Eureka Math™ Assessment Packet

Algebra I Module 1

Module 1

Mid-Module Assessment	Qty: 30
End-of-Module Assessment	Qty: 30

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A STORY OF FUNCTIONS

ALGEBRA I

Name _____

Date _____

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

At 1:00 p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late and drives at a constant speed of 30 mph to the grocery store, arriving at 1:28 p.m. He then spends 20 minutes buying groceries.

a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).



b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.

c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.



- 2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and that the digit in the **tens** place changes every second or so.
 - a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?) Explain how you arrived at your estimate.

b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.

c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, and every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage **rate** with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.



- 3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys *B* gallons, and each time it places an order for black ink, it buys *K* gallons. Over a one-month period, the company places *m* orders of blue ink and *n* orders of black ink.
 - a. What quantities could the following expressions represent in terms of the problem context?

m + n

mB + nK

 $\frac{mB+nK}{m+n}$

b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

$$\frac{m}{m+n}$$
 and $\frac{n}{m+n}$,

and explain which expression must be greater using those interpretations.



4. Sam says that he knows a clever set of steps to rewrite the expression

(x+3)(3x+8) - 3x(x+3)

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

- 5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, $1 + ((2 + 3) \cdot 4)$ is one such expression.
 - a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.



b. In both of your expressions, replace 1 with a, 2 with b, 3 with c, and 4 with d to get two algebraic expressions. For example, $a + ((b + c) \cdot d)$ shows the replacements for the example given.

Are your algebraic expressions equivalent? Circle: Yes No

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide two examples:
 - (1) Find four different numbers (other than 0, 1, 2, 3, 4) that when substituted for *a*, *b*, *c*, and *d* into each expression, the expressions evaluate to **different numbers**, and

(2) Find four different, nonzero numbers that when substituted into each expression, the expressions evaluate to the **same number**.



6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for associative property and **C** for commutative property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.



- a. Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.
- b. Using the diagram above to help guide you, give *two different* proofs that $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$.



- 7. Ahmed learned: "To multiply a whole number by ten, just place a zero at the end of the number." For example, 2813×10 , he says, is 28,130. He doesn't understand why this rule is true.
 - a. What is the product of the polynomial $2x^3 + 8x^2 + x + 3$ times the polynomial x?
 - b. Use part (a) as a hint. Explain why the rule Ahmed learned is true.

8.

- a. Find the following products:
 - i. (x-1)(x+1)
 - ii. $(x-1)(x^2 + x + 1)$
 - iii. $(x-1)(x^3 + x^2 + x + 1)$
 - iv. $(x-1)(x^4 + x^3 + x^2 + x + 1)$



- v. $(x-1)(x^n + x^{n-1} + \dots + x^3 + x^2 + x + 1)$
- b. Substitute x = 10 into each of the products from parts (i) through (iv) and your answers to show how each of the products appears as a statement in arithmetic.

c. If we substituted x = 10 into the product $(x - 2)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ and computed the product, what number would result?

d. Multiply (x - 2) and $(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$, and express your answer in standard form.

Substitute x = 10 into your answer, and see if you obtain the same result that you obtained in part (c).



- e. Francois says $(x 9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ must equal $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ because when x = 10, multiplying by x 9 is the same as multiplying by 1.
 - i. Multiply $(x 9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$.

ii. Put x = 10 into your answer.

Is it the same as $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ with x = 10?

iii. Was Francois right?



M1

Name	_ Date	

- 1. Solve the following equations for *x*. Write your answer in set notation.
 - a. 3x 5 = 16
 - b. 3(x+3) 5 = 16

c. 3(2x-3) - 5 = 16

d. 6(x+3) - 10 = 32

e. Which two equations above have the same solution set? Write a sentence explaining how the properties of equality can be used to determine the pair without having to find the solution set for each.



- 2. Let *c* and *d* be real numbers.
 - a. If c = 42 + d is true, then which is greater: c or d, or are you not able to tell? Explain how you know your choice is correct.

b. If c = 42 - d is true, then which is greater: c or d, or are you not able to tell? Explain how you know your choice is correct.

3. If a < 0 and c > b, circle the expression that is greater:

a(b-c) or a(c-b)

Use the properties of inequalities to explain your choice.



