. All of the inside dimensions of the planters are whole numbers.

Problem 1

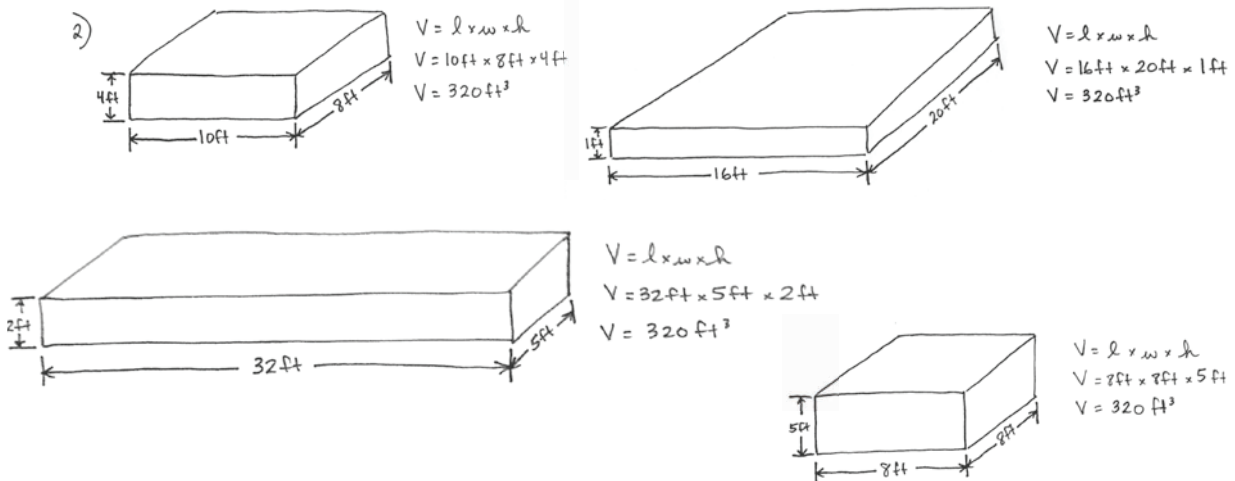
Geoffrey's first planter is 8 feet long and 2 feet wide. The container is filled with soil to a height of 3 feet in the planter. What is the volume of soil in the planter? Explain your work using a diagram.

In this problem, students are given three dimensions (length, width, and height) and are asked to find the volume of the soil in the planter. The use of the volume formula allows students to find the number of cubic feet of soil in the planter. A non-scaled illustration of the planter is the most logical diagram to accompany this work.



Problem 2

Geoffrey wants to grow some tomatoes in four large planters. He wants each planter to have a volume of 320 cubic feet, but he wants them all to be different. Show four different ways Geoffrey can make these planters, and draw diagrams with the planters' measurements on them.



In Problem 2, students are asked to come up with four sets of differing dimensions that all result in a volume of 320 cubic feet. This problem requires students to think in terms of whole to part. They need to find factors of 32 tens to generate the dimensions. The illustrations are just four such examples. Encourage students to come up with different values for each dimension rather than just changing the name of the same dimensions (length, width, and height) repeatedly, although that method would result in the same volume. Simply changing the shape of the planter can provide opportunities for discussion of which shape might be best to use for different situations. For example, in a small yard, planting a tree that would need more depth than length might be the best option. There are a wide variety of dimensions that would be acceptable here, but be sure to have students check their multiplication using the volume formula.

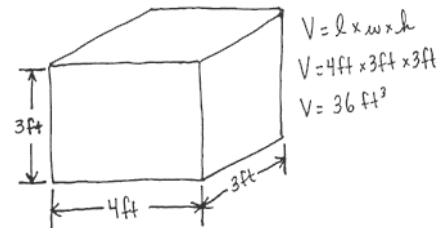
Problem 3

Geoffrey wants to make one planter that extends from the ground to just below his back window. The window starts 3 feet off the ground. If he wants the planter to hold 36 cubic feet of soil, name one way he could build the planter so it is not taller than 3 feet. Explain how you know.

Since Geoffrey wants to build a planter with a height of 3 feet & a volume of 36 cubic feet, the base of the planter should have an area of 12 sq. ft. I drew a planter with a length of 4 ft, a width of 3 ft, & a height of 3 ft.

$$36 \div 3 = 12$$

$$12 = 4 \times 3$$



This problem requires students to work backward and reason that since Geoffrey needs 36 cubic feet of volume and that the height of Geoffrey's planter is 3 feet, then division shows that the base of the planter must have an area of 12 square feet. From there, students have the freedom to design a planter with a base measuring 12 ft \times 1 ft, 6 ft \times 2 ft, or 4 ft \times 3 ft.

(Note: These dimensions may represent either the length or the width.) Again, this presents an opportunity to discuss the connection between the areas of the base and the shape the planter takes. Extend the problem by asking students to choose the planter that would not extend beyond a 5-foot window. (Any of the previously mentioned planters could work in this case, depending on which side is turned toward the window.) These discussions are sure to invite construction of viable arguments and the opportunity to critique the reasoning of classmates.

Problem 4

After all of this gardening work, Geoffrey decides he needs a new shed to replace the old one. His current shed is a rectangular prism that measures 6 feet long by 5 feet wide by 8 feet high. He realizes he needs a shed with 480 cubic feet of storage.



NOTES ON MULTIPLE MEANS OF EXPRESSION:

If students have difficulty drawing the rectangular prisms freehand, isometric dot paper can be used as a scaffold. Students may also like to use a computer to draw their figures to be printed out. Calculations can be done alongside.

- a. Will he achieve his goal if he doubles each dimension? Why or why not?

4a) SHED: $V = 6\text{ ft} \times 5\text{ ft} \times 8\text{ ft}$
 $V = 240\text{ ft}^3$

SHED
 DIMENSIONS:
 DOUBLED $V = 240\text{ ft}^3 \times 8$
 $V = 1,920\text{ ft}^3$

Geoffrey's current shed has a volume of 240 cubic ft, which is half the volume he needs. By doubling each dimension of the shed, Geoffrey will get a shed that is 8 times the current size (because $2 \times 2 \times 2 = 8$). To double the volume he needs only to double one dimension, not all three.

This part of Problem 4 gives students a chance to explore the exponential growth potential of doubling all three dimensions simultaneously. Doubling the length, width, and height of Geoffrey's shed results in a volume that is 8 times that of his current shed $(l \times 2) \times (w \times 2) \times (h \times 2) = (l \times w \times h) \times 8$. While this size shed certainly provides the 480 cubic feet he is looking for, students can reason that doubling each dimension would lead to a shed that is far larger than Geoffrey needs. This may lead to students trying to double only two of the dimensions and then realizing that simply doubling one of the dimensions of his shed gives Geoffrey double the volume. This discussion can also include an exploration of which dimension makes the most sense to double given that this is a garden shed. Would doubling the height give more usable space for gardening equipment? Does it make more sense to double either the length or the width? Challenge: Is there a way to change two dimensions and still simply double the space?

- b. If he wants to keep the height the same, what other dimensions could he use to reach his target volume?

4b) Since Geoffrey wants to double the volume of his shed & keep the height the same, he could double the length & keep the width the same too. Or he could double the width & keep the length the same.

$$l = 12\text{ ft}$$

$$w = 5\text{ ft}$$

$$h = 8\text{ ft}$$

OR

$$l = 6\text{ ft}$$

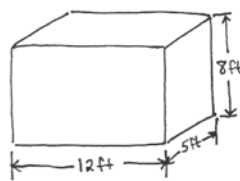
$$w = 10\text{ ft}$$

$$h = 8\text{ ft}$$

This problem builds on students' thinking from the previous problem and asks them to identify dimensions that would yield a shed volume of 480 cubic feet while maintaining a height of 8 feet. Most students will correctly reason that they simply need to double one of the other dimensions (the length or the width) in order to create a doubled volume. However, there are additional ways to create a volume of 480 cubic feet with a height of 8 feet, including halving the length and quadrupling the width. Engage students in a discussion about why this is possible (think back to Problem 3), and have them share their alternate solutions.

- c. If he uses the dimensions in part (b), what would be the area of the new shed's floor?

4c)



$$A = \ell \times w$$

$$A = 12\text{ ft} \times 5\text{ ft}$$

$$A = 60\text{ ft}^2$$

The floor would have an area of 60 square feet.

Part (c) requires students to remember their work from Lesson 4 and multiply the length times the width to find the area of the shed floor. Since students use their varied answers from part (b) to answer this question, expect to find variety in the responses here as well. However, this is another opportunity to engage students in a discussion about why the area must be 60 square feet, despite using different dimensions from part (b).

Student Debrief (10 minutes)

Lesson Objective: Solve word problems involving the volume of rectangular prisms with whole number edge lengths.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check their work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What effect does doubling one dimension have on the volume? Doubling two dimensions? Doubling all dimensions? Why?
- What effect would doubling one dimension while halving another have on the volume? Why?
- How many prisms can you think of that have a volume of 100 cm^3 ?
- If Geoffrey had been using fractional lengths for the dimensions for his planters, how would that have changed the possible answers to these questions?

Name Joe Date _____

Solve:

Geoffrey builds rectangular planters.

1. Geoffrey's first planter is 8 feet long and 2 feet wide. The container is filled with soil to a height of 3 feet in the planter. What is the volume of soil in the planter? Explain your work using a diagram.

$$V = L \times W \times H$$

$$= 8\text{ ft} \times 2\text{ ft} \times 3\text{ ft}$$

$$= 48\text{ ft}^3$$

There is 48 ft^3 of soil in the planter.

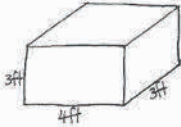
2. Geoffrey wants to grow some tomatoes in four large planters. He wants each planter to have a volume of 320 cubic feet, but he wants them all to be different. Show four different ways Geoffrey can make these planters, and draw diagrams with the planters' measurements on them.

<p>Planter A</p> $V = L \times W \times H$ $= 16\text{ ft} \times 4\text{ ft} \times 5\text{ ft}$ $= 320\text{ ft}^3$	<p>Planter B</p> $V = L \times W \times H$ $= 16\text{ ft} \times 10\text{ ft} \times 1\text{ ft}$ $= 320\text{ ft}^3$
<p>Planter C</p> $V = L \times W \times H$ $= 32\text{ ft} \times 5\text{ ft} \times 2\text{ ft}$ $= 320\text{ ft}^3$	<p>Planter D</p> $V = L \times W \times H$ $= 8\text{ ft} \times 8\text{ ft} \times 5\text{ ft}$ $= 320\text{ ft}^3$

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

3. Geoffrey wants to make one planter that extends from the ground to just below his back window. The window starts 3 feet off the ground. If he wants the planter to hold 36 cubic feet of soil, name one way he could build the planter so it is not taller than 3 feet. Explain how you know.



$$36 \div 3 = 12$$

$$12 \div 4 = 3$$

$$V = L \times W \times H$$

$$= 4 \text{ ft} \times 3 \text{ ft} \times 3 \text{ ft}$$

$$= 36 \text{ ft}^3$$

Since Geoffrey wants to build a planter with a height of 3 ft & a volume of 36 ft³, the base of the planter should have an area of 12 ft². I drew a planter with L=4 ft, W=3 ft, H=3 ft.

4. After all of this gardening work, Geoffrey decides he needs a new shed to replace the old one. His current shed is a rectangular prism that measures 6 feet long by 5 feet wide by 8 feet high. He realizes he needs a shed with 480 cubic feet of storage.

a. Will he achieve his goal if he doubles each dimension? Why or why not?

Shed: $V = 6 \text{ ft} \times 5 \text{ ft} \times 8 \text{ ft}$
 $= 240 \text{ ft}^3$

By doubling each dimension of the shed, Geoffrey will get a shed that is 8 times the current size because $(2 \times 2 \times 2 = 8)$. To double the volume he needs only to double one dimension, not all three.

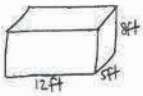
shed dimensions doubled: $V = 240 \text{ ft}^3 \times 8$
 $= 1,920 \text{ ft}^3$

b. If he wants to keep the height the same, what could the other dimensions be for him to get the volume he wants?

He could double the length and keep the width the same. OR he could double the width and keep the length the same.

$L = 12 \text{ ft}$ OR $L = 6 \text{ ft}$
 $W = 5 \text{ ft}$ OR $W = 10 \text{ ft}$
 $H = 8 \text{ ft}$ OR $H = 8 \text{ ft}$

c. If he uses the dimensions in part (b), what could be the area of the new shed's floor?



$$A = L \times W$$

$$= 12 \text{ ft} \times 5 \text{ ft}$$

$$= 60 \text{ ft}^2$$

The floor could have an area of 60 ft².

A

Number Correct: _____

Multiply Fractions

1.	$\frac{1}{2} \times \frac{1}{2} =$	
2.	$\frac{1}{2} \times \frac{1}{3} =$	
3.	$\frac{1}{2} \times \frac{1}{4} =$	
4.	$\frac{1}{2} \times \frac{1}{7} =$	
5.	$\frac{1}{7} \times \frac{1}{2} =$	
6.	$\frac{1}{3} \times \frac{1}{2} =$	
7.	$\frac{1}{3} \times \frac{1}{3} =$	
8.	$\frac{1}{3} \times \frac{1}{6} =$	
9.	$\frac{1}{3} \times \frac{1}{5} =$	
10.	$\frac{1}{5} \times \frac{1}{3} =$	
11.	$\frac{1}{5} \times \frac{2}{3} =$	
12.	$\frac{2}{5} \times \frac{2}{3} =$	
13.	$\frac{1}{4} \times \frac{1}{3} =$	
14.	$\frac{1}{4} \times \frac{2}{3} =$	
15.	$\frac{3}{4} \times \frac{2}{3} =$	
16.	$\frac{1}{6} \times \frac{1}{3} =$	
17.	$\frac{5}{6} \times \frac{1}{3} =$	
18.	$\frac{5}{6} \times \frac{2}{3} =$	
19.	$\frac{5}{4} \times \frac{2}{3} =$	
20.	$\frac{1}{5} \times \frac{1}{5} =$	
21.	$\frac{2}{5} \times \frac{2}{5} =$	
22.	$\frac{2}{5} \times \frac{3}{5} =$	

23.	$\frac{2}{5} \times \frac{5}{3} =$	
24.	$\frac{3}{5} \times \frac{5}{2} =$	
25.	$\frac{1}{3} \times \frac{1}{3} =$	
26.	$\frac{1}{3} \times \frac{2}{3} =$	
27.	$\frac{2}{3} \times \frac{2}{3} =$	
28.	$\frac{2}{3} \times \frac{3}{2} =$	
29.	$\frac{2}{3} \times \frac{4}{3} =$	
30.	$\frac{2}{3} \times \frac{5}{3} =$	
31.	$\frac{3}{2} \times \frac{3}{5} =$	
32.	$\frac{3}{4} \times \frac{1}{5} =$	
33.	$\frac{3}{4} \times \frac{4}{5} =$	
34.	$\frac{3}{4} \times \frac{5}{5} =$	
35.	$\frac{3}{4} \times \frac{6}{5} =$	
36.	$\frac{1}{4} \times \frac{6}{5} =$	
37.	$\frac{1}{7} \times \frac{1}{7} =$	
38.	$\frac{1}{8} \times \frac{3}{5} =$	
39.	$\frac{5}{6} \times \frac{1}{4} =$	
40.	$\frac{3}{4} \times \frac{3}{4} =$	
41.	$\frac{2}{3} \times \frac{6}{6} =$	
42.	$\frac{3}{4} \times \frac{6}{2} =$	
43.	$\frac{7}{8} \times \frac{7}{9} =$	
44.	$\frac{7}{12} \times \frac{9}{8} =$	

B

Number Correct: _____

Improvement: _____

Multiply Fractions

1.	$\frac{1}{2} \times \frac{1}{3} =$	
2.	$\frac{1}{2} \times \frac{1}{4} =$	
3.	$\frac{1}{2} \times \frac{1}{5} =$	
4.	$\frac{1}{2} \times \frac{1}{9} =$	
5.	$\frac{1}{9} \times \frac{1}{2} =$	
6.	$\frac{1}{5} \times \frac{1}{2} =$	
7.	$\frac{1}{5} \times \frac{1}{3} =$	
8.	$\frac{1}{5} \times \frac{1}{7} =$	
9.	$\frac{1}{5} \times \frac{1}{3} =$	
10.	$\frac{1}{3} \times \frac{1}{5} =$	
11.	$\frac{1}{3} \times \frac{2}{5} =$	
12.	$\frac{2}{3} \times \frac{2}{5} =$	
13.	$\frac{1}{3} \times \frac{1}{4} =$	
14.	$\frac{1}{3} \times \frac{3}{4} =$	
15.	$\frac{2}{3} \times \frac{3}{4} =$	
16.	$\frac{1}{3} \times \frac{1}{6} =$	
17.	$\frac{2}{3} \times \frac{1}{6} =$	
18.	$\frac{2}{3} \times \frac{5}{6} =$	
19.	$\frac{3}{2} \times \frac{3}{4} =$	
20.	$\frac{1}{5} \times \frac{1}{5} =$	
21.	$\frac{3}{5} \times \frac{3}{5} =$	
22.	$\frac{3}{5} \times \frac{4}{5} =$	

23.	$\frac{3}{5} \times \frac{5}{4} =$	
24.	$\frac{4}{5} \times \frac{5}{3} =$	
25.	$\frac{1}{4} \times \frac{1}{4} =$	
26.	$\frac{1}{4} \times \frac{3}{4} =$	
27.	$\frac{3}{4} \times \frac{3}{4} =$	
28.	$\frac{3}{4} \times \frac{4}{3} =$	
29.	$\frac{3}{4} \times \frac{5}{4} =$	
30.	$\frac{3}{4} \times \frac{6}{4} =$	
31.	$\frac{4}{3} \times \frac{4}{6} =$	
32.	$\frac{2}{3} \times \frac{1}{5} =$	
33.	$\frac{2}{3} \times \frac{4}{5} =$	
34.	$\frac{2}{3} \times \frac{5}{5} =$	
35.	$\frac{2}{3} \times \frac{6}{5} =$	
36.	$\frac{1}{3} \times \frac{6}{5} =$	
37.	$\frac{1}{9} \times \frac{1}{9} =$	
38.	$\frac{1}{5} \times \frac{3}{8} =$	
39.	$\frac{3}{4} \times \frac{1}{6} =$	
40.	$\frac{2}{3} \times \frac{2}{3} =$	
41.	$\frac{3}{4} \times \frac{8}{8} =$	
42.	$\frac{2}{3} \times \frac{6}{3} =$	
43.	$\frac{6}{7} \times \frac{8}{9} =$	
44.	$\frac{7}{12} \times \frac{8}{7} =$	

Name _____

Date _____

Geoffrey builds rectangular planters.

1. Geoffrey's first planter is 8 feet long and 2 feet wide. The container is filled with soil to a height of 3 feet in the planter. What is the volume of soil in the planter? Explain your work using a diagram.
2. Geoffrey wants to grow some tomatoes in four large planters. He wants each planter to have a volume of 320 cubic feet, but he wants them all to be different. Show four different ways Geoffrey can make these planters, and draw diagrams with the planters' measurements on them.

Planter A	Planter B
Planter C	Planter D

3. Geoffrey wants to make one planter that extends from the ground to just below his back window. The window starts 3 feet off the ground. If he wants the planter to hold 36 cubic feet of soil, name one way he could build the planter so it is not taller than 3 feet. Explain how you know.
4. After all of this gardening work, Geoffrey decides he needs a new shed to replace the old one. His current shed is a rectangular prism that measures 6 feet long by 5 feet wide by 8 feet high. He realizes he needs a shed with 480 cubic feet of storage.
- a. Will he achieve his goal if he doubles each dimension? Why or why not?
- b. If he wants to keep the height the same, what could the other dimensions be for him to get the volume he wants?
- c. If he uses the dimensions in part (b), what could be the area of the new shed's floor?

Name _____

Date _____

A storage shed is a rectangular prism and has dimensions of 6 meters by 5 meters by 12 meters. If Jean were to double these dimensions, she believes she would only double the volume. Is she correct? Explain why or why not. Include a drawing in your explanation.

Name _____

Date _____

Wren makes some rectangular display boxes.

1. Wren's first display box is 6 inches long, 9 inches wide, and 4 inches high. What is the volume of the display box? Explain your work using a diagram.
2. Wren wants to put some artwork into three shadow boxes. She knows they all need a volume of 60 cubic inches, but she wants them all to be different. Show three different ways Wren can make these boxes by drawing diagrams and labeling the measurements.

Shadow Box A	Shadow Box B
Shadow Box C	

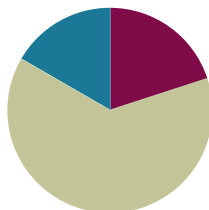
3. Wren wants to build a box to organize her scrapbook supplies. She has a stencil set that is 12 inches wide that needs to lay flat in the bottom of the box. The supply box must also be no taller than 2 inches. Name one way she could build a supply box with a volume of 72 cubic inches.
4. After all of this organizing, Wren decides she also needs more storage for her soccer equipment. Her current storage box measures 1 foot long by 2 feet wide by 2 feet high. She realizes she needs to replace it with a box with 12 cubic feet of storage, so she doubles the width.
- Will she achieve her goal if she does this? Why or why not?
 - If she wants to keep the height the same, what could the other dimensions be for a 12-cubic-foot storage box?
 - If she uses the dimensions in part (b), what is the area of the new storage box's floor?
 - How has the area of the bottom in her new storage box changed? Explain how you know.

Lesson 8

Objective: Apply concepts and formulas of volume to design a sculpture using rectangular prisms within given parameters.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Concept Development	(38 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Multiply Whole Numbers and Decimals (4 minutes)
- Mixed Numbers to Improper Fractions (4 minutes)
- Multiply Mixed Numbers (4 minutes)

Multiply Whole Numbers and Decimals (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews Module 4 Lesson 17.

T: (Write $3 \times 2 = \underline{\quad}$.) Say the complete number sentence.

S: $3 \times 2 = 6$.

$$3 \times 2 = 6 \quad 3 \times 0.2 = 0.6 \quad 0.3 \times 0.2 = 0.06 \quad 0.03 \times 0.2 = 0.006$$

$$2 \times 7 = 14 \quad 2 \times 0.7 = 1.4 \quad 0.2 \times 0.7 = 0.14 \quad 0.02 \times 0.7 = 0.014$$

$$5 \times 3 = 15 \quad 0.5 \times 3 = 1.5 \quad 0.5 \times 0.3 = 0.15 \quad 0.5 \times 0.03 = 0.015$$

T: (Write $3 \times 0.2 = \underline{\quad}$.) On your personal white board, write the complete number sentence.

S: (Write $3 \times 0.2 = 0.6$.)

T: (Write $0.3 \times 0.2 = \underline{\quad}$.) Write the complete number sentence.

S: (Write $0.3 \times 0.2 = 0.06$.)

T: (Write $0.03 \times 0.2 = \underline{\quad}$.) Write the complete number sentence.

S: (Write $0.03 \times 0.2 = 0.006$.)

Continue with the following possible sequence: 2×7 , 2×0.7 , 0.2×0.7 , 0.02×0.7 , 5×3 , 0.5×3 , 0.5×0.3 , and 0.5×0.03 .

Mixed Numbers to Improper Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for Lesson 10.

T: How many halves are in 1?

S: 2.

T: How many halves are in 2?

S: 4.

T: How many halves are in 3?

S: 6.

T: (Write $3\frac{1}{2} = \underline{\hspace{1cm}}$.) How many halves are in $3\frac{1}{2}$?
Write $3\frac{1}{2}$ as an improper fraction on your personal white board.

S: (Write $3\frac{1}{2} = \frac{7}{2}$.)

Continue with the following possible sequence: $5\frac{1}{2}$, $4\frac{1}{3}$, $4\frac{2}{3}$, $2\frac{1}{4}$, $2\frac{3}{4}$, and $4\frac{5}{6}$.

Multiply Mixed Numbers (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for Lesson 10.

Format as illustrated to the right.

T: (Write $3\frac{1}{2} \times 2$, and below it, $(3 \times 2) + (\frac{1}{2} \times 2)$.)

T: (Point to 3×2 .) Tell me the complete multiplication sentence.

S: $3 \times 2 = 6$.

T: (Point to $\frac{1}{2} \times 2$.) Tell me the complete multiplication sentence.

S: $\frac{1}{2} \times 2 = 1$.

T: Tell me the addition sentence combining the products.

S: $6 + 1 = 7$.

Continue with the following possible sequence: $4\frac{1}{3} \times 2$, $3\frac{1}{5} \times 4$, and $4\frac{2}{3} \times 3$.



**NOTES ON
MULTIPLE MEANS
OF REPRESENTATION:**

If students are having trouble with fractions, give them fraction tiles to work with. Have them build 1 whole from halves, then 2 wholes, and so on. Then, have them build 3 wholes plus 1 half, and have them count how many halves are in $3\frac{1}{2}$. Continue with other fractional units.

$$\begin{aligned} 3\frac{1}{2} \times 2 &= \\ &= (3 \times 2) + (\frac{1}{2} \times 2) \\ &= 6 + 1 \\ &= 7 \end{aligned}$$

Concept Development (38 minutes)

Materials: (S) Problem Set, project requirements (Template 1), box patterns (a–c) (Templates 2–4), lid patterns (Template 5) (at least three of each template per group), evaluation rubric (Template 6), scissors, tape, rulers

Note: The time for the Application Problem has been allocated to the Concept Development for this lesson.

Copy the flattened boxes on the thickest paper available. Each student or group needs three to five copies of each box, but they may not all be used.

Students cut the templates to form boxes of a certain volume by adjusting the height of the sides. They should construct the boxes by taping edges together and then turn the boxes open side down to create their sculptures. They may also tape lids on the open ends of their boxes to make construction easier.

Please also note the evaluation rubric included in this lesson. The rubric can be shared with students so they understand how their work will be judged.

This activity can be done individually or in pairs.

- T:** Today, we will be putting our math sense and geometric skills to work as each of you designs a sculpture created from a collection of rectangular prisms. Read the requirements and the rubric with a neighbor.

Distribute the project requirements (Template 1), the Problem Set, and the evaluation rubric (Template 6). Allow students time to read all three.

- T:** Now that you've had an opportunity to read the requirements and the way your work will be evaluated, share your ideas about what you might like to design.
- S:** I want to make a shape using five prisms and make it as random as possible. → I want to see if I can do a capital F, for my name. → I was going to do a scaled version of my tree house, but I'm not sure if I'll be able to scale the dimensions right.
- T:** Here are the boxes, like the ones we used in Lesson 2, that you can use to build your rectangular boxes. There are three different bases to choose from. You may adjust the volume by adjusting the height of the sides of your box. Watch me cut one of the box patterns and make a box.

**NOTE ON MATERIALS:**

When printing the box and lid patterns, be sure the printer is set to *actual size*.

**NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**

Some students may be overwhelmed by the amount of reading and interpretation of directions required for the project. Reading the requirements as a class and having discussion after each one can be helpful. Or place accomplished readers with those who struggle.

Some students may benefit from having cubes to actually construct a model of their structures first.

**NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**

Students who struggle may be encouraged to use only three prisms or be given more latitude in total volume or in the relationships between the prisms. Alternatively, those whose spatial skills are well developed may be given additional restrictions to meet or requirements to fulfill or may be asked to use more prisms to construct their designs.

(Demonstrate cutting the 6 cm × 3 cm box pattern, Template 2.) If I want to build this first prism to have a volume of 36 cubic centimeters, what height will I need to measure and cut the sides?

S: They would need to be 2 cm high.

T: Yes. I'll measure all my side flaps 2 cm from the base. Then, I'll cut, fold, and tape them. (Demonstrate.) Talk with your neighbor about how you'll construct your first box and calculate the volume.

S: Cut the base with rectangles attached on each side that are the same height, and fold them up. Then, calculate the volume. → Decide on the volume, and find the area of the base. Then, cut the height to give the volume you need.

T: It might be a good idea to draw a very rough sketch of the design you're thinking of creating.

S: (Draw.)

T: Reread the third prompt with a friend. Share your ideas about how you'll meet its requirements.

S: (Share.)

T: What were your ideas?

S: I'm going to make Prism A and then try to cut one of the other prisms to make it half the volume of Prism A and call it Prism D. → I can make the biggest prism possible and then divide the volume in half and try to make another prism one-half of that volume. → I'll make a prism and then use the same base to make another prism but cut the height in half. That will give me half of the original volume.

T: Reread the fourth prompt with a friend. Share your ideas about how you'll meet its requirements.

S: I will take Prism B and cut it up to create a prism with one-third the volume. → I can use the same big prism as before and divide the volume by 3 and find a prism with dimensions that will equal that number. → I can take one of the first three prisms and make the height one-third of the original height, and this will give me one-third of the volume.

T: The final prompt says that the total volume of your design must not exceed 1,000 cubic centimeters. Share your ideas.

S: What's the biggest prism I can do? → We can do a total of five prisms, but they can't be more than 200 cubic centimeters. One has to be half of another. Another has to be a third of another; this is going to require some thinking! → Let's just say we do one prism that's 420 cubic cm and one that's half of that (210 cubic cm) and then one that's a third of that (140 cubic cm). That's 770 cubic centimeters. That means I still have 230 cubic units and two prisms to play with. I can make one 3 cm by 6 cm by 4 cm and one 5 cm x 5 cm x 4 cm for a total of 942 cubic centimeters.

T: Once you've finalized your boxes, cut a lid with tabs that will fit, and tape it to your box. This will give your boxes stability, so they'll be easier to tape together. I can tell that you're excited to get started. Be sure to check your math as you progress, and feel free to share your ideas with a neighbor as you work. (Circulate around the room to ask clarifying questions or provide support to struggling learners as students work.)



NOTES ON MULTIPLE MEANS OF EXPRESSION:

Students whose fine motor skills are less developed may enjoy producing a virtual version of this project. They might use computer-based drawing tools to draw the prisms (e.g., Google's SketchUp). These can be printed and then measured to fulfill the requirements of the project.

Student Debrief (10 minutes)

Lesson Objective: Apply concepts and formulas of volume to design a sculpture using rectangular prisms within given parameters.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check their work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What was your thought process as you designed your sculpture? Was your sculpture inspired by something you have seen or owned?
- Which figure did you cut into halves or thirds when creating another shape? What was your thought process as you created a shape one-half or one-third the size? Did you cut one dimension into halves or thirds, or did you scale the entire volume first and then select dimensions to meet that volume?
- What was your biggest challenge in designing your sculpture? Explain.

Name: Alex Date: _____

Using the box patterns, construct a sculpture containing at least 5, but not more than 7, rectangular prisms that meets the following requirements in the table below.

1.	My sculpture has 5 to 7 rectangular prisms.	Number of prisms: <u>5</u>
2.	Each prism is labeled with a letter, dimensions, and volume.	
	Prism A: <u>10 cm</u> by <u>7 cm</u> by <u>4 cm</u>	Volume = <u>280 cm³</u>
	Prism B: <u>5 cm</u> by <u>5 cm</u> by <u>6 cm</u>	Volume = <u>150 cm³</u>
	Prism C: <u>6 cm</u> by <u>3 cm</u> by <u>5 cm</u>	Volume = <u>90 cm³</u>
	Prism D: <u>10 cm</u> by <u>7 cm</u> by <u>2 cm</u>	Volume = <u>140 cm³</u>
	Prism E: <u>5 cm</u> by <u>5 cm</u> by <u>2 cm</u>	Volume = <u>50 cm³</u>
	Prism _____ by _____ by _____	Volume = _____
	Prism _____ by _____ by _____	Volume = _____
3.	Prism D has $\frac{1}{2}$ the volume of prism A.	Prism D Volume = <u>140 cm³</u> Prism A Volume = <u>280 cm³</u>
4.	Prism E has $\frac{1}{3}$ the volume of prism B.	Prism E Volume = <u>50 cm³</u> Prism B Volume = <u>150 cm³</u>
5.	The total volume of all the prisms is 1,000 cubic centimeters or less.	Total volume: <u>710 cm³</u> 280 150 90 140 50 <u>710</u>

Yes!

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____

Date _____

Using the box patterns, construct a sculpture containing at least 5, but not more than 7, rectangular prisms that meets the following requirements in the table below.

1.	My sculpture has 5 to 7 rectangular prisms. Number of prisms: _____	
2.	Each prism is labeled with a letter, dimensions, and volume.	
	Prism A _____ by _____ by _____ Volume = _____ Prism B _____ by _____ by _____ Volume = _____ Prism C _____ by _____ by _____ Volume = _____ Prism D _____ by _____ by _____ Volume = _____ Prism E _____ by _____ by _____ Volume = _____ Prism ____ _____ by _____ by _____ Volume = _____ Prism ____ _____ by _____ by _____ Volume = _____	
3.	Prism D has $\frac{1}{2}$ the volume of Prism ____.	Prism D Volume = _____ Prism ____ Volume = _____
4.	Prism E has $\frac{1}{3}$ the volume of Prism ____.	Prism E Volume = _____ Prism ____ Volume = _____
5.	The total volume of all the prisms is 1,000 cubic centimeters or less.	Total volume: _____ Show calculations:

Name _____

Date _____

Sketch a rectangular prism that has a volume of 36 cubic cm. Label the dimensions of each side on the prism. Fill in the blanks that follow.

Height: _____ cm

Length: _____ cm

Width: _____ cm

Volume: _____ cubic cm

Name _____

Date _____

1. I have a prism with the dimensions of 6 cm by 12 cm by 15 cm. Calculate the volume of the prism, and then give the dimensions of three different prisms that each have $\frac{1}{3}$ of the volume.

	Length	Width	Height	Volume
Original Prism	6 cm	12 cm	15 cm	
Prism 1				
Prism 2				
Prism 3				

2. Sunni's bedroom has the dimensions of 11 ft by 10 ft by 10 ft. Her den has the same height but double the volume. Give two sets of the possible dimensions of the den and the volume of the den.

Project Requirements

1. Each project must include 5 to 7 rectangular prisms.
 2. All prisms must be labeled with a letter (beginning with A), dimensions, and volume.
 3. Prism D must be $\frac{1}{2}$ the volume of another prism.
 4. Prism E must be $\frac{1}{3}$ the volume of another prism.
 5. The total volume of all of the prisms must be 1,000 cubic centimeters or less.
-

Project Requirements

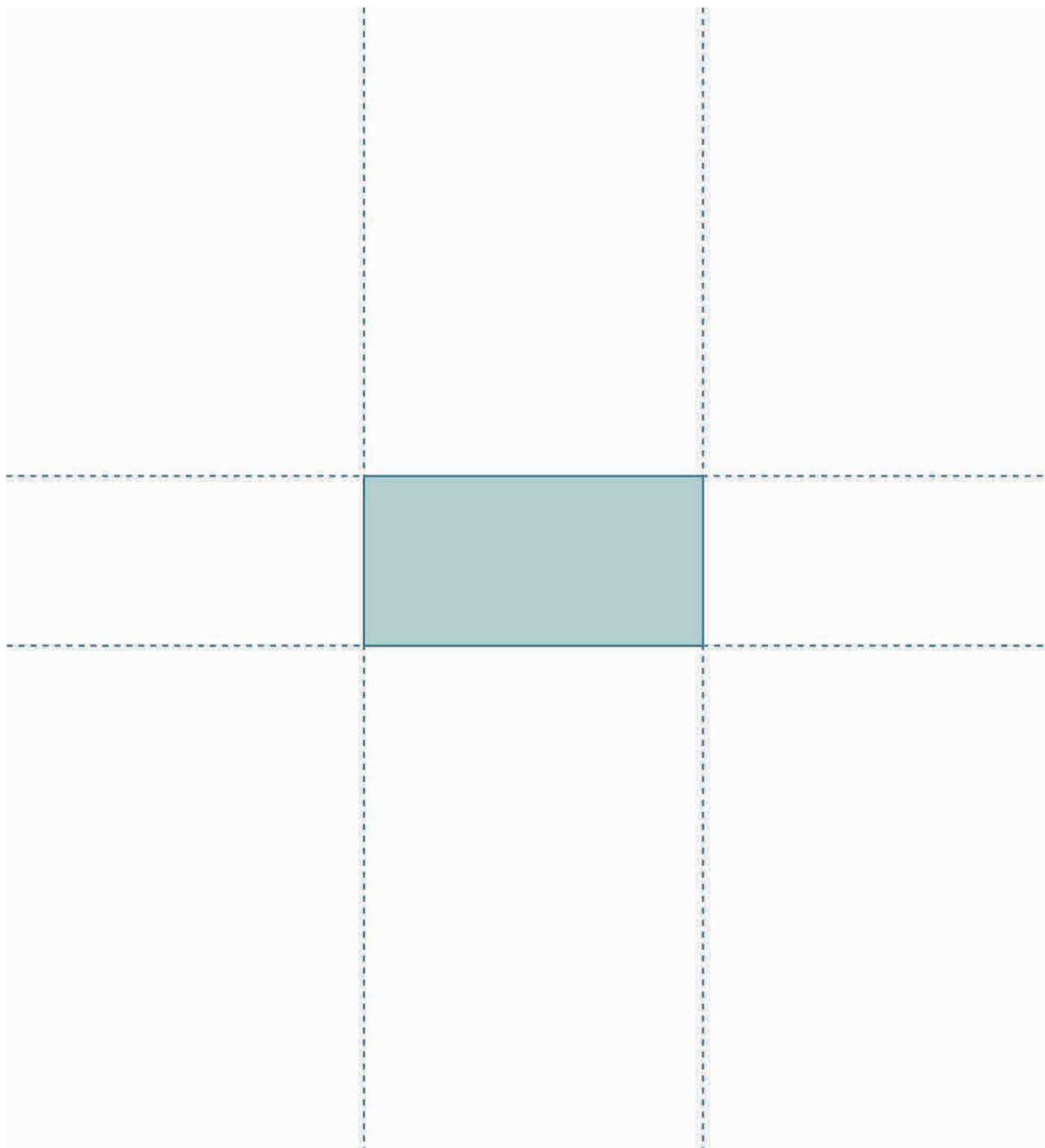
1. Each project must include 5 to 7 rectangular prisms.
 2. All prisms must be labeled with a letter (beginning with A), dimensions, and volume.
 3. Prism D must be $\frac{1}{2}$ the volume of another prism.
 4. Prism E must be $\frac{1}{3}$ the volume of another prism.
 5. The total volume of all of the prisms must be 1,000 cubic centimeters or less.
-

Project Requirements

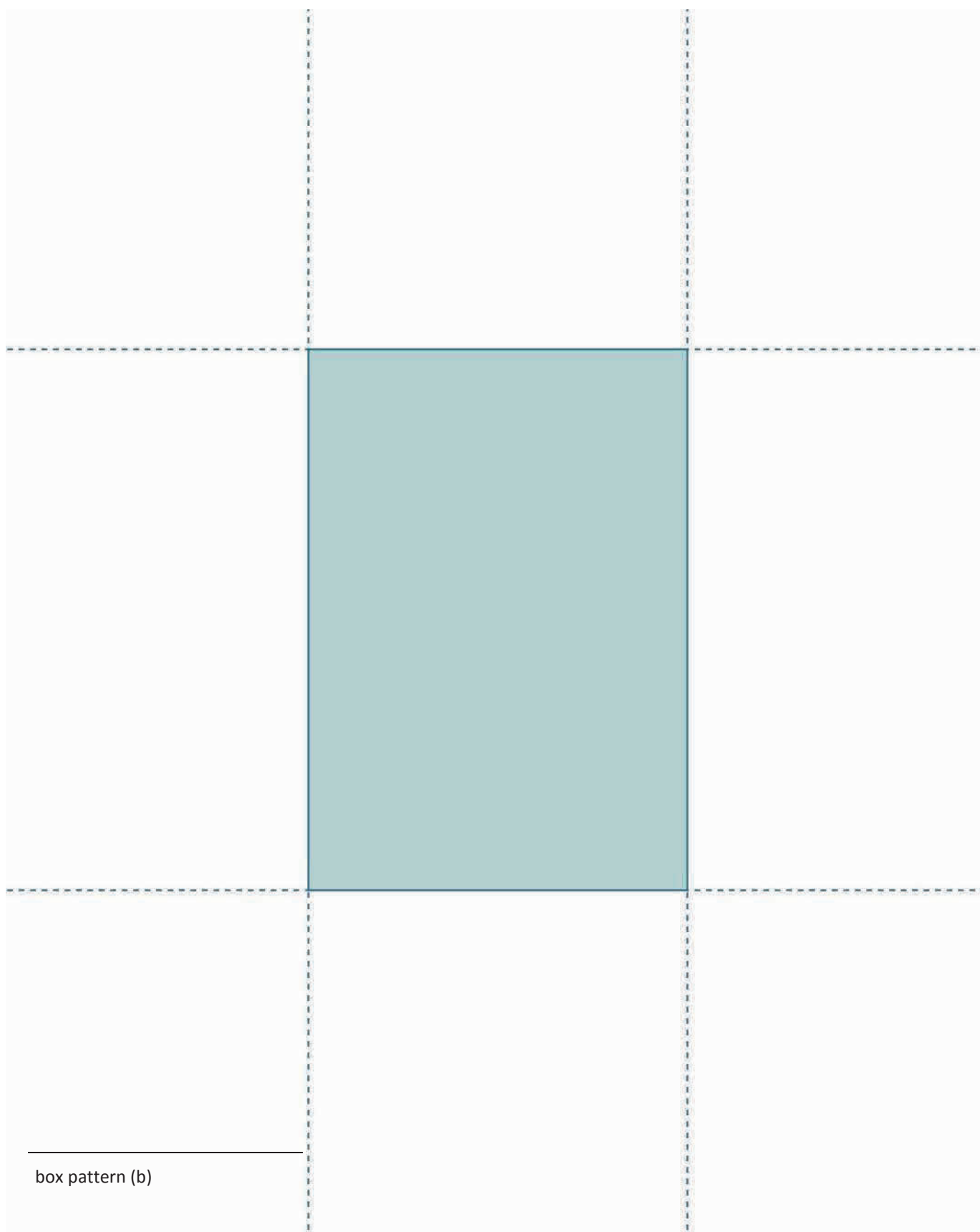
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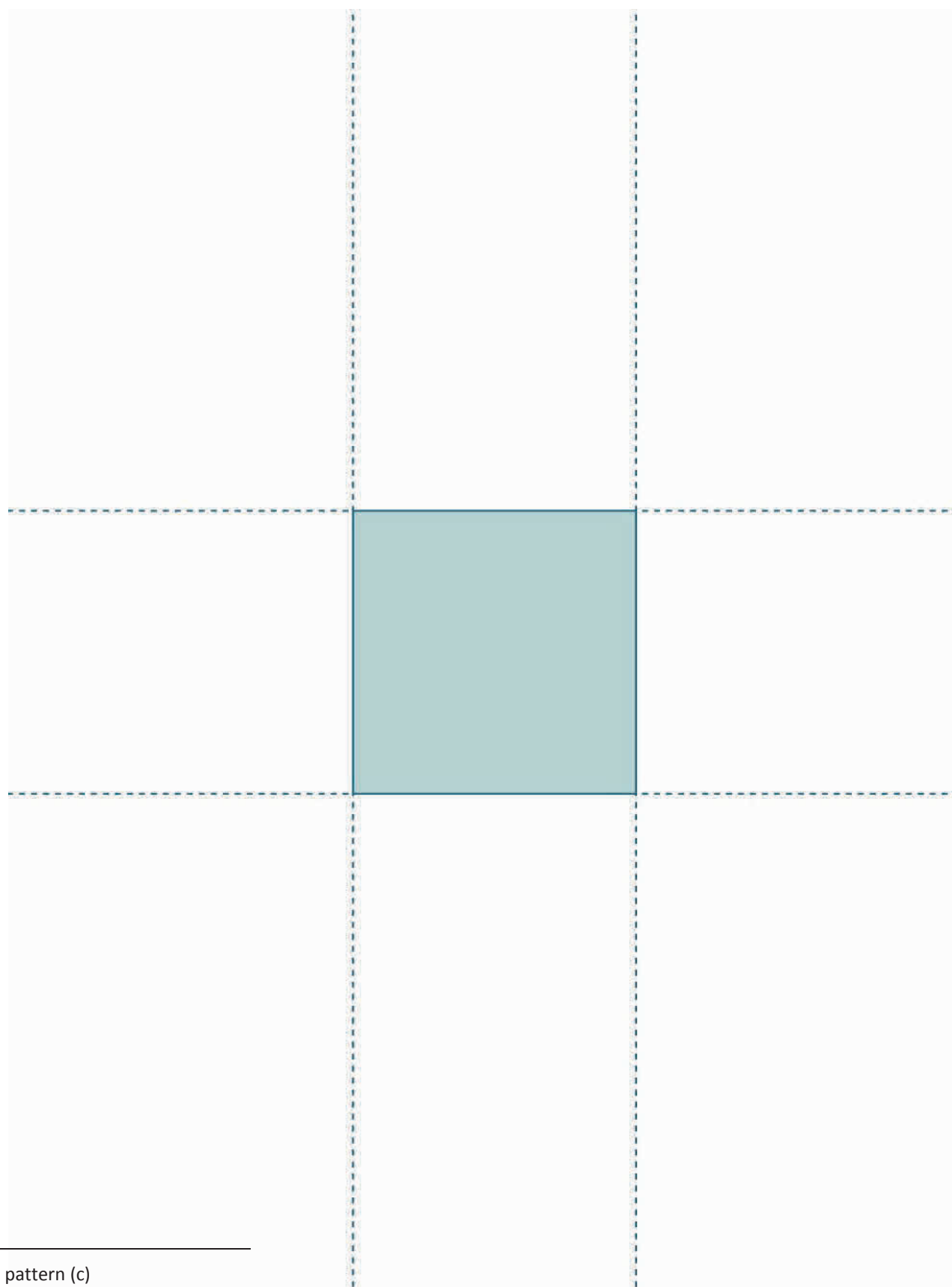
project requirements

Note: Be sure to set printer to *actual size* before printing.

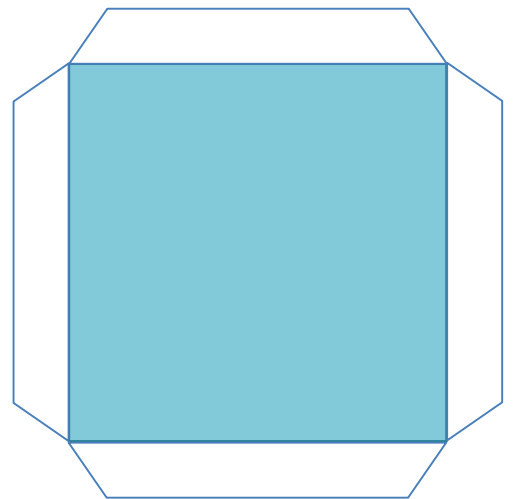
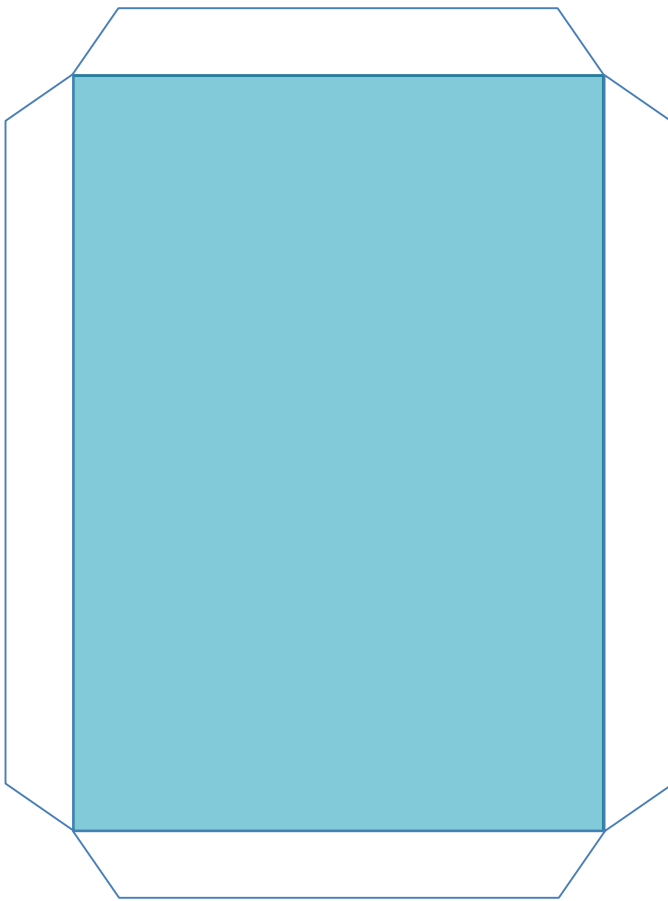


box pattern (a)





box pattern (c)



lid patterns

Name _____

Date _____

Evaluation Rubric

CATEGORY	4	3	2	1	Subtotal
Completeness of Personal Project and Classmate Evaluation	All components of the project are present and correct, and a detailed evaluation of a classmate's project has been completed.	Project is missing 1 component, and a detailed evaluation of a classmate's project has been completed.	Project is missing 2 components, and an evaluation of a classmate's project has been completed.	Project is missing 3 or more components, and an evaluation of a classmate's project has been completed.	(× 4) _____/16
Accuracy of Calculations	Volume calculations for all prisms are correct.	Volume calculations include 1 error.	Volume calculations include 2–3 errors.	Volume calculations include 4 or more errors.	(× 5) _____/20
Neatness and Use of Color	All elements of the project are carefully and colorfully constructed.	Some elements of the project are carefully and colorfully constructed.	Project lacks color or is not carefully constructed.	Project lacks color and is not carefully constructed.	(× 2) _____/4
					TOTAL: _____/40

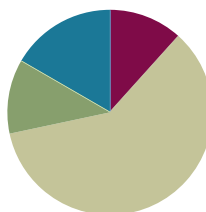
evaluation rubric

Lesson 9

Objective: Apply concepts and formulas of volume to design a sculpture using rectangular prisms within given parameters.

Suggested Lesson Structure

Fluency Practice	(7 minutes)
Application Problem	(7 minutes)
Concept Development	(36 minutes)
Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (7 minutes)

- Multiply Decimals (3 minutes)
- Multiply Mixed Numbers (4 minutes)

Multiply Decimals (3 minutes)

Materials: (S) Personal white board

Note: This fluency exercise reviews Module 4's Lessons 17 and 18.

T: (Write $4 \times 2 = \underline{\quad}$.) Say the number sentence.

S: $4 \times 2 = 8$.

$4 \times 2 = 8$	$4 \times 0.2 = 0.8$	$0.4 \times 0.2 = 0.08$	$0.04 \times 0.2 = 0.008$
$2 \times 9 = 18$	$2 \times 0.9 = 1.8$	$0.2 \times 0.9 = 0.18$	$0.02 \times 0.9 = 0.018$
$4 \times 3 = 12$	$0.4 \times 3 = 1.2$	$0.4 \times 0.3 = 0.12$	$0.4 \times 0.03 = 0.012$

T: (Write $4 \times 0.2 = \underline{\quad}$.) On your personal white board, write the number sentence.

S: (Write $4 \times 0.2 = 0.8$.)

T: (Write $0.4 \times 0.2 = \underline{\quad}$.) Write the number sentence.

S: (Write $0.4 \times 0.2 = 0.08$.)

T: (Write $0.04 \times 0.2 = \underline{\quad}$.) Write the number sentence.

S: (Write $0.04 \times 0.2 = 0.008$.)

Continue with the following possible sequence: 2×9 , 2×0.9 , 0.2×0.9 , 0.02×0.9 , 4×3 , 0.4×3 , 0.4×0.3 , and 0.4×0.03 .

Multiply Mixed Numbers (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise prepares students for Lesson 10. Format writing as illustrated to the right.

T: (Write $4\frac{1}{3} \times 2$ and below it $(4 \times 2) + (\frac{1}{3} \times 2)$.)

T: (Point to 4×2 .) Tell me the complete multiplication sentence.

S: $4 \times 2 = 8$.

T: (Point to $\frac{1}{3} \times 2$.) Tell me the complete multiplication sentence.

S: $\frac{1}{3} \times 2 = \frac{2}{3}$.

T: Tell me the addition sentence combining the two products.

S: $8 + \frac{2}{3} = 8\frac{2}{3}$.

$$\begin{aligned} 4\frac{1}{3} \times 2 \\ &= (4 \times 2) + (\frac{1}{3} \times 2) \\ &= 8 + \frac{2}{3} \\ &= 8\frac{2}{3} \end{aligned}$$

Continue with the following possible sequence: $7\frac{1}{5} \times 3$ and $5\frac{2}{3} \times 2$.

Application Problem (7 minutes)

The chart below shows the dimensions of various rectangular packing boxes. If possible, answer the following without calculating the volume.

- Which box will provide the greatest volume?
- Which box has a volume that is equal to the volume of the book box? How do you know?
- Which box is $\frac{1}{3}$ the volume of the lamp box?

Box Type	Dimensions (l × w × h)
Book Box	12 in × 12 in × 12 in
Picture Box	36 in × 12 in × 36 in
Lamp Box	12 in × 9 in × 48 in
The Flat	12 in × 6 in × 24 in

a) The picture box has the greatest volume because it has the largest dimensions of all the boxes.

b) The flat and book box have the same volume. Their lengths are the same. The flat's width is $\frac{1}{2}$ the book's, and its height is double the book's, so they have the same volume.

c) The book box is $\frac{1}{3}$ the volume of the lamp box. The lengths are the same. If you factor the width and height:

Book: 12×12 Lamp: 9×48

$\begin{array}{c} 3 \quad 4 \quad 3 \quad 4 \\ 12 \times 12 \end{array}$
 $\begin{array}{c} 3 \quad 3 \quad 4 \quad 4 \quad 3 \\ 9 \times 48 \end{array}$

The lamp has an extra factor of 3.

Note: This Application Problem builds on students' understanding of a scaling principle. Students can use their sense of part-whole and scaling knowledge to answer these questions without finding the volume of the boxes.

Concept Development (36 minutes)

Materials: (T) Copy of student work from Lesson 8, evaluation rubric (Lesson 8 Template 6) (S) Rulers, 2 copies of Problem Set (1 for use during Concept Development and 1 for independent work)

Note: Before class, the projects should be labeled only with a number and no student names. The review process in today's lesson should proceed anonymously.

T: (Post the image of the shape below on the board.)
Here is a student's project designed according to the directions we used yesterday. I've measured the boxes, and the measurements that you see on the diagram are correct. The volume of Prism A is given. (Distribute a copy of the Problem Set to each student.) Your job is to use the rubric to see if this student met all the requirements of the assignment.

T: Before we can do that, we must confirm the volumes that the student recorded are correct. Work with a neighbor to check the work this student did to find the volumes of the prisms. (Allow students time to work and share their results.)

T: What did you find? Are the recorded volumes correct?

S: They are correct. → Prisms A and C have volumes of 36 cm^3 . → Prism B has a volume of 420 cm^3 .
→ Prism E has a volume of 140 cm^3 . → Prism D's volume is 18 cm^3 .

T: Now we are ready to begin our review. Look at the first item on the list. How many prisms are in this design?

