EUREKA MATH™

From the nonprofit Great Minds

A Story of Units®

Solving Word Problems
Grades K–5

Virtual Engagement Materials

May 2020

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Professional Reading—What is a Model?

Read the excerpt from the K-12 Modeling Progressions document[1]. Highlight sentences or phrases that resonate with you.

The word "model" can be used as a noun, verb, or adjective. As an adjective, "model" often signifies an ideal, as in "model student." In this progression, "model" will be a noun or a verb.

In elementary mathematics, a model might be a representation such as a math drawing or a situation equation[2], line plot, picture graph, or bar graph, or building made of blocks. In Grades 6-7. a model could be a table or plotted line or box plot, scatter plot, or histogram. In Grade 8, students begin to use functions to model relationships between quantities.

In high school, modeling becomes more complex, building on what students learned in K-8. Representations such as tables or scatter plots are often intermediate steps rather than the models themselves.

The same representations and concrete objects used as models of real-life situations are used to understand mathematical or statistical concepts. In elementary grades, students use rows of dots or tape diagrams to represent addition and subtraction of numbers as well as to model quantities in real-life situations. Later they use tape diagrams, arrays, and area diagrams to represent multiplication and division of numbers and model quantities. In Grade 6 geometry, nets may represent a three-dimensional mathematical object (e.g., a prism) as well as a design for a real-world object (e.g., a gingerbread house). In Grade 8, students use physical objects (e.g., paper triangles), transparencies, or geometry software to understand congruence and similarity. In Grades 6-8 statistics, simulations help students to understand what can happen during statistical sampling.

The use of representations and physical objects to understand mathematics is sometimes referred to as "modeling mathematics," and the associated representations and objects are sometimes called "models." For example, an area diagram is often called an "area model" when it is used to describe 2×3 as well as when it is used to describe a rectangular garden that is 2 feet by 3 feet. Students need not make distinctions such as "area diagram" and "area model." However, teachers should be aware that "modeling" as used in the Standards is primarily about using mathematics to describe the real world.

[2]Situation Equations and Solution Equations

Consider the following Grade 2 problem.

Susan has 57 cents in her piggy bank. If she just put in 30 cents today, how much did she have yesterday?

This is an *add to with start unknown* problem. A situation equation that could be used to represent this problem might look like:

$$? + 30 = 57$$

1. Common Core Standards Writing Team. 2019, February 8. *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-12, Modeling*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. http://mathematicalmusings.org/wp-content/uploads/2019/02/modeling_02082019.pdf

This equation denotes the order in which quantities are joined within the situation: Susan started with an unknown amount of money, represented using a question mark, then added 30 cents to it, resulting in a total of 57 cents.

To solve this problem, however, students may see that they can reverse the actions of the situation, or subtract 30 cents from 57 cents, essentially taking the 30 cents back out of the piggy bank.

$$57 - 30 = 27$$

This would be considered a solution equation because it is strategically used to help solve the problem even though it does not represent the situation.

Drawing math pictures such as tape diagrams can greatly increase students' ability to identify the part-whole relationships that exist in a given situation, guiding them to first write situation equations and representing the unknown quantity symbolically. They then are better able to manipulate the parts and whole to determine an efficient solution pathway.

1. Common Core Standards Writing Team. 2019, February 8. *Progressions for the Common Core State Standards in Mathematics (draft). Grades K-12, Modeling*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. http://mathematicalmusings.org/wp-content/uploads/2019/02/modeling_02082019.pdf

There are $\frac{3}{4}$ as many boys as girls in a class of fifth-graders. If there are 35 students in the class, how many are girls?[1]

[1] G5 M4 L24 Problem Set #5

Notes on Pedagogy: The RDW Process

Sequence

Read the problem,

Draw and label a model as you reread,

- Can I draw something?
- What can I draw?
- What conclusions can I make from my drawing?

Write an equation or equations that help solve the problem.

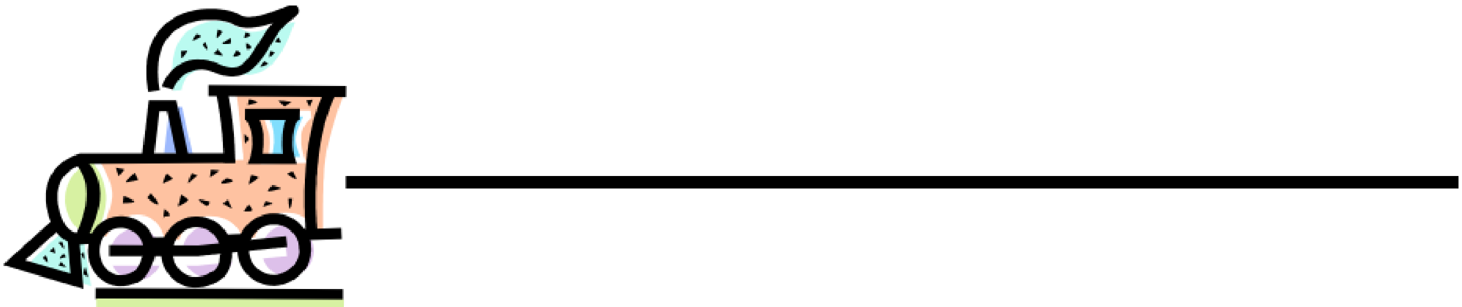
Write a statement of the answer to the question.

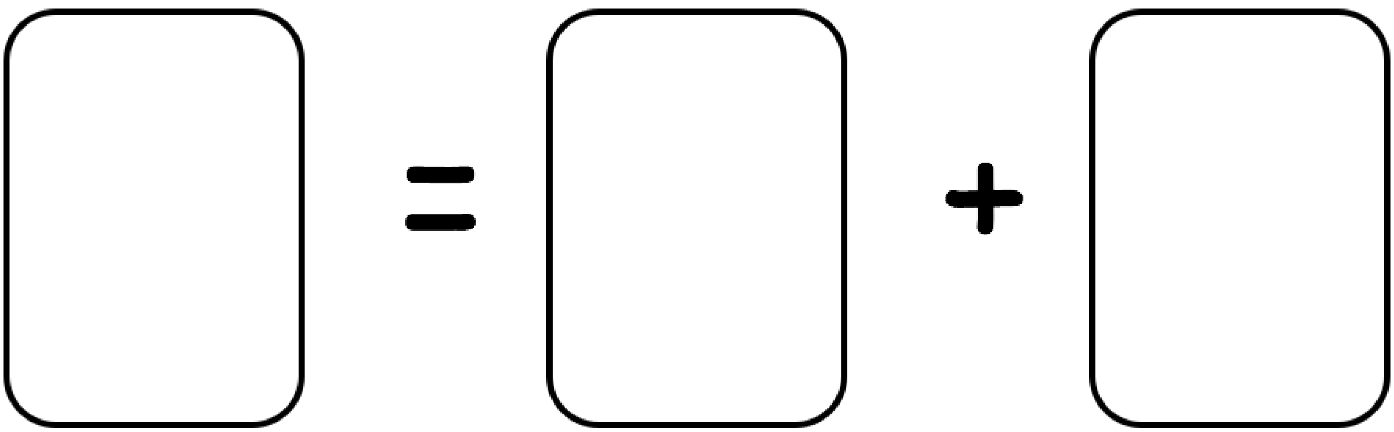
Customize your facilitation of the RDW process.

As you make decisions, consider the following:

- How does the problem align with your students' needs? (e.g., does it introduce new complexities and lend itself to teacher-led facilitation, or is less scaffolding appropriate?)
- Second to the mathematics, what is your goal for this segment of instruction? (e.g., informal assessment, guided practice with a new strategy, developing language for share and critique, etc.)
- Will you encourage students to use a specific model, or intentionally give open-ended directions?
- Will students work independently, in pairs, or in cooperative groups?
- Will students share their work? For what purpose? (e.g., for pairs to correct their work, to highlight exemplars or a particular strategy for the whole class, to develop oral language for share and critique, to expose and discuss a common error, etc.)

