A Story of Units®

Eureka Math™

Grade 4, Module 3

Teacher Edition

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Grade 4 • Module 3

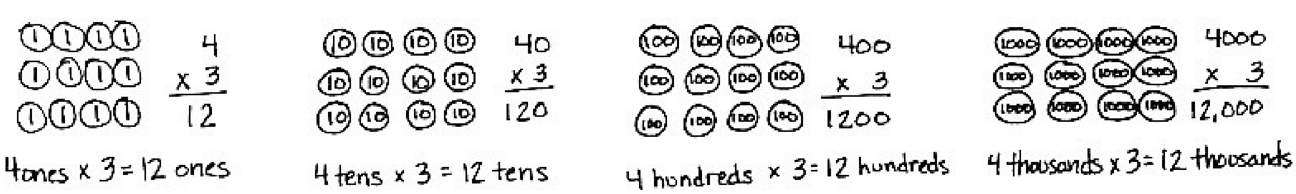
Multi-Digit Multiplication and Division

OVERVIEW

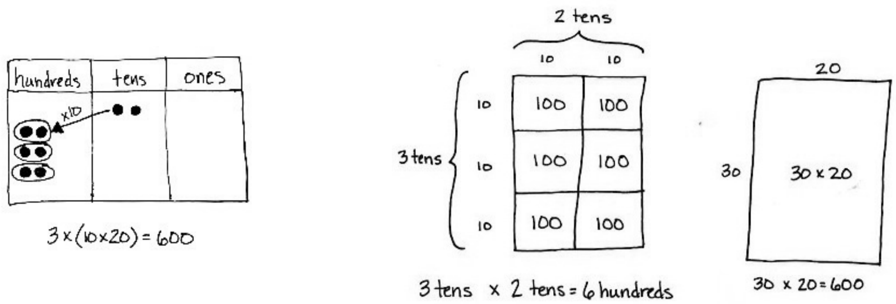
In this 43-day module, students use place value understanding and visual representations to solve multiplication and division problems with multi-digit numbers. As a key area of focus for Grade 4, this module moves slowly but comprehensively to develop students' ability to reason about the methods and models chosen to solve problems with multi-digit factors and dividends.

Students begin in Topic A by investigating the formulas for area and perimeter. They then solve multiplicative comparison problems including the language of *times as much as* with a focus on problems using area and perimeter as a context (e.g., "A field is 9 feet wide. It is 4 times as long as it is wide. What is the perimeter of the field?"). Students create diagrams to represent these problems as well as write equations with symbols for the unknown quantities (**4.OA.1**). This is foundational for understanding multiplication as scaling in Grade 5 and sets the stage for proportional reasoning in Grade 6. This Grade 4 module, beginning with area and perimeter, allows for new and interesting word problems as students learn to calculate with larger numbers and interpret more complex problems (**4.OA.2**, **4.OA.3**, **4.MD.3**).

In Topic B, students use place value disks to multiply single-digit numbers by multiples of 10, 100, and 1,000 and two-digit multiples of 10 by two-digit multiples of 10 (**4.NBT.5**). Reasoning between arrays and written numerical work allows students to see the role of place value units in multiplication (as pictured below). Students also practice the language of units to prepare them for multiplication of a single-digit factor by a factor with up to four digits and multiplication of two two-digit factors.

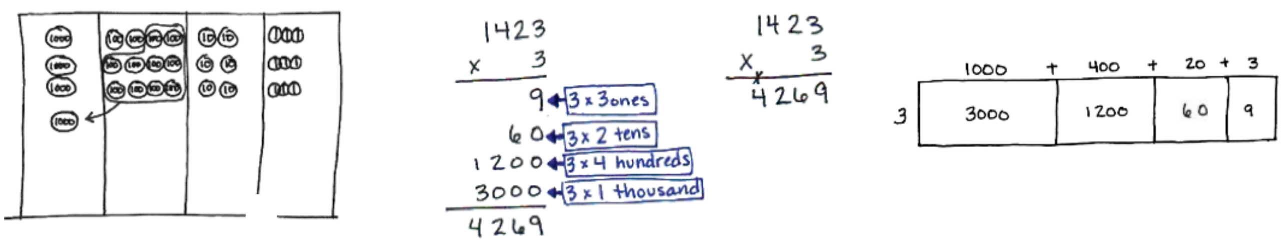


In preparation for two-digit by two-digit multiplication, students practice the new complexity of multiplying two two-digit multiples of 10. For example, students have multiplied 20 by 10 on the place value chart and know that it shifts the value one place to the left, 10 × 20 = 200. To multiply 20 by 30, the associative property allows for simply tripling the product, 3 × (10 × 20), or multiplying the units, 3 tens × 2 tens = 6 hundreds (alternatively, (3 × 10) × (2 × 10) = (3 × 2) × (10 × 10)). Introducing this early in the module allows students to practice during fluency so that, by the time it is embedded within the two-digit by two-digit multiplication in Topic H, understanding and skill are in place.



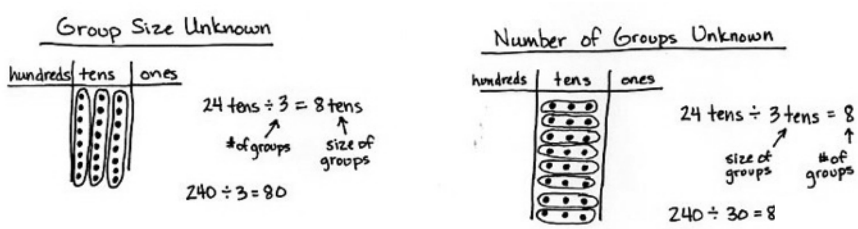
Building on their work in Topic B, students begin in Topic C decomposing numbers into base ten units in order to find products of single-digit by multi-digit numbers. Students use the distributive property and multiply using place value disks to model. Practice with place value disks is used for two-, three-, and four-digit by one-digit multiplication problems with recordings as partial products. Students bridge partial products to the recording of multiplication via the standard algorithm.[1] Finally, the partial products method, the standard algorithm, and the area model are compared and connected by the distributive property (**4.NBT.5**).

1,423 x 3



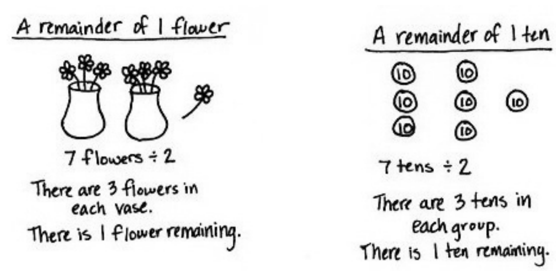
Topic D gives students the opportunity to apply their new multiplication skills to solve multi-step word problems (**4.OA.3**, **4.NBT.5**) and multiplicative comparison problems (**4.OA.2**). Students write equations from statements within the problems (**4.OA.1**) and use a combination of addition, subtraction, and multiplication to solve.

In Topic E, students synthesize their Grade 3 knowledge of division types (*group size unknown* and *number of groups unknown*) with their new, deeper understanding of place value.

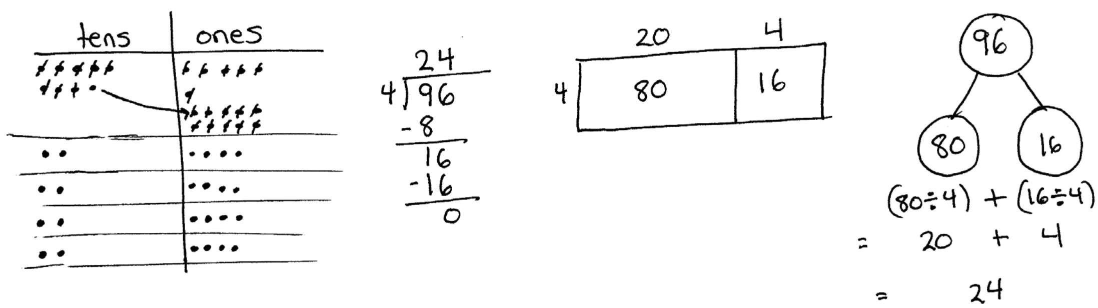


[1]Students become fluent with the standard algorithm for multiplication in Grade 5 (5.NBT.5). Grade 4 students are introduced to the standard algorithm in preparation for fluency and as a general method for solving multiplication problems based on place value strategies, alongside place value disks, partial products, and the area model. Students are not assessed on the standard algorithm in Grade 4.

Students focus on interpreting the remainder within division problems, both in word problems and long division (**4.OA.3**). A remainder of 1, as exemplified below, represents a leftover flower in the first situation and a remainder of 1 ten in the second situation.[2]



While we have no reason to subdivide a remaining flower, there are good reasons to subdivide a remaining ten. Students apply this simple idea to divide two-digit numbers unit by unit: dividing the tens units first, finding the remainder (the number of tens unable to be divided), and decomposing remaining tens into ones to then be divided. Students represent division with single-digit divisors using arrays and the area model before practicing with place value disks. The standard division algorithm[3] is practiced using place value knowledge, decomposing unit by unit. Finally, students use the area model to solve division problems, first with and then without remainders (**4.NBT.6**).