S: 5.

T: Check the Element Present? box, and record the number of prisms used under Specifics of Element.

S: (Check the box and record 5 prisms.)

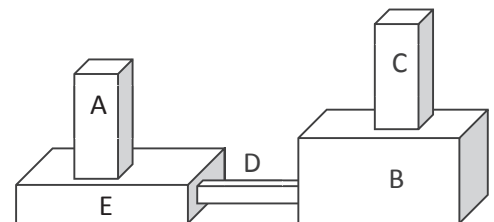
T: The Notes box is for any positive comments we might like to give to this student on this particular element. This is also the place to tell him anything that might be missing in the design. Since this student has met this requirement, turn and talk to your partner about what positive comment you might make.

S: I like the way the prisms are sort of symmetrical. → The way the boxes are stacked from big to little looks good. → Putting the skinny box in the middle makes the design look really big even though he only used 5 boxes.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

The high number of measurements recorded on the diagram may be overwhelming to students with visual acuity difficulties. These students may benefit from a second diagram with figures slightly separated and units listed on each dimension or a larger print version of the diagram.



Prism A: $6 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm} = 36 \text{ cm}^3$
Prism B: $10 \text{ cm} \times 7 \text{ cm} \times 6 \text{ cm} = 420 \text{ cm}^3$
Prism C: $6 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm} = 36 \text{ cm}^3$
Prism D: $6 \text{ cm} \times 3 \text{ cm} \times 1 \text{ cm} = 18 \text{ cm}^3$
Prism E: $10 \text{ cm} \times 7 \text{ cm} \times 2 \text{ cm} = 140 \text{ cm}^3$

- T: Let's look at our next requirement. Are all the parts labeled with a letter? Record your answer.
- S: Yes. They all have letters.
- T: Are all prisms labeled with their dimensions and volume? Record your response.
- S: Yes. All the prisms have dimensions and volume recorded.
- T: Do all recorded measurements have the correct units? What are the units for the dimensions and volume?
- S: Yes. All dimensions are in centimeters, and the volumes are in cubic centimeters.
- T: Write that down. What's next on our list? How will we find out if this student met the requirements? Turn and talk.
- S: We need to find out if Prism D is one-half of one of the other prisms and if Prism E is one-third of another prism. → We need to calculate the volume of all of the prisms first and then check if Prism D has one-half the volume of one of the other prisms and if Prism E has one-third the volume of one prism. → Prism D has a volume of 18 cm^3 , which is one-half of Prism A's volume. → Prism E has a volume of 140 cm^3 , which is one-third of Prism B's volume.
- T: Record your findings. Check the Requirement boxes, and use the second page to record your calculations. (Circulate to make sure students are using the correct parts of the rubric to record the information.)
- S: (Record.)
- T: What is the total volume of this shape?
- S: (Work and show that $36 \text{ cm}^3 + 36 \text{ cm}^3 + 18 \text{ cm}^3 + 140 \text{ cm}^3 + 420 \text{ cm}^3 = 650 \text{ cm}^3$.)
- T: Did this student meet all the requirements of the assignment? Tell me how you know.
- S: Yes, she did. The volume is 650 cm^3 , which is less than $1,000 \text{ cm}^3$. → There are 5 prisms, and they had to have 5 to 7 prisms. → The volume of Prism D is one-half the volume of Prism A. → The volume of Prism E is one-third the volume of Prism B.
- T: Remember, if there's something that doesn't meet a requirement in the project that you review, you will record that in the Notes column. You may also use the Notes box to say something that you notice about their work.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Have students who easily grasp this concept and move quickly through the Problem Set double one or more dimensions and calculate the new volume of the figure. Another option is to ask them if the units were centimeters instead of inches, how many liters of liquid the structure would hold.

Name: Roberta Date: _____

I reviewed project number 21.

Use the rubric below to evaluate your friend's project. Ask questions and measure the parts to determine whether your friend has all the required elements. Respond to the prompt in italics in the third column. The final column can be used to write something you find interesting about that element if you like.

Space is provided beneath the rubric for your calculations.

Requirement	Element present?	Specifics of Element	Notes
1. Sculpture has 5 to 7 prisms.	✓	# of prisms: <u>5</u>	
2. All prisms are labeled with a letter.	✓	Write letters used: <u>A-D</u>	
3. All prisms have correct dimensions with units written on the top.	✓	List any prisms with incorrect dimensions or units:	
4. All prisms have correct volume with units written on top.	✓	List any prism with incorrect dimensions or units:	
5. Prism D has $\frac{1}{2}$ the volume of another prism.	✓	Record on next page:	
6. Prism E has $\frac{1}{3}$ the volume of another prism.	✓	Record on next page:	
7. The total volume of all the parts together is 1,000 cubic units or less.	✓	Total volume: <u>650 cm^3</u>	

Calculations:

$36 \text{ cm}^3 + 420 \text{ cm}^3 + 36 \text{ cm}^3 + 18 \text{ cm}^3 + 140 \text{ cm}^3 = 650 \text{ cm}^3$

T: I'm assigning each of you the sculpture of a fellow classmate (or pair) to review independently just as we did this one. Write the number of the project that you review on your Problem Set. Begin by confirming the measurements and volumes calculated by your classmate. (Distribute one sculpture to each student, and circulate to answer questions that arise.)

Student Debrief (10 minutes)

Lesson Objective: Apply concepts and formulas of volume to design a sculpture using rectangular prisms within given parameters.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check their work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How was the student work you assessed similar to and different from the design you created?
- If the work that you assessed did not meet the requirements, what feedback did you provide to help the student be successful?
- How was assessing student work different from creating your design yesterday? If you could go back and change your design, would you? In what ways?
- Students might enjoy investigating the sculptures of David Smith, particularly his *Cubi* series (as shown to the right). Many in this series of sculptures are composed exclusively of rectangular prisms.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

8. Measure the dimensions of each prism. Calculate the volume of each prism and the total volume. Record that information in the table below. If your measurements or volume differ from those listed on the project, put a star by the prism label in the table below, and record on the rubric.

Prism	Dimensions	Volume
A	6cm by 3cm by 2cm	36cm ³
B	10cm by 7cm by 6cm	420cm ³
C	6cm by 3cm by 2cm	36cm ³
D	6cm by 3cm by 1cm	18cm ³
E	10cm by 7cm by 2cm	140cm ³
	by by	
	by by	

9. Prism D's volume is $\frac{1}{2}$ that of Prism A or C.

Show calculations below.

$36\text{cm}^3 \div 2 = 18\text{cm}^3$. Both Prisms A and C have the same volume of 36cm^3 .

Prism D's volume of 18cm^3 is $\frac{1}{2}$ of Prism A or C's volume.

10. Prism E's volume is $\frac{1}{3}$ that of Prism B.

Show calculations below.

$420\text{cm}^3 \div 3 = 140\text{cm}^3$

Prism E's volume of 140cm^3 is $\frac{1}{3}$ of Prism B's volume.

11. Total volume of sculpture: 650cm^3

Show calculations below.

$$\begin{array}{r} 36 \\ + 420 \\ \hline 456 \end{array} \quad \begin{array}{r} 456 \\ + 36 \\ \hline 492 \end{array} \quad \begin{array}{r} 492 \\ + 18 \\ \hline 510 \end{array} \quad \begin{array}{r} 510 \\ + 140 \\ \hline 650 \end{array}$$



David Smith (1906-1965), *Cubi VI*, 1963, stainless steel. Art © [Estate of David Smith/Licensed by VAGA, New York, NY](#). Photo: Yair Talmor.

Name _____

Date _____

I reviewed project number _____.

Use the rubric below to evaluate your friend's project. Ask questions and measure the parts to determine whether your friend has all the required elements. Respond to the prompt in italics in the third column. The final column can be used to write something you find interesting about that element if you like.

Space is provided beneath the rubric for your calculations.

	Requirement	Element Present? (✓)	Specifics of Element	Notes
1.	The sculpture has 5 to 7 prisms.		<i># of prisms:</i>	
2.	All prisms are labeled with a letter.		<i>Write letters used:</i>	
3.	All prisms have correct dimensions with units written on the top.		<i>List any prisms with incorrect dimensions or units:</i>	
4.	All prisms have correct volume with units written on the top.		<i>List any prism with incorrect dimensions or units:</i>	
5.	Prism D has $\frac{1}{2}$ the volume of another prism.		<i>Record on next page:</i>	
6.	Prism E has $\frac{1}{3}$ the volume of another prism.		<i>Record on next page:</i>	
7.	The total volume of all the parts together is 1,000 cubic units or less.		<i>Total volume:</i>	

Calculations:

8. Measure the dimensions of each prism. Calculate the volume of each prism and the total volume. Record that information in the table below. If your measurements or volume differ from those listed on the project, put a star by the prism label in the table below, and record on the rubric.

Prism	Dimensions	Volume
A	_____ by _____ by _____	
B	_____ by _____ by _____	
C	_____ by _____ by _____	
D	_____ by _____ by _____	
E	_____ by _____ by _____	
	_____ by _____ by _____	
	_____ by _____ by _____	

9. Prism D's volume is $\frac{1}{2}$ that of Prism _____.
Show calculations below.

10. Prism E's volume is $\frac{1}{3}$ that of Prism _____.
Show calculations below.

11. Total volume of sculpture: _____.
Show calculations below.

Name _____

Date _____

A student designed this sculpture. Using the dimensions on the sculpture, find the dimensions of each rectangular prism. Then, calculate the volume of each prism.

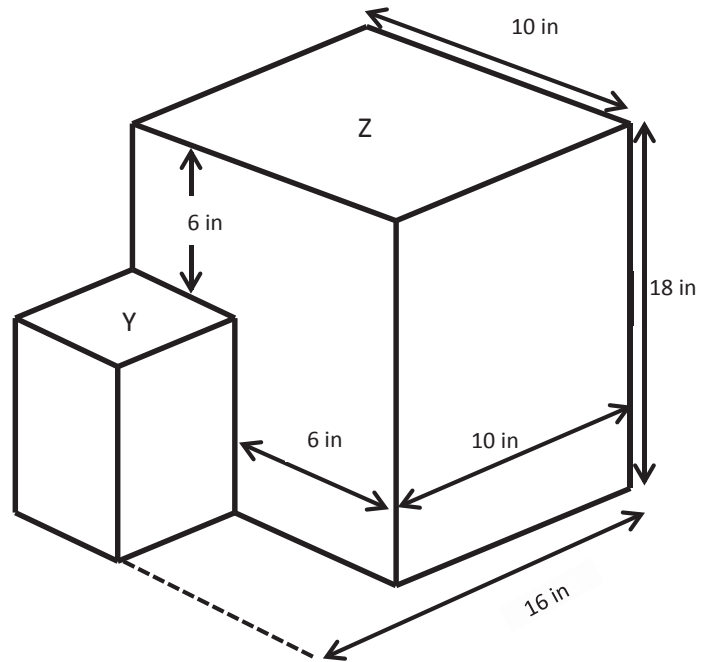
- a. Rectangular Prism Y

Height: _____ inches

Length: _____ inches

Width: _____ inches

Volume: _____ cubic inches



- b. Rectangular Prism Z

Height: _____ inches

Length: _____ inches

Width: _____ inches

Volume: _____ cubic inches

- c. Find the total volume of the sculpture. Label the answer.

Name _____

Date _____

1. Find three rectangular prisms around your house. Describe the item you are measuring (cereal box, tissue box, etc.), and then measure each dimension to the nearest whole inch, and calculate the volume.

- a. Rectangular Prism A

Item:

Height: _____ inches

Length: _____ inches

Width: _____ inches

Volume: _____ cubic inches

- b. Rectangular Prism B

Item:

Height: _____ inches

Length: _____ inches

Width: _____ inches

Volume: _____ cubic inches

- c. Rectangular Prism C

Item:

Height: _____ inches

Length: _____ inches

Width: _____ inches

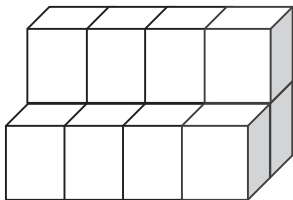
Volume: _____ cubic inches

Name _____

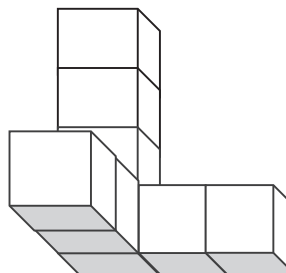
Date _____

1. Tell the volume of each solid figure made of 1-inch cubes. Specify the correct unit of measure.

a.

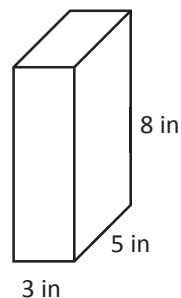


b.

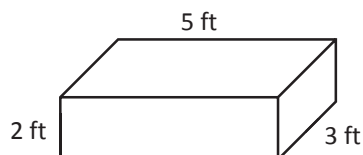


2. Jack found the volume of the prism pictured to the right by multiplying 5×8 and then adding $40 + 40 + 40 = 120$. He says the volume is 120 cubic inches.

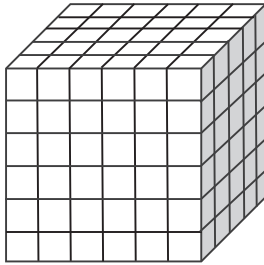
- a. Jill says he did it wrong. He should have multiplied the bottom first (3×5) and then multiplied by the height. Explain to Jill why Jack's method works and is equivalent to her method.



- b. Use Jack's method to find the volume of this right rectangular prism.

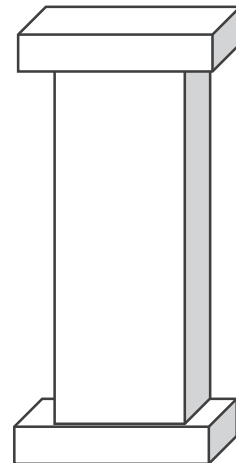


3. If the figure below is made of cubes with 2 cm side lengths, what is its volume? Explain your thinking.

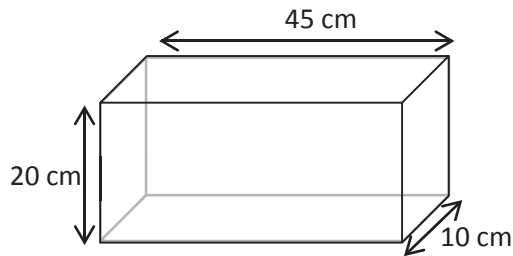


4. The volume of a rectangular prism is 840 in^3 . If the area of the base is 60 in^2 , find its height. Draw and label a model to show your thinking.

5. The following structure is composed of two right rectangular prisms that each measure 12 inches by 10 inches by 5 inches and one right rectangular prism that measures 10 inches by 8 inches by 36 inches. What is the total volume of the structure? Explain your thinking.

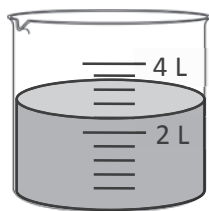


6. a. Find the volume of the rectangular fish tank. Explain your thinking.

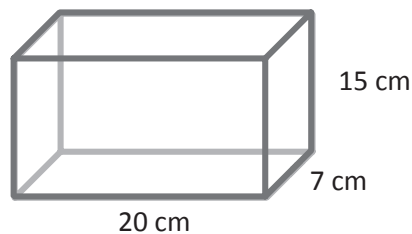


- b. If the fish tank is completely filled with water and then 900 cubic centimeters are poured out, how high will the water be? Give your answer in centimeters, and show your work.

7. Juliet wants to know if the chicken broth in this beaker will fit into this rectangular food storage container. Explain how you would figure it out without pouring the contents in. If it will fit, how much more broth could the storage container hold? If it will not fit, how much broth will be left over? (Remember: $1 \text{ cm}^3 = 1 \text{ mL}$.)



Beaker



Storage Container

**Mid-Module Assessment Task
Standards Addressed**
Topics A–B

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
 - A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
 - A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
 - Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
 - Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
 - Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency*. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

A Progression Toward Mastery

Assessment Task Item	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1	The student has neither the correct volume nor the correct unit of measure for either figure.	The student calculates the volume incorrectly for both figures but uses the correct unit of measure.	The student calculates the volume for one figure correctly and uses the correct unit of measure for both.	The student correctly calculates the volume and uses the correct unit of measure for both: a. 12 in^3 b. 8 in^3
2	The student is unable to explain the equivalence of the two approaches and is unable to find the volume of the prism in part (b) using Jack's method.	The student makes an attempt to explain the equivalence of the two approaches but uses faulty logic and is unable to find the volume of the prism in part (b) using Jack's method.	The student explains the equivalence between the two approaches by explaining that the sides may be multiplied in any order because any face can be used as the base of the figure but is unable to use Jack's method to calculate the volume of the prism in part (b). The student may not score more than 3 points if the student does not label part (b) as cubic feet (ft^3).	In part (a), the student correctly explains the equivalence between the two approaches by explaining the sides may be multiplied in any order because any face can be used as the base of the figure. The student also uses Jack's method to correctly calculate the area of the prism in part (b) as $(3 \text{ ft} \times 2 \text{ ft}) \times 5 \text{ ft} = 30 \text{ ft}^3$.



A Progression Toward Mastery

3	The student is neither able to calculate the volume of the figure nor explain the reasoning used.	The student uses a correct method for finding the volume of the cube but does not consider the size of the cubes (2 cm side lengths) and does not explain the reasoning used.	The student uses a correct method for finding the volume of the cube and is able to explain the reasoning used but does not consider the size of the cubes (2 cm side lengths). The student may not score more than 3 points if the student does not correctly identify the units as cubic centimeters (cm^3).	The student correctly answers $1,440 \text{ cm}^3$ and is able to explain the reasoning used.
4	The student is neither able to calculate the height of the prism nor able to draw and label a model.	The student is able to either find the missing height or draws an unlabeled model.	The student draws a labeled model showing 60 in^2 and some inches high but makes a calculation error when finding the height of the prism.	The student clearly does the following: <ul style="list-style-type: none"> ▪ Draws a three-dimensional rectangular prism as a model. ▪ Labels the model, including inches for height and inches squared for the base. ▪ Calculates the height of the prism as 14 in.
5	The student is able to calculate the volume of one part of the figure but is unable to explain the reasoning used.	The student explains the reasoning used but makes more than one calculation error.	The student explains the reasoning used but makes one calculation error. The student may not score more than 3 points if the student does not correctly identify the units as cubic inches (in^3).	The student does the following: <ul style="list-style-type: none"> ▪ Correctly calculates the volume of the prism as $4,080 \text{ in}^3$. ▪ Clearly explains the reasoning used.



A Progression Toward Mastery

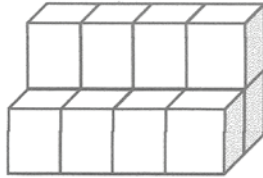
6	The student is unable to correctly answer any part of the task.	The student correctly answers either part (a) or part (b) but does not explain the reasoning used.	The student calculates either part (a) or part (b) correctly and explains the reasoning used. The student may not score more than 3 points if the student does not correctly identify the units as cubic centimeters (cm^3).	The student correctly does the following: a. Calculates $9,000 \text{ cm}^3$ and clearly explains the reasoning used. b. Calculates 18 cm and shows correct work and reasoning.
7	The student attempts a calculation but does not achieve an answer or explain her thinking.	The student attempts part of the answer but miscalculates or does not explain her thinking.	The student explains her thinking and correctly calculates either the volume in the beaker or the volume of the container but makes a mistake in the other, leading to incorrect answers.	The student does the following: ▪ Clearly explains a method for determining if the contents will fit without pouring—the volume of the beaker is $2,400 \text{ mL} = 2,400 \text{ cm}^3$. The volume of the storage container is $2,100 \text{ cm}^3$. ▪ Correctly answers that the broth will not fit. ▪ Correctly answers that Juliet will have 300 mL more (or 300 cm^3), or 0.3 L more, broth that will not fit in the storage container.

Name Jane

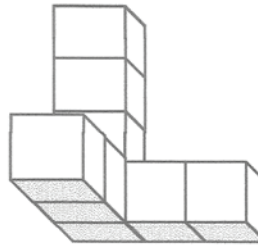
Date _____

1. Tell the volume of each solid figure made of 1-inch cubes. Specify the correct unit of measure.

a.

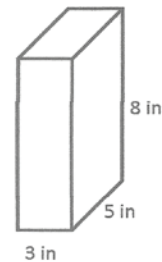
 12 in^3

b.

 8 in^3

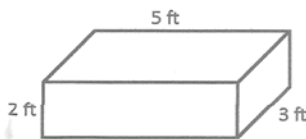
2. Jack found the volume of the prism pictured to the right by multiplying 5×8 and then adding: $40 + 40 + 40 = 120$. He says the volume is 120 cubic inches.

- a. Jill says he did it wrong. He should have multiplied the bottom first (3×5) and then multiplied by the height. Explain to Jill why Jack's method works and is equivalent to her method.



Jack thought of it like slices. He figured the area of one slice (8×5). Then he visualized 2 more slices, so he added $40 + 40 + 40$ which is 120. This is the same answer he would have gotten if he multiplied $(3 \times 5) \times 8$.

- b. Use Jack's method to find the volume of this right rectangular prism.

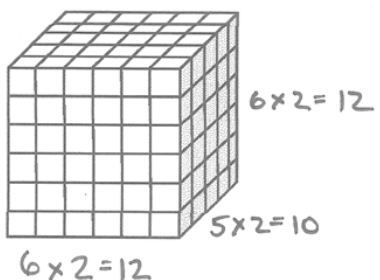


$$3 \times 2 = 6$$

$$6 + 6 + 6 + 6 + 6 = 30$$

The volume of this right rectangular prism is 30 ft^3 .

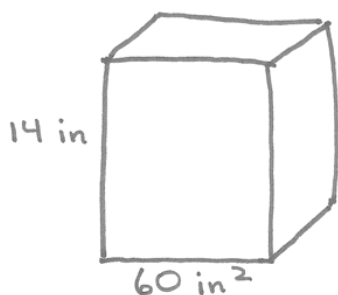
3. If the figure below is made of cubes with 2 cm side lengths, what is its volume? Explain your thinking.



The volume is
 $1,440 \text{ cm}^3$.

$12 \times 12 \times 10 = 144 \times 10 = 1,440$
First I counted the cubes. Since each cube is worth 2 cm each, I doubled the number on each side. Then I could have added the layers, but multiplying is faster.

4. The volume of a rectangular prism is 840 in^3 . If the area of the base is 60 in^2 , find its height. Draw and label a model to show your thinking.



$$V = 840 \text{ in}^3$$

$$\frac{840}{60} = \frac{84}{6} = 14$$

The height is 14 inches.

5. The following structure is composed of two right rectangular prisms that each measure 12 inches by 10 inches by 5 inches and one right rectangular prism that measures 10 inches by 8 inches by 36 inches. What is the total volume of the structure? Explain your thinking.

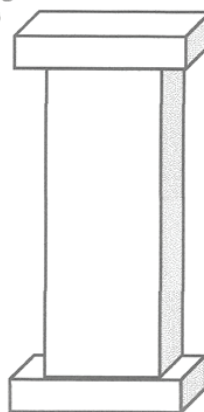
$$12 \text{ in} \times 10 \text{ in} \times 5 \text{ in} = 120 \text{ in}^2 \times 5 = 600 \text{ in}^3$$

$$600 \text{ in}^3 \times 2 = 1,200 \text{ in}^3$$

$$10 \text{ in} \times 8 \text{ in} \times 36 \text{ in} = 360 \text{ in}^2 \times 8 \text{ in} = 2,880 \text{ in}^3$$

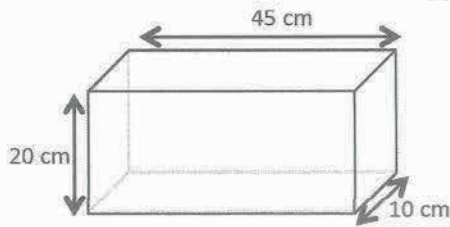
$$\begin{array}{r} 360 \\ \times 8 \\ \hline 2,880 \\ 2,880 \\ + 1,200 \\ \hline 4,080 \end{array}$$

I found the volume of the top piece, then doubled it. I added that to the volume of the middle piece.



The volume of the structure is $4,080 \text{ in}^3$.

6. a. Find the volume of the rectangular fish tank. Explain your thinking.

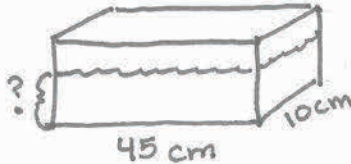


$$45 \text{ cm} \times 20 \text{ cm} \times 10 \text{ cm} = 900 \text{ cm}^2 \times 10 \text{ cm} = 9,000 \text{ cm}^3$$

I multiplied all the sides to get the volume.

The volume of the fish tank is $9,000 \text{ cm}^3$.

- b. If the fish tank is completely filled with water, and then 900 cubic centimeters are poured out, how high will the water be? Give your answer in centimeters, and show your work.



$$\begin{array}{r} 9,000 \text{ cm}^3 \\ - 900 \text{ cm}^3 \\ \hline 8,100 \text{ cm}^3 \end{array}$$

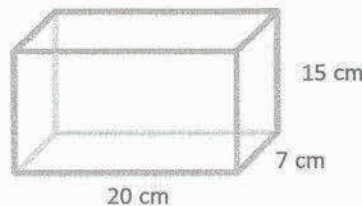
$$\begin{array}{r} 18 \\ 450 \overline{) 8,100} \\ \underline{- 4,50} \\ 3,600 \\ \underline{- 3,600} \\ 0 \end{array}$$

The water is 18 cm high.

7. Juliet wants to know if the chicken broth in this beaker will fit into this rectangular food storage container. Explain how you would figure it out without pouring the contents in. If it will fit, how much more broth could the storage container hold? If it will not fit, how much broth will be left over? (Remember $1 \text{ cm}^3 = 1 \text{ mL}$.)



beaker



storage container

First, I found the volume of the storage container.

$$20 \text{ cm} \times 15 \text{ cm} \times 7 \text{ cm} = 300 \text{ cm}^2 \times 7 \text{ cm} = 2,100 \text{ cm}^3 = 2.1 \text{ L}$$

Since each line on the beaker is 400 mL, the beaker is holding 2.4 L of broth. The broth will not fit in the container. $2.4 \text{ L} - 2.1 \text{ L} = 0.3 \text{ L}$ Juliet will have 0.3 L or 300 mL of broth left over.



Topic C

Area of Rectangular Figures with Fractional Side Lengths

- Focus Standards:**
- Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
 - Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
 - Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Instructional Days: 6

Coherence

-Links from:	G4–M4	Angle Measure and Plane Figures
-Links to:	G6–M2	Arithmetic Operations Including Division of Fractions

In Topic C, students extend their understanding of area as they use rulers and right angle templates to construct and measure rectangles with fractional side lengths and find their areas. They apply their extensive knowledge of fraction multiplication to interpret areas of rectangles with fractional side lengths and solve real-world problems involving these figures, including reasoning about scaling through contexts in which areas are compared. Visual models and equations are used to represent the problems through the Read-Draw-Write (RDW) protocol.

A Teaching Sequence Toward Mastery of Area of Rectangular Figures with Fractional Side Lengths

Objective 1: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing, and relate to fraction multiplication.
(Lesson 10)

Objective 2: Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.
(Lesson 11)

Objective 3: Measure to find the area of rectangles with fractional side lengths.
(Lesson 12)

Objective 4: Multiply mixed number factors, and relate to the distributive property and the area model.
(Lesson 13)

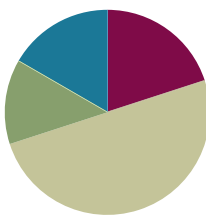
Objective 5: Solve real-world problems involving area of figures with fractional side lengths using visual models and/or equations.
(Lessons 14–15)

Lesson 10

Objective: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing, and relate to fraction multiplication.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(8 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Multiply Decimals (4 minutes)
- Change Mixed Numbers to Fractions (4 minutes)
- Multiply Mixed Numbers and Fractions (4 minutes)

Multiply Decimals (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 4 Lessons 17–18.

T: (Write $2 \times 2 = \underline{\quad}$.) Say the multiplication sentence with the answer.

S: $2 \times 2 = 4$.

T: (Write $2 \times 0.2 = \underline{\quad}$.) On your personal white board, write the number sentence and the answer.

S: (Write $2 \times 0.2 = 0.4$.)

T: (Write $0.2 \times 0.2 = \underline{\quad}$.) Try this problem.

S: (Write $0.2 \times 0.2 = 0.04$.)

T: (Write $0.02 \times 0.2 = \underline{\quad}$.) Try this problem.

S: (Write $0.02 \times 0.2 = 0.004$.)

$2 \times 2 = 4$	$2 \times 0.2 = 0.4$	$0.2 \times 0.2 = 0.04$	$0.02 \times 0.2 = 0.004$
$3 \times 4 = 12$	$3 \times 0.4 = 1.2$	$0.3 \times 0.4 = 0.12$	$0.03 \times 0.4 = 0.012$
$5 \times 7 = 35$	$0.5 \times 7 = 3.5$	$0.5 \times 0.7 = 0.35$	$0.5 \times 0.07 = 0.035$

Continue with the following possible sequence: 3×4 , 3×0.4 , 0.3×0.4 , 0.03×0.4 , 5×7 , 0.5×7 , 0.5×0.7 , and 0.5×0.07 .

Change Mixed Numbers to Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for today's lesson.

T: How many fourths are in 1?

S: 4.

T: How many fourths are in 2?

S: 8.

T: (Write $2\frac{1}{4} = \frac{\quad}{4}$.) How many fourths are in $2\frac{1}{4}$? Write $2\frac{1}{4}$ as an improper fraction on your personal white board.

S: (Write $2\frac{1}{4} = \frac{9}{4}$.)

Continue with the following possible sequence: $2\frac{3}{4}$, $2\frac{1}{2}$, $4\frac{2}{3}$, $3\frac{3}{4}$, $2\frac{5}{6}$, $3\frac{3}{8}$, $4\frac{5}{8}$, and $5\frac{7}{8}$.

Multiply Mixed Numbers and Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for today's lesson.

T: (Write $3\frac{1}{2} \times 2\frac{1}{3} = \underline{\quad}$. Point to $3\frac{1}{2}$.) Say $3\frac{1}{2}$ as a fraction greater than one.

S: $\frac{7}{2}$.

T: (Write $= \frac{7}{2} \times 2\frac{1}{3}$. Point to $2\frac{1}{3}$.) Say $2\frac{1}{3}$ as a fraction greater than one.

S: $\frac{7}{3}$.

T: (Write $= \frac{7}{2} \times \frac{7}{3}$. Beneath it, write $= \underline{\quad}$. Beneath it, write $= \underline{\quad}$.)

Multiply the fractions. Then, write the answer as a mixed number.

S: (Write $3\frac{1}{2} \times 2\frac{1}{3} = \underline{\quad}$. Beneath it, write $= \frac{7}{2} \times \frac{7}{3}$. Beneath it, write $= \frac{49}{6}$. Beneath it, write $= 8\frac{1}{6}$.)

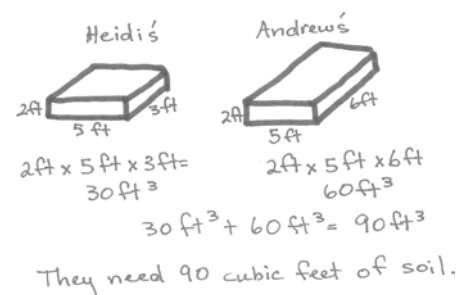
$$\begin{aligned} 3\frac{1}{2} \times 2\frac{1}{3} &= \underline{\quad} \\ &= \frac{7}{2} \times \frac{7}{3} \\ &= \frac{49}{6} \\ &= 8\frac{1}{6} \end{aligned}$$

Continue with the following possible sequence: $3\frac{1}{3} \times 2\frac{3}{4}$ and $3\frac{4}{5} \times 4\frac{2}{3}$.

Application Problem (8 minutes)

Heidi and Andrew designed two raised flowerbeds for their garden. Heidi's flowerbed was 5 feet long by 3 feet wide, and Andrew's flowerbed was the same length but twice as wide. Calculate how many cubic feet of soil they need to buy to have soil to a depth of 2 feet in both flowerbeds.

Note: This Application Problem reviews the volume work from Topic B.



Concept Development (30 minutes)

Materials: (T) 3-unit \times 2-unit rectangle, patty paper (units for tiling), large chart paper (for recording dimensions of rectangles), personal white board (S) 5 large mystery rectangles lettered A–E (1 of each size per group), patty paper (units for tiling), Problem Set

Note: The lesson is written such that the length of one standard patty paper ($5\frac{1}{2}$ inches by $5\frac{1}{2}$ inches) is one unit. Hamburger patty paper (available from big box discount stores in boxes of 1,000) is the ideal square unit for this lesson due to its translucence and size. Measurements for the mystery rectangles are given in generic units so that any size square unit may be used to tile, as long as the tiling units can be folded. Any paper may be used if patty paper is not available. Consider color-coding Rectangles A–E for easy reference.

Preparation: Each group needs one copy of Rectangles A–E. The most efficient way of producing these rectangles is to use the patty paper to measure and trace the outer dimensions of one rectangle. Then, use that rectangle as a template to cut the number required for the class. Rectangles should measure as follows:

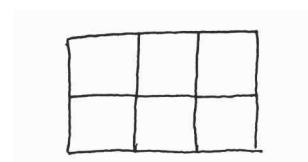
Demo Rectangle A: 3 units \times 2 units

Rectangle B: 3 units \times $2\frac{1}{2}$ units

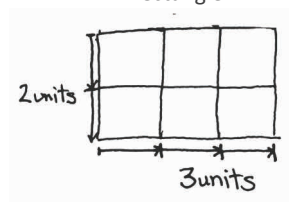
Rectangle C: $1\frac{1}{2}$ units \times 5 units

Rectangle D: 2 units \times $1\frac{3}{4}$ units

Rectangle E: $\frac{3}{4}$ unit \times 5 units



Rectangle A



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Folding the square units allows students to clearly see the relationship of the fractional square unit while maintaining the relationship to the whole square unit. Consequently, if students become confused about the size of the fractional square unit, the paper may be easily unfolded as a reminder.



- T: We want to determine the areas of some mystery rectangles today. Find the rectangle at your table labeled A. (Allow students time to find the rectangle.)
- T: If we want to find the area of this mystery rectangle, what kind of units would we use to measure it?
- S: Square units.
- T: (Hold up a patty paper tile.) This will be the square unit we will use to find the area of Rectangle A. Work with your partner to find the number of squares that will cover this rectangle with no space between the units and no overlaps. Please start at the top left-hand corner to place your first tile. (Allow students time to work.)

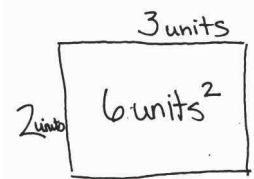
- T: How many square units covered the rectangle?
- S: 6 square units.
- T: Let's sketch a picture of what our tiling looks like. Draw the outside of your rectangle first. (Model as students draw.)
- T: Now, show the six tiles. (Allow students time to draw.)
- T: Look at the longest side of your rectangle. If we wanted to measure this side with a piece of string, how many units long would the string need to be? Explain how you know to your partner.
- S: It is 3 units long. I can look at the edge of the units and count. → To measure the length of the side, I'm not looking at the whole tile; I only need to count the length of each unit. There are 3 equal units on the edge, so the string would need to be 3 units long.
- T: Let's record that. (Write the length of Rectangle A in the chart.) What is the length of the shorter side?
- S: 2 units.
- T: Let's record that in our chart.
- T: What is the area of Rectangle A?
- S: 6 square units.
- T: If we had only labeled the length and the width in our sketch, could we still know the area? Why or why not?
- S: Yes. We know the square units are there even if we do not draw them all. → We still just multiply the sides together. We can imagine the tiles.
- T: What would a sketch of this look like? Draw it with your partner. (Allow students time to draw.)
- T: Now, find Rectangle B. Compare its size to Rectangle A. Will its area be greater than or less than that of Rectangle A?
- S: Greater.
- T: We see that Rectangles A and B are the same length. What about the width?
- S: Rectangle B is wider than two tiles but not as wide as three tiles.
- T: Fold your tiles to decide what fraction of another tile we need to cover the extra width. Work with your partner. (Allow students time to fold.)



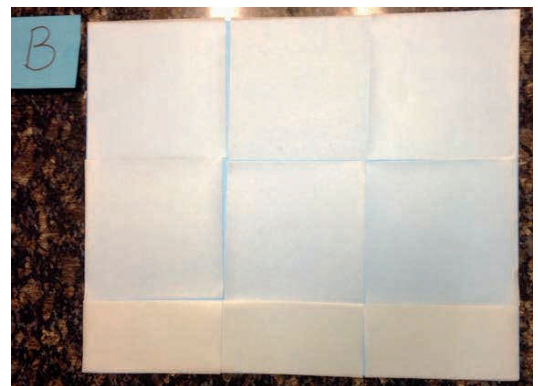
NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Students may use the tiles to measure the outside dimensions of the rectangle before tiling. For some, marking the length and width with tick marks to show the lengths of the units may help them visualize the linear measurement more easily.

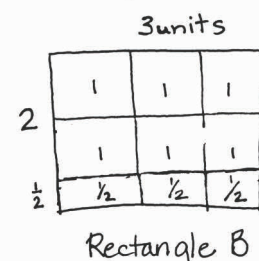
The dimensions can then be recorded on the Problem Set prior to drawing the rectangle and partial products.



Rectangle	Length	Width	Area
A	3 units	2 units	6 units ²
B	3 units	$2\frac{1}{2}$ units	$7\frac{1}{2}$ units ²



- T: What fraction of the tile do you need to cover this part of the rectangle? How do you know?
- S: I need half a tile. I laid a whole tile over the extra part, and it looked like half to me. → After I folded up the part of the tile that was hanging off the rectangle, I could see that the fold split the tile into two equal parts. That means it is halves.
- T: Finish folding enough tiles to completely cover the width of Rectangle B.
- S: (Fold to cover the rectangle completely.)
- T: Let's record by sketching and filling in the blanks on the Problem Set. I will record in the chart. What is the length of Rectangle B?
- S: 3 units. (Record on the Problem Set.)
- T: What is the width?
- S: $2\frac{1}{2}$ units.
- T: What is the area? How do you know?
- S: The area is $7\frac{1}{2}$ units squared. I counted all of the whole square units first and then added on the halves.
 → I knew it was at least 6 square units, and then we had 3 more halves, so that's $7\frac{1}{2}$ square units.
 → $3 \times 2\frac{1}{2} = 6\frac{3}{2} = 7\frac{1}{2}$.
- T: When we record our tiling, is it necessary to sketch each tile? Why or why not?
- S: No. We can just write down how many there are.
 → We can show just the side lengths of 3 and $2\frac{1}{2}$.
 I'll know that means three squares across and two and a half squares down. → It is like the area model with whole numbers. If I know the sides, I can show the total area by just multiplying.
- T: Let's sketch this rectangle again but without the individual tiles. Draw the rectangle, and label the length. (Allow students time to draw.)
- T: Now, let's decompose the $2\frac{1}{2}$ units on the width as $2 + \frac{1}{2}$. (Label and draw a horizontal line across the rectangle as pictured. Allow students time to draw.)
- T: Let's record the first partial product. (Point.) Three units long by 2 units wide is what area?
- S: 6 square units.
- T: Let's record the second partial product. (Point.) What is the length of this portion?
- S: 3 units.
- T: What is the width of this portion?
- S: 1 half unit.




NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

The spatial and visualization skills involved in Lessons 10 and 11 are quite natural for some students and quite challenging for others. Consequently, the time needed to accomplish the tasks varies, but all students should be given the opportunity to tile all the rectangles. Both lessons offer two challenging questions at the end of the Problem Sets for those who finish the tiling quickly.

- T: What is the area of this part? How do you know?
- S: The area is $1\frac{1}{2}$ square units because 3 copies of $\frac{1}{2}$ is 3 halves. \rightarrow 3 units long by $\frac{1}{2}$ unit wide is $1\frac{1}{2}$ square units.
- T: Does this $7\frac{1}{2}$ unit squared area make sense given our prediction? Why or why not?
- S: It does make sense. It is only a little wider than the first rectangle, and $7\frac{1}{2}$ is not that much more than 6. \rightarrow You can see the first rectangle inside this one. There was a part that was 3 units by 2 units, and then a smaller part was added on that was 3 units by just half another unit. That is where the extra $1\frac{1}{2}$ square units come from. \rightarrow Three times two was easy, and then I know that half of 3 is $1\frac{1}{2}$. By decomposing the mixed number, it was easy to find the total area.
- T: Work with your partner to find the length, width, and area of Rectangles C, D, and E using the patty paper and recording with the area model. Record your findings on your Problem Set, and then answer the last two questions in the time remaining. You may record your tiling without drawing each tile if you wish.
- S: (Work.)

Problem Set (5 minutes)

Students should do their personal best to complete the remainder of the Problem Set within the allotted five minutes if they have finished the tiling problems. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing, and relate to fraction multiplication.


The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Name JJ Date _____


Sketch the rectangles and your tiling. Write the dimensions and the units you counted in the blanks. Then use multiplication to confirm the area. Show your work. We will do Rectangles A and B together.

1. Rectangle A:




3 units long 2 units wide
Area = 6 units²

2. Rectangle B:



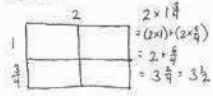
3 units long $2\frac{1}{2}$ units wide
Area = $7\frac{1}{2}$ units²

3. Rectangle C:




5 units long $1\frac{1}{2}$ units wide
Area = $7\frac{1}{2}$ units²

4. Rectangle D:



2 units long $1\frac{3}{4}$ units wide
Area = $3\frac{1}{2}$ units²

5. Rectangle E:



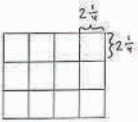
5 units long $\frac{3}{4}$ units wide
Area = $3\frac{3}{4}$ units²

Any combination of the questions below may be used to lead the discussion.

Record students' answers to Problem 1 to complete the class chart as answers are reviewed.

- What relationship did you notice between the areas of Rectangle C and Rectangle E? What accounts for this relationship?
- How was Rectangle E different from the other rectangles you tiled? Describe how you tiled it.
- How did you determine the area of Rectangle E? Did you count the single units? Add repeatedly? Multiply the sides?
- Could you place these rectangles in order of greatest to least area by using relationships among the dimensions but without actually performing the calculations? Why or why not?
- How did you determine the area of the rectangle in Problem 6?
- Analyze and compare different solution strategies for Problem 7.

6. The rectangle to the right is composed of squares that measure $2\frac{1}{4}$ inches on each side. What is its area in square inches? Explain your thinking using pictures and numbers.



Q: $2\frac{1}{4} \text{ in.} \times 4 = 9 \text{ in.}$
 A: $2\frac{1}{4} \text{ in.} \times 3 = 6\frac{3}{4} \text{ in.}$

9 in

6 in

54 in^2

$A = 54 \text{ in}^2 + 6\frac{3}{4} \text{ in}^2$
 $A = 60\frac{3}{4} \text{ in}^2$

$\frac{3}{4} \text{ in.}$ $\frac{2\frac{1}{4} \text{ in.}}{2} = 6\frac{3}{4} \text{ in.}$

7. A rectangle has a perimeter of $35\frac{1}{2}$ feet. If the length is 12 feet, what is the area of the rectangle?

12 ft

A = ?

12 ft

Perimeter: $35\frac{1}{2} \text{ ft}$
 $35\frac{1}{2} \text{ ft} - 24 \text{ ft} = 11\frac{1}{2} \text{ ft}$
 $11\frac{1}{2} \text{ ft} \div 2 = \frac{23}{2} \text{ ft} \times \frac{1}{2} = \frac{23}{4} \text{ ft} = 5\frac{3}{4} \text{ ft}$

Area: $12 \text{ ft} \times 5\frac{3}{4} \text{ ft}$
 $= 60 \text{ ft}^2 + \frac{3(12)}{4} \text{ ft}^2$
 $= 60 \text{ ft}^2 + 9 \text{ ft}^2$
 $= 69 \text{ ft}^2$

The area of the rectangle is 69 ft^2 .

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____ Date _____

Sketch the rectangles and your tiling. Write the dimensions and the units you counted in the blanks. Then, use multiplication to confirm the area. Show your work. We will do Rectangles A and B together.

1. **Rectangle A:**

Rectangle A is

_____ units long _____ units wide

Area = _____ units²

2. **Rectangle B:**

Rectangle B is

_____ units long _____ units wide

Area = _____ units²

3. **Rectangle C:**

Rectangle C is

_____ units long _____ units wide

Area = _____ units²

4. **Rectangle D:**

Rectangle D is

_____ units long _____ units wide

Area = _____ units²

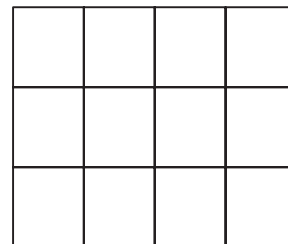
5. **Rectangle E:**

Rectangle E is

_____ units long _____ units wide

Area = _____ units²

6. The rectangle to the right is composed of squares that measure $2\frac{1}{4}$ inches on each side. What is its area in square inches? Explain your thinking using pictures and numbers.

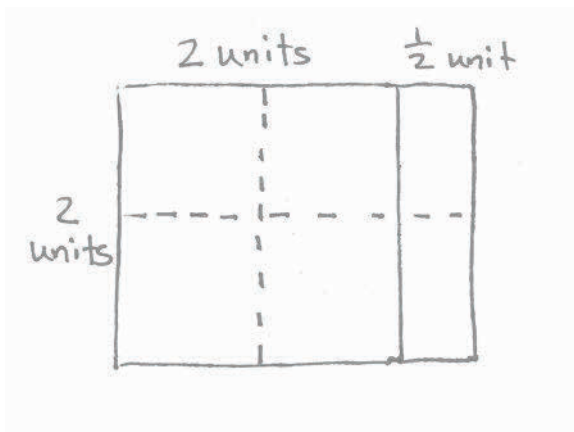


7. A rectangle has a perimeter of $35\frac{1}{2}$ feet. If the length is 12 feet, what is the area of the rectangle?

Name _____

Date _____

Emma tiled a rectangle and then sketched her work. Fill in the missing information, and multiply to find the area.

**Emma's Rectangle:**

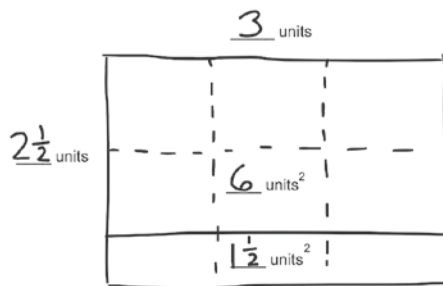
_____ units long _____ units wide

Area = _____ units²

Name _____

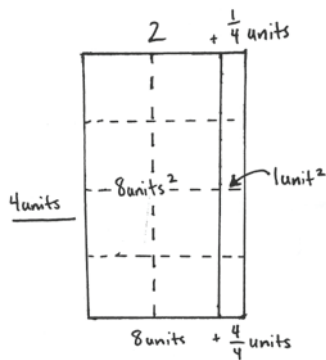
Date _____

1. John tiled some rectangles using square units. Sketch the rectangles if necessary. Fill in the missing information, and then confirm the area by multiplying.

a. **Rectangle A:**

Rectangle A is

3 units long $2\frac{1}{2}$ units wide

Area = _____ units²b. **Rectangle B:**

Rectangle B is

_____ units long _____ units wide

Area = _____ units²c. **Rectangle C:**

Rectangle C is

$\frac{3}{4}$ units long 4 units wide

Area = _____ units²

d. **Rectangle D:**

Rectangle D is

2 units long $1\frac{3}{4}$ units wide

Area = _____ units²

2. Rachel made a mosaic from different color rectangular tiles. Three tiles measured $3\frac{1}{2}$ inches \times 3 inches. Six tiles measured 4 inches \times $3\frac{1}{4}$ inches. What is the area of the whole mosaic in square inches?

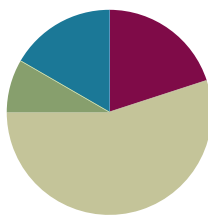
3. A garden box has a perimeter of $27\frac{1}{2}$ feet. If the length is 9 feet, what is the area of the garden box?

Lesson 11

Objective: Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Sprint: Multiply Decimals (8 minutes)
- Multiplying Fractions (4 minutes)

Sprint: Multiply Decimals (8 minutes)

Materials: (S) Multiply Decimals Sprint

Note: This fluency activity reviews Module 4.

Multiplying Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for Lesson 13.

T: (Write $\frac{1}{2} \times \frac{1}{3} = \underline{\hspace{1cm}}$.) Say the multiplication equation with the answer.

S: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

T: (Write $\frac{1}{2} \times \frac{3}{4} = \underline{\hspace{1cm}}$.) Say the multiplication equation with the answer.

S: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.

T: (Write $\frac{2}{5} \times \frac{2}{3} = \underline{\hspace{1cm}}$.) On your personal white board, write the multiplication equation and the answer.

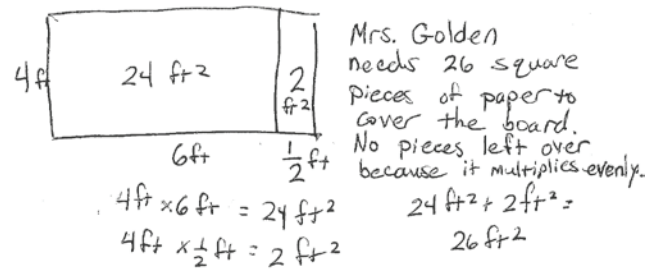
S: (Write $\frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$.)

Continue with the following possible sequence: $\frac{1}{2} \times \frac{1}{5}$, $\frac{1}{2} \times \frac{3}{5}$, $\frac{3}{4} \times \frac{3}{5}$, $\frac{4}{5} \times \frac{2}{3}$, and $\frac{3}{4} \times \frac{5}{6}$.

Application Problem (5 minutes)

Mrs. Golden wants to cover her 6.5-foot by 4-foot bulletin board with silver paper that comes in 1-foot squares. How many squares does Mrs. Golden need to cover her bulletin board? Will there be any fractional pieces of silver paper left over? Explain why or why not. Draw a sketch to show your thinking.

Note: This Application Problem reviews Lesson 10's concept of tiling, now with one dimension as a decimal fraction.



Concept Development (33 minutes)

Materials: (T) Rectangles, patty paper units for tiling, personal white board (S) 1 demonstration and 5 mystery rectangles lettered A–E (1 of each size per group), patty paper units for tiling, Problem Set

Note: Today's lesson parallels the structure of Lesson 10. Rectangles for each group should be prepared in advance following yesterday's instructions. The dimensions of today's rectangles are given below.

Rectangle A: $4\frac{1}{2}$ units \times $2\frac{1}{2}$ units

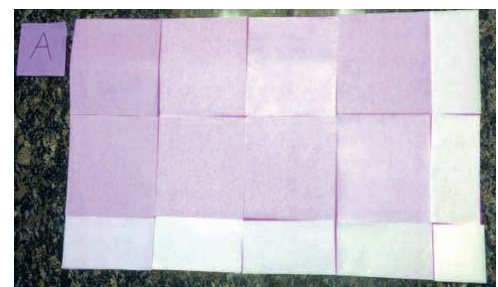
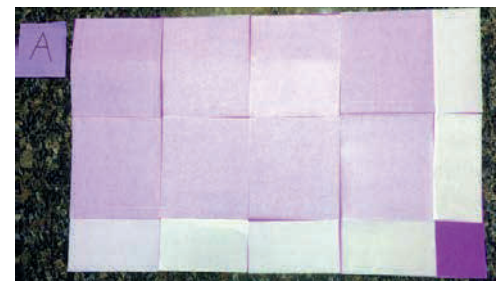
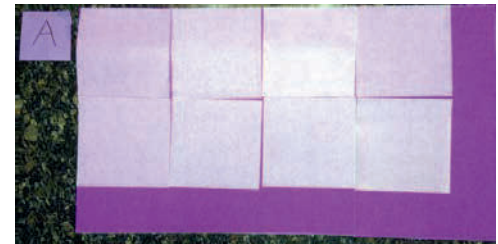
Rectangle B: $1\frac{3}{4}$ units \times $3\frac{3}{4}$ units

Rectangle C: $\frac{3}{4}$ unit \times $1\frac{1}{2}$ units

Rectangle D: $\frac{3}{4}$ unit \times $\frac{1}{2}$ unit

Rectangle E: $\frac{2}{3}$ unit \times $\frac{2}{3}$ unit

The added complexities of today's lesson involve the inclusion of two mixed number or fractional side lengths. This is an application of the fraction multiplication lessons of Module 4. Students record partial products rather than draw individual tiles.



- T: Let's start with Rectangle A. Work with your partner to place as many whole tiles on Rectangle A as you can. Remember to start at the top left corner of the rectangle.
- S: (Place whole tiles on Rectangle A.)
- T: How many whole tiles fit?
- S: 8.
- T: Is this the area of the rectangle?
- S: No.
- T: Fold some of your square units to cover the rest of the rectangle's length. (Allow time for students to work.)
- T: What fractional unit do you need to do this? How many?
- S: I needed 2 half units. → The unit was halves. I needed 2 of them.
- T: Now, fold some units to cover the rest of the rectangle's width. (Allow time for students to work.)
- T: What fractional unit did you use this time, and how many?
- S: I needed halves again. This time, it was 4. → It was 4 half units.
- T: I see that we have covered almost all of the rectangle, but there seems to be a part at the bottom that is even smaller than the halves we just placed. How can we find the fractional unit that will fit here? Turn and talk.
- S: I can see that if I fold a square unit in half, it fits in one direction, but it is too long in the other direction. Maybe if I fold it again, it will fit. → If I fold it in half, it fits the length. Then, if I fold that half in half again, it fits perfectly in the space. → The part is half the size of half a square unit. Half of a half is 1 fourth of a square unit.
- T: Unfold the paper that you have made to fit in this part. What fraction of a whole square unit covers this part?
- S: 1 fourth of a square unit.
- T: Work with your partner to count the tiles to determine the area.
- S: (Count the tiles with partners.)



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

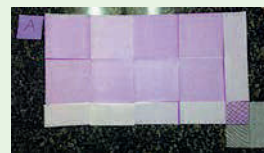
Include Rectangle E as an optional challenge. Folding and tiling Rectangle E requires students to fold thirds and to reason about another area less than 1 square tile. Recording for Rectangle E should be done on a separate piece of paper, as it is not included on the Problem Set.

The last two problems on the Problem Set also offer extensions for students who finish the tiling and multiplication quickly.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

For some students, it may be more effective to place a whole square unit over the last corner of the rectangle and then trace the outline or shade the corner of the rectangle on the patty paper. (Because the patty paper is translucent, the edge of the rectangle is clearly visible.) Students may then fold until only the outlined portion of the paper is visible. When the paper is unfolded, only 1 of the 4 equal parts is bordered (or shaded).



