EUREKA
MATH™

A Story of Units™

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Grade 7 Supplemental Materials

April 2020

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Module and Topic Level Materials

Lesson Materials

Grade 7 • Module 3

Expressions and Equations

OVERVIEW

In Grade 6, students interpreted expressions and equations as they reasoned about one-variable equations (**6.EE.A.2**). This module consolidates and expands upon students' understanding of equivalent expressions as they apply the properties of operations (associative, commutative, and distributive) to write expressions in both standard form (by expanding products into sums) and in factored form (by expanding sums into products). They use linear equations to solve unknown angle problems and other problems presented within context to understand that solving algebraic equations is all about the numbers. It is assumed that a number already exists to satisfy the equation and context; we just need to discover it. A number sentence is an equation that is said to be true if both numerical expressions evaluate to the same number; it is said to be false otherwise. Students use the number line to understand the properties of inequality and recognize when to *preserve the inequality* and when to *reverse the inequality* when solving problems leading to inequalities. They interpret solutions within the context of problems. Students extend their sixth-grade study of geometric figures and the relationships between them as they apply their work with expressions and equations to solve problems involving area of a circle and composite area in the plane, as well as volume and surface area of right prisms. In this module, students discover the most famous ratio of all, π, and begin to appreciate why it has been chosen as the symbol to represent the Grades 6–8 mathematics curriculum, *A Story of Ratios*.

To begin this module, students will generate equivalent expressions using the fact that addition and multiplication can be done in any order with any grouping and will extend this understanding to subtraction (adding the inverse) and division (multiplying by the multiplicative inverse, also known as the reciprocal) (**7.EE.A.1**). They extend the properties of operations with numbers (learned in earlier grades) and recognize how the same properties hold true for letters that represent numbers. Knowledge of rational number operations from Module 2 is demonstrated as students collect like terms containing both positive and negative integers.

An area model is used as a tool for students to rewrite products as sums and sums as products and to provide a visual representation leading students to recognize the repeated use of the distributive property in factoring and expanding linear expressions (**7.EE.A.1**). Students examine situations where more than one form of an expression may be used to represent the same context, and they see how looking at each form can bring a new perspective (and thus deeper understanding) to the problem. Students recognize and use the identity properties and the existence of additive inverses to efficiently write equivalent expressions in standard form, for example, $2x + (-2x) + 3 = 0 + 3 = 3$ (**7.EE.A.2**). By the end of the topic, students have the opportunity to practice knowledge of operations with rational numbers gained in Module 2 (**7.NS.A.1**, **7.NS.A.2**) as they collect like terms with rational number coefficients (**7.EE.A.1**).

In Topic B, students use linear equations and inequalities to solve problems (**7.EE.B.4**). They continue to use tape diagrams from earlier grades where they see fit, but will quickly discover that some problems would more reasonably be solved algebraically (as in the case of large numbers). Guiding students to arrive at this realization on their own develops the need for algebra. This algebraic approach builds upon work in Grade 6 with equations (**6.EE.B.6**, **6.EE.B.7**) to now include multi-step equations and inequalities containing rational numbers (**7.EE.B.3**, **7.EE.B.4**). Students solve problems involving consecutive numbers; total cost; age comparisons; distance, rate, and time; area and perimeter; and missing angle measures. Solving equations with a variable is all about numbers, and students are challenged with the goal of finding the number that makes the equation true. When given in context, students recognize that a value exists, and it is simply their job to discover what that value is. Even the angles in each diagram have a precise value, which can be checked with a protractor to ensure students that the value they find does indeed create a true number sentence.

In Topic C, students continue work with geometry as they use equations and expressions to study area, perimeter, surface area, and volume. This final topic begins by modeling a circle with a bicycle tire and comparing its perimeter (one rotation of the tire) to the length across (measured with a string) to allow students to discover the most famous ratio of all, pi. Activities in comparing circumference to diameter are staged precisely for students to recognize that this symbol has a distinct value and can be approximated by $\frac{22}{7}$, or 3.14, to give students an intuitive sense of the relationship that exists. In addition to representing this value with the π symbol, the fraction and decimal approximations allow for students to continue to practice their work with rational number operations. All problems are crafted in such a way as to allow students to practice skills in reducing within a problem, such as using $\frac{22}{7}$ for finding circumference with a given diameter length of 14 cm, and recognize what value would be best to approximate a solution. This understanding allows students to accurately assess work for reasonableness of answers. After discovering and understanding the value of this special ratio, students will continue to use pi as they solve problems of area and circumference (**7.G.B.4**).

In this topic, students derive the formula for area of a circle by dividing a circle of radius r into pieces of pi and rearranging the pieces so that they are lined up, alternating direction, and form a shape that resembles a rectangle. This “rectangle” has a length that is $\frac{1}{2}$ the circumference and a width of r. Students determine that the area of this rectangle (reconfigured from a circle of the same area) is the product of its length and its width: $\frac{1}{2}C \cdot r = \frac{1}{2}2\pi r \cdot r = \pi r^2$ (**7.G.B.4**). The precise definitions for diameter, circumference, pi, and circular region or disk will be developed during this topic with significant time being devoted to students’ understanding of each term.

Students build upon their work in Grade 6 with surface area and nets to understand that surface area is simply the sum of the area of the lateral faces and the base(s) (**6.G.A.4**). In Grade 7, they continue to solve real-life and mathematical problems involving area of two-dimensional shapes and surface area and volume of prisms (e.g., rectangular, triangular), focusing on problems that involve fractional values for length (**7.G.B.6**). Additional work (examples) with surface area will occur in Module 6 after a formal definition of rectangular pyramid is established.

This module is comprised of 26 lessons; 9 days are reserved for administering the Mid-Module and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B, and the End-of-Module Assessment follows Topic C.

Topic A

Use Properties of Operations to Generate Equivalent Expressions

7.EE.A.1, 7.EE.A.2

Focus Standards:	7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
	7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* $a + 0.05a = 1.05a$ *means that "increase by* 5%*" is the same as "multiply by* 1.05*."*
Instructional Days:	6	
Lessons 1–2:	Generating Equivalent Expressions (P)[1]	
Lessons 3–4:	Writing Products as Sums and Sums as Products (P)	
Lesson 5:	Using the Identity and Inverse to Write Equivalent Expressions (P)	
Lesson 6:	Collecting Rational Number Like Terms (P)	

In Lesson 1 of Topic A, students write equivalent expressions by finding sums and differences extending the *any order* (commutative property) and *any grouping* (associative property) to collect like terms and rewrite algebraic expressions in standard form (**7.EE.A.1**). In Lesson 2, students rewrite products in standard form by applying the commutative property to rearrange like items (numeric coefficients, like variables) next to each other and rewrite division as multiplying by the multiplicative inverse. Lessons 3 and 4 have students using a rectangular array and the distributive property as they first multiply one term by a sum of two or more terms to expand a product to a sum, and then reverse the process to rewrite the sum as a product of the GCF and a remaining factor. Students model real-world problems with expressions and see how writing in one form versus another helps them to understand how the quantities are related (**7.EE.A.2**). In Lesson 5, students recognize that detecting inverses and the identity properties of 0 for addition and 1 for multiplication allows for ease in rewriting equivalent expressions. This topic culminates with Lesson 6 with students applying repeated use of the distributive property as they collect like terms containing fractional coefficients to rewrite rational number expressions.