In Topic F, armed with an understanding of remainders, students explore factors, multiples, and prime and composite numbers within 100 (**4.OA.4**), gaining valuable insights into patterns of divisibility as they test for primes and find factors and multiples. This prepares them for Topic G's work with multi-digit dividends.

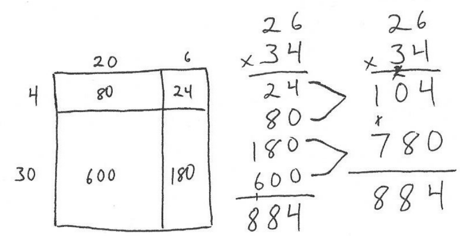
Topic G extends the practice of division with three- and four-digit dividends using place value understanding. A connection to Topic B is made initially with dividing multiples of 10, 100, and 1,000 by single-digit numbers. Place value disks support students visually as they decompose each unit before dividing. Students then practice using the standard algorithm to record long division. They solve word problems and make connections to the area model as was done with two-digit dividends (**4.NBT.6**, **4.OA.3**).

---

[2]Note that care must be taken in the interpretation of remainders. Consider the fact that 7 ÷ 3 is not equal to 5 ÷ 2 because the remainder of 1 is in reference to a different whole amount ($2\frac{1}{3}$ is not equal to $2\frac{1}{2}$).

[3]Students become fluent with the standard division algorithm in Grade 6 (6.NS.2). For adequate practice in reaching fluency, students are introduced to, but not assessed on, the division algorithm in Grade 4 as a general method for solving division problems.

The module closes as students multiply two-digit by two-digit numbers. Students use their place value understanding and understanding of the area model to empower them to multiply by larger numbers (as pictured to the right). Topic H culminates at the most abstract level by explicitly connecting the partial products appearing in the area model to the distributive property and recording the calculation vertically (**4.NBT.5**). Students see that partial products written vertically are the same as those obtained via the distributive property: 4 twenty-sixes + 30 twenty-sixes = 104 + 780 = 884.

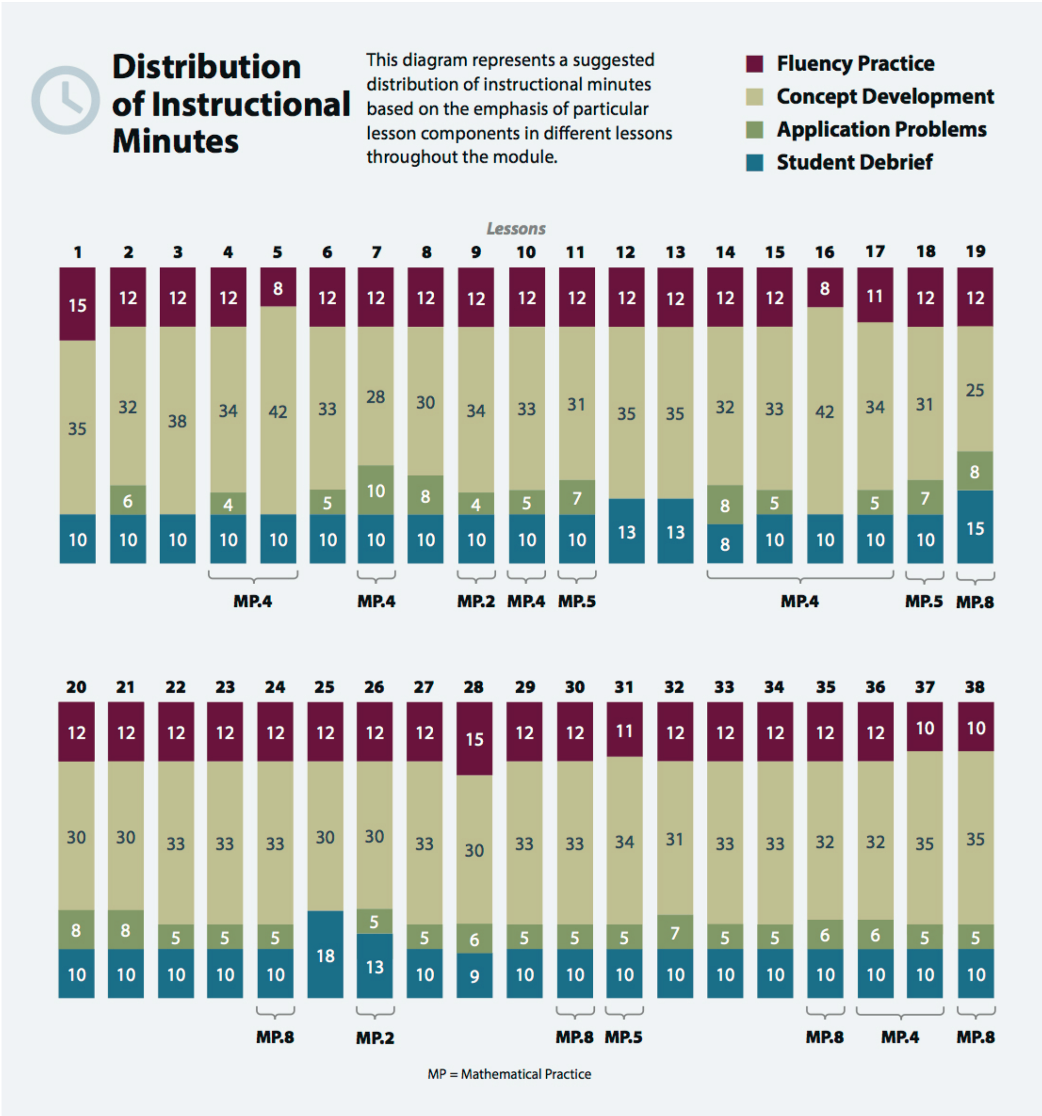


As students progress through this module, they are able to apply the multiplication and division algorithms because of their in-depth experience with the place value system and multiple conceptual models. This helps to prepare them for fluency with the multiplication algorithm in Grade 5 and the division algorithm in Grade 6. Students are encouraged in Grade 4 to continue using models to solve when appropriate.

Notes on Pacing for Differentiation

Within this module, if pacing is a challenge, consider the following omissions. In Lesson 1, omit Problem 1 if you embedded it into Module 2, and omit Problem 4, which can be used for a center activity. In Lesson 8, omit the drawing of models in Problems 2 and 4 of the Concept Development and in Problem 2 of the Problem Set. Instead, have students think about and visualize what they would draw. Omit Lesson 10 because the objective for Lesson 10 is the same as that for Lesson 9. Omit Lesson 19, and instead, embed discussions of interpreting remainders into other division lessons. Omit Lesson 21 because students solve division problems using the area model in Lesson 20. Using the area model to solve division problems with remainders is not specified in the Progressions documents. Omit Lesson 31, and instead, embed analysis of division situations throughout later lessons. Omit Lesson 33, and embed into Lesson 30 the discussion of the connection between division using the area model and division using the algorithm.

Look ahead to the Pacing Suggestions for Module 4. Consider partnering with the art teacher to teach Module 4's Topic A simultaneously with Module 3.



Focus Grade Level Standards

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See CCLS Glossary, Table 2.)

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.[5]

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Foundational Standards

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See CCLS Glossary, Table 2.)

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?.*

[4] 4.NBT.4 is addressed in Module 1 and is then reinforced throughout the year.

[5] 4.MD.1 is addressed in Modules 2 and 7; 4.MD.2 is addressed in Modules 2, 5, 6, and 7.

3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

3.MD.7 Relate area to the operations of multiplication and addition.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** Students solve multi-step word problems using the four operations by writing equations with a letter standing in for the unknown quantity.

MP.4 **Model with mathematics.** Students apply their understanding of place value to create area models and rectangular arrays when performing multi-digit multiplication and division. They use these models to illustrate and explain calculations.

MP.5 **Use appropriate tools strategically.** Students use mental computation and estimation strategies to assess the reasonableness of their answers when solving multi-step word problems. They draw and label bar and area models to solve problems as part of the RDW process. Additionally, students select an appropriate place value strategy when solving multiplication and division problems.

MP.8 **Look for and express regularity in repeated reasoning.** Students express the regularity they notice in repeated reasoning when they apply place value strategies in solving multiplication and division problems. They move systematically through the place values, decomposing or composing units as necessary, applying the same reasoning to each successive unit.

