

## Solving Word Problems: Building Algebraic Fluency through Tape Diagrams

# A Story of Ratios<sup>®</sup> A Story of Functions<sup>®</sup>

Sample Work



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#### **Opening Examples**

**Directions:** Complete the two problems below by drawing a model and using algebra. Try to keep your work in the left-hand column as the facilitator will also model these problems.

1. At a country concert, the ratio of the number of boys to the number of girls is 2 : 7. If there are 250 more girls than boys, how many boys are at the concert?



2. Find three consecutive integers such that their sum is 1,623.





#### Systematic Modeling of Tape Diagrams





**Building Algebraic Fluency through Tape Diagrams** 







Group 2

EUREKA MATH





Building Algebraic Fluency through Tape Diagrams





**Building Algebraic Fluency through Tape Diagrams** 

#### **Thinking Flexibly with Models**

EUREKA MATH



$$f + 154 + f = 180$$
  
 $|80 - 154 = 216$   
 $20nits = 216$   
 $10nit = \frac{216}{2} = 13$   
 $f = 13$ 

Directions: Model each equation.





**Directions:** Practice solving these expressions and equations using tape diagrams

#### **Tape Diagram Practice**



EUREKASample WorkMATHBuilding Algebraic Fluency through Tape Diagrams

#### **Transition to Word Problems**

**Directions:** Create models for the four word problems presented below. Use these models to help you complete the matching activity.

1. Molly has 9 cups of flour. If this is  $\frac{3}{4}$  of the amount she needs to make bread, how many cups does she need?



3units = 9 1unit =  $\frac{9}{3}$  = 3 X=4units = 4(3) = 12 X= 12

- Molly needs a total of 12 cups of flour.
- 2. Peter's Pants Palace advertises the following sale: Pants are  $\frac{1}{3}$  off their original price. If a pair of pants costs \$33 originally, what is the sale price?



3units = 33 1unit = <u>33</u> = 11 X= 2units = 2(11)=22 X=22 The sale price is  $\frac{422}{22}$ 

Units = 36

1 unit = 34 = 9

3. The sum of the ages of two brothers is 46. The younger brother is 10 years older than a third of the older brother's age. How old is the younger brother? U|q-10=3|q



The yanger brother is 19 9+10=19
4. Gia had 25 songs in a playlist composed of songs from her two favorite artists, Beyonce and Jennifer Lopez. Jennifer Lopez has 4 more songs than 1 sixth of the number of Beyonce songs. How many songs did she have by each artist in the playlist?





25-4=21  
70 nits = 21  
10 nit = 
$$\frac{21}{7}$$
 = 3  
Beyonce = 60 nits = 6(3)=18  
Jennifer = 10 nit + 4 = 3+4=7



#### **Coherent Strand of Word Problems**

**Grade 7:** Henry is using a total of 16 ft. of lumber to make a bookcase. The left and right sides are each 4 ft. high. The top, bottom, and two shelves are all the same length. How long is each shelf?



**Grade 8:** Given a right triangle, find the degree measures of the angles if one angle is ten degrees more than four times the degree measure of the other angle and the third angle is the right angle.

Angle 1  
Angle 2  
Angle 2  
Angle 3  
Angle 4  
Angle 3  

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23 hours as a cashier.



## **STATION ONE**

Josie took a long multiple-choice, end-of-year vocabulary test. The ratio of the number of problems Josie got incorrect to the number of problems she got correct is 2 : 9. If Josie missed 8 questions, how many did she get correct? (G6 M1 L3)



Josie got 36 questions correct.

## **STATION TWO**

Kurt has driven 276 miles of his road trip but has 70% of the trip left to go. How many more miles does Kurt have to drive to get to his destination? (G7 M4 L5)



30% = 276  $10\% = \frac{276}{3} = 92$  $70\% = 92 \cdot 7 = 644$ 

Kurt has 644 miles to get to the destination.



## **STATION THREE**

Find two consecutive even numbers such that the sum of the smaller number and twice the greater number is 100. (G9 M1 L25)



The even integers are 32 and 34.

### **STATION FOUR**

A far Let f represent the number of five-dollar bills. 18 bills Let d represent the number of one-dollar bills.

lollar bills in her wallet. Altogether she has >> she have? (G8 M4 L29)



There are 7 one-dollar bills and 11 five-dollar bills.



#### **Grade Level Bank of Problems**

Directions: Review and highlight the problems that may support your students' mathematical thinking.

Grade 6

1. Pam and her brother both open savings accounts. Each begins with a balance of zero dollars. For every two dollars that Pam saves in her account, her brother saves five dollars in his account. If Pam has 40 dollars in her account, how much money does her brother have in his account? (G6 M1 L3)



Pam's brother has \$100 in his account.

2. Last summer, at Camp Okey-Fun-Okey, the ratio of the number of boy campers to the number of girl campers was 8: 7. If there were a total of 195 campers, how many boy campers were there? How many girl campers? (G6 M1 L5)



There were 104 boy campers and 91 girl campers.