- 4. Solve for x in each of the equations or inequalities below, and name the property and/or properties used:
 - a. $\frac{3}{4}x = 9$
 - b. 10 + 3x = 5x
 - c. a + x = b
 - d. cx = d

e.
$$\frac{1}{2}x - g < m$$

f.
$$q + 5x = 7x - r$$



g.
$$\frac{3}{4}(x+2) = 6(x+12)$$

h. 3(5-5x) > 5x

- 5. The equation 3x + 4 = 5x 4 has the solution set {4}.
 - a. Explain why the equation (3x + 4) + 4 = (5x 4) + 4 also has the solution set $\{4\}$.



b. In part (a), the expression (3x + 4) + 4 is equivalent to the expression 3x + 8. What is the definition of equivalent expressions? Why does changing an expression on one side of an equation to an equivalent expression leave the solution set unchanged?

c. When we square both sides of the original equation, we get the following new equation:

$$(3x+4)^2 = (5x-4)^2.$$

Show that 4 is still a solution to the new equation. Show that 0 is also a solution to the new equation but is not a solution to the original equation. Write a sentence that describes how the solution set to an equation may change when both sides of the equation are squared.



d. When we replace x by x^2 in the original equation, we get the following new equation:

$$3x^2 + 4 = 5x^2 - 4.$$

Use the fact that the solution set to the original equation is $\{4\}$ to find the solution set to this new equation.

6. The Zonda Information and Telephone Company (ZI&T) calculates a customer's total monthly cell phone charge using the formula,

$$C = (b + rm)(1 + t),$$

where C is the total cell phone charge, b is a basic monthly fee, r is the rate per minute, m is the number of minutes used that month, and t is the tax rate.

Solve for *m*, the number of minutes the customer used that month.



7. Students and adults purchased tickets for a recent basketball playoff game. All tickets were sold at the ticket booth—season passes, discounts, etc., were not allowed.

Student tickets cost \$5 each, and adult tickets cost \$10 each. A total of \$4,500 was collected. 700 tickets were sold.

a. Write a system of equations that can be used to find the number of student tickets, *s*, and the number of adult tickets, *a*, that were sold at the playoff game.

b. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the ticket booth charged students and adults the same price of \$10 per ticket?

c. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the student price was kept at \$5 per ticket and adults were charged \$15 per ticket instead of \$10?



- 8. Alexus is modeling the growth of bacteria for an experiment in science. She assumes that there are *B* bacteria in a Petri dish at 12:00 noon. In reality, each bacterium in the Petri dish subdivides into two new bacteria approximately every 20 minutes. However, for the purposes of the model, Alexus assumes that each bacterium subdivides into two new bacteria exactly every 20 minutes.
 - a. Create a table that shows the total number of bacteria in the Petri dish at $\frac{1}{3}$ hour intervals for 2 hours starting with time 0 to represent 12:00 noon.

- b. Write an equation that describes the relationship between total number of bacteria T and time h in hours, assuming there are B bacteria in the Petri dish at h = 0.
- c. If Alexus starts with 100 bacteria in the Petri dish, draw a graph that displays the total number of bacteria with respect to time from 12:00 noon (h = 0) to 4:00 p.m. (h = 4). Label points on your graph at time h = 0, 1, 2, 3, 4.



d. For her experiment, Alexus plans to add an anti-bacterial chemical to the Petri dish at 4:00 p.m. that is supposed to kill 99.9% of the bacteria instantaneously. If she started with 100 bacteria at 12:00 noon, how many live bacteria might Alexus expect to find in the Petri dish right after she adds the anti-bacterial chemical?

- 9. Jack is 27 years older than Susan. In 5 years, he will be 4 times as old as she is.
 - a. Find the present ages of Jack and Susan.

b. What calculations would you do to check if your answer is correct?



10.

a. Find the product: $(x^2 - x + 1)(2x^2 + 3x + 2)$.

b. Use the results of part (a) to factor 21,112 as a product of a two-digit number and a three-digit number.

11. Consider the following system of equations with the solution x = 3, y = 4.

Equation A1: y = x + 1

Equation A2: y = -2x + 10



a. Write a unique system of two linear equations with the same solution set. This time make both linear equations have positive slope.

Equation B1:

Equation B2:



b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1: y = x + 1

Equation C2: 3y = -3x + 21

What multiple of A2 was added to A1?

c. What is the solution to the system given in part (b)?

d. For any real number *m*, the line y = m(x - 3) + 4 passes through the point (3,4).

Is it certain, then, that the system of equations

Equation D1: y = x + 1

Equation D2: y = m(x - 3) + 4

has only the solution x = 3, y = 4? Explain.



12. The local theater in Jamie's home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.



a. Venus objected and said there was more than one reason that Jamie's thinking was flawed. What reasons could Venus be thinking of?



b. Use equations, inequalities, graphs, and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.

c. The theater charges \$9 for each adult ticket and \$6 for each child ticket. The theater sold 144 tickets for the first showing of the new release. The total money collected from ticket sales for that show was \$1,164. Write a system of equations that could be used to find the number of child tickets and the number of adult tickets sold, and solve the system algebraically. Summarize your findings using the context of the problem.