Overview of Module Topics and Lesson Objectives

Standards		Topics and Objectives	Days
4.OA.1 **4.OA.2** **4.MD.3** 4.OA.3	A	**Multiplicative Comparison Word Problems** Lesson 1: Investigate and use the formulas for area and perimeter of rectangles. Lesson 2: Solve multiplicative comparison word problems by applying the area and perimeter formulas. Lesson 3: Demonstrate understanding of area and perimeter formulas by solving multi-step real world problems.	3
4.NBT.5 4.OA.1 4.OA.2 4.NBT.1	B	**Multiplication by 10, 100, and 1,000** Lesson 4: Interpret and represent patterns when multiplying by 10, 100, and 1,000 in arrays and numerically. Lesson 5: Multiply multiples of 10, 100, and 1,000 by single digits, recognizing patterns. Lesson 6: Multiply two-digit multiples of 10 by two-digit multiples of 10 with the area model.	3
4.NBT.5 4.OA.2 4.NBT.1	C	**Multiplication of up to Four Digits by Single-Digit Numbers** Lesson 7: Use place value disks to represent two-digit by one-digit multiplication. Lesson 8: Extend the use of place value disks to represent three- and four-digit by one-digit multiplication. Lessons 9–10: Multiply three- and four-digit numbers by one-digit numbers applying the standard algorithm. Lesson 11: Connect the area model and the partial products method to the standard algorithm.	5
4.OA.1 **4.OA.2** **4.OA.3** **4.NBT.5**	D	**Multiplication Word Problems** Lesson 12: Solve two-step word problems, including multiplicative comparison. Lesson 13: Use multiplication, addition, or subtraction to solve multi-step word problems.	2
		Mid-Module Assessment: Topics A–D (review 1 day, assessment ½ day, return ½ day)	2

Standards	Topics and Objectives			Days
4.NBT.6 4.OA.3	E	**Division of Tens and Ones with Successive Remainders**		8
		Lesson 14:	Solve division word problems with remainders.	
		Lesson 15:	Understand and solve division problems with a remainder using the array and area models.	
		Lesson 16:	Understand and solve two-digit dividend division problems with a remainder in the ones place by using place value disks.	
		Lesson 17:	Represent and solve division problems requiring decomposing a remainder in the tens.	
		Lesson 18:	Find whole number quotients and remainders.	
		Lesson 19:	Explain remainders by using place value understanding and models.	
		Lesson 20:	Solve division problems without remainders using the area model.	
		Lesson 21:	Solve division problems with remainders using the area model.	
4.OA.4	F	**Reasoning with Divisibility**		4
		Lesson 22:	Find factor pairs for numbers to 100, and use understanding of factors to define prime and composite.	
		Lesson 23:	Use division and the associative property to test for factors and observe patterns.	
		Lesson 24:	Determine if a whole number is a multiple of another number.	
		Lesson 25:	Explore properties of prime and composite numbers to 100 by using multiples.	
4.OA.3 **4.NBT.6** 4.NBT.1	G	**Division of Thousands, Hundreds, Tens, and Ones**		8
		Lesson 26:	Divide multiples of 10, 100, and 1,000 by single-digit numbers.	
		Lesson 27:	Represent and solve division problems with up to a three-digit dividend numerically and with place value disks requiring decomposing a remainder in the hundreds place.	
		Lesson 28:	Represent and solve three-digit dividend division with divisors of 2, 3, 4, and 5 numerically.	

Standards		Topics and Objectives	Days
		Lesson 29: Represent numerically four-digit dividend division with divisors of 2, 3, 4, and 5, decomposing a remainder up to three times. Lesson 30: Solve division problems with a zero in the dividend or with a zero in the quotient. Lesson 31: Interpret division word problems as either *number of groups unknown* or *group size unknown*. Lesson 32: Interpret and find whole number quotients and remainders to solve one-step division word problems with larger divisors of 6, 7, 8, and 9. Lesson 33: Explain the connection of the area model of division to the long division algorithm for three- and four-digit dividends	
4.NBT.5 4.OA.3 4.MD.3	H	**Multiplication of Two-Digit by Two-Digit Numbers** Lesson 34: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart. Lesson 35: Multiply two-digit multiples of 10 by two-digit numbers using the area model. Lesson 36: Multiply two-digit by two-digit numbers using four partial products. Lessons 37–38: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.	5
		End-of-Module Assessment: Topics A–H (review 1 day, assessment ½ day, return ½ day, remediation or further applications 1 day)	3
Total Number of Instructional Days			**43**

Terminology

New or Recently Introduced Terms

- Associative property (e.g., 96 = 3 × (4 × 8) = (3 × 4) × 8)
- Composite number (positive integer having three or more whole number factors)
- Distributive property (e.g., 64 × 27 = (60 × 20) + (60 × 7) + (4 × 20) + (4 × 7))
- Divisible
- Divisor (the number by which another number is divided)
- Formula (a mathematical rule expressed as an equation with numbers and/or variables)
- Long division (process of dividing a large dividend using several recorded steps)

- Partial product (e.g., 24 × 6 = (20 × 6) + (4 × 6) = 120 + 24)
- Prime number (positive integer greater than 1 having whole number factors of only 1 and itself)
- Remainder (the number left over when one integer is divided by another)

Familiar Terms and Symbols[6]

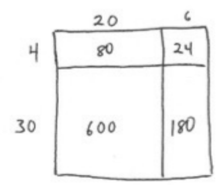
- Algorithm (steps for base ten computations with the four operations)
- Area (the amount of two-dimensional space in a bounded region)
- Area model (a model for multiplication and division problems that relates rectangular arrays to area, in which the length and width of a rectangle represent the factors for multiplication, and for division, the width represents the divisor and the length represents the quotient)
- Array (a set of numbers or objects that follow a specific pattern, a matrix)
- Bundling, grouping, renaming, changing (compose or decompose a 10, 100, etc.)
- Compare (to find the similarity or dissimilarity between)
- Distribute (decompose an unknown product in terms of two known products to solve)
- Divide, division (e.g., 15 ÷ 5 = 3)
- Equation (a statement that the values of two mathematical expressions are equal using the = sign)
- Factors (numbers that can be multiplied together to get other numbers)
- Mixed units (e.g., 1 ft 3 in, 4 lb 13 oz)
- Multiple (product of a given number and any other whole number)
- Multiply, multiplication (e.g., 5 × 3 = 15)
- Perimeter (length of a continuous line forming the boundary of a closed geometric figure)
- Place value (the numerical value that a digit has by virtue of its position in a number)
- Product (the result of multiplication)
- Quotient (the result of division)
- Rectangular array (an arrangement of a set of objects into rows and columns)
- Rows, columns (e.g., in reference to rectangular arrays)
- ___ *times as many* ___ *as* ___ (multiplicative comparative sentence frame)

[6]These are terms and symbols students have used or seen previously.

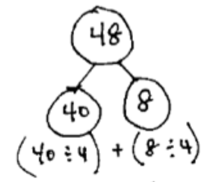
Suggested Tools and Representations

- Area model
- Grid paper
- Number bond
- Place value disks: suggested minimum of 1 set per pair of students (18 ones, 18 tens, 18 hundreds, 18 thousands, 1 ten thousand)

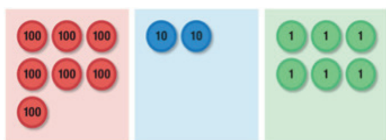
Area Model



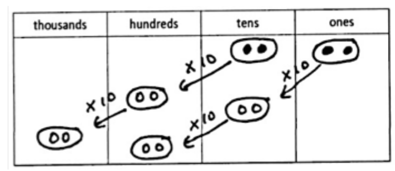
Number Bond



Place Value Disks



Thousands Place Value Chart



- Tape diagram
- Ten thousands place value chart (Lesson 7 Template)
- Thousands place value chart (Lesson 4 Template)

Scaffolds[7]

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*."

[7]Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website www.p12.nysed.gov/specialed/aim for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic D	Constructed response with rubric	4.OA.1 4.OA.2 4.OA.3 4.NBT.5 4.MD.3
End-of-Module Assessment Task	After Topic H	Constructed response with rubric	4.OA.1 4.OA.2 4.OA.3 4.OA.4 4.NBT.5 4.NBT.6 4.MD.3

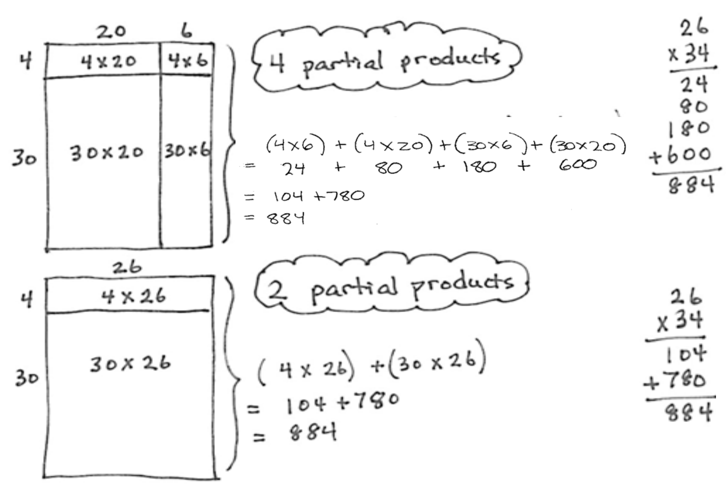
Topic H

Multiplication of Two-Digit by Two-Digit Numbers

4.NBT.5, 4.OA.3, 4.MD.3

Focus Standard:	4.NBT.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Instructional Days:	5	
Coherence -Links from:	G3–M1	Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
	G3–M3	Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10
-Links to:	G5–M2	Multi-Digit Whole Number and Decimal Fraction Operations

Module 3 closes with Topic H as students multiply two-digit by two-digit numbers.