Also, guide students to isolate the last corner of the rectangle and use a single piece of patty paper to model the multiplication of a fraction by a fraction to produce a double-shaded area (as in Module 4). This double-shaded portion can then be laid on top of the rectangle's corner, fitting perfectly.

T: What is the area? How did you count it?

S: I counted the 8 squares first. I added 6 halves or 3 more squares. Then, I added $\frac{1}{4}$ to 11. That's $11\frac{1}{4}$ square units. \rightarrow I could see 2 rows of $4\frac{1}{2}$ units. That is 9. Then, there were 4 halves and $\frac{1}{4}$ in a row. That is $2\frac{1}{4} + 9 = 11\frac{1}{4}$. The area is $11\frac{1}{4}$ square units.

T: Let's record our thinking. I will work on the board. You record on your Problem Set. Sketch the rectangle first. Decompose the length and width into ones and fractional parts.

S: (Sketch and decompose the length and width.)

T: How did you decompose the length?

S: $4 + \frac{1}{2}$.

T: (Record in the algorithm.) The width?

S: $2 + \frac{1}{2}$.

T: (Record in the algorithm.) Let's use multiplication to confirm the area we found with counting. Let's start with the ones. (Point, and then record each partial product in the rectangle and in the algorithm.)
2 units \times 4 units equals ...?

S: 8 square units.

T: (Point and record.) 2 units $\times \frac{1}{2}$ unit equals...?

S: 2 half square units. \rightarrow 1 square unit.

T: (Point and record.) $\frac{1}{2}$ unit \times 4 units equals...?

S: 4 half square units. \rightarrow 2 square units.

T: (Point and record.) $\frac{1}{2}$ unit $\times \frac{1}{2}$ unit equals...?

S: $\frac{1}{4}$ square unit.

T: Find the sum.

S: (Work to find $11\frac{1}{4}$ units².)

T: Was the area the same using multiplication and the area model?

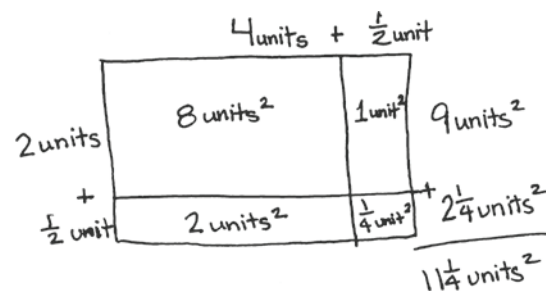
S: Yes!

T: Use your tiles to determine the area and dimensions of the other rectangles. Record your findings on your Problem Set. Then, multiply to confirm the area.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Please note that the algorithm is provided so that students are exposed to a more formal representation of the distribution. However, students should not be required to be as formal in their own calculations. Using an area model to keep track of students' thinking is sufficient.



$$\begin{aligned}
 4\frac{1}{2} \times 2\frac{1}{2} &= (4 + \frac{1}{2}) \times (2 + \frac{1}{2}) \\
 &= (2 \times 4) + (2 \times \frac{1}{2}) + (\frac{1}{2} \times 4) + (\frac{1}{2} \times \frac{1}{2}) \\
 &= 8 + 1 + 2 + \frac{1}{4} \\
 &= 11\frac{1}{4}
 \end{aligned}$$

Problem Set (10 minutes)

Students should do their personal best to complete the remainder of the Problem Set within the allotted 10 minutes if they have finished the tiling problems. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

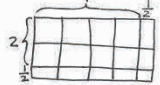
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

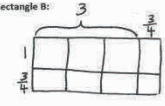
Any combination of the questions below may be used to lead the discussion.


- Compare the rectangles we tiled today to the rectangles we tiled yesterday. What do you notice? How did that change the way we had to tile?
- Which rectangle was the easiest to tile? Which was the hardest? Why?
- Explain your strategy for tiling Rectangle D (and Rectangle E, where applicable). How was finding the area of this rectangle similar to the fraction multiplication we did in Module 4? How was it different?
- Explain your strategy for finding the areas of the rectangles in Problem 5 and how you compared them.
- How is Problem 6 in today's Problem Set like Problem 7 in yesterday's Problem Set (Lesson 10), and how is it different? Yesterday's problem read: A rectangle has a perimeter of $35\frac{1}{2}$ feet. If the length is 12 feet, what is the area of the rectangle?


Name Tia Date _____

Draw the rectangle and your tiling.
Write the dimensions and the units you counted in the blanks.
Then, use multiplication to confirm the area. Show your work.

1. Rectangle A is: 
Rectangle A is: $4\frac{1}{2}$ units long $2\frac{1}{2}$ units wide
Area = $11\frac{1}{4}$ units²
 $4\frac{1}{2} \times 2\frac{1}{2} = (2 \times 4) + (2 \times \frac{1}{2}) + (\frac{1}{2} \times 4) + (\frac{1}{2} \times \frac{1}{2})$
 $= 8 + 1 + 2 + \frac{1}{4}$
 $= 11\frac{1}{4}$


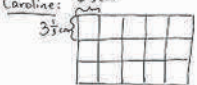
2. Rectangle B is: 
Rectangle B is: $3\frac{3}{4}$ units long $1\frac{3}{4}$ units wide
Area = $6\frac{9}{16}$ units²
 $3\frac{3}{4} \times 1\frac{3}{4} = (1 \times 3) + (1 \times \frac{3}{4}) + (\frac{3}{4} \times 3) + (\frac{3}{4} \times \frac{3}{4})$
 $= 3 + \frac{3}{4} + 2\frac{3}{4} + \frac{9}{16}$
 $= 6\frac{9}{16}$

3. Rectangle C is: 
Rectangle C is: $1\frac{1}{2}$ units long $\frac{3}{4}$ units wide
Area = $1\frac{1}{8}$ units²
 $1\frac{1}{2} \times \frac{3}{4} = (\frac{3}{4} \times 1) + (\frac{3}{4} \times \frac{1}{2})$
 $= \frac{3}{4} + \frac{3}{8} = \frac{6}{8} + \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}$

4. Rectangle D is: 
Rectangle D is: $\frac{3}{4}$ units long $\frac{1}{2}$ units wide
Area = $\frac{3}{8}$ units²
 $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

5. Colleen and Caroline each built a rectangle out of square tiles placed in 3 rows of 5. Colleen used tiles that measured $1\frac{1}{2}$ cm in length. Caroline used tiles that measured $3\frac{1}{2}$ cm in length.

a. Draw the girls' rectangles, and label the lengths and widths of each.

Colleen: 
Caroline: 

b. What are the areas of the rectangles in square centimeters?

Colleen: $l: 1\frac{1}{2} \text{ cm} \times 5 = 5\frac{1}{2} \text{ cm} = 8\frac{1}{2} \text{ cm}$
 $w: 1\frac{1}{2} \text{ cm} \times 3 = 3\frac{1}{2} \text{ cm} = 5 \text{ cm}$
 $A: 8\frac{1}{2} \text{ cm} \times 5 \text{ cm} = (8 \text{ cm} \times 5 \text{ cm}) + (5 \text{ cm} \times \frac{1}{2} \text{ cm})$
 $= 40 \text{ cm}^2 + \frac{5}{2} \text{ cm}^2$
 $A = 41\frac{1}{2} \text{ cm}^2$

Caroline: $l: 3\frac{1}{2} \text{ cm} \times 5 = 15\frac{5}{2} \text{ cm} = 16\frac{5}{2} \text{ cm}$
 $w: 3\frac{1}{2} \text{ cm} \times 3 = 9\frac{3}{2} \text{ cm} = 10 \text{ cm}$
 $A: 16\frac{5}{2} \text{ cm} \times 10 \text{ cm} = (16 \text{ cm} \times 10 \text{ cm}) + (\frac{5}{2} \text{ cm} \times 10 \text{ cm})$
 $= 160 \text{ cm}^2 + \frac{25}{2} \text{ cm}^2$
 $A = 166\frac{1}{2} \text{ cm}^2$

c. Compare the areas of the rectangles.

The area of Colleen's rectangle is smaller than Caroline's.
 $41\frac{1}{2} \text{ cm}^2 < 166\frac{1}{2} \text{ cm}^2$

6. A square has a perimeter of 51 inches. What is the area of the square?

$? = 12\frac{3}{4} \text{ in}$
 $P: 51 \text{ in}$
 $51 \div 4 = \frac{51}{4} = 12\frac{3}{4}$

The area of the square is $162\frac{9}{16} \text{ in}^2$.

$A: 12\frac{3}{4} \text{ in} \times 12\frac{3}{4} \text{ in}$
 $= (12 \text{ in} \times 12 \text{ in}) + (12 \text{ in} \times \frac{3}{4} \text{ in}) + (\frac{3}{4} \text{ in} \times 12 \text{ in}) + (\frac{3}{4} \text{ in} \times \frac{3}{4} \text{ in})$
 $= 144 \text{ in}^2 + 9 \text{ in}^2 + 9 \text{ in}^2 + \frac{9}{16} \text{ in}^2$
 $= 162\frac{9}{16} \text{ in}^2$

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

A

Number Correct: _____

Multiply Decimals

1.	$3 \times 2 =$	
2.	$3 \times 0.2 =$	
3.	$3 \times 0.02 =$	
4.	$3 \times 3 =$	
5.	$3 \times 0.3 =$	
6.	$3 \times 0.03 =$	
7.	$2 \times 4 =$	
8.	$2 \times 0.4 =$	
9.	$2 \times 0.04 =$	
10.	$5 \times 3 =$	
11.	$5 \times 0.3 =$	
12.	$5 \times 0.03 =$	
13.	$7 \times 2 =$	
14.	$7 \times 0.2 =$	
15.	$7 \times 0.02 =$	
16.	$4 \times 3 =$	
17.	$4 \times 0.3 =$	
18.	$0.4 \times 3 =$	
19.	$0.4 \times 0.3 =$	
20.	$0.4 \times 0.03 =$	
21.	$0.3 \times 0.04 =$	
22.	$6 \times 2 =$	

23.	$0.6 \times 2 =$	
24.	$0.6 \times 0.2 =$	
25.	$0.6 \times 0.02 =$	
26.	$0.2 \times 0.06 =$	
27.	$5 \times 7 =$	
28.	$0.5 \times 7 =$	
29.	$0.5 \times 0.7 =$	
30.	$0.5 \times 0.07 =$	
31.	$0.7 \times 0.05 =$	
32.	$2 \times 8 =$	
33.	$9 \times 0.2 =$	
34.	$3 \times 7 =$	
35.	$8 \times 0.03 =$	
36.	$4 \times 6 =$	
37.	$0.6 \times 7 =$	
38.	$0.7 \times 0.7 =$	
39.	$0.8 \times 0.06 =$	
40.	$0.09 \times 0.6 =$	
41.	$6 \times 0.8 =$	
42.	$0.7 \times 0.9 =$	
43.	$0.08 \times 0.8 =$	
44.	$0.9 \times 0.08 =$	

B

Number Correct: _____

Improvement: _____

Multiply Decimals

1.	$4 \times 2 =$	
2.	$4 \times 0.2 =$	
3.	$4 \times 0.02 =$	
4.	$2 \times 3 =$	
5.	$2 \times 0.3 =$	
6.	$2 \times 0.03 =$	
7.	$3 \times 3 =$	
8.	$3 \times 0.3 =$	
9.	$3 \times 0.03 =$	
10.	$4 \times 3 =$	
11.	$4 \times 0.3 =$	
12.	$4 \times 0.03 =$	
13.	$9 \times 2 =$	
14.	$9 \times 0.2 =$	
15.	$9 \times 0.02 =$	
16.	$5 \times 3 =$	
17.	$5 \times 0.3 =$	
18.	$0.5 \times 3 =$	
19.	$0.5 \times 0.3 =$	
20.	$0.5 \times 0.03 =$	
21.	$0.3 \times 0.05 =$	
22.	$8 \times 2 =$	

23.	$0.8 \times 2 =$	
24.	$0.8 \times 0.2 =$	
25.	$0.8 \times 0.02 =$	
26.	$0.2 \times 0.08 =$	
27.	$5 \times 9 =$	
28.	$0.5 \times 9 =$	
29.	$0.5 \times 0.9 =$	
30.	$0.5 \times 0.09 =$	
31.	$0.9 \times 0.05 =$	
32.	$2 \times 6 =$	
33.	$7 \times 0.2 =$	
34.	$3 \times 8 =$	
35.	$9 \times 0.03 =$	
36.	$4 \times 8 =$	
37.	$0.7 \times 6 =$	
38.	$0.6 \times 0.6 =$	
39.	$0.6 \times 0.08 =$	
40.	$0.06 \times 0.9 =$	
41.	$8 \times 0.6 =$	
42.	$0.9 \times 0.7 =$	
43.	$0.07 \times 0.7 =$	
44.	$0.8 \times 0.09 =$	

Name _____

Date _____

Draw the rectangle and your tiling.

Write the dimensions and the units you counted in the blanks.

Then, use multiplication to confirm the area. Show your work.

1. **Rectangle A:**2. **Rectangle B:**

Rectangle A is

_____ units long _____ units wide

Area = _____ units²

Rectangle B is

_____ units long _____ units wide

Area = _____ units²3. **Rectangle C:**4. **Rectangle D:**

Rectangle C is

_____ units long _____ units wide

Area = _____ units²

Rectangle D is

_____ units long _____ units wide

Area = _____ units²

5. Colleen and Caroline each built a rectangle out of square tiles placed in 3 rows of 5. Colleen used tiles that measured $1\frac{2}{3}$ cm in length. Caroline used tiles that measured $3\frac{1}{3}$ cm in length.
- a. Draw the girls' rectangles, and label the lengths and widths of each.
- b. What are the areas of the rectangles in square centimeters?
- c. Compare the areas of the rectangles.
6. A square has a perimeter of 51 inches. What is the area of the square?

Name _____

Date _____

To find the area, Andrea tiled a rectangle and sketched her answer. Sketch Andrea's rectangle, and find the area. Show your multiplication work.

Rectangle is

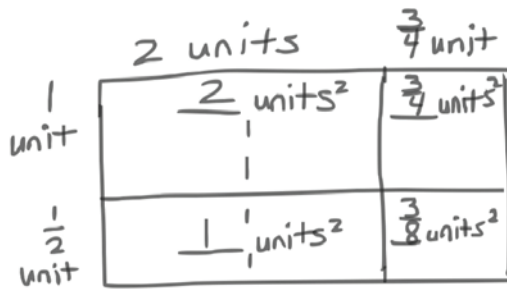
$$2\frac{1}{2} \text{ units} \times 2\frac{1}{2} \text{ units}$$

Area = _____

Name _____

Date _____

1. Kristen tiled the following rectangles using square units. Sketch the rectangles, and find the areas. Then, confirm the area by multiplying. Rectangle A has been sketched for you.

a. **Rectangle A:**

Rectangle A is

_____ units long \times _____ units wideArea = _____ units²b. **Rectangle B:**

Rectangle B is

 $2\frac{1}{2}$ units long \times $\frac{3}{4}$ unit wideArea = _____ units²c. **Rectangle C:**

Rectangle C is

 $3\frac{1}{3}$ units long \times $2\frac{1}{2}$ units wideArea = _____ units²

d. **Rectangle D:**

Rectangle D is

$3\frac{1}{2}$ units long \times $2\frac{1}{4}$ units wide

Area = _____ units²

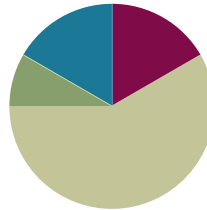
2. A square has a perimeter of 25 inches. What is the area of the square?

Lesson 12

Objective: Measure to find the area of rectangles with fractional side lengths.

Suggested Lesson Structure

■ Fluency Practice	(10 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(35 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Multiplying Fractions (4 minutes)
- Find the Volume (6 minutes)

Multiplying Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for Lesson 13.

T: (Write $\frac{1}{3} \times \frac{1}{4} = \underline{\hspace{1cm}}$.) Complete the multiplication number sentence.

S: $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.

T: (Write $\frac{1}{3} \times \frac{2}{5} = \underline{\hspace{1cm}}$.) Complete the multiplication number sentence.

S: $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$.

T: (Write $\frac{3}{5} \times \frac{2}{3} = \underline{\hspace{1cm}}$. Beneath it, write = $\underline{\hspace{1cm}}$.) On your personal white board, write the complete multiplication number sentence. Then, simplify the fraction.

S: (Write $\frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$. Beneath it, write = $\frac{2}{5}$.)

Continue with the following possible sequence: $\frac{1}{2} \times \frac{1}{4}$, $\frac{1}{2} \times \frac{3}{4}$, $\frac{3}{4} \times \frac{2}{3}$, $\frac{5}{6} \times \frac{2}{3}$, and $\frac{3}{4} \times \frac{5}{6}$.

Find the Volume (6 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews volume concepts and formulas.

T: (Project a prism $5 \text{ units} \times 2 \text{ units} \times 4 \text{ units}$. Write $V = \underline{\hspace{1cm}} \text{ units} \times \underline{\hspace{1cm}} \text{ units} \times \underline{\hspace{1cm}} \text{ units}$.) Find the volume.

S: (Write $5 \text{ units} \times 2 \text{ units} \times 4 \text{ units} = 40 \text{ units}^3$.)

T: How many layers of 10 cubes are in the prism?

S: 4 layers.

T: (Write $4 \times 10 \text{ units}^3 = \underline{\hspace{1cm}}$.) Four copies of 10 cubic units is...?

S: 40 cubic units.

T: How many layers of 8 cubes are there?

S: 5 layers.

T: (Write $5 \times 8 \text{ units}^3 = \underline{\hspace{1cm}}$.) Five copies of 8 cubic units is...?

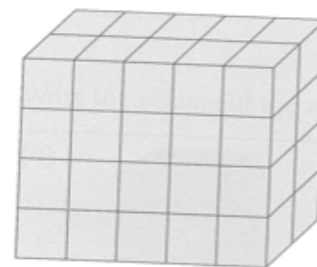
S: 40 cubic units.

T: How many layers of 20 cubes are there?

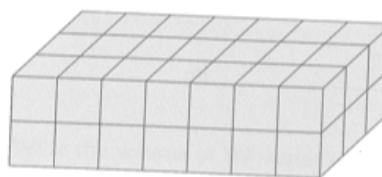
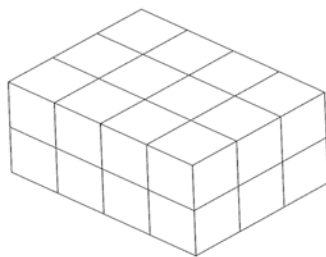
S: 2 layers.

T: Write a multiplication sentence to find the volume of the prism, starting with the number of layers. (Point.)

S: (Write $2 \text{ units} \times 20 \text{ units}^2 = 40 \text{ units}^3$.)



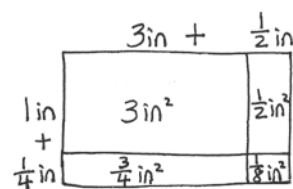
Repeat the process with the following prisms.



Application Problem (5 minutes)

Margo is designing a label. The dimensions of the label are $3\frac{1}{2}$ inches by $1\frac{1}{4}$ inches. What is the area of the label? Use the RDW process.

Note: Students can use the area model used in Module 4 and in Module 5 Lessons 10–11 to solve. This bridges to today's lesson, which extends the use of the area model.



$$\begin{aligned} & 3 + \frac{1}{2} + \frac{3}{4} + \frac{1}{8} \\ &= 3 + \frac{4}{8} + \frac{6}{8} + \frac{1}{8} \\ &= 4\frac{3}{8} \end{aligned}$$

The area of the label is $4\frac{3}{8} \text{ in}^2$.

Concept Development (35 minutes)

Materials: (T) Ruler, projector (S) Ruler, Problem Set

Problem 1(a)

Project the first rectangle in the Problem Set.

T: We will find the areas of more mystery rectangles today. What was the relationship between the areas we found using square tiles and the areas we found using multiplication?

S: We got the same answers. → Tiling or finding partial products using multiplication will always give the same area because the rectangle we are using is the same.

T: Today, we will use a ruler to help us find area. Turn and talk to your partner about how you think a ruler might be useful in finding the area of a rectangle.

S: It's not square units, but we can measure the edges. → The ruler lets us measure the sides to find out the lengths we need to multiply.

T: Work with your partner to measure in inches the lengths of the first rectangle of the Problem Set. Compare your measurements.

S: (Measure the first rectangle.)

T: What are the lengths of the sides?

S: 2 inches and $2\frac{1}{2}$ inches.

T: Estimate the area of this rectangle. Turn and talk.

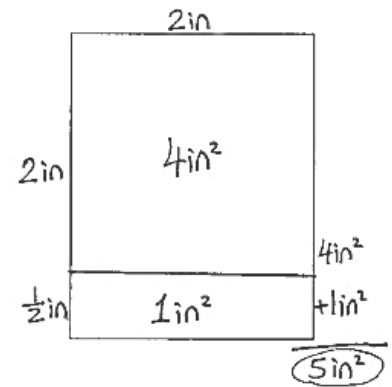
S: If this was just a 2-inch square, the area would be 4 square inches. It's a little longer than that, so it will be a little more than 4. → The longer side is between 2 and 3 inches, so the area should be somewhere between 4 square inches and 6 square inches.

T: Let's find the actual area. Decompose the longer side by marking the end of the 2 whole inches and labeling the wholes and the half inch on our rectangles. (Model on the board as shown.)

S: (Decompose and label.)

T: Now, let's use this decomposition to find the area of smaller parts of the rectangle. Using your ruler, draw a line separating the ones from the fractional units. (Model.)

S: (Separate the ones with a line.)



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Some students will benefit from drawing each square inch as a tile, connecting back to the tiling process. Others may need to use inch tile manipulatives to understand this process. (Remember that concrete materials should be foldable.)

Encourage students to return to pictorial or concrete representations as needed during any lesson to scaffold understanding.

- T: Now, let's multiply to find the areas of these sections. Let's start with the ones by ones part. Talk with your partner. What is the area of the part that is 2 inches by 2 inches? If it helps, imagine or draw tiles in your rectangle.
- S: There are 2 going across and 2 rows of them, so 4 altogether. → I remember that I can multiply the sides, so 2 inches \times 2 inches is 4 square inches.
- T: What is the area?
- S: 4 square inches.
- T: Record that.
- T: Turn and talk. What is the area of the smaller part? How do you know?
- S: Half of 2, so 1. → Two times $\frac{1}{2}$. Two halves make 1, so 1. → 1 square inch.
- T: Yes, the area is 1 square inch. Let's write that, too. (Model as shown in the image on the previous page.)
- T: What is the total area of the rectangle? Does our answer make sense?
- S: 5 square inches. → It makes sense because we said the area should be between 4 and 6 square inches, and it is.

Problem 1(b)

Project the second rectangle in the Problem Set.

- T: Measure the next rectangle with your ruler.
- T: What is the length?
- S: $1\frac{3}{4}$ inches.
- T: And the width?
- S: $1\frac{3}{4}$ inches. → This is a square, so the width is also $1\frac{3}{4}$ inches.
- T: Estimate the area with your partner.
- S: It's almost 2 inches by 2 inches. The area should be less than 4 square inches. → The area will be between 1 square inch and 4 square inches but closer to 4 because the sides are almost 2 inches long.
- T: Decompose the sides into ones and fractional parts, and record that on your Problem Set.

	1 in.	$\frac{3}{4}$ in.
1 in.	1 in^2	$\frac{3}{4} \text{ in}^2$
$\frac{3}{4}$ in.	$\frac{3}{4} \text{ in}^2$	$\frac{9}{16} \text{ in}^2$

$$\begin{aligned}
 &1 + \frac{3}{4} + \frac{3}{4} + \frac{9}{16} \\
 &= 1 + \frac{12}{16} + \frac{12}{16} + \frac{9}{16} \\
 &= 1 + \frac{33}{16} \\
 &= 3\frac{1}{16}
 \end{aligned}$$

Area is $3\frac{1}{16} \text{ in}^2$

Circulate and assist students. Then, project a student's work, or record on the board as shown.

- T: Work with your partner to find the area of each of these four parts.
- S: (Find the area of each of the four parts.)
- T: What is the area of the part that is 1 inch on each side?
- S: 1 square inch.



**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

For students who need to review fraction multiplication, model the shaded area models from Module 4 to show a fraction times a fraction or a fraction of a fraction.

- T: Then, we have two parts with 1 inch on one side and $\frac{3}{4}$ inch on the other. What is the area of each of those parts? How do you know?
- S: It is not a whole square inch. → A whole tile would not fit in either of these places. We would have to fold it to make it fit. → Three-fourths of a square inch because 3 fourths times 1 is 3 fourths.
- T: (Record the measurements in each part of the area model.) Now we are left with the last little square. It is $\frac{3}{4}$ of an inch on each side. Is this area greater or less than the other parts? How do you know?
- S: It is smaller because both sides are shorter than the other parts. → It is only part of an inch on each side, so it will be less area. → The area is a fraction of a fraction. We want 3 fourths of 3 fourths. It is a fraction of an inch on each side. Three-fourths of a square inch would be like splitting a whole into 4 parts and taking 1 part off.
- T: What do we need to do to find the area of this last section of our square?
- S: Just like before, we need to multiply the length times the width. → We need to multiply $\frac{3}{4}$ by $\frac{3}{4}$.
- T: What is the area of the small square?
- S: $\frac{9}{16}$ square inch.
- T: How will we find the total area?
- S: Add all the parts. → Add across each row, and then add the rows together.

Circulate and support students as they add the partial products. Review the need for common denominators as necessary.

T: What is the total area of the square?

S: $3\frac{1}{16}$ square inches!

Repeat this sequence of questioning with each problem as necessary. As students understand the concept, release them to work independently.

Problem Set (10 minutes)

Students should do their personal best to complete the remainder of the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Handwritten student work showing four area models (a, b, c, d) for finding the area of rectangles with fractional side lengths. Each model is divided into smaller squares, and the area is calculated by summing the areas of the parts.

Model a: A rectangle with length 2 in and width $1\frac{1}{2}$ in. It is divided into a 2×1 section (area 4 in^2) and a $2 \times \frac{1}{2}$ section (area 1 in^2). Total area $A = 5 \text{ in}^2$.

Model b: A rectangle with length $1\frac{1}{4}$ in and width $3\frac{3}{4}$ in. It is divided into a 1×3 section (area 3 in^2), a $1 \times \frac{3}{4}$ section (area $\frac{3}{4} \text{ in}^2$), a $\frac{1}{4} \times 3$ section (area $\frac{3}{4} \text{ in}^2$), and a $\frac{1}{4} \times \frac{3}{4}$ section (area $\frac{9}{16} \text{ in}^2$). Total area $A = 3\frac{3}{16} \text{ in}^2$.

Model c: A rectangle with length 2 in and width $2\frac{3}{4}$ in. It is divided into a 2×2 section (area 4 in^2) and a $2 \times \frac{3}{4}$ section (area $1\frac{3}{2} \text{ in}^2$). Total area $A = 5\frac{1}{2} \text{ in}^2$.

Model d: A rectangle with length $2\frac{1}{4}$ in and width $3\frac{1}{4}$ in. It is divided into a 2×3 section (area 6 in^2), a $2 \times \frac{1}{4}$ section (area $\frac{1}{2} \text{ in}^2$), a $\frac{1}{4} \times 3$ section (area $\frac{3}{4} \text{ in}^2$), and a $\frac{1}{4} \times \frac{1}{4}$ section (area $\frac{1}{16} \text{ in}^2$). Total area $A = 7\frac{5}{16} \text{ in}^2$.

Student Debrief (10 minutes)

Lesson Objective: Measure to find the area of rectangles with fractional side lengths.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Look back at the area model we did together in Problem 1(b) ($1\frac{3}{4} \times 1\frac{3}{4}$). How many squares do you see in your area model? What patterns do you see whenever you have an area model of a square?
- What is the relationship between Problem 1(e) and Problem 1(f) in the Problem Set? (Both rectangles have the same area. The length of Problem 1(f) is 5 times the length of Problem 1(e). The width of Problem 1(f) is one-fifth the width of Problem 1(e).)
- Using mental math, how can you find $\frac{1}{2}$ times any fraction? (Double the denominator.)
- How is Problem 2(b) like the example we did together, $1\frac{3}{4} \times 1\frac{3}{4}$? (Both have two factors that are the same.)

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

2. Find the area of rectangles with the following dimensions. Explain your thinking using the area model.

a. $1\frac{3}{4} \text{ ft} \times 1\frac{3}{4} \text{ ft}$
 $A = 1\frac{7}{8} \text{ ft}^2$

b. $1\frac{3}{4} \text{ yd} \times 1\frac{3}{4} \text{ yd}$
 $A = 1\frac{7}{8} \text{ yd}^2$

c. $2\frac{1}{4} \text{ yd} \times 1\frac{1}{2} \text{ yd}$
 $A = 2\frac{1}{4} \text{ yd}^2$

d. $1\frac{1}{4} \text{ yd} \times 1\frac{1}{2} \text{ yd}$
 $A = 2\frac{1}{4} \text{ yd}^2$

3. Hanley is putting carpet in her house. She wants to carpet her living room, which measures $15 \text{ ft} \times 12\frac{1}{2} \text{ ft}$. She also wants to carpet her dining room, which is $10\frac{1}{4} \text{ ft} \times 10\frac{1}{4} \text{ ft}$. How many square feet of carpet will she need to cover both rooms?

a. What is the area of the living room?
 $A = 187\frac{1}{2} \text{ ft}^2$

b. What is the area of the dining room?
 $A = 105\frac{1}{4} \text{ ft}^2$

c. What is the total area of both rooms?
 $A = 292\frac{3}{4} \text{ ft}^2$

d. Fred cut a $3\frac{1}{4}$ inch square of construction paper for an art project. He cut a square from the edge of the big rectangle whose sides measured $3\frac{1}{4}$ inches. (See the picture below.)

e. What is the area of the smaller square that Fred cut out?
 $A = 10\frac{9}{16} \text{ in}^2$

f. What is the area of the remaining paper?
 $A = 84\frac{1}{2} \text{ in}^2$

Name _____

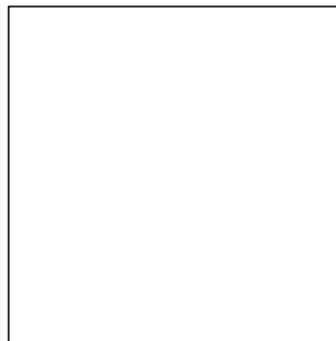
Date _____

1. Measure each rectangle to the nearest $\frac{1}{4}$ inch with your ruler, and label the dimensions. Use the area model to find each area.

a.



b.



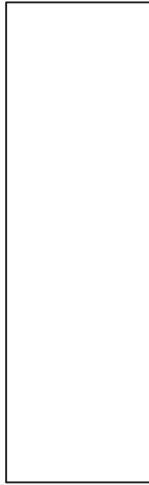
c.



d.



e.



f.



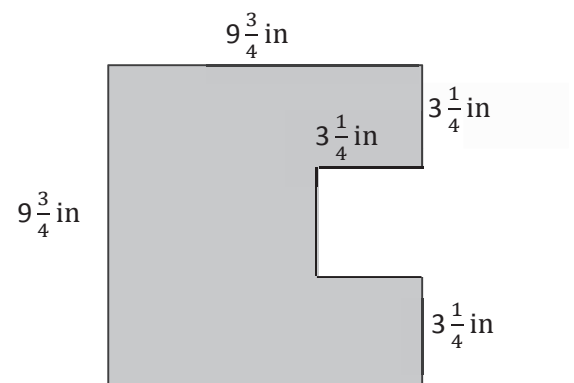
2. Find the area of rectangles with the following dimensions. Explain your thinking using the area model.

a. $1 \text{ ft} \times 1\frac{1}{2} \text{ ft}$

b. $1\frac{1}{2} \text{ yd} \times 1\frac{1}{2} \text{ yd}$

c. $2\frac{1}{2} \text{ yd} \times 1\frac{3}{16} \text{ yd}$

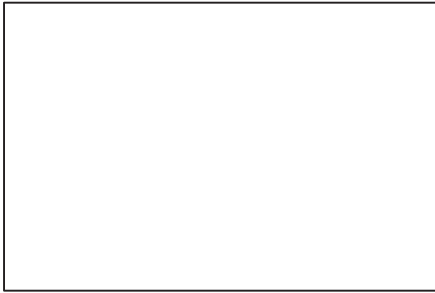
3. Hanley is putting carpet in her house. She wants to carpet her living room, which measures $15 \text{ ft} \times 12\frac{1}{3} \text{ ft}$. She also wants to carpet her dining room, which is $10\frac{1}{4} \text{ ft} \times 10\frac{1}{3} \text{ ft}$. How many square feet of carpet will she need to cover both rooms?
4. Fred cut a $9\frac{3}{4}$ -inch square of construction paper for an art project. He cut a square from the edge of the big rectangle whose sides measured $3\frac{1}{4}$ inches. (See the picture below.)
- a. What is the area of the smaller square that Fred cut out?
- b. What is the area of the remaining paper?



Name _____

Date _____

Measure the rectangle to the nearest $\frac{1}{4}$ inch with your ruler, and label the dimensions. Find the area.

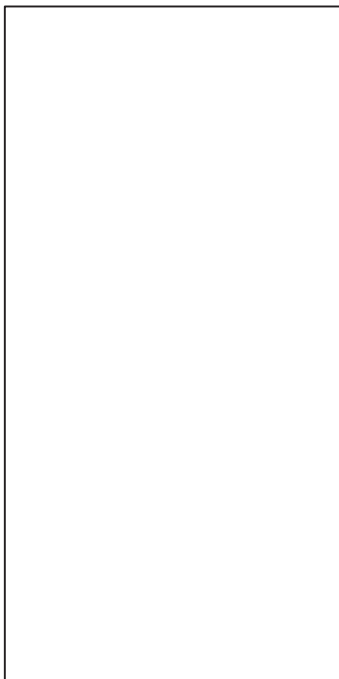


Name _____

Date _____

1. Measure each rectangle to the nearest $\frac{1}{4}$ inch with your ruler, and label the dimensions. Use the area model to find the area.

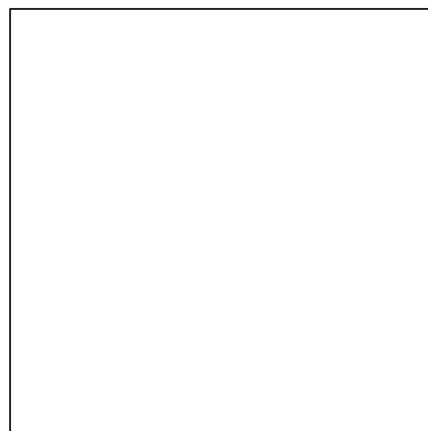
a.



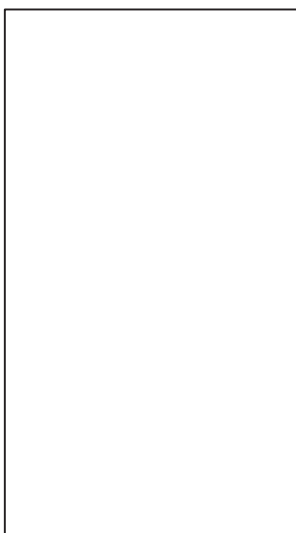
b.



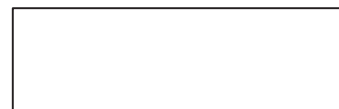
c.



d.



e.



2. Find the area of rectangles with the following dimensions. Explain your thinking using the area model.

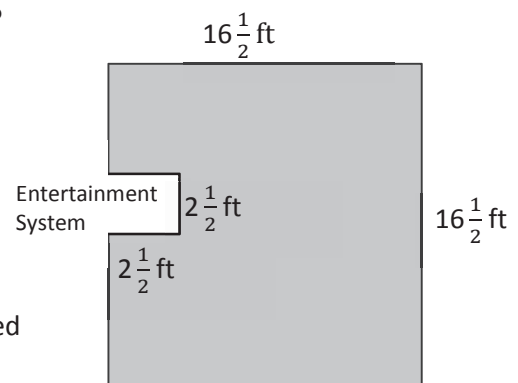
a. $2\frac{1}{4}\text{ yd} \times \frac{1}{4}\text{ yd}$

b. $2\frac{1}{2}\text{ ft} \times 1\frac{1}{4}\text{ ft}$

3. Kelly buys a tarp to cover the area under her tent. The tent is 4 feet wide and has an area of 31 square feet. The tarp she bought is $5\frac{1}{3}$ feet by $5\frac{3}{4}$ feet. Can the tarp cover the area under Kelly's tent? Draw a model to show your thinking.

4. Shannon and Leslie want to carpet a $16\frac{1}{2}$ -ft by $16\frac{1}{2}$ -ft square room. They cannot put carpet under an entertainment system that juts out. (See the drawing below.)

- a. In square feet, what is the area of the space with no carpet?



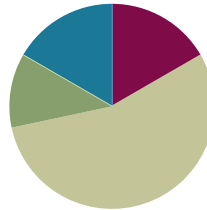
- b. How many square feet of carpet will Shannon and Leslie need to buy?

Lesson 13

Objective: Multiply mixed number factors, and relate to the distributive property and the area model.

Suggested Lesson Structure

■ Fluency Practice	(10 minutes)
■ Application Problem	(7 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Multiplying Fractions (4 minutes)
- Find the Volume (6 minutes)

Multiplying Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for today's lesson.

T: (Write $\frac{1}{3} \times \frac{1}{5} = \underline{\hspace{1cm}}$.) Say the complete multiplication number sentence..

S: $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$.

T: (Write $\frac{2}{3} \times \frac{2}{5} = \underline{\hspace{1cm}}$.) Say the complete multiplication number sentence.

S: $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$.

T: (Write $\frac{3}{4} \times \frac{2}{3} = \underline{\hspace{1cm}}$. Beneath it, write $= \underline{\hspace{1cm}}$.) On your personal white board, write the complete multiplication sentence. Then, simplify the fraction.

S: (Write $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$. Beneath it, write $= \frac{1}{2}$.)

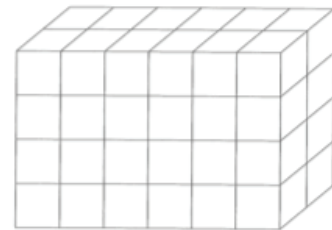
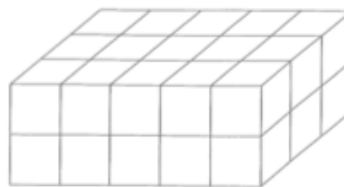
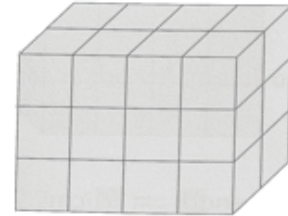
Continue with the following possible sequence: $\frac{1}{2} \times \frac{3}{4}$, $\frac{2}{3} \times \frac{3}{5}$, $\frac{3}{4} \times \frac{4}{5}$, $\frac{5}{6} \times \frac{3}{4}$, and $\frac{3}{5} \times \frac{5}{6}$.

Find the Volume (6 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews volume concepts and formulas.

- T: (Project a prism 4 units \times 2 units \times 3 units.
Write $V = \underline{\hspace{1cm}}$ units \times $\underline{\hspace{1cm}}$ units \times $\underline{\hspace{1cm}}$ units.) Find the volume.
- S: (Write $4 \text{ units} \times 2 \text{ units} \times 3 \text{ units} = 24 \text{ units}^3$.)
- T: How many layers of 6 cubes are in the prism?
- S: 4 layers.
- T: (Write $4 \times 6 \text{ units}^3$.) Four copies of 6 cubic units is...?
- S: 24 cubic units.
- T: How many layers of 8 cubes are there?
- S: 3 layers.
- T: (Write $3 \times 8 \text{ units}^3$.) Three copies of 8 cubic units is...?
- S: 24 cubic units.
- T: How many layers of 12 cubes are there?
- S: 2 layers.
- T: Write a multiplication equation to find the volume of the prism, starting with the number of layers.
- S: (Write $2 \times 12 \text{ units}^3 = 24 \text{ units}^3$.)



Repeat the process for the prisms pictured.

Application Problem (7 minutes)

The Colliers want to put new flooring in a $6\frac{1}{2}$ -foot by $7\frac{1}{3}$ -foot bathroom. The tiles they want come in 12-inch squares. What is the area of the bathroom floor? If the tiles cost \$3.25 per square foot, how much will they spend on the flooring?

Note: This type of tiling applies the work from Lessons 10–12 and bridges to today's lesson on the distributive property.

$$\begin{array}{r}
 7 + \frac{1}{3} \\
 6 \times 42 = 252 \\
 + \frac{1}{3} \times 2 = \frac{2}{3} \\
 \hline
 252 + \frac{2}{3} = 252\frac{2}{3}
 \end{array}
 \qquad
 \begin{array}{r}
 3.25 \\
 \times 48 \\
 \hline
 2600 \\
 + 13000 \\
 \hline
 156.00
 \end{array}$$

The area of the bathroom is $252\frac{2}{3} \text{ ft}^2$. Since they can't buy a partial tile, they need to buy 48 tiles. The flooring will cost \$156.

Concept Development (33 minutes)

Materials: (S) Personal white board

In this lesson, students reason about the most efficient strategy to use for multiplying mixed numbers: distributing with the area model or multiplying improper fractions and canceling to simplify.

Problem 1

Find the area of a rectangle $1\frac{1}{3}$ inches \times $3\frac{3}{4}$ inches, and discuss strategies for solving.

$1\frac{1}{3}$ in

T: (Project Rectangle 1.) How is this rectangle different from the rectangles we have been working with?

S: We know the dimensions of this one. → The side lengths are given to us, so we don't need to tile or measure.

T: Find the area of this rectangle. Use an area model to show your thinking.

S: (Find the area using a model.)

T: What is the area of this rectangle?

S: 5 inches squared.

T: We have used the area model many times in Grade 5 to help us multiply numbers with mixed units. How are these side lengths like multi-digit numbers? Turn and talk.

S: A two-digit number has two different size units in it. The ones are the smaller units, and the tens are the bigger units. These mixed numbers are like that. The ones are the bigger units, and the fractions are the smaller units.
→ Mixed numbers are another way to write decimals. Decimals have ones and fractions, and so do these.

T: (Point to the model and calculations.) When we add partial products, what property of multiplication are we using?

S: The distributive property.

T: Let's find the area of this rectangle again. This time, let's use a single unit to express each of the side lengths. What is $1\frac{1}{3}$ expressed in thirds?

S: 4 thirds.

T: (Record on the rectangle.) Express $3\frac{3}{4}$ using only fourths.

S: 15 fourths.

$3\frac{3}{4}$ in

$$\begin{aligned}
 & (1 \times 3) + \left(\frac{1}{3} \times 3\right) \\
 &= 3 + 1 \\
 &= 4 \\
 & \left(1 \times \frac{3}{4}\right) + \left(\frac{1}{3} \times \frac{3}{4}\right) \\
 &= \frac{3}{4} + \frac{1}{4} \\
 &= 1 \\
 & 4 + 1 = 5 \\
 & A = 5 \text{ square inches}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{4}{3} \times \frac{15}{4} \\
 &= \frac{4 \times 15}{3 \times 4} \\
 &= \frac{15}{3} \\
 &= 5 \\
 & A = 5 \text{ in}^2
 \end{aligned}$$

- T: (Record on the rectangle.) Multiply these fractions to find the area.
- S: (Multiply to find the area.)
- T: What is the area?
- S: 5 in^2 .
- T: Which strategy did you find to be more efficient? Why?
- S: I like the strategy of expressing the dimensions as a fraction greater than one. This way was a lot faster for me! \rightarrow These fractions were easy to simplify before I multiplied, so there were fewer calculations to do to find the area.
- T: Do you think it will always be true that multiplying the fractions will be the most efficient? Why or why not?
- S: This seems easier because it's multiplying whole numbers. \rightarrow I like the distributive property better because the numbers stay smaller doing one part at a time. \rightarrow I'm not sure—some larger mixed numbers might be a lot more challenging.
- T: There are lots of different viewpoints here. Let's try another example to test these strategies again.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Some students may need a quick refresher on changing mixed numbers to improper fractions or vice versa. Students should be reminded that a mixed number is an addition sentence. So, when converting to an improper fraction, the whole number can be expressed in the unit of the fractional part and then both like fractions added.

Problem 2

Determine when the distributive property or the multiplication of fractions is more efficient to solve for area.

- T: (Draw a rectangle with side lengths $16\frac{1}{2}$ inches and $4\frac{1}{4}$ inches.) Which strategy do you think might be more efficient to find the area of this rectangle? Turn and talk.
- S: The fractions are pretty easy, so I think the distributive property will be quicker. \rightarrow The numerators will be big. I think distributing will be easier. \rightarrow I like to simplify fractions, so I think it would be easier to work with improper fractions.
- T: Work with a partner to find the area of this rectangle. Partner A, use the distributive property with an area model. Partner B, express the sides using fractions greater than 1. (Allow students time to work.)
- T: What is the area? Which strategy was more efficient?
- S: The improper fractions were messy. When I converted to improper fractions, the numerators were 33 and 17, and there weren't any common factors to help me simplify. The area is $\frac{561}{8} \text{ in}^2$, which is right, but it's weird. I had to use long division to figure out that the area was $70\frac{1}{8}$ square inches. \rightarrow The distributive property was much easier on this one. The partial products were all easy to do in my head. I just added the sums of the rows and got $70\frac{1}{8}$ square inches.

Area Model:

16 in.	$\frac{1}{2}$ in.	
4 in.	64 in^2	2 in^2
$\frac{1}{4}$ in.	4 in^2	$\frac{1}{8} \text{ in}^2$
		66 in^2
		$70\frac{1}{8} \text{ in}^2$

Calculations:

$$16\frac{1}{2} \text{ in} \times 4\frac{1}{4} \text{ in} = \frac{33}{2} \text{ in} \times \frac{17}{4} \text{ in} = \frac{561}{8} \text{ in}^2 = 70\frac{1}{8} \text{ in}^2$$

Long Division:

$$\begin{array}{r} 70\frac{1}{8} \\ 8 \overline{) 561} \\ \underline{56} \\ 01 \\ \underline{0} \\ 1 \end{array}$$

T: Does the method that you choose matter? Why or why not? Turn and talk.

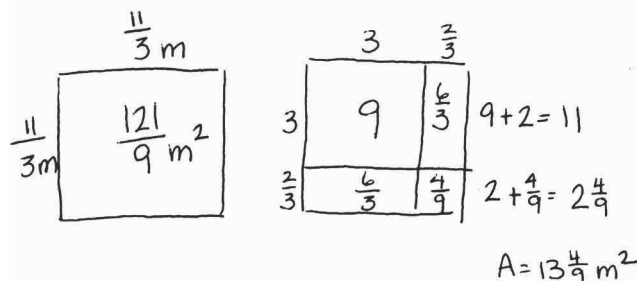
S: Either way, we got the right answer. → Depending on the numbers, sometimes distributing is easier, and sometimes just multiplying the improper fractions is easier.

Repeat the process to find the area of a square with side length $3\frac{2}{3}$ m.

T: When should you use each strategy?

Talk to your partner.

S: If the numbers are small, fraction multiplication might be better, especially if some factors can be simplified. → For large mixed numbers, I think the area model is easier, especially if some of the partial products are whole numbers or have common denominators. → You can always start with one strategy and change to the other if it gets too hard.



Problem 3

An 8-inch by 10-inch picture is resting on a mat. Three-fourths inch of the mat shows around the entire edge of the picture. Find the area of the mat not covered by the picture.

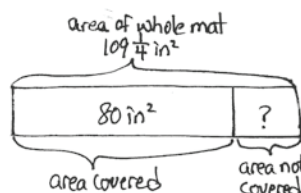
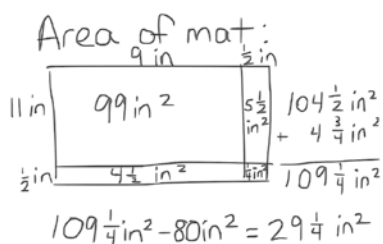
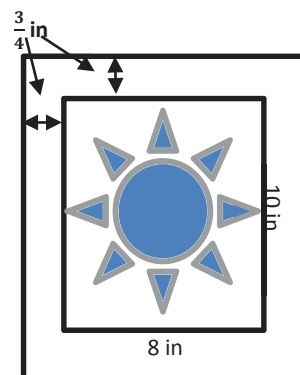
T: Compare this problem to others we have done. Turn and talk.

S: There are two rectangles to think about here. → We have to think about how to get just the part that is the mat—not the area of the whole thing. → It is a little bit of a mystery rectangle because they are asking about the mat. They gave us the measurements of the picture and only what we see of the mat.

T: Work with your partner, and use RDW to solve. (Allow students time to work.)

T: What did you think about to solve this problem?

S: I started by imagining the mat without the picture on top. I added the extra part of the mat ($1\frac{1}{2}$ inches) to the picture to find the length and width of the mat. Then, I multiplied and found the area of the mat. I subtracted the picture's area from the mat and got the answer. → I started to use improper fractions, but the numbers were really large, so I used the area model. → I used the area model for the mat's area because I saw the measurements were going to have fractions. Then, I just multiplied 8×10 to find the area of the picture. → After I figured out the area of the mat, I drew a tape diagram to show the part I knew and the part I needed to find. → I visualized 4 rectangles and then added their areas.



$109\frac{1}{4} - 80 = 29\frac{1}{4}$
The area of the mat not covered by the picture is $29\frac{1}{4} \text{ in}^2$.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Multiply mixed number factors, and relate to the distributive property and the area model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What are the strategies that we have used to find the area of a rectangle? Which one do you find the easiest? The most difficult? How do you decide which strategy you will use for a given problem? What kinds of things do you think about when deciding?
- In the Problem Set, when did you use the distributive property, and when did you multiply improper fractions? Why did you make those choices?
- How did you solve Problem 3?
- What are some situations in real life where finding the area of something would be needed or useful?

Name: Tien Date: _____

1. Find the area of the following rectangles. Draw an area model if it helps you.

a. $\frac{5}{4} \text{ km} \times \frac{12}{5} \text{ km}$
 $\frac{5}{4} \times \frac{12}{5} = \frac{60}{20} = 3$
 $A = 3 \text{ km}^2$

b. $16\frac{1}{2} \text{ m} \times 4\frac{1}{2} \text{ m}$

 $A = 69 \text{ m}^2$

c. $2\frac{1}{2} \text{ yd} \times 9\frac{1}{2} \text{ yd}$
 $2\frac{1}{2} \times 9\frac{1}{2} = \frac{5}{2} \times \frac{19}{2} = \frac{95}{2} = 47\frac{1}{2}$
 $A = 47\frac{1}{2} \text{ yd}^2$

d. $\frac{7}{8} \text{ mi} \times 4\frac{1}{2} \text{ mi}$
 $\frac{7}{8} \times \frac{9}{2} = \frac{63}{16} = 3\frac{15}{16}$
 $A = 3\frac{15}{16} \text{ mi}^2$

2. Julie is cutting rectangles out of fabric to make a quilt. If the rectangles are $2\frac{1}{2}$ inches wide and $3\frac{1}{5}$ inches long, what is the area of four such rectangles?

Area of 4 rect:
 $9\frac{3}{5} \text{ in}^2 \times 4$
 $= (9 \text{ in}^2 \times 4) + (\frac{3}{5} \text{ in}^2 \times 4)$
 $= 36 \text{ in}^2 + \frac{12}{5} \text{ in}^2$
 $= 37\frac{2}{5} \text{ in}^2$
 The area of 4 rectangles is $37\frac{2}{5} \text{ in}^2$.

3. Mr. Howard's pool is connected to his pool house by a sidewalk as shown. He wants to buy sod for the lawn, shown in gray. How much sod does he need to buy?

lawn: $24\frac{1}{2} \times 24\frac{1}{2}$
 $= (24 \times 24) + (\frac{1}{2} \times 24) + (24 \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2})$
 $= 576 + 12 + 12 + \frac{1}{4}$
 $= 600\frac{1}{4}$
 $A = 600\frac{1}{4} \text{ yd}^2$

pool house: $A = 16 \text{ yd}^2$

sidewalk: $A = 3 \text{ yd}^2$

pool: $2\frac{1}{2} \times 7\frac{1}{2}$
 $= (2 \times 7) + (\frac{1}{2} \times 7) + (2 \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2})$
 $= 14 + 3\frac{1}{2} + 1 + \frac{1}{4}$
 $= 18\frac{3}{4}$
 $A = 18\frac{3}{4} \text{ yd}^2$

Mr. Howard will need to buy $562\frac{1}{4} \text{ yd}^2$ of sod.

lawn = $600\frac{1}{4} \text{ yd}^2$

 $600\frac{1}{4} - 16 - 3 - 18\frac{3}{4}$
 $= 581\frac{1}{4} - 18\frac{3}{4}$
 $= 580\frac{3}{4} - 18\frac{3}{4}$
 $= 562\frac{1}{4} = 562\frac{1}{4}$

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____ Date _____

1. Find the area of the following rectangles. Draw an area model if it helps you.

a. $\frac{5}{4} \text{ km} \times \frac{12}{5} \text{ km}$

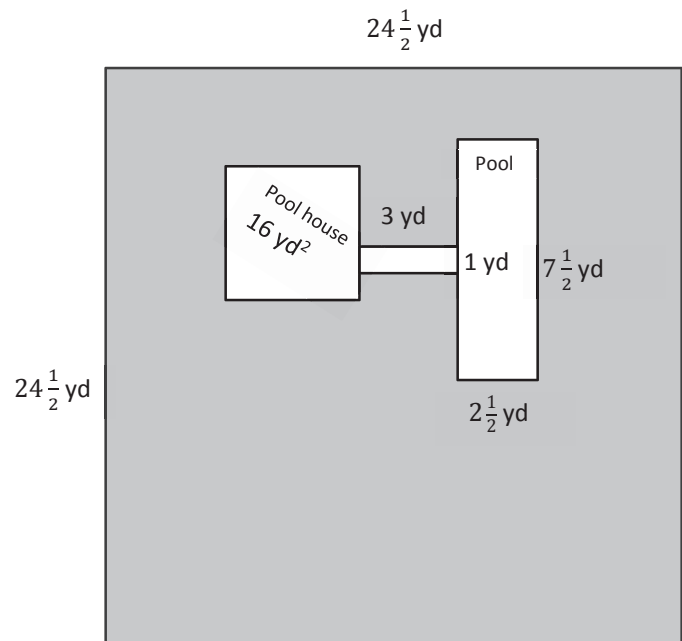
b. $16\frac{1}{2} \text{ m} \times 4\frac{1}{5} \text{ m}$

c. $4\frac{1}{3} \text{ yd} \times 5\frac{2}{3} \text{ yd}$

d. $\frac{7}{8} \text{ mi} \times 4\frac{1}{3} \text{ mi}$

2. Julie is cutting rectangles out of fabric to make a quilt. If the rectangles are $2\frac{3}{5}$ inches wide and $3\frac{2}{3}$ inches long, what is the area of four such rectangles?

3. Mr. Howard's pool is connected to his pool house by a sidewalk as shown. He wants to buy sod for the lawn, shown in gray. How much sod does he need to buy?