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[1] Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

Lesson 3: Writing Products as Sums and Sums as Products

Student Outcomes

- Students use area and rectangular array models and the distributive property to write products as sums and sums as products.
- Students use the fact that the opposite of a number is the same as multiplying by -1 to write the opposite of a sum in standard form.
- Students recognize that rewriting an expression in a different form can shed light on the problem and how the quantities in it are related.

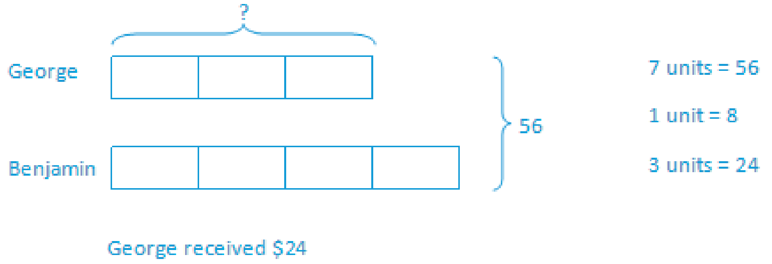
Classwork

Opening Exercise (4 minutes)

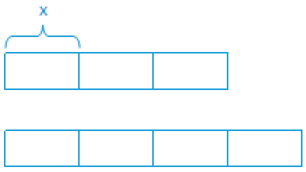
Students create tape diagrams to represent the problem and solution.

Opening Exercise

Solve the problem using a tape diagram. A sum of money was shared between George and Benjamin in a ratio of $3 : 4$. If the sum of money was $\$56.00$, how much did George get?



Have students label one unit as x in the diagram.

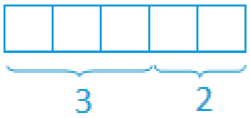


- What does the rectangle labeled x represent?
  - $\$8.00$

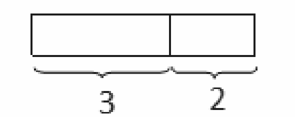
Example 1 (3 minutes)

Example 1

Represent $3 + 2$ using a tape diagram.



Represent it also in this fashion:

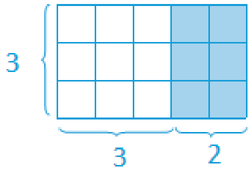


- Now, let's represent another expression, $x + 2$. Make sure the units are the same size when you are drawing the known 2 units.

Represent $x + 2$ using a tape diagram.

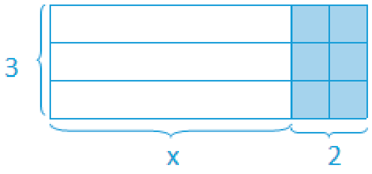
- Note the size of the units that represent 2 in the expression $x + 2$. Using the size of these units, can you predict what value x represents?
    - *Approximately six units*

Draw a rectangular array for $3(3 + 2)$.



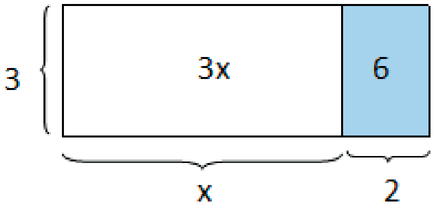
Then, have students draw a similar array for $3(x + 2)$.

Draw an array for $3(x + 2)$.



- Determine the area of the shaded region.
  - 6
- Determine the area of the unshaded region.
  - $3x$

Record the areas of each region:



Introduce the term *distributive property* in the Key Terms box from the Student Materials.

Key Terms

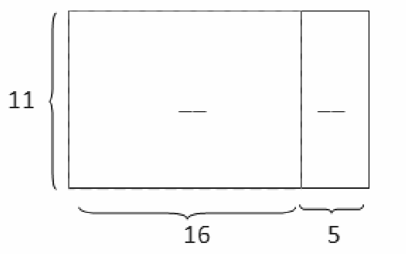
DISTRIBUTIVE PROPERTY: The *distributive property* can be written as the identity

$$a(b+c) = ab + ac \text{ for all numbers } a, b, \text{ and } c.$$

Exercise 1 (3 minutes)

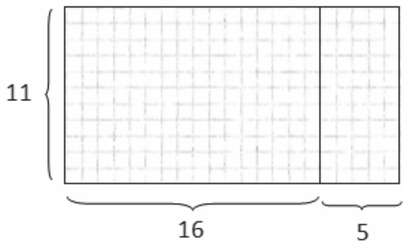
Exercise 1

Determine the area of each region using the distributive property.



***Answers:* 176, 55**

Draw in the units in the diagram for students.



- Is it easier to just imagine the 176 and 55 square units?
    - *Yes*

Example 2 (5 minutes)

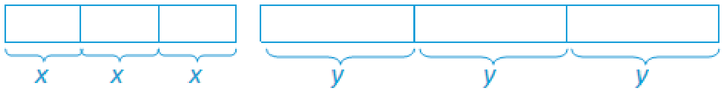
Model the creation of the tape diagrams for the following expressions. Students draw the tape diagrams on the student pages and use the models for discussion.

Example 2

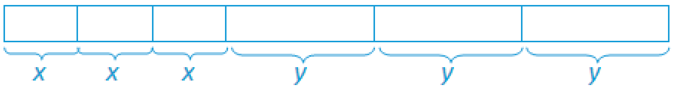
Draw a tape diagram to represent each expression.

a. $(x + y) + (x + y) + (x + y)$

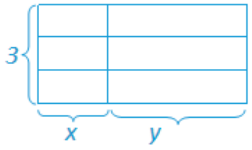
b. $(x + x + x) + (y + y + y)$



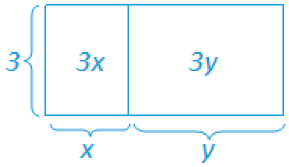
c. $3x + 3y$



Or

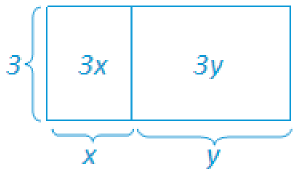


d. $3(x + y)$



Ask students to explain to their neighbors why all of these expressions are equivalent.

Discuss how to rearrange the units representing x and y into each of the configurations on the previous page.

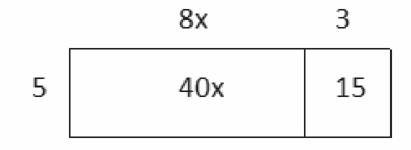
- What can we conclude about all of these expressions?
  - *They are all equivalent.*
- How does $3(x + y) = 3x + 3y$?
  - *Three groups of* $(x + y)$ *is the same as multiplying* 3 *with the* x *and the* y*.*
- How do you know the three representations of the expressions are equivalent?
  - *The arithmetic, algebraic, and graphic representations are equivalent. Problem (c) is the standard form of problems (b) and (d). Problem (a) is the equivalent of problems (b) and (c) before the distributive property is applied. Problem (b) is the expanded form before collecting like terms.*
- Under which conditions would each representation be most useful?
  - *Either* $3(x + y)$ *or* $3x + 3y$ *because it is clear to see that there are* 3 *groups of* $(x + y)$*, which is the product of the sum of* x *and* y*, or that the second expression is the sum of* $3x$ *and* $3y$*.*
- Which model best represents the distributive property?
  - 

Summarize the distributive property.

