

A Story of UnitsTM

Preparation and Customization of a *Eureka Math*® Lesson

Virtual Engagement Materials

April 2020



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Professional Reading 1

The textbook is the material on which Chinese teachers spend most of their time and devote most of their efforts to "study intensively." They study it constantly throughout the school year when they teach it. First of all, they work for an understanding of "what it is." They study how it interprets and illustrates the ideas in the [standards], why the authors structured the book in a certain way, what the connections among the contents are, what the connections are between the content of a certain textbook compared with an old version and why changes have been made, and so on. At a more detailed level, they study how each [module] of the textbook is organized, how the content was presented by the authors, and why. They study what examples are in a [topic], why these examples were selected, and why the examples were presented in a certain order. They review the exercises in each [lesson] of a [topic], the purpose for each exercise . . . and so on. Indeed, they conduct a very careful and critical investigation of the textbook. Although teachers usually find the authors' ideas ingenious and inspiring, they also sometimes find parts of the textbook that from their perspective are unsatisfactory, or inadequate illustrations of ideas in the [standards].

Excerpted from Ma, Liping. 1999. *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*, 131–132. Mahwah, NJ: Lawrence Erlbaum.

Directions for Debrief

Read the following quotes from Professional Reading 1. What thoughts do they trigger about your current process for preparing a lesson?

- 1. "At a more detailed level, they study how each [module] of the textbook is organized, how the content was presented by the authors, and why. They study what examples are in a [topic]...."
- 2. "They review the exercises in each [lesson] of a [topic], the purpose for each exercise . . . and so on."
- 3. "Although teachers usually find the authors' ideas ingenious and inspiring, they also sometimes find parts of the textbook that from their perspective are unsatisfactory, or inadequate illustrations of ideas in the [standards].



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GRADE 3 • MODULE 2

Place Value and Problem Solving with Units of Measure

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Directions: Highlight and annotate how you notice the number line developing in this module, along with any other reactions you had.

Grade 3 • Module 2

Place Value and Problem Solving with Units of Measure

OVERVIEW

In this 25-day module, students explore measurement using kilograms, grams, liters, milliliters, and intervals of time in minutes. Students begin by learning to tell and write time to the nearest minute using analog and digital clocks in Topic A (**3.MD.1**). They understand time as a continuous measurement through exploration with stopwatches, and use the number line, a continuous measurement model, as a tool for counting intervals of minutes within 1 hour (**3.MD.1**). Students see that an analog clock is a portion of the number line shaped into a circle. They use both the number line and clock to represent addition and subtraction problems involving intervals of minutes within 1 hour (**3.MD.1**).

Introduced in Topic B, kilograms and grams are measured using digital and spring scales. Students use manipulatives to build a kilogram and then decompose it to explore the relationship between the size and weight of kilograms and grams (**3.MD.2**). An exploratory lesson relates metric weight and liquid volume measured in liters and milliliters, highlighting the coherence of metric measurement. Students practice measuring liquid volume using the vertical number line and a graduated beaker (**3.MD.2**). Building on the estimation skills with metric length gained in Grade 2, students in Grade 3 use kilograms, grams, liters, and milliliters to estimate the weights and liquid volumes of familiar objects. Finally, they use their estimates to reason about solutions to one-step addition, subtraction, multiplication, and division word problems involving metric weight and liquid volume given in the same units (**3.MD.2**).

Now more experienced with measurement and estimation using different units and tools, students further develop their skills by learning to round in Topic C (**3.NBT.1**). They measure and then use place value understandings and the number line as tools to round two-, three-, and four-digit measurements to the nearest ten or hundred (**3.NBT.1**, **3.MD.1**, **3.MD.2**).

Students measure and round to solve problems in Topics D and E (**3.NBT.1**, **3.MD.1**, **3.MD.2**). In these topics, they use estimations to test the reasonableness of sums and differences precisely calculated using standard algorithms. From their work with metric measurement, students have a deeper understanding of the composition and decomposition of units. They demonstrate this understanding in every step of the addition and subtraction algorithms with two- and three-digit numbers, as 10 units are changed for 1 larger unit or 1 larger unit is changed for 10 smaller units (**3.NBT.2**). Both topics end in problem solving involving metric units or intervals of time. Students round to estimate, and then calculate precisely using the standard algorithm to add or subtract two- and three-digit measurements given in the same units (**3.NBT.1**, **3.NBT.2**, **3.MD.1**, **3.MD.2**).