Name _____

Date _____

Find the area of the following rectangles. Draw an area model if it helps you.

1. $\frac{7}{2} \text{ mm} \times \frac{14}{5} \text{ mm}$

2. $5\frac{7}{8} \text{ km} \times \frac{18}{4} \text{ km}$

Name _____

Date _____

1. Find the area of the following rectangles. Draw an area model if it helps you.

a. $\frac{8}{3} \text{ cm} \times \frac{24}{4} \text{ cm}$

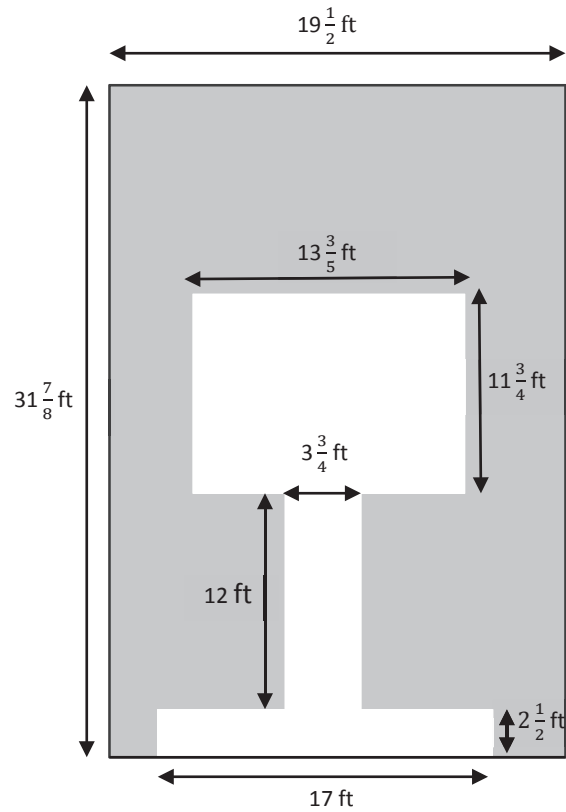
b. $\frac{32}{5} \text{ ft} \times 3\frac{3}{8} \text{ ft}$

c. $5\frac{4}{6} \text{ in} \times 4\frac{3}{5} \text{ in}$

d. $\frac{5}{7} \text{ m} \times 6\frac{3}{5} \text{ m}$

2. Chris is making a tabletop from some leftover tiles. He has 9 tiles that measure $3\frac{1}{8}$ inches long and $2\frac{3}{4}$ inches wide. What is the greatest area he can cover with these tiles?

3. A hotel is recarpeting a section of the lobby. Carpet covers the part of the floor as shown below in gray. How many square feet of carpeting will be needed?

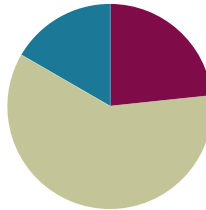


Lesson 14

Objective: Solve real-world problems involving area of figures with fractional side lengths using visual models and/or equations.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Concept Development	(38 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Multiply Fractions (4 minutes)
- Find the Volume (5 minutes)
- Physiometry (3 minutes)

Multiply Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 4 Lessons 13–16.

T: (Write $\frac{1}{2} \times \frac{1}{4} = \underline{\hspace{1cm}}$.) Complete the multiplication number sentence.

S: $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

Continue the process with $\frac{1}{2} \times \frac{1}{5}$ and $\frac{1}{2} \times \frac{1}{9}$.

T: (Write $\frac{1}{2} \times \frac{1}{8} = \underline{\hspace{1cm}}$.) On your personal white board, complete the number sentence.

S: (Write $\frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$.)

T: (Write $\frac{1}{2} \times \frac{5}{8} = \underline{\hspace{1cm}}$.) Say the complete multiplication sentence.

S: $\frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$.

Repeat the process with $\frac{1}{4} \times \frac{5}{5}$, $\frac{1}{3} \times \frac{9}{8}$, and $\frac{3}{4} \times \frac{1}{7}$.

T: (Write $\frac{3}{4} \times \frac{3}{5} = \underline{\hspace{1cm}}$.) Complete the multiplication sentence.

S: (Write $\frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$.)

Continue the process with $\frac{2}{3} \times \frac{3}{8}$.

T: (Write $\frac{1}{4} \times \frac{5}{8} = \underline{\hspace{1cm}}$.) Complete the number sentence.

S: (Write $\frac{1}{4} \times \frac{5}{8} = \frac{5}{32}$.)

T: (Write $\frac{2}{3} \times \frac{3}{2} = \underline{\hspace{1cm}}$.) Try this problem.

S: (Write $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$.)

Find the Volume (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews volume concepts and formulas.

T: (Project a prism 3 units \times 2 units \times 7 units.
Write $V = \underline{\hspace{1cm}}$ units \times $\underline{\hspace{1cm}}$ units \times $\underline{\hspace{1cm}}$ units.) Find the volume.

S: (Write 3 units \times 2 units \times 7 units = 42 units³.)

T: How many layers of 6 cubes are in the prism?

S: 7 layers.

T: Write a multiplication sentence to find the volume starting with the number of layers.

S: (Write $7 \times 6 \text{ units}^3 = 42 \text{ units}^3$.)

T: How many layers of 21 cubes are there?

S: 2 layers.

T: Write a multiplication sentence to find the volume starting with the number of layers.

S: (Write $2 \times 21 \text{ units}^3 = 42 \text{ units}^3$.)

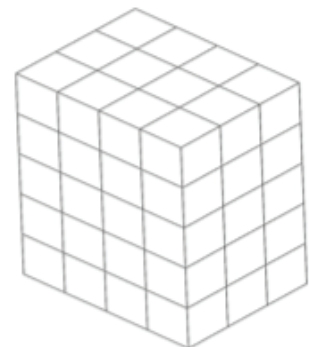
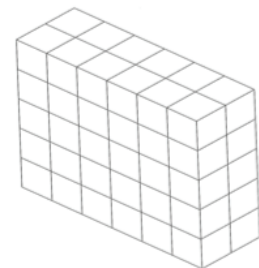
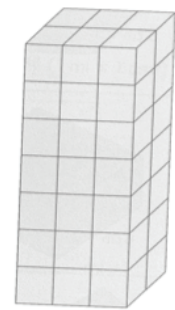
T: How many layers of 14 cubes are there?

S: 3 layers.

T: Write a multiplication sentence to find the volume starting with the number of layers.

S: (Write $3 \times 14 \text{ units}^3 = 42 \text{ units}^3$.)

Repeat the process for the other prisms.



Physiometry (3 minutes)

Materials: (S) Personal white board

Note: Kinesthetic memory is strong memory. This fluency activity prepares students for Lesson 16.

T: Stand up.

S: (Stand up.)

T: (Point at the side wall.) Point to the wall that runs parallel to the one I am pointing to.

S: (Point to the opposite wall.)

T: (Point to the back wall.)

S: (Point to the front wall.)

T: (Point to the side wall.)

S: (Point to the opposite side wall.)

T: (Point at the front wall.)

S: (Point at the back wall.)

T: (Stretch one arm up, directly at the ceiling. Stretch the other arm directly toward a wall, parallel to the floor.) What type of angle do you think I am modeling with my arms?

S: A right angle.

T: Model a right angle with your arms.

S: (Stretch one arm up, directly at the ceiling. Stretch another arm directly toward a wall, parallel to the floor.)

T: (Stretch the arm pointing toward a wall directly up toward the ceiling. Move the arm pointing toward the ceiling so that it points directly toward the opposite wall.) Model another right angle.

S: (Stretch the arm pointing toward a wall directly up toward the ceiling. Move the arm pointing toward the ceiling so that it points directly toward the opposite wall.)

Concept Development (38 minutes)

Materials: (S) Problem Set

Note: The Problem Set has been incorporated into the Concept Development. The problems in today's lesson can be time intensive. It may be that only two or three problems can be solved in the time allowed. Students will approach representing these problems from many perspectives. Allow students the flexibility to use the approach that makes the most sense to them.

Suggested Delivery of Instruction for Solving This Lesson's Word Problems**1. Model the problem.**

Have two pairs of students who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something? This may or may not be a tape diagram today. An area model may be more appropriate.
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give everyone two minutes to finish working on that question, sharing his work and thinking with a peer. All students should write their equations and statements of the answer.

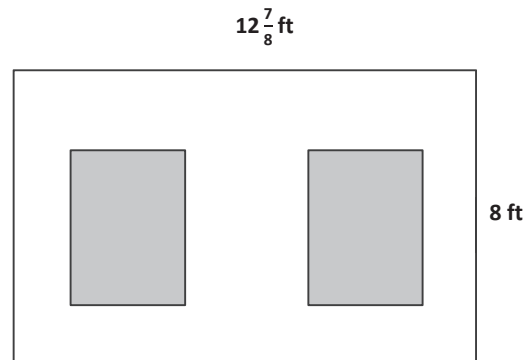
3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solutions.

Problem 1

George decided to paint a wall with two windows. Both windows are $3\frac{1}{2}$ -ft by $4\frac{1}{2}$ -ft rectangles. Find the area the paint needs to cover.

Students must keep track of three different areas to solve Problem 1. Using a part-whole tape diagram to represent these areas may be helpful to some students, while others may find using the area model to be more helpful. Students have choices in the strategy for computing the areas as well. Some may choose to use the distributive property. Others may choose to multiply improper fractions. Once students have solved, ask them to justify their choice of strategy. Were they able to tell which strategy to use from the beginning? Did they change direction once they began? If so, why? Flexibility in thinking about these types of problems should be a focus.



Area of wall

$3\frac{1}{2}$?
windows	paint

$$103 - 31\frac{1}{2} = 71\frac{1}{2}$$

Area to be painted is $71\frac{1}{2}\text{ft}^2$

Wall: $12 + \frac{7}{8}$

8	96	7	103ft^2
---	----	---	------------------

Window:

4	$\frac{1}{2}$
3	12
$\frac{1}{2}$	2

$$12 + 1\frac{1}{2} = 13\frac{1}{2}$$

$$2 + \frac{1}{4} = 2\frac{1}{4}$$

$$15\frac{3}{4}$$

$$15\frac{3}{4} \times 2 = 30\frac{6}{4}$$

window area = $31\frac{1}{2}\text{ft}^2$

Area of windows:

$$3\frac{1}{2} \times 4\frac{1}{2} = 12 + 2 + 1\frac{1}{2} + \frac{1}{4} = 15\frac{3}{4}$$

$$2 \times 15\frac{3}{4} = 30\frac{6}{4} = 31\frac{1}{2}$$

Area of wall:

$$8 \times 12\frac{7}{8} = 96 + 7 = 103$$

Area to paint:

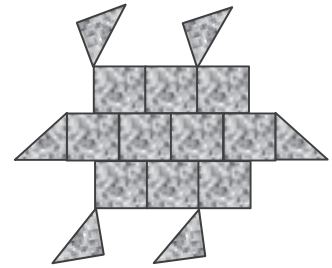
$$103 - 31\frac{1}{2} = 71\frac{1}{2}$$

The paint needs to cover $71\frac{1}{2}$ square feet.

Problem 2

Joe uses square tiles, some of which he cuts in half, to make the figure pictured to the right. If each square tile has a side length of $2\frac{1}{2}$ inches, what is the total area of the figure?

The presence of the triangles in the design may prove challenging for some students. Students who understand area as a procedure of multiplying sides—but do not understand the meaning of area—may need scaffolding to help them reason about mentally reassembling the 6 halves to find 3 whole tiles.



10 whole tiles + 6 half tiles = 13 tiles
 Area of a tile: $2\frac{1}{2}\text{in} \times 2\frac{1}{2}\text{in}$
 $13 \times 6\frac{1}{4}\text{in}^2 = 81\frac{1}{4}\text{in}^2$
 $\frac{13}{4} = 3\frac{1}{4}$
 The total area is $81\frac{1}{4}$ square inches.

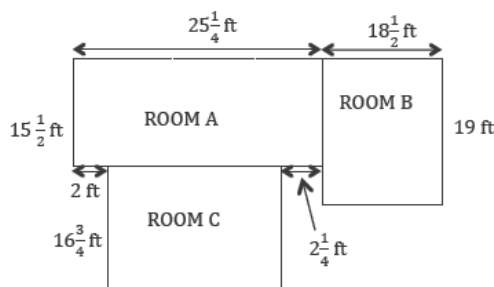


**NOTES ON
MULTIPLE MEANS
OF ACTION AND
EXPRESSION:**

If students struggle with this problem, give them 13 square units, and allow them to make designs with the tiles and find the areas. They quickly see that the layout of the tiles does not change the area the tiles cover. They can then re-create the design in Problem 2, physically reassembling the half tiles as necessary to reason about the wholes.

Problem 3

All-In-One Carpets is installing carpeting in three rooms. How many square feet of carpet are needed to carpet all three rooms?



**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

Problem 3 might be extended by inviting students to research actual carpet prices from local ads or the Internet and calculate what such a project might cost in real life. Comparison between the costs of using different types of flooring (hardwood versus carpet, for example) may also be made.

While this problem is a fairly straightforward, additive area problem, an added complexity occurs in finding the dimensions of Room C. The complexity of this problem also lies in the need to keep three different areas organized before finding the total area. Again, once students have had an opportunity to work through the protocol, discuss the pros and cons of various approaches, including the reasoning for their choice of strategy.

$$\begin{aligned} \text{Room A: } 25\frac{1}{4} \times 15\frac{1}{2} &= 375 + \frac{25}{2} + \frac{15}{4} + \frac{1}{8} \\ &= 375 + 12\frac{1}{2} + 3\frac{3}{4} + \frac{1}{8} \\ &= 390 + \frac{11}{8} \\ &= 391\frac{3}{8} \end{aligned}$$

$$\begin{array}{r} 25 \\ \times 15 \\ \hline 125 \\ + 250 \\ \hline 375 \end{array}$$

$$A_{\text{Room A}} = 391\frac{3}{8} \text{ ft}^2$$

$$\begin{aligned} \text{Room B: } 18\frac{1}{2} \times 19 &= 342 + \frac{19}{2} \\ &= 342 + 9\frac{1}{2} \\ &= 351\frac{1}{2} \end{aligned}$$

$$\begin{array}{r} 19 \\ \times 18 \\ \hline 152 \\ + 190 \\ \hline 342 \end{array}$$

$$A_{\text{Room B}} = 351\frac{1}{2} \text{ ft}^2$$

$$\begin{aligned} 391\frac{3}{8} + 351\frac{1}{2} + 351\frac{3}{4} \\ = 1,093 + \frac{3}{8} + \frac{4}{8} + \frac{6}{8} \end{aligned}$$

$$= 1,093 + \frac{13}{8}$$

$$= 1,094\frac{5}{8}$$

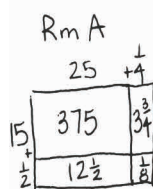
$1,094\frac{5}{8} \text{ ft}^2$ of carpet is needed.

$$\begin{aligned} \text{Room C: } 21 \times 16\frac{3}{4} &= 336 + \frac{21 \times 3}{4} \\ &= 336 + \frac{63}{4} \end{aligned}$$

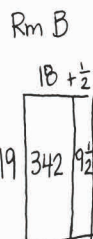
$$\begin{array}{r} 21 \\ \times 16 \\ \hline 126 \\ + 210 \\ \hline 336 \end{array}$$

$$\begin{aligned} &= 336 + 15\frac{3}{4} \\ &= 351\frac{3}{4} \end{aligned}$$

$$A_{\text{Room C}} = 351\frac{3}{4} \text{ ft}^2$$

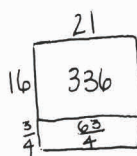


$$\begin{aligned} &375 + 12\frac{1}{2} \\ &= 375 + 12\frac{5}{8} \\ &= 390 + \frac{11}{8} \\ &= 391\frac{3}{8} \end{aligned}$$



$$= 351\frac{1}{2}$$

Rm C $25\frac{1}{4} - 4\frac{1}{4} = 21$



$$\begin{aligned} &= 336 + 15\frac{3}{4} \\ &= 351\frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{Total: } &391\frac{3}{8} \\ &351\frac{4}{8} \\ &+ 351\frac{6}{8} \\ &\hline &1093\frac{13}{8} \\ &= 1,094\frac{5}{8} \end{aligned}$$

All-in-One needs $1,094\frac{5}{8} \text{ ft}^2$ of carpet.

Problem 4

Mr. Johnson needs to buy sod for his front lawn.

- If the lawn measures $36\frac{2}{3}$ ft by $45\frac{1}{6}$ ft, how many square feet of sod will he need?
- If sod is only sold in whole square feet, how much will Mr. Johnson have to pay?

Sod Prices

Area	Price per Square Foot
First 1,000 sq ft	\$0.27
Next 500 sq ft	\$0.22
Additional square feet	\$0.19

The dimensions of the yard are larger than any others in the Problem Set to encourage students to use the distributive property to find the total area. Because the total area ($1,656\frac{1}{9}$ ft²) is numerically closer to 1,656, students may be tempted to round down. Reasoning about the $\frac{1}{9}$ ft² area can provide an opportunity to discuss the pros and cons of sodding that last fraction of a square foot. In the final component of the protocol, ask the following or similar questions:

- Is it worth the extra money for such a small amount of area left to cover? While 19 cents is a small cost, what if the sod had been more expensive?
- What if the costs had been structured so that the last whole square foot of sod had lowered the price of the entire amount?
- What could Mr. Johnson do with the other 8 ninths?

a. $36\frac{2}{3} \times 45\frac{1}{6}$

$$\begin{array}{r} 45 \\ \times 36 \\ \hline 270 \\ 1350 \\ \hline 1620 \end{array}$$

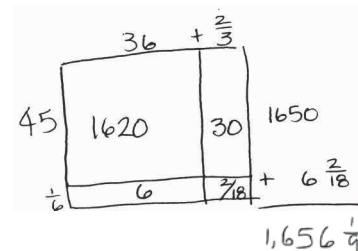
He needs $1,656\frac{1}{9}$ ft² of sod.

b. 1657 whole s.f.:

$$\begin{array}{l} 1000 \times \$0.27 = \$270.00 \\ 500 \times \$0.22 = \$110.00 \\ 157 \times \$0.19 = \$29.83 \\ \hline \$409.83 \end{array}$$

\$409.83

He will have to pay \$409.83.



Mr Johnson needs to buy 1657 sq.ft of sod and it will cost \$409.83.

Problem 5

Jennifer's class decides to make a quilt. Each of the 24 students will make a quilt square that is 8 inches on each side. When they sew the quilt together, every edge of each quilt square will lose $\frac{3}{4}$ of an inch.

- Draw one way the squares could be arranged to make a rectangular quilt. Then, find the perimeter of your arrangement.
- Find the area of the quilt.

There are many ways to lay out the quilt squares. Allow students to draw their layouts, and then compare the perimeters. Ask the following questions:

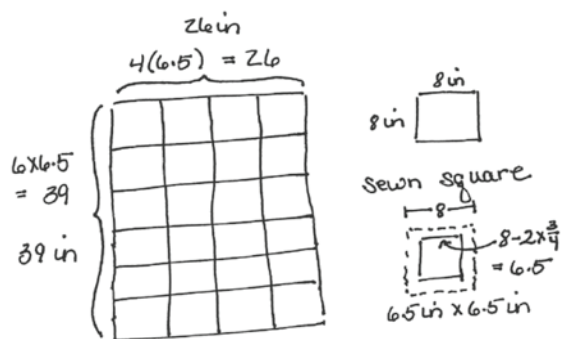
- Does the difference in perimeter affect the area? Why or why not?
- Are there advantages to one arrangement of the blocks over another (e.g., lowering the cost for an edging by minimizing the perimeter or fitting the dimensions of the quilt to a specific wall or bed size)?

Problem 5 harkens back to Problem 2 but with an added layer of complexity. Students might be asked to compare and contrast the two problems. In this problem, students must account for the seam allowances on all four sides of the quilt squares before finding the area. Students find that each quilt block becomes $42\frac{1}{4}$ -inches square after sewing and may simply multiply this area by 24.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

This problem may be extended for students who finish early. Ask them to find the arrangement that gives the largest perimeter and then the smallest. The problem can also be changed to having seams only between squares so there are three different square areas to calculate. Another extension could be offered by asking students to find the area of the seams. (Find the unfinished area of the 24 squares, and subtract the finished area.)



a. $8 \times 6\frac{1}{2}$

The perimeter of my arrangement is 143 inches.

$$\frac{3 \times 13}{2} = \frac{39}{2} = 19\frac{1}{2}$$

$$\frac{8 \times 13}{2} = \frac{104}{2} = 52$$

$$P = 19\frac{1}{2} + 19\frac{1}{2} + 52 + 52$$

$$= 39 + 104$$

$$= 143$$

b. Each square's area:

$$8 \text{ in} - 1\frac{1}{2} \text{ in} = 6\frac{1}{2} \text{ in}$$

All the squares:

$$42\frac{1}{4} \times 24$$

$$\frac{169}{4} \times \frac{24}{1}$$

$$= 1,014$$

The quilt's area is $1,014 \text{ in}^2$.

Student Debrief (10 minutes)

Lesson Objective: Solve real-world problems involving area of figures with fractional side lengths using visual models and/or equations.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Do these problems remind you of any others that we have seen in this module? In what ways are they like other problems? In what ways are they different?
- What did you learn from looking at your classmates' drawings? Did that support your understanding of the problems in a deeper way? When you checked for reasonableness, what process was used?
- When finding the areas, which strategy did you use more often—distribution or improper fractions? Is there a pattern to when you used which? How did you decide? What advice would you give a student who was not sure what to do?
- Which problems did you find the most difficult? Which one was easiest for you? Why?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name: Charles Date: _____

1. George decided to paint a wall with two windows. Both windows are $3\frac{1}{2}$ -ft by $4\frac{1}{2}$ -ft rectangles. Find the area the paint needs to cover.

Windows: $3\frac{1}{2} \times 4\frac{1}{2} = \frac{7}{2} \times \frac{9}{2} = \frac{63}{4} = 15\frac{3}{4}$
 $15\frac{3}{4} \times 2 = 30\frac{3}{4} = 31\frac{1}{2}$
 $A = 31\frac{1}{2} \text{ ft}^2$

Wall: $12\frac{7}{8} \times 8 = 96 + 7 = 103$
 $A = 103 \text{ ft}^2$

Paint: $103 - 31\frac{1}{2} = 71\frac{1}{2}$
The paint needs to cover $71\frac{1}{2} \text{ ft}^2$.

2. Joe uses square tiles, some of which he cuts in half, to make the figure below. If each square tile has a side length of $2\frac{1}{2}$ inches, what is the total area of the figure?

10 whole tiles + 6 half tiles = 13 whole tiles

Tile: $2\frac{1}{2} \times 2\frac{1}{2} = \frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = 6\frac{1}{4}$
 $A = 6\frac{1}{4} \text{ in}^2$

Figure: $13 \times 6\frac{1}{4} \text{ in}^2 = 78\frac{13}{4} \text{ in}^2 = 81\frac{1}{4} \text{ in}^2$
The total area of the figure is $81\frac{1}{4} \text{ in}^2$.

3. All-in-One Carpets is installing carpeting in three rooms. How many square feet of carpet are needed to carpet all three rooms?

Room A: $25\frac{1}{4} \times 15\frac{1}{2} = 391\frac{3}{8}$ $A = 391\frac{3}{8} \text{ ft}^2$
 Room B: $17 \times 18\frac{1}{2} = 313\frac{1}{2}$ $A = 313\frac{1}{2} \text{ ft}^2$
 Room C: $21 \times 16\frac{3}{4} = 351\frac{3}{4}$ $A = 351\frac{3}{4} \text{ ft}^2$
 $A + B + C = 391\frac{3}{8} + 313\frac{1}{2} + 351\frac{3}{4} = 1,093 + \frac{13}{8} = 1,094\frac{1}{8}$
1,094 $\frac{1}{8}$ ft² are needed to carpet all 3 rooms.

4. Mr. Johnson needs to buy sod for his front lawn.

a. If the lawn measures $36\frac{3}{4}$ ft by $45\frac{1}{2}$ ft, how many square feet of sod will he need?

$36\frac{3}{4} \times 45\frac{1}{2} = (36 \times 45) + (\frac{3}{4} \times 45) + (36 \times \frac{1}{2}) + (\frac{3}{4} \times \frac{1}{2})$
 $= 1,620 + \frac{27}{2} + 18 + \frac{3}{8} = 1,620 + 13\frac{1}{2} + 18 + \frac{3}{8}$
 $= 1,656\frac{1}{4}$
He needs $1,656\frac{1}{4} \text{ ft}^2$ of sod.

b. If sod is only sold in whole square feet, how much will Mr. Johnson have to pay?

1,657 whole feet needed.

1,000 \times \$0.21 = \$210.00
 500 \times \$0.22 = \$110.00
 157 \times \$0.19 = \$29.83
\$409.83

Area	Price per Square Foot
First 1,000 sq ft	\$0.21
Next 500 sq ft	\$0.22
Additional square feet	\$0.19

He will have to pay \$409.83

5. Jennifer's class decides to make a quilt. Each of the 24 students will make a quilt square that is 8 inches on each side. When they sew the quilt together, every edge of each quilt square will use $\frac{1}{4}$ of an inch.

a. Draw one way the squares could be arranged to make a rectangular quilt. Then, find the perimeter of your arrangement.

8 in

8 \times $6\frac{1}{2}$ in

3 \times $4\frac{1}{2}$ in

8 \times $6\frac{1}{2}$ = $48 + 4 = 52$
 3 \times $4\frac{1}{2}$ = $18 + 1\frac{1}{2} = 19\frac{1}{2}$
 $P = 52 + 52 + 19\frac{1}{2} + 19\frac{1}{2}$
 $= 104 + 39 = 143$
The perimeter of this arrangement is 143 in.

b. Find the area of the quilt.

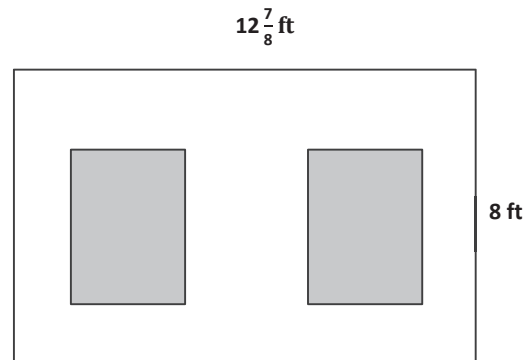
quilt square: $6\frac{1}{2} \times 6\frac{1}{2}$
 $= (6 \times 6) + (\frac{1}{2} \times 6) + (6 \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2})$
 $= 36 + 3 + 3 + \frac{1}{4}$
 $= 42\frac{1}{4}$ $A = 42\frac{1}{4} \text{ in}^2$

quilt: $24 \times 42\frac{1}{4} \text{ in}^2$
 $= (24 \times 42) + (24 \times \frac{1}{4})$
 $= 1,008 + 6 = 1,014$
The quilt's area is $1,014 \text{ in}^2$.

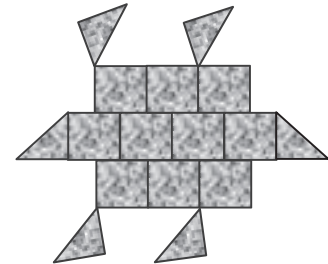
Name _____

Date _____

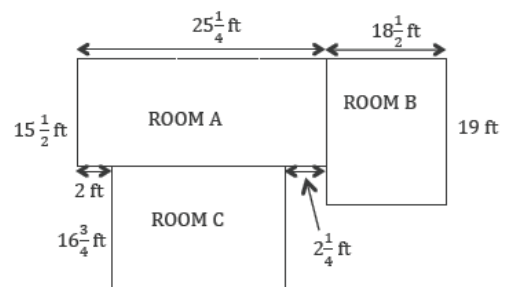
1. George decided to paint a wall with two windows. Both windows are $3\frac{1}{2}$ -ft by $4\frac{1}{2}$ -ft rectangles. Find the area the paint needs to cover.



2. Joe uses square tiles, some of which he cuts in half, to make the figure below. If each square tile has a side length of $2\frac{1}{2}$ inches, what is the total area of the figure?



3. All-In-One Carpets is installing carpeting in three rooms. How many square feet of carpet are needed to carpet all three rooms?



4. Mr. Johnson needs to buy sod for his front lawn.
- a. If the lawn measures $36\frac{2}{3}$ ft by $45\frac{1}{6}$ ft, how many square feet of sod will he need?
- b. If sod is only sold in whole square feet, how much will Mr. Johnson have to pay?

Sod Prices

Area	Price per Square Foot
First 1,000 sq ft	\$0.27
Next 500 sq ft	\$0.22
Additional square feet	\$0.19

5. Jennifer's class decides to make a quilt. Each of the 24 students will make a quilt square that is 8 inches on each side. When they sew the quilt together, every edge of each quilt square will lose $\frac{3}{4}$ of an inch.
- a. Draw one way the squares could be arranged to make a rectangular quilt. Then, find the perimeter of your arrangement.
- b. Find the area of the quilt.

Name _____

Date _____

Mr. Klimek made his wife a rectangular vegetable garden. The width is $5\frac{3}{4}$ ft, and the length is $9\frac{4}{5}$ ft. What is the area of the garden?

3. A-Plus Glass is making windows for a new house that is being built. The box shows the list of sizes they must make.

15 windows $4\frac{3}{4}$ -ft long and $3\frac{3}{5}$ -ft wide

7 windows $2\frac{4}{5}$ -ft wide and $6\frac{1}{2}$ -ft long

How many square feet of glass will they need?

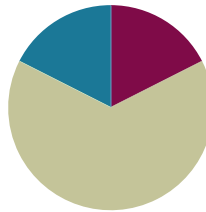
4. Mr. Johnson needs to buy seed for his backyard lawn.
- If the lawn measures $40\frac{4}{5}$ ft by $50\frac{7}{8}$ ft, how many square feet of seed will he need to cover the entire area?
 - One bag of seed will cover 500 square feet if he sets his seed spreader to its highest setting and 300 square feet if he sets the spreader to its lowest setting. How many bags of seed will he need if he uses the highest setting? The lowest setting?

Lesson 15

Objective: Solve real-world problems involving area of figures with fractional side lengths using visual models and/or equations.

Suggested Lesson Structure

■ Fluency Practice	(10 minutes)
■ Concept Development	(40 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Divide Whole Numbers by Unit Fractions and Unit Fractions by Whole Numbers (6 minutes)
- Quadrilaterals (4 minutes)

Divide Whole Numbers by Unit Fractions and Unit Fractions by Whole Numbers (6 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 4.

T: (Write $1 \div \frac{1}{2}$.) Say the division expression.

S: $1 \div \frac{1}{2}$.

T: How many halves are in 1?

S: 2.

T: (Write $1 \div \frac{1}{2} = 2$. Beneath it, write $2 \div \frac{1}{2}$.) How many halves are in 2?

S: 4.

T: (Write $2 \div \frac{1}{2} = 4$. Beneath it, write $3 \div \frac{1}{2}$.) How many halves are in 3?

S: 6.

T: (Write $3 \div \frac{1}{2} = 6$. Beneath it, write $7 \div \frac{1}{2}$.) On your personal white board, write the complete division sentence.

S: (Write $7 \div \frac{1}{2} = 14$.)

Continue with the following possible sequence: $1 \div \frac{1}{4}$, $2 \div \frac{1}{4}$, $9 \div \frac{1}{4}$, and $3 \div \frac{1}{5}$.

T: (Write $\frac{1}{2} \div 2 = \underline{\hspace{1cm}}$.) Say the complete division sentence.

S: $\frac{1}{2} \div 2 = \frac{1}{4}$.

T: (Write $\frac{1}{2} \div 2 = \frac{1}{4}$. Beneath it, write $\frac{1}{2} \div 3 = \underline{\hspace{1cm}}$.) Say the complete division sentence.

S: $\frac{1}{2} \div 3 = \frac{1}{6}$.

T: (Write $\frac{1}{2} \div 3 = \frac{1}{6}$. Beneath it, write $\frac{1}{2} \div 4 = \underline{\hspace{1cm}}$.) Say the complete division sentence.

S: $\frac{1}{2} \div 4 = \frac{1}{8}$.

T: (Write $\frac{1}{2} \div 9 = \underline{\hspace{1cm}}$.) On your personal white board, write the complete division sentence.

S: (Write $\frac{1}{2} \div 9 = \frac{1}{18}$.)

Continue with the following possible sequence: $\frac{1}{5} \div 2$, $\frac{1}{5} \div 3$, $\frac{1}{5} \div 5$, and $\frac{1}{8} \div 4$.

Quadrilaterals (4 minutes)

Materials: (T) Shape sheet (Template)

Note: This fluency activity reviews Grade 3 geometry concepts in anticipation of Module 6 content. The sheet can be duplicated if preferred.

T: (Project the shape sheet template and the list of attributes.) Take one minute to discuss the attributes of the shapes you see. You can use the list to support you.

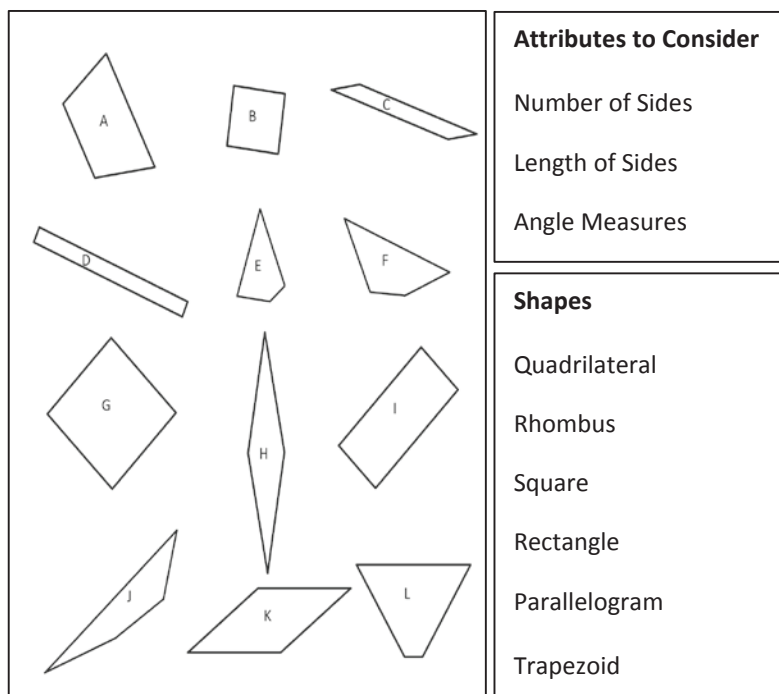
S: Some have right angles. → All have straight sides. → They all have four sides. → B and G, and maybe H and K, have all equal sides. I am not really sure.

T: If we wanted to verify whether the sides are equal, what could we do?

S: Measure with a ruler.

T: What about the angles? How could you verify that they are right angles?

S: I could compare it to something that I know is a right angle. → I could use a set square. → I could use a protractor to measure.



- T: Now, look at the shape names. Determine which shapes might fall into each category. (Post the shape names.)
- S: B and G might be squares. → All of them are quadrilaterals. → H and K might be rhombuses. It is hard to know if their sides are equal. → D and I are rectangles. Oh yeah, and B and G are, too. → L and A look like trapezoids.
- T: Which are quadrilaterals?
- S: All of them.
- T: Which shapes appear to be rectangles?
- S: B, D, G, and I.
- T: Which appear to have opposite sides of equal length but are not rectangles?
- S: C, H, and K. → A and L have one pair of opposite sides that look the same.
- T: Squares are rhombuses with right angles. Do you see any other shapes that might have four equal sides without right angles?
- S: H and K.

Concept Development (40 minutes)

Materials: (S) Problem Set

Note: The Problem Set has been incorporated into the Concept Development. While there are only four problems, most are multi-step and require time to solve.

Suggested Delivery of Instruction for Solving This Lesson's Word Problems

1. Model the problem.

Have two pairs of students who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something? This might be a tape diagram or an area model.
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.


Give everyone two minutes to finish working on that question, sharing her work and thinking with a peer. All students should write their equations and statements of the answer.

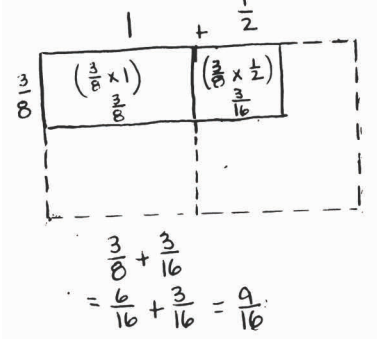
3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solutions.

Problem 1

The length of a flowerbed is 4 times as long as its width. If the width is $\frac{3}{8}$ meter, what is the area?

width $\boxed{\frac{3}{8}}$
 length 
 $\text{length} = \frac{3}{8} \text{ m} \times 4 = \frac{12}{8} \text{ m} = \frac{3}{2} \text{ m}$
 $\text{area} = \frac{3}{8} \text{ m} \times \frac{3}{2} \text{ m} = \frac{9}{16} \text{ m}^2$

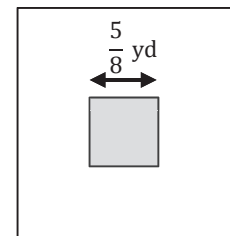

 $\frac{3}{8} + \frac{3}{16}$
 $= \frac{6}{16} + \frac{3}{16} = \frac{9}{16}$
 The flowerbed's area is $\frac{9}{16} \text{ m}^2$.

While this problem is quite simple to calculate, two complexities must be navigated. First, the length is not given. Second, the resulting area is less than a whole square meter. Once students have arrived at a solution, ask if their results make sense and why. If students need support, discuss what this might look like if the flowerbed was tiled with 1-meter square tiles. The length of the bed necessitates that 2 whole tiles be used. (How is it that the area is less than 1?) Students might draw or represent the problem with concrete materials to explain their thinking. The folding for these units may prove challenging but worthwhile because these types of problems are often done procedurally by students rather than with a deep understanding of what their answers represent. As in Lesson 14, continue to have students explain their choice of strategy in terms of efficiency. As students share their approaches with the class, encourage those who had difficulty to ask how the presenters got started with their drawings and calculations. Also, encourage students to explain any false starts they experienced when solving and how and why their thinking changed.

Problem 2

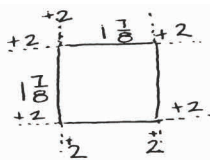
Mrs. Johnson grows herbs in square plots. Her basil plot measures $\frac{5}{8}$ yd on each side.

- Find the total area of the basil plot.
- Mrs. Johnson puts a fence around the basil. If the fence is 2 ft from the edge of the garden on each side, what is the perimeter of the fence in feet?
- What is the total area, in square feet, that the fence encloses?



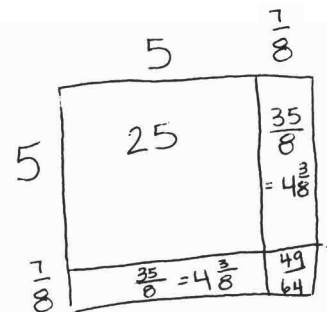
a. $\frac{5}{8} \text{ yd}$
 $\frac{5}{8} \text{ yd}$ 
 The total area of the basil plot is $\frac{25}{64} \text{ yd}^2$.

b. $\frac{5}{8} \text{ yd} = \frac{5}{8} \times 1 \text{ yd}$
 $= \frac{5}{8} \times 3 \text{ ft}$
 $= \frac{15}{8} \text{ ft}$
 $= 1\frac{7}{8} \text{ ft}$



$$1\frac{7}{8} \text{ ft} \times 4 = 20\frac{28}{8} \text{ ft} = 23\frac{1}{2} \text{ ft}$$

The perimeter of the fence is $23\frac{1}{2} \text{ ft}$.

c. 

$$25 + 4\frac{3}{8} + 4\frac{3}{8} + \frac{1}{64} = 34\frac{33}{64} \text{ ft}^2$$

The fenced area is $34\frac{33}{64} \text{ ft}^2$.
 That's a little more than $34\frac{1}{2} \text{ ft}^2$.

As in Problem 1, the fraction multiplication involved in completing Part (a) is not rigorous. However, this problem offers an opportunity to explore the relationships of square yards to square feet and the importance of understanding the actual size of such units. The expression of the area as $\frac{25}{64} \text{ yd}^2$ may be conceptually challenging for students. They might be encouraged to relate this to a benchmark of 1 half or 1 third square yard (which is 3 square feet). Students might be asked to show what a tiling of this garden plot would look like.

It may even be helpful to use yardsticks to show the actual size of the herb plot. The area expressed as a bit more than 5 square feet may be surprising to students. Help students make connections to the shading models of Module 4 and how the representation of the area model for the basil plot compares and contrasts to the representation of fraction multiplication. Part (b) offers a bit of complexity in that the dimensions of the garden are given in yards, yet the distance from the garden to the fence is given in feet. Part (c) requires that students use the fence measurements to find the total area enclosed by the fence. Multiple methods may be used to accomplish this.



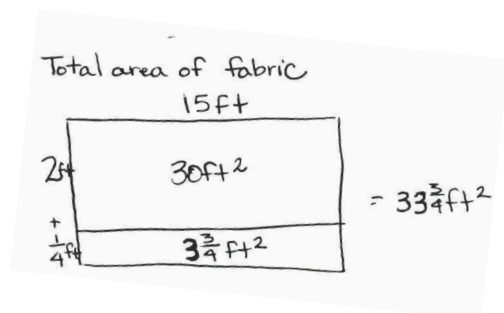
NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Problem 2 may be extended by having students convert the fence perimeter back into yards and the fenced area back into square yards. The understanding that $1 \text{ yd}^2 = 9 \text{ ft}^2$ makes this an interesting challenge.

Problem 3

Janet bought 5 yards of fabric $2\frac{1}{4}$ -feet wide to make curtains. She used $\frac{1}{3}$ of the fabric to make a long set of curtains and the rest to make 4 short sets.

- Find the area of the fabric she used for the long set of curtains.
- Find the area of the fabric she used for each of the short sets.



$$\begin{aligned} \text{a. } 33\frac{3}{4} \times \frac{1}{3} \\ &= (33 \times \frac{1}{3}) + (\frac{3}{4} \times \frac{1}{3}) \\ &= 11 + \frac{1}{4} \\ &= 11\frac{1}{4} \end{aligned}$$

The area of the long curtain is $11\frac{1}{4} \text{ ft}^2$

$$\begin{aligned} \text{b. } 33\frac{3}{4} - 11\frac{1}{4} &= 22\frac{2}{2} \\ 22\frac{2}{2} \times \frac{1}{4} &= (22 \times \frac{1}{4}) + (\frac{2}{2} \times \frac{1}{4}) \\ &= 5\frac{1}{2} + \frac{1}{8} \\ &= 5\frac{5}{8} \end{aligned}$$

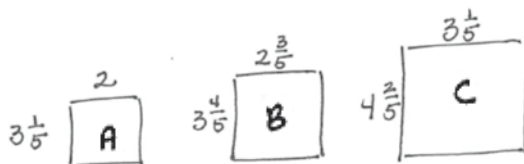
The area of each set of short curtains is $5\frac{5}{8} \text{ ft}^2$

As in Problem 2, there are different units within a multi-step problem. After students have solved the problem, allow them to share whether they converted both measurements to feet or yards and the advantages and disadvantages of both. A discussion of the relationship of the square yards to the square feet may also be fruitful. Discuss the various strategies students may have used to find the fabric left for the shorter curtains.

Problem 4

Some wire is used to make 3 rectangles: A, B, and C. Rectangle B's dimensions are $\frac{3}{5}$ cm larger than Rectangle A's dimensions, and Rectangle C's dimensions are $\frac{3}{5}$ cm larger than Rectangle B's dimensions. Rectangle A is 2 cm by $3\frac{1}{5}$ cm.

- What is the total area of all three rectangles?
- If a 40-cm coil of wire was used to form the rectangles, how much wire is left?



$$\begin{aligned} \text{Area of A: } 2 \times 3\frac{1}{5} &= 6\frac{2}{5} \text{ cm}^2 \\ \text{Area of B: } 2\frac{3}{5} \times 3\frac{4}{5} &= 7\frac{4}{5} \text{ cm}^2 + 2\frac{2}{25} \text{ cm}^2 = 9\frac{22}{25} \text{ cm}^2 \\ \text{Area of C: } 3\frac{1}{5} \times 4\frac{2}{5} &= 12\frac{4}{5} \text{ cm}^2 + 1\frac{2}{25} \text{ cm}^2 = 14\frac{2}{25} \text{ cm}^2 \end{aligned}$$

The complexity of this problem stems from students making sense of the way each rectangle increases in dimension. As always, encourage students to start by drawing each of the three rectangles. Since the denominators are easily expressed as hundredths, some students may use decimals to calculate these areas. Should this occur, help make the connection for other students to that learning from Module 4.

For part (b), students must shift their thinking to the perimeter and use the outer dimensions of the rectangles to find the total amount of wire used. Again, students may use decimals for the calculations. Be sure to have students compare their decimal and fraction solutions with one another for equivalence and explain why they chose each type of fraction.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Encourage students to look for patterns in the growth of the perimeters. As a challenge, ask students to tell the perimeter of the fifth or tenth rectangle in the series.

$$\begin{aligned} \text{a Total area:} \\ 6\frac{2}{5} + 9\frac{22}{25} + 14\frac{2}{25} \\ = 6\frac{10}{25} + 9\frac{22}{25} + 14\frac{2}{25} \\ = 29\frac{34}{25} \\ = 30\frac{9}{25} \\ = 30.36 \end{aligned}$$

The total area is $30\frac{9}{25}\text{cm}^2$.

$$\begin{aligned} \text{a } 2 \times 3.2 &= 6.4 \\ 2.6 \times 3.8 &= 9.88 \\ 3.2 \times 4.4 &= 14.08 \\ \hline &30.36 \end{aligned}$$

The total area is 30.36cm^2 .

$$\begin{aligned} \text{b. Perimeter:} \\ \#1 \quad (2 \times 2) + (2 \times 3.2) \\ = 4 + 6.4 \\ = 10.4 \\ \#2 \quad (2 \times 2.6) + (2 \times 3.8) \\ = 5.2 + 7.6 \\ = 12.8 \\ \#3 \quad (2 \times 3.2) + (2 \times 4.4) \\ = 6.4 + 8.8 \\ = 15.2 \end{aligned}$$

$$\begin{aligned} \text{Perimeter:} \\ \#1 \quad 2 \times 5\frac{1}{5} &= 10\frac{2}{5} \\ \#2 \quad 2 \times 6\frac{2}{5} &= 12\frac{4}{5} \\ \#3 \quad 2 \times 7\frac{3}{5} &= 15\frac{6}{5} \\ \hline \text{Total Perimeter} &38\frac{2}{5} \\ \text{There was } 1\frac{3}{5}\text{ cm of wire left.} \end{aligned}$$

$$\begin{aligned} 40 - 38\frac{2}{5} \\ = 1\frac{3}{5} \end{aligned}$$

$$\begin{aligned} \text{Total perimeter:} \\ 10.4\text{cm} + 12.8\text{cm} + 15.2\text{cm} &= 38.4\text{cm} \end{aligned}$$

$$40\text{cm} - 38.4\text{cm} = 1.6\text{cm}$$

There was 1.6cm of wire left

Student Debrief (10 minutes)

Lesson Objective: Solve real world problems involving area of figures with fractional side lengths using visual models and/or equations.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Compare the problems for which the distributive property seems most efficient and the problems for which multiplying improper fractions (or using decimals to multiply) seems so. What influences your choice of strategy?
- Sort problems from yesterday's and today's lessons (Lessons 14 and 15) from simple to complex. What do the problems have in common? Have students compare their sort to a partner's.

Name: Ting Date: _____

1. The length of a flowerbed is 4 times as long as its width. If the width is $\frac{3}{8}$ meter, what is the area?

Width: $\frac{3}{8}$ m
 Length: $4 \times \frac{3}{8} \text{ m} = \frac{12}{8} \text{ m} = \frac{3}{2} \text{ m}$

Area: $\frac{3}{8} \text{ m} \times \frac{3}{2} \text{ m} = \frac{9}{16} \text{ m}^2$

The flower bed's area is $\frac{9}{16} \text{ m}^2$.

2. Mrs. Johnson grows herbs in square plots. Her basil plot measures $\frac{5}{8}$ yd on each side.

a. Find the total area of the basil plot.

$\frac{5}{8} \text{ yd}$ $\frac{5}{8} \text{ yd}$
 Area: $\frac{5}{8} \text{ yd} \times \frac{5}{8} \text{ yd} = \frac{25}{64} \text{ yd}^2$

The total area of the basil plot is $\frac{25}{64} \text{ yd}^2$.

b. Mrs. Johnson puts a fence around the basil. If the fence is 2 ft from the edge of the garden on each side, what is the perimeter of the fence in feet?

$\frac{5}{8} \text{ yd} = \frac{5}{8} \times 3 \text{ ft} = \frac{15}{8} \text{ ft} = 1\frac{7}{8} \text{ ft}$

Perimeter: $4 \times 1\frac{7}{8} \text{ ft} = 4 \times \frac{15}{8} \text{ ft} = 20\frac{15}{2} \text{ ft} = 20 + 3\frac{3}{4} \text{ ft} = 23\frac{3}{4} \text{ ft}$

The perimeter of the fence is $23\frac{3}{4} \text{ ft}$.

c. What is the total area, in square feet, that the fence encloses?

$\frac{35}{8} = 4\frac{3}{8}$
 $25 + 4\frac{3}{8} + 4\frac{3}{8} = \frac{119}{4}$
 $= 33 + \frac{3}{4} + \frac{3}{4} = \frac{48}{4}$
 $= 33 + \frac{24}{24} + \frac{24}{24} = \frac{49}{64}$
 $= 33 + \frac{49}{64} = 34\frac{49}{64}$

The fence encloses an area of $34\frac{49}{64} \text{ ft}^2$.

3. Janet bought 5 yards of fabric $2\frac{1}{4}$ feet wide to make curtains. She used $\frac{1}{3}$ of the fabric to make a long set of curtains and the rest to make 4 short sets.

a. Find the area of the fabric she used for the long set of curtains.

$5 \text{ yd} = 15 \text{ ft}$
 $\frac{1}{3} \text{ of } 15 \text{ ft} = 5 \text{ ft}$

Area: $5 \text{ ft} \times 2\frac{1}{4} \text{ ft} = 10\frac{5}{4} \text{ ft}^2 = 11\frac{1}{4} \text{ ft}^2$

The area of the long set of curtains is $11\frac{1}{4} \text{ ft}^2$.

b. Find the area of the fabric she used for each of the short sets.

$15 \text{ ft} - 5 \text{ ft} = 10 \text{ ft}$
 $\frac{1}{4} \text{ of } 10 \text{ ft} = \frac{10}{4} \text{ ft} = 2\frac{3}{4} \text{ ft} = 2\frac{3}{4} \text{ ft}$

Area: $2\frac{3}{4} \text{ ft} \times 2\frac{1}{4} \text{ ft} = \frac{25}{4} \text{ ft} \times \frac{5}{4} \text{ ft} = \frac{125}{16} \text{ ft}^2 = 7\frac{13}{16} \text{ ft}^2$

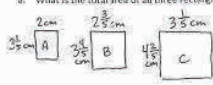
The area of each set of short curtains is $7\frac{13}{16} \text{ ft}^2$.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

4. Some wire is used to make 3 rectangles: A, B, and C. Rectangle B's dimensions are $\frac{3}{5}$ cm larger than Rectangle A's dimensions, and Rectangle C's dimensions are $\frac{1}{5}$ cm larger than Rectangle B's dimensions. Rectangle A is 2 cm by $3\frac{1}{5}$ cm.

a. What is the total area of all three rectangles?



$A = 2 \text{ cm} \times 3\frac{1}{5} \text{ cm}$
 $= 6\frac{2}{5} \text{ cm}^2$

$A_B = 2\frac{3}{5} \text{ cm} \times 3\frac{4}{5} \text{ cm}$
 $= (2 \text{ cm} + \frac{3}{5} \text{ cm}) \times (3 \text{ cm} + \frac{4}{5} \text{ cm})$
 $= (2 \text{ cm} \times 3 \text{ cm}) + (2 \text{ cm} \times \frac{4}{5} \text{ cm}) + (\frac{3}{5} \text{ cm} \times 3 \text{ cm}) + (\frac{3}{5} \text{ cm} \times \frac{4}{5} \text{ cm})$
 $= 6 \text{ cm}^2 + \frac{16}{5} \text{ cm}^2 + \frac{9}{5} \text{ cm}^2 + \frac{12}{25} \text{ cm}^2$
 $= 6 \text{ cm}^2 + \frac{16}{5} \text{ cm}^2 + \frac{9}{5} \text{ cm}^2 + \frac{12}{25} \text{ cm}^2 = 8 \text{ cm}^2 + \frac{37}{25} \text{ cm}^2 = 9\frac{37}{25} \text{ cm}^2$

$A_C = 3\frac{1}{5} \text{ cm} \times 4\frac{1}{5} \text{ cm}$
 $= (3 \text{ cm} + \frac{1}{5} \text{ cm}) \times (4 \text{ cm} + \frac{1}{5} \text{ cm})$
 $= (3 \text{ cm} \times 4 \text{ cm}) + (3 \text{ cm} \times \frac{1}{5} \text{ cm}) + (\frac{1}{5} \text{ cm} \times 4 \text{ cm}) + (\frac{1}{5} \text{ cm} \times \frac{1}{5} \text{ cm})$
 $= 12 \text{ cm}^2 + \frac{3}{5} \text{ cm}^2 + \frac{4}{5} \text{ cm}^2 + \frac{1}{25} \text{ cm}^2$
 $= 12 \text{ cm}^2 + \frac{7}{5} \text{ cm}^2 + \frac{1}{25} \text{ cm}^2$
 $= 12 \text{ cm}^2 + \frac{28}{25} \text{ cm}^2 + \frac{1}{25} \text{ cm}^2 = 12 \text{ cm}^2 + \frac{29}{25} \text{ cm}^2 = 12\frac{29}{25} \text{ cm}^2$

$\text{Total Area} = 6\frac{2}{5} \text{ cm}^2 + 9\frac{37}{25} \text{ cm}^2 + 12\frac{29}{25} \text{ cm}^2$
 $= 30\frac{3}{25} \text{ cm}^2$
 The total area is $30\frac{3}{25} \text{ cm}^2$.

b. If a 40 cm coil of wire was used to form the rectangles, how much wire is left?

$A: 2 + 2 + 3\frac{1}{5} + 3\frac{1}{5}$
 $= 10\frac{2}{5}$
 $P_A = 10\frac{2}{5} \text{ cm}$

$B: 2\frac{3}{5} + 2\frac{3}{5} + 3\frac{4}{5} + 3\frac{4}{5}$
 $= 10\frac{14}{5} = 12\frac{4}{5}$
 $P_B = 12\frac{4}{5} \text{ cm}$

$C: 3\frac{1}{5} + 3\frac{1}{5} + 4\frac{1}{5} + 4\frac{1}{5}$
 $= 14\frac{2}{5} = 15\frac{2}{5}$
 $P_C = 15\frac{2}{5} \text{ cm}$

$\text{Total Perimeter} = 10\frac{2}{5} \text{ cm} + 12\frac{4}{5} \text{ cm} + 15\frac{2}{5} \text{ cm}$
 $= 37\frac{7}{5} \text{ cm} = 38\frac{2}{5} \text{ cm}$

$40 \text{ cm} - 38\frac{2}{5} \text{ cm} = 1\frac{3}{5} \text{ cm}$
 There was $1\frac{3}{5} \text{ cm}$ of wire left.