Larry the Train is pulling 7 cars. 3 cars are full, and 4 cars are empty.[2]





[2] GK M4 L14

Analyze Kindergarten Word Problems

1. *There are 4 snakes on a rock. 2 more snakes slither over. How many snakes are on the rock now?*[3]

2. *There were 6 girls playing soccer. A boy came to play. How many children were playing soccer then?*[4]

3. *Dominic has 6 yellow stickers and 2 blue stickers. How many stickers does he have in all?*[5]

4. *The students were playing with 7 balls on the playground. They kicked some into a puddle and now some are muddy! What is one way the balls might look now?*[6]

5. *Five green frogs were sitting on the side of the pond. It was so hot that 2 of the frogs went for a swim! How many frogs were still sitting on the side of the pond?*[7]

[3] GK M4 L16
[4] GK M4 L16
[5] GK M4 L17
[6] GK M4 L18
[7] GK M4 L21

Common addition and subtraction situations*

Note: The shading shows the grade level where the problem type is introduced.

Kindergarten	Grade 1	Grade 2

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 - 3 = ?, ? + 3 = 5

**Table 1: Common addition and subtraction situations* is excerpted from the Glossary of the CCSSM. Authors: National Governors Association Center for Best Practices, Council of Chief State School Officers. Title: Common Core State Standards for Mathematics. Publisher: National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C. Copyright Date: 2010.

Ideas Set in Motion

Kindergarten:

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Grade 1:

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Grade 2:

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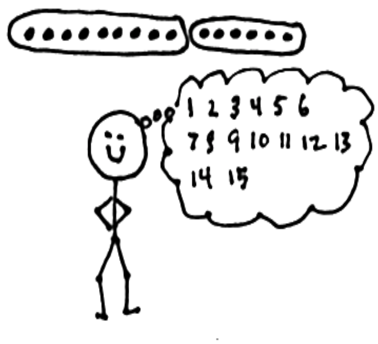
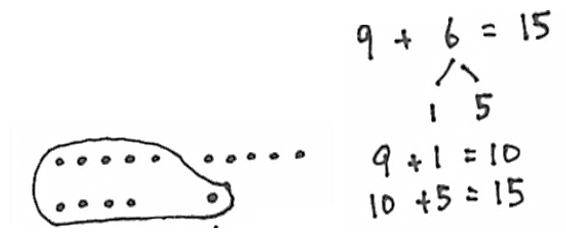
Directions: Study the following drawings from the G1 M2 Module Overview.

Level 1: Count all

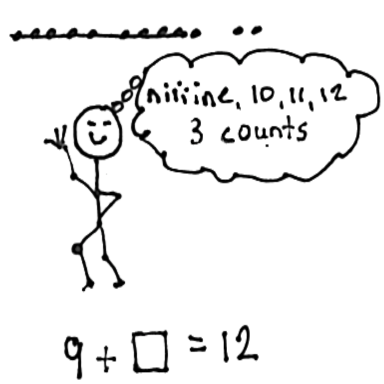


Level 2: Count on

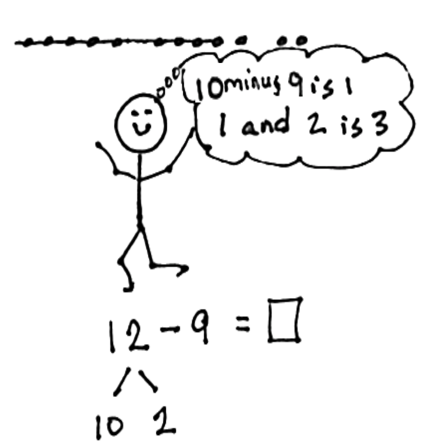
Level 3: Decompose an addend to compose ten



Level 2: Count on



Level 3: Decompose ten; add what is left to the ones.



For a more detailed description of each level, see Appendix A.

Grade 1 Word Problems

Directions: Solve the 3 problems below using the RDW process.

1. *Nine dogs were playing at the park. Some more dogs ran in. Then there were 12 dogs in all. How many dogs ran in?*[8]

2. *Ben and Peter caught 17 tadpoles. They gave some to Anton. They have 4 tadpoles left. How many tadpoles did they give to Anton?*[9]

3. *It snowed 14 days. Some snowy days, we stayed home. Nine snowy days we were in school. How many snowy days did we stay home?*[10]

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[8] G1 M4 L20
[9] G1 M4 L21
[10] G1 M5 L11

Introduce Comparison Word Problems (Grades 1 and 2)

Rose wrote 8 letters. Nikil wrote 12 letters. How many more letters did Nikil write than Rose?[11]

[11] G1 M6 L1 Concept Development

Grade 2 Comparison Word Problems

Directions: Work independently to solve the problems below using the RDW process.[12]

1. *Caleb has* 37 *more pennies than Richard. Richard has* 40 *pennies. Joe has* 25 *pennies. How many pennies does Caleb have?*

2. *Luigi has* 9 *more books than Mario. Luigi has* 52 *books. How many books does Mario have?*

[12] Problems 1-2 are the Application Problems from G2 M2.

Introduce 2-Step Word Problems (Grade 2)

Lee's fish tank has 24 goldfish and some silver fish. In all, there are 59 fish in the aquarium. Lee puts in some more silver fish. Now, there are 51 silver fish. How many silver fish did Lee put in the tank?[13]

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[13] G2 M4 L31

Analyze 2-Step Word Problems (Grade 2)

Directions: Read each problem and study its sample solution. After you have read and studied a problem, pause and "Say Something" about it (e.g., question, observation of new complexity, key point, new connection/insight).[14]

Problem 1

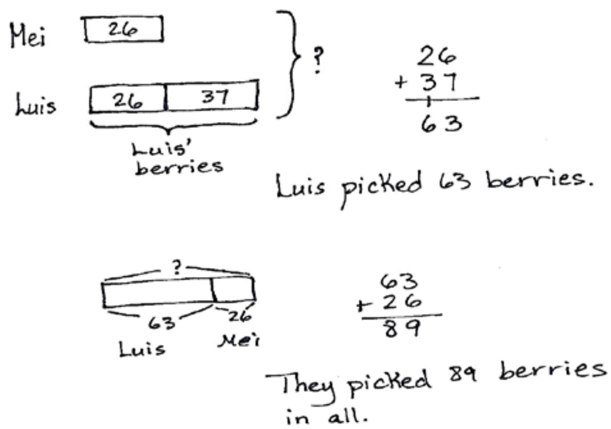
Solve a two-step *add to with result unknown* word problem using a tape diagram.

Mei picked 26 berries. Luis picked 37 more berries than Mei.

a. How many berries did Luis pick?

b. How many berries did they pick in all?

Circulate and ask guiding questions as needed to help students identify the steps in the problem and to determine if they are looking for the whole or a missing part. Once they draw their tape diagram, they may solve using any written method that they can explain and relate to their drawings.

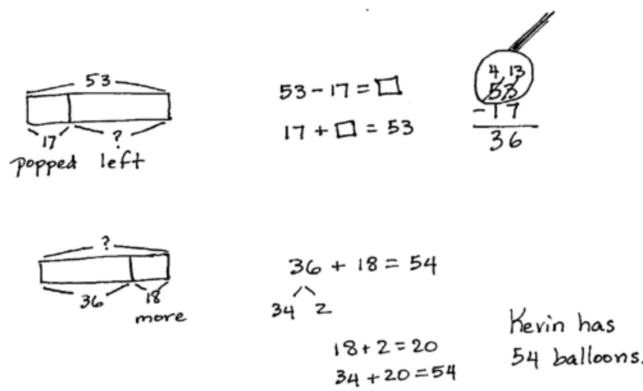


Problem 2

Solve a two-step *take from/add to with result unknown* word problem by drawing a tape diagram. Then, students may use any strategy they have learned to solve.