Lesson 34 begins this topic by having students use the area model to represent and solve the multiplication of two-digit multiples of 10 by two-digit numbers using a place value chart. Practice with this model helps to prepare students for two-digit by two-digit multiplication and builds the understanding of multiplying units of 10. In Lesson 35, students extend their learning to represent and solve the same type of problems using area models and partial products.

In Lesson 36, students make connections to the distributive property and use both the area model and four partial products to solve problems. Lesson 37 deepens students' understanding of multi-digit multiplication by transitioning from four partial products with representation of the area model to two partial products with representation of the area model and finally to two partial products without representation of the area model.

Topic H culminates at the most abstract level with Lesson 38 as students are introduced to the multiplication algorithm for two-digit by two-digit numbers. Knowledge from Lessons 34–37 provides a firm foundation for understanding the process of the algorithm as students make connections from the area model to partial products to the standard algorithm (**4.NBT.5**). Students see that partial products written vertically are the same as those obtained via the distributive property: 4 twenty-sixes + 30 twenty-sixes = 104 + 780 = 884.

A Teaching Sequence Toward Mastery of Multiplication of Two-Digit by Two-Digit Numbers	
Objective 1:	**Multiply two-digit multiples of 10 by two-digit numbers using a place value chart.** **(Lesson 34)**
Objective 2:	**Multiply two-digit multiples of 10 by two-digit numbers using the area model.** **(Lesson 35)**
Objective 3:	**Multiply two-digit by two-digit numbers using four partial products.** **(Lesson 36)**
Objective 4:	**Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.** **(Lessons 37–38)**

Name ____ Date ____________________

1. Use the associative property to rewrite each expression. Solve using disks, and then complete the number sentences.

20 × 41

____ × ____ × ____ = ____

hundreds	tens	ones

2. Distribute 32 as 30 + 2 and solve.

60 × 32

Name ______________________________ Date ________________

Use an area model to represent the following expressions. Then, record the partial products and solve.

1. 30 × 93

```
    9 3
  × 3 0
  -----

+ -----
  =====
```

2. 40 × 76

```
    7 6
  × 4 0
  -----

+ -----
  =====
```

Name ____ Date ____________________

Record the partial products to solve.

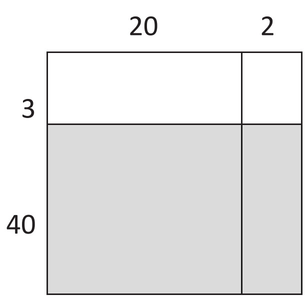
Draw an area model first to support your work, or draw the area model last to check your work.

1. 26×43

2. 17×55

Name ____ Date ________________

1. Solve 43 × 22 using 4 partial products and 2 partial products. Remember to think in terms of units as you solve. Write an expression to find the area of each smaller rectangle in the area model.



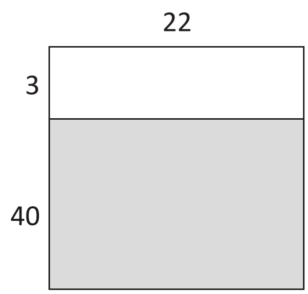
$$\begin{array}{r} 2\ 2 \\ \times\ 4\ 3 \\ \hline \end{array}$$

3 ones × 2 ones

3 ones × 2 tens

4 tens × 2 ones

4 tens × 2 tens

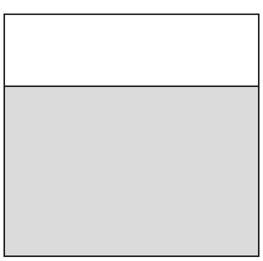


$$\begin{array}{r} 2\ 2 \\ \times\ 4\ 3 \\ \hline \end{array}$$

3 ones × 22 ones

4 tens × 22 ones

2. Solve the following using 2 partial products.



$$\begin{array}{r} 6\ 4 \\ \times\ 1\ 5 \\ \hline \end{array}$$

5 ones × 64 ones

1 ten × 64 ones

Name ______________________________ Date ________________

Solve using the multiplication algorithm.

1.

$$\begin{array}{r} 72 \\ \times\ 43 \\ \hline \\ \hline \\ \hline \end{array}$$

____ × ____

____ × _____

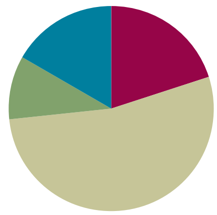
2. 35 × 53

Lesson 36

Objective: Multiply two-digit by two-digit numbers using four partial products.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(6 minutes)
■ Concept Development	(32 minutes)
■ Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Draw a Unit Fraction **3.G.2** (4 minutes)
- Divide Three Different Ways **4.NBT.6** (4 minutes)
- Multiply by Multiples of 10 Written Vertically **4.NBT.5** (4 minutes)

Draw a Unit Fraction (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Grade 3 geometry and fraction concepts in anticipation of Modules 4 and 5. Accept reasonable drawings. Using rulers is not necessary to review the concept and takes too long.

T: On your personal white boards, write the name for any four-sided figure.
S: (Write *quadrilateral*.)
T: Draw a quadrilateral that has 4 right angles and 4 equal sides.
S: (Draw a square.)
T: Partition the square into 4 equal parts.
S: (Partition.)
T: Shade in 1 of the parts.
S: (Shade.)
T: Write the fraction of the square that you shaded.
S: (Write $\frac{1}{4}$.)

Continue with the following possible sequence: Partition a rectangle into 5 equal parts, shading $\frac{1}{5}$; partition a rhombus into 2 equal parts, shading $\frac{1}{2}$; partition a square into 12 equal parts, shading $\frac{1}{12}$; and partition a rectangle into 8 equal parts, shading $\frac{1}{8}$.

Divide Three Different Ways (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 32 and 33.

T: (Write 406 ÷ 7.) Find the quotient using place value disks.
T: Find the quotient using the area model.
T: Find the quotient using the standard algorithm.

Repeat using 3,168 ÷ 9.

Multiply by Multiples of 10 Written Vertically (4 minutes)

Materials: (S) Personal white board

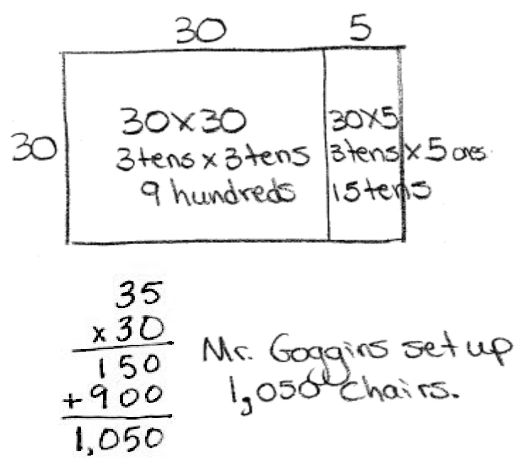
Note: This fluency activity reviews Lesson 35's content.

T: (Write 30 × 23 vertically.) When I write 30 × 23, you say "3 tens times 3 ones plus 3 tens times 2 tens." (Point to the corresponding expressions as students speak.)
S: 3 tens times 3 ones + 3 tens times 2 tens.
T: Write and solve the entire equation vertically.
T: What is 30 times 23?
S: 690.

Continue with the following possible sequence: 30 × 29, 40 × 34, and 50 × 45.

Application Problem (6 minutes)

Mr. Goggins set up 30 rows of chairs in the gymnasium. If each row had 35 chairs, how many chairs did Mr. Goggins set up? Draw an area model to represent and to help solve this problem. Discuss with a partner how the area model can help you solve 30 × 35.



Note: This Application Problem builds on prior learning from Lesson 35 where students used an area model and partial products to multiply a two-digit multiple of 10 by a two-digit number using an area model. This Application Problem also helps bridge to today's lesson in that students apply prior knowledge of the area model and partial products to represent and solve two-digit by two-digit multiplication.

Concept Development (32 minutes)

> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
>
> Lead a discussion with students in order to deepen their understanding of representing expressions in numerical form and unit form. Be sure that students understand that there are different ways to express numbers in both written and oral form.

Materials: (S) Personal white board

Problem 1: Use the distributive property to represent and solve two-digit by two-digit multiplication.