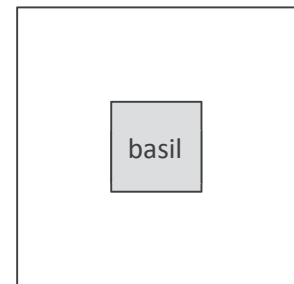
Name _____

Date _____

1. The length of a flowerbed is 4 times as long as its width. If the width is $\frac{3}{8}$ meter, what is the area?

2. Mrs. Johnson grows herbs in square plots. Her basil plot measures $\frac{5}{8}$ yd on each side.

- a. Find the total area of the basil plot.



- b. Mrs. Johnson puts a fence around the basil. If the fence is 2 ft from the edge of the garden on each side, what is the perimeter of the fence in feet?

- c. What is the total area, in square feet, that the fence encloses?
3. Janet bought 5 yards of fabric $2\frac{1}{4}$ -feet wide to make curtains. She used $\frac{1}{3}$ of the fabric to make a long set of curtains and the rest to make 4 short sets.
- a. Find the area of the fabric she used for the long set of curtains.
- b. Find the area of the fabric she used for each of the short sets.

4. Some wire is used to make 3 rectangles: A, B, and C. Rectangle B's dimensions are $\frac{3}{5}$ cm larger than Rectangle A's dimensions, and Rectangle C's dimensions are $\frac{3}{5}$ cm larger than Rectangle B's dimensions. Rectangle A is 2 cm by $3\frac{1}{5}$ cm.
- a. What is the total area of all three rectangles?
- b. If a 40-cm coil of wire was used to form the rectangles, how much wire is left?

Name _____

Date _____

Wheat grass is grown in planters that are $3\frac{1}{2}$ inch by $1\frac{3}{4}$ inch. If there is a 6×6 array of these planters with no space between them, what is the area covered by the planters?

Name _____

Date _____

1. The width of a picnic table is 3 times its length. If the length is $\frac{5}{6}$ -yd long, what is the area of the picnic table in square feet?

2. A painting company will paint this wall of a building. The owner gives them the following dimensions:

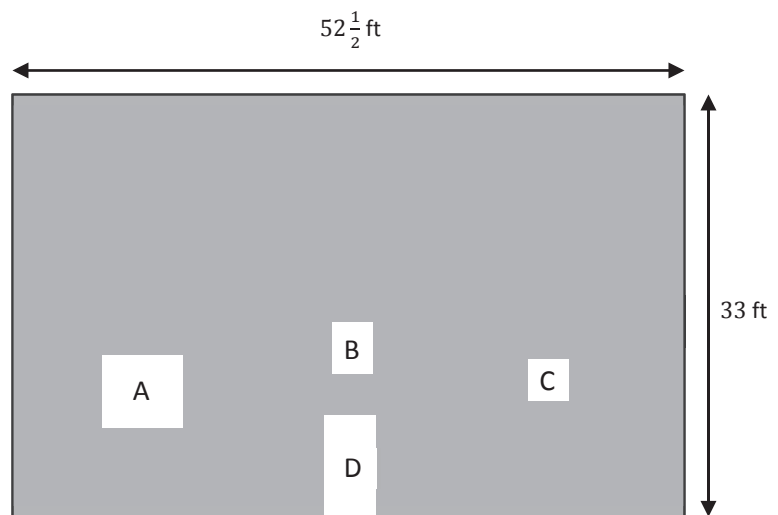
Window A is $6\frac{1}{4}$ ft \times $5\frac{3}{4}$ ft.

Window B is $3\frac{1}{8}$ ft \times 4 ft.

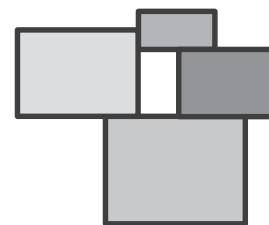
Window C is $9\frac{1}{2}$ ft².

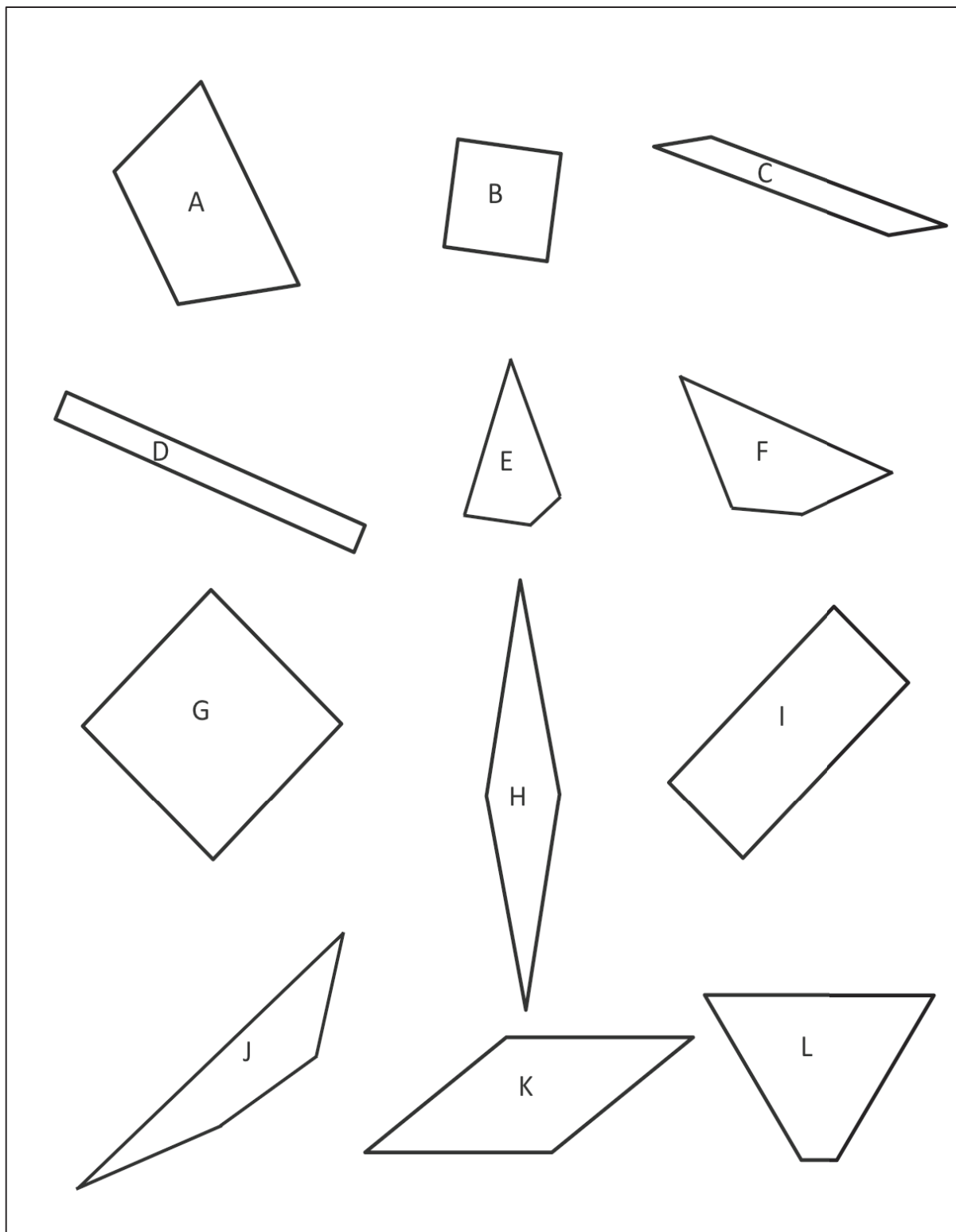
Door D is 4 ft \times 8 ft.

What is the area of the painted part of the wall?



3. A decorative wooden piece is made up of four rectangles as shown to the right. The smallest rectangle measures $4\frac{1}{2}$ inches by $7\frac{3}{4}$ inches. If $2\frac{1}{4}$ inches are added to each dimension as the rectangles get larger, what is the total area of the entire piece?





shape sheet



Topic D

Drawing, Analysis, and Classification of Two-Dimensional Shapes

Focus Standards:	■	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. <i>For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</i>	
	■	Classify two-dimensional figures in a hierarchy based on properties.	
Instructional Days:	6		
Coherence -Links from:	G3–M7	Geometry and Measurement Word Problems	
	G4–M4	Angle Measure and Plane Figures	
-Links to:	G6–M4	Expressions and Equations	

In Topic D, students draw two-dimensional shapes to analyze their attributes and then use those attributes to classify them. Familiar figures, such as parallelograms, rhombuses, squares, and trapezoids, were defined in earlier grades. By Grade 4, students gained an understanding of shapes beyond the intuitive level. Grade 5 extends this understanding through an in-depth analysis of the properties and defining attributes of quadrilaterals.

Grade 4's work with the protractor is applied in this topic to construct various quadrilaterals. Using measurement tools illuminates the attributes used to define and recognize each quadrilateral. Students see, for example, that the same process they used to construct a parallelogram also produces a rectangle when all angles are constructed to measure 90° . Students then analyze defining attributes and create a hierarchical classification of quadrilaterals.

A Teaching Sequence Toward Mastery of Drawing, Analysis, and Classification of Two-Dimensional Shapes

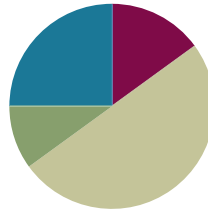
- Objective 1:** Draw trapezoids to clarify their attributes, and define trapezoids based on those attributes.
(Lesson 16)
- Objective 2:** Draw parallelograms to clarify their attributes, and define parallelograms based on those attributes.
(Lesson 17)
- Objective 3:** Draw rectangles and rhombuses to clarify their attributes, and define rectangles and rhombuses based on those attributes.
(Lesson 18)
- Objective 4:** Draw kites and squares to clarify their attributes, and define kites and squares based on those attributes.
(Lesson 19)
- Objective 5:** Classify two-dimensional figures in a hierarchy based on properties.
(Lesson 20)
- Objective 6:** Draw and identify varied two-dimensional figures from given attributes.
(Lesson 21)

Lesson 16

Objective: Draw trapezoids to clarify their attributes, and define trapezoids based on those attributes.

Suggested Lesson Structure

■ Fluency Practice	(9 minutes)
■ Application Problem	(6 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(15 minutes)
Total Time	(60 minutes)



Fluency Practice (9 minutes)

- Divide Whole Numbers by Unit Fractions and Unit Fractions by Whole Numbers (5 minutes)
- Quadrilaterals (4 minutes)

Divide Whole Numbers by Unit Fractions and Unit Fractions by Whole Numbers (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 4.

T: (Write $1 \div \frac{1}{2}$.) Say the division expression.

S: $1 \div \frac{1}{2}$.

T: How many halves are in 1 one?

S: 2.

T: (Write $1 \div \frac{1}{2} = 2$. Beneath it, write $2 \div \frac{1}{2} = \underline{\hspace{1cm}}$.) How many halves are in 2 ones?

S: 4.

T: (Write $2 \div \frac{1}{2} = 4$. Beneath it, write $3 \div \frac{1}{2} = \underline{\hspace{1cm}}$.) How many halves are in 3 ones?

S: 6.

T: (Write $3 \div \frac{1}{2} = 6$. Beneath it, write $7 \div \frac{1}{2} = \underline{\hspace{1cm}}$.) Complete the division sentence.

S: (Write $7 \div \frac{1}{2} = 14$.)

Continue with the following possible problems: $1 \div \frac{1}{4}$, $2 \div \frac{1}{4}$, $9 \div \frac{1}{4}$, and $3 \div \frac{1}{5}$.

T: (Write $\frac{1}{2} \div 2 = \underline{\hspace{1cm}}$.) Say the division sentence with the answer.

S: $\frac{1}{2} \div 2 = \frac{1}{4}$.

T: (Write $\frac{1}{2} \div 2 = \frac{1}{4}$. Beneath it, write $\frac{1}{2} \div 3 = \underline{\hspace{1cm}}$.) Say the division sentence with the answer.

S: $\frac{1}{2} \div 3 = \frac{1}{6}$.

T: (Write $\frac{1}{2} \div 3 = \frac{1}{6}$. Beneath it, write $\frac{1}{2} \div 4 = \underline{\hspace{1cm}}$.) Say the division sentence with the answer.

S: $\frac{1}{2} \div 4 = \frac{1}{8}$.

T: (Write $\frac{1}{2} \div 9 = \underline{\hspace{1cm}}$.) Complete the number sentence.

S: (Write $\frac{1}{2} \div 9 = \frac{1}{18}$.)

Continue with the following possible sequence: $\frac{1}{5} \div 2$, $\frac{1}{5} \div 3$, $\frac{1}{5} \div 5$, and $\frac{1}{8} \div 4$.

Quadrilaterals (4 minutes)

Materials: (T) Shape sheet (Lesson 15 Template) (S) Personal white board

Note: This fluency activity reviews Grade 3 geometry concepts in anticipation of Module 5 content.

T: (Project the shape sheet that includes the following: square; rhombus that is not a square; rectangle that is not a square; and several quadrilaterals that are not squares, rhombuses, or rectangles.) How many sides are in each polygon?

S: 4.

T: On your personal white board, write down the name for any four-sided polygon.

S: (Write quadrilateral.)

T: (Point to Shape B.) This quadrilateral has four equal sides and four right angles. On your board, write the name of this quadrilateral that is the most specific.

S: (Write square.)

T: Rhombuses are parallelograms with four equal sides. (Point to Shape G.) Is this polygon a rhombus?

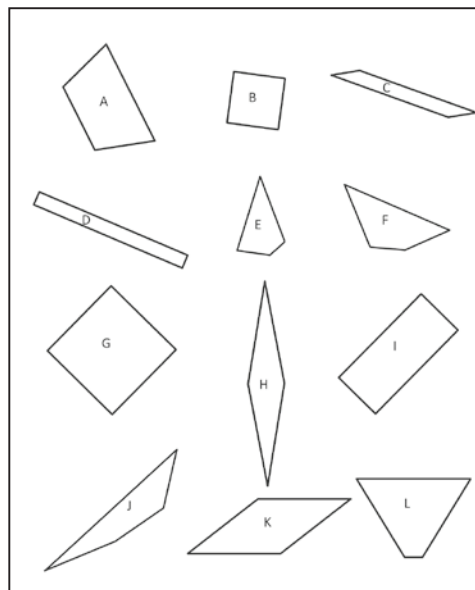
S: Yes.

T: Is it a rectangle?

S: Yes.

T: Is a square also a rhombus?

S: Yes!

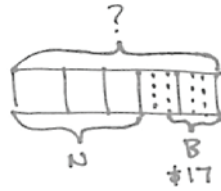


- T: (Point to Shape K.) This polygon has four equal sides. Is it a square?
- S: No.
- T: Is a rhombus always a square?
- S: No!
- T: (Point to Shape I.) This polygon has four equal angles, but the sides are not equal. Write the name of this quadrilateral. There is more than one answer.
- S: (Write rectangle. → Parallelogram. → Trapezoid.)
- T: Draw a quadrilateral that is not a square, rhombus, or rectangle.
- S: (Draw.)

Application Problem (6 minutes)

Kathy spent 3 fifths of her money on a necklace and 2 thirds of the remainder on a bracelet. If the bracelet cost \$17, how much money did she have at first?

Note: Today's Application Problem should be a quick review of a fraction of a set, as well as decimal multiplication from Modules 2 and 4.



$$1 \text{ unit} = 17 \div 4 = 4.25$$

$$15 \text{ units} = 15 \times 4.25 = 60 + 3.75 = 63.75$$

She had \$63.75 at first.

Concept Development (30 minutes)

Materials: (T) Collection of polygons (Template 1), ruler, protractor, set square (or right angle template), quadrilateral hierarchy: color (Template 3) (S) Collection of polygons (Template 1, 1 per pair of students); ruler; protractor; set square (or right angle template); scissors; crayons, markers, or colored pencils; blank paper for drawing; quadrilateral hierarchy (Template 2)

Note: The color hierarchy of quadrilaterals may be used to make color-coded cut-outs of each quadrilateral.

Problem 1

- Sort polygons by the number of sides.
- Sort quadrilaterals into trapezoids and non-trapezoids.

T: Work with your partner to sort the collection of polygons by the number of sides they have.

S: (Sort.)

T: What are polygons with four sides called?

S: Quadrilaterals.

T: Which shapes are quadrilaterals?

S: Shapes A, B, D, F, G, H, J, K, and N.



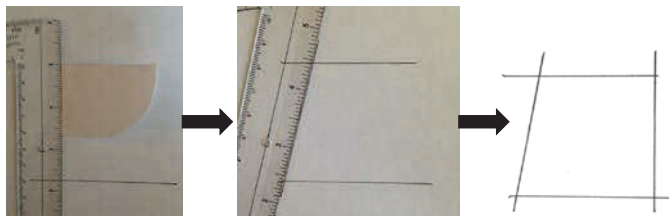
NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Problem 1 in the Concept Development provides an opportunity for a quick formative assessment. If students have difficulty sorting and articulating attributes, consider a review of concepts from Grade 4 Module 4.

- T: What attribute do you need to consider to separate the quadrilaterals into two groups—trapezoids and non-trapezoids?
- S: We look for sides that are parallel. → Trapezoids have at least one set of parallel sides, so quadrilaterals with parallel sides go in the trapezoid pile.
- T: Separate the trapezoids in your collection of quadrilaterals from the non-trapezoids.
- S: (Sort. The trapezoid set includes Shapes A, B, D, J, and K.)
- T: Talk to your partner. How are the shapes in the trapezoid group alike? How are they different?
- S: They all have four straight sides, but they don't all look the same. → They are all quadrilaterals, but they have different side lengths and angle measures. → Some of the trapezoids are rhombuses, rectangles, or squares. → They all have at least one pair of sides that are parallel.

Problem 2

- a. Draw a trapezoid according to the definition of a trapezoid.
- b. Measure and label its angles to explore their relationships.
- T: Look at the sorted shapes. I am going to ask you to draw a trapezoid in a minute. What attributes do you need to include? Turn and talk.
- S: We have to draw four straight sides.
→ Two of the sides need to be parallel to each other. → We could draw any of the shapes in our trapezoid pile. → If we have one set of parallel sides, it will be a trapezoid.
- T: Use your ruler and set square (or right angle template) to draw a pair of parallel lines on your blank paper positioned at any angle on the sheet.
- S: (Draw.)
- T: Finish your trapezoid by drawing a third and fourth segment that cross the parallel pair of lines. Make sure they do not cross each other.
- S: (Draw.)



NOTES ON DRAWING FIGURES:

If students need specific scaffolding for drawing figures, please see Grade 4 Module 4.

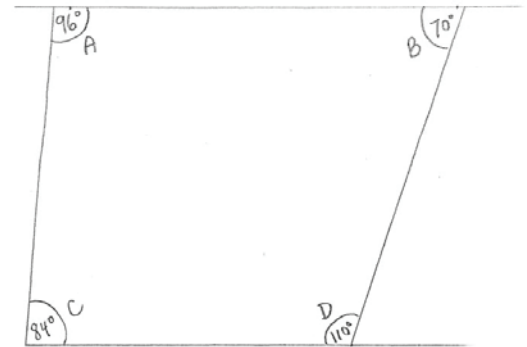
To draw parallel lines with a set square:

1. Draw a line.
2. Line up one side of the set square on the line.
3. Line up a ruler with the perpendicular side of the set square.
4. Slide the set square along the ruler until the desired place for a second line is reached.
5. Draw along the side of the set square to mark the second, parallel line.
6. Remove the set square, and extend the second line with the ruler, if necessary.

T: Compare your trapezoid with your partner's. What is alike? What is different?

S: My horizontal parallel sides are closer together than my partner's. → My trapezoid is a rectangle, but my partner's isn't. → My trapezoid is taller than my partner's. → I have right angles in mine, but my partner does not. → My trapezoid is a square, but my partner's is not.

T: Label the angles of one of the trapezoids as A, B, C, and D on the inside of the shape. Label the top left angle as angle A. Label the top right angle as angle B. On the bottom, label the left angle as angle C, and finally, label the last angle on the bottom right as angle D. Now, measure these four interior angles of your trapezoid with your protractor, and write the measurements inside each angle.



S: (Measure.)

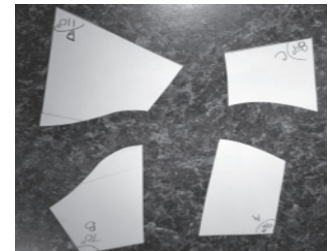
T: Cut out your trapezoid.

S: (Cut.)

T: Cut your trapezoid into two parts by cutting between the parallel sides with a wavy cut (as shown to the right).

S: (Cut.)

T: Place $\angle A$ alongside $\angle C$. (See the image.) What do you notice in your trapezoid and in your partner's?



S: The angles line up. → The two angles make a straight line.
→ If I add the angles together, it is 180 degrees. → It is a straight line, but my angles only add up to something close to 180 degrees. → My partner's trapezoid did the same thing.

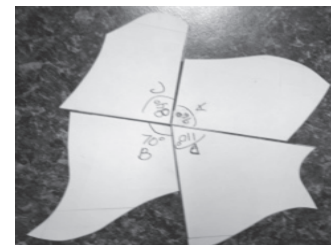
T: I heard you say that the angles make a straight line. What is the measure of a straight angle?

S: 180 degrees.

T: I also heard a few of you say that your angles did not add up to exactly 180 degrees. How do you explain that?

S: It sure looked like a straight line, so maybe we read our protractor a little bit wrong. → I might not have lined up the protractor exactly with the line I was using to measure.

T: Place $\angle B$ alongside $\angle D$. (See the image.) What do you notice?



S: It's the same as before. The angles make a straight line. → These angles add up to 180 degrees, too.

T: How many pairs of angles add up to 180 degrees?

S: Two pairs.

- T: Cut each part of your trapezoid into two pieces using a wavy cut.
- S: (Cut.)
- T: Place all four of your angles together at a point. (Demonstrate. See the image.) What do you notice about the angles?
- S: They all fit together like a puzzle. → The angles go all the way around. → Angles A and C made a straight line, and angles B and D made a straight line. I could put the straight lines together. The two straight angles make 360 degrees.
- T: How does this compare to your partner's trapezoid? Turn and talk.
- S: It's the same in my partner's trapezoid. → The angles in my partner's trapezoid weren't the same size as mine, but they still all fit together all the way around.
- T: (Distribute the Problem Set to students.) Let's practice drawing more trapezoids and thinking about their attributes by completing the Problem Set.
- S: (Complete the Problem Set.)

Please note the extended time designated for the Student Debrief of today's lesson.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Note: Today's Problem Set should be kept for use in the Lesson 17 Student Debrief.

Student Debrief (15 minutes)

Lesson Objective: Draw trapezoids to clarify their attributes, and define trapezoids based on those attributes.

Name Zikera Date _____

1. Draw a pair of parallel lines in each box. Then use the parallel lines to draw a trapezoid with the following:

a. No right angles	b. Only 1 obtuse angle
c. 2 obtuse angles	d. At least 1 right angle

2. Use the trapezoids you drew to complete the tasks below.

- Measure the angles of the trapezoid with your protractor, and record the measurements on the figures.
- Use a marker or crayon to circle pairs of angles inside each trapezoid with a sum equal to 180° . Use a different color for each pair.

3. List the properties that are shared by all the trapezoids that you worked with today.

They have 4 straight sides, but they don't all look the same.

They are quadrilaterals. They have different side lengths and angle measures.

They have at least one pair of sides that are parallel.

4. When can a quadrilateral also be called a trapezoid?

A quadrilateral can also be called a trapezoid when it has at least one pair of opposite, parallel sides.

5. Follow the directions to draw one last trapezoid.

- Draw a segment \overline{AB} parallel to the bottom of this page that is 5 cm long.
- Draw two 55° angles with vertices at A and B so that an isosceles triangle is formed with \overline{AB} as the base of the triangle.
- Label the top vertex of your triangle as C .
- Use your set square to draw a line parallel to \overline{AB} that intersects both \overline{AC} and \overline{BC} .
- Shade the trapezoid that you drew.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- (Allow students to share the myriad of trapezoids that are produced in Problem 1 of the Problem Set. Consolidate the lists of attributes students generated for trapezoids in Problem 3.) Where do these pairs seem to occur consistently? Is this true for all quadrilaterals? Just trapezoids?
- (Use the trapezoids that students produce in Problem 1 to articulate the formal definition of trapezoids. Post these definitions in the classroom for reference as Topic D proceeds.)

A trapezoid:

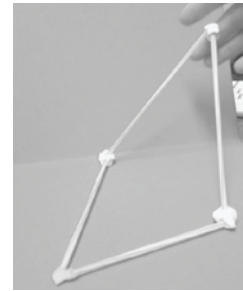
- Is a quadrilateral in which at least one pair of opposite sides is parallel.
- (Begin the construction of the hierarchy diagram, see Template 2 following this lesson. Students might draw or glue examples of trapezoids and quadrilaterals and/or list attributes within the diagram.) Explain how you decided to place each of the figures on the hierarchy. What are questions you asked yourself as you classified them?
 - Respond to the following statements with *true* or *false*. Explain your reasoning.
 - All trapezoids are quadrilaterals. (True. All trapezoids have 4 straight sides.)
 - All quadrilaterals are trapezoids. (False. A trapezoid must have at least one pair of opposite, parallel sides, but quadrilaterals need only 4 sides. No side needs to be parallel to another.)
 - The trapezoid you drew in Problem 5 is called an *isosceles trapezoid*. Think back to what you know about isosceles triangles. Why is this a good name for this quadrilateral? (Isosceles triangles have 2 equal sides. The quadrilateral drawn in Problem 5 has 2 equal sides.) How is it like some of the other trapezoids that you drew today? How is it different?
 - (Over the course of several days, students explore the formal definition of a quadrilateral, see the boxed text below, through the examination of counter examples.)

Formal Definition of a Quadrilateral:

(Only the first bullet is introduced today.)

- Consists of four different points (A, B, C, D) in the plane and four segments $(\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA})$.
- Is arranged so that the segments intersect only at their endpoints.
- Has no two adjacent segments that are collinear.

- Step 1: Ask students to tell what they know about a quadrilateral. The response is most likely a polygon with four straight sides.
- Step 2: Use straws joined with sticky tack to represent two segments *on the plane* and two segments *off the plane*. Ask: “Is this figure also a quadrilateral? What must we add to our definition to eliminate the possibility of this figure?”
- Step 3: Lead students to see that a four-sided figure is only a quadrilateral if all four segments lie in the same plane. Then, provide only the first part of the formal definition (first bullet above).



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____

Date _____

1. Draw a pair of parallel lines in each box. Then, use the parallel lines to draw a trapezoid with the following:

a. No right angles.

b. Only 1 obtuse angle.

c. 2 obtuse angles.

d. At least 1 right angle.

2. Use the trapezoids you drew to complete the tasks below.
 - a. Measure the angles of the trapezoid with your protractor, and record the measurements on the figures.
 - b. Use a marker or crayon to circle pairs of angles inside each trapezoid with a sum equal to 180° . Use a different color for each pair.
3. List the properties that are shared by all the trapezoids that you worked with today.
4. When can a quadrilateral also be called a trapezoid?
5. Follow the directions to draw one last trapezoid.
 - a. Draw a segment \overline{AB} parallel to the bottom of this page that is 5 cm long.
 - b. Draw two 55° angles with vertices at A and B so that an isosceles triangle is formed with \overline{AB} as the base of the triangle.
 - c. Label the top vertex of your triangle as C .
 - d. Use your set square to draw a line parallel to \overline{AB} that intersects both \overline{AC} and \overline{BC} .
 - e. Shade the trapezoid that you drew.

Name _____

Date _____

a. Use a ruler and a set square to draw a trapezoid.

b. What attribute must be present for a quadrilateral to also be a trapezoid?

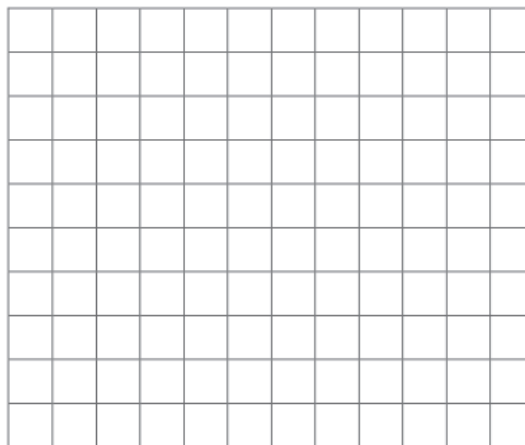
Name _____

Date _____

1. Use a straightedge and the grid paper to draw:

a. A trapezoid with exactly 2 right angles.

b. A trapezoid with no right angles.



2. Kaplan incorrectly sorted some quadrilaterals into trapezoids and non-trapezoids as pictured below.

a. Circle the shapes that are in the wrong group, and tell why they are sorted incorrectly.

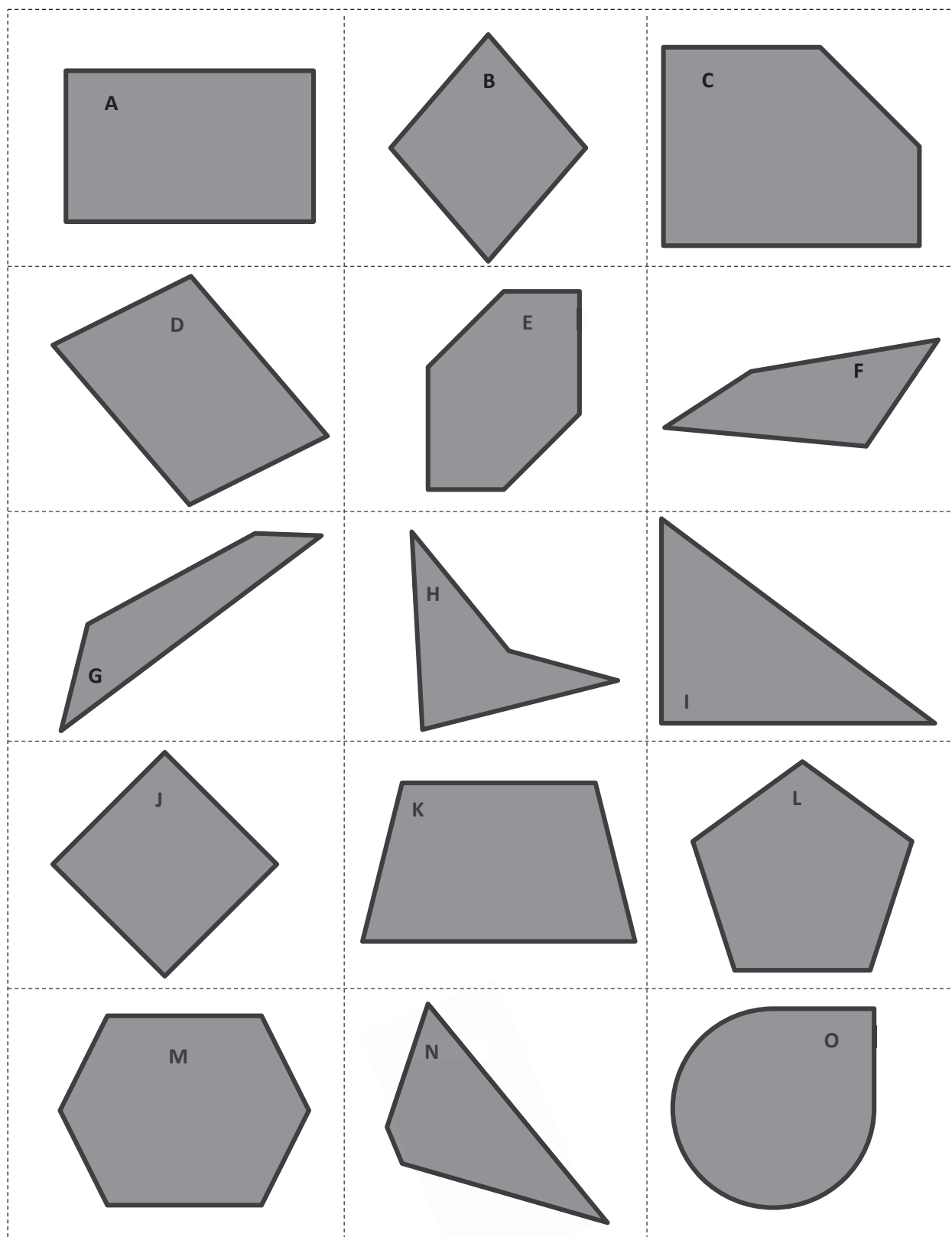
Trapezoids	Non-Trapezoids

b. Explain what tools would be necessary to use to verify the placement of all the trapezoids.

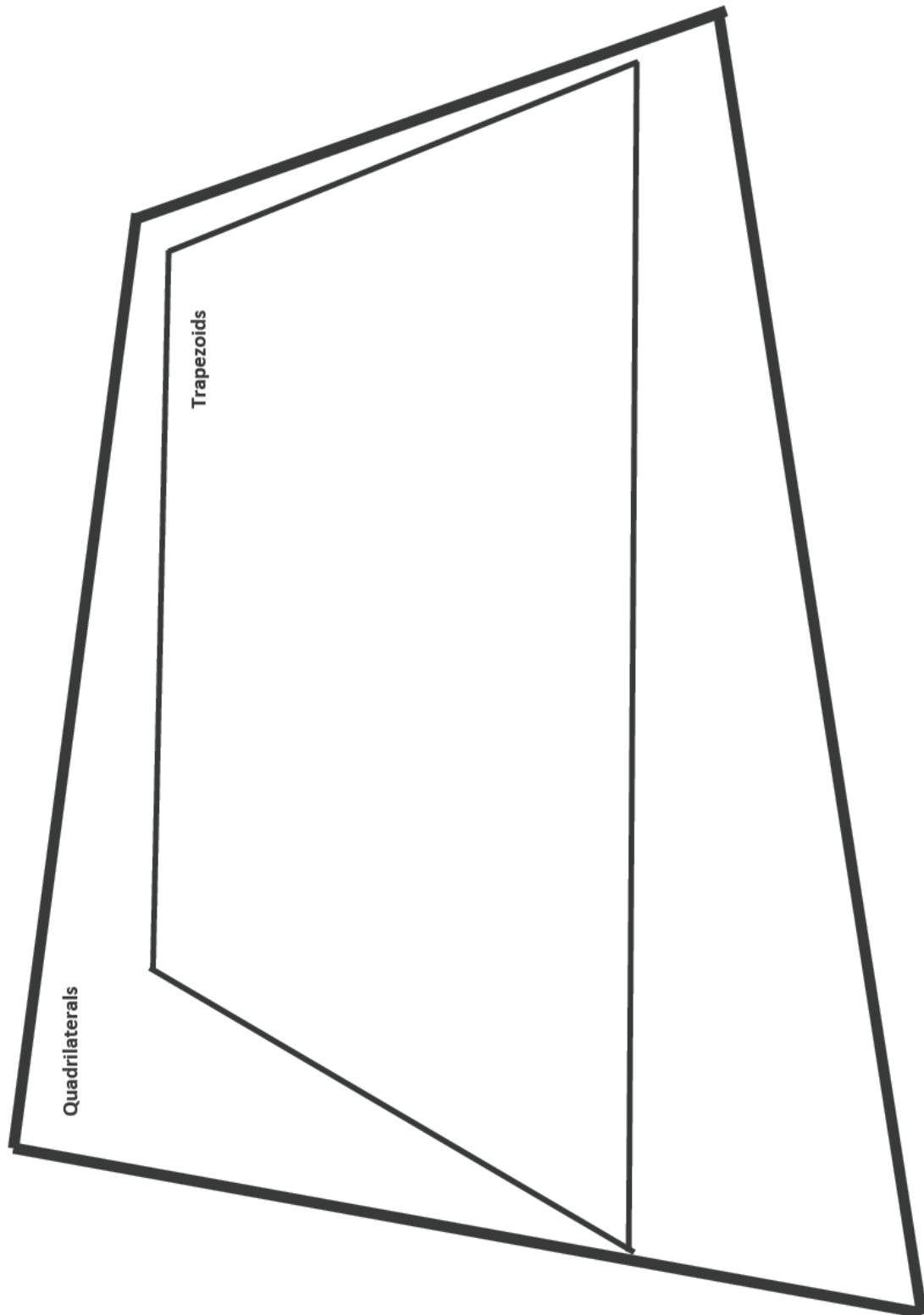
3. a. Use a straightedge to draw an isosceles trapezoid on the grid paper.



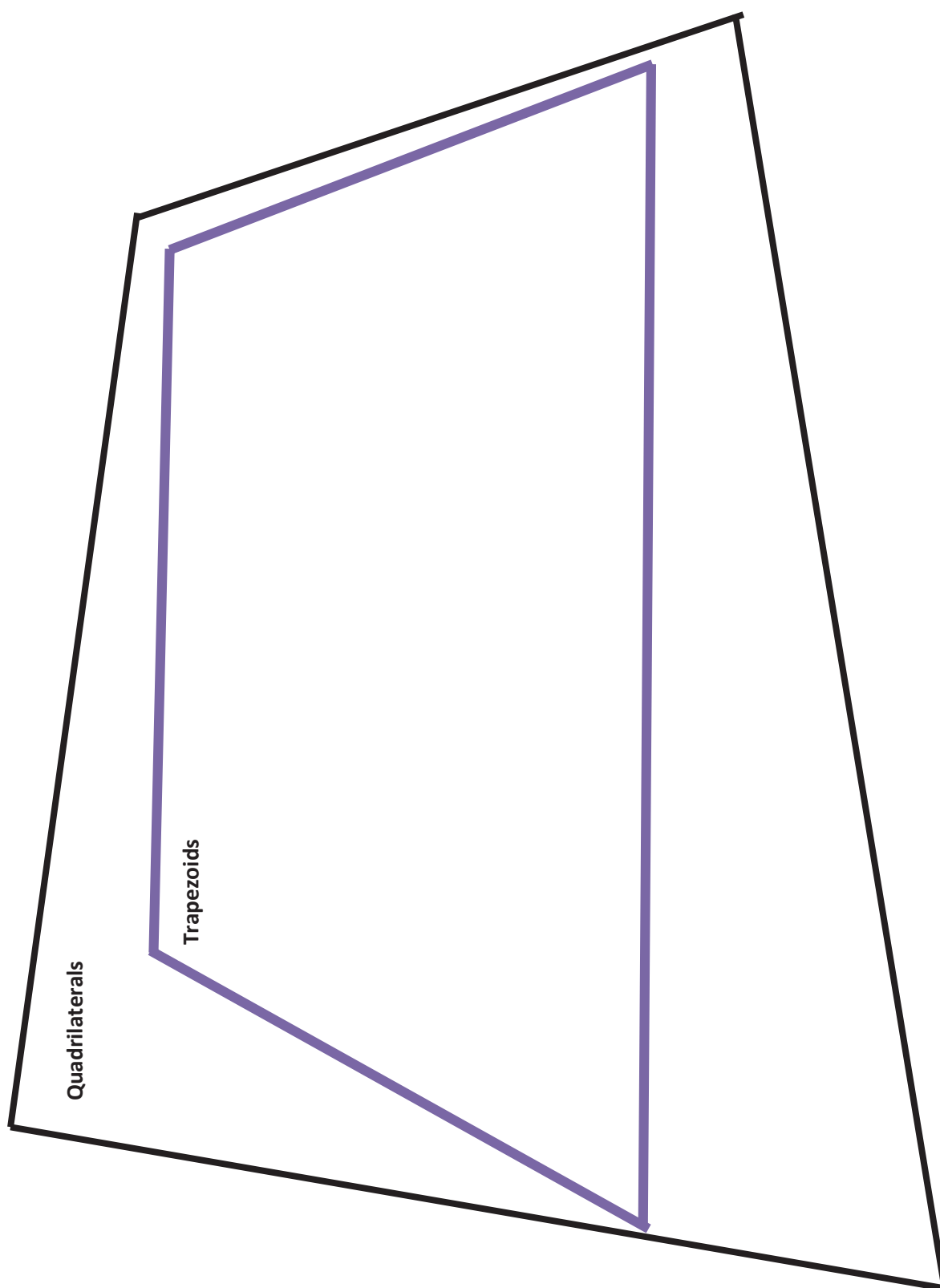
- b. Why is this shape called an isosceles trapezoid?



collection of polygons



quadrilateral hierarchy



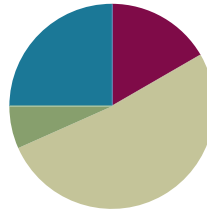
quadrilateral hierarchy: color

Lesson 17

Objective: Draw parallelograms to clarify their attributes, and define parallelograms based on those attributes.

Suggested Lesson Structure

■ Fluency Practice	(10 minutes)
■ Application Problem	(4 minutes)
■ Concept Development	(31 minutes)
■ Student Debrief	(15 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Multiply by Multiples of 10 and 100 (4 minutes)
- Divide Whole Numbers by Unit Fractions and Fractions by Whole Numbers (6 minutes)

Multiply by Multiples of 10 and 100 (4 minutes)

Note: This fluency activity reviews Modules 1 and 2.

T: (Write $31 \times 10 = \underline{\hspace{1cm}}$.) Say the multiplication sentence.

S: $31 \times 10 = 310$.

T: (Write $31 \times 10 = 310$. Below it, write $310 \times 2 = \underline{\hspace{1cm}}$.) Say the multiplication sentence.

S: $310 \times 2 = 620$.

T: (Write $310 \times 2 = 620$. Below it, write $31 \times 20 = 31 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.) Say 31×20 as a three-factor multiplication sentence with 10 as one factor.

S: $31 \times 10 \times 2 = 620$.

Follow the same process for 21×40 .

T: (Write $32 \times 30 = 32 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.) Write 32×30 as a three-factor multiplication sentence, and solve.

S: (Write $32 \times 30 = 32 \times 10 \times 3 = 960$.)

Repeat the process for 241×20 .

T: (Write $21 \times 100 = \underline{\hspace{1cm}}$.) Say the multiplication sentence.

S: $21 \times 100 = 2,100$.

T: (Write $21 \times 100 = 2,100$. Below it, write $2,100 \times 3 = \underline{\hspace{1cm}}$.) Say the multiplication sentence.

S: $2,100 \times 3 = 6,300$.

T: (Write $2,100 \times 3 = 6,300$. Below it, write $21 \times 300 = \underline{\hspace{2cm}}$.) Say 21×300 as a three-factor multiplication sentence with 100 as one factor.

S: $21 \times 3 \times 100 = 6,300$.

T: (Write $21 \times 300 = 6,300$.)

Direct students to solve using the same method for 42×400 and 34×300 .

Divide Whole Numbers by Unit Fractions and Fractions by Whole Numbers (6 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 4 Lessons 25–27 and prepares students for today’s lesson.

T: (Write $2 \div \frac{1}{3} = \underline{\hspace{2cm}}$.) Say the complete division sentence.

S: $2 \div \frac{1}{3} = 6$.

T: (Write $2 \div \frac{1}{3} = 6$. Beneath it, write $3 \div \frac{1}{5} = \underline{\hspace{2cm}}$.) Say the complete division sentence.

S: $3 \div \frac{1}{5} = 15$.

T: (Write $3 \div \frac{1}{5} = 15$. Beneath it, write $6 \div \frac{1}{2} = \underline{\hspace{2cm}}$.) On your personal white board, complete the division sentence.

S: (Write $6 \div \frac{1}{2} = 12$.)

Continue this process for $3 \div \frac{1}{4}$, $9 \div \frac{1}{3}$, $1 \div \frac{1}{10}$, $2 \div \frac{1}{10}$, $8 \div \frac{1}{10}$, and $10 \div \frac{1}{10}$.

T: (Write $\frac{1}{2} \div 2 = \underline{\hspace{2cm}}$.) Say the complete division sentence.

S: $\frac{1}{2} \div 2 = \frac{1}{4}$.

T: (Write $\frac{1}{2} \div 2 = \frac{1}{4}$. Beneath it, write $\frac{1}{3} \div 5 = \underline{\hspace{2cm}}$.) Say the complete division sentence.

S: $\frac{1}{3} \div 5 = \frac{1}{15}$.

T: (Write $\frac{1}{3} \div 5 = \frac{1}{15}$. Beneath it, write $\frac{1}{6} \div 5 = \underline{\hspace{2cm}}$.) On your personal white board, write the complete division sentence.

S: (Write $\frac{1}{6} \div 5 = \frac{1}{30}$.)

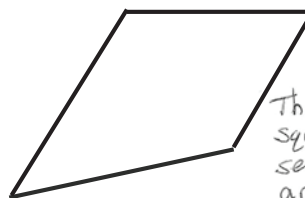
T: (Write $\frac{1}{4} \div 4 = \underline{\hspace{2cm}}$.) Say the complete division sentence.

S: $\frac{1}{4} \div 4 = \frac{1}{16}$.

Continue this process for the following possible sequence: $9 \div \frac{1}{4}$, $\frac{1}{4} \div 9$, $5 \div \frac{1}{8}$, $\frac{1}{8} \div 5$, $\frac{1}{7} \div 8$, and $8 \div \frac{1}{7}$.

Application Problem (4 minutes)

Ava drew the quadrilateral to the right and called it a trapezoid. Adam said Ava is wrong. Explain to your partner how a set square can be used to determine who is correct. Support your answer using the properties of trapezoids.



They can use a set square and ruler to see if 2 of the lines are parallel. If so, then it is a trapezoid.

Concept Development (31 minutes)

Materials: (T) Ruler, protractor, set square (or right angle template), quadrilateral hierarchy with parallelogram: color (Template 2) (S) Ruler; protractor; set square (or right angle template); scissors; crayons, markers, or colored pencils; blank paper for drawing; quadrilateral hierarchy with parallelogram (Template 1)



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

If students need specific scaffolding for drawing figures, please see Grade 4 Module 4.

Problem 1

- Draw a parallelogram, and articulate the definition.
- Measure and label its angles to explore their relationships.
- Measure to explore diagonals of parallelograms.

T: What name could we give to all of the shapes we drew yesterday?

S: Quadrilateral. → Trapezoid.

T: Use your ruler and set square to draw a pair of parallel lines on your blank paper positioned at any angle on the sheet.

S: (Draw.)

T: Because we are about to draw a quadrilateral beginning with one pair of parallel sides, what name can we give every figure we will draw today?

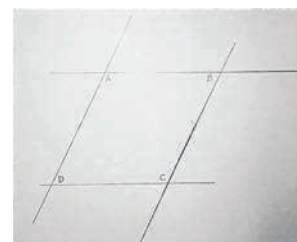
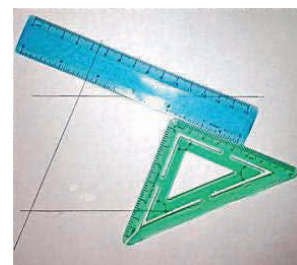
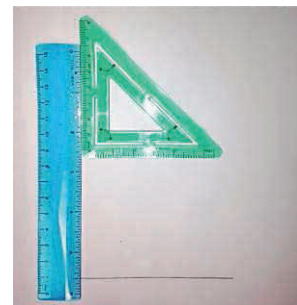
S: Trapezoid.

T: If I want to draw a trapezoid that can also be called a parallelogram, what will I need to draw next?

S: Parallelograms have two sets of parallel sides, so you have to draw another pair of parallel lines. → Draw another pair of parallel lines that cross the first ones.

T: Use your tools to draw a second pair of parallel lines that intersect your first pair.

S: (Draw.)



- T: Measure the sides of your parallelogram, and compare your parallelogram with your partner's. What is alike? What is different?
- S: The opposite sides in my parallelogram are the same lengths, and my partner's are, too. → My parallel sides are closer together than my partner's. → My parallelogram is a rectangle, but my partner's isn't. → My parallelogram has four equal sides, and my partner's has two different pairs of equal sides. → I have right angles in mine, but my partner does not. → I drew a square, and my partner drew a rectangle.
- T: Label the angles of your parallelogram inside the shape. Label the upper angle on the left as angle *A* and the upper angle on the right as angle *B*. Continue clockwise by labeling the lower right angle as angle *C* and the angle directly below *A* as angle *D*. Then, measure the angles of your parallelogram with your protractor, and write the measurements inside each angle.
- S: (Measure.)
- T: Cut out your parallelogram.
- S: (Cut.)
- T: Make a copy of your parallelogram on another blank sheet by tracing it and labeling the vertices with the same letters.
- S: (Trace and label.)
- T: Cut your first parallelogram into four parts by cutting between each set of parallel sides with a wavy cut.
- S: (Cut.)
- T: Put angle *A* on top of angle *C*, and put angle *B* on top of angle *D*. What do you notice about your parallelogram's angles and about your partner's? Turn and talk.
- S: Angles *A* and *C* are the same size, and so are angles *B* and *D*. → Our parallelograms don't look anything alike, but the angles opposite each other in each of our parallelograms are equal.
- T: Place your angles alongside each other, and find as many combinations that form straight lines as you can.
- S: (Work.)
- T: Compare your findings with your partner's. What do you notice? How many pairs did you find?
- S: We both found four pairs that make straight lines. → Yesterday, some of us only had two pairs of angles that made straight lines. Today, all of us have four pairs.
- T: So, thinking about what we drew and what we have discovered about these angles, when can a trapezoid also be called a parallelogram? Turn and talk.
- S: When a trapezoid has more than one pair of parallel sides, it can be called a parallelogram. → Trapezoids have at least two pairs of angles that add up to 180 degrees. When they have more than that, they can also be called a parallelogram.

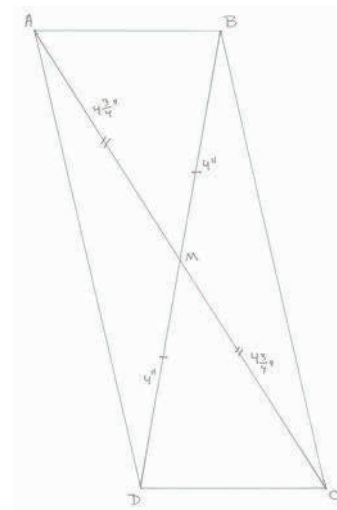


NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

The discussion of parallelograms as special trapezoids is based on the inclusive definition of a trapezoid as a quadrilateral with *at least* one set of parallel sides. That is, trapezoids may have more than one set of parallel sides.

Also note that the dialogue as written here assumes recall of Grade 4 geometric concepts. Additionally, scaffolded questions may be necessary for students to verbalize the conditions necessary to classify a trapezoid as a parallelogram.

- T: Place all four of your angles together at a point. What do you notice about the angles in your parallelogram and in your partner's?
- S: (Work.) All four of my angles add up to 360 degrees. → My parallelogram was different than my neighbors, but the angles go all the way around again, like they did with our trapezoids.
- T: Use your ruler to draw the diagonals on the copy you made of your parallelogram.
- S: (Draw.)
- T: Measure each diagonal, and record the measurements on your paper. Are these segments equal to each other?
- S: I drew a long skinny parallelogram, and my diagonals aren't the same length. But my partner drew a square, and his are the same length. → Some people have equal diagonals, and some people don't.
- T: I hear you saying that the diagonals of a parallelogram may or may not be equal to each other. Label the point where your diagonals intersect as point M .
- S: (Draw and label.)
- T: Measure from each corner of your parallelogram to point M . Record all of the measurements on the figure. Compare your measurements to those of your partner.
- S: (Measure and compare.)
- T: What do you notice about the diagonals of your parallelogram now?
- S: The length from opposite corners to point M on the same diagonal is equal. → The diagonals cut each other into two equal parts. → One diagonal crosses the other at its midpoint. → M is the midpoint of both diagonals. → Even though our parallelograms look really different, our diagonals still cross at their midpoints.
- T: The diagonals of a parallelogram bisect each other. Say, "bisect."
- S: Bisect.
- T: Let's break down this word into parts. Think about the first part, bi-. How many wheels are on a bicycle?
- S: Two.
- T: What does the word section mean?
- S: Parts of something.
- T: Sect also means to cut. Turn and talk to your partner about why bisect is a good name for what you see in all the parallelograms' diagonals.
- S: Bi- means two. These segments are cut in two equal parts. → Bi- means two, and sect means cut, so bisect means to cut in two equal parts.
- T: (Distribute the Problem Set to students.) Let's practice drawing more parallelograms and thinking about their attributes by completing the Problem Set.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

For early finishers of the Problem Set, offer this challenge:

In parallelogram $ABCD$, $\angle A$ is $\frac{1}{3}$ the measure of $\angle B$. Find the measures of all the angles of the figure.

Please note the extended time designated for the Student Debrief of today's lesson.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Note: Today's Problem Set from Lessons 16 and 17 should be kept for use in Lesson 18's Student Debrief as well.

Student Debrief (15 minutes)

Lesson Objective: Draw parallelograms to clarify their attributes, and define parallelograms based on those attributes.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- (Allow students to share all the different parallelograms that are produced in Problem 1 of the Problem Set. Consolidate the lists of attributes students generated for parallelograms in Problem 4.) What attributes do all parallelograms share? Where do these pairs seem to occur consistently? Is this true for all quadrilaterals? Trapezoids? Parallelograms?
- (Use the parallelograms that students produce in Problem 1 to articulate the formal definition of a parallelogram. Continue posting these definitions in the classroom for reference as Topic D proceeds.)

Name Carter Date _____

1. Draw a parallelogram in each box with the attributes listed.

a. No right angles. 	b. At least 2 right angles.
c. Equal sides with no right angles. 	d. All sides equal with at least 2 right angles.

In each parallelogram there are 2 pairs of angles that add up to 180° .

2. Use the parallelograms you drew to complete the tasks below.

- Measure the angles of the parallelogram with your protractor, and record the measurements on the figures.
- Use a marker or crayon to circle pairs of angles inside each parallelogram with a sum equal to 180° . Use a different color for each pair.

3. Draw another parallelogram below.

- Draw the diagonals, and measure their lengths. Record the measurements to the side of your figure.
- Measure the length of each of the four segments of the diagonals from the vertices to the point of intersection of the diagonals. Color the segments that have the same length the same color. What do you notice? The diagonals bisect each other. Each diagonal cuts the parallelogram into two equal triangles.

4. List the properties that are shared by all of the parallelograms that you worked with today.

Parallelograms have 4 sides.
They have 2 sets of opposite sides parallel.
Their opposite angles are equal in measure.
The diagonals bisect each other.
Opposite sides are equal in length.

- When can a quadrilateral also be called a parallelogram?
When a quadrilateral has 2 sets of opposite sides parallel, it can also be called a parallelogram.
- Can a trapezoid also be called a parallelogram?
In order for a trapezoid to also be called a parallelogram, it must have 2 sets of opposite sides parallel.