Example 3 (5 minutes)

Example 3

Find an equivalent expression by modeling with a rectangular array and applying the distributive property to the expression $5(8x + 3)$.



Distribute the factor to all the terms.	$\mathbf{5\ (8x + 3)}$
Multiply.	$\mathbf{5(8x) + 5(3)}$
	$\mathbf{40x + 15}$

Substitute given numerical value to demonstrate equivalency. Let $\boldsymbol{x = 2}$

$\mathbf{5(8x + 3) = 5\ (8(2) + 3) = 5(16 + 3) = 5(19) = 95}$ $\mathbf{40x + 15 = 40(2) + 15 = 80 + 15 = 95}$

Both equal $\mathbf{95}$***, so the expressions are equal.***

Scaffolding:

For the struggling student, draw a rectangular array for $5(3)$. The number of squares in the rectangular array is the product because the factors are 5 and 3. Therefore, $5(3) = 15$ is represented.

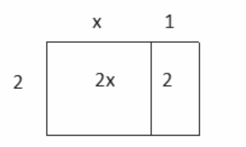
Exercise 2 (3 minutes)

Allow students to work on the problems independently and share aloud their equivalent expressions. Substitute numerical values to demonstrate equivalency.

Exercise 2

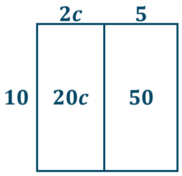
For parts (a) and (b), draw an array for each expression and apply the distributive property to expand each expression. Substitute the given numerical values to demonstrate equivalency.

a. $2(x+1), x=5$



$2x+2,\ 12$

b. $10(2c+5), c=1$



$20c+50,\ 70$

For parts (c) and (d), apply the distributive property. Substitute the given numerical values to demonstrate equivalency.

c. $3(4f-1), f=2$

$12f-3,\ 21$

d. $9(-3r-11), r=10$

$-27r-99,\ -369$

Example 4 (3 minutes)

Example 4

Rewrite the expression $(6x+15)\div 3$ in standard form using the distributive property.

$$(6x+15)\times\frac{1}{3}$$
$$(6x)\frac{1}{3}+(15)\frac{1}{3}$$
$$2x+5$$

- How can we rewrite the expression so that the distributive property can be used?
  - *We can change from dividing by 3 to multiplying by* $\frac{1}{3}$.

Exercise 3 (3 minutes)

Exercise 3

Rewrite the expressions in standard form.

a. $(2b + 12) \div 2$

$\frac{1}{2}(2b + 12)$

$\frac{1}{2}(2b) + \frac{1}{2}(12)$

$b + 6$

b. $(20r - 8) \div 4$

$\frac{1}{4}(20r - 8)$

$\frac{1}{4}(20r) - \frac{1}{4}(8)$

$5r - 2$

c. $(49g - 7) \div 7$

$\frac{1}{7}(49g - 7)$

$\frac{1}{7}(49g) - \frac{1}{7}(7)$

$7g - 1$

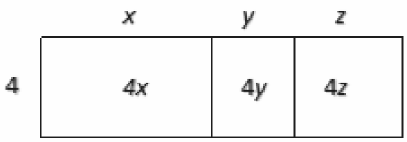
Example 5 (3 minutes)

Model the following exercise with the use of rectangular arrays. Discuss:

- What is a verbal explanation of $4(x + y + z)$?
  - *There are* 4 *groups of the sum of* x, y, *and* z.

Example 5

Expand the expression $4(x + y + z)$**.**



The expanded expression is 4x + 4y + 4z

Exercise 4 (3 minutes)

Instruct students to complete the exercise individually.

Exercise 4

Expand the expression from a product to a sum by removing grouping symbols using an area model and the repeated use of the distributive property: $3(x + 2y + 5z)$.

Repeated use of the distributive property:

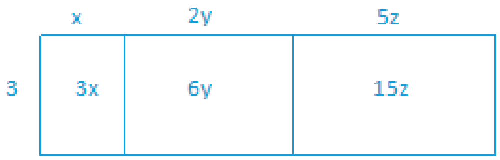
$3(x + 2y + 5z)$

$3 \cdot x + 3 \cdot 2y + 3 \cdot 5z$

$3x + 3 \cdot 2 \cdot y + 3 \cdot 5 \cdot z$

$3x + 6y + 15z$

Visually:



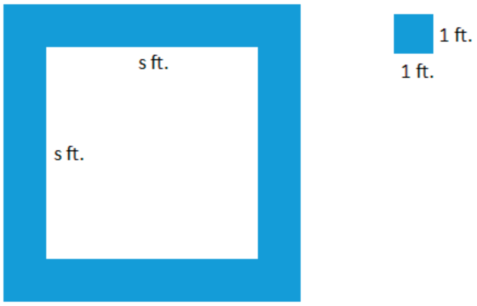
The expanded expression is 3x + 6y + 15z.

Example 6 (5 minutes)

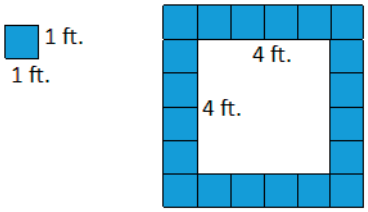
After reading the problem aloud with the class, use different lengths to represent s in order to come up with expressions with numerical values.

Example 6

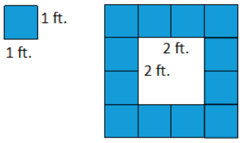
A square fountain area with side length s ft. is bordered by a single row of square tiles as shown. Express the total number of tiles needed in terms of s three different ways.



- What if $s = 4$? How many tiles would you need to border the fountain?
  - *I would need* 20 *tiles to border the fountain—four for each side and one for each corner.*



- What if $s = 2$? How many tiles would you need to border the fountain?
    - *I would need 12 tiles to border the fountain—two for each side and one for each corner.*

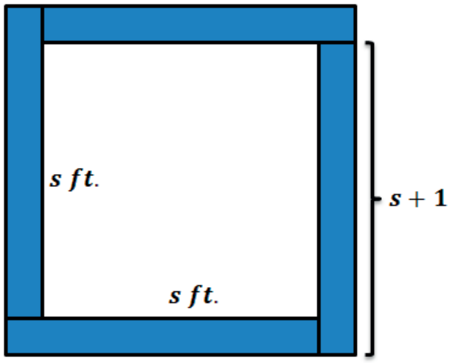


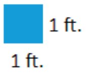
- What pattern or generalization do you notice?
    - *Answers may vary. Sample response: There is one tile for each corner and four times the number of tiles to fit one side length.*

After using numerical values, allow students two minutes to create as many expressions as they can think of to find the total number of tiles in the border in terms of s. Reconvene by asking students to share their expressions with the class from their seat.