3. The Superintendent of Highways is further interested in the numbers of commercial vehicles that frequently use the county's highways. He obtains information from the Department of Motor Vehicles for the month of September and finds that for every 14 noncommercial vehicles, there were 5 commercial vehicles. If there were 108 more noncommercial vehicles than commercial vehicles, how many of each type of vehicle frequently use the county's highways during the month of September? (G6 M1 L5)



There were 60 commercial vehicles and 168 non-commercial vehicles.

4. The Business Direct Hotel caters to people who travel for different types of business trips. On Saturday night, the ratio of the number of occupied rooms to the number of unoccupied rooms is 2 : 5. However, on Sunday night the ratio of the number of occupied rooms to the number of unoccupied rooms is 6 : 1. If the hotel has 432 occupied rooms on Sunday night, how many unoccupied rooms does it have on Saturday night? (derived from G6 M1 L6)



There were 360 unoccupied rooms on Saturday night.



5. At the beginning of Grade 6, the ratio of the number of advanced math students to the number of regular math students was 3 : 8. However, after taking placement tests, students were moved around changing the ratio of the number of advanced math students to the number of regular math students to 4 : 7. How many students started in regular math and advanced math if there were 92 students in advanced math after the placement tests? (G6 M1 L6)



There were 69 students in Advanced Math and 184 students in Regular Math before the placement test.

6. Maria has  $\frac{3}{4}$  lb. of trail mix. She needs to share it equally among 6 friends. How much will each friend be given? (G6 M2 L1)



Each friend gets  $\frac{1}{8}$  lb of trail mix.



7. The Lopez family adopted 6 miles of trail on the Erie Canal. If each family member can clean up  $\frac{3}{4}$  of a mile, how many family members are needed to clean the adopted section? (G6 M2 L2)



8 family members are needed.

8. Margo is freezing 8 cups of strawberries. If this is  $\frac{2}{3}$  of the total strawberries that were picked, how many cups of strawberries did Margo pick? (G6 M2 L2)



<sup>12</sup> cups of strawberries were picked.

9. Xavier, Molly's friend, purchased  $\frac{11}{8}$  cups of strawberries. If he eats  $\frac{3}{4}$  of a cup of strawberries per serving, how many servings will he have? (G6 M2 L5)



- 10. Tina uses  $\frac{1}{8}$  oz. of cinnamon each time she makes a batch of coffee cake topping. How many batches
- can she make if she has  $\frac{1}{2}$  oz. left in her spice jar? (G6 M2 L5)



She can make 4 batches of topping.



11. Zolanda spent  $\frac{5}{8}$  of her class period taking notes. If Zolanda spent 25 minutes taking notes, how long is her class period? (G6 M2 L6)





12. Juan has gained 20 lb. since last year. He now weighs 120 lb. Rashod is 15 lb. heavier than Diego. Rashod weighs the same amount that Juan weighed last year. How much does Diego weigh? Let *j* represent Juan's weight last year in pounds, and let *d* represent Diego's weight in pounds. (G6 M4 L28)





13. Marisa has twice as much money as Frank. Christina has \$20 more than Marisa. If Christina has \$100, how much money does Frank have? Let *f* represent the amount of money Frank has in dollars and *m* represent the amount of money Marisa has in dollars. (G6 M4 L28)







14. Raeana is twice as old as Madeline, and Laura is 10 years older than Raeana. If Laura is 50 years old, how old is Madeline? Let *m* represent Madeline's age in years, and let *r* represent Raeana's age in years. (G6 M4 L28)



Madeline is 20 years old.

15. The measures of two angles have a sum of 180 degrees. The measures of the angles are in a ratio of 5 : 1. Determine the measures of both angles. (G6 M4 L30)



The angles measure  $30^{\circ}$  and  $150^{\circ}$ .

#### Grade 7

1. A hand-held digital music player was marked down by  $\frac{1}{4}$  of the original price. If the sales price is \$128.00, what is the original price? (G7 M1 L14)



The original price is about \$170.67.



2. Joanna ran a mile in physical education class. After resting for one hour, her heart rate was 60 beats per minute. If her heart rate decreased by  $\frac{2}{5}$ , what was her heart rate immediately after she ran the mile? (G7 M1 L14)



Joanna's original heart beat was 100 beats per minute.

3. A motorcycle dealer paid a certain price for a motorcycle and marked it up by  $\frac{1}{5}$  of the price he paid. Later he sold it for \$14,000. What was the original price? (G7 M1 L14)



The original price of the motorcycle was about \$11,666.67.

4. Eric's father works two part-time jobs, one in the morning and one in the afternoon, and works a total of 40 hours each five-day workweek. If his schedule is the same each day, and he works 3 hours each morning, how many hours does Eric's father work each afternoon? (G7 M2 L17)



Eric's father works 5 hours each afternoon.



5. The cost of a babysitting service on a cruise is \$10 for the first hour and \$12 for each additional hour. If the total cost of babysitting baby Aaron was \$58, how many hours was Aaron at the sitter? (G7 M2 L17)



Aaron's parents paid \$12 for four hours, plus the \$10 for the initial hour. Therefore, Aaron was at the sitter for 5 hours.