- When can a quadrilateral also be called a parallelogram? When can a trapezoid also be called a parallelogram? Respond to the following statements with *true* or *false*. Explain your reasoning.
 - All parallelograms are trapezoids. (True. The defining attribute of a trapezoid is that it has at least one set of parallel sides. Since parallelograms have two sets of parallel sides, they also fit the definition of a trapezoid.)
 - All trapezoids are parallelograms. (False. If the trapezoid has only one set of parallel sides, it is not also a parallelogram.)
 - All parallelograms are quadrilaterals. (True. All parallelograms have four straight sides.)
 - All quadrilaterals are parallelograms. (False. There are many four-sided shapes that do not have two pairs of parallel sides.)

A parallelogram:

- Is a quadrilateral in which both pairs of opposite sides are parallel.

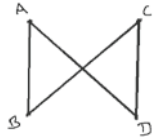
A quadrilateral:

- Consists of four different points A, B, C, D in the plane and four segments, $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$,
- Is arranged so that the segments intersect only at their endpoints, and
- Has no two adjacent segments that are collinear.

- (Continue the construction of the hierarchy diagram from Lesson 16. Students might draw or glue examples of parallelograms and/or list attributes within the diagram. Encourage students to explain their placements of the figures in the hierarchy.)
- (Continue exploring the formal definition of a quadrilateral through the examination of counter-examples. See the boxed text above.)

Step 1: Begin by asking students to tell what they know about a quadrilateral. Today's response should be a polygon with four straight sides that *lie in the same plane*.

Step 2: Follow the first bullet in the definition verbatim to draw four straight segments in the same plane, but have the segments intersect as shown to the right.



Ask, "Is this figure also a quadrilateral? What must we add to our definition to eliminate the possibility of this figure?"

Step 3: Lead students to see that a four-sided figure is only a quadrilateral if all four segments lie in the same plane *and the segments intersect only at their endpoints*. Then, add the second bullet of the definition to that written in Lesson 16.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____

Date _____

1. Draw a parallelogram in each box with the attributes listed.

a. No right angles.

b. At least 2 right angles.

c. Equal sides with no right angles.

d. All sides equal with at least 2 right angles.

2. Use the parallelograms you drew to complete the tasks below.
 - a. Measure the angles of the parallelogram with your protractor, and record the measurements on the figures.
 - b. Use a marker or crayon to circle pairs of angles inside each parallelogram with a sum equal to 180° . Use a different color for each pair.
3. Draw another parallelogram below.
 - a. Draw the diagonals, and measure their lengths. Record the measurements to the side of your figure.
 - b. Measure the length of each of the four segments of the diagonals from the vertices to the point of intersection of the diagonals. Color the segments that have the same length the same color. What do you notice?
4. List the properties that are shared by all of the parallelograms that you worked with today.
 - a. When can a quadrilateral also be called a parallelogram?
 - b. When can a trapezoid also be called a parallelogram?

Name _____

Date _____

1. Draw a parallelogram.

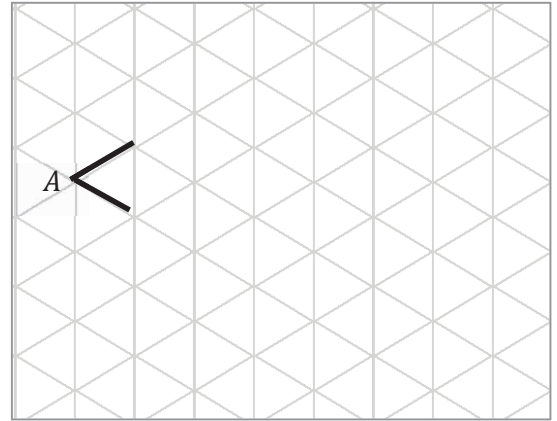
2. When is a trapezoid also called a parallelogram?

Name _____

Date _____

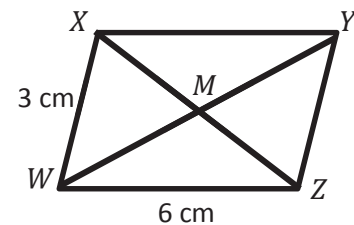
- 1.
- $\angle A$
- measures
- 60°
- .

- a. Extend the rays of $\angle A$, and draw parallelogram $ABCD$ on the grid paper.
- b. What are the measures of $\angle B$, $\angle C$, and $\angle D$?



- 2.
- $WXYZ$
- is a parallelogram not drawn to scale.

- a. Using what you know about parallelograms, give the measure of sides XY and YZ .
- b. $\angle WXY = 113^\circ$. Use what you know about angles in a parallelogram to find the measure of the other angles.



$\angle XYZ = \underline{\hspace{2cm}}^\circ$

$\angle YZW = \underline{\hspace{2cm}}^\circ$

$\angle ZWX = \underline{\hspace{2cm}}^\circ$

3. Jack measured some segments in Problem 2. He found that
- $\overline{WY} = 8$
- cm and
- $\overline{MZ} = 3$
- cm.

Give the lengths of the following segments:

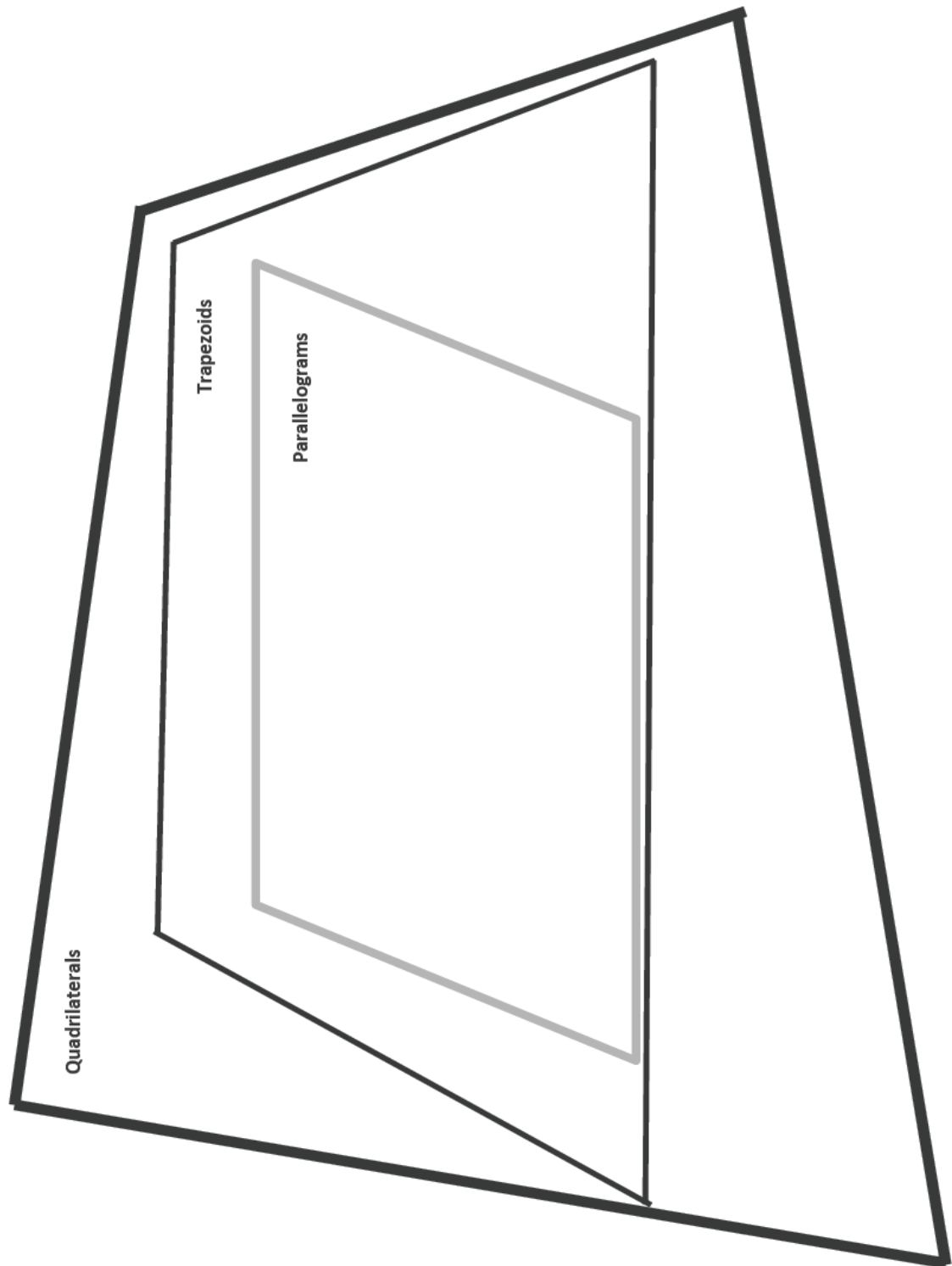
$WM = \underline{\hspace{2cm}} \text{ cm}$

$MY = \underline{\hspace{2cm}} \text{ cm}$

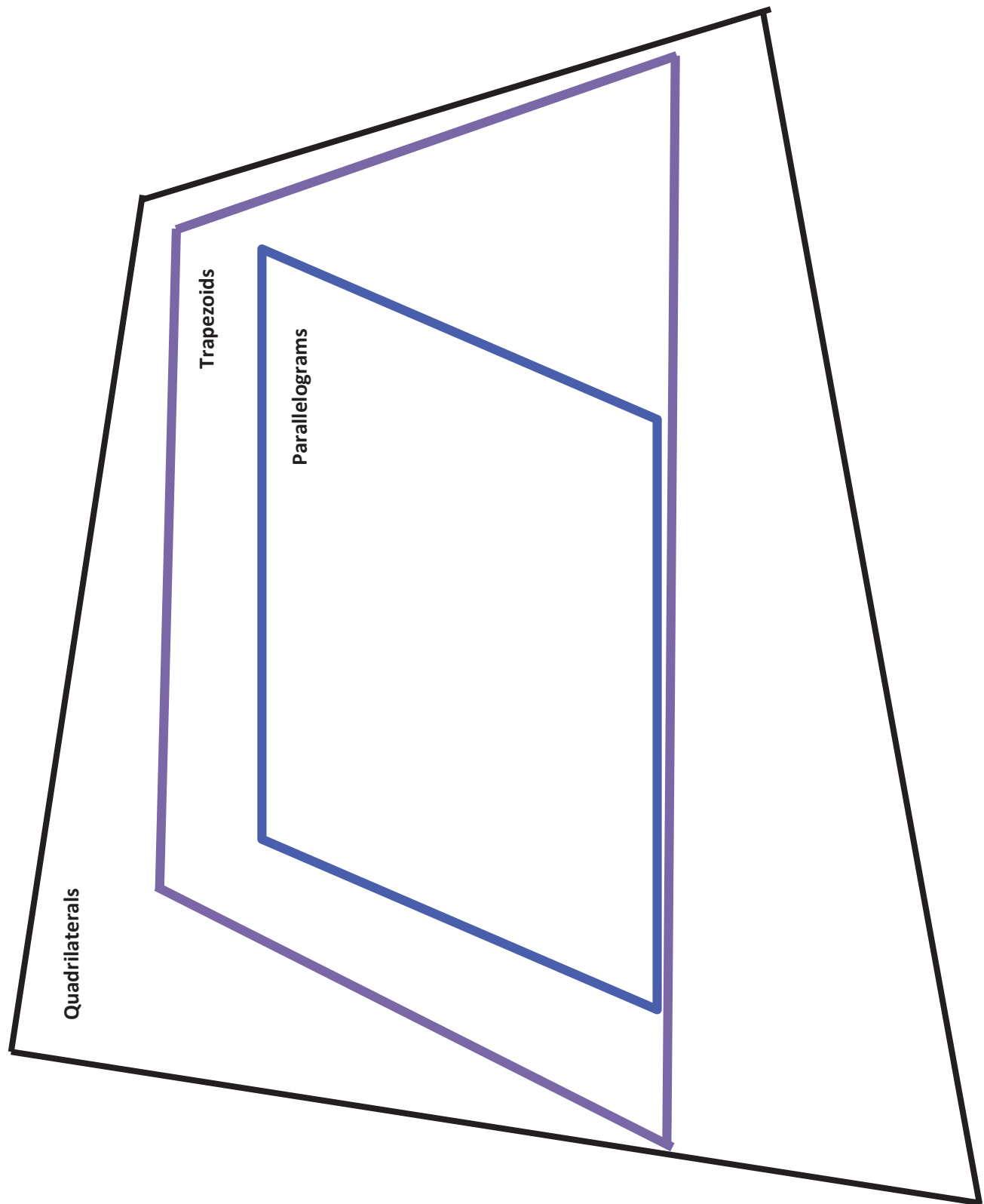
$XM = \underline{\hspace{2cm}} \text{ cm}$

$XZ = \underline{\hspace{2cm}} \text{ cm}$

- Using the properties of shapes, explain why all parallelograms are trapezoids.
- Teresa says that because the diagonals of a parallelogram bisect each other, if one diagonal is 4.2 cm, the other diagonal must be half that length. Use words and pictures to explain Teresa's error.



quadrilateral hierarchy with parallelogram



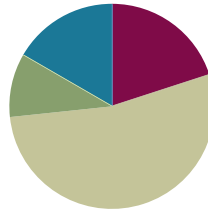
quadrilateral hierarchy with parallelogram: color

Lesson 18

Objective: Draw rectangles and rhombuses to clarify their attributes, and define rectangles and rhombuses based on those attributes.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(6 minutes)
■ Concept Development	(32 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Sprint: Divide Whole Numbers by Fractions and Fractions by Whole Numbers (9 minutes)
- Multiply by Multiples of 10 and 100 (3 minutes)

Sprint: Divide Whole Numbers by Fractions and Fractions by Whole Numbers (9 minutes)

Materials: (S) Divide Whole Numbers by Fractions and Fractions by Whole Numbers Sprint

Note: This fluency activity reviews Module 4.

Multiply by Multiples of 10 and 100 (3 minutes)

Note: This fluency activity reviews Modules 1–2.

T: (Write $42 \times 10 = \underline{\hspace{1cm}}$.) Say the multiplication sentence.

S: $42 \times 10 = 420$.

T: (Write $42 \times 10 = 420$. Below it, write $420 \times 2 = \underline{\hspace{1cm}}$.) Say the multiplication sentence.

S: $420 \times 2 = 840$.

T: (Write $420 \times 2 = 840$. Below it, write $42 \times 20 = 42 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.) Say 42×20 as a three-factor multiplication sentence with 10 as one of the factors.

S: $42 \times 10 \times 2 = 840$.

Follow the same process for 23×30 .

T: (Write $213 \times 30 = 213 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.) Write 213×30 as a three-factor multiplication sentence with 10 as one of the factors, and solve.

S: (Write $213 \times 30 = 213 \times 10 \times 3 = 6,390$.)

Repeat the process for $4,213 \times 20$.

T: (Write $31 \times 100 = \underline{\hspace{2cm}}$.) Say the multiplication sentence.

S: $31 \times 100 = 3,100$.

T: (Write $31 \times 100 = 3,100$. Below it, write $3,100 \times 3 = \underline{\hspace{2cm}}$.) Say the multiplication sentence.

S: $3,100 \times 3 = 9,300$.

T: (Write $3,100 \times 3 = 9,300$. Below it, write $31 \times 300 = \underline{\hspace{2cm}}$.) Say 31×300 as a three-factor multiplication sentence with 100 as one of the factors.

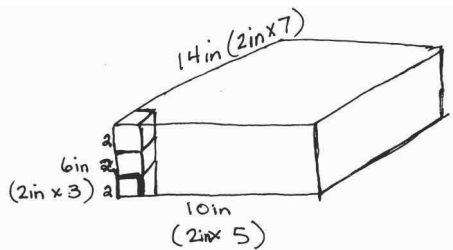
S: $31 \times 100 \times 3 = 9,300$.

T: (Write $31 \times 300 = 9,300$.)

Direct students to solve 43×300 using the same method.

Application Problem (6 minutes)

How many 2-inch cubes are needed to build a rectangular prism that measures 10 inches by 14 inches by 6 inches?



$5 \times 7 \times 3 = 105$
105 2-inch cubes
are needed to build
the prism.

$$\begin{aligned} \text{OR} \\ V &= 10 \text{ in} \times 14 \text{ in} \times 6 \text{ in} = 840 \text{ in}^3 \\ V_{\text{cube}} &= 2 \text{ in} \times 2 \text{ in} \times 2 \text{ in} = 8 \text{ in}^3 \\ \frac{840}{8} &= 105 \end{aligned}$$

Note: Today's Application Problem requires that students reason about volume concepts from earlier in this module.

Concept Development (32 minutes)

Materials: (T) Quadrilateral hierarchy with square: color (Template 2) (S) Ruler, set square or square template, protractor, scissors, quadrilateral hierarchy with square (Template 1)

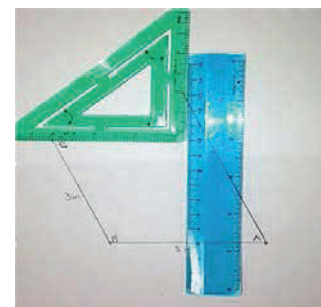
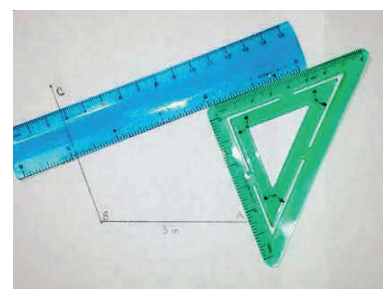
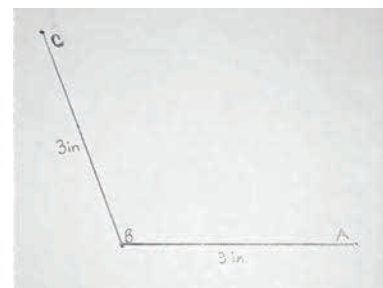
Problem 1

- Draw a rhombus, and articulate the definition.
- Measure and label its angles to explore their relationships.
- Measure to explore diagonals of rhombuses.

T: Give the least specific name for all the shapes we have drawn so far.

S: Quadrilaterals.

- T: Tell your partner a more specific name for a shape we have drawn, and explain what property it has that gives it that name.
- S: Trapezoids because we have drawn shapes with at least one pair of parallel sides. → Some of the quadrilaterals could be called trapezoids and parallelograms. Parallelograms have two pairs of parallel sides.
- T: How did we start drawing the trapezoids and parallelograms?
- S: By drawing a pair of parallel sides.
- T: If we wanted to draw a parallelogram that is also a rhombus, what would we need to think about?
- S: It would need to have four sides the same length. → It would need another pair of parallel sides, but we would need to measure to be sure we drew all the sides the same length.
- T: Draw an angle with sides that are equal length. Then, label the vertex as B and the endpoints of the sides as A and C .
- S: (Draw an angle.)
- T: Draw a line parallel to one of the sides through the endpoint of the other side.
- S: (Draw a parallel line.)
- T: Now, do the same for the second side.
- S: (Draw a second parallel line.)
- T: Label the last angle as D .
- S: (Label the angle.)
- T: Measure the sides, and compare your figure with your partner's. What is the most specific name for this shape? How do you know?
- S: My sides were two inches long. My partner's were three inches long, but they both have two sets of parallel sides, and the sides are all the same length. So, we both drew a rhombus. → It's a parallelogram with four equal sides. → Mine is a parallelogram with equal sides, but my partner's is a square. We both drew a rhombus with four equal sides, even though I started with an acute angle and he started with a right angle.
- T: Measure the angles, and mark them inside the rhombus.
- S: (Measure and mark the angles.)
- T: What do you notice? Turn and talk.
- S: The angles that are beside each other add up to a straight angle. → There are two pairs of angles. Each pair adds up to 180° . → Angles between parallel lines equal 180° . → The opposite angles are the same size.
- T: Use your ruler to draw the diagonals of your rhombus. Then, measure them and the distance from each corner to the point where they intersect. Tell your partner what you notice.
- S: These diagonals are equal. → The diagonals bisect each other. → The point where they cross is the midpoint of both diagonals.



T: Now, measure the angles formed by the diagonals. What is the measure?

S: They are right angles. → The angles are all 90° .

T: What is the name for lines that intersect at a right angle?

S: Perpendicular lines.

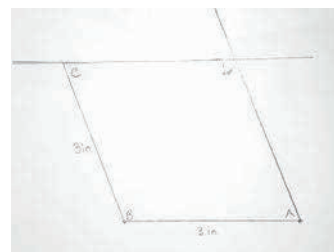
T: Because they bisect each other at a 90° angle, we call these diagonals **perpendicular bisectors**.

T: From our drawing, what attribute needs to be present to call this parallelogram a rhombus?

S: All four sides must be equal.

T: What else did we discover about the diagonals of a rhombus?

S: The diagonals are perpendicular bisectors.



Problem 2

a. Draw a rectangle according to the definition of a rectangle.

b. Measure and label its angles to explore their relationships.

c. Measure to explore the diagonals of rectangles.

T: If I want to draw a parallelogram that is also a rectangle, what must I include in my drawing?

S: Rectangles are parallelograms, so they need two sets of parallel sides. → Rectangles have right angles and opposite sides that are parallel and equal.

T: Use your ruler and set square to draw a rectangle.

S: (Draw a rectangle.)

T: Cut out your rectangle, and by folding, confirm that the angles are all 90° and the opposite sides are the same lengths.

S: (Cut and fold the rectangle.)

T: Now, measure the diagonals, the segments of the diagonals, and the angles around the intersection point. Record your measurements on the figure.

S: (Measure and record on the figure.)

T: What do you notice? Turn and talk.

S: The diagonals are equal lengths. → The segments of the diagonals are equal. → The angles between the parallel lines equal 180° . → The diagonals are equal and bisect each other.

T: Are the diagonals perpendicular bisectors? How do you know?

S: They are not perpendicular bisectors because they don't form right angles.

T: What properties must be present for a parallelogram to also be a rectangle?

S: The sides across from each other have to be the same length. → All angles are 90° . → Diagonals bisect each other.

- T: (Distribute the Problem Set to students.)
Let's practice drawing more rhombuses and rectangles and think about their attributes by completing the Problem Set.
- S: (Complete the Problem Set.)

Problem Set

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Note: Along with today's Problem Set, the Problem Sets from Lessons 16 and 17 should be used in today's Student Debrief.

Student Debrief (10 minutes)

Lesson Objective: Draw rectangles and rhombuses to clarify their attributes, and define rectangles and rhombuses based on those attributes.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Name Lisa Date _____

1. Draw the figures in each box with the attributes listed.

a. Rhombus with no right angles	b. Rectangle with not all sides equal
c. Rhombus with 1 right angle	d. Rectangle with all sides equal

2. Use the figures you drew to complete the tasks below:

- Measure the angles of the figures with your protractor, and record the measurements on the figures.
- Use a marker or crayon to circle pairs of angles inside each figure with a sum equal to 180° . Use a different color for each pair.

3. Draw a rhombus and a rectangle below

- Draw the diagonals and measure their lengths. Record the measurements on the figure.
- Measure the length of each segment of the diagonals from the vertex to the intersection point of the diagonals. Using a marker or crayon, color segments that have the same length. Use a different color for each different length.

4. List the properties that are shared by all of the rhombuses that you worked with today.

Rhombuses have 4 straight sides of equal lengths. Opposite sides are parallel. The diagonals are perpendicular bisectors. The angles next to each other add up to 180° .

5. List the properties that are shared by all of the rectangles that you worked with today.

Rectangles have 4 straight sides. Opposite sides are parallel and equal in length. Rectangles have 4 right angles. Diagonals are equal and bisect each other.

6. When can a trapezoid also be called a rhombus?

A trapezoid is also a rhombus when all 4 sides are the same length.

7. When can a parallelogram also be called a rectangle?

A parallelogram is also a rectangle when the 4 angles are 90° .

8. When can a quadrilateral also be called a rhombus?

A quadrilateral can also be called a rhombus when all 4 sides are equal in length.

Any combination of the questions below may be used to lead the discussion.

- (Allow students to share all the different rhombuses and rectangles that are produced in Problem 1 of the Problem Set.) What attributes do all rhombuses share? What attributes appear on the rhombus list that were not on the list for parallelograms? What attributes do all rectangles share? Is this true for all quadrilaterals? Rhombuses? Rectangles? (Use the rhombuses and rectangles produced in Problem 1 to articulate the formal definitions. Continue posting definitions for comparisons.)
- When can a quadrilateral also be called a rhombus? When can a quadrilateral also be called a rectangle?
- Respond to the following statements with true or false. Explain your reasoning.
 - All parallelograms are rhombuses. (False. Parallelograms have pairs of opposite sides that are the same length, but both pairs do not necessarily have the same length.)
 - All rhombuses are parallelograms. (True. Rhombuses have all sides that are equal in length, and the opposite sides are also parallel.)
 - All parallelograms are rectangles. (False. It is possible to have a quadrilateral with two pairs of parallel sides that do not have four right angles.)
 - All rectangles are parallelograms. (True. If a quadrilateral has four right angles, it must have two pairs of parallel sides.)
 - All trapezoids are rhombuses. (False. Trapezoids have at least one pair of parallel sides, but the lengths of the sides do not have to be equal.)
 - All rhombuses are trapezoids. (True. If a quadrilateral has four equal sides, then the opposite sides will be parallel.)
 - All trapezoids are rectangles. (False. All rectangles must have four right angles. The defining attribute of a trapezoid is that it must have at least one pair of parallel sides.)
 - All rectangles are trapezoids. (True. In order to be a trapezoid, a quadrilateral needs to have at least one pair of parallel sides, and all rectangles have two pairs of parallel sides.)
- Continue the construction of the hierarchy diagram from Lessons 16 and 17. Students might draw or glue examples of rhombuses and rectangles and list attributes within the diagram. Encourage them to explain their placement of the figures in the hierarchy.

A rhombus:

- Is a quadrilateral with all sides of equal length.

A rectangle:

- Is a quadrilateral with four right angles.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

If students are confused about the segments of a quadrilateral lying in the same plane or intersecting only at their endpoints, use the straws from Lesson 16 to demonstrate counter examples.

- Continue exploring the formal definition of a quadrilateral through the examination of counter-examples. (See the text box with the definition.)

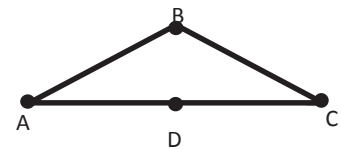
A quadrilateral:

- Consists of four different points, A, B, C, D , in the plane and four segments, $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$,
- Is arranged so that the segments intersect only at their endpoints, and
- Has no two adjacent segments that are collinear.

Step 1: Begin by asking students to tell what they know about a quadrilateral. Today's response should be a polygon with four straight sides that lie in the same plane and segments that only intersect at their endpoints.

Step 2: Follow the first and second bullets in the definition verbatim to draw four straight segments in the same plane that only intersect at their endpoints but have collinear endpoints as shown to the right.

Ask: "Is this figure also a quadrilateral? What must we add to our definition to eliminate the possibility of this figure?"



Step 3: Lead students to see that a four-sided figure is only a quadrilateral if all four segments lie in the same plane, the segments intersect only at their endpoints, *and no two segments are collinear*. Then, add the third bullet of the definition to that written in Lesson 17.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

A

Number Correct: _____

Divide Whole Numbers by Fractions and Fractions by Whole Numbers

1.	$\frac{1}{2} \div 2 =$	
2.	$\frac{1}{2} \div 3 =$	
3.	$\frac{1}{2} \div 4 =$	
4.	$\frac{1}{2} \div 7 =$	
5.	$7 \div \frac{1}{2} =$	
6.	$6 \div \frac{1}{2} =$	
7.	$5 \div \frac{1}{2} =$	
8.	$3 \div \frac{1}{2} =$	
9.	$2 \div \frac{1}{5} =$	
10.	$3 \div \frac{1}{5} =$	
11.	$4 \div \frac{1}{5} =$	
12.	$7 \div \frac{1}{5} =$	
13.	$\frac{1}{5} \div 7 =$	
14.	$\frac{1}{3} \div 2 =$	
15.	$2 \div \frac{1}{3} =$	
16.	$\frac{1}{4} \div 2 =$	
17.	$2 \div \frac{1}{4} =$	
18.	$\frac{1}{5} \div 2 =$	
19.	$2 \div \frac{1}{5} =$	
20.	$3 \div \frac{1}{4} =$	
21.	$\frac{1}{4} \div 3 =$	
22.	$\frac{1}{4} \div 4 =$	

23.	$4 \div \frac{1}{4} =$	
24.	$\frac{1}{3} \div 3 =$	
25.	$\frac{2}{3} \div 3 =$	
26.	$\frac{1}{4} \div 2 =$	
27.	$\frac{3}{4} \div 2 =$	
28.	$\frac{1}{5} \div 2 =$	
29.	$\frac{3}{5} \div 2 =$	
30.	$\frac{1}{6} \div 2 =$	
31.	$\frac{5}{6} \div 2 =$	
32.	$\frac{5}{6} \div 3 =$	
33.	$\frac{1}{6} \div 3 =$	
34.	$3 \div \frac{1}{6} =$	
35.	$6 \div \frac{1}{6} =$	
36.	$7 \div \frac{1}{7} =$	
37.	$8 \div \frac{1}{8} =$	
38.	$9 \div \frac{1}{9} =$	
39.	$\frac{1}{8} \div 7 =$	
40.	$9 \div \frac{1}{8} =$	
41.	$\frac{1}{8} \div 7 =$	
42.	$7 \div \frac{1}{6} =$	
43.	$9 \div \frac{1}{7} =$	
44.	$\frac{1}{8} \div 9 =$	

B

Number Correct: _____

Improvement: _____

Divide Whole Numbers by Fractions and Fractions by Whole Numbers

1.	$\frac{1}{2} \div 2 =$	
2.	$\frac{1}{5} \div 3 =$	
3.	$\frac{1}{5} \div 4 =$	
4.	$\frac{1}{5} \div 7 =$	
5.	$7 \div \frac{1}{5} =$	
6.	$6 \div \frac{1}{5} =$	
7.	$5 \div \frac{1}{5} =$	
8.	$3 \div \frac{1}{5} =$	
9.	$2 \div \frac{1}{2} =$	
10.	$3 \div \frac{1}{2} =$	
11.	$4 \div \frac{1}{2} =$	
12.	$7 \div \frac{1}{2} =$	
13.	$\frac{1}{2} \div 7 =$	
14.	$\frac{1}{4} \div 2 =$	
15.	$2 \div \frac{1}{4} =$	
16.	$\frac{1}{3} \div 2 =$	
17.	$2 \div \frac{1}{3} =$	
18.	$\frac{1}{2} \div 2 =$	
19.	$2 \div \frac{1}{2} =$	
20.	$4 \div \frac{1}{3} =$	
21.	$\frac{1}{3} \div 4 =$	
22.	$\frac{1}{3} \div 3 =$	

23.	$3 \div \frac{1}{3} =$	
24.	$\frac{1}{4} \div 4 =$	
25.	$\frac{3}{4} \div 4 =$	
26.	$\frac{1}{3} \div 3 =$	
27.	$\frac{2}{3} \div 3 =$	
28.	$\frac{1}{6} \div 2 =$	
29.	$\frac{5}{6} \div 2 =$	
30.	$\frac{1}{5} \div 5 =$	
31.	$\frac{3}{5} \div 5 =$	
32.	$\frac{3}{5} \div 4 =$	
33.	$\frac{1}{5} \div 6 =$	
34.	$6 \div \frac{1}{5} =$	
35.	$6 \div \frac{1}{4} =$	
36.	$7 \div \frac{1}{6} =$	
37.	$8 \div \frac{1}{7} =$	
38.	$9 \div \frac{1}{8} =$	
39.	$\frac{1}{8} \div 8 =$	
40.	$9 \div \frac{1}{9} =$	
41.	$\frac{1}{9} \div 8 =$	
42.	$7 \div \frac{1}{7} =$	
43.	$9 \div \frac{1}{6} =$	
44.	$\frac{1}{8} \div 6 =$	

Name _____

Date _____

1. Draw the figures in each box with the attributes listed.

a. Rhombus with no right angles

b. Rectangle with not all sides equal

c. Rhombus with 1 right angle

d. Rectangle with all sides equal

2. Use the figures you drew to complete the tasks below.

- Measure the angles of the figures with your protractor, and record the measurements on the figures.
- Use a marker or crayon to circle pairs of angles inside each figure with a sum equal to 180° . Use a different color for each pair.

3. Draw a rhombus and a rectangle below.
 - a. Draw the diagonals, and measure their lengths. Record the measurements on the figure.
 - b. Measure the length of each segment of the diagonals from the vertex to the intersection point of the diagonals. Using a marker or crayon, color segments that have the same length. Use a different color for each different length.
4.
 - a. List the properties that are shared by all of the rhombuses that you worked with today.
 - b. List the properties that are shared by all of the rectangles that you worked with today.
 - c. When can a trapezoid also be called a rhombus?
 - d. When can a parallelogram also be called a rectangle?
 - e. When can a quadrilateral also be called a rhombus?

Name _____

Date _____

1. Draw a rhombus.

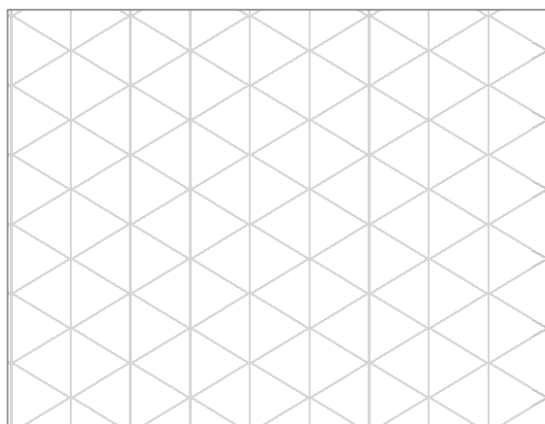
2. Draw a rectangle.

Name _____

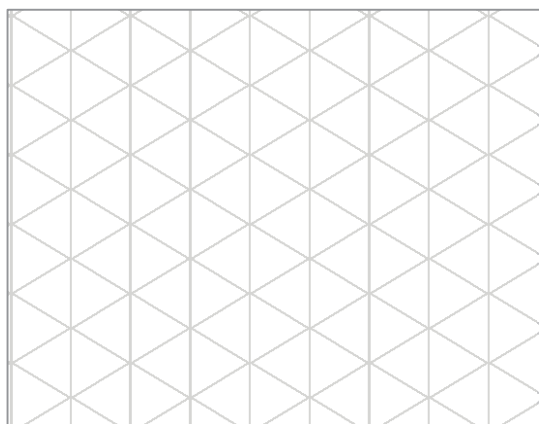
Date _____

1. Use the grid paper to draw.

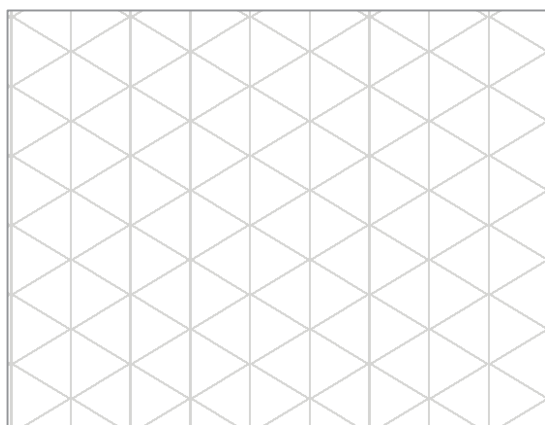
a. A rhombus with no right angles



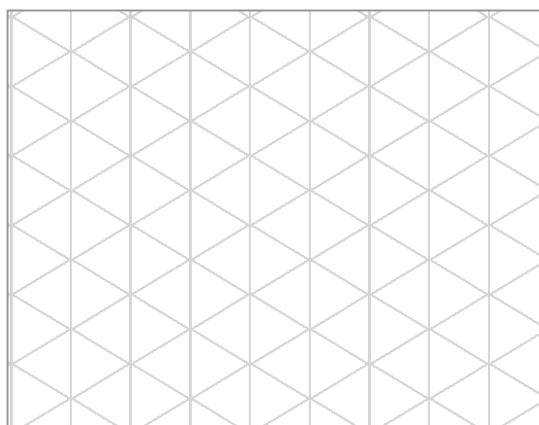
b. A rhombus with 4 right angles



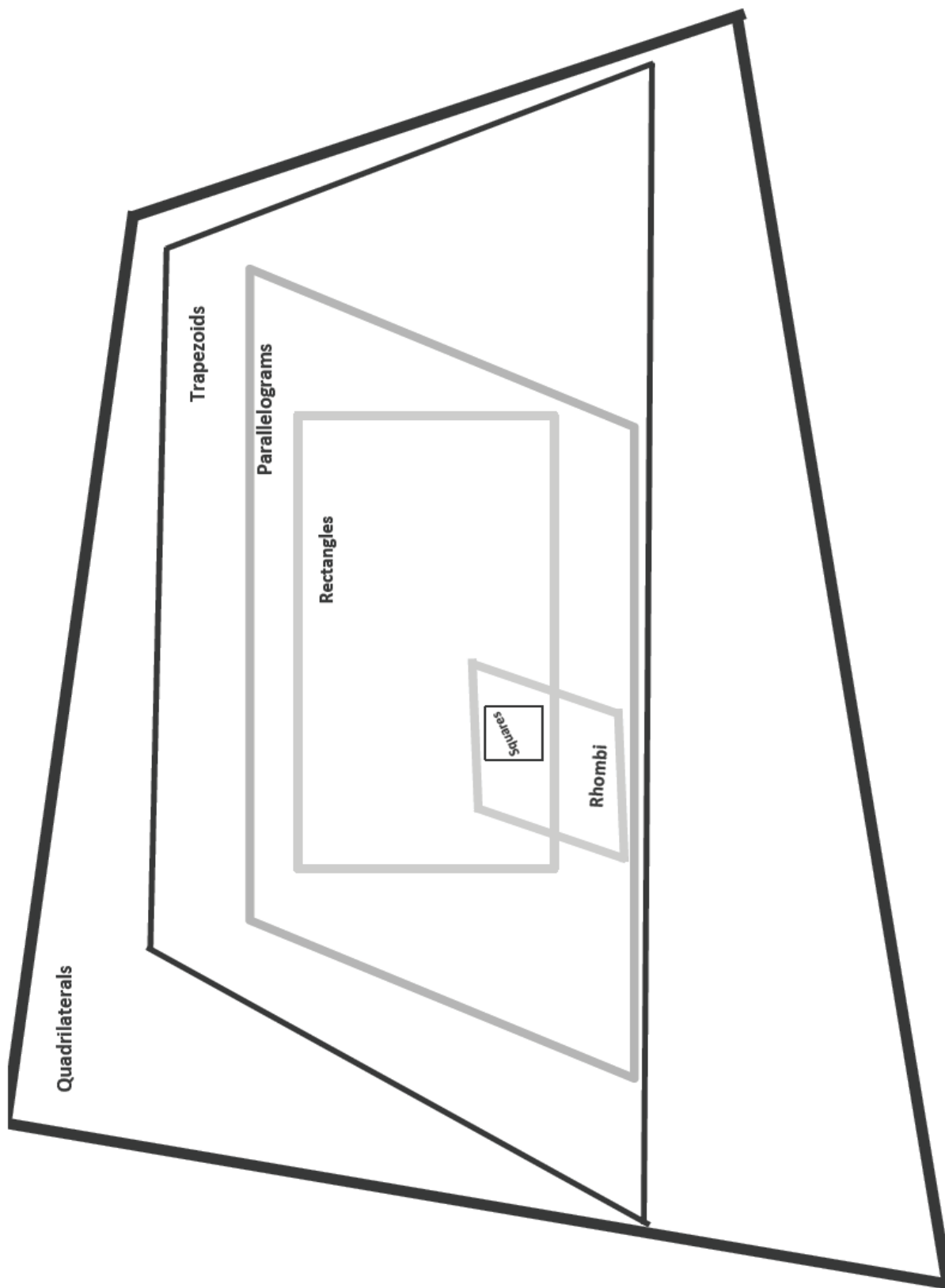
c. A rectangle with not all sides equal



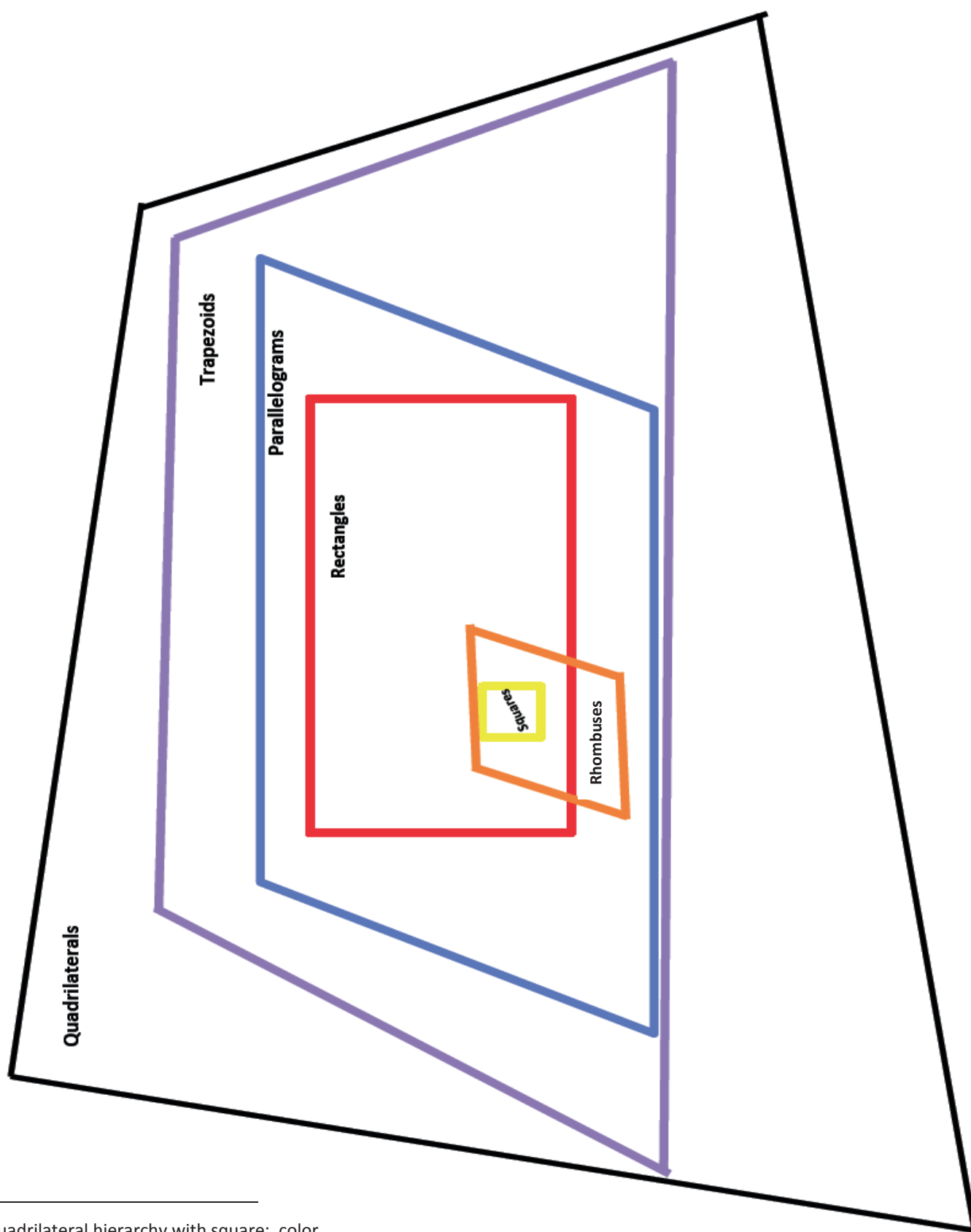
d. A rectangle with all sides equal



2. A rhombus has a perimeter of 217 cm. What is the length of each side of the rhombus?
3. List the properties that all rhombuses share.
4. List the properties that all rectangles share.



quadrilateral hierarchy with square



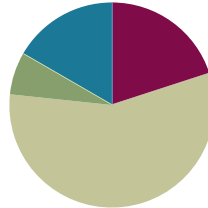
quadrilateral hierarchy with square: color

Lesson 19

Objective: Draw kites and squares to clarify their attributes, and define kites and squares based on those attributes.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(4 minutes)
■ Concept Development	(34 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Sprint: Multiply by Multiples of 10 and 100 (8 minutes)
- Divide by Multiples of 10 and 100 (4 minutes)

Sprint: Multiply by Multiples of 10 and 100 (8 minutes)

Materials: (S) Multiply by Multiples of 10 and 100 Sprint

Note: This fluency activity reviews Module 2.

Divide by Multiples of 10 and 100 (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 2.

T: (Write $240 \div 10 = \underline{\quad}$.) Say the division sentence.

S: $240 \div 10 = 24$.

T: (Write $240 \div 10 = 24$. To the right, write $24 \div 2 = \underline{\quad}$.) Say the division sentence.

S: $24 \div 2 = 12$.

T: (Write $24 \div 2 = 12$. Below it, write $240 \div 20 = \underline{\quad}$.) Say $240 \div 20$ as a division sentence, but divide first by 10 and then by 2 rather than by 20.

S: $240 \div 10 \div 2 = 12$.

T: (Write $240 \div 20 = 12$.)

$$240 \div 10 = 24$$

$$24 \div 2 = 12$$

$$240 \div 20 = 12$$

$$\begin{array}{ccc} & \swarrow & \searrow \\ 10 & \times & 2 \end{array}$$

Continue the process for the following possible sequence: $690 \div 30$, $8,600 \div 20$, $4,800 \div 400$, and $9,600 \div 300$.

Application Problem (4 minutes)

The teacher asked her class to draw parallelograms that are rectangles. Kylie drew Figure 1, and Zach drew Figure 2. Zach agrees that Kylie has drawn a parallelogram but says that it is not a rectangle. Is he correct? Use properties to justify your answer.

Note: Today's Application Problem gives students another opportunity to verbalize the hierarchical nature of the relationships between types of quadrilaterals.

Zach is not correct. Kylie's figure is a rectangle. The opposite sides are equal lengths, parallel, and all 4 angles are 90° .

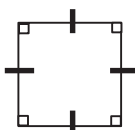


Figure 1

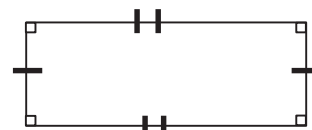


Figure 2

Concept Development (34 minutes)

Materials: (T) Quadrilateral hierarchy with kite: color (Template 2) (S) Ruler, set square or square template, protractor, scissors, quadrilateral hierarchy with kite (Template 1)

Problem 1

- Draw a square, and articulate the definition.
- Measure and label its angles to explore their relationships.
- Measure to explore diagonals of squares.

T: What shapes have we drawn so far?

S: Quadrilaterals. → Rhombuses and rectangles. → Trapezoids and parallelograms, but in a rhombus all sides are the same length.

T: Can a rectangle ever be a rhombus? Can a rhombus ever be a rectangle? Turn and talk.

S: Well, a rectangle and a rhombus are both parallelograms. A rectangle must have right angles, but a rhombus can have angles that are not right angles. → Rhombuses must have four equal sides, but rectangles might not. → A square is a rhombus and a rectangle at the same time.

T: Let's see if we can answer this question by drawing.

T: Draw a segment 3 inches long on your blank paper, and label the endpoints A and B .

S: (Draw a segment.)

T: (Demonstrate.) Now, using your set square, draw three-inch segments from both point A and point B at a 90° angle to \overline{AB} . Draw both line segments toward the bottom of your paper.

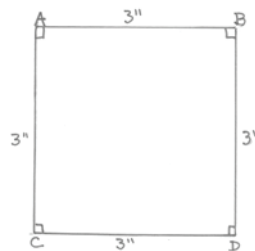
S: (Draw additional line segments.)

T: Label the endpoints as C and D . Are \overline{AC} and \overline{BD} parallel? How do you know?

S: I checked with my set square. They are parallel. → They must be parallel because we drew them both as right angles to the same segment.

T: Use your straightedge to connect points C and D .

- T: Measure segment \overline{CD} . What is its length?
- S: \overline{CD} is also 3 inches long.
- T: What have we drawn? How do you know?
- S: A square. It has four right angles and four equal sides.
- T: Based on the properties of parallel sides, tell your partner another name for this shape, and justify your choice.
- S: It is a trapezoid. It has a pair of parallel sides. → I can call this a parallelogram because there are two sets of parallel sides.
- T: Use your protractor to measure angles C and D . What are their measures?
- S: 90° . → All of the angles are 90° .
- T: Since this is a parallelogram with four right angles and two sets of opposite equal sides, what can we call it?
- S: A rectangle.
- T: Since this is a parallelogram with four equal sides, what can we call it?
- S: A rhombus.
- T: Let's return to our question, can a rhombus ever be a rectangle? Can a rectangle ever be a rhombus? Why or why not?
- S: Yes. A square is a rhombus and a rectangle at the same time. → A rectangle can be a rhombus if it is a square. → A rhombus can be a rectangle if it is a square.
- T: Using what you just drew, list the attributes of a square with your partner.
- S: A square has four sides that are equal and four right angles. → A square has opposite sides that are parallel, four right angles, and sides that are all equal in length. → A square is a rectangle with four sides that are equal length. → A square is a rhombus with four right angles.
- T: Draw the diagonals of the square. Before we measure them, predict whether the diagonals will bisect each other, and justify your predictions using properties. Turn and talk.
- S: The parallelograms we drew had bisecting diagonals, and this is definitely a parallelogram. I think the diagonals will bisect. → I think they will bisect each other because a square is a rectangle, and all the rectangles' diagonals we measured bisected each other. → We drew rhombuses yesterday, and all those diagonals bisected each other. A square is a rhombus, so that should be true in a square, too.
- T: Measure the lengths of the diagonals. Then, to test your prediction, measure the distance from each corner to the point where they intersect.
- S: (Draw and measure.)



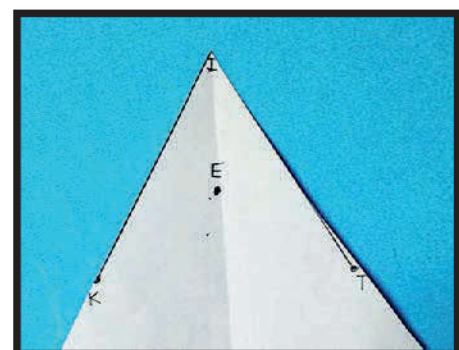
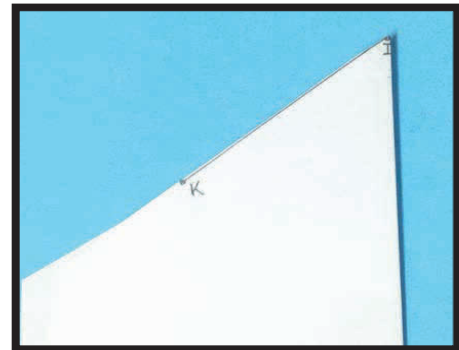
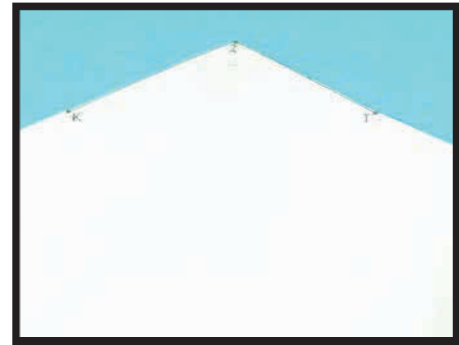
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

English language learners and others may feel overwhelmed distinguishing terms in this lesson. To support understanding, point to a picture or make gestures to clarify the meaning of *parallel*, *rhombus*, *attribute*, etc., each time they are mentioned. Building additional checks for understanding into instruction may also prove helpful, as might recording student observations of shape attributes and definitions in a list, table, or graphic organizer.

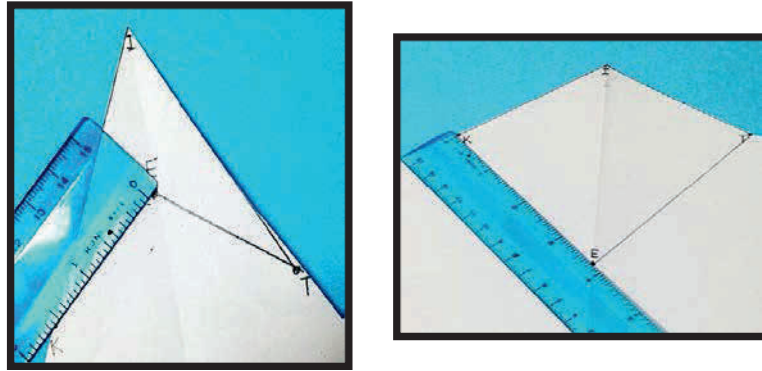
- T: What did you find?
- S: The diagonals do bisect each other.
- T: Now, use the protractor to measure the angles where the diagonals intersect.
- S: (Measure the intersecting angles.)
- T: What did you find?
- S: They intersect at right angles. → All the angles are 90° .
→ The diagonals are perpendicular to each other.
- T: When the diagonals of a quadrilateral bisect each other at a 90° angle, we say the diagonals are perpendicular bisectors.

Problem 2

- Draw a kite, and articulate the definition.
 - Measure and label its sides and angles to explore their relationships.
 - Measure to explore diagonals of kites.
- T: We have one more quadrilateral to explore. Let's see if you can guess the figure if I give you some real-world clues. It works best outside on windy days, and it is flown with a string. (Give clues until the figure is named.)
- S: A kite.
- T: Sketch a kite.
- S: (Sketch.)
- T: Compare your kite to your neighbor's. How are they alike? How are they different? Turn and talk.
- S: Mine is narrow, and my partner's is wider. → Mine is taller, and my partner's is shorter. → They all have four sides.
- T: Let's draw a kite using our tools. Draw an angle of any measure with two sides that are the same length but at least two inches long. Mark the vertex as I and the endpoints of the segments as K and T .
- S: (Draw a kite.)
- T: Use your scissors to cut along the rays of your angle.
- S: (Cut along the rays.)
- T: Fold your angle in half matching points K and T . (Refer to the image.) Open it, and mark a point on the fold, and label it E .
- S: (Fold and label.)
- T: Use your ruler to connect your point to the ends of the other segments. Then, cut out your kite.
- S: (Cut out the kite.)



- T: Measure the two sides that you just drew. What do you notice about the sides? How are they different from parallelograms?



- S: There are two sets of sides that are equal to each other, but they are next to each other, not across from each other. → Opposite sides are not equal on mine, but adjacent sides are. → None of these sides are parallel to each other.
- T: Use your protractor to measure the angles of your kite, and record the measurements on your figure.
- S: (Measure and record the angles of the kite.)
- T: What do you notice? Turn and talk. (Allow students time to share with a partner.)
- S: There are always at least two angles of equal measure. They are across from each other where the unequal sides meet. → My kite is also a square. It has two pairs of equal angles.
- T: Now, draw the diagonals of the kite. Measure the length of the diagonals, the segments of the diagonals, and the angles where the diagonals intersect.
- S: (Draw and measure the diagonals, segments, and angles.)
- T: What can you say about the diagonals of a kite? Turn and talk.
- S: My diagonals cross outside my kite, but they are still perpendicular. → The diagonals are not the same length. → The diagonals meet at 90° angles; they are perpendicular. → One diagonal bisects the other, but they are not both bisected.
- T: What are the attributes of a kite? Tell your partner.
- S: A kite is a quadrilateral with equal adjacent sides. → It is a quadrilateral with two pairs of adjacent sides that have equal lengths.