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives D			Days
3.NBT.2	А	Time Measurement and Problem Solving		5
3.MD.1		Lesson 1:	Explore time as a continuous measurement using a stopwatch.	
		Lesson 2:	Relate skip-counting by fives on the clock and telling time to a continuous measurement model, the number line.	
		Lesson 3:	Count by fives and ones on the number line as a strategy to tell time to the nearest minute on the clock.	
		Lesson 4:	Solve word problems involving time intervals within 1 hour by counting backward and forward using the number line and clock.	
		Lesson 5:	Solve word problems involving time intervals within 1 hour by adding and subtracting on the number line.	
3.NBT.2	В	Measuring Weight and Liquid Volume in Metric Units		6
3.MD.2		Lesson 6:	Build and decompose a kilogram to reason about the size and weight of 1 kilogram, 100 grams, 10 grams, and 1 gram.	
		Lesson 7:	Develop estimation strategies by reasoning about the weight in kilograms of a series of familiar objects to establish mental benchmark measures.	
		Lesson 8:	Solve one-step word problems involving metric weights within 100 and estimate to reason about solutions.	
		Lesson 9:	Decompose a liter to reason about the size of 1 liter, 100 milliliters, 10 milliliters, and 1 milliliter.	
		Lesson 10:	Estimate and measure liquid volume in liters and milliliters using the vertical number line.	
		Lesson 11:	Solve mixed word problems involving all four operations with grams, kilograms, liters, and milliliters given in the same units.	
		Mid-Module Assessment: Topics A–B (assessment ½ day, return ½ day, remediation or further applications 1 day)		2



Standards	Topics and Objectives Da			Days
3.NBT.1 3.MD.1 3.MD.2	С	Rounding to the Lesson 12: Lesson 13:	e Nearest Ten and Hundred Round two-digit measurements to the nearest ten on the vertical number line. Round two- and three-digit numbers to the nearest ten on the	3
		Lesson 14:	vertical number line. Round to the nearest hundred on the vertical number line.	
3.NBT.2 3.NBT.1 3.MD.1 3.MD.2	D	Two- and Three Lesson 15: Lesson 16: Lesson 17:	 Digit Measurement Addition Using the Standard Algorithm Add measurements using the standard algorithm to compose larger units once. Add measurements using the standard algorithm to compose larger units twice. Estimate sums by rounding and apply to solve measurement word problems. 	3
3.NBT.2 3.NBT.1 3.MD.1 3.MD.2	E	Two- and Three Lesson 18: Lesson 19: Lesson 20: Lesson 21:	 Digit Measurement Subtraction Using the Standard Algorithm Decompose once to subtract measurements including three- digit minuends with zeros in the tens or ones place. Decompose twice to subtract measurements including three- digit minuends with zeros in the tens and ones places. Estimate differences by rounding and apply to solve measurement word problems. Estimate sums and differences of measurements by rounding, and then solve mixed word problems. 	4
		End-of-Module remediation or	Assessment: Topics A–E (assessment ½ day, return ½ day, further applications 1 day)	2
Total Number of Instructional Days25			25	



Virtual Engagement Materials

Preparation and Customization of a Eureka Math Lesson

Directions: While reading, annotate and consider your responses to the following Guiding Questions: 1. Highlight and annotate how you notice the number line being used in this module, along with any other reactions you had.

2. Complete the Topic C Exit Tickets on the next three pages.

Topic C Overview

Rounding to the Nearest Ten and Hundred

3.NBT.1, 3.MD.1, 3.MD.2

Focus Standard:	3.NBT.1	Use place value understanding to round whole numbers to the nearest 10 or 100.
	3.MD.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
	3.MD.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one- step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
Instructional Days:	3	
Coherence -Links from:	G2-M2	Addition and Subtraction of Length Units
-Links to:	G4–M2	Unit Conversions and Problem Solving with Metric Measurement

Topic C builds on students' Grade 2 work with comparing numbers according to the value of digits in the hundreds, tens, and ones places (**2.NBT.4**). Lesson 12 formally introduces rounding two-digit numbers to the nearest ten. Rounding to the leftmost unit usually presents the least challenging type of estimate for students, and so here the sequence begins. Students measure two-digit intervals of minutes and metric measurements, and then use place value understanding to round. They understand that when moving to the right across the places in a number, the digits represent smaller units. Intervals of minutes and metric measurements provide natural contexts for estimation. The number line, presented vertically, provides a new perspective on a familiar tool.

Students continue to use the vertical number line in Lessons 13 and 14. Their confidence with this tool by the end of Topic C lays the foundation for further work in Grades 4 and 5 (4.NBT.3, 5.NBT.4). In Lesson 13, the inclusion of rounding three-digit numbers to the nearest ten adds new complexity to the previous day's learning. Lesson 14 concludes the module as students round three- and four-digit numbers to the nearest hundred.

A Teaching Sequence Toward Mastery of Rounding to the Nearest Ten and Hundred

Objective 1: Round two-digit measurements to the nearest ten on the vertical number line. (Lesson 12)

Objective 2: Round two- and three-digit numbers to the nearest ten on the vertical number line. (Lesson 13)

Objective 3: Round to the nearest hundred on the vertical number line. (Lesson 14)



Lesson 12 Exit Ticket

The weight of a golf ball is shown below.