- Which expressions would you use and why?
    - *Although all the expressions are equivalent, $4(s + 1)$, or $4s + 4$, is useful because it is the most simplified, concise form. It is in standard form with all like terms collected.*

Sample Responses:



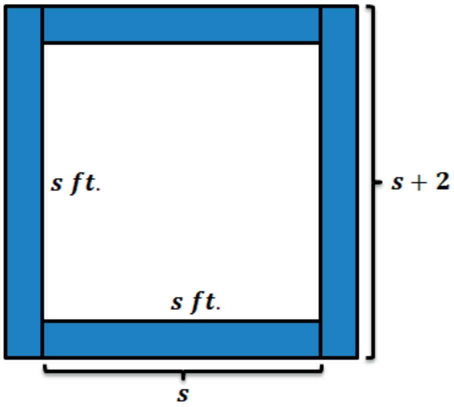


$s + s + s + s + 4.$

***Explanation:** There are 4 sides of s tiles and 4 extra tiles for the corners.*

$4(s + 1)$

***Explanation:** There are four groups of s tiles plus 1 corner tile.*



$2s + 2(s + 2)$

***Explanation:** There are 2 opposite sides of s tiles plus 2 groups of a side of s tiles plus 2 corner tiles.*

Closing (3 minutes)

- What are some of the methods used to write products as sums?
    - *We used the distributive property and rectangular arrays.*
- In terms of a rectangular array and equivalent expressions, what does the product form represent, and what does the sum form represent?
    - *The total area represents the expression written in sum form, and the length and width represent the expressions written in product form.*

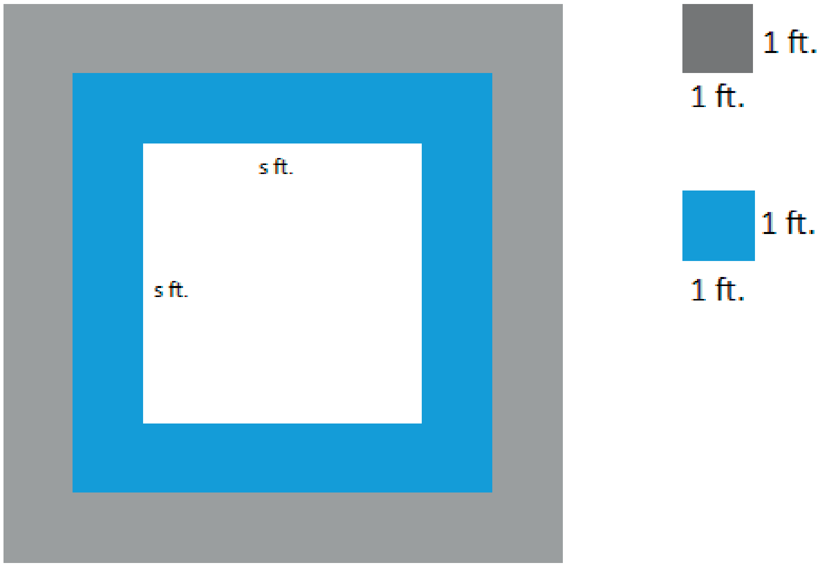
Exit Ticket (3 minutes)

Name ____ Date____________________

Lesson 3: Writing Products as Sums and Sums as Products

Exit Ticket

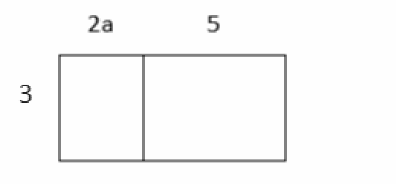
A square fountain area with side length s ft. is bordered by two rows of square tiles along its perimeter as shown. Express the total number of grey tiles (the second border of tiles) needed in terms of s three different ways.



Problem Set Sample Solutions

1\.

a. **Write two equivalent expressions that represent the rectangular array below.**



$3(2a+5)$ or $6a+15$

b. **Verify informally that the two expressions are equivalent using substitution.**

Let $a=4$.

$3(2a+5)$	$6a+15$
$3(2(4)+5)$	$6(4)+15$
$3(8+5)$	$24+15$
$3(13)$	39
39	

2\. **You and your friend made up a basketball shooting game. Every shot made from the free throw line is worth 3 points, and every shot made from the half-court mark is worth 6 points. Write an equation that represents the total number of points, P, if f represents the number of shots made from the free throw line, and h represents the number of shots made from half-court. Explain the equation in words.**

$P=3f+6h$ or $P=3(f+2h)$

The total number of points can be determined by multiplying each free throw shot by 3 and then adding that to the product of each half-court shot multiplied by 6.

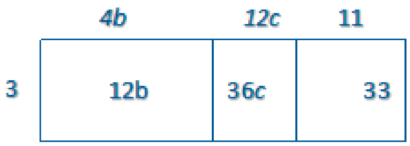
The total number of points can also be determined by adding the number of free throw shots to twice the number of half-court shots and then multiplying the sum by three.

3\. **Use a rectangular array to write the products in standard form.**

a. $2(x+10)$

$2x+20$

b. $3(4b+12c+11)$



$12b+36c+33$

4. **Use the distributive property to write the products in standard form.**

a. $3(2x-1)$

$6x-3$

b. $10(b+4c)$

$10b+40c$

c. $9(g-5h)$

$9g-45h$

d. $7(4n-5m-2)$

$28n-35m-14$

e. $a(b+c+1)$

$ab+ac+a$

f. $(8j-3l+9)6$

$48j-18l+54$

g. $(40s+100t)\div 10$

$4s+10t$

h. $(48p+24)\div 6$

$8p+4$

i. $(2b+12)\div 2$

$b+6$

j. $(20r-8)\div 4$

$5r-2$

k. $(49g-7)\div 7$

$7g-1$

l. $(14g+22h)\div \frac{1}{2}$

$28g+44h$

5. **Write the expression in standard form by expanding and collecting like terms.**

a. $4(8m-7n)+6(3n-4m)$

$8m-10n$

b. $9(r-s)+5(2r-2s)$

$19r-19s$

c. $12(1-3g)+8(g+f)$

$-28g+8f+12$

Name ______________________________ Date ______________

1. Use the following expression below to answer parts (a) and (b).

$$4x - 3(x - 2y) + \frac{1}{2}(6x - 8y)$$

a. Write an equivalent expression in standard form, and collect like terms.

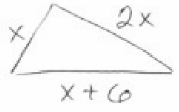
$$4x - 3(x - 2y) + \frac{1}{2}(6x - 8y)$$
$$4x - 3x + 6y + 3x - 4y$$
$$4x - 3x + 3x + 6y - 4y$$
$$4x + 2y$$

b. Express the answer from part (a) as an equivalent expression in factored form.

$$4x + 2y$$
$$2(2x + y)$$

2. Use the following information to solve the problems below.

a. The longest side of a triangle is six more units than the shortest side. The third side is twice the length of the shortest side. If the perimeter of the triangle is 25 units, write and solve an equation to find the lengths of all three sides of the triangle.