Kevin had 53 balloons. His cat popped 17 of them. His father gives him 18 more balloons. How many balloons does Kevin have now?

Drawing tape diagrams is essential to understanding the relationships within the problem. Equally important is that teachers encourage students to be flexible in their thinking while solving. A student might recognize, for example, that 17 balloons were popped and 18 given, so Kevin has 1 more than he started with.



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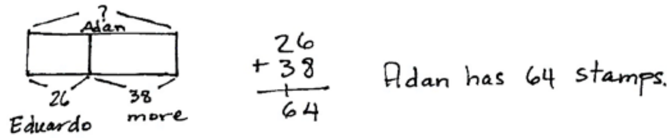
[14] Problems 1-3 are from the G2 M4 L31 Concept Development

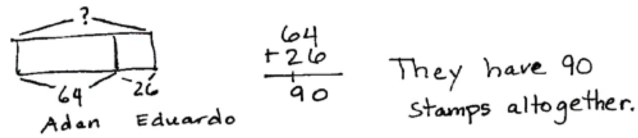
Problem 3

Solve a two-step comparison problem by drawing a tape diagram and using a preferred method to solve.

Eduardo collects 26 stamps. Adan collects 38 more than Eduardo. How many stamps do they have altogether?

Circulate and encourage students to use their favorite method to solve. Remind them to be prepared to explain their strategy using place value language.

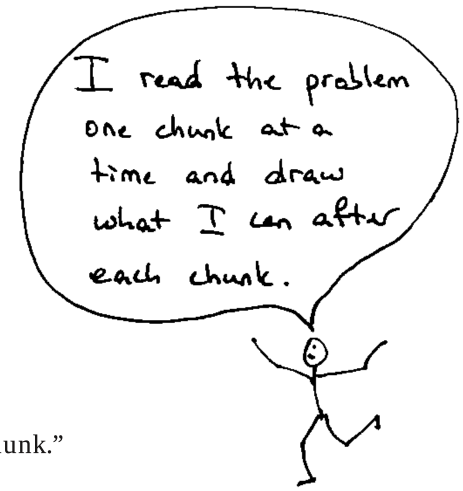




Notes on Pedagogy: The RDW Process and the Standards for Mathematical Practice (MP)

The RDW process often involves moving back and forth between reading and drawing. Students might first read the problem entirely then reread the first sentence. Draw and label. Reread the second sentence. Draw and label, etc. Consider the following example:

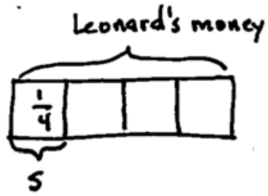
Leonard spent $\frac{1}{4}$ of his money on a sandwich. He spent 2 times as much on a gift for his brother as on some comic books. He had $\frac{3}{8}$ of his money left. What fraction of his money did he spend on comic books?



Read:

Leonard spent $\frac{1}{4}$ of his money on a sandwich.

Draw:



Read:

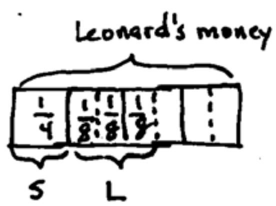
He spent 2 times as much on a gift for his brother as on some comic books.

Draw: "Hmmm. I really can't draw anything here. Let me move to the next chunk."

Read:

He had $\frac{3}{8}$ of his money left.

Draw:

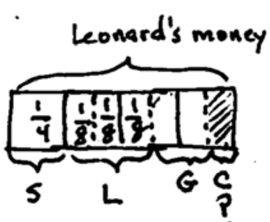


The RDW process culminates with a statement and a labeled drawing, an illustration of the story. The statement puts the answer back into context. Does the statement make sense? Does it correspond correctly to the drawing? Does the drawing tell the story? This is MP.2 in action, "reasoning abstractly and quantitatively." The drawing precipitates the reasoning. The student does not figure out the problem and then draw but rather decodes the relationships through the drawing. The abstract numbers are manipulated in the calculation and restored as quantities in the statement.

Read:

What fraction of his money did he spend on comic books?

Draw and write:



$\frac{1}{4} = \frac{2}{8}$ and $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$ and $1 - \frac{5}{8} = \frac{3}{8}$

$\frac{3}{8}$ = 3 units because $\frac{1}{4} = \frac{2}{8}$

$\frac{1}{8}$ = 1 unit

Leonard spent $\frac{1}{8}$ of his money on comic books.

Directions:

Read the MPs. Make sure you are able to summarize the MP and share evidence of what that MP would look like in action in a classroom.

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

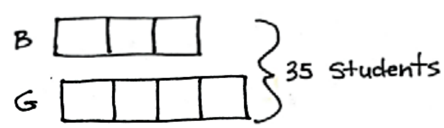
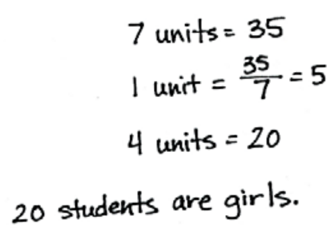
MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.1 Make sense of problems and persevere in solving them.

Statement from MP.1	Example of Student Thinking with “– as many boys as girls”
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.	e.g., “Hmm. I can draw a tape to represent the boys. There are more girls than boys, so I can draw a second longer tape to represent the girls.”
They analyze givens, constraints, relationships, and goals.	e.g., “I’m looking for the number of girls. I know the value of the total number of students, and the relationship between the boys and girls. I also know you can’t have half a person! My answer will be a whole number.”
They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.	e.g., “I can see that I have a total of 7 equal units. If there are 35 students in the class, I can find the value of 1 unit using 35 ÷ 7.” 1 unit = 35 ÷ 7 = 5 4 units = 4 × 5 = 20
They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.	e.g., “This is like the problem we solved last week...”
They monitor and evaluate their progress and change course if necessary.	e.g., “Oops. Have to erase this. The parts need to be equal. I’m confusing myself!”
Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.	e.g., “Hmm. I see that 35 ÷ 7 is the same as finding 1 unit when 35 is divided into 7 equal parts. I can see that in the tape, too.”
Younger students might rely on using concrete e.g. objects or pictures to help conceptualize and solve a problem.	e.g., “Let me use linker cubes to solve this.”
Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”	e.g., “If I label each unit in my tape diagrams with 5, I can skip-count to find the total. 7 units is the same as 35 students, and 4 units is the same as 20 girls. And my answer isn’t a fraction – I can’t have part of a person!”
They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	e.g., “Monica used a different equation than me and Jamal. – × 35 × 4. It is correct, too, because finding a seventh of a number is the same as dividing by 7.”

MP.2 Reason abstractly and quantitatively.

Statement from MP.2	Example of Student Thinking with “– as many boys as girls”
Mathematically proficient students make sense of quantities and their relationships in problem situations.	e.g., “Hmm. I have a total number of students, but that number includes both girls and boys. So I can draw two tapes, 1 to show the girls and the other to show the boys.”
They bring two complementary abilities to bear on problems involving quantitative relationships:	
the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—	e.g., “I can divide by 7 to find the value of one part, and then multiply the value of one part by 4.” 1 unit = $35 \div 7 = 5$ 4 units = $4 \times 5 = 20$
and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.	e.g., “I know that there are – as many boys as girls, which means for every – of a boy there is 1 girl. But we can’t have – of a boy! So I can multiply– $\times$ 4 to make a whole number: 3 boys. But then I also have to multiply the number of girls by 4: $1 \times 4 = 4$. Now I can see that there are 3 boys for every 4 girls.”
Quantitative reasoning entails habits of creating a coherent representation of the problem at hand;	e.g., 1 unit = $35 \div 7 = 5$ 4 units = $4 \times 5 = 20$ or 
considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	e.g., 

MP.4 Model with mathematics.