- a. The golf ball weighs ______.
- b. Round the weight of the golf ball to the nearest ten grams. Model your thinking on the number line.

c. The golf ball weighs about _____.

d. Explain how you used the halfway point on the number line to round to the nearest ten grams.



Lesson 13 Exit Ticket

1. Round to the nearest ten. Use the number line to model your thinking.



2. Bobby rounds 603 to the nearest ten. He says it is 610. Is he correct? Why or why not? Use a number line and words to explain your answer.



Lesson 14 Exit Ticket

1. Round to the nearest hundred. Use the number line to model your thinking.



2. There are 685 people at the basketball game. Draw a vertical number line to round the number of people to the nearest hundred people.



G3 M2 End-of-Module Assessment

1. Paul is moving to Australia. The total weight of his 4 suitcases is shown on the scale to the right. On a number line, round the total weight to the nearest 100 kilograms.



2. Paul buys snacks for his flight. He compares cashews to yogurt raisins. The cashews weigh 205 grams, and the yogurt raisins weigh 186 grams. What is the difference between the weight of the cashews and yogurt raisins?



- 3. The clock to the right shows what time it is now.
 - a. Estimate the time to the nearest 10 minutes.

- Time Right Now
- b. The clock to the right show Paul's departure time. Estimate the time to the nearest 10 minutes.

Departure Time



c. Use your answers from Parts (a) and (b) to estimate how long Paul has before his flight leaves.



- 4. A large airplane uses about 256 liters of fuel every minute.
 - a. Round to the nearest ten liters to estimate how many liters of fuel get used every minute.

b. Use your estimate to find about how many liters of fuel are used every 2 minutes.

c. Calculate precisely how many liters of fuel are used every 2 minutes.

d. Draw a tape diagram to find the difference between your estimate and the precise calculation.



Preparation and Customization of a Eureka Math Lesson

- 5. Baggage handlers lift heavy luggage into the plane. The weight of one bag is shown on the scale to the right.
 - a. One baggage handler lifts 3 bags of the same weight. Round to estimate the total weight he lifts. Then, calculate exactly.



b. Another baggage handler lifts luggage that weighs a total of 200 kilograms. Write and solve an equation to show how much more weight he lifts than the first handler in Part (a).

c. The baggage handlers load luggage for 18 minutes. If they start at 10:25 p.m., what time do they finish?

d. One baggage handler drinks the amount of water shown below every day at work. How many liters of water does he drink during all 7 days of the week?





Lesson 13

Objective: Round two- and three-digit numbers to the nearest ten on the vertical number line.

Suggested Lesson Structure

Total Time	(60 minutes)
Student Debrief	(10 minutes)
Concept Development	(30 minutes)
Application Problem	(7 minutes)
Fluency Practice	(13 minutes)

Fluency Practice (13 minutes)

- Group Counting 3.0A.1
- Rename the Tens 3.NBT.3
- Halfway on the Number Line 3.NBT.1

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. It reviews foundational strategies for multiplication from Module 1 and anticipates Module 3.

(4 minutes)

(4 minutes)

(5 minutes)

Direct students to count forward and backward, occasionally changing the direction of the count:

- Threes to 30
- Fours to 40
- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

As students' fluency with skip-counting improves, help them make a connection to multiplication by tracking the number of groups they count using their fingers.



Rename the Tens (4 minutes)

Note: This activity prepares students for rounding in this lesson and anticipates the work in Lesson 14 where students round numbers to the nearest hundred on the number line.

- T: (Write 9 tens = ____.) Say the number.
- S: 90.

Continue with the following possible sequence: 10 tens, 20 tens, 80 tens, 63 tens, and 52 tens.

Halfway on the Number Line (5 minutes)

Note: This activity reviews rounding using a vertical number line from Lesson 12.

- T: (Project a vertical line with endpoints labeled 30 and 40.) What number is halfway between 3 tens and 4 tens?
- S: 35.
- T: (Write 35 halfway between 30 and 40.)

Continue with the following possible sequence: 130 and 140, 830 and 840, and 560 and 570.

Application Problem (7 minutes)



Note: This problem reviews finding intervals of minutes from Topic A and leads directly into rounding intervals of minutes to the nearest ten in this lesson. Encourage students to share and discuss simplifying strategies they may have used to solve. Possible strategies:

- Count by ones from 12:17 to 12:20 and then by fives to 12:45.
- Count by tens and ones, 12:27, 12:37, plus 8 minutes.
- Subtract 17 minutes from 45 minutes.



Materials: (T) Place value cards (S) Personal white board

Problem 1: Round two-digit measurements to the nearest ten.