NOTES ON KITES:

If no student produces a concave kite (an arrowhead) through the process of drawing in the lesson, draw one for students to consider. It is important to note that although the diagonals do not intersect within the kite, the same relationships hold true. The lines containing the diagonals intersect at a right angle, and only one bisects the other. Students who produce such a kite may need help drawing the diagonals.

- T: A kite is a quadrilateral that has adjacent sides, or sides next to each other, that are equal. Can a kite ever be a parallelogram? Can a parallelogram ever be a kite? Why or why not? Turn and talk.
- S: Yes, a parallelogram can be a kite. A square and a rhombus both have all equal sides, so that fits the definition of a kite. → Squares and rhombuses have sides next to each other that are equal. They are the only parallelograms that could also be called kites. → Any quadrilateral with all sides equal would have adjacent sides equal, so a rhombus and a square are kites.
- T: (Distribute the Problem Set to students.) Let's practice drawing more squares and kites and thinking about their attributes by completing the Problem Set.
- S: (Complete the Problem Set.)

Problem Set

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Draw kites and squares to clarify their attributes, and define kites and squares based on those attributes.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

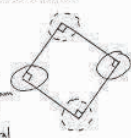
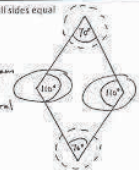
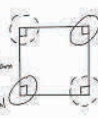
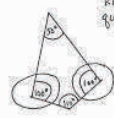
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- (Allow students to share the myriad of squares and kites that are produced in Problem 1 of the Problem Set.) Compare and contrast these quadrilaterals.
- (Use the figures produced in Problem 1 to articulate the formal definitions of both squares and kites. Continue to post the definitions.)

Name: Lisa Date: _____

1. Draw the figures in each box with the attributes listed. If your figure has more than one name, write it in the box.

<p>a. Rhombus with 2 right angles</p> <p>rhombus kite square rectangle parallelogram trapezoid quadrilateral</p> 	<p>b. Kite with all sides equal</p> <p>kite rhombus parallelogram trapezoid quadrilateral</p> 
<p>c. Kite with 4 right angles</p> <p>kite square rhombus rectangle parallelogram trapezoid quadrilateral</p> 	<p>d. Kite with 2 pairs of adjacent sides equal (The pairs are not equal to each other.)</p> <p>kite quadrilateral</p> 

2. Use the figures you drew to complete the tasks below.

a. Measure the angles of the figures with your protractor, and record the measurements on the figures.

b. Use a marker or crayon to circle pairs of angles that are equal in measure, inside each figure. Use a different color for each pair.

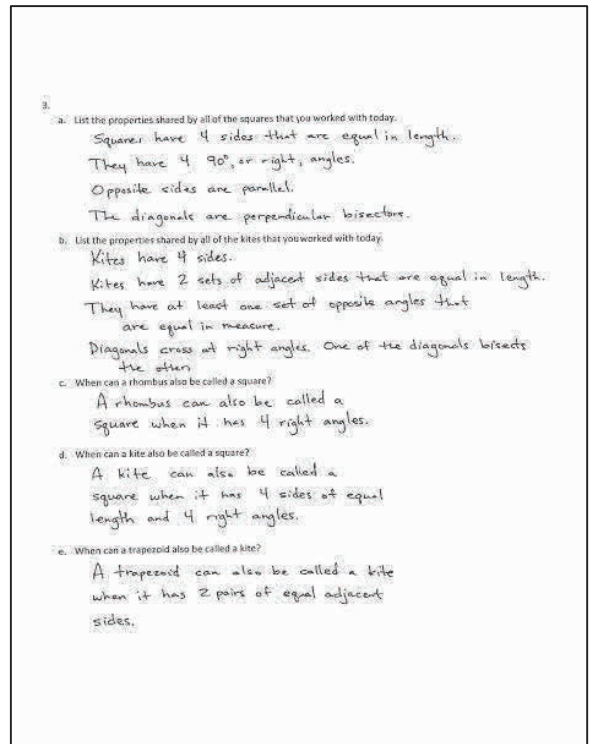
A square:

- Is a rhombus with four right angles.
- Is a rectangle with four equal sides.

A kite:

- Is a quadrilateral in which two consecutive sides have equal length.
- Has two remaining sides of equal length.

- (Consolidate the lists of attributes students generated for squares and kites in Problem 3.) What attributes do all squares share? What attributes do all kites share? When is a quadrilateral a kite but not a square or rhombus?
- When can a quadrilateral also be called a square?
- Respond to the following statements with *true* or *false*. Explain your reasoning.
 - All squares are quadrilaterals. (True. All squares have four sides.)
 - All quadrilaterals are squares. (False. Not all quadrilaterals have equal side lengths.)
 - All rhombuses are squares. (False. All rhombuses must have four equal sides, but the angles do not have to be 90° .)
 - All squares are rhombuses. (True. The defining attribute of a rhombus is that its four sides must be equal in length. Squares also have four sides of equal length.)
 - All rectangles are squares. (False. Rectangles do not have to have four sides of equal length.)
 - All squares are rectangles. (True. The defining attribute of a rectangle is that it must have four right angles. All squares have four right angles.)
 - All squares are parallelograms. (True. All parallelograms have two sets of parallel sides. All squares have two sets of parallel sides.)
 - All parallelograms are squares. (False. Squares must have four right angles and four equal sides. If a shape has four right angles, it will also have two sets of parallel sides. Parallelograms only need to have two sets of parallel sides.)
 - All kites are quadrilaterals. (True. All kites are closed, four-sided figures.)
 - All quadrilaterals are kites. (False. There are many four-sided shapes that do not have two sets of equal adjacent sides.)
 - All kites are squares. (False. Squares must have 4 equal sides, but kites only need to have 2 pairs of equal sides.)
 - All squares are kites. (True. Squares have 2 pairs of equal sides.)
- (Finish the construction of the hierarchy diagram. (See Templates 1 and 2 at the end of this lesson.) Students might draw or glue examples of squares and kites or list attributes within the diagram. Encourage students to explain their placement of the figures in the hierarchy.)



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

A

Number Correct: _____

Multiply by Multiples of 10 and 100

1.	$2 \times 10 =$	
2.	$12 \times 10 =$	
3.	$12 \times 100 =$	
4.	$4 \times 10 =$	
5.	$34 \times 10 =$	
6.	$34 \times 100 =$	
7.	$7 \times 10 =$	
8.	$27 \times 10 =$	
9.	$27 \times 100 =$	
10.	$3 \times 10 =$	
11.	$3 \times 2 =$	
12.	$3 \times 20 =$	
13.	$13 \times 10 =$	
14.	$13 \times 2 =$	
15.	$13 \times 20 =$	
16.	$13 \times 100 =$	
17.	$13 \times 200 =$	
18.	$2 \times 4 =$	
19.	$22 \times 4 =$	
20.	$22 \times 40 =$	
21.	$22 \times 400 =$	
22.	$33 \times 2 =$	

23.	$33 \times 20 =$	
24.	$33 \times 200 =$	
25.	$24 \times 10 =$	
26.	$24 \times 20 =$	
27.	$24 \times 100 =$	
28.	$24 \times 200 =$	
29.	$23 \times 30 =$	
30.	$23 \times 300 =$	
31.	$71 \times 2 =$	
32.	$71 \times 20 =$	
33.	$14 \times 2 =$	
34.	$14 \times 3 =$	
35.	$14 \times 30 =$	
36.	$14 \times 300 =$	
37.	$82 \times 20 =$	
38.	$15 \times 300 =$	
39.	$71 \times 600 =$	
40.	$18 \times 40 =$	
41.	$75 \times 30 =$	
42.	$84 \times 300 =$	
43.	$87 \times 60 =$	
44.	$79 \times 800 =$	

B

Number Correct: _____

Improvement: _____

Multiply by Multiples of 10 and 100

1.	$3 \times 10 =$	
2.	$13 \times 10 =$	
3.	$13 \times 100 =$	
4.	$5 \times 10 =$	
5.	$35 \times 10 =$	
6.	$35 \times 100 =$	
7.	$8 \times 10 =$	
8.	$28 \times 10 =$	
9.	$28 \times 100 =$	
10.	$4 \times 10 =$	
11.	$4 \times 2 =$	
12.	$4 \times 20 =$	
13.	$14 \times 10 =$	
14.	$14 \times 2 =$	
15.	$14 \times 20 =$	
16.	$14 \times 100 =$	
17.	$14 \times 200 =$	
18.	$2 \times 3 =$	
19.	$22 \times 3 =$	
20.	$22 \times 30 =$	
21.	$22 \times 300 =$	
22.	$44 \times 2 =$	

23.	$44 \times 20 =$	
24.	$44 \times 200 =$	
25.	$42 \times 10 =$	
26.	$42 \times 20 =$	
27.	$42 \times 100 =$	
28.	$42 \times 200 =$	
29.	$32 \times 30 =$	
30.	$32 \times 300 =$	
31.	$81 \times 2 =$	
32.	$81 \times 20 =$	
33.	$13 \times 3 =$	
34.	$13 \times 4 =$	
35.	$13 \times 40 =$	
36.	$13 \times 400 =$	
37.	$72 \times 30 =$	
38.	$15 \times 300 =$	
39.	$81 \times 600 =$	
40.	$16 \times 40 =$	
41.	$65 \times 30 =$	
42.	$48 \times 300 =$	
43.	$89 \times 60 =$	
44.	$76 \times 800 =$	

Name _____

Date _____

1. Draw the figures in each box with the attributes listed. If your figure has more than one name, write it in the box.

a. Rhombus with 2 right angles

b. Kite with all sides equal

c. Kite with 4 right angles

d. Kite with 2 pairs of adjacent sides equal
(The pairs are not equal to each other.)

2. Use the figures you drew to complete the tasks below.

- Measure the angles of the figures with your protractor, and record the measurements on the figures.
- Use a marker or crayon to circle pairs of angles that are equal in measure, inside each figure. Use a different color for each pair.

3. a. List the properties shared by all of the squares that you worked with today.
- b. List the properties shared by all of the kites that you worked with today.
- c. When can a rhombus also be called a square?
- d. When can a kite also be called a square?
- e. When can a trapezoid also be called a kite?

Name _____

Date _____

1. List the property that must be present to call a rectangle a square.

2. Excluding rhombuses and squares, explain the difference between parallelograms and kites.

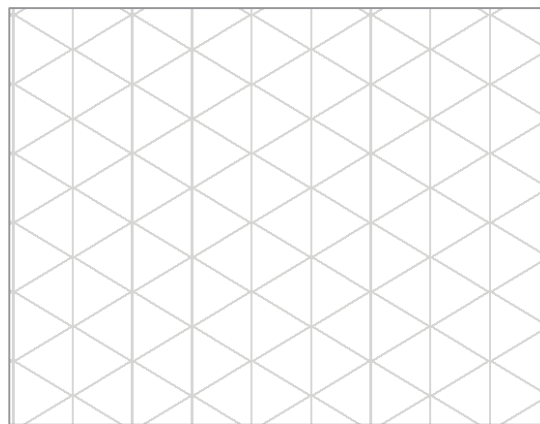
Name _____

Date _____

1. a. Draw a kite that is not a parallelogram on the grid paper.

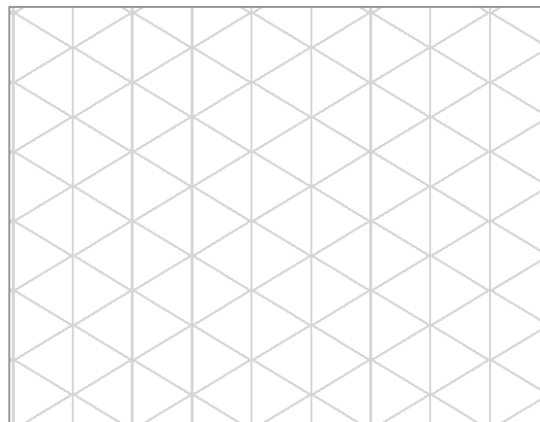
- b. List all the properties of a kite.

- c. When can a parallelogram also be a kite?

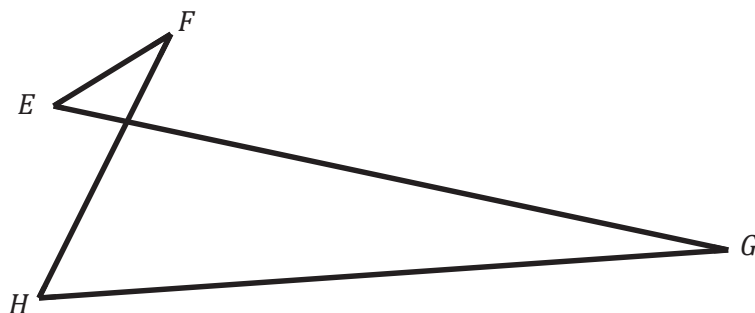


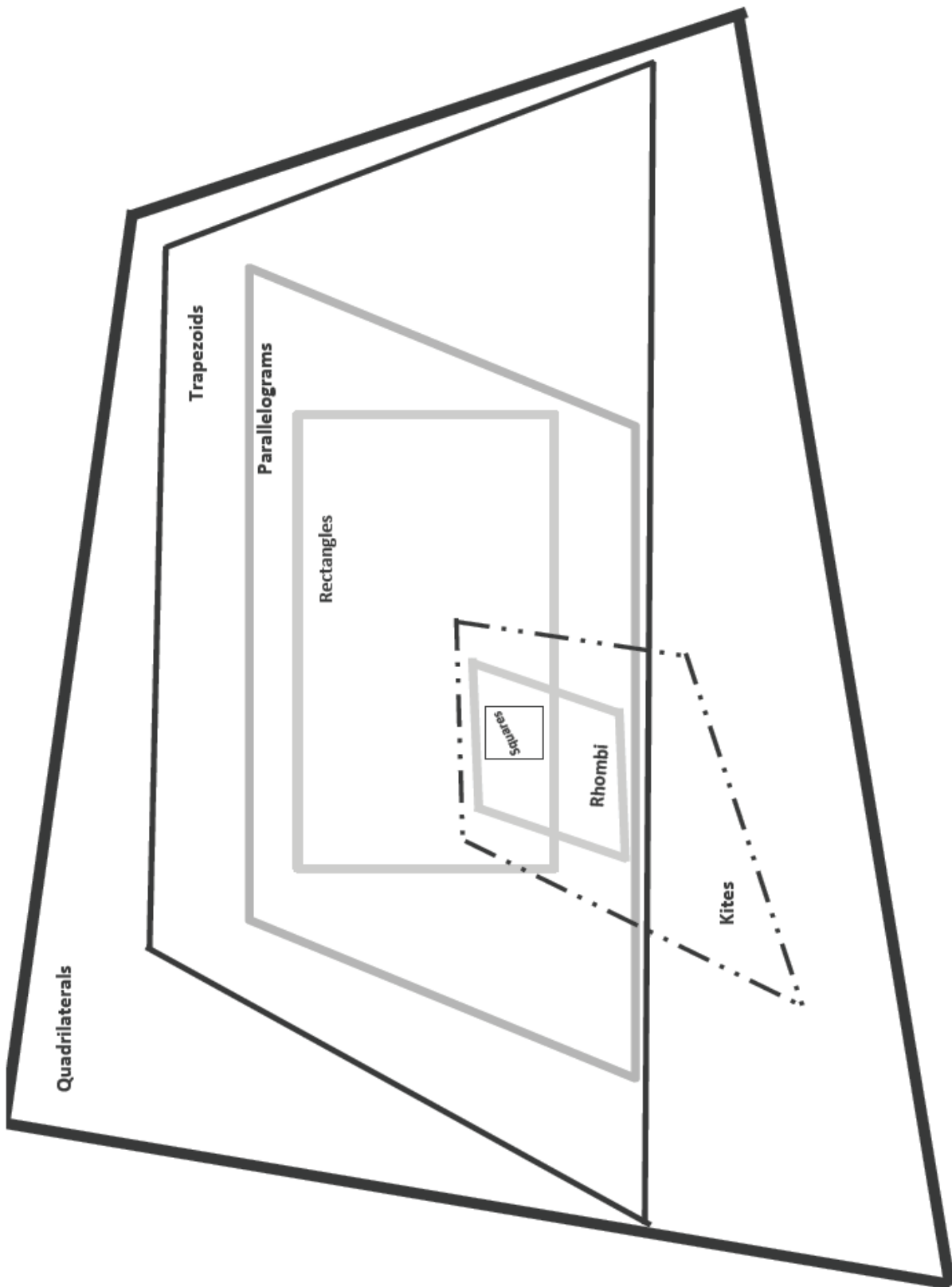
2. If rectangles must have right angles, explain how a rhombus could also be called a rectangle.

3. Draw a rhombus that is also a rectangle on the grid paper.

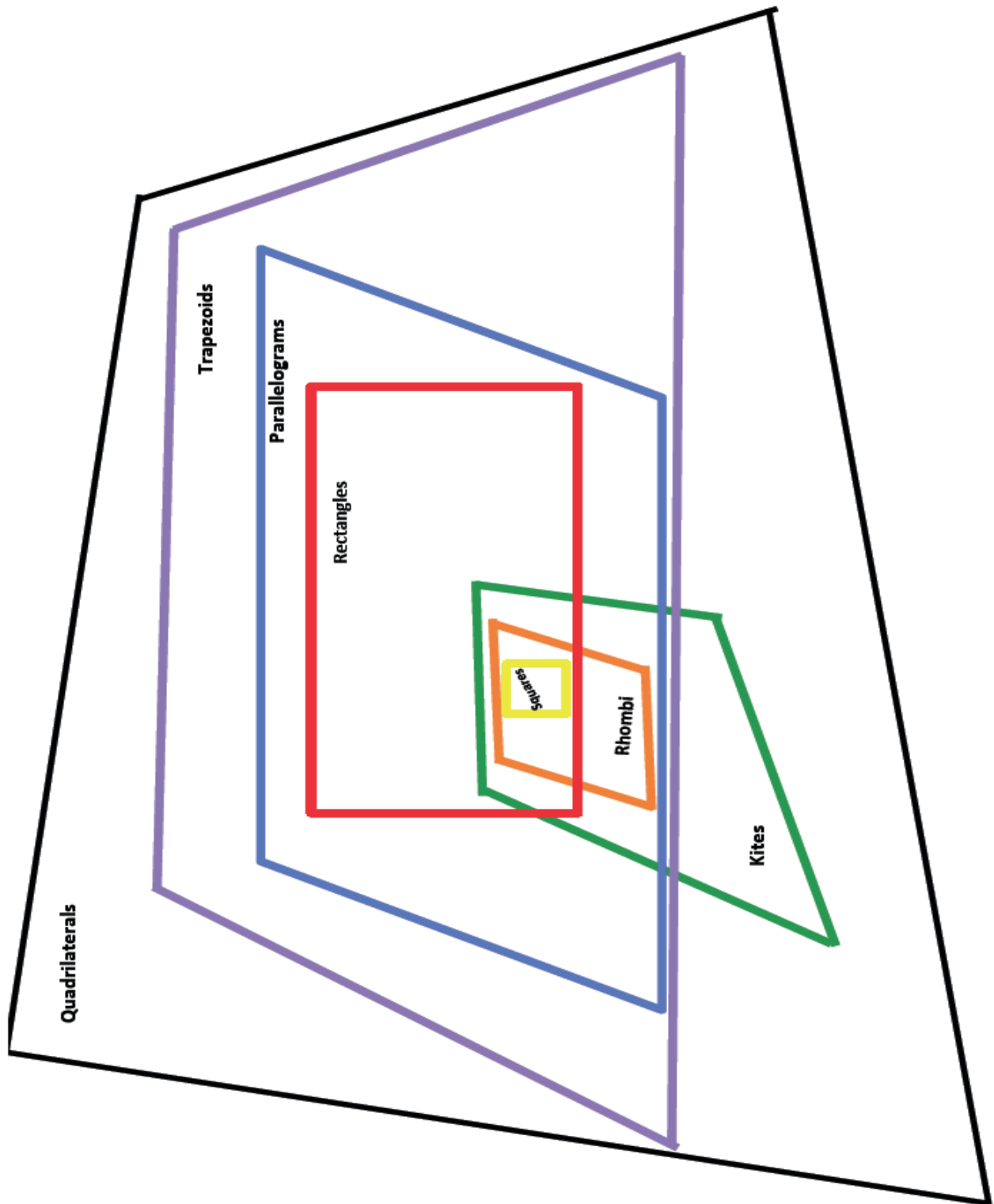


4. Kirkland says that figure $EFGH$ below is a quadrilateral because it has four points in the same plane and four segments with no three endpoints collinear. Explain his error.





quadrilateral hierarchy with kite



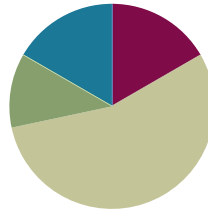
quadrilateral hierarchy with kite: color

Lesson 20

Objective: Classify two-dimensional figures in a hierarchy based on properties.

Suggested Lesson Structure

■ Fluency Practice	(10 minutes)
■ Application Problem	(7 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Divide by Multiples of 10 and 100 (4 minutes)
- Find the Volume (6 minutes)

Divide by Multiples of 10 and 100 (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Module 2.

T: (Write $930 \div 10 = \underline{\quad}$.) Say the completed division sentence.

S: $930 \div 10 = 93$.

T: (Write $930 \div 10 = \underline{\quad}$. To the right, write $93 \div 3 = \underline{\quad}$.) Say the complete division sentence.

S: $93 \div 3 = 31$.

T: (Write $93 \div 3 = 31$. Below it, write $930 \div 30 = \underline{\quad}$.) Say $930 \div 30$ as a division sentence, but divide first by 10 and then by 3.

S: $930 \div 10 \div 3 = 31$.

T: (Write $930 \div 30 = 31$.)

$$930 \div 10 = 93$$

$$93 \div 3 = 31$$

$$930 \div 30 = 31$$

$$\begin{array}{c} \swarrow \quad \searrow \\ 10 \quad \times \quad 3 \end{array}$$

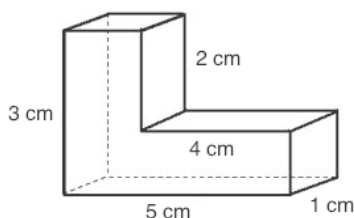
Continue the process for the following possible sequence: $420 \div 20$, $4,800 \div 40$, $8,400 \div 400$, and $6,900 \div 300$.

Find the Volume (6 minutes)

Materials: (S) Personal white board

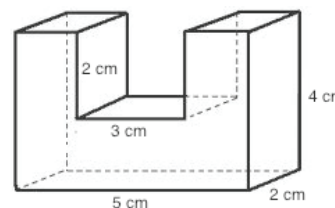
Note: This fluency activity reviews Topic B.

- T: Say the formula for finding the volume of a rectangular prism.
- S: Length times width times height.
- T: Visualize a line that breaks the figure into two rectangular prisms.



- T: Find the volume of the composite figure by adding the volumes of each rectangular prism.
- S: (Write $3 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 3 \text{ cubic cm}$, $4 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 4 \text{ cubic cm}$, and $3 \text{ cubic cm} + 4 \text{ cubic cm} = 7 \text{ cubic cm}$.)

Continue the process for the composite figure to the right.

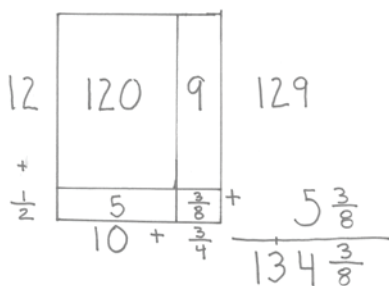


**NOTES ON
MULTIPLE MEANS
OF ACTION AND
EXPRESSION:**

Some students may benefit from building the composite three-dimensional figures for the Find the Volume fluency activity. Consider offering cubic units, and then, have them reanalyze the drawing after having built the figure.

Application Problem (7 minutes)

Nita buys a rug that is $10\frac{3}{4}$ feet \times $12\frac{1}{2}$ feet. What is the area of the rug? Show your thinking with an area model and a multiplication sentence.



$$\begin{aligned}
 &10\frac{3}{4} \times 12\frac{1}{2} \\
 &= (10 \times 12) + (10 \times \frac{1}{2}) + (\frac{3}{4} \times 12) + (\frac{3}{4} \times \frac{1}{2}) \\
 &= 120 + 5 + 9 + \frac{3}{8} \\
 &= 134\frac{3}{8}
 \end{aligned}$$

The area of the rug is $134\frac{3}{8} \text{ ft}^2$.

Note: Today's Application Problem reviews areas of regions with fractional sides from earlier in this module.

Concept Development (33 minutes)

Materials: (T) Quadrilateral hierarchy with kite: color (Lesson 19 Template 2), image of a trapezoid that is not a parallelogram (S) Personal white board, shape name cards (Template 1, 1 per pair of students), shapes for sorting (Template 2, 1 per pair of students), protractor, ruler, set square, quadrilateral hierarchy with kite (Lesson 19 Template 1, 1 per pair of students), scissors, glue

Part 1

Justify responses to true or false statements about quadrilaterals based on properties.

- a. Trapezoids are always quadrilaterals.
- b. Quadrilaterals are always trapezoids.

T: (Project Sentence (a) on the board.) Talk to your partner about whether the statement is true or false. Justify your answer using properties of the shapes.

S: This is true because all trapezoids have the properties of quadrilaterals. They just have an extra property; they have at least one set of parallel sides. → Look at this trapezoid I drew. It has four segments in the same plane that only intersect at their endpoints. You cannot draw a trapezoid without these properties of quadrilaterals. It is true that trapezoids are always quadrilaterals.

T: (Project Sentence (b) on the board.) What about this statement? Trapezoids are always quadrilaterals. Are quadrilaterals always trapezoids? Why or why not? Turn and talk.

S: This is not true. There are lots of quadrilaterals that do not have any parallel sides. → If a quadrilateral does not have parallel sides, it cannot be a trapezoid. This statement is false.

T: (Write on the board: _____ are always _____. Give pairs of students one copy of the shape name cards.) Write this sentence frame on your personal white boards, and turn all your cards facedown on your table.

S: (Write the sentence frame.)

T: Each partner should choose a shape name card and place it in one of the blanks in the sentence frame. Work together to decide whether your statement is true or false, and use the properties of the figures to justify your answer. Then, switch the cards in the frame, and repeat the sequence. Finally, put the cards back on the table facedown. (Allow students time to work.)

**NOTES ON
MULTIPLE MEANS
OF REPRESENTATION:**

Depending on the English proficiency level of English language learners, it might be helpful to demonstrate how to justify responses to true or false statements, give extra response time, or provide sentence frames or starters, such as the following:

- The statement is true or false because ...
- I disagree because ...

Part 2

Classify two-dimensional figures in a hierarchy using tools to confirm properties.

- T: (Project an image of a trapezoid that is not a parallelogram and the quadrilateral hierarchy with kite: color (Lesson 19 - Template 2) for students to see.) What does this shape look like?
- S: Quadrilateral. → A trapezoid.
- T: How could I use my tools to be sure of these classifications? What properties would I need to confirm in order to classify this shape as a trapezoid? Turn and talk.
- S: It is two-dimensional, and it has four sides, so we know it is a quadrilateral. → I can see it is a quadrilateral, but to be sure it is a trapezoid, I could use my set square to check if it has at least one pair of parallel sides.
- T: I will confirm for you that this figure does have four segments in the same plane, and they only intersect at their endpoints. None of the endpoints are collinear, and it has one pair of parallel sides. With that information, could I place this figure inside the quadrilateral on the hierarchy diagram? Why or why not?
- S: Yes. It is a quadrilateral.
- T: (Place the figure on the diagram inside the quadrilateral only.) Could I place it inside the trapezoid on the hierarchy diagram? Why or why not?
- S: Yes, it can go there because it has one set of parallel sides.
- T: Can I place it inside the parallelogram on the hierarchy? Why or why not?
- S: No. It does not have two sets of parallel sides, so it cannot go inside the parallelogram.
- T: This figure is inside the quadrilateral ring and the trapezoid ring. What does that mean for its properties?
- S: It has all the properties of a quadrilateral and all the properties of a trapezoid.
- T: (Give a copy of quadrilateral hierarchy with kite (Lesson 19 – Template 1) and one shapes for sorting sheet to each pair of students.) Work with your partner to classify the shapes on your sheet. Use your tools to confirm their properties. Then, cut out the shapes, and glue them on the hierarchy diagram. Be prepared to defend their placements.
- S: (Work.)

Circulate and ask questions of students as they confirm properties and sort. Encourage students to verbalize that attributes belonging to a category of figures also belong to all subcategories of the figure. The following sentence frame might be used: Because a _____ is a _____, it must have _____. (For example: Because a rhombus is a trapezoid, it must have at least one set of parallel sides.)

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Classify two-dimensional figures in a hierarchy based on properties.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- (The false statements in Problem 1 may be corrected in many ways. Allow students the opportunity to share approaches.)
- (Students might be challenged to draw counter-examples for false statements in Problem 1.)
- (Ask students to draw other quadrilaterals with the same attributes as those in Problem 2.) What is the most specific name for the shape? What is the least specific name?
- (Review the formal definitions of all the quadrilaterals from the topic. Compare them with a view toward noticing the hierarchical nature. For example, a rhombus is a parallelogram with four equal sides. Point out that because a rhombus *is* a parallelogram, it has all the attributes of a parallelogram *and* four equal sides.)
- As the most specific quadrilateral that we have explored, can a square can be correctly classified as any of the quadrilaterals on the hierarchy? (Making a list of all of a square's attributes using sides, angles, and diagonals, can help drive home the understanding of the hierarchy.)

Name Jeremy Date _____

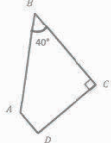
1. True-False. If the statement is false, rewrite it to make it true.

	T	F
a. All trapezoids are quadrilaterals.	✓	
b. All parallelograms are rhombuses. <u>Some parallelograms are rhombuses.</u>		✓
c. All squares are trapezoids.	✓	
d. All rectangles are squares. <u>All squares are rectangles.</u>		✓
e. Rectangles are always parallelograms.	✓	
f. All parallelograms are trapezoids.	✓	
g. All rhombuses are rectangles. <u>Some rhombuses are rectangles.</u>		✓
h. Kites are never rhombuses. <u>Kites are sometimes rhombuses.</u>		✓
i. All squares are kites.	✓	
j. All kites are squares. <u>Some kites are squares.</u>		✓
k. All rhombuses are squares. <u>All squares are rhombuses.</u>		✓

2. Fill in the blanks.

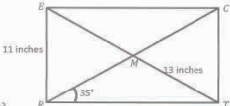
a. $ABCD$ is a trapezoid. Find the measurements listed below.

$\angle A =$ 140°
 $\angle D =$ 90°
 What other names does this figure have?
Quadrilateral



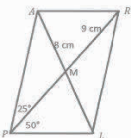
b. $RECT$ is a rectangle. Find the measurements listed below.

Line $TE =$ 26 inches
 Line $RC =$ 26 inches
 Line $CT =$ 11 inches
 $\angle ERM =$ 55°
 $\angle CTR =$ 90°
 What other names does this figure have?
Quadrilateral
Trapezoid
Parallelogram



c. $PARL$ is a parallelogram. Find the measurements listed below.

Line $AL =$ 16 inches
 Line $PR =$ 18 inches
 $\angle ARL =$ 75°
 $\angle PAR =$ 105°
 $\angle RLP =$ 105°
 What other names does this figure have?
Quadrilateral
Trapezoid



Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____

Date _____

1. True or false. If the statement is false, rewrite it to make it true.

	T	F
a. All trapezoids are quadrilaterals.		
b. All parallelograms are rhombuses.		
c. All squares are trapezoids.		
d. All rectangles are squares.		
e. Rectangles are always parallelograms.		
f. All parallelograms are trapezoids.		
g. All rhombuses are rectangles.		
h. Kites are never rhombuses.		
i. All squares are kites.		
j. All kites are squares.		
k. All rhombuses are squares.		

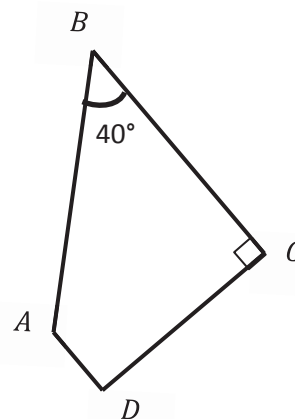
2. Fill in the blanks.

- a. $ABCD$ is a trapezoid. Find the measurements listed below.

$$\angle A = \underline{\hspace{2cm}}^\circ$$

$$\angle D = \underline{\hspace{2cm}}^\circ$$

What other names does this figure have?



- b. $RECT$ is a rectangle. Find the measurements listed below.

$$\text{Line } TE = \underline{\hspace{2cm}}$$

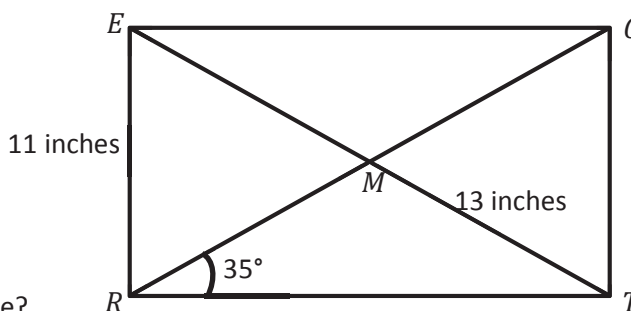
$$\text{Line } RC = \underline{\hspace{2cm}}$$

$$\text{Line } CT = \underline{\hspace{2cm}}$$

$$\angle ERM = \underline{\hspace{2cm}}^\circ$$

$$\angle CTR = \underline{\hspace{2cm}}^\circ$$

What other names does this figure have?



- c. $PARL$ is a parallelogram. Find the measurements listed below.

$$\text{Line } AL = \underline{\hspace{2cm}}$$

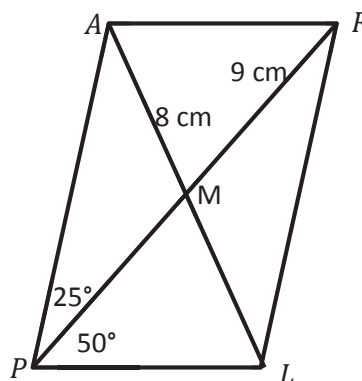
$$\text{Line } PR = \underline{\hspace{2cm}}$$

$$\angle ARL = \underline{\hspace{2cm}}^\circ$$

$$\angle PAR = \underline{\hspace{2cm}}^\circ$$

$$\angle RLP = \underline{\hspace{2cm}}^\circ$$

What other names does this figure have?



Name _____

Date _____

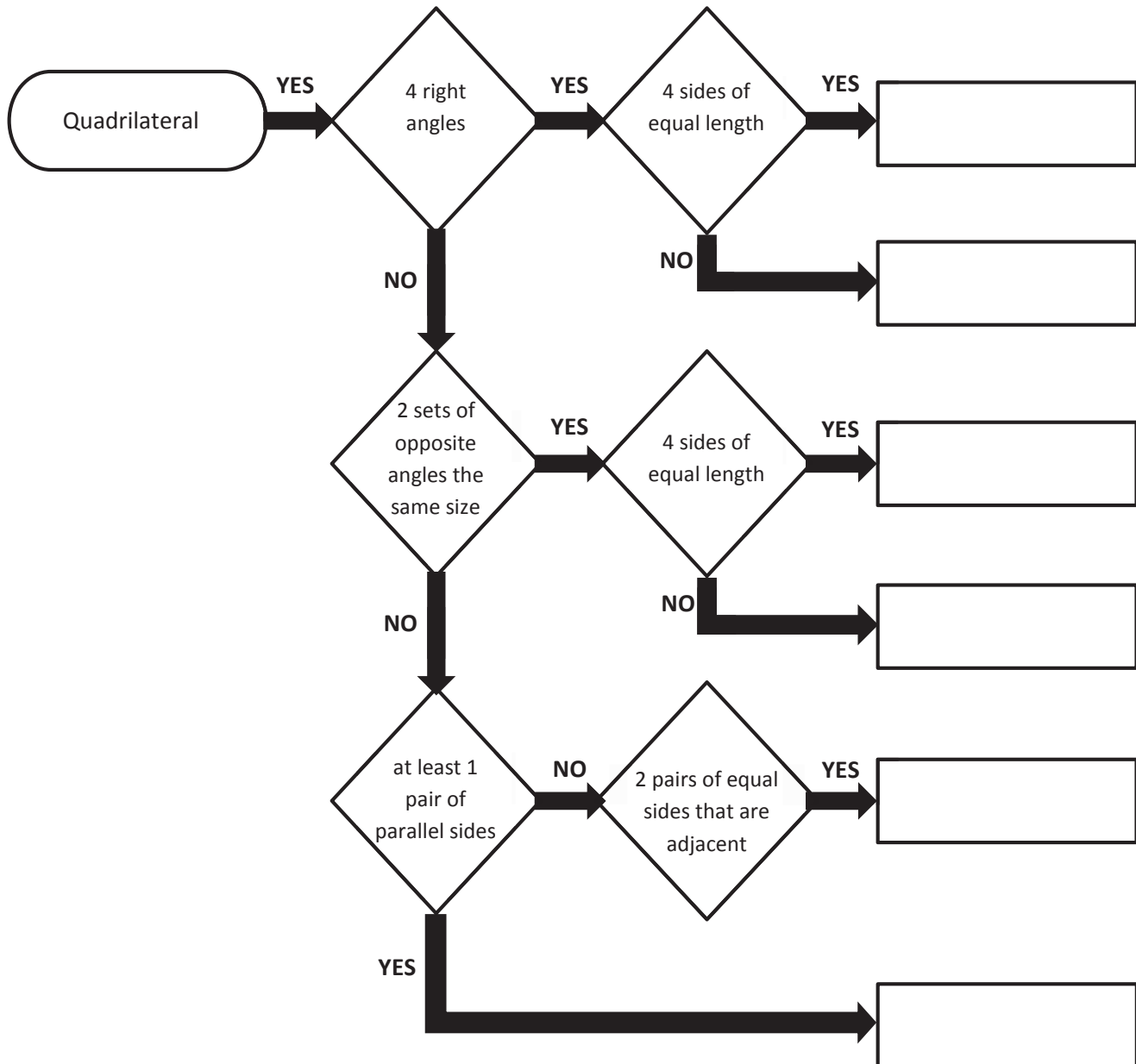
Use your tools to draw a square in the space below. Then, fill in the blanks with an attribute. There is more than one answer to some of these.

- a. Because a square is a kite, it must have _____.
- b. Because a square is a rhombus, it must have _____.
- c. Because a square is a rectangle, it must have _____.
- d. Because a square is a parallelogram, it must have _____.
- e. Because a square is a trapezoid, it must have _____.
- f. Because a square is a quadrilateral, it must have _____.

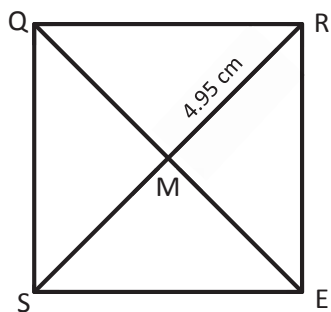
Name _____

Date _____

1. Follow the flow chart, and put the name of the figure in the boxes.



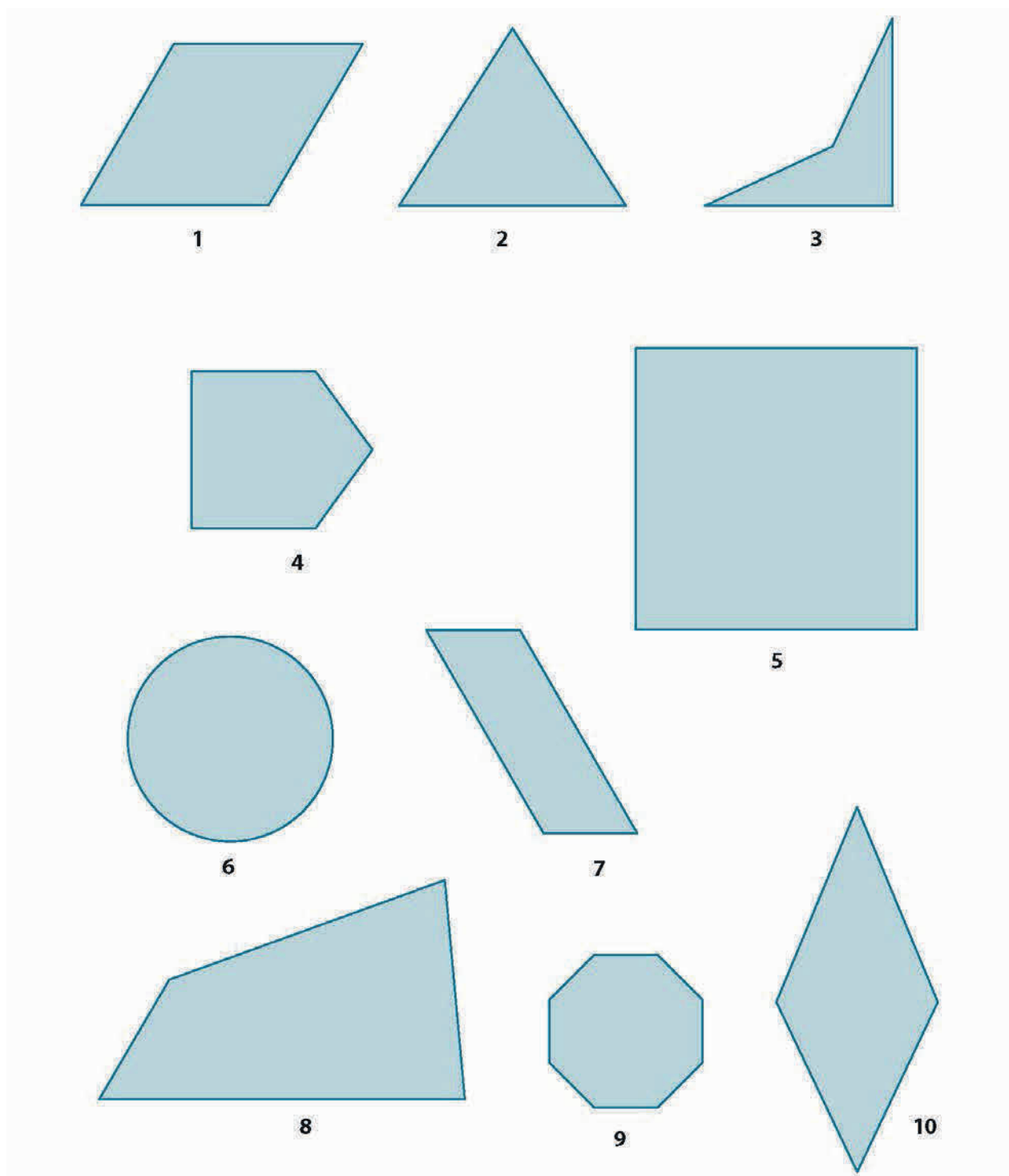
2. $SQRE$ is a square with an area of 49 cm^2 , and $RM = 4.95 \text{ cm}$. Find the measurements using what you know about the properties of squares.



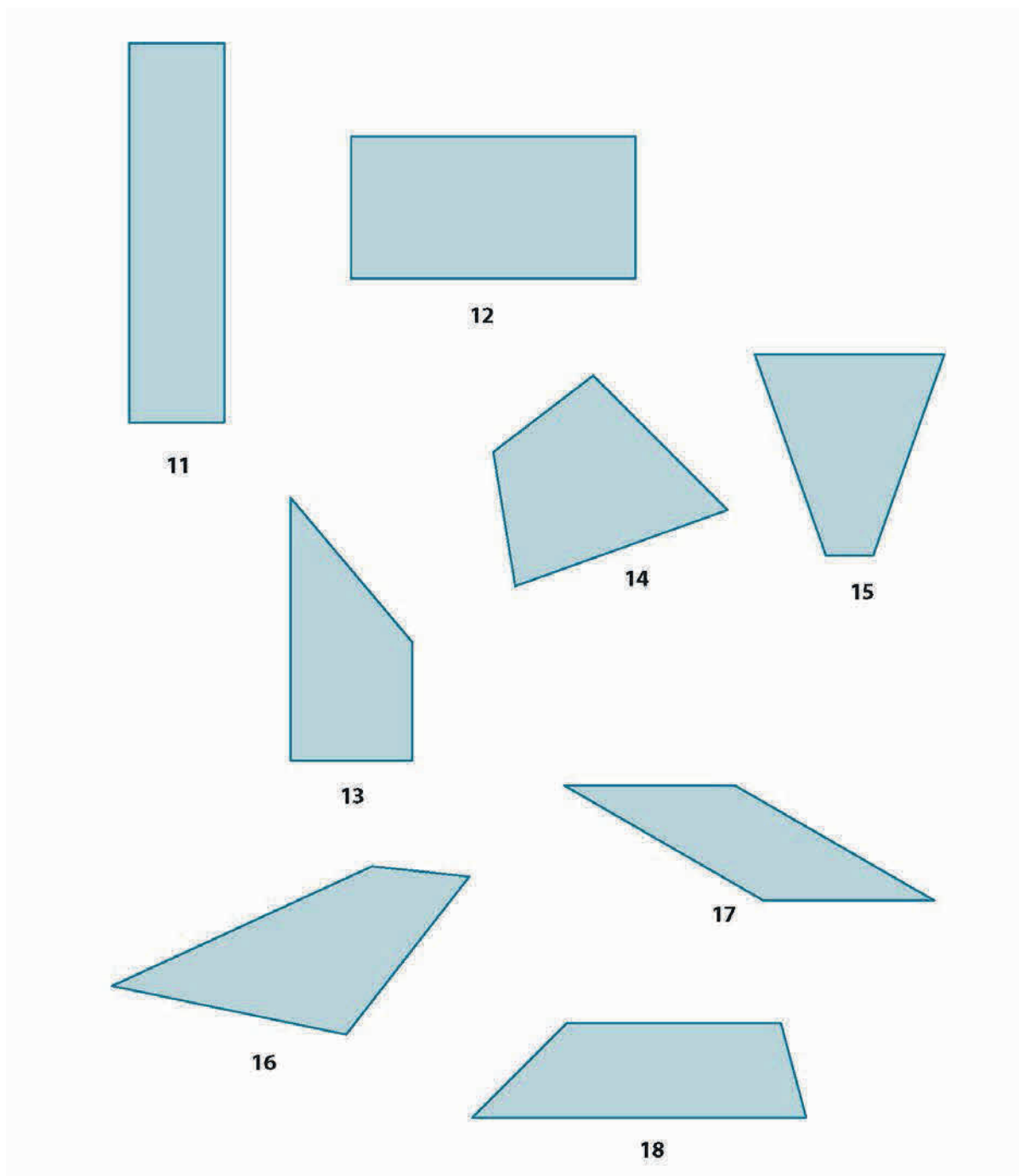
- a. $RS =$ _____ cm
- b. $QE =$ _____ cm
- c. Perimeter = _____ cm
- d. $m\angle QRE =$ _____ $^\circ$
- e. $m\angle RMQ =$ _____ $^\circ$

Quadrilaterals	Trapezoids
Parallelograms	Rectangles
Rhombuses	Kites
Squares	Polygons

shape name cards



shapes for sorting (page 1)



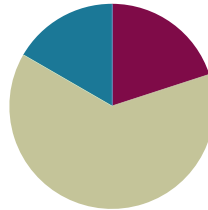
shapes for sorting (page 2)

Lesson 21

Objective: Draw and identify varied two-dimensional figures from given attributes.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Concept Development	(38 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Sprint: Divide by Multiples of 10 and 100 (8 minutes)
- Find the Volume (4 minutes)

Sprint: Divide by Multiples of 10 and 100 (8 minutes)

Materials: (S) Divide by Multiples of 10 and 100 Sprint

Note: This fluency activity reviews Module 2.

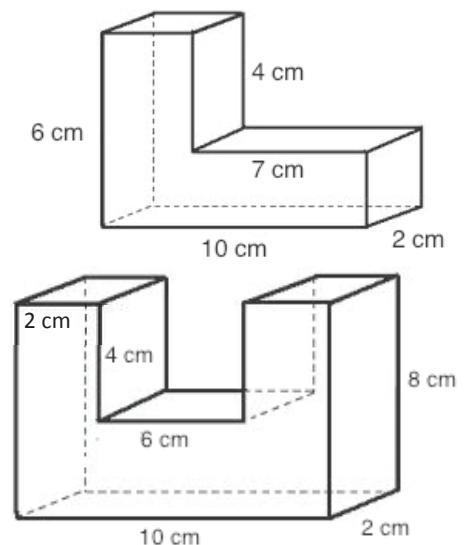
Find the Volume (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Topic B.

- T: Say the formula for finding the volume of a rectangular prism.
- S: Length times width times height.
- T: (Project a figure as shown to the right.) Visualize a line that breaks the figure into two rectangular prisms.
- T: Find the volume of the composite figure by adding the volumes of each rectangular prism.
- S: (Write $6\text{ cm} \times 3\text{ cm} \times 2\text{ cm} = 36\text{ cubic cm}$.
 $7\text{ cm} \times 2\text{ cm} \times 2\text{ cm} = 28\text{ cubic cm}$.
 $36\text{ cubic cm} + 28\text{ cubic cm} = 64\text{ cubic cm}$.)

Allow early finishers to continue on to the next figure.



Concept Development (38 minutes)

Materials: (S) Task cards, 6 for each pair of students (Templates 1–4), ruler, set square, protractor, Problem Set (or blank paper)

Note: The drawing tasks in the Concept Development are time-consuming. In order to give students ample time, no Application Problem is included in today's lesson.

Note: Today's Concept Development asks students to apply the nested relationships among quadrilaterals that have been explored throughout this topic. It should be conducted following a protocol similar to that of a problem-solving lesson involving word problems. Allow students to wrestle with the drawing tasks and then share the work during the Student Debrief. Allow students to redraw, as necessary, after the Student Debrief discussion. Task cards (24 per set) should be copied in sufficient quantity that pairs of students can share six cards.

- T: (Project on the board: Draw a quadrilateral that has two pairs of equal sides. Tell as many names as you can for this shape. Circle the most specific name.) What shape could you draw to satisfy the attributes of this task? Turn and talk. Then, draw your shape.
- S: I could draw a parallelogram. It has two pairs of equal sides. → A rectangle would work because it has two pairs of equal sides. → It says "two pairs of equal sides." I would draw a square. It has two sets of equal sides. The two sets also happen to be equal to each other. → A rhombus would work, too, because it is like a square. It has two sets of equal sides. → I could draw a kite. It has two pairs of equal sides. The sides that are equal are just next to each other rather than across from each other.
- T: Compare your shape with your neighbor's. Did we all draw the same shape? Is there only one shape that would be correct for this task?
- S: (Share work with partner.)
- T: This is the shape I drew. (Project a rectangle.) Name this shape.
- S: Rectangle. → Parallelogram. → Quadrilateral. → Polygon. → Trapezoid.
- T: (Record student responses.) Which of the names we listed is the most specific?
- S: Rectangle.
- T: (Circle *rectangle* on the board.) Is there a quadrilateral that we should not construct for this task? Why not?
- S: A trapezoid that is not a parallelogram because it would not have two pairs of equal sides. → An isosceles trapezoid would not work for this task because there would only be one set of equal sides.

**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

The task cards for today's lesson are numbered from simplest to most complex. Differentiate instruction by assigning tasks based on student need.

**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

The relationships between sides and angles in quadrilaterals can serve as an interesting extension. Students can explore the effects of changing side lengths on angle size, and vice versa, with online tools like Interactive Quadrilaterals at:
<http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html>.

T: Pull six task cards from the envelope on your table. Record the number of the task and a brief summary of the task in the boxes on your Problem Set. Follow the directions on the cards to draw the shapes in the boxes.

S: (Work.)

The Problem Set serves as a recording sheet for the drawings in the lesson. Time should be given for students to share their approaches to constructing the figures on the task cards.

Student Debrief (10 minutes)

Lesson Objective: Draw and identify varied two-dimensional figures from given attributes.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.



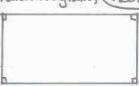
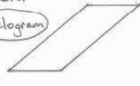
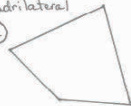

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Find someone who completed two of the same tasks you did. Compare the shapes that you drew. Must they be the same shape to correctly follow the directions on the card? Why or why not?
- Which tasks produced quadrilaterals with the same specific name on everyone's Problem Set? Which tasks produced the most varied quadrilaterals?
- Choose three of your quadrilaterals, and paste them in the correct part of the hierarchy diagram. Explain why they belong there.
- Explain to your partner how you corrected John's error in Problem 2.

Name Baxter Date _____

1. Write the number on your task card and a summary of the task in the blank. Then draw the figure in the box. Label your figure with as many names as you can. Circle the most specific name.

Task #17: Parallelogram with 60° angle Quadrilateral, Trapezoid <u>Parallelogram</u> 	Task #7: Quadrilateral with 4 equal sides Quadrilateral, Trapezoid, Parallelogram, Kite, Rhombus <u>Rhombus</u> 
Task #2: Rectangle with a length twice its width Quadrilateral, Trapezoid, Parallelogram, <u>Rectangle</u> 	Task #11: Parallelogram with no right angles Quadrilateral, Trapezoid, <u>Parallelogram</u> 
Task #21: Kite that is not a parallelogram Quadrilateral, <u>Kite</u> 	Task #24: Quadrilateral whose diagonals do not bisect each other <u>Quadrilateral</u> 

2. John says that because rhombuses do not have perpendicular sides, they cannot be rectangles. Explain his error in thinking.

In order to be a rhombus a quadrilateral needs 4 equal sides. Some rhombuses do have perpendicular sides. These are squares, and squares are rectangles.

3. Jack says that because kites don't have parallel sides, a square is not a kite. Explain his error in thinking.

A Kite needs to have 2 pairs of equal adjacent sides. If the 2 pairs are the same length, and the angles are all 90°, then the Kite could be a square.