Statement from MP.4	Example of Student Thinking
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.	Walking home, Marta quickly does some mental math. “If I spend $12 on 3 CDs, I’ll have about $14 left of the $50 I got for my birthday.” The use of tape diagram in math class has helped her to be able to reason without it.
In early grades, this might be as simple as writing an addition equation to describe a situation.	3 cookies + 7 cookies = 10 cookies
In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.	Antonio is deciding where to purchase 2 shirts. The retail price is the same at 3 stores. The first store offers 20% off; the second store offers 30% off the second shirt; and the third store gives half off of the second shirt. Which offer is the best deal for Antonio? Explain your reasoning algebraically.
Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.	At first, Antonio decides to solve the shirt problem by assuming the shirts cost $10 each. He then checks to see if he gets the same result when they cost $60 each.
They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.	Antonio uses a chart to organize his thinking about the three different sale options.
By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.	Ben is using geometry to calculate the distance of the earth from the moon. To support his reasoning, he models using isosceles triangles.
They can analyze those relationships mathematically to draw conclusions.	Ben’s moon problem is an example of this, too!
They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	Jamal reasons that his answer makes sense. He can see from his model that 15 boys is – as many as 20 girls, and that together, 15 boys and 20 girls make 35 students.

Stages of Model Drawing

The foundation of the tape diagram begins in Kindergarten. As students advance there are different ways to model and scaffold the labeling, as you can see in the sequence below. The primary concern is that the model supports the student in decoding the situation, and makes sense in relationship to the problem.

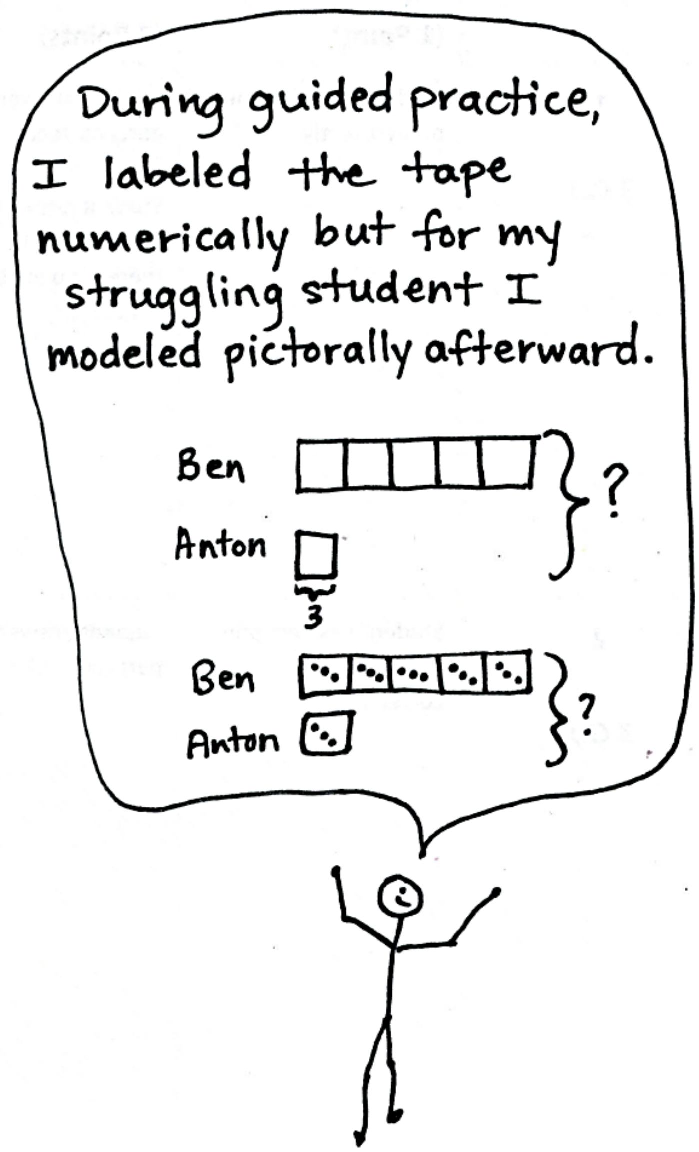
In deciding how to model for a whole class the question is, "What is the most sophisticated method I can use that students can readily access?" To differentiate you might label pictorially, or write the label inside the tape, or write the label outside the tape.

In the sequence on the next page, the whole is labeled at the bottom of the tapes in Stages 1-5. The advantage of labeling the whole at the bottom is that it readily transitions to the number line model as students get older. Does this mean it is always correct? Not necessarily.

Consider this situation: *Jordan uses* 3 *lemons to make* 1 *pitcher of lemonade. He makes* 4 *pitchers.* It might be natural to first draw 1 unit and label it as having a value of 3, and then draw 3 more like units. If labeling outside the tape, it makes sense to write '3' on the bottom rather than the top. Then the student can see the unit being labeled, rather than covering it with his hand to write. As she draws, the student might then label the unknown at the top of the tape.

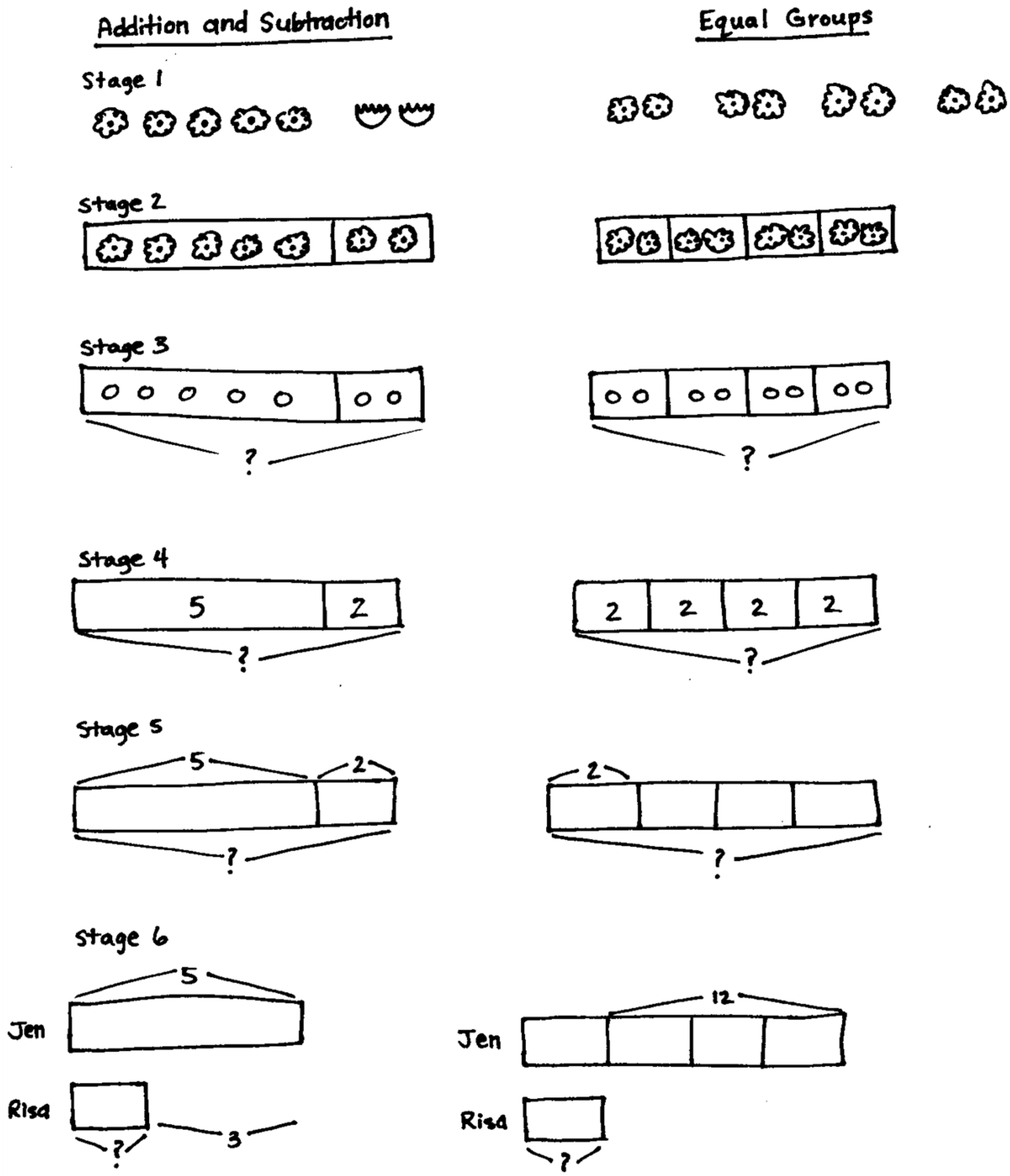
Although variations in labeling are not necessarily incorrect, consistent approaches can be helpful as students learn to model. However, take care that initial consistencies or restrictions do not become rigid rules. Some flexibility allows the model to function as a tool for reasoning about relationships, rather than a procedure to be followed.[15]

A rule of thumb might be to let the situation guide the drawing as you work chunk by chunk, with an eye toward labeling so that students will understand. As you use the tape diagram be aware of the choices you make. Over time, expect students to label and model in their own ways when working independently.