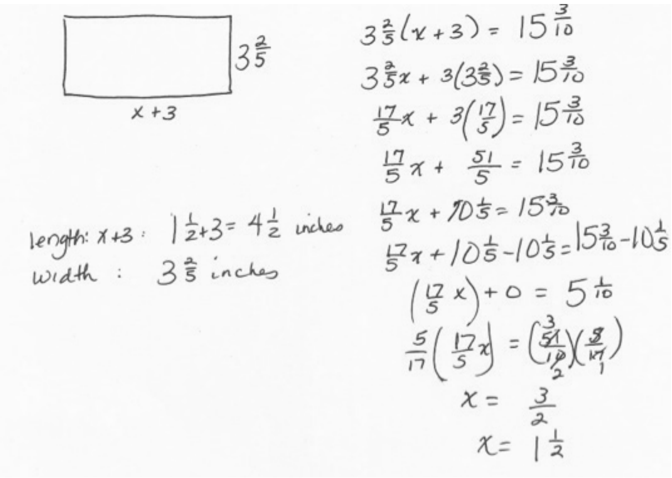


$$2x + x + x + 6 = 25$$
$$4x + 6 = 25$$
$$4x + 6 - 6 = 25 - 6$$
$$4x + 0 = 19$$
$$\frac{1}{4}(4x) = \frac{1}{4}(19)$$
$$x = \frac{19}{4}$$
$$x = 4\frac{3}{4}$$

smallest side:
$x = 4\frac{3}{4}$
largest side:
$x + 6 = 10\frac{3}{4}$
third side:
$2x = 9\frac{1}{2}$

3 sides are:
$4\frac{3}{4}$ units, $10\frac{3}{4}$ units, $9\frac{1}{2}$ units

b. The length of a rectangle is $(x+3)$ inches long, and the width is $3\frac{2}{5}$ inches. If the area is $15\frac{3}{10}$ square inches, write and solve an equation to find the length of the rectangle.

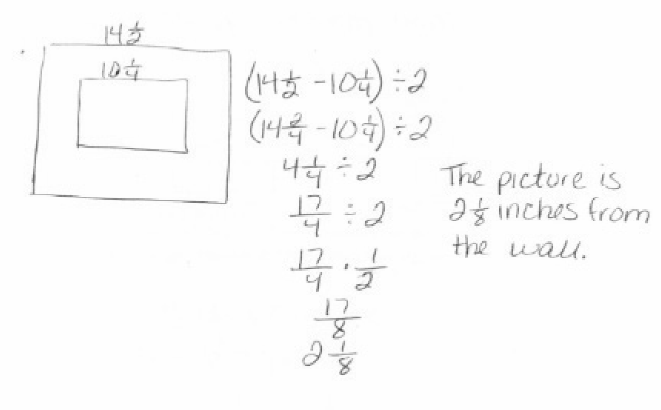


$$3\tfrac{2}{5}(x+3) = 15\tfrac{3}{10}$$
$$3\tfrac{2}{5}x + 3(3\tfrac{2}{5}) = 15\tfrac{3}{10}$$
$$\tfrac{17}{5}x + 3(\tfrac{17}{5}) = 15\tfrac{3}{10}$$
$$\tfrac{17}{5}x + \tfrac{51}{5} = 15\tfrac{3}{10}$$
$$\tfrac{17}{5}x + 10\tfrac{1}{5} = 15\tfrac{3}{10}$$
$$\tfrac{17}{5}x + 10\tfrac{1}{5} - 10\tfrac{1}{5} = 15\tfrac{3}{10} - 10\tfrac{1}{5}$$
$$(\tfrac{17}{5}x) + 0 = 5\tfrac{1}{10}$$
$$\tfrac{5}{17}(\tfrac{17}{5}x) = (\tfrac{\overset{3}{\cancel{51}}}{\underset{2}{\cancel{10}}})(\tfrac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{17}}})$$
$$x = \tfrac{3}{2}$$
$$x = 1\tfrac{1}{2}$$

length: $x+3$: $1\frac{1}{2}+3 = 4\frac{1}{2}$ inches
width: $3\frac{2}{5}$ inches

3. A picture $10\frac{1}{4}$ feet long is to be centered on a wall that is $14\frac{1}{2}$ feet long. How much space is there from the edge of the wall to the picture?

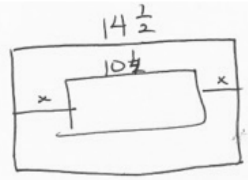
a. Solve the problem arithmetically.



$$(14\tfrac{1}{2} - 10\tfrac{1}{4}) \div 2$$
$$(14\tfrac{2}{4} - 10\tfrac{1}{4}) \div 2$$
$$4\tfrac{1}{4} \div 2$$
$$\tfrac{17}{4} \div 2$$
$$\tfrac{17}{4} \cdot \tfrac{1}{2}$$
$$\tfrac{17}{8}$$
$$2\tfrac{1}{8}$$

The picture is $2\frac{1}{8}$ inches from the wall.

b. Solve the problem algebraically.



Let x: distance from one side to the picture

$x + 10\frac{1}{4} + x = 14\frac{1}{2}$

$2x + 10\frac{1}{4} = 14\frac{1}{2}$

$2x + 10\frac{1}{4} - 10\frac{1}{4} = 14\frac{1}{2} - 10\frac{1}{4}$

$2x + 0 = 4\frac{1}{4}$

$\left(\frac{1}{2}\right)(2x) = \left(4\frac{1}{4}\right)\left(\frac{1}{2}\right)$

$x = \left(\frac{17}{4}\right)\left(\frac{1}{2}\right)$

$x = \frac{17}{8} = 2\frac{1}{8}$

The picture is $2\frac{1}{8}$ inches from the wall.

c. Compare the approaches used in parts (a) and (b). Explain how they are similar.

The solutions are the same. The actual operations performed in the equation are the same operations done arithmetically.

4. In August, Cory begins school shopping for his triplet daughters.

a. One day, he bought 10 pairs of socks for $\$2.50$ each and 3 pairs of shoes for d dollars each. He spent a total of $\$135.97$. Write and solve an equation to find the cost of one pair of shoes.