T: (Use the context of the Application Problem to continue with today's lesson.) Mr. Goggins set up an additional 4 rows of chairs with 35 chairs in each row. Let's change our area model to represent the additional rows. (Revise the area model.)

T: What is the length of this entire side? (Point to the vertical length.)

S: 34.

T: And the length of this side? (Point to the horizontal length.)

S: 35.

T: Use the area formula. What expression is shown by the area model now?

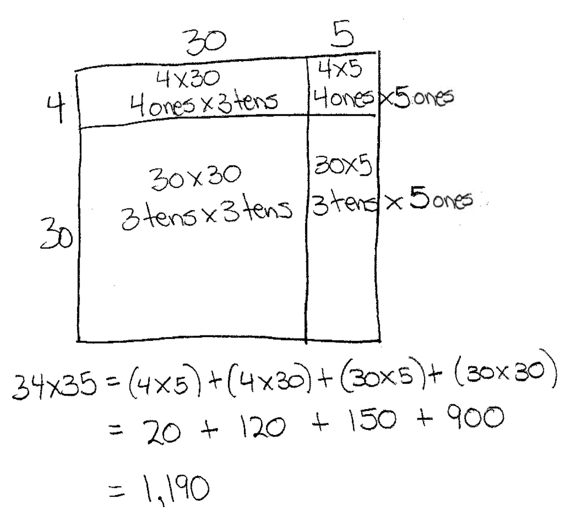
S: 34 × 35.

MP.4

T: We can use the area model to help us represent two-digit times two-digit multiplication. Write the expressions that represent the areas of the two smaller rectangles that we just created.

S: 4 × 5 and 4 × 30.

T: Let's say the expressions in unit form to help us understand their value. Using the units for each factor, say 4 × 5 and 4 × 30 in unit form.



S: 4 ones × 5 ones and 4 ones × 3 tens.

T: Write those unit expressions in each rectangle. How can we use these expressions and the expressions of the other two rectangles to find the area of the whole rectangle?

S: We can find the sum of all of the smaller areas.

T: Let's represent this using the distributive property. We are going to move from top to bottom, right to left to represent the areas of the smaller rectangles. You tell me the numerical expressions as I point to each of the smaller rectangles. I will write what you say. 34 × 35 equals…?

S: 34 × 35 = (4 × 5) + (4 × 30) + (30 × 5) + (30 × 30).

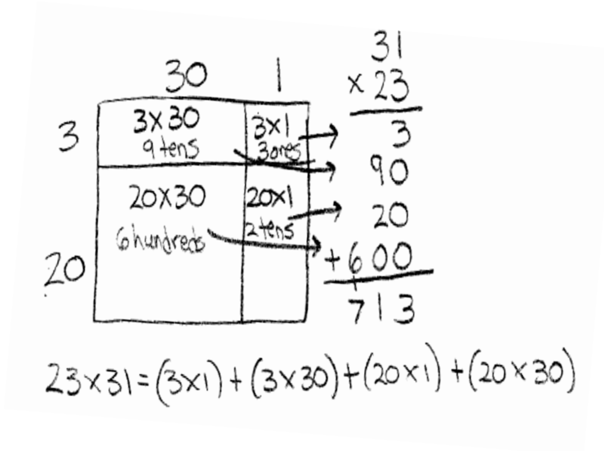
T: Now, express this same number sentence in unit form (without rewriting).

S: 34 × 35 = (4 ones × 5 ones) + (4 ones × 3 tens) + (3 tens × 5 ones) + (3 tens × 3 tens).

T: Now, we are ready to solve! First, let's find each of the four partial products. Then, we can add the four partial products to find 34 × 35.

S: 20 ones + 12 tens + 15 tens + 9 hundreds = 20 + 120 + 150 + 900 = 1,190.

Problem 2: Find the product of 23 and 31 using an area model and partial products to solve.



T: Let's solve 23 × 31 using area to model the product.

T: (Draw a rectangle.) Break down the length and width according to place value units.

S: 2 tens 3 ones and 3 tens 1 one. → 20 and 3. 30 and 1.

T: (Draw one vertical and one horizontal line subdividing the rectangle.) Turn and tell your partner the length and width of each of the 4 smaller rectangles we just created.

S: 3 and 1, 3 and 30, 20 and 1, and 20 and 30.

T: Using the area model that you just drew, write an equation that represents the product of 23 and 31 as the sum of those four areas.

S: 23 × 31 = (3 × 1) + (3 × 30) + (20 × 1) + (20 × 30).

T: Now, we are ready to solve!

T: Let's look at a way to record the partial products. (Write 23 × 31 vertically.) Recall that when we multiplied a one-digit number by a two-, three-, or four-digit number, we recorded the partial products. We also recorded partial products when we multiplied a two-digit number by a multiple of 10. Let's put it all together and do precisely the same thing here.

T: (Point to the area model and the expression showing the distributive property.) What is the product of 3 ones and 1 one?

S: 3 ones.

T: Record the product below. Draw an arrow connecting the rectangle with the corresponding partial product. How about 3 ones times 3 tens?

S: 9 tens.

T: Record the product below the first partial product. Draw an arrow connecting the rectangle with the corresponding partial product. What is 2 tens times 1 one?

S: 2 tens or 20.

T: As before, record the partial product below the other two and do the same with 2 tens times 3 tens.

T: Draw arrows to connect the new partial products with the corresponding rectangles. Now, let's add the partial products together. What is the sum?

S: The sum is 713. That means that 23 × 31 = 713.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Students working below grade level may benefit from continuing to write out the expressions used to find each of the partial products. Students may write the expressions in numerical form or in unit form.

To help solidify place value, it might also be helpful to have students shade, in different colors, the rectangles that represent the ones, tens, and hundreds.

Students working above grade level may be ready to use the four partial product algorithm and can be encouraged to do so.

Problem 3: Find the product of 26 and 34 using partial products. Verify partial products using the area model.

T: Draw an area model to represent 26 × 34.

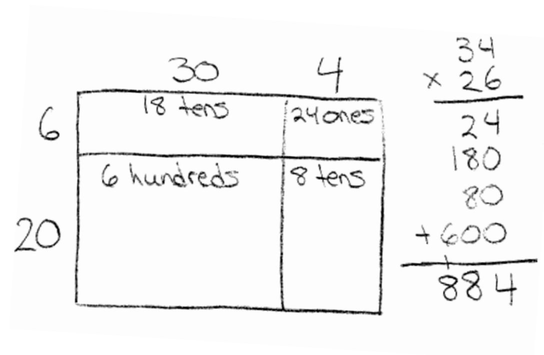
T: How do I find the area of the smallest rectangle?

S: Multiply 6 ones times 4 ones.

T: Point to 6 ones times 4 ones in the algorithm. What is 6 ones times 4 ones?

S: 24 ones.

T: Record 24 beneath the expression and in the corresponding area.



T: Point to the next area to solve for. Tell me the expression.

S: 6 ones times 3 tens.

T: Locate those numbers in the algorithm. Solve for 6 ones times 3 tens.

S: 18 tens.

T: Record 18 tens under the expression.

S: We can also record 18 tens in this rectangle.

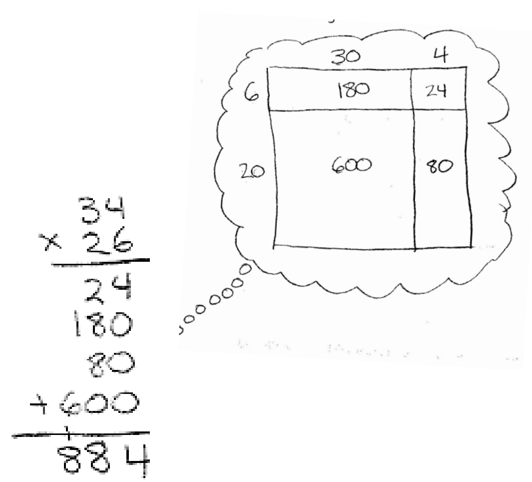
Continue connecting the width and length of each rectangle in the model to the location of those units in the algorithm. Record the partial products first under the expression and then inside the area.

T: What is the last step?

S: Add together all of the partial products. 24 + 180 + 80 + 600 = 884. → 26 × 34 = 884.

Problem 4: Find the product of 26 and 34 without using an area model. Record the partial products to solve.

T: Take a mental picture of your area model before you erase it, the partial products, and the final product.



T: When we multiplied these numbers before, with what did we start?

S: 6 ones × 4 ones.

T: Do you see 6 ones × 4 ones?

S: Yes.

Students point to 6 ones × 4 ones. You might model on the board as students also record.

T: What is 6 ones × 4 ones?

S: 24 ones.

T: Record 24 ones as a partial product.

T: What did we multiply next?

S: 6 ones × 3 tens. That's 18 tens or 180.

T: Where do we record 180?

S: Below the 24.

T: Now what?