- T: Let's round 28 minutes to the nearest 10 minutes.
- T: How many tens are in 28? (Show place value cards for 28.)
- S: 2 tens! (Pull apart the cards to show the 2 tens as 20. Perhaps cover the zero in the ones to clarify the interpretation of 20 as 2 tens.)
- T: Draw a tick mark near the bottom of the number line. To the right, label it 20 = 2 tens.
- S: (Draw and label 20 = 2 tens.)
- T: What is 1 more ten than 2 tens?
- S: 3 tens! (Show the place value card for 30 or 3 tens. Again, cover the zero to help clarify.)
- T: Draw a tick mark near the top of the number line. To the right, label it 30 = 3 tens.
- S: (Draw and label 30 = 3 tens.)
- T: What number is halfway between 20 and 30?
- S: 25.
- T: In unit form, what number is halfway between 2 tens and 3 tens?
- S: 2 tens 5 ones.
- T: (Show 2 tens 5 ones with the place value cards.) Estimate to draw a tick mark halfway between 20 and 30. Label it 25 = 2 tens 5 ones.
- S: (Draw and label 25 = 2 tens 5 ones.)
- T: When you look at your vertical number line, is 28 more than halfway or less than halfway between 20 and 30? Turn and talk to a partner about how you know. Then plot it on the number line.
- S: 28 is more than halfway between 2 tens and 3 tens. \rightarrow I know because 28 is more than 25, and 25 is halfway. \rightarrow I know because 5 ones is halfway, and 8 is more than 5.
- T: What is 28 rounded to the nearest ten?
- S: 30.
- T: Tell me in unit form.
- S: 2 tens 8 ones rounded to the nearest ten is 3 tens.
- T: Let's go back to our Application Problem. How would you round to answer the question, "About how long was the ballet recital?" Discuss with a partner.
- S: The ballet recital took about 30 minutes. \rightarrow Rounded to the nearest ten, the ballet recital took 30 minutes.

Continue with rounding 17 milliliters to the nearest ten. (Leave the number line used for this on the board. It will be used in Problem 2.)



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Alternatively, challenge students who round with automaticity to quickly round 28 minutes to the nearest 10 minutes (without the number line). Students can then write their own word problem for rounding 17 milliliters or 17 minutes.

30=3 tens 28=2 tens 8 ones +25=2 tens 5 ones -20=2 tens







Problem 2: Round three-digit measurements of milliliters to the nearest ten.

- T: To round 17 milliliters to the nearest ten, we drew a number line with **endpoints** 1 ten and 2 tens. How will our endpoints change to round 1 *hundred* 17 to the nearest ten? Turn and talk.
- S: Each endpoint has to grow by 1 hundred.
- T: How many tens are in 1 hundred? (Show the place value card of 100.)
- S: 10 tens.
- T: When I cover the ones, we see the 10 tens. (Put your hand over the zero in the ones place.)
- T: What is 1 more ten than 10 tens?
- S: 11 tens.

MP.6

- T: (Show the place value cards for 10 tens and then 11 tens, that is, 100 and 110.)
- T: (Show 117 with the place value cards.)
- T: How many tens are in 117? Turn and talk about how you know.
- S: (Track on fingers.) 10, 20, 30, 40, 50, ..., 110. Eleven tens. → 17 has 1 ten, so 117 has 10 tens, plus 1 ten makes 11 tens. → 110 has 11 tens. → 100 has 10 tens and one more ten is 11 tens.
- T: What is 1 more ten than 11 tens?
- S: 12 tens.
- T: What is the value of 12 tens?
- S: 120.
- T: What will we label our bottom endpoint on the number line when we round 117 to the nearest ten?
- S: 110 = 11 tens.
- T: The top endpoint?
- S: 120 = 12 tens.
- T: (Draw and label endpoints on the vertical number line.)
- T: How should we label our halfway point?
- S: 115 = 11 tens 5 ones.
- T: (Show 11 tens 5 ones with the place value cards.) On your number line, mark and label the halfway point.
- S: (Mark and label the halfway point.)
- T: Is 117 more or less than halfway between 110 and 120? Tell your partner how you know.
- S: It's closer to 120. 17 is only 3 away from 20, but 7 away from 10. → It's more than halfway between 110 and 120.











Preparation and Customization of a Eureka Math Lesson

- T: Label 117 on your number line now. (Allow time for students to label 117.) What is 117 rounded to the nearest ten? Use a complete sentence.
- S: 117 rounded to the nearest ten is 120.
- T: Tell me in unit form with tens and ones.
- S: 11 tens 7 ones rounded to the nearest ten is 12 tens.
- T: What is 17 rounded to the nearest ten?
- S: 20.
- T: Again, what is 117 rounded to the nearest ten?
- S: 120.
- T: Remember from telling time that a number line is continuous. The models we drew to round 17 milliliters and 117 milliliters were the same, even though they showed different portions of the number line; corresponding points are 1 hundred milliliters apart. Discuss the similarities and differences between rounding within those two intervals with your partner.



Reduce the small motor demands of plotting points on a number line by enlarging the number line and offering alternatives to marking with a pencil, such as placing stickers or blocks. Additionally, connect back to yesterday's lesson by using beakers or scales with water or rice.