[15] The outlined progression is derived from and inspired by: Huat, Julianan Ng Chye and Lim Kian Huat. *A Handbook for Mathematics Teachers in Primary School*. Tarrytown: Marshall Cavendish Education, 2003. Print.

Stages of Model Drawing



Equal Groups Problems - Division (Grade 3)

Problem A

Mr. Lawton picks 12 tomatoes from his garden. He divides them equally among 3 bags. How many tomatoes are in each bag?

Math Drawing

Number Sentence

Answer Statement

Problem B

Mr. Lawton picks 12 tomatoes from his garden. He divides them into bags of 3. How many bags does he pack?

Math Drawing

Number Sentence

Answer Statement

Multiplication and Division Problem Types

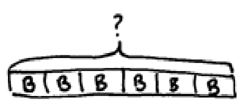
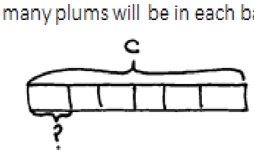
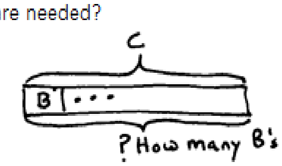
Directions

Step 1 Study the three equal group word problem types below.

Step 2 Analyze the relationship between the question in each story problem and the representation of the unknown in the tape diagram.

Step 3 Read each problem listed below the chart. Imagine how a student might represent the situation.

Step 4 Match each problem to a problem type in the chart.

	Unknown Product	**Unknown Group Size (Partitive Division)**	**Unknown Number of Groups (Measurement Division)**
	$A \times B = x$ $AB = x$	$Ax = C$ and $\frac{C}{A} = x$	$Bx = C$ and $\frac{C}{B} = x$
Equal Groups of Objects	There are A bags with B plums in each bag. How many plums are there in all? 	C plums are shared equally into A bags. How many plums will be in each bag? 	C plums are to be packed B to a bag. How many bags are needed? 

1. Daniel's fish tank holds 24 liters of water. He uses a 4-liter bucket to fill the tank. How many buckets of water are needed to fill the tank? (G3 M2 L9)

2. Mr. Doyle shares 1 roll of bulletin board paper equally with 8 teachers. The total length of the roll is 72 meters. How much bulletin board paper does each teacher get? (G3 M3 L15)

3. Every day at the bagel factory, Cyndi makes 5 different kinds of bagels. If she makes 144 of each kind, what is the total number of bagels that she makes? (G4 M3 L8)

4. 2,365 books were donated to an elementary school. If 5 classrooms shared the books equally, how many books did each class receive? (G4 M3 L31)

5. Mrs. Mayuko paid $40.68 for 3 kg of shrimp. What's the cost of 1 kilogram of shrimp? (G5 M1 L14)

6. 12.48 mL of medicine were separated into doses of 4 mL each. How many doses were made? (G5 M1 L13)

Measurement and Partitive Division with Fractional Divisors

1. Anouk has 3 pounds of ground beef. She uses it all to make $\frac{1}{4}$ pound patties. How many patties does Anouk make?

2. Anouk has 3 pounds of ground beef, which is $\frac{1}{4}$ of what she needs. How many pounds of ground beef does Anouk need?

Exploring Part–Whole Relationships

Directions:

Step 1 Use the red and blue strips to model each scenario. Strips may not overlap.

Step 2 After modeling with the strips, draw a tape diagram (in the space below) to represent your work.

Problem 1

a. The red strip is twice as long as the blue strip.

b. The red strip is $\frac{1}{2}$ as long as the blue strip.

c. The blue strip is 3 times as long as the red strip.

d. The blue strip is $\frac{1}{3}$ as long as the red strip.

e. The blue strip is $\frac{2}{3}$ as long as the red strip.

f. The red strip is $1\frac{1}{2}$ times as long as the blue strip.

2. Work independently and draw the following ribbons. When finished, compare your work to your partner's.

   a. 1 ribbon. The piece shown below represents $\frac{1}{3}$. Complete the drawing to show the whole ribbon.

   b. 1 ribbon. The piece shown below represents $\frac{1}{4}$. Complete the drawing to show the whole ribbon.

   c. 1 ribbon. The piece shown below represents $\frac{3}{4}$. Complete the drawing to show the whole ribbon.

   d. Two ribbons, A and B. One-third of A is equal to all of B. Draw a picture of the ribbons.

A:

B:

   e. 3 ribbons, C, D, and E. C is $\frac{1}{4}$ the length of D. E is twice as long as D. Draw a picture of the ribbons.

Multiplicative Comparison Word Problems

1. Morgan is 23 years old. Her grandfather is 4 times as old. How old is her grandfather?

2. Ivy has 4 times as many stickers as Adrian. Ivy has 320 stickers. How many stickers does Adrian have?

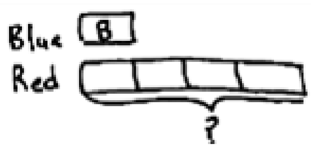
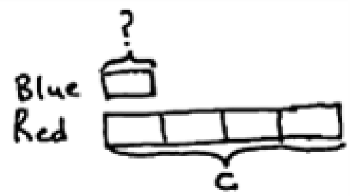
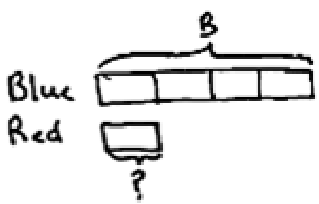
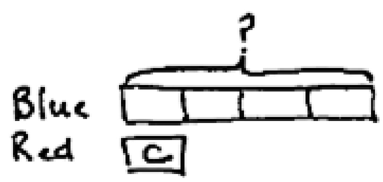
3. Julia's rope is 8 meters long. Sue's rope is $\frac{1}{4}$ as long as Julia's. How many more meters of rope does Julia have than Sue?

Multiplicative Comparison Word Problem Types

Directions:

Step 1 Study the chart below.

Step 2 Classify Problems 1–3 on the previous page by multiplicative comparison word problem type.

Compare	A >1 Larger unknown A blue hat costs $B. A red hat costs **A times as much** as the blue hat. How much does the red hat cost? 	A >1 Smaller Unknown A red hat costs $C and that is **A times as much** as a blue hat costs. How much does a blue hat cost? 
	A <1 Smaller Unknown A blue hat costs $B. A red hat costs **A as much** as the blue hat. How much does the red hat cost? 	A <1 Larger Unknown A red hat costs $C and that is **A of the cost of** a blue hat. How much does a blue hat cost? 

Adapted from *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team 2011, 23).

Modes of Instructional Delivery of Word Problems

Teachers must make a decision about the mode of instructional delivery when problem solving to determine what is best for a particular class of students.