- What part of a kite's definition did Jack not understand in Problem 3? How did you correct his thinking?
- How do all the shapes that were drawn today fit the definition of a quadrilateral?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

A

Number Correct: _____

Divide by Multiples of 10 and 100

1.	$30 \div 10 =$	
2.	$430 \div 10 =$	
3.	$4,300 \div 10 =$	
4.	$4,300 \div 100 =$	
5.	$43,000 \div 100 =$	
6.	$50 \div 10 =$	
7.	$850 \div 10 =$	
8.	$8,500 \div 10 =$	
9.	$8,500 \div 100 =$	
10.	$85,000 \div 100 =$	
11.	$600 \div 10 =$	
12.	$60 \div 3 =$	
13.	$600 \div 30 =$	
14.	$4,000 \div 100 =$	
15.	$40 \div 2 =$	
16.	$4,000 \div 200 =$	
17.	$240 \div 10 =$	
18.	$24 \div 2 =$	
19.	$240 \div 20 =$	
20.	$3,600 \div 100 =$	
21.	$36 \div 3 =$	
22.	$3,600 \div 300 =$	

23.	$480 \div 4 =$	
24.	$480 \div 40 =$	
25.	$6,300 \div 3 =$	
26.	$6,300 \div 30 =$	
27.	$6,300 \div 300 =$	
28.	$8,400 \div 2 =$	
29.	$8,400 \div 20 =$	
30.	$8,400 \div 200 =$	
31.	$96,000 \div 3 =$	
32.	$96,000 \div 300 =$	
33.	$96,000 \div 30 =$	
34.	$900 \div 30 =$	
35.	$1,200 \div 30 =$	
36.	$1,290 \div 30 =$	
37.	$1,800 \div 300 =$	
38.	$8,000 \div 200 =$	
39.	$12,000 \div 200 =$	
40.	$12,800 \div 200 =$	
41.	$2,240 \div 70 =$	
42.	$18,400 \div 800 =$	
43.	$21,600 \div 90 =$	
44.	$25,200 \div 600 =$	

B

Number Correct: _____

Improvement: _____

Divide by Multiples of 10 and 100

1.	$20 \div 10 =$	
2.	$420 \div 10 =$	
3.	$4,200 \div 10 =$	
4.	$4,200 \div 100 =$	
5.	$42,000 \div 100 =$	
6.	$40 \div 10 =$	
7.	$840 \div 10 =$	
8.	$8,400 \div 10 =$	
9.	$8,400 \div 100 =$	
10.	$84,000 \div 100 =$	
11.	$900 \div 10 =$	
12.	$90 \div 3 =$	
13.	$900 \div 30 =$	
14.	$6,000 \div 100 =$	
15.	$60 \div 2 =$	
16.	$6,000 \div 200 =$	
17.	$240 \div 10 =$	
18.	$24 \div 2 =$	
19.	$240 \div 20 =$	
20.	$6,300 \div 100 =$	
21.	$63 \div 3 =$	
22.	$6,300 \div 300 =$	

23.	$840 \div 4 =$	
24.	$840 \div 40 =$	
25.	$3,600 \div 3 =$	
26.	$3,600 \div 30 =$	
27.	$3,600 \div 300 =$	
28.	$4,800 \div 2 =$	
29.	$4,800 \div 20 =$	
30.	$4,800 \div 200 =$	
31.	$69,000 \div 3 =$	
32.	$69,000 \div 300 =$	
33.	$69,000 \div 30 =$	
34.	$800 \div 40 =$	
35.	$1,200 \div 40 =$	
36.	$1,280 \div 40 =$	
37.	$1,600 \div 400 =$	
38.	$8,000 \div 200 =$	
39.	$14,000 \div 200 =$	
40.	$14,600 \div 200 =$	
41.	$2,560 \div 80 =$	
42.	$16,100 \div 700 =$	
43.	$14,400 \div 60 =$	
44.	$37,800 \div 900 =$	

Name _____

Date _____

1. Write the number on your task card and a summary of the task in the blank. Then, draw the figure in the box. Label your figure with as many names as you can. Circle the most specific name.

Task #__ : _____	Task #__ : _____
Task #__ : _____	Task #__ : _____
Task #__ : _____	Task #__ : _____

2. John says that because rhombuses do not have perpendicular sides, they cannot be rectangles. Explain his error in thinking.
3. Jack says that because kites do not have parallel sides, a square is not a kite. Explain his error in thinking.

Name _____

Date _____

1. Use the word bank to fill in the blanks.

trapezoids parallelograms

All _____ are _____, but not all _____ are _____.

2. Use the word bank to fill in the blanks.

kites rhombuses

All _____ are _____, but not all _____ are _____.

Name _____

Date _____

1. Answer the questions by checking the box.

Sometimes**Always**

- a. Is a square a rectangle?
- b. Is a rectangle a kite?
- c. Is a rectangle a parallelogram?
- d. Is a square a trapezoid?
- e. Is a parallelogram a trapezoid?
- f. Is a trapezoid a parallelogram?
- g. Is a kite a parallelogram?

- h. For each statement that you answered with *sometimes*, draw and label an example that justifies your answer.

2. Use what you know about quadrilaterals to answer each question below.

- a. Explain when a trapezoid is not a parallelogram. Sketch an example.

- b. Explain when a kite is not a parallelogram. Sketch an example.

<p>Task 1: Draw a trapezoid with a right angle.</p>	<p>Task 2: Draw a rectangle with a length that is twice its width.</p>	<p>Task 3: Draw a quadrilateral with 2 pairs of equal sides and no parallel sides.</p>
<p>Task 4: Draw a rhombus with right angles.</p>	<p>Task 5: Draw a parallelogram with two pairs of perpendicular sides.</p>	<p>Task 6: Draw a rhombus with 4 equal angles.</p>

task cards (1–6)

<p>Task 7: Draw a quadrilateral with four equal sides.</p>	<p>Task 8: Draw a parallelogram with right angles.</p>	<p>Task 9: Draw a parallelogram with a side of 4 cm and a side of 6 cm.</p>
<p>Task 10: Draw an isosceles trapezoid.</p>	<p>Task 11: Draw a parallelogram with no right angles.</p>	<p>Task 12: Draw a rectangle that is also a rhombus.</p>

task cards (7–12)

<p>Task 13: Draw a quadrilateral that has at least one pair of equal opposite angles.</p>	<p>Task 14: Draw a quadrilateral that has only one pair of equal opposite angles.</p>	<p>Task 15: Draw a trapezoid with four right angles.</p>
<p>Task 16: Draw a kite that is also a parallelogram.</p>	<p>Task 17: Draw a parallelogram with a 60° angle.</p>	<p>Task 18: Draw a rectangle that is not a rhombus.</p>

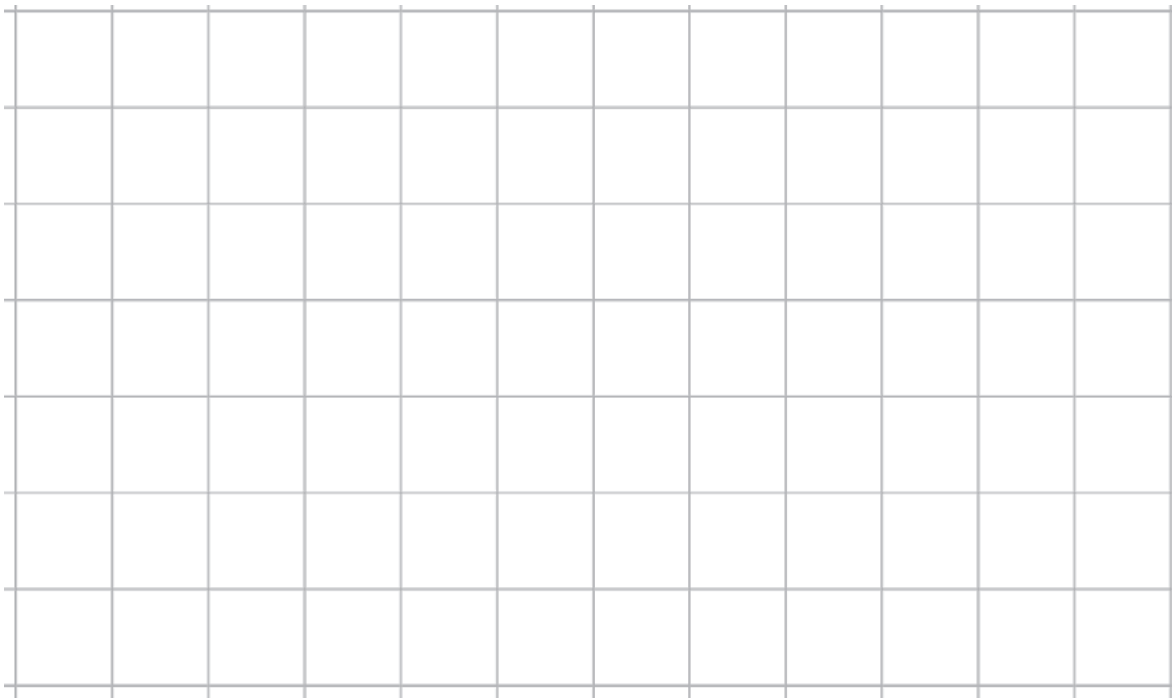
task cards (13–18)

<p>Task 19: Draw a rhombus that is not a rectangle.</p>	<p>Task 20: Draw a parallelogram that is not a rectangle.</p>	<p>Task 21: Draw a kite that is not a parallelogram.</p>
<p>Task 22: Draw a quadrilateral whose diagonals bisect each other at a right angle.</p>	<p>Task 23: Draw a trapezoid that is not a parallelogram.</p>	<p>Task 24: Draw a quadrilateral whose diagonals do not bisect each other.</p>

task cards (19–24)

Name _____ Date _____

1. Use your ruler to draw a rectangle that measures $4\frac{1}{2}$ by $2\frac{3}{4}$ inches, and find its area.



2. Heather has a rectangular yard. She measures it and finds out it is $24\frac{1}{2}$ feet long by $12\frac{4}{5}$ feet wide.
- She wants to know how many square feet of sod she will need to completely cover the yard. Draw the yard, and label the measurements.
 - How much sod will Heather need to cover the yard?
 - If each square foot of sod costs 65 cents, how much will she have to pay to cover her yard?

3. A rectangular container that has a length of 30 cm, a width of 20 cm, and a height of 24 cm is filled with water to a depth of 15 cm. When an additional 6.5 liters of water are poured into the container, some water overflows. How many liters of water overflow the container? Use words, pictures, and numbers to explain your answer. (Remember: $1 \text{ cm}^3 = 1 \text{ mL}$.)
4. Jim says that a $2\frac{1}{2}$ inch by $3\frac{1}{4}$ inch rectangle has a section that is 2 inches \times 3 inches and a section that is $\frac{1}{2}$ inch \times $\frac{1}{4}$ inch. That means the total area is just the sum of these two smaller areas, or $6\frac{1}{8} \text{ in}^2$. Why is Jim incorrect? Use an area model to explain your thinking. Then, give the correct area of the rectangle.
5. Miguel and Jacqui built towers out of craft sticks. Miguel's tower had a 4-inch square base. Jacqui's tower had a 6-inch square base. If Miguel's tower had a volume of 128 cubic inches and Jacqui's had a volume of 288 cubic inches, whose tower was taller? Explain your reasoning.

6. Read the statements. Circle True or False. Explain your choice for each using words and/or pictures.

a. All parallelograms are quadrilaterals.

True

False

b. All squares are rhombuses.

True

False

c. Squares are rhombuses but not rectangles.

True

False

d. The opposite angles in a parallelogram have the same measure.

True

False



e. Because the angles in a rectangle are 90° , it is not a parallelogram.

True

False

f. The sum of the angle measures of any trapezoid is greater than the sum of the angle measures of any parallelogram.

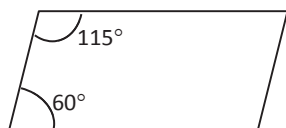
True

False

g. The following figure is a parallelogram.

True

False



End-of-Module Assessment Task Standards Addressed

Topics A–D

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
 - Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
- Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
 - A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
 - A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
 - Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
 - Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
 - Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.



Classify two-dimensional figures into categories based on their properties.

- Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
- Classify two-dimensional figures in a hierarchy based on properties.

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency*. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

A Progression Toward Mastery

Assessment Task Item	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1	The student is unable to draw the rectangle and unable to find the area.	The student draws one dimension accurately but is unable to find the area.	The student accurately draws both dimensions of the rectangle but makes a calculation error when finding the area.	The student correctly does the following: <ul style="list-style-type: none"> Draws the rectangle. Calculates the area as $12\frac{3}{8}$ in².
2	The student is unable to draw the yard, calculate the area using appropriate units, or calculate the cost of the sod.	The student does one of the following: <ul style="list-style-type: none"> Draws and labels the yard. Calculates the area of the yard with or without the correct units (square feet). Finds the cost of the sod. 	The student is able to correctly perform two of the following actions in any combination: <ul style="list-style-type: none"> Draws and labels the yard. Calculates the area of the yard with the correct units (square feet). Finds the cost of the sod. 	The student correctly does the following: <ul style="list-style-type: none"> Draws the yard and labels correctly with the length as $24\frac{1}{2}$ ft and the width as $12\frac{4}{5}$ ft. Calculates the area of the yard using appropriate units as $313\frac{6}{10}$ ft² or $313\frac{3}{5}$ ft². Finds the cost of the sod to be \$203.84.
3	The student is unable to find the volume of the water that has overflowed and is unable to explain the reasoning used.	The student finds the volume of the water that has overflowed but is unable to explain the reasoning used.	The student makes a calculation error in finding the volume of the water that has overflowed but is able to clearly explain the reasoning used.	The student finds the volume of the water that has overflowed to be 1.1 L and uses words, numbers, and pictures to clearly explain the reasoning used.



A Progression Toward Mastery

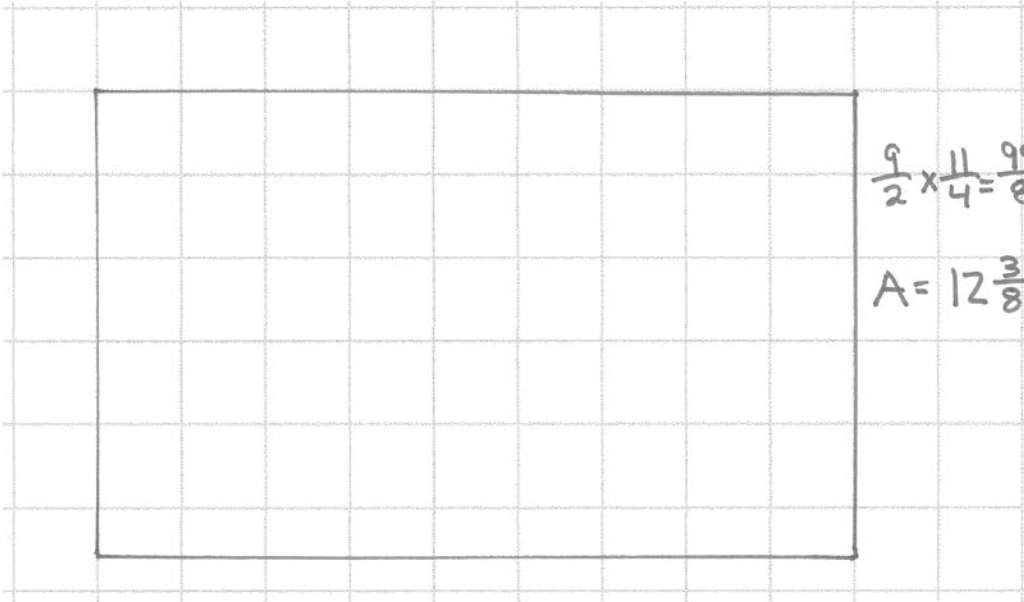
4	The student is not able to draw an area model, provide an explanation of Jim's error, or give the correct area.	<p>The student does one of the following:</p> <ul style="list-style-type: none"> ▪ Accurately partitions the area model in both dimensions. ▪ Provides a clear explanation of Jim's error. ▪ Calculates the correct area of the rectangle. 	<p>The student does two of the following:</p> <ul style="list-style-type: none"> ▪ Accurately partitions the area model in both dimensions. ▪ Provides a clear explanation of Jim's error. ▪ Calculates the correct area of the rectangle. 	<p>The student does the following:</p> <ul style="list-style-type: none"> ▪ Accurately partitions the area model in both dimensions. ▪ Provides a clear explanation of Jim's error. ▪ Calculates the correct area of the rectangle as $8\frac{1}{8}$ in².
5	The student is neither able to find the heights of the towers nor able to answer which tower is taller.	<p>The student makes an attempt to calculate the towers' heights but makes errors in both calculations. Explanation of the reasoning used is unclear.</p>	The student calculates the heights of the towers but makes a calculation error that causes an error in the determination of the taller tower. However, the explanation of the reasoning used is clear.	<p>The student does the following:</p> <ul style="list-style-type: none"> ▪ Accurately calculates the heights of both towers (8 inches). ▪ Explains clearly that the towers are equal in height.
6	The student provides a combination of at least three correct true or false responses and/or explanations.	The student provides a combination of at least six correct true or false responses and/or explanations.	The student provides a combination of at least seven correct true or false responses and/or explanations.	<p>The student provides seven correct true or false responses and clear explanations for all seven items.</p> <ol style="list-style-type: none"> True True False True False False False

Name

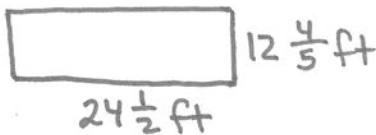
Jean

Date

1. Use your ruler to draw a rectangle that measures $4\frac{1}{2}$ by $2\frac{3}{4}$ inches, and find its area.



2. Heather has a rectangular yard. She measures it and finds out it is $24\frac{1}{2}$ feet long by $12\frac{4}{5}$ feet wide.
 a. She wants to know how many square feet of sod she will need to completely cover the yard.
 Draw the yard, and label the measurements.



$$\begin{array}{r} 64 \\ \times 49 \\ \hline 576 \\ + 2560 \\ \hline 3,136 \end{array}$$

$$\begin{array}{r} 1568 \\ 2 \overline{) 3136} \\ \underline{-2} \\ 11 \\ \underline{-10} \\ 13 \\ \underline{-12} \\ 16 \\ \underline{-16} \\ 0 \end{array}$$

$$\begin{array}{r} 313\frac{3}{5} \\ 5 \overline{) 1568} \\ \underline{-15} \\ 6 \\ \underline{-5} \\ 18 \\ \underline{-15} \\ 3 \end{array}$$

- b. How much sod will Heather need to cover the yard?

$$12\frac{4}{5} \times 24\frac{1}{2} = \frac{64}{5} \times \frac{49}{2} = \frac{3136}{10} = \frac{1568}{5} = 313\frac{3}{5}$$

She'll need $313\frac{3}{5} \text{ ft}^2$ of sod to cover her yard.

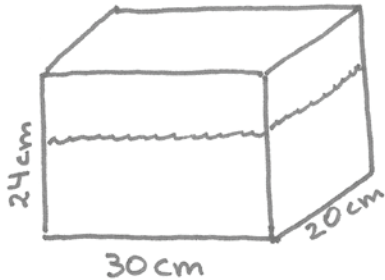
- c. If each square foot of sod costs 65 cents, how much will she have to pay to cover her yard?

$$313\frac{3}{5} = 313.6$$

$$\begin{array}{r} 313.6 \\ \times .65 \\ \hline 15680 \\ + 188160 \\ \hline 203840 \end{array}$$

Heather will have to pay \$203.84 to cover her yard.

3. A rectangular container that has a length of 30 cm, a width of 20 cm, and a height of 24 cm is filled with water to a depth of 15 cm. When an additional 6.5 liters of water is poured into the container, some water overflows. How many liters of water overflow the container? Use words, pictures, and numbers to explain your answer. (Remember $1 \text{ cm}^3 = 1 \text{ mL}$.)



$$30 \times 20 \times 24 = 720 \times 20 = 14,400$$

$$\text{Volume of the container} = 14,400 \text{ cm}^3$$

$$30 \times 20 \times 15 = 450 \times 20 = 9,000$$

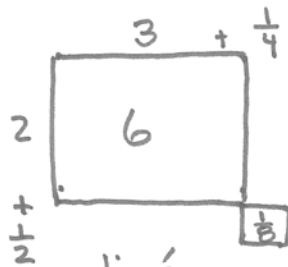
$$\text{Volume of water } 9,000 \text{ cm}^3$$

$$14,400 - 9,000 = 5,400$$

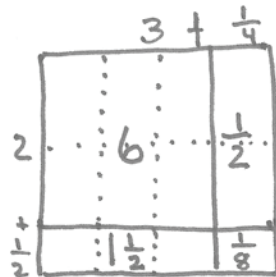
$$\text{Room left in the container} = 5,400 \text{ cm}^3 \text{ or } 5.4 \text{ L}$$

$$6.5 \text{ L} - 5.4 \text{ L} = 1.1 \text{ L} \quad \text{The water overflowed by } 1.1 \text{ L or } 1,100 \text{ cm}^3.$$

4. Jim says that a $2\frac{1}{2}$ inch by $3\frac{1}{4}$ inch rectangle has a section that is 2 inches \times 3 inches and a section that is $\frac{1}{2}$ inch \times $\frac{1}{4}$ inches. That means the total area is just the sum of these two smaller areas, or $6\frac{1}{8} \text{ in}^2$. Why is Jim incorrect? Use an area model to explain your thinking. Then, give the correct area of the rectangle.



Jim's
Incorrect Model



Correct Model

In order to find the area, all sections of the area model must be calculated and added.

$$6 + \frac{1}{2} + 3\frac{1}{2} + \frac{1}{8} = 8\frac{1}{8}$$

The area of the rectangle is $8\frac{1}{8} \text{ in}^2$.

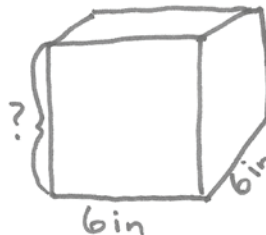
5. Miguel and Jacqui built towers out of craft sticks. Miguel's tower had a 4-inch square base. Jacqui's tower had a 6-inch square base. If Miguel's tower had a volume of 128 cubic inches and Jacqui's had a volume of 288 cubic inches, whose tower was taller? Explain your reasoning.



Miguel

$$V = 128 \text{ in}^3$$

$$\begin{array}{r} 8 \\ 16 \overline{) 128} \\ \underline{-128} \\ 0 \end{array}$$



$$V = 288 \text{ in}^3$$

$$\begin{array}{r} 8 \\ 36 \overline{) 288} \\ \underline{-288} \\ 0 \end{array}$$

Both towers have the same height of 8 in. I divided the volumes by the bases and got a height of 8 in.

6. Read the statements. Circle *True* or *False*. Explain your choice for each using words and/or pictures.

- a. All parallelograms are quadrilaterals.

True

False

All parallelograms have 4 straight sides, so all parallelograms are a type of quadrilateral.

- b. All squares are rhombuses.

True

False

All rhombuses have 4 equal sides, and so do all squares. Some rhombuses do not have 4 right angles, so not all rhombuses are squares.

- c. Squares are rhombuses, but not rectangles.

True


False

All squares are both rhombuses and rectangles. Squares and rhombuses both have 4 equal sides. Squares and rectangles both have 4 right angles.

- d. The opposite angles in a parallelogram have the same measure.

True

False

 The opposite sides of parallelograms are parallel and equal in length. The four angles always add up to 360° . Opposite angles are always equal.

- e. Because the angles in a rectangle are 90° , it is not a parallelogram.

True

False

All rectangles are parallelograms because all rectangles have 2 pairs of parallel sides.

- f. The sum of the angle measures of any trapezoid is greater than the sum of the angle measures of any parallelogram.

True

False

The sum of the 4 angles of any quadrilateral, including trapezoids and parallelograms, is always 360° .

- g. The following figure is a parallelogram.

True

False



Opposite angles in a parallelogram are always equal. If you add up these angles ($60^\circ + 60^\circ + 115^\circ + 115^\circ$) the sum is only 350° . Therefore, the opposite angles can't be equal, and this isn't a parallelogram. The angles need to add up to 360° .

Answer Key

Eureka Math Grade 5 Module 5

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Answer Key

GRADE 5 • MODULE 5

Addition and Multiplication with Volume and Area

Lesson 1

Problem Set

1. Explanations will vary.
 - a. 1 cm^3
 - b. 5 cm^3
 - c. 9 cm^3
 - d. 9 cm^3
 - e. 12 cm^3
 - f. 20 cm^3
2. Answers will vary.
3.
 - a. Explanations will vary.
 - b. 10 cm^3 ; explanations will vary.

Exit Ticket

1.
 - a. 5 cm^3
 - b. 12 cm^3
2. Answers will vary.

Homework

1. Explanations will vary
 - a. 2 cm^3
 - b. 4 cm^3
 - c. 6 cm^3
 - d. 6 cm^3
 - e. 12 cm^3
 - f. 16 cm^3
2. Answers will vary.
3. A cube is hidden under the second layer, explanations will vary.
4. Answers will vary.

Lesson 2

Problem Set

1.
 - a. 8
 - b. 16
 - c. 48
2.
 - a. Predictions will vary; 8 cm^3
 - b. Predictions will vary; 16 cm^3
 - c. Predictions will vary; 40 cm^3
3.
 - a. Predictions will vary.
 - b. Explanations will vary.
 - c. 27; answers will vary.

Exit Ticket

1. 8
2. Predictions will vary; 12 cm^3

Homework

1. Explanations will vary.
 - a. 4
 - b. 12
 - c. 24
2. Explanations will vary.
 - a. 6
 - b. 12
 - c. 32
3. Answers will vary.

Lesson 3

Sprint

Note: Depending on the skill that teachers wish to develop, students can be asked to express their answers as fractions or as whole numbers. However, directions should be clearly stated prior to beginning Sprint routine to avoid confusion when calling out answers.

Side A

- | | | | |
|--------------|---------------------|---------------------|------------------------|
| 1. 2 fifths | 12. 2 | 23. 60 sixths or 10 | 34. 90 sixths or 15 |
| 2. 3 fifths | 13. 4 halves or 2 | 24. 15 thirds or 5 | 35. 24 fourths or 6 |
| 3. 4 fifths | 14. 2 | 25. 30 thirds or 10 | 36. 72 fourths or 18 |
| 4. 4 fifths | 15. 6 thirds or 2 | 26. 30 thirds or 10 | 37. 32 eighths or 4 |
| 5. 3 eighths | 16. 2 | 27. 15 fifths or 3 | 38. 96 eighths or 12 |
| 6. 5 eighths | 17. 10 fifths or 2 | 28. 30 fifths or 6 | 39. 160 eighths or 20 |
| 7. 7 eighths | 18. 9 thirds or 3 | 29. 60 fifths or 12 | 40. 224 eighths or 28 |
| 8. 7 eighths | 19. 18 thirds or 6 | 30. 45 fifths or 9 | 41. 270 ninths or 30 |
| 9. 3 tenths | 20. 8 fourths or 2 | 31. 45 fifths or 9 | 42. 441 ninths or 49 |
| 10. 7 tenths | 21. 24 fourths or 6 | 32. 18 sixths or 3 | 43. 168 sevenths or 24 |
| 11. 7 tenths | 22. 12 sixths or 2 | 33. 90 sixths or 15 | 44. 294 sevenths or 42 |

Side B

- | | | | |
|---------------|---------------------|----------------------|------------------------|
| 1. 2 sevenths | 12. 2 | 23. 24 fourths or 6 | 34. 120 sixths or 20 |
| 2. 3 sevenths | 13. 10 fifths or 2 | 24. 15 fifths or 3 | 35. 20 fourths or 5 |
| 3. 4 sevenths | 14. 3 | 25. 30 fifths or 6 | 36. 60 fourths or 15 |
| 4. 4 sevenths | 15. 9 thirds or 3 | 26. 60 fifths or 12 | 37. 24 eighths or 3 |
| 5. 3 tenths | 16. 5 | 27. 45 fifths or 9 | 38. 72 eighths or 9 |
| 6. 7 tenths | 17. 10 halves or 5 | 28. 45 fifths or 9 | 39. 120 eighths or 15 |
| 7. 9 tenths | 18. 6 thirds or 2 | 29. 15 thirds or 5 | 40. 168 eighths or 21 |
| 8. 9 tenths | 19. 12 thirds or 4 | 30. 30 thirds or 10 | 41. 315 ninths or 35 |
| 9. 3 eighths | 20. 12 sixths or 2 | 31. 30 thirds or 10 | 42. 378 ninths or 42 |
| 10. 5 eighths | 21. 60 sixths or 10 | 32. 24 sixths or 4 | 43. 147 sevenths or 21 |
| 11. 5 eighths | 22. 8 fourths or 2 | 33. 120 sixths or 20 | 44. 336 sevenths or 48 |

Problem Set

1. Answers will vary.
2. Jonah is correct; explanations will vary.
3. 50 in^3 ; 100 in^3 ; 175 in^3 ; explanations will vary.
4. 4; 12; 48 m^3

Exit Ticket

1. Answers will vary; Answers will vary; 12
2. 24

Homework

1. Answers will vary.
2. Explanations will vary.
3. 48 in^3 ; 80 in^3 ; 112 in^3 ; explanations will vary.
4. 3; 8; 24 m^3

Lesson 4

Problem Set

1.
 - a. 5; 2; 2; 20
 - b. 3; 2; 4; 24
 - c. 4; 2; 4; 32
 - d. 4; 3; 3; 36
2.
 - a. $5\text{ cm} \times 2\text{ cm} \times 2\text{ cm} = 20\text{ cm}^3$ (or variant)
 - b. $3\text{ cm} \times 2\text{ cm} \times 4\text{ cm} = 24\text{ cm}^3$ (or variant)
 - c. $4\text{ cm} \times 2\text{ cm} \times 4\text{ cm} = 32\text{ cm}^3$ (or variant)
 - d. $4\text{ cm} \times 3\text{ cm} \times 3\text{ cm} = 36\text{ cm}^3$ (or variant)
3.
 - a. $4\text{ in} \times 3\text{ in} \times 4\text{ in} = 48\text{ in}^3$
 - b. $3\text{ m} \times 2\text{ m} \times 6\text{ m} = 36\text{ m}^3$
4. 560 cm^3
5. Explanations will vary.
 - a. 300 cm^3
 - b. 240 in^3

Exit Ticket

1. 2; 2; 4; 16; $2\text{ mm} \times 2\text{ mm} \times 4\text{ mm} = 16\text{ mm}^3$ (or variant)
2. 100 ft^3

Homework

1.
 - a. 5; 2; 4; 40
 - b. 3; 2; 5; 30
 - c. 4; 2; 4; 32
 - d. 8; 3; 3; 72
2.
 - a. $5\text{ cm} \times 2\text{ cm} \times 4\text{ cm} = 40\text{ cm}^3$ (or variant)
 - b. $3\text{ cm} \times 2\text{ cm} \times 5\text{ cm} = 30\text{ cm}^3$ (or variant)
 - c. $4\text{ cm} \times 2\text{ cm} \times 4\text{ cm} = 32\text{ cm}^3$ (or variant)
 - d. $8\text{ cm} \times 3\text{ cm} \times 3\text{ cm} = 72\text{ cm}^3$ (or variant)
3.
 - a. $8\text{ in} \times 4\text{ in} \times 8\text{ in} = 256\text{ in}^3$
 - b. $10\text{ m} \times 3\text{ m} \times 7\text{ m} = 210\text{ m}^3$
4. $20,160\text{ in}^3$
5.
 - a. 224 m^3
 - b. $2,366\text{ in}^3$

Lesson 5

Problem Set

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. $1 \text{ cm}^3 = 1 \text{ mL}$, explanations will vary.
5. No, the beaker holds 40 mL less than the cube.
6.
 - a. 7,800 mL
 - b. 7.8 L
 - c. 1,560 mL; explanations will vary.
7. 2 cm

Exit Ticket

- a. 225 cm^3
- b. Beaker shaded to line between 200 mL and 250 mL

Homework

1. Beaker shaded to line between 20 mL and 40 mL; explanations will vary.
2. A; C; explanations will vary.
3. Answers will vary.

Lesson 6

Problem Set

1.
 - a. 420 cm^3 ; strategies will vary.
 - b. 444 in^3 ; strategies will vary.
 - c. 108 cm^3 ; strategies will vary.
 - d. 324 m^3 ; strategies will vary.
2. $3,840 \text{ in}^3$
3. 4 cm
4. 120 cm^3 or 120 mL
5.
 - a. A: 144 ft^3 ; B: 288 ft^3
 - b. 6 ft
 - c. 18 ft

Exit Ticket

303 in^3

Homework

1.
 - a. 72 in^3 ; strategies will vary.
 - b. $1,431 \text{ cm}^3$; strategies will vary.
 - c. 249 mm^3 ; strategies will vary.
 - d. 472 m^3 ; strategies will vary.
2. $1,254 \text{ in}^3$
3. 5 cm
4. 585 cm^3 or 585 mL
5. A: 74 ft^3 ; B and C: 222 ft^3

Lesson 7

Sprint

Side A

1.	$\frac{1}{4}$	12.	$\frac{4}{15}$	23.	$\frac{10}{15}$	34.	$\frac{15}{20}$
2.	$\frac{1}{6}$	13.	$\frac{1}{12}$	24.	$\frac{15}{10}$	35.	$\frac{18}{20}$
3.	$\frac{1}{8}$	14.	$\frac{2}{12}$	25.	$\frac{1}{9}$	36.	$\frac{6}{20}$
4.	$\frac{1}{14}$	15.	$\frac{6}{12}$	26.	$\frac{2}{9}$	37.	$\frac{1}{49}$
5.	$\frac{1}{14}$	16.	$\frac{1}{18}$	27.	$\frac{4}{9}$	38.	$\frac{3}{40}$
6.	$\frac{1}{6}$	17.	$\frac{5}{18}$	28.	$\frac{6}{6}$	39.	$\frac{5}{24}$
7.	$\frac{1}{9}$	18.	$\frac{10}{18}$	29.	$\frac{8}{9}$	40.	$\frac{9}{16}$
8.	$\frac{1}{18}$	19.	$\frac{10}{12}$	30.	$\frac{10}{9}$	41.	$\frac{12}{18}$
9.	$\frac{1}{15}$	20.	$\frac{1}{25}$	31.	$\frac{9}{10}$	42.	$\frac{18}{8}$
10.	$\frac{1}{15}$	21.	$\frac{4}{25}$	32.	$\frac{3}{20}$	43.	$\frac{49}{72}$
11.	$\frac{2}{15}$	22.	$\frac{6}{25}$	33.	$\frac{12}{20}$	44.	$\frac{63}{96}$

Side B

1.	$\frac{1}{6}$	12.	$\frac{4}{15}$	23.	$\frac{15}{20}$	34.	$\frac{10}{15}$
2.	$\frac{1}{8}$	13.	$\frac{1}{12}$	24.	$\frac{20}{15}$	35.	$\frac{12}{15}$
3.	$\frac{1}{10}$	14.	$\frac{3}{12}$	25.	$\frac{1}{16}$	36.	$\frac{6}{15}$
4.	$\frac{1}{18}$	15.	$\frac{6}{12}$	26.	$\frac{3}{16}$	37.	$\frac{1}{81}$
5.	$\frac{1}{18}$	16.	$\frac{1}{18}$	27.	$\frac{9}{16}$	38.	$\frac{3}{40}$
6.	$\frac{1}{10}$	17.	$\frac{2}{18}$	28.	$\frac{12}{12}$	39.	$\frac{3}{24}$
7.	$\frac{1}{15}$	18.	$\frac{10}{18}$	29.	$\frac{15}{16}$	40.	$\frac{4}{9}$
8.	$\frac{1}{35}$	19.	$\frac{9}{8}$	30.	$\frac{18}{16}$	41.	$\frac{24}{32}$
9.	$\frac{1}{15}$	20.	$\frac{1}{25}$	31.	$\frac{16}{18}$	42.	$\frac{12}{9}$
10.	$\frac{1}{15}$	21.	$\frac{9}{25}$	32.	$\frac{2}{15}$	43.	$\frac{48}{63}$
11.	$\frac{2}{15}$	22.	$\frac{12}{25}$	33.	$\frac{8}{15}$	44.	$\frac{56}{84}$

Problem Set

- 48 ft³; diagrams will vary.
- Four different diagrams drawn
- Answers and explanations will vary.
- No, explanations will vary.
 - Explanations will vary.
 - 60 ft²

Exit Ticket

No; explanations and drawings will vary.

Homework

1. 216 in³; diagrams will vary.
2. Three different diagrams drawn
3. Answers will vary.
4.
 - a. No; explanations will vary.
 - b. Answers will vary.
 - c. Answers will vary.
 - d. Answers and explanations will vary.

Lesson 8

Problem Set

Parameters will vary.

Exit Ticket

Prism sketches and dimensions will vary.

Homework

1. $1,080 \text{ cm}^3$; answers will vary.
2. Answers will vary.

Lesson 9

Problem Set

Measurements and calculations will vary.

Exit Ticket

- a. 12; 6; 4; 288
- b. 18; 10; 10; 1,800
- c. 2,088

Homework

Answers will vary.

Lesson 10

Problem Set

1. Rectangle 3 units by 2 units drawn with tiles; 3; 2; 6
2. Rectangle 3 units by $2\frac{1}{2}$ units drawn with tiles; 3; $2\frac{1}{2}$; $7\frac{1}{2}$
3. Rectangle $1\frac{1}{2}$ units by 5 units drawn with tiles; 5; $1\frac{1}{2}$; $7\frac{1}{2}$
4. Rectangle 2 units by $1\frac{3}{4}$ units drawn with tiles; 2; $1\frac{3}{4}$; $3\frac{1}{2}$
5. Rectangles $\frac{3}{4}$ unit by 5 units drawn with tiles; 5; $\frac{3}{4}$; $3\frac{3}{4}$
6. $60\frac{3}{4}\text{ in}^2$; explanations will vary.
7. 69 ft^2

Exit Ticket

$$2\frac{1}{2}; 2; 5$$

Homework

1. a. $7\frac{1}{2}$
b. 4; $2\frac{1}{4}$; 9
c. Rectangle $\frac{3}{4}$ units by 4 units is drawn and tiled; 3
d. Rectangle 2 units by $1\frac{3}{4}$ units is drawn and tiled; $3\frac{1}{2}$
2. $109\frac{1}{2}\text{ in}^2$
3. $42\frac{3}{4}\text{ ft}^2$

Lesson 11

Sprint

Side A

- | | | | |
|---------|-----------|-----------|-----------|
| 1. 6 | 12. 0.15 | 23. 1.2 | 34. 21 |
| 2. 0.6 | 13. 14 | 24. 0.12 | 35. 0.24 |
| 3. 0.06 | 14. 1.4 | 25. 0.012 | 36. 24 |
| 4. 9 | 15. 0.14 | 26. 0.012 | 37. 4.2 |
| 5. 0.9 | 16. 12 | 27. 35 | 38. 0.49 |
| 6. 0.09 | 17. 1.2 | 28. 3.5 | 39. 0.048 |
| 7. 8 | 18. 1.2 | 29. 0.35 | 40. 0.054 |
| 8. 0.8 | 19. 0.12 | 30. 0.035 | 41. 4.8 |
| 9. 0.08 | 20. 0.012 | 31. 0.035 | 42. 0.63 |
| 10. 15 | 21. 0.012 | 32. 16 | 43. 0.064 |
| 11. 1.5 | 22. 12 | 33. 1.8 | 44. 0.072 |

Side B

- | | | | |
|---------|-----------|-----------|-----------|
| 1. 8 | 12. 0.12 | 23. 1.6 | 34. 24 |
| 2. 0.8 | 13. 18 | 24. 0.16 | 35. 0.27 |
| 3. 0.08 | 14. 1.8 | 25. 0.016 | 36. 32 |
| 4. 6 | 15. 0.18 | 26. 0.016 | 37. 4.2 |
| 5. 0.6 | 16. 15 | 27. 45 | 38. 0.36 |
| 6. 0.06 | 17. 1.5 | 28. 4.5 | 39. 0.048 |
| 7. 9 | 18. 1.5 | 29. 0.45 | 40. 0.054 |
| 8. 0.9 | 19. 0.15 | 30. 0.045 | 41. 4.8 |
| 9. 0.09 | 20. 0.015 | 31. 0.045 | 42. 0.63 |
| 10. 12 | 21. 0.015 | 32. 12 | 43. 0.049 |
| 11. 1.2 | 22. 16 | 33. 1.4 | 44. 0.072 |

Problem Set

1. $4\frac{1}{2}$; $2\frac{1}{2}$; $11\frac{1}{4}$
2. $3\frac{3}{4}$; $1\frac{3}{4}$; $6\frac{9}{16}$
3. $1\frac{1}{2}$; $\frac{3}{4}$; $1\frac{1}{8}$
4. $\frac{3}{4}$; $\frac{1}{2}$; $\frac{3}{8}$
5. a. Rectangles drawn
- b. Colleen's: $41\frac{2}{3}\text{ cm}^2$; Caroline's: $166\frac{2}{3}\text{ cm}^2$
- c. Answers will vary.
6. $162\frac{9}{16}\text{ in}^2$

Exit Ticket

Rectangle $2\frac{1}{2}$ square units sketched; $6\frac{1}{4}$ square units

Homework

1. a. $2\frac{3}{4}$; $1\frac{1}{2}$; $4\frac{1}{8}$
- b. Rectangle $2\frac{1}{2}$ by $\frac{3}{4}$ units drawn and tiled; $1\frac{7}{8}$
- c. Rectangle $3\frac{1}{3}$ by $2\frac{1}{2}$ units drawn and tiled; $8\frac{1}{3}$
- d. Rectangle $3\frac{1}{2}$ by $2\frac{1}{4}$ units drawn and tiled; $7\frac{7}{8}$
2. $39\frac{1}{16}\text{ in}^2$

Lesson 12

Problem Set

1.
 - a. Rectangle labeled; 5 in^2
 - b. Rectangle labeled; $3 \frac{1}{16} \text{ in}^2$
 - c. Rectangle labeled; $5 \frac{1}{2} \text{ in}^2$
 - d. Rectangle labeled; $7 \frac{5}{16} \text{ in}^2$
 - e. Rectangle labeled; $1 \frac{7}{8} \text{ in}^2$
 - f. Rectangle labeled; $1 \frac{7}{8} \text{ in}^2$
2.
 - a. $1 \frac{1}{2} \text{ ft}^2$, explanations will vary.
 - b. $2 \frac{1}{4} \text{ yd}^2$, explanations will vary.
 - c. $2 \frac{31}{32} \text{ yd}^2$, explanations will vary.
3. $290 \frac{11}{12} \text{ ft}^2$
4.
 - a. $10 \frac{9}{16} \text{ in}^2$
 - b. $84 \frac{1}{2} \text{ in}^2$

Exit Ticket

$2 \frac{1}{4} \text{ in}$, $1 \frac{1}{2} \text{ in}$; area = $3 \frac{3}{8} \text{ in}^2$

Homework

1.
 - a. Rectangle labeled; $6 \frac{1}{8} \text{ in}^2$
 - b. Rectangle labeled; $1 \frac{11}{16} \text{ in}^2$
 - c. Rectangle labeled; $5 \frac{1}{16} \text{ in}^2$
 - d. Rectangle labeled; $4 \frac{1}{8} \text{ in}^2$
 - e. Rectangle labeled; $\frac{7}{8} \text{ in}^2$
2.
 - a. Area model drawn; $\frac{9}{16} \text{ yd}^2$
 - b. Area model drawn; $3 \frac{1}{8} \text{ ft}^2$
3. No; answers will vary.
4.
 - a. $6 \frac{1}{4} \text{ ft}^2$
 - b. 266 ft^2

Lesson 13

Problem Set

1. a. 3 km^2
b. $69\frac{3}{10} \text{ m}^2$
c. $24\frac{5}{9} \text{ yd}^2$
d. $3\frac{19}{24} \text{ mi}^2$
2. $38\frac{2}{15} \text{ in}^2$
3. $562\frac{1}{2} \text{ yd}^2$

Exit Ticket

1. $9\frac{4}{5} \text{ mm}^2$
2. $26\frac{7}{16} \text{ km}^2$

Homework

1. a. 16 cm^2
b. $21\frac{3}{5} \text{ ft}^2$
c. $26\frac{1}{15} \text{ in}^2$
d. $4\frac{5}{7} \text{ m}^2$
2. $77\frac{11}{32} \text{ in}^2$
3. $374\frac{21}{80} \text{ ft}^2$

Lesson 14

Problem Set

- $71\frac{1}{2}\text{ ft}^2$
- $81\frac{1}{4}\text{ in}^2$
- $1,094\frac{5}{8}\text{ ft}^2$
- $1,656\frac{1}{9}\text{ ft}^2$
 - \$409.83
- Answers will vary.
 - The area of the quilt is $1,014\text{ in}^2$.

Exit Ticket

$$56\frac{7}{20}\text{ ft}^2.$$

Homework

- 180 ft^2
 $161\frac{2}{3}\text{ ft}^2$
- $307\frac{1}{16}\text{ in}^2$
- $383\frac{9}{10}\text{ ft}^2$
- $2,075\frac{7}{10}\text{ ft}^2$
5 bags for the highest setting;
7 bags for the lowest

Lesson 15

Problem Set

1. $\frac{9}{16} \text{ m}^2$
2. a. $\frac{25}{64} \text{ yd}^2$
b. $23\frac{1}{2} \text{ ft}$
c. $34\frac{33}{64} \text{ ft}^2$
3. a. $11\frac{1}{4} \text{ ft}^2$
b. $5\frac{5}{8} \text{ ft}^2$
4. a. $30\frac{9}{25} \text{ cm}^2$
b. $1\frac{3}{5} \text{ cm}$

Exit Ticket

$$220\frac{1}{2} \text{ in}^2$$

Homework

1. $18\frac{3}{4} \text{ ft}^2$
2. $1,642\frac{9}{16} \text{ ft}^2$
3. $375\frac{3}{4} \text{ in}^2$

Lesson 16

Problem Set

1. Drawings will vary.
2. Answers will vary.
3. Answers will vary but should include that they all have at least one pair of parallel lines.
4. When it has at least one pair of parallel lines
5. Drawings will vary.

Exit Ticket

- a. Trapezoids will vary.
- b. Answers will vary.

Homework

1.
 - a. Drawings will vary.
 - b. Drawings will vary.
2.
 - a. Shapes in the wrong groups circled; explanations will vary.
 - b. Explanations will vary.
3.
 - a. Trapezoids will vary.
 - b. Answers will vary.

Lesson 17

Problem Set

1. Parallelograms will vary.
2. Answers will vary.
 - a. Answers will vary.
 - b. Answers will vary.
3. Parallelograms will vary.
 - a. Answers will vary.
 - b. Answers will vary.
4. Answers will vary.
 - a. Answers will vary.
 - b. Answers will vary.

Exit Ticket

1. Parallelograms will vary.
2. When it has two pairs of parallel lines

Homework

1.
 - a. Parallelograms will vary.
 - b. 120° ; 60° ; 120°
2.
 - a. 6 cm; 3 cm
 - b. 67° ; 113° ; 67°
3. 4; 4; 3; 6
4. Answers will vary.
5. Answers will vary.

Lesson 18

Sprint

Side A

1.	$\frac{1}{4}$	12.	35	23.	16	34.	18
2.	$\frac{1}{6}$	13.	$\frac{1}{35}$	24.	$\frac{1}{9}$	35.	36
3.	$\frac{1}{8}$	14.	$\frac{1}{6}$	25.	$\frac{2}{9}$	36.	49
4.	$\frac{1}{14}$	15.	6	26.	$\frac{1}{8}$	37.	64
5.	14	16.	$\frac{1}{8}$	27.	$\frac{3}{8}$	38.	81
6.	12	17.	8	28.	$\frac{1}{10}$	39.	$\frac{1}{56}$
7.	10	18.	$\frac{1}{10}$	29.	$\frac{3}{10}$	40.	72
8.	6	19.	10	30.	$\frac{1}{12}$	41.	$\frac{1}{56}$
9.	10	20.	12	31.	$\frac{5}{12}$	42.	42
10.	15	21.	$\frac{1}{12}$	32.	$\frac{5}{18}$	43.	63
11.	20	22.	$\frac{1}{16}$	33.	$\frac{1}{18}$	44.	$\frac{1}{72}$

Side B

1.	$\frac{1}{4}$	12.	14	23.	9	34.	30
2.	$\frac{1}{15}$	13.	$\frac{1}{14}$	24.	$\frac{1}{16}$	35.	24
3.	$\frac{1}{20}$	14.	$\frac{1}{8}$	25.	$\frac{3}{16}$	36.	42
4.	$\frac{1}{35}$	15.	8	26.	$\frac{1}{9}$	37.	56
5.	35	16.	$\frac{1}{6}$	27.	$\frac{2}{9}$	38.	72
6.	30	17.	6	28.	$\frac{1}{12}$	39.	$\frac{1}{64}$
7.	25	18.	$\frac{1}{4}$	29.	$\frac{5}{12}$	40.	81
8.	15	19.	4	30.	$\frac{1}{25}$	41.	$\frac{1}{72}$
9.	4	20.	12	31.	$\frac{3}{25}$	42.	49
10.	6	21.	$\frac{1}{12}$	32.	$\frac{3}{20}$	43.	54
11.	8	22.	$\frac{1}{9}$	33.	$\frac{1}{30}$	44.	$\frac{1}{48}$

Problem Set

- Drawings will vary.
- Answers will vary.
- Rhombus and rectangle will vary.
 - Measurements will vary.
 - Drawings will vary.
- Answers will vary.
 - Answers will vary.
 - When all four sides are equal
 - When all the angles are 90°
 - When all sides are equal and pairs of sides are parallel to each other

Exit Ticket

1. Rhombuses will vary.
2. Rectangles will vary.

Homework

1.
 - a. Rhombuses will vary.
 - b. Rhombuses will vary.
 - c. Rectangles will vary.
 - d. Rectangles will vary.
2. 54.25 cm or $54\frac{1}{4}$ cm
3. Answers will vary.
4. Answers will vary.

Lesson 19

Sprint

Side A

- | | | | |
|----------|-----------|-----------|------------|
| 1. 20 | 12. 60 | 23. 660 | 34. 42 |
| 2. 120 | 13. 130 | 24. 6,600 | 35. 420 |
| 3. 1,200 | 14. 26 | 25. 240 | 36. 4,200 |
| 4. 40 | 15. 260 | 26. 480 | 37. 1,640 |
| 5. 340 | 16. 1,300 | 27. 2,400 | 38. 4,500 |
| 6. 3,400 | 17. 2,600 | 28. 4,800 | 39. 42,600 |
| 7. 70 | 18. 8 | 29. 690 | 40. 720 |
| 8. 270 | 19. 88 | 30. 6,900 | 41. 2,250 |
| 9. 2,700 | 20. 880 | 31. 142 | 42. 25,200 |
| 10. 30 | 21. 8,800 | 32. 1,420 | 43. 5,220 |
| 11. 6 | 22. 66 | 33. 28 | 44. 63,200 |

Side B

- | | | | |
|----------|-----------|-----------|------------|
| 1. 30 | 12. 80 | 23. 880 | 34. 52 |
| 2. 130 | 13. 140 | 24. 8,800 | 35. 520 |
| 3. 1,300 | 14. 28 | 25. 420 | 36. 5,200 |
| 4. 50 | 15. 280 | 26. 840 | 37. 2,160 |
| 5. 350 | 16. 1,400 | 27. 4,200 | 38. 4,500 |
| 6. 3,500 | 17. 2,800 | 28. 8,400 | 39. 48,600 |
| 7. 80 | 18. 6 | 29. 960 | 40. 640 |
| 8. 280 | 19. 66 | 30. 9,600 | 41. 1,950 |
| 9. 2,800 | 20. 660 | 31. 162 | 42. 14,400 |
| 10. 40 | 21. 6,600 | 32. 1,620 | 43. 5,340 |
| 11. 8 | 22. 88 | 33. 39 | 44. 60,800 |

Problem Set

1. Figures drawn.
2.
 - a. Answers will vary.
 - b. Answers will vary.
3.
 - a. Answers will vary.
 - b. Answers will vary.
 - c. When all four angles are 90°
 - d. When both pairs of adjacent sides are equal, and when all four angles are 90°
 - e. When all four sides and/or all four angles are equal.

Exit Ticket

1. All four sides are equal.
2. Kites have equal adjacent sides, but parallelograms have equal opposite sides.

Homework

1.
 - a. Kites will vary.
 - b. Quadrilateral that has two pairs of equal adjacent sides
 - c. Both pairs of sides are equal, or when it is a rhombus
2. It has right angles.
3. Squares will vary.
4. Explanations will vary.

Lesson 20

Problem Set

1.
 - a. T
 - b. F; answers will vary.
 - c. T
 - d. F; answers will vary.
 - e. T
 - f. T
 - g. F; answers will vary.
 - h. F; answers will vary.
 - i. T
 - j. F; answers will vary.
 - k. F; answers will vary.
2.
 - a. 140; 90; quadrilateral
 - b. 26 in; 26 in; 11 in; 55; 90; trapezoid, parallelogram, quadrilateral
 - c. 16 cm; 18 cm; 75; 105; 105; quadrilateral, trapezoid

Exit Ticket

Squares will vary.

- a. Equal adjacent sides
- b. All four sides equal
- c. All four right angles
- d. Both pairs of opposite sides equal and parallel
- e. At least one set of opposite parallel sides
- f. Four sides

Homework

1. Square; rectangle; rhombus; parallelogram; kite; trapezoid
2. 9.9; 9.9; 28; 90; 90

Lesson 21

Sprint

Side A

- | | | | |
|---------|--------|------------|---------|
| 1. 3 | 12. 20 | 23. 120 | 34. 30 |
| 2. 43 | 13. 20 | 24. 12 | 35. 40 |
| 3. 430 | 14. 40 | 25. 2,100 | 36. 43 |
| 4. 43 | 15. 20 | 26. 210 | 37. 6 |
| 5. 430 | 16. 20 | 27. 21 | 38. 40 |
| 6. 5 | 17. 24 | 28. 4,200 | 39. 60 |
| 7. 85 | 18. 12 | 29. 420 | 40. 64 |
| 8. 850 | 19. 12 | 30. 42 | 41. 32 |
| 9. 85 | 20. 36 | 31. 32,000 | 42. 23 |
| 10. 850 | 21. 12 | 32. 320 | 43. 240 |
| 11. 60 | 22. 12 | 33. 3,200 | 44. 42 |

Side B

- | | | | |
|---------|--------|------------|---------|
| 1. 2 | 12. 30 | 23. 210 | 34. 20 |
| 2. 42 | 13. 30 | 24. 21 | 35. 30 |
| 3. 420 | 14. 60 | 25. 1,200 | 36. 32 |
| 4. 42 | 15. 30 | 26. 120 | 37. 4 |
| 5. 420 | 16. 30 | 27. 12 | 38. 40 |
| 6. 4 | 17. 24 | 28. 2,400 | 39. 70 |
| 7. 84 | 18. 12 | 29. 240 | 40. 73 |
| 8. 840 | 19. 12 | 30. 24 | 41. 32 |
| 9. 84 | 20. 63 | 31. 23,000 | 42. 23 |
| 10. 840 | 21. 21 | 32. 230 | 43. 240 |
| 11. 90 | 22. 21 | 33. 2,300 | 44. 42 |

Problem Set

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.

Exit Ticket

1. Parallelograms; trapezoids; trapezoids; parallelograms
2. Rhombuses; kites; kites; rhombuses

Homework

1.
 - a. Always
 - b. Sometimes
 - c. Always
 - d. Always
 - e. Always
 - f. Sometimes
 - g. Sometimes
 - h. Drawings will vary.
2.
 - a. Explanations will vary.
 - b. Explanations will vary.

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