S: All the numbers went in the same place, we just wrote a 1 in front of them all to show they were 1 hundred more. → We still just paid attention to the number of tens. We thought about if 17 was more or less than halfway between 10 and 20.

Continue with rounding the following possible measurements to the nearest ten: 75 mL, 175 mL, 212 g, 315 mL, and 103 kg.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Depending on your class, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

NOTES ON
SYMBOLS

This symbol is used to show that the answer is an approximate: \approx . Before students start work on the Problem Set, call their attention to it and point out the difference between \approx and =.



Student Debrief (10 minutes)

Lesson Objective: Round two- and three-digit numbers to the nearest ten on the vertical number line.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What is the same and different about Problems 1(c) and 1(d)? Did you solve the problems differently? Why or why not?
- Look at Problem 1(f). Did the zero in 405 make the problem challenging? Why?
- How did our fluency activities Rename the Ten and Halfway on the Number Line help with our rounding work today?
- Think back to yesterday's activity where we measured and then rounded at stations. How did that work help you envision the units we worked with today on the number line?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.







Directions: Please complete the problems for Lesson 13

Lesson 13 Problem Set





2. Round the weight of each item to the nearest 10 grams. Draw number lines to model your thinking.

ltem	Number Line	Round to the nearest 10 grams
36 grams		
52 grams		
142 grams		

3. Carl's basketball game begins at 3:03 p.m. and ends at 3:51 p.m. How many minutes did Carl's basketball game last?

Round the total number of minutes in the game to the nearest 10 minutes.



Preparation and Customization of a Eureka Math Lesson

Lesson 13 Homework





2. Round the weight of each item to the nearest 10 grams. Draw number lines to model your thinking.

Item	Number Line	Round to the nearest 10 grams
Cereal bar: 45 grams		
Loaf of bread: 673 grams		

3. The Garden Club plants rows of carrots in the garden. One seed packet weighs 28 grams. Round the total weight of 2 seed packets to the nearest 10 grams. Model your thinking using a number line.



Chatting Teachers

Ideas for "Must Do" Customizations "Zero Problems" "Rapid white Board Exchanges" I want to use a " Extra Dees" (or Aize, Bees, etc.) "Rapid White Board I'm going to tell my students to do these "Zero Problems" written on Exchange" to practice related pairs with the board before they do #1. immediate feedback. 26 -> 126 Zero. $45 \rightarrow 145$ How many tens are in each number? $97 \rightarrow 297$ @ 37 _____ 137 ____ 140 ____ $104 \rightarrow 504$ 6 65 _____ 365 ____ 370 ____ © 390 ____ 400 ____ 402 _ I'm going to insert "Extra Dees" after problem (a). They need more related problems and more with '5' in the ones. I'll write them on the board labeled d1, d2, d3 and d4.



Scaffolding for Must Do Problems

Anticipated Difficulty	Must Do Problems Remedial Suggestion
The first problem of the Problem Set is too challenging.	Write a short sequence of problems on the board that provides a ladder to Problem 1. Direct the class or small group to complete those first problems to empower them to begin the Problem Set. Consider labeling these problems <i>Zero Problems</i> because students do them before Problem 1.
The jump in complexity between two problems is too big.	Provide a problem or set of problems that creates a bridge between the two problems. Label them with the number of the problem they follow. For example, if the challenging jump is between Problems B and C, consider labeling the bridging problems <i>Extra Bs</i> .
Students lack fluency or foundational skills necessary for the lesson.	Before beginning the Problem Set, do a quick, engaging fluency exercise, such as a Rapid White Board Exchange or Sprint. Before beginning any fluency activity for the first time, assess that students are set up for success with the easiest problem in the set.
Students need more work at the concrete or pictorial level.	Provide manipulatives or the opportunity to draw solution strategies. Especially in Kindergarten, at times the Problem Set or pencil and paper aspect might be completely excluded, allowing students simply to work with materials.
Students need more work at the abstract level.	Hone the Problem Set to reduce the amount of drawing as appropriate for certain students or the whole class.



Preparation and Customization of a Eureka Math Lesson

Professional Reading 2

Directions: Read the selection. Highlight at least one statement or phrase that stands out to you.

Probably anyone who knows Chinese and English would translate the Chinese term *jiaocai* as "teaching materials" because *jiao* literally means "teaching" and *cai* means "materials." But I would say that in fact *jiaocai* is more like what "curriculum" means in the United States. Generally, when Chinese teachers refer to *zuanyan jiaocai*, the term consists of three main components—the [standards] (*jiaoxue dagang*), textbooks (*keben*), and teacher's manuals (*beike fudao cailiao*) (Ma 130).