d: cost of shoes

$10(2.50) + 3d = 135.97$

$25 + 3d \neq 135.97$

$3d + 25 = 135.97$

$3d + 25 - 25 = 135.97 - 25$

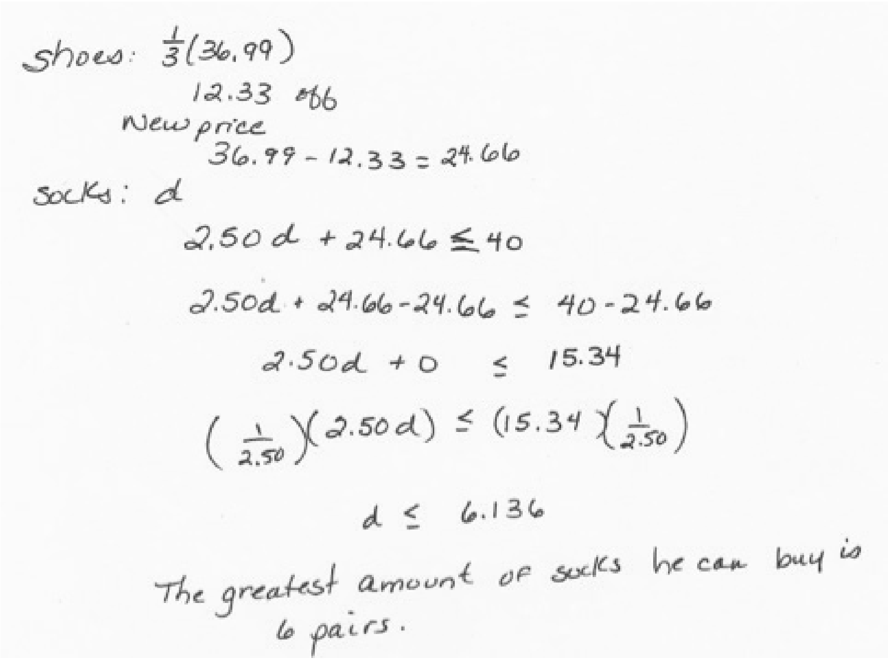
$3d + 0 = 110.97$

$\left(\frac{1}{3}\right)(3d) = (110.97)\left(\frac{1}{3}\right)$

$d = 36.99$

The cost of one pair of shoes is $36.99

b. The following day Cory returned to the store to purchase some more socks. He had $\$40$ to spend. When he arrived at the store, the shoes were on sale for $\frac{1}{3}$ off. What is the greatest amount of pairs of socks Cory can purchase if he purchases another pair of shoes in addition to the socks?



shoes: $\frac{1}{3}(36.99)$

12.33 off

New price

$36.99 - 12.33 = 24.66$

socks: d

$$2.50d + 24.66 \le 40$$

$$2.50d + 24.66 - 24.66 \le 40 - 24.66$$

$$2.50d + 0 \le 15.34$$

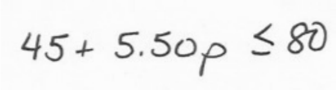
$$\left(\frac{1}{2.50}\right)(2.50d) \le (15.34)\left(\frac{1}{2.50}\right)$$

$$d \le 6.136$$

The greatest amount of socks he can buy is 6 pairs.

5. Ben wants to have his birthday at the bowling alley with a few of his friends, but he can spend no more than $\$80$. The bowling alley charges a flat fee of $\$45$ for a private party and $\$5.50$ per person for shoe rentals and unlimited bowling.

a. Write an inequality that represents the total cost of Ben's birthday for p people given his budget.



$$45 + 5.50p \le 80$$

b. How many people can Ben pay for (including himself) while staying within the limitations of his budget?

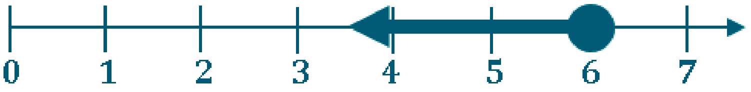
p: number of people invited +

$$45 + 5.50p \le 80$$
$$5.50p + 45 \le 80$$
$$5.50p + 45 - 45 \le 80 - 45$$
$$\left(\frac{1}{5.50}\right)(5.50p) \le (35)\left(\frac{1}{5.50}\right)$$
$$p \le \frac{350}{55}$$
$$p \le \frac{70}{11}$$
$$p \le 6\frac{4}{11}$$

6 people can attend the party.

$p \le 6$

c. Graph the solution of the inequality from part (a).



6. Jenny invited Gianna to go watch a movie with her family. The movie theater charges one rate for 3D admission and a different rate for regular admission. Jenny and Gianna decided to watch the newest movie in 3D. Jenny's mother, father, and grandfather accompanied Jenny's little brother to the regular admission movie.

a. Write an expression for the total cost of the tickets. Define the variables.

d: cost in dollars of 3D admission
r: Cost in dollars of regular admission

Jenny	Gianna	Mother	Father	Grandfather	Brother
d +	d +	r +	r +	r +	r

$2d + 4r$

b. The cost of the 3D ticket was double the cost of the regular admission. Write an equation to represent the relationship between the two types of tickets.

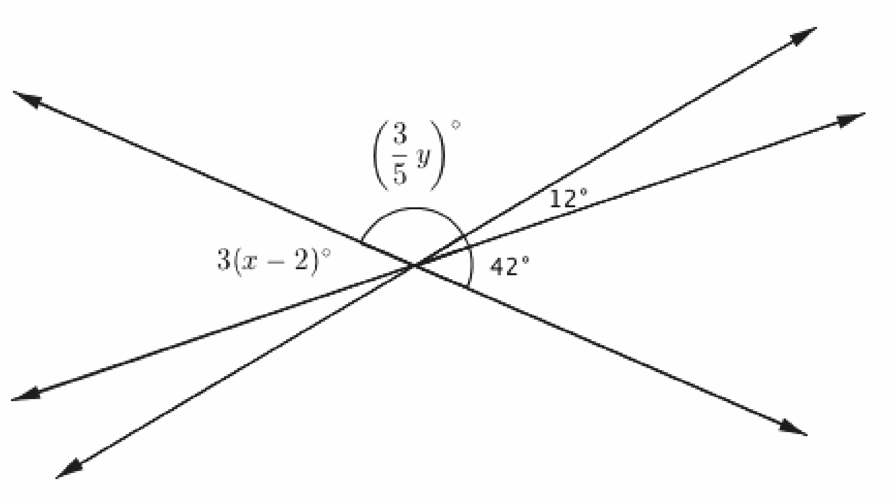
$$d = 2r$$

c. The family purchased refreshments and spent a total of $18.50. If the total amount of money spent on tickets and refreshments was $94.50, use an equation to find the cost of one regular admission ticket.

$$2d + 4r + 18.50 = 94.50$$
$$2(2r) + 4r + 18.50 = 94.50$$
$$4r + 4r + 18.50 = 94.50$$
$$8r + 18.50 = 94.50$$
$$8r + 18.50 - 18.50 = 94.50 - 18.50$$
$$8r + 0 = 76$$
$$\left(\frac{1}{8}\right)(8r) = (76)\left(\frac{1}{8}\right)$$
$$r = 9.5$$

The cost of one regular admission ticket is $9.50

7. The three lines shown in the diagram below intersect at the same point. The measures of some of the angles in degrees are given as $3(x-2)°$, $\left(\frac{3}{5}y\right)°$, $12°$, $42°$.



a. Write and solve an equation that can be used to find the value of x.

$3(x-2) = 42$

$3x - 6 = 42$

$3x - 6 + 6 = 42 + 6$

$3x + 0 = 48$

$\left(\frac{1}{3}\right)(3x) = (48)\left(\frac{1}{3}\right)$

$x = 16$

or

$\frac{1}{3}(\cancel{3}(x-2)) = (42)\frac{1}{3}$

$x - 2 = 14$

$x - 2 + 2 = 14 + 2$

$x + 0 = 16$

$x = 16$

b. Write and solve an equation that can be used to find the value of y.