S: We multiply the tens. 2 tens × 4 ones and then 2 tens × 3 tens.

T: What are 2 tens × 4 ones and 2 tens × 3 tens?

S: 8 tens and 6 hundreds.

T: Let's record these as partial products. Notice that we have four partial products. Let's again identify from where they came. (Point to each part of the algorithm as students chorally read the expressions used to solve the two-digit by two-digit multiplication.)

S: 6 ones × 4 ones = 24 ones. 6 ones × 3 tens = 18 tens. 2 tens × 4 ones = 8 tens.
2 tens × 3 tens = 6 hundreds.

T: What is their sum?

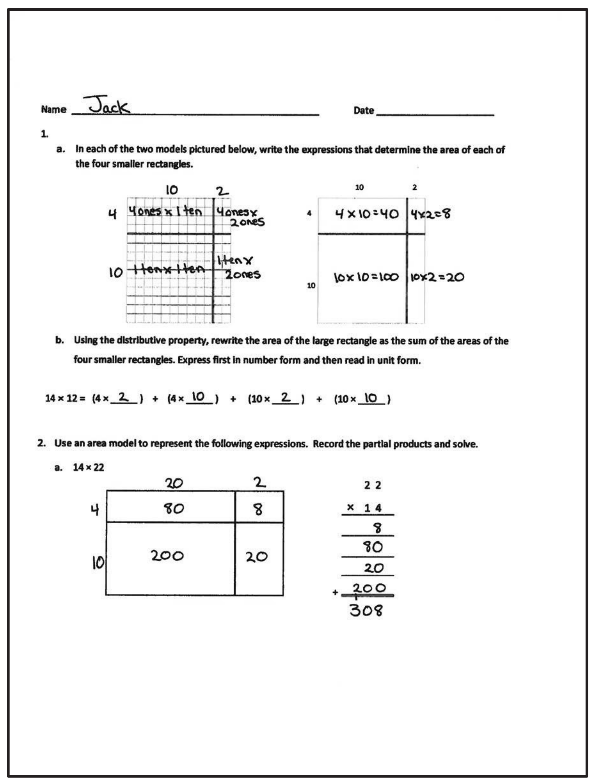
S: 24 + 180 + 80 + 600 = 884. → 26 × 34 = 884.

T: Visualize to relate this back to the area model that we drew earlier.

Repeat for 38 × 43. You might first draw the area model (without multiplying out the partial products) and then erase it so that students again visualize the connection.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


Name Jack Date

1.

a. In each of the two models pictured below, write the expressions that determine the area of each of the four smaller rectangles.

b. Using the distributive property, rewrite the area of the large rectangle as the sum of the areas of the four smaller rectangles. Express first in number form and then read in unit form.

14 × 12 = (4 × 2) + (4 × 10) + (10 × 2) + (10 × 10)

2. Use an area model to represent the following expressions. Record the partial products and solve.

a. 14 × 22

Student Debrief (10 minutes)

Lesson Objective: Multiply two-digit by two-digit numbers using four partial products.

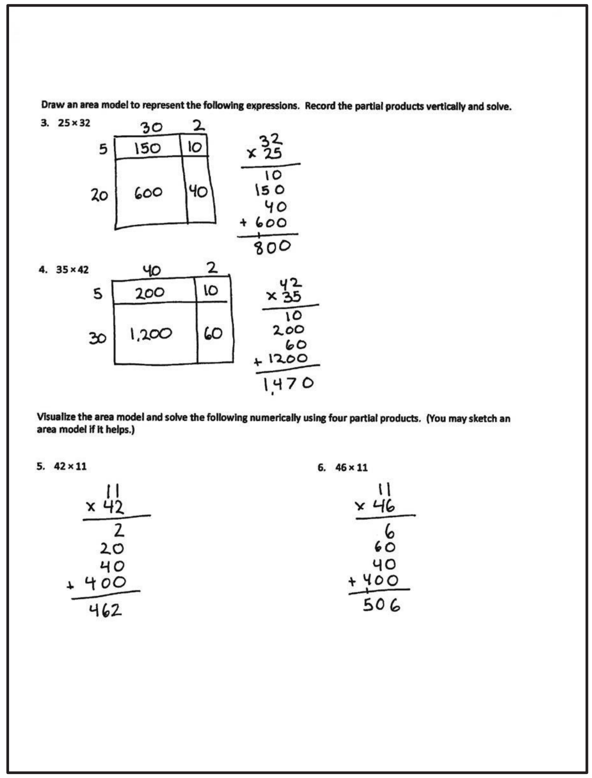
The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How does Problem 1(a) support your understanding of the distributive property and partial products?

- How do Problems 1 and 2 help to prepare you to solve Problems 3, 4, 5, and 6?
- How did our previous work with area models and partial products help us to be ready to solve two-digit by two-digit multiplication problems using partial products?
- How is it helpful to think about the areas of each rectangle in terms of *units*?
- How could you explain to someone that *ones* × *tens* equals *tens* but *tens* × *tens* equals *hundreds*?
- What significant math vocabulary did we use today to communicate precisely?
- How did the Application Problem connect to today's lesson?


Draw an area model to represent the following expressions. Record the partial products vertically and solve.

3. 25 × 32

4. 35 × 42

Visualize the area model and solve the following numerically using four partial products. (You may sketch an area model if it helps.)

5. 42 × 11

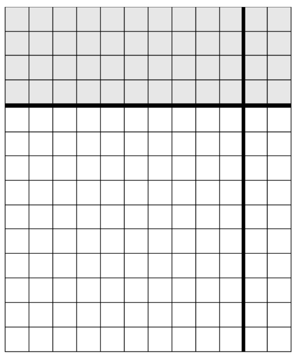
6. 46 × 11

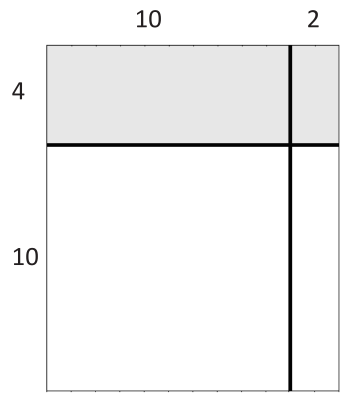
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ____ Date _______________________

1. a. In each of the two models pictured below, write the expressions that determine the area of each of the four smaller rectangles.



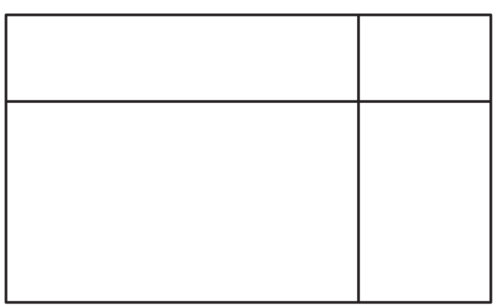


b. Using the distributive property, rewrite the area of the large rectangle as the sum of the areas of the four smaller rectangles. Express first in number form, and then read in unit form.

14 × 12 = (4 × _____) + (4 × _____) + (10 × _____) + (10 × _____)

2. Use an area model to represent the following expression. Record the partial products and solve.

14 × 22



```
    2 2
×   1 4
_______
_______
_______
_______
+ _______
```

Draw an area model to represent the following expressions. Record the partial products vertically and solve.

3. 25 × 32

4. 35 × 42

Visualize the area model and solve the following numerically using four partial products. (You may sketch an area model if it helps.)

5. 42 × 11

6. 46 × 11

Name ____ Date ____________________

Record the partial products to solve.

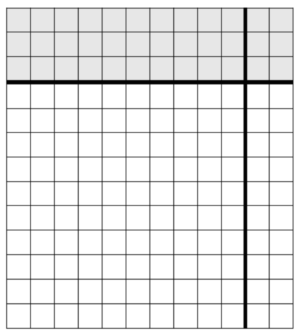
Draw an area model first to support your work, or draw the area model last to check your work.

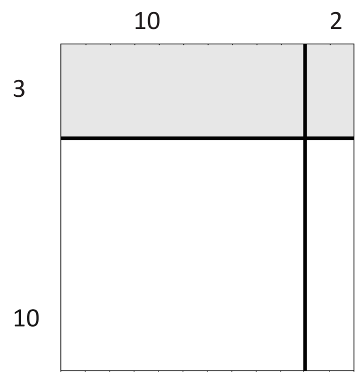
1. 26 × 43

2. 17 × 55

Name ____ Date ______________________

1. a. In each of the two models pictured below, write the expressions that determine the area of each of the four smaller rectangles.



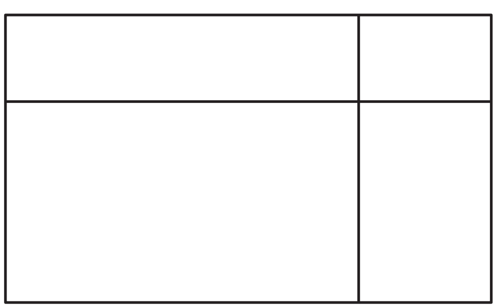


b. Using the distributive property, rewrite the area of the large rectangle as the sum of the areas of the four smaller rectangles. Express first in number form, and then read in unit form.