To study teaching materials is extremely important. To study teaching materials is to study what we are to teach and how to teach it to our students; in other words, to find links between the knowledge and the students. The student teachers from normal schools doing their student teaching with me usually can't understand why we spend so much time studying teaching materials and what we can learn from studying. For them, it seems to be too simple and too plain to study: there are just several example problems, one of which you can solve in a minute and explain to students in two minutes. But I told them that even after teaching for more than thirty years, every time I study a textbook I see something new. How to inspire students' minds, how to explain in a clear way, how to spend less time and let students benefit more, how to motivate students to learn these topics. . . . Your answers for all these questions are supported by a deep and broad understanding of what the teaching material is about. And every time you study it, you get a better idea of what it is and how to teach it. You never will feel you have nothing more to learn from studying teaching materials. (Tr. Mao qtd. in Ma 135).

Excerpted from Ma, Liping. Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States. Mahwah, NJ: Lawrence Erlbaum, 1999. Print.



Preparing to Teach a Module from A Story of Units®

Preparing lessons will be more effective and efficient if the teacher analyzes the module first. Each module in *A Story of Units* can be compared to a chapter in a book. How does the module move the plot, the mathematics, forward? What new learning takes place? How do the topics and objectives build on one another? The following is a suggested process for preparing to teach a module. Similar to preparing to teach a *Eureka Math* lesson, preparing to teach a module is a three-step process: Step 1, Preview the plot; Step 2, Dig into the details; and Step 3, Summarize the story.

Step 1: Preview the plot.

- A: Read the Table of Contents. At a high level, what is the module's plot? How does the story develop across the topics?
- B: Preview the module's Exit Tickets¹ to see the trajectory of the mathematics and the nature of the work students are expected to be able to do.

Step 2: Dig into the details.

- A: Dig into a careful reading of the Module Overview. Read the narrative and refer liberally to the lessons and topic overviews to clarify meaning. The lessons demonstrate strategies, models, the use of vocabulary, and concept development.
- B: Having thoroughly investigated the Module Overview, read through the Overview of Module Topics and Lesson Objectives chart to discern the module's plot further. How do the topics flow and tell a coherent story? How do the objectives move from simple to complex?

Step 3: Summarize the story.

Complete the Mid- and End-of-Module Assessments. Use the strategies and models the module presents to explain the thinking involved. Again, refer liberally to the work students do in the lessons to see how students who are learning with the curriculum might respond

¹ Conduct a more in-depth preview by searching Problem Sets rather than Exit Tickets. Use this same process to preview the coherence or flow of any component of the curriculum, such as Fluency Practice or Application Problems.

Preparing to Teach a Lesson from A Story of Units

We recommend a three-step process to prepare a lesson. The process takes into account that, at times, teachers may need to adjust (customize) lessons to fit the time constraints and unique needs of their students.

Step 1: Discern the plot.

- A: Briefly review the module's Table of Contents, recalling the overall story of the module and analyzing the role of this lesson in the module.
- B: Read the Topic Overview related to the lesson; then review the Exit Ticket of each lesson in the topic.
- C: Review the assessment following the topic, keeping in mind that assessments can be found midway through the module and at the end of the module.

Step 2: Find the ladder.

The ladder is a metaphor for the teaching sequence. The sequence is evident at the macro level in the role that this lesson plays in the overall story, but it's also evident on the lesson level. On the lesson level, each rung on the ladder represents the next step in understanding, or the next skill needed to reach the objective.

- A: Complete the lesson's Problem Set.
- B: Analyze and write notes about the new complexities each problem presents as well as the sequences and progressions you notice across problems (e.g., pictorial to abstract, smaller to larger numbers, single- to multi-step problems). The new complexities are the rungs of the ladder.
- C: Anticipate where students might stumble, and note the potential cause of the struggle.
- D: Answer the Student Debrief questions, anticipating how students will respond.



Continued on the next page



Step 3: Hone the lesson.

At times, the lesson and Problem Set are appropriate for all students and the day's schedule. At other times they need customizing. If the decision is to customize based on the needs of students or scheduling constraints, then you might designate Must Do, Could Do, and/or Extension problems.

- A: Select Must Do problems from the Problem Set that meet the objective and provide a coherent experience for students (think about the rungs on a ladder). The expectation is that the majority of the class will complete the Must Do problems within the allotted time. While choosing Must Do problems, keep in mind the need for a balance of calculations, various word problem types², and work at both the pictorial and abstract levels.
- B: Must Do problems might also include remedial work as necessary for the whole class, a small group, or individual students. Depending on anticipated difficulties, those problems might take different forms as shown in the Scaffolding for Must Do Problems chart in this packet.
- C: Could Do problems are for students who work with greater fluency and understanding and who can, therefore, complete more work within a given time frame.
- D: At times, a particularly tricky problem might be designated as an Extension problem. This problem can be motivating, especially for advanced students. Consider creating the opportunity for students to share their Extension solutions with the class.
- E: Consider how best to use the vignettes of the Concept Development section of the lesson. Read through them, and highlight critical questions or parts to include in your lesson delivery so students will be independently successful on the assigned task.
- F: Pay close attention to the questions chosen for the Student Debrief. Regularly ask students, "What was our learning goal today?" Help them articulate the goal.
- G: Adjust the lesson components (e.g., Fluency, Application Problem, Homework, Exit Ticket) as necessary to support the work students are expected to do on the Problem Set.