- Will instruction encourage students to use a specific model (an array, area model, or tape diagram) to reason about the relationships in the problem, or will instruction allow any math drawing that makes sense?
- Will it be better to use a step-by-step guided approach with this problem because of new complexities, or will students work independently and then share their strategies?
- Will students work independently or in pairs? Will they work in cooperative groups with a protocol or solve solo and then share with a partner?

The chart below lays out three modes of delivery of instruction. Note: Many gradations exist within and between each mode of delivery.

Modes of Delivery of Instruction of Word Problems		
Modeling with Interactive Questioning (MP.2 and MP.4)	**Guided Instruction (MP.2 and MP.4)**	**Independent Practice (MP.2, MP.4, and MP.1)**
The teacher models the whole RDW process with interactive questioning and some choral response, such as "What did Monique say, everyone?" After completing the problems, students might reflect with a partner on the steps they used to solve the problem: "Students, think back on what we did to solve this problem. What did we do first?" Students might then be given the same or a similar problem to solve for homework.	Each student has a copy of the question. Though guided by the teacher, they work independently at times and then come together again. Timing is important. Students might hear, "You have 2 minutes to do your drawing." Or "Put your pencils down. Time to work together again." The Debrief might include selecting different student work to share.	The students are given a word problem to solve and a more extended amount of time to solve it. The teacher circulates, observes, lightly supports, and thinks about which student work to show to support the mathematical objectives of the lesson and success with problem solving. When sharing student work, encourage students to think about the work with questions such as "What do you notice about Jeremy's work? What is the same about Jeremy's work and Sara's work?"

Debrief:

- Discuss the strengths and weaknesses of each mode of instructional delivery.
- When might each mode be the right choice? When is it best to control more? Less?

Discuss what you would hear, see, and experience during each mode of delivery.

Suggested Delivery of Instruction for Solving Word Problem Lessons

Suggested Delivery of Instruction for Solving Word Problem Lessons

1. Model the problem.

Have two pairs of students model the problem at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something?
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give students two minutes to finish their work on that question, sharing their work and thinking with a peer. All should then write their equations and statements of the answer.

3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solutions.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Consider selecting students who may have difficulty grasping the Problem Set to work at the board. Support them as they work, and praise their effort and perseverance. This approach can help improve classroom climate, where top students are usually selected to work at the board, and reinforce the notion that determination and persistence are key to learning math skills.

Debrief:

- What are the benefits of this mode of delivery and of its effect on problem solving?
- What obstacles do you anticipate encountering? What can you do to address these obstacles proactively?
- Compare and contrast this method of delivery with your current practice.

As time allows, begin reading "Struggle for Smarts? How Eastern and Western Cultures Tackle Learning" by Alix Spiegel in Appendix D.

SPOT Analysis

STRENGTHS	PROBLEMS
OPPORTUNITIES	TO-DO

Appendix A: Level 1, 2 and 3 Counting Strategies

Excerpted from: Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K–5 Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona, pp. 36-38.

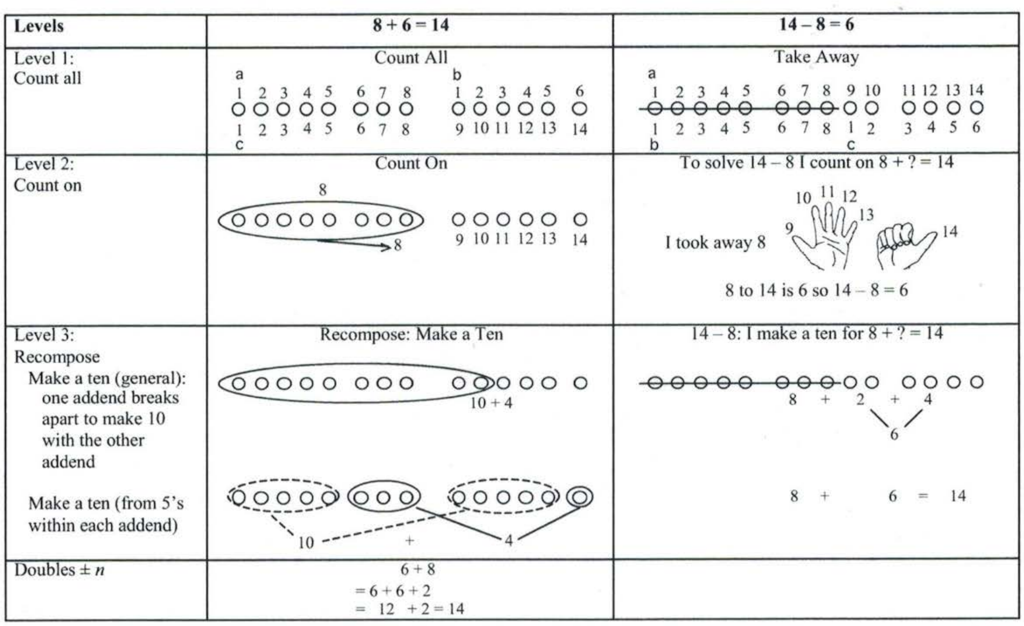
Appendix. Methods used for solving single-digit addition and subtraction problems

Level 1. Direct Modeling by Counting All or Taking Away.

Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Adding (8 + 6 = □): Represent each addend by a group of objects. Put the two groups together. Count the total. Use this strategy for Add To/Result Unknown and Put Together/Total Unknown.

Subtracting (14 − 8 = □): Represent the total by a group of objects. Take the known addend number of objects away. Count the resulting group of objects to find the unknown added. Use this strategy for Take From/Result Unknown.



Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Level 2. Counting On.

Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).

Counting on can be used to find the total or to find an addend. These look the same to an observer. The difference is what is monitored: the total or the known addend. Some students count down to solve subtraction problems, but this method is less accurate and more difficult than counting on. Counting on is not a rote method. It requires several connections between cardinal and counting meanings of the number words and extended experience with Level 1 methods in Kindergarten.

> Adding (e. g., $8 + 6 = \square$) uses counting on to find a total: One counts on from the first addend (or the larger number is taken as the first addend). Counting on is monitored so that it stops when the second addend has been counted on. The last number word is the total.
>
> Finding an unknown addend (e.g., $8 + \square = 14$): One counts on from the known addend. The keeping track method is monitored so that counting on stops when the known total has been reached. The keeping track method tells the unknown addend.
>
> Subtracting ($14 - 8 = \square$): One thinks of subtracting as finding the unknown addend, as $8 + \square = 14$ and uses counting on to find an unknown addend (as above).

The problems in Table 2 which are solved by Level 1 methods in Kindergarten can also be solved using Level 2 methods: counting on to find the total (adding) or counting on to find the unknown addend (subtracting).

The middle difficulty (lightly shaded) problem types in Table 2 for Grade 1 are directly accessible with the embedded thinking of Level 2 methods and can be solved by counting on.

> Finding an unknown addend (e.g., $8 + \square = 14$) is used for Add To/Change Unknown, Put Together/Take Apart/Addend Unknown, and Compare/Difference Unknown. It is also used for Take From/Change Unknown ($14 - \square =$

8) after a student has decomposed the total into two addends, which means they can represent the situation as $14 - 8 = \square$.

Adding or subtracting by counting on is used by some students for each of the kinds of Compare problems (see the equations in Table 2). Grade 1 students do not necessarily master the Compare Bigger Unknown or Smaller Unknown problems with the misleading language in the bottom row of Table 2.

Solving an equation such as $6 + 8 = \square$ by counting on from 8 relies on the understanding that $8 + 6$ gives the same total, an implicit use of the commutative property without the accompanying written representation $6 + 8 = 8 + 6$.

Level 3. Convert to an Easier Equivalent Problem.

Decompose an addend and compose a part with another addend.

These methods can be used to add or to find an unknown addend (and thus to subtract). These methods implicitly use the associative property.

Adding

Make a ten. E.g, for $8 + 6 = \square$,

$$8 + \underline{6} = 8 + \underline{2 + 4} = 10 + 4 = 14,$$

so $8 + 6$ becomes $10 + 4$.