13 × 12 = (3 × _____) + (3 × _____) + (10 × _____) + (10 × _____)

Use an area model to represent the following expression. Record the partial products and solve.

2. 17 × 34



```
    3 4
  × 1 7
  _____
  _____
  _____
+ _____
  =====
```

Draw an area model to represent the following expressions. Record the partial products vertically and solve.

3. 45 × 18

4. 45 × 19

Visualize the area model and solve the following numerically using four partial products. (You may sketch an area model if it helps.)

5. 12 × 47

6. 23 × 93

7. 23 × 11

8. 23 × 22

Name Jack Date

1. What is the greatest multiple of 7 that is less than 60?

7, 14, 21, 28, 35, 42, 49, (56), 63

56 is the greatest multiple of 7 that is less than 60.

2. Identify each number as prime or composite. Then list all of its factors.

a. 3 prime 1, 3

b. 6 composite 1, 2, 3, 6

c. 15 composite 1, 3, 5, 15

d. 24 composite 1, 2, 3, 4, 6, 8, 12, 24

e. 29 prime 1, 29

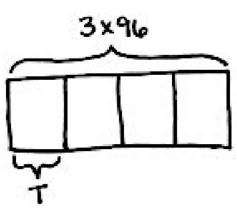
3. Use any place value strategy to divide.

a. 3,600 ÷ 9

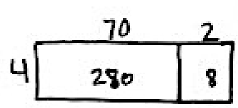
36 hundreds ÷ 9 = 4 hundreds

= 400

b. 96 pencils come in a box. If 4 teachers share 3 boxes equally, how many pencils does each teacher receive?



96
x 3
288



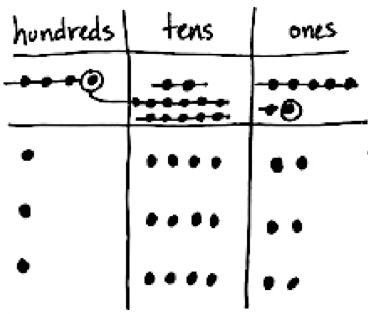
4) 288 = 70 + 2
−280
8
−8
0

70 + 2 = 72

Each teacher receives 72 pencils.

4. 427 ÷ 3

a. Solve by drawing place value disks:



} 1 hundred 4 tens 2 ones = 142 with a remainder of 1

b. Solve numerically:

142 R1
3) 427
−3
12
−12
07
−6
1

✓ 142 × 3 = 426
426 + 1 = 427

5. Use any place value strategy to multiply or divide.

a. 5316 ÷ 3

1,772
3) 5316
−3
23
−21
21
−21
06
−6
0

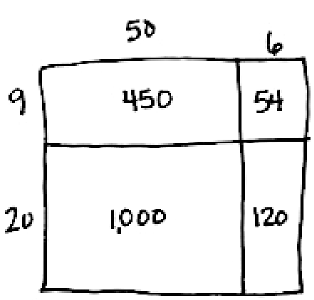
✓ 1,772 × 3 = 5,316

b. 3,809 ÷ 5

761 R4
5) 3809
−35
30
−30
09
−5
4

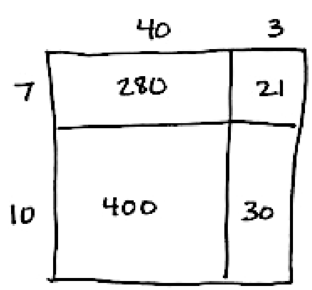
✓ 761 × 5 = 3,805
3,805 + 4 = 3,809

c. 29 × 56



56 × 29: 54 + 450 + 120 + 1000 = 1,624

d. 17 × 43

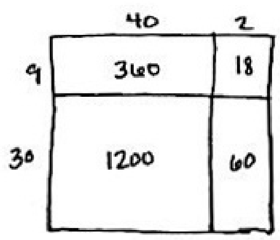


43 × 17: 21 + 280 + 30 + 400 = 731

Directions: Solve using a model or equation. Show your work and write your answer as a statement.

6. A new grocery store is opening next week.

a. The store's rectangular floor is 42 meters long and 39 meters wide. How many square meters of flooring do they need? Use estimation to assess the reasonableness of your answer.



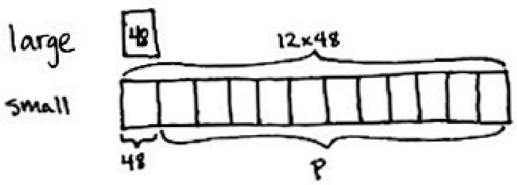
$42 \times 39 = 18 + 360 + 60 + 1200 = 1,638$

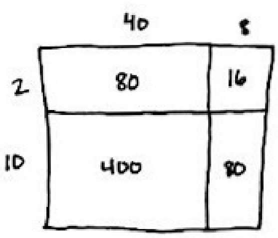
They need 1,638 square meters of flooring. My answer is reasonable because it is close to my estimate of 1,600 square meters.

$42 \times 39 \approx 40 \times 40$

$40 \times 40 = 1,600$

b. The store ordered small posters and large posters to promote their opening. 12 times as many small posters were ordered as large posters. If there were 48 large posters, how many more small posters were ordered than large posters?





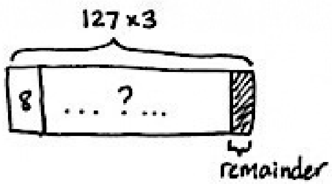
$48 \times 12 = 16 + 80 + 80 + 400 = 576$

$576 - 48 = 528$

P = 528

528 more small posters were ordered than large posters.

c. Uniforms are sold in packages of 8. The store's 127 employees will each be given 3 uniforms. How many packages will the store need to order?



127 × 3 = 381

381 ÷ 8 = 47 R5
381 − 32 = 61 (after first step)
61 − 56 = 5

47 × 8 = 376

376 + 5 = 381

The store needs to order 48 packages. If they order 47 packages, only 376 uniforms will come and they will need 5 more uniforms.

d. There are 3 numbers for the combination to the store's safe. The first number is 17. The other 2 numbers can be multiplied together to give a product of 28. What are all of the possibilities for the other two numbers? Write your answers as multiplication equations, and then write all of the possible combinations to the safe.

28 = 1 × 28
28 = 28 × 1
28 = 2 × 14
28 = 14 × 2
28 = 4 × 7
28 = 7 × 4

The combination possibilities are:

17, 1, 28
17, 28, 1
17, 2, 14
17, 14, 2
17, 4, 7
17, 7, 4

End-of-Module Assessment Task Standards Addressed	Topics A–H

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency*. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

A Progression Toward Mastery				
Assessment Task Item	**STEP 1 Little evidence of reasoning without a correct answer. (1 Point)**	**STEP 2 Evidence of some reasoning without a correct answer. (2 Points)**	**STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)**	**STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)**
1 **4.OA.4**	The student answers incorrectly with a number that is not a multiple of 7.	The student answers incorrectly with a number that is a multiple of 7 but greater than 60.	The student answers with a multiple of 7 that is less than 60 but not 56.	The student correctly answers: The greatest multiple of 7 that is less than 60 is 56.
2 **4.OA.4**	The student is unable to complete the majority of Parts (a)–(e).	The student correctly answers prime or composite for three parts and misses more than a total of three factors.	The student correctly answers prime or composite for four of the five parts and misses three or fewer factors.	The student correctly answers: a. Prime; 1, 3 b. Composite; 1, 2, 3, 6 c. Composite; 1, 3, 5, 15 d. Composite; 1, 2, 3, 4, 6, 8, 12, 24 e. Prime; 1, 29

A Progression Toward Mastery

3 **4.OA.3** **4.NBT.5** **4.NBT.6**	The student incorrectly answers both parts and shows no reasoning.	The student correctly answers one part and shows little reasoning.	The student answers one part correctly but shows solid reasoning in both problems, or the student shows some reasoning with correct answers for both parts.	The student correctly answers using any place value strategy: a. 400 b. Each teacher received 72 pencils.
4 **4.NBT.6**	The student incorrectly represents division using place value disks and incorrectly solves numerically.	The student incorrectly solves the numeric equation but shows some understanding of the place value chart and use of the algorithm.	The student decomposes incorrectly in one place value or does not include the remainder.	The student correctly decomposes and divides using the place value disks and provides a numerical answer of 142 with a remainder of 1.
5 **4.NBT.6**	The student answers fewer than two parts correctly, showing little to no evidence of place value strategies.	The student correctly solves two parts, showing little evidence of place value strategies.	The student correctly solves three parts with understanding of place value strategies, or the student correctly solves all four parts but does not show solid evidence of place value understanding.	The student solves all parts correctly using any place value strategy: a. 1,772 b. 761 with a remainder of 4 c. 1,624 d. 731
6 **4.MD.3** **4.OA.1** **4.OA.2** **4.OA.3** **4.NBT.5** **4.NBT.6**	The student incorrectly answers two or more of the four parts, showing little to no reasoning.	The student correctly answers two of four parts, showing some reasoning.	The student answers all four parts correctly but shows little reasoning in Part (a), or the student answers three of four parts correctly showing solid reasoning and understanding mathematically.	The student correctly answers all four parts, showing solid evidence of place value understanding: a. 1,638 square meters of flooring (estimate 40 × 40 = 1,600 square m). It is a reasonable because the answer and estimate have a difference of only 38 square meters. b. 528 more small posters than large posters.