² For more information, see the Progressions Documents for the Common Core Math Standards. Bibliographic reference: Common Core Standards Writing Team. (2011, May 29). Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; K–5 Operations and Algebraic Thinking. Tucson, AZ: Institute for Mathematics and Education, University of Arizona, p. 9 and 23, respectively.



Preparing to Teach a Lesson by Extending the Action of the Concept Development

If teachers decide to extend the actions of the Concept Development as independent work rather than use the Problem Set, we recommend a three-step process to prepare a lesson. It is understood that at times teachers may need to make adjustments (customizations) to lessons to fit the time constraints and unique needs of their students. The recommended planning process is outlined below.

Note: The ladder of Step 2 is a metaphor for the teaching sequence. The sequence can be seen not only at the macro level in the role that this lesson plays in the overall story, but also at the lesson level, where each rung in the ladder represents the next objective in the teaching sequence. As *A Story of Units* moves into the elementary grades, the ladder is also evident between selected problems, where each rung in the ladder represents the next skill needed to reach the objective. To reach the objective, or the top of the ladder, all students must be able to access the first rung and each successive rung.

Step 1: Discern the plot.

- A: Briefly review the module's Table of Contents, recalling the overall story of the module and analyzing the role of this lesson in the module.
- B: Read the Topic Overview related to the lesson, and then review the Concept Development of each lesson in the topic.
- C: Review the assessment tasks for the topic, keeping in mind that assessments can be found midway through the module and at the end of the module.

Step 2: Find the ladder.

- A: Gather the necessary materials, and do the actions of the Concept Development for each objective (lesson) in the topic and adjacent topics as necessary (e.g., if preparing the first lesson of a topic, return to the last lesson of the prior topic to make sense of the rungs between the lessons).
- B: Analyze and write notes on the new complexities of each objective in the topic (e.g., smaller to larger numbers, simple to challenging configurations, concrete to pictorial to abstract). The new complexities are the rungs of the ladder.
- C: Anticipate where students might struggle, and write a note about the potential cause of the struggle.
- D: Answer the Student Debrief questions, always anticipating how students will respond.





Preparation and Customization of a Eureka Math Lesson

Step 3: Hone the lesson.

At times, the lesson and the accompanying materials are appropriate for all students and the day's schedule. At others, they may need customizing. If the decision is to customize based on the needs of students or scheduling constraints, a suggestion is to create sets of materials that allow students to move through the concept from simple to complex.

A: Having anticipated where students might struggle or need an extra challenge, identify appropriate customizations as detailed in the chart below.

Anticipated Difficulty	Customization Suggestion
The task of the Practice ³ is too challenging.	Depending on the Practice task, consider using smaller quantities, different colors, or templates to support students working independently on the task. Once students feel confident in the task, remove the scaffolds. For example, if the task is to count 9 cubes, use 3 red, 3 yellow, and 3 blue cubes instead of mixed colors or all one color.
The jump in complexity between two lessons is too big.	Provide workstations for students to visit that repeat skills and objectives students have previously learned. For example, if students have trouble counting with one-to-one correspondence with numbers above 5, provide the supportive structure of a cropped egg carton with 10 slots to bridge to the task.
Students lack fluency or foundational skills necessary for the lesson.	Before beginning independent work on a Practice task, do a quick, engaging fluency exercise. Before beginning any fluency activity for the first time, assess that the sequence of questions begins by developing the concept at the simplest level before advancing.
Students need more work at the concrete or pictorial level.	Create sets of materials that allow students more practice with manipulatives or pictures before moving to a more complex skill. For example, continue to work with counters in a circle before asking students to count images printed in a circular configuration.
Students need more work moving from abstract to concrete or abstract to pictorial.	Hone the Practice to reduce the amount of drawing or use of manipulatives as appropriate for certain students or the whole class. For example, if students have already mastered counting a group and matching a numeral, give them a numeral, and ask them to create a matching group.

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³The Practice task refers to the actions of the Concept Development.



- B: Adjust the Practice to reflect the customizations or to address scheduling constraints (e.g., use more challenging Problem Sets such as those that have been omitted for use by the whole class for students who work with greater fluency and understanding and who, therefore, can complete more work within a given time frame).
- C: Consider how to best use the vignettes of the Concept Development section of the lesson. Read through the vignettes, and highlight selected parts to be included in the delivery of instruction so that students can be independently successful in their Practice.
- D: Pay close attention to the questions chosen for the Student Debrief. Regularly ask students, "What math are we learning today?" "Could you teach what you learned to someone else?" Help them articulate the goal to encourage metacognition and use of vocabulary.