$\frac{3}{5}y + 12 + 42 = 180$

$\frac{3}{5}y + 54 = 180$

$\frac{3}{5}y + 54 - 54 = 180 - 54$

$\frac{3}{5}y + 0 = 126$

$\left(\frac{5}{3}\right)\left(\frac{3}{5}y\right) = \left(\overset{42}{\cancel{126}}\right)\left(\frac{5}{\cancel{3}_1}\right)$

$y = (42)(5)$

$y = 210$

A Progression Toward Mastery

Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem**
1	**a** **7.EE.A.1**	Student demonstrates a limited understanding of writing the expression in standard form. OR Student makes a conceptual error, such as dropping the parentheses or adding instead of multiplying.	Student makes two or more computational errors.	Student demonstrates a solid understanding but makes one computational error and completes the question by writing a correct equivalent expression in standard form. OR Student answers correctly, but no further correct work is shown.	Student writes the expression correctly in standard form, $4x + 2y$. Student shows appropriate work, such as using the distributive property and collecting like terms.
	b **7.EE.A.1**	Student demonstrates a limited understanding of writing the expression in factored form.	Student writes the expression correctly in factored form but has part (a) incorrect.	Student demonstrates a solid understanding by writing a correct equivalent expression in factored form based on the answer from part (a) but makes one computational error.	Student uses the correct expression from part (a). Student writes an equivalent expression in factored form.

Grade 7 Module 3 Lesson 3 Teacher A

Lesson Type (circle one): [✓ **Problem Set**] **Exploration** **Socratic** **Modeling Cycle**

Context: These students used *Eureka Math* for the first time in Grade 6. They are familiar with the tape diagram model, but not the area model. Analysis of Exit Tickets from previous lessons show that many students still struggle with multiplying expressions that contain a variable, particularly when one of the factors is a fraction.

Student Outcome(s)	▪ Students use an area model and distributive property to write products as sums and sums as products. ▪ Students recognize that rewriting an expression in a different form can shed light on the problem and how the quantities in it are related.	
Materials	Personal whiteboard materials	
	Customizations	**Rationale**
Fluency Activity (as needed)	$2 \cdot x = 2x$ $2 \cdot 3 \cdot x = 6x$ $2 \cdot 3x = 6x$ $5 \cdot x = 5x$ $5 \cdot 8 \cdot x = 40x$ $5 \cdot 8x = 40x$ $\frac{1}{2} \cdot 2b = b$ $\frac{1}{3} \cdot 6x = 2x$	Whiteboard exchange allows my students to practice the more complicated math in a quick way and allows me to see which students continue to struggle with multiplying whole numbers and variable expressions.
Classwork <u>**Customization Key**</u> ▪ **Must Do problem (M)** ▪ **Could Do problem (C)** ▪ **Extension problem (E)**	Omit Opening Exercise. Replace with Grade 4 Module 3 Lesson 11 Problem Set 1. (M) Example 1: Show numeric and symbolic side-by-side. (C) Exercise 1 (M) Example 2: Use as a guided example. (M) Example 3: Do $5(x + 3)$ first with explicit directions; then guide students with $5(8x + 3)$, as needed. (M) Exercise 2(a), 2(c) (C) Exercise 2(b), 2(d) (M) Example 4: Use as a guided example. (M) Exercise 3(a) (C) Exercise 3(b)–(c) (E) Example 5 (E) Exercise 4 Omit Example 6.	Omitted original opening exercise and replaced it with one that uses the area model in a more comfortable setting—whole numbers. Because students are not familiar with the area model, starting with whole numbers will make the transition to using variable expressions easier. Inserted an example, $5(x + 3)$, before Example 3 as a scaffold to the more complex expression $5(8x + 3)$. I can provide direct instruction about how to use the model with the simpler example and allow students to work independently on the given example. It will allow me time to work with students who may have struggled with the fluency activity. Made parts of Exercises 2 and 3 Could Do problems because they are similar to the Must Do problems. We can use them for extra practice as needed. Omitted Example 6 to make time for the fluency activity.
Closing	Describe to an elbow partner how we used the area model to write products as sums. Sentence starter: We used the area model to _______.	Revised the closing to just one prompt to focus on the use of the model to write products as sums. Many students struggle to start talking so the sentence starter helps begin the conversation.

Formative Assessment	Use whiteboards: Draw an area model to find an equivalent expression for $\boldsymbol{2(3x+4)}$. Draw an area model to find an equivalent expression for $\boldsymbol{3(4x-1)}$.	The existing Exit Ticket relied on the work of Example 6. For that reason, I changed the Exit Ticket to a less formal whiteboard check for understanding.
Problem Set	(M) 1, 3(a), 4(a)–(c), 4(g), 4(j)	Reduced the number and complexity of problems to match what I did in class.
Summative Assessment	Topic Quiz should focus on $a(b+c)$ type problems.	
Lesson Notes	Need to pull Grade 4 Module 3 Lesson 11 Problem Set. Could Do exercises listed in Classwork are similar to Must Do exercises and could be assigned to early finishers or used as second chance after additional instruction.	

Grade 7 Module 3 Lesson 3 Administrator A

Lesson Type (circle one): [✓ **Problem Set**] **Exploration** **Socratic** **Modeling Cycle**

Student Outcome(s)	▪ Students use an area model and distributive property to write products as sums and sums as products. ▪ Students recognize that rewriting an expression in a different form can shed light on the problem and how the quantities in it are related.	
Materials	Personal whiteboard materials	
	Customizations	**Questions**
Fluency Activity (as needed)	$2 \cdot x = 2x$ $2 \cdot 3 \cdot x = 6x$ $2 \cdot 3x = 6x$ $5 \cdot x = 5x$ $5 \cdot 8 \cdot x = 40x$ $5 \cdot 8x = 40x$ $\frac{1}{2} \cdot 2b = b$ $\frac{1}{3} \cdot 6x = 2x$	I noticed you added a fluency activity to this lesson. Why did you choose to do so and why these problems?
Classwork **<u>Customization Key</u>** ▪ **Must Do problem (M)** ▪ **Could Do problem (C)** ▪ **Extension problem (E)**	Omit Opening Exercise. Replace with Grade 4 Module 3 Lesson 11 Problem Set 1. (M) Example 1: Show numeric and symbolic side-by-side. (C) Exercise 1 (M) Example 2: Use as a guided example. (M) Example 3: Do $5(x+3)$ first with explicit directions; then guide students with $5(8x+3)$, as needed. (M) Exercise 2(a), 2(c) (C) Exercise 2(b), 2(d) (M) Example 4: Use as a guided example. (M) Exercise 3(a) (C) Exercise 3(b)–(c) (E) Example 5 (E) Exercise 4 Omit Example 6.	Talk to me about your choice to replace the Opening Exercise. In Example 3, I see that you added the problem $5(x+3)$, which is quite similar to what's already there. Why not just do the Example as is? Tell me about your choice to make some parts of the exercises Must Do's and others Could Do's. Why omit Example 6?
Closing	Describe to an elbow partner how we used the area model to write products as sums. Sentence starter: We used the area model to _______.	The lesson has two prompts in the Closing and no sentence starter. Talk to me about the adjustments you made to the Closing.
Formative Assessment	Use whiteboards: Draw an area model to find an equivalent expression for $2(3x+4)$. Draw an area model to find an equivalent expression for $3(4x-1)$.	Help me understand why you didn't use the Exit Ticket from the lesson.
Problem Set	(M) 1, 3(a), 4(a)–(c), 4(g), 4(j)	The Problem Set has other problems. How did you decide which to assign?
Summative Assessment	Topic Quiz should focus on $a(b+c)$ type problems.	
Lesson Notes	Need to pull Grade 4 Module 3 Lesson 11 Problem Set. Could Do exercises listed in Classwork are similar to Must Do exercises and could be assigned to early finishers or used as second chance after additional instruction.	