A Progression Toward Mastery				
				c. 48 packages. d. Equations of 1 × 28 = 28 28 × 1 = 28 2 × 14 = 28 14 × 2 = 28 4 × 7 = 28 7 × 4 = 28 Combinations of 17, 1, 28 17, 28, 1 17, 2, 14 17, 14, 2 17, 4, 7 17, 7, 4

Preparing to Teach a Module

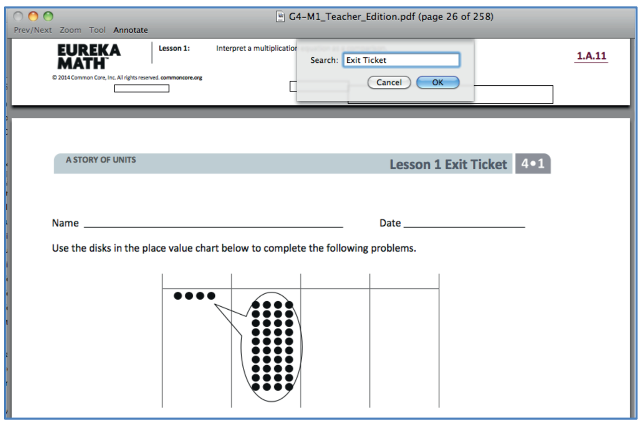
Preparation of lessons will be more effective and efficient if there has been an adequate analysis of the module first. Each module in *A Story of Units* can be compared to a chapter in a book. How is the module moving the plot, the mathematics, forward? What new learning is taking place? How are the topics and objectives building on one another? The following is a suggested process for preparing to teach a module.

Step 1: Get a preview of the plot.

A: Read the Table of Contents. At a high level, what is the plot of the module? How does the story develop across the topics?

B: Preview the module's Exit Tickets[7] to see the trajectory of the module's mathematics and the nature of the work students are expected to be able to do.

Note: When studying a PDF file, enter "Exit Ticket" into the search feature to navigate from one Exit Ticket to the next.



Step 2: Dig into the details.

A: Dig into a careful reading of the Module Overview. While reading the narrative, *liberally* reference the lessons and Topic Overviews to clarify the meaning of the text—the lessons demonstrate the strategies, show how to use the models, clarify vocabulary, and build understanding of concepts. Consider searching the video gallery on *Eureka Math*'s website to watch demonstrations of the use of models and other teaching techniques.

B: Having thoroughly investigated the Module Overview, read through the chart entitled Overview of Module Topics and Lesson Objectives to further discern the plot of the module. How do the topics flow and tell a coherent story? How do the objectives move from simple to complex?

Step 3: Summarize the story.

Complete the Mid- and End-of-Module Assessments. Use the strategies and models presented in the module to explain the thinking involved. Again, liberally reference the work done in the lessons to see how students who are learning with the curriculum might respond.

[7] A more in-depth preview can be done by searching the Problem Sets rather than the Exit Tickets. Furthermore, this same process can be used to preview the coherence or flow of any component of the curriculum, such as Fluency Practice or Application Problems.

Preparing to Teach a Lesson

A three-step process is suggested to prepare a lesson. It is understood that at times teachers may need to make adjustments (customizations) to lessons to fit the time constraints and unique needs of their students. The recommended planning process is outlined below. Note: The ladder of Step 2 is a metaphor for the teaching sequence. The sequence can be seen not only at the macro level in the role that this lesson plays in the overall story, but also at the lesson level, where each rung in the ladder represents the next step in understanding or the next skill needed to reach the objective. To reach the objective, or the top of the ladder, all students must be able to access the first rung and each successive rung.

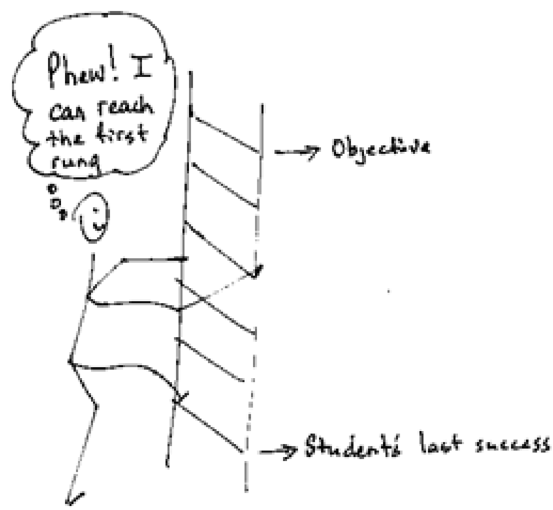
Step 1: Discern the plot.

A: Briefly review the Table of Contents for the module, recalling the overall story of the module and analyzing the role of this lesson in the module.

B: Read the Topic Overview of the lesson, and then review the Problem Set and Exit Ticket of each lesson of the topic.

C: Review the assessment following the topic, keeping in mind that assessments can be found midway through the module and at the end of the module.

Step 2: Find the ladder.



A: Complete the lesson's Problem Set.

B: Analyze and write notes on the new complexities of each problem as well as the sequences and progressions throughout problems (e.g., pictorial to abstract, smaller to larger numbers, single- to multi-step problems). The new complexities are the rungs of the ladder.

C: Anticipate where students might struggle, and write a note about the potential cause of the struggle.

D: Answer the Student Debrief questions, always anticipating how students will respond.

Step 3: Hone the lesson.

At times, the lesson and Problem Set are appropriate for all students and the day's schedule. At others, they may need customizing. If the decision is to customize based on either the needs of students or scheduling constraints, a suggestion is to decide upon and designate "Must Do" and "Could Do" problems.

A: Select "Must Do" problems from the Problem Set that meet the objective and provide a coherent experience for students; reference the ladder. The expectation is that the majority of the class will complete the "Must Do" problems within the allocated time. While choosing the "Must Do" problems, keep in mind the need for a balance of calculations, various word problem types[8], and work at both the pictorial and abstract levels.

[8] See the Progression Documents "K, Counting and Cardinality" and "K–5, Operations and Algebraic Thinking" pp. 9 and 23, respectively.

B: "Must Do" problems might also include remedial work as necessary for the whole class, a small group, or individual students. Depending on anticipated difficulties, those problems might take different forms as shown in the chart below.

Anticipated Difficulty	"Must Do" Remedial Problem Suggestion
The first problem of the Problem Set is too challenging.	Write a short sequence of problems on the board that provides a ladder to Problem 1. Direct the class or small group to complete those first problems to empower them to begin the Problem Set. Consider labeling these problems "Zero Problems" since they are done prior to Problem 1.
There is too big of a jump in complexity between two problems.	Provide a problem or set of problems that creates a bridge between the two problems. Label them with the number of the problem they follow. For example, if the challenging jump is between Problems 2 and 3, consider labeling these problems "Extra 2s."
Students lack fluency or foundational skills necessary for the lesson.	Before beginning the Problem Set, do a quick, engaging fluency exercise, such as a Rapid White Board Exchange, "Thrilling Drill," or Sprint. Before beginning any fluency activity for the first time, assess that students are poised for success with the easiest problem in the set.
More work is needed at the concrete or pictorial level.	Provide manipulatives or the opportunity to draw solution strategies. Especially in Kindergarten, at times the Problem Set or pencil and paper aspect might be completely excluded, allowing students to simply work with materials.
More work is needed at the abstract level.	Hone the Problem Set to reduce the amount of drawing as appropriate for certain students or the whole class.

C: "Could Do" problems are for students who work with greater fluency and understanding and can, therefore, complete more work within a given time frame. Adjust the Exit Ticket and Homework to reflect the "Must Do" problems or to address scheduling constraints.

D: At times, a particularly tricky problem might be designated as a "Challenge!" problem. This can be motivating, especially for advanced students. Consider creating the opportunity for students to share their "Challenge!" solutions with the class at a weekly session or on video.

E: Consider how to best use the vignettes of the Concept Development section of the lesson. Read through the vignettes, and highlight selected parts to be included in the delivery of instruction so that students can be independently successful on the assigned task.

F: Pay close attention to the questions chosen for the Student Debrief. Regularly ask students, "What was the lesson's learning goal today?" Hone the goal with them.