Preparing to Teach a Module from *A Story of Ratios*[®] or *A Story of Functions*[®]

Preparing lessons will be more effective and efficient if the teacher analyzes the module first. Each module in *A Story of Ratios* or *A Story of Functions* can be compared to a chapter in a book. How does the module move the plot, the mathematics, forward? What new learning takes place? How do the topics and objectives build on one another? The following is a suggested process for preparing to teach a module.

Step 1: Preview the plot.

- A: Read the Table of Contents. At a high level, what is the plot of the module? How does the story develop across the topics?
- B: Preview the module's Exit Tickets to see the trajectory of the module's mathematics and the nature of the work students are expected to be able to do.

Step 2: Dig into the details.

- A: Dig into a careful reading of the Module Overview. While reading the narrative, refer liberally to the lessons and topic overviews to clarify the meaning of the text. The lessons demonstrate strategies, models, the use of vocabulary, and concept development.
- B: Having thoroughly investigated the Module Overview, read through the Student Outcomes of each lesson (in order) to discern the plot of the module further. How do the topics flow and tell a coherent story? How do the outcomes move students to new understandings?

Step 3: Summarize the story.

Complete the Mid- and End-of-Module Assessments. Use the strategies and models the module presents to explain the thinking involved. Again, refer liberally to the lessons to anticipate how students who are learning with the curriculum might respond.



Preparing to Teach a Lesson from *A Story of Ratios®* or *A Story of Functions®*

We recommend a three-step process to prepare a lesson. The process takes into account that, at times, teachers may need to adjust (customize) lessons to fit the time constraints and unique needs of their students.

Step 1: Discern the plot.

- A: Briefly review the module's Table of Contents, recalling the overall story of the module and analyzing the role of this lesson in the module.
- B: Read the Topic Overview related to the lesson, and then review the Student Outcome(s) and Exit Ticket of each lesson in the topic.
- C: Review the assessment following the topic, keeping in mind that assessments can be found midway through the module and at the end of the module.

Step 2: Find the ladder.

The ladder is a metaphor for the teaching sequence. The sequence is evident at the macro level in the role that this lesson plays in the overall story, but it's also evident on the lesson level. On the lesson level, each rung on the ladder represents the next step in understanding, or the next skill needed to reach the objective.

- A: Work through the lesson, answering and completing each question, example, exercise, and challenge.
- B: Analyze the new complexities or concepts each question or problem introduces; these notes on the sequence of new complexities and concepts are the rungs of the ladder.
- C: Anticipate where students might stumble, and note the potential cause of the struggle.
- D: Answer the Closing questions, anticipating how students will respond.



Step 3: Hone the lesson.

Teachers may need to customize lessons if the class period is not long enough to do all the lesson presents, and/or if students lack prerequisite skills and understanding to move through the entire lesson in the time allotted. A suggestion for customizing the lesson is to designate problems, questions, examples, exercises, or challenges as Must Do, Could Do, and/or Extensions.

Continued on the next page



- A: Select Must Do questions and problems that meet the Student Outcome(s) while still providing a coherent experience for students (think about the rungs on a ladder). The expectation should be that the majority of the class will be able to complete the Must Do portions of the lesson within the allocated time. While choosing the Must Do portions of the lesson, keep in mind the need for a balance of dialogue and conceptual questioning, application problems, and abstract problems and a balance between students using pictorial/graphical representations and abstract representations. Highlight dialogue to include in the delivery of instruction so students have a chance to articulate and consolidate understanding as they move through the lesson.
- B: Must Do portions might also include remedial work as necessary for the whole class, a small group, or individual students. Depending on the anticipated difficulties, the remedial work might take on different forms as suggested in the chart below.

Anticipated Difficulty	Must Do Problems Remedial Suggestion
The first problem of the lesson is too challenging.	Write a short sequence of problems on the board that provides a ladder to Problem 1. Direct students to complete those first problems to empower them to begin the lesson.
The jump in complexity between two problems is too big.	Provide a problem or set of problems that bridge student understanding from one problem to the next.
Students lack fluency or foundational skills necessary for the lesson.	Before beginning the lesson, do a quick, engaging fluency exercise, such as a Rapid White Board Exchange or Sprint. Before beginning any fluency activity for the first time, assess that students have conceptual understanding of the problems in the set and they will find success with the easiest problem in the set.
Students need more work at the concrete or pictorial level.	Provide manipulatives or the opportunity to draw solution strategies.
Students need more work at the abstract level.	Add a Rapid White Board Exchange of abstract problems for students to complete toward the end of the lesson.

- C: Could Do problems are for students who work with greater fluency and understanding and who can, therefore, complete more work within a given time frame.
- D: At times, a particularly complex problem might be designated as an Extension problem for advanced students. Consider creating the opportunity for students to share Extension solutions with the class.
- E: If the lesson is customized, carefully select Closing questions that reflect such decisions, and adjust the Exit Ticket as necessary.



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