Grade 7 Module 3 Lesson 3 Teacher B

Lesson Type (circle one): [✓ **Problem Set**] **Exploration** **Socratic** **Modeling Cycle**

Context: Most students began using *Eureka Math* in primary grades, meaning they are proficient in their use of the area model. Analysis of short skill inventory assessment administered before starting the module, showed that many students should be capable of completing some of the simpler Examples and Exercises independently.

Student Outcome(s)	▪ Students use an area model and distributive property to write products as sums and sums as products. ▪ Students recognize that rewriting an expression in a different form can shed light on the problem and how the quantities in it are related.	
Materials	Personal whiteboard materials	
	Customizations	**Rationale**
Fluency Activity (as needed)	N/A	
Classwork **Customization Key** ▪ **Must Do problem (M)** ▪ **Could Do problem (C)** ▪ **Extension problem (E)**	Omit Opening Exercise and instead use Example 2(a)–(d) as Opening with pair work; then debrief the equivalent expressions shown in the models. Omit Exercise 1.	In previous modules, students have demonstrated proficiency with using tape diagrams and they are also proficient with the area model. Omitting the original Opening Exercise and Exercise 1 frees up 8 minutes from the lesson, which provides time for students to explore, in pairs, the models in Example 2.
	(C) Example 1: Start with $x + 2$ and then go to $3(x + 2)$. Ask, how is the work in this Example similar to what we did in the Opening Exercise?	I can use Example 1 to continue the discussion about equivalent expressions and prepare students for the work of Example 3.
	(M) Example 3: Students work independently with whiteboards. Can use similar problems $4(x + 3)$, $4(5x + 3)$, $5(9x + 2)$ as needed. (C) Exercise 2(a)–(d): for additional practice	I can check for understanding as students do Example 3 independently then provide a few similar problems as needed for students who may struggle or need additional instruction. I can assign Exercise 2(a)–(d) to students who do not require additional instruction, while I work with students who do.
	(M) Example 4: Use as guided example. (M) Exercise 3(a)–(b) (C) Exercise 3(c)	Example 4 is more complex because students must rewrite the division problem as multiplication. I will guide them through that part and then allow them to find the equivalent expression by using the area model independently. Students will practice with Exercise 3(a) and (b). I will assign Part (c) if needed.
	(C) Example 5: Challenge students to complete independently (C) Exercise 4	I can differentiate for my early finishers or those students who need a challenge by having them do Example 5 and Exercise 4 while others practice with Exercise 3(a) and (b).
	(M) Example 6: Use as guided example.	Example 6 is the most challenging of the lesson and, because my students are familiar with the model, this problem will advance their understanding of product form and sum form. We will use the extra time gained from omitting earlier parts of the lesson here.

Closing	▪ What are some of the methods we used to write products as sums? ▪ In terms of equivalent expressions, what does the product form represent and what does the sum form represent? product form: $4(s+1)$ sum form: $4s+4$	My students have difficulty starting a discussion without an example so I will display the product form and sum form we developed from Example 6 to get them started.
Formative Assessment	Add $\frac{1}{5}(10x+20)$ to existing Exit Ticket.	The Exit Ticket is similar to Example 6 and uses only whole numbers. I want to gather data regarding my students' ability to work with fractions also.
Problem Set	(M) 3, 4 (C) 1 (E) 2, 5	All students should be able to complete Problem Sets 3 and 4 successfully. I will assign Problem Set 1 to students who may have struggled and Problem Sets 2 and 5 to students who need a challenge.
Summative Assessment	If not assigned, use Problem Set 2 on a quiz or as partner work for the Opening Exercise of Lesson 4.	
Lesson Notes		

Grade 7 Module 3 Lesson 3 Administrator B

Lesson Type (circle one):	[✓ **Problem Set**] **Exploration** **Socratic** **Modeling Cycle**	
Student Outcome(s)	▪ Students use an area model and distributive property to write products as sums and sums as products. ▪ Students recognize that rewriting an expression in a different form can shed light on the problem and how the quantities in it are related.	
Materials	Personal whiteboard materials	
	Customizations	**Questions**
Fluency Activity (as needed)	N/A	
Classwork **Customization Key** ▪ **Must Do problem (M)** ▪ **Could Do problem (C)** ▪ **Extension problem (E)**	Omit Opening Exercise and instead use Example 2(a)–(d) as Opening with pair work; then debrief the equivalent expressions shown in the models. Omit Exercise 1.	I noticed you omitted the Opening Exercise and replaced it with an Example. What informed your choice for that customization?
	(C) Example 1: Start with $x + 2$ and then go to $3(x + 2)$. Ask, how is the work in this Example similar to what we did in the Opening Exercise?	Tell me about your choice to make Example 1(a) a Could Do problem.
	(M) Example 3: Students work independently with whiteboards. Can use similar problems $4(x + 3)$, $4(5x + 3)$, $5(9x + 2)$ as needed.	Talk to me about having students do Example 3 independently.
	(C) Exercise 2(a)–(d): for additional practice	Why did you include similar problems and then note Exercise 2(a)–(d) as a Could Do?
	(M) Example 4: Use as guided example. (M) Exercise 3(a)–(b) (C) Exercise 3(c)	If students can do Example 3 independently, shouldn't they also be able to do Example 4 independently? How will you know whether they can perform the task if it's guided?
	(C) Example 5: Challenge students to complete independently (C) Exercise 4 (M) Example 6: Use as guided example.	Why choose to make Example 5 a Could Do and Example 6 a Must Do?
Closing	▪ What are some of the methods we used to write products as sums? ▪ In terms of equivalent expressions, what does the product form represent and what does the sum form represent? product form: $4(s + 1)$ sum form: $4s + 4$	I don't see product form and sum form examples in the Closing. Why did you include them?
Formative Assessment	Add $\frac{1}{5}(10x + 20)$ to existing Exit Ticket.	I'm surprised to see that you've added something to the Exit Ticket. Why was that necessary?
Problem Set	(M) 3, 4 (C) 1 (E) 2, 5	Tell me about your choice to identify the tasks in the Problem Set this way.
Summative Assessment	If not assigned, use Problem Set 2 on a quiz or as partner work for the Opening Exercise of Lesson 4.	
Lesson Notes		