Doubles plus or minus 1. E.g., for $6 + 7 = \square$,

$$6 + \underline{7} = 6 + \underline{6 + 1} = 12 + 1 = 13,$$

so $6 + 7$ becomes $12 + 1$.

Finding an unknown addend

Make a ten. E. g., for $8 + \square = 14$,

$$8 + \underline{2} = 10 \text{ and } \underline{4} \text{ more makes } 14.\ \underline{2 + 4} = 6.$$

So $8 + \square = 14$ is done as two steps: how many up to ten and how many over ten (which can be seen in the ones place of 14).

Doubles plus or minus 1. E.g., for $6 + \square = 13$,

$$6 + \underline{6 + 1} = 12 + 1.\ \underline{6 + 1} = 7.$$

So $6 + \square = 13$ is done as two steps: how many up to 12 $(6 + 6)$ and how many from 12 to 13.

Subtracting

Thinking of subtracting as finding an unknown addend. E.g., solve $14-8=\square$ or $13-6=\square$ as $8+\square=14$ or $6+\square=13$ by the above methods (make a ten or doubles plus or minus 1).

Make a ten by going down over ten. E.g., $14-8=\square$ can be done in two steps by going down over ten: $14-4$ (to get to 10) $-4=6$.

The Level 1 and Level 2 problem types can be solved using these Level 3 methods.

Level 3 problem types can be solved by representing the situation with an equation or drawing, then re-representing to create a situation solved by adding, subtracting, or finding an unknown addend as shown above by methods at any level, but usually at Level 2 or 3. Many students only show in their writing part of this multi-step process of re-representing the situation.

> Students re-represent Add To/Start Unknown $\square+6=14$ situations as $6+\square=14$ by using the commutative property (formally or informally).
>
> Students re-represent Take From/Start Unknown $\square-8=6$ situations by reversing as $6+8=\square$, which may then be solved by counting on from 8 or using a Level 3 method.

At Level 3, the Compare misleading language situations can be solved by representing the known quantities in a diagram that shows the bigger quantity in relation to the smaller quantity. The diagram allows the student to find a correct solution by representing the difference between quantities and seeing the relationship among the three quantities. Such diagrams are the same diagrams used for the other versions of compare situations; focusing on which quantity is bigger and which is smaller helps to overcome the misleading language.

Some students may solve Level 3 problem types by doing the above re-representing but use Level 2 counting on.

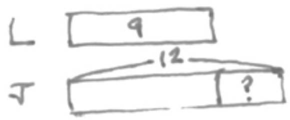
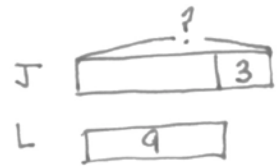
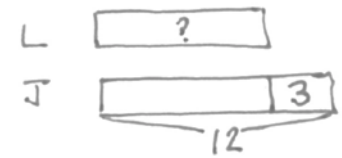
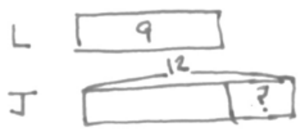
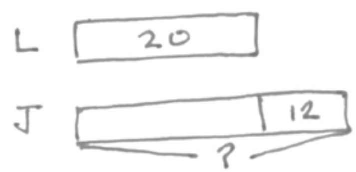
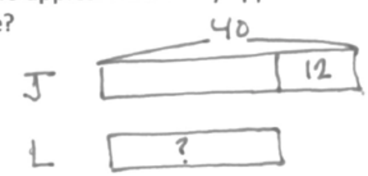
As students move through levels of solution methods, they increasingly use equations to represent problem situations as situation equations and then to re-represent the situation with a solution equation or a solution computation. They relate equations to diagrams, facilitating such re-representing. Labels on diagrams can help connect the parts of the diagram to the corresponding parts of the situation. But students may know and understand things that they may not use for a given solution of a problem as they increasingly do various representing and re-representing steps mentally.

Appendix B: Problem Types with Math Drawings

Addition and Subtraction Situations[i] with Possible Math Drawings[ii]

	Result Unknown	Change Unknown	Start Unknown
Add to	*4* bunnies sat on the grass. *5* more bunnies hopped there. How many bunnies are on the grass now? 4 + 5 = 9	*12* bunnies were sitting on the grass. Some more bunnies hopped over Then there were *15* bunnies. How many bunnies hopped over? 12 + □ = 15; 15 − 12 = 3	Some bunnies were sitting on the grass. *12* more bunnies hopped there. Then there were *37* bunnies. How many bunnies were on the grass before? □ + 12 = 37; 37 − 12 = 25
Take from	*6* apples were on a plate. I ate *2* apples. How many apples are on the table now? 6 − 2 = 4	*12* apples were on the table. I ate some apples. Then there were *8* apples. How many apples did I eat? 12 − □ = 8; 12 − 8 = 4	Some cookies were on a plate. I ate *3* cookies. Then there were *19* cookies. How many cookies were there to begin with? □ − 3 = 19; 19 + 3 = 22

	Total Unknown	Both Addends Unknown	Addend Unknown
Put Together/ Take Apart	*2* red apples and *6* green apples are on the table. How many apples are on the table? R G 2 + 6 = 8	Grandma has *5* flowers. How many can she put in her red vase and how many in her blue vase? R B 1 + 4; 2 + 3; 3 + 2; 4 + 1	*15* apples are on the table. *7* are red and the rest are green. How many apples are green? G R 15 − 7 = 8

	Result Unknown	Change Unknown	Start Unknown
Compare	Lucy has *9* apples. Julie has *12* apples. *How many more* apples does Julie have than Lucy?  12 - 9 = 3	Julie has *3 more* apples than Lucy. Lucy has *9* apples. How many apples does Julie have?  9 + 3 = 12	Lucy has *3* fewer apples than Julie. Julie has *12* apples. How many apples does Lucy have?  12 - 3 = 9
	Lucy has *9* apples. Julie has *12* apples. *How many fewer* apples does Lucy have than Julie?  12 - 9 = 3	"Fewer" version suggests addition. Lucy has *12* fewer apples than Julie. Lucy has *20* apples. How many apples does Julie have?  20 + 12 = 32	"More" version suggests subtraction. Julie has *12* more apples than Lucy. Julie has *40* apples. How many apples does Lucy have?  40 - 12 = 28

[i] Derived from: *K, Counting and Cardinality; K–5, Operations and Algebraic Thinking*
https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

[ii] Kindergarten types are done with intuitive pictorial models and have totals of 10 and less. Grade 1 situations are done with pictorial tape diagrams and some numerical and internal and external labeling and use totals of 20 and less. Grade 2 situations are done without pictorial images and have totals of 100 or less. This is not to suggest that these are the correct models but rather to present different levels of complexity in math drawings and to signal the grade level when the Common Core introduces certain situation types.

Appendix C

Table 3: Multiplication and division situations

	$A \times B = \square$	$A \times \square = C$ and $C \div A = \square$	$\square \times B = C$ and $C \div B = \square$
Equal Groups of Objects	Unknown Product There are *A* bags with *B* plums in each bag. How many plums are there in all?	Group Size Unknown If *C* plums are shared equally into *A* bags, then how many plums will be in each bag?	Number of Groups Unknown If *C* plums are to be packed *B* to a bag, then how many bags are needed?
Arrays of Objects		*Equal groups language*	
	Unknown Product There are *A* rows of apples with *B* apples in each row. How many apples are there?	Unknown Factor If *C* apples are arranged into *A* equal rows, how many apples will be in each row?	Unknown Factor If *C* apples are arranged into equal rows of *B* apples, how many rows will there be?
		Row and column language	
	Unknown Product The apples in the grocery window are in *A* rows and *B* columns. How many apples are there?	Unknown Factor If *C* apples are arranged into an array with *A* rows, how many columns of apples are there?	Unknown Factor If *C* apples are arranged into an array with *B* columns, how many rows are there?
Compare		$A > 1$	
	Larger Unknown A blue hat costs \$*B*. A red hat costs *A* times as much as the blue hat. How much does the red hat cost?	Smaller Unknown A red hat costs \$*C* and that is *A* times as much as a blue hat costs. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$*C* and a blue hat costs \$*B*. How many times as much does the red hat cost as the blue hat?
		$A < 1$	
	Smaller Unknown A blue hat costs \$*B*. A red hat costs *A* as much as the blue hat. How much does the red hat cost?	Larger Unknown A red hat costs \$*C* and that is *A* of the cost of a blue hat. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$*C* and a blue hat costs \$*B*. What fraction of the cost of the blue hat is the cost of the red hat?

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes
Equal groups problems can also be stated in terms of columns, exchanging the order of *A* and B, so that the same array is described. For example: There are *B* columns of apples with *A* apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

Multiplicative Compare problems appear first in Grade 4, with whole-number values for A, B, and C, and with the "times as much" language in the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., "A red hat costs *A* times as much as the blue hat" results in the same comparison as "A blue hat costs $1/A$ times as much as the red hat," but has a different subject.

Source: ***Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking*** **(Common Core Standards Writing Team 2011, 23).**

Appendix D

Struggle for Smarts? How Eastern and Western Cultures Tackle Learning

By Alix Spiegel

In 1979, when Jim Stigler was still a graduate student at the University of Michigan, he went to Japan to research teaching methods and found himself sitting in the back row of a crowded fourth-grade math class.

"The teacher was trying to teach the class how to draw three-dimensional cubes on paper," Stigler explains, "and one kid was just totally having trouble with it. His cube looked all cockeyed, so the teacher said to him, 'Why don't you go put yours on the board?' So right there I thought, 'That's interesting! He took the one who can't do it and told him to go and put it on the board.' "

Stigler knew that in American classrooms, it was usually the best kid in the class who was invited to the board. And so he watched with interest as the Japanese student dutifully came to the board and started drawing, but still couldn't complete the cube. Every few minutes, the teacher would ask the rest of the class whether the kid had gotten it right, and the class would look up from their work, and shake their heads no. And as the period progressed, Stigler noticed that he—Stigler—was getting more and more anxious.

"I realized that I was sitting there starting to perspire," he says, "because I was really empathizing with this kid. I thought, 'This kid is going to break into tears!' "

But the kid didn't break into tears. Stigler says the child continued to draw his cube with equanimity. "And at the end of the class, he did make his cube look right! And the teacher said to the class, 'How does that look, class?' And they all looked up and said, 'He did it!' And they broke into applause." The kid smiled a huge smile and sat down, clearly proud of himself.

Stigler is now a professor of psychology at UCLA who studies teaching and learning around the world, and he says it was this small experience that first got him thinking about how differently East and West approach the experience of intellectual struggle.

"I think that from very early ages we [in America] see struggle as an indicator that you're just not very smart," Stigler says. "It's a sign of low ability—people who are smart don't struggle, they just naturally get it, that's our folk theory. Whereas in Asian cultures they tend to see struggle more as an opportunity."

In Eastern cultures, Stigler says, it's just assumed that struggle is a predictable part of the learning process. Everyone is expected to struggle in the process of learning, and so struggling becomes a chance to show that you, the student, have what it takes emotionally to resolve the problem by persisting through that struggle.

"They've taught them that suffering can be a good thing," Stigler says. "I mean it sounds bad, but I think that's what they've taught them."

Granting that there is a lot of cultural diversity within East and West and it's possible to point to counterexamples in each, Stigler still sums up the difference this way: For the most part in American culture, intellectual struggle in schoolchildren is seen as an indicator of weakness, while in Eastern cultures it is not only tolerated but is often used to measure emotional strength.

It's a small difference in approach that Stigler believes has some very big implications.

'Struggle'

Stigler is not the first psychologist to notice the difference in how East and West approach the experience of intellectual struggle.

Jin Li is a professor at Brown University who, like Stigler, compares the learning beliefs of Asian and U.S. children. She says that to understand why these two cultures view struggle so differently, it's good to step back and examine how they think about where academic excellence comes from.

For the past decade or so, Li has been recording conversations between American mothers and their children, and Taiwanese mothers and their children. Li then analyzes those conversations to see how the mothers talk to the children about school.

She shared with me one conversation that she had recorded between an American mother and her 8-year-old son.

The mother and the son are discussing books. The son, though young, is a great student who loves to learn. He tells his mother that he and his friends talk about books even during recess, and she responds with this:

> **Mother:** Do you know that's what smart people do, smart grown-ups?
>
> **Child:** I know ... talk about books.
>
> **Mother:** Yeah. So that's a pretty smart thing to do to talk about a book.
>
> **Child:** Hmmm mmmm.

It's a small exchange—a moment. But Li says, this drop of conversation contains a world of cultural assumptions and beliefs.

Essentially, the American mother is communicating to her son that the cause of his success in school is his intelligence. He's smart—which, Li says, is a common American view.

"The idea of intelligence is believed in the West as a cause," Li explains. "She is telling him that there is something in him, in his mind, that enables him to do what he does."

But in many Asian cultures, Li says, academic excellence isn't linked with intelligence in the same way. "It resides in what they do, but not who they are, what they're born with," she says.

She shares another conversation, this time between a Taiwanese mother and her 9-year-old son. They are talking about the piano—the boy won first place in a competition, and the mother is explaining to him why.

"You practiced and practiced with lots of energy," she tells him. "It got really hard, but you made a great effort. You insisted on practicing yourself."

"So the focus is on the process of persisting through it despite the challenges, not giving up, and that's what leads to success," Li says.

All of this matters because the way you conceptualize the act of struggling with something profoundly affects your actual behavior.

Obviously if struggle indicates weakness—a lack of intelligence—it makes you feel bad, and so you're less likely to put up with it. But if struggle indicates strength—an ability to face down the challenges that inevitably occur when you are trying to learn something—you're more willing to accept it.

And Stigler feels in the real world it is easy to see the consequences of these different interpretations of struggle.

"We did a study many years ago with first-grade students," he tells me. "We decided to go out and give the students an impossible math problem to work on, and then we would measure how long they worked on it before they gave up."

The American students "worked on it less than 30 seconds on average and then they basically looked at us and said, 'We haven't had this,' " he says.

But the Japanese students worked for the entire hour on the impossible problem. "And finally we had to stop the session because the hour was up. And then we had to debrief them and say, 'Oh, that was not a possible problem; that was an impossible problem!' and they looked at us like, 'What kind of animals are we?' " Stigler recalls.

"Think about that [kind of behavior] spread over a lifetime," he says. "That's a big difference."

Not East Versus West

This is not to imply that the Eastern way of interpreting struggle—or anything else—is better than the Western way, or vice versa. Each has its strengths and weaknesses, which both sides know. Westerns tend to worry that their kids won't be able to compete against Asian kids who excel in many areas but especially in math and science. Li says that educators from Asian countries have their own set of worries.

" 'Our children are not creative. Our children do not have individuality. They're just robots.' You hear the educators from Asian countries express that concern, a lot," she notes.

So, is it possible for one culture to adopt the beliefs of another culture if they see that culture producing better results?

Both Stigler and Li think that changing culture is hard, but that it's possible to think differently in ways that can help. "Could we change our views of learning and place more emphasis on struggle?" Stigler asks. "Yeah."

For example, Stigler says, in the Japanese classrooms that he's studied, teachers consciously design tasks that are slightly beyond the capabilities of the students they teach, so the students can actually experience struggling with something just outside their reach. Then, once the task is mastered, the teachers actively point out that the student was able to accomplish it through hard work and struggle.

"And I just think that especially in schools, we don't create enough of those experiences, and then we don't point them out clearly enough."

But we can, Stigler says.

In the meantime, he and the other psychologists doing this work say there are more differences to map—differences that allow both cultures to more clearly see who they are.

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