6. A taxi cab in Myrtle Beach charges \$2 per mile and \$1 for every person. If a taxi cab ride for two people costs \$12, how far did the taxi cab travel? (G7 M2 L17)



7. Jenny is on the local swim team for the summer and has swim practice four days per week. The schedule is the same each day. The team swims in the morning and then again for 2 hours in the evening. If she swims 12 hours per week, how long does she swim each morning? (G7 M2 L22)





8. Charlotte receives a weekly allowance from her parents. She spent half of this week's allowance at the movies but earned an additional \$4 for performing extra chores. If she did not spend any additional money and finished the week with \$12, what is Charlotte's weekly allowance? (G7 M2 L23)



Charlotte's weekly allowance is \$16.



9. The sum of two consecutive even numbers is 54. Find the numbers. (G7 M3 L7)



The two numbers are 26 and 28.

10. Barry's mountain bike weighs 6 pounds more than Andy's. If their bikes weigh 42 pounds altogether, how much does Barry's bike weigh? (G7 M3 L8)



Barry's bike weighs 24 pounds.

11. The measures of three angles at a point are in the ratio of 2 : 3 : 5. Find the measures of the angles. (G7 M3 L11)



The measures of the three angles are  $72^{\circ}$ ,  $108^{\circ}$ , and  $180^{\circ}$ .

12. In Ty's math class, 20% of students got an A on a test. If there were 30 students in the class, how many got an A? (G7 M4 L2)



Six students got an A.



13. Lu's math score on her achievement test in seventh grade was a 650. Her math teacher told her that her test level went up by 25% from her Grade 6 test score level. What was Lu's test score level in Grade 6? (G7 M4 L4)



Lu's test score level was 520 in sixth grade.

14. Erin and Sasha went to a candy shop. Sasha bought 50% more candies than Erin. After Erin bought 8 more candies, Sasha had 20% more. How many candies did Erin and Sasha have at first? (G7 M4 L9)



The number of candies Sasha has does not change. Therefore, she has 48 candies. Each unit in the 'Before' model is double the size of each unit in the 'After' model. Therefore, each unit in the 'Before' model represents 16, which means Erin started with 16 · 2 or 32 candies.

15. Kimberly and Mike have an equal amount of money. After Kimberly spent \$50 and Mike spent \$25, Mike's money was 50% more than Kimberly's. How much did Kimberly and Mike have at first? (G7 M4 L9)





16. In a pair of complementary angles, the measurement of the larger angle is three times that of the smaller angle. Find the measurements of the two angles. (G7 M6 L1)



The measurements of the two angles are  $22.5^{\circ}$  and  $67.5^{\circ}$ .

17. The measure of a supplement of an angle is 6° more than twice the measure of the angle. Find the measurements of the two angles. (G7 M6 L1)



The measurements of the two angles are  $58^{\circ}$  and  $122^{\circ}$ .

18. Three adjacent angles are at a point. The second angle is 20° more than the first, and the third is 20° more than the second angle. Find the measurements of all three angles. (G7 M6 L3)



The first angle has a measurement of  $100^{\circ}$ . The second angle has a measurement of  $100^{\circ} + 20^{\circ}$  or  $120^{\circ}$ . The third angle has a measurement of  $100^{\circ} + 20^{\circ} + 20^{\circ}$  or  $140^{\circ}$ .

#### Grade 8

1. The sum of three consecutive integers is 372. What is the smallest integer? (G8 M4 L1)





2. The sum of three consecutive odd integers is 93. What is the largest integer? (G8 M4 L1)



3. Bruce bought two books. One book costs \$4.00 more than three times the other. Together, the two books cost him \$72. How much does each book cost? (Modified, G8 M4 L1)



- Book 1 costs \$17.  $3 \bullet 17 = 51$  and 51 + 4 = 55, so book 2 costs \$55.
- 4. The measure of one angle is described as twelve more than four times a number. Its supplement is twice as large. Find the measure of each angle in degrees. (G8 M4 L5)



48 + 12 = 60, so Angle 1 measures  $60^{\circ}$ . 96 + 12 + 12 = 120, so its supplement measures  $120^{\circ}$ .



5. The sum of the measures of angles x and y is 127°. If the measure of  $\angle x$  is 34° more than half the measure of  $\angle y$ , what is the measure of each angle? (G8 M4 L29)



Angle y measures  $62^{\circ}$ . 31 + 34 = 65, so angle x measures  $65^{\circ}$ .

6. Paulo has a certain amount of money. If he spends \$6.00, then he has  $\frac{1}{4}$  of the original amount left. How much money did he have to begin with? (Modified, G8 M4 L1)





7. Steven has some money. If he spends \$9.00, then he will have  $\frac{3}{5}$  of the amount he started with. How much money did he start with? (Modified, G8 M4 L1)



Steven started with \$22.50.

8. When you add 18 to  $\frac{1}{4}$  of a number, you get the number itself. What is the number? (Modified, G8 M4 L1)



The number is 24.



9. Small boxes contain DVDs, and large boxes contain one gaming machine. Three boxes of gaming machines and a box of DVDs weigh 48 pounds. Three boxes of gaming machines and 5 boxes of DVDs weigh 72 pounds. How much does each box weigh? (G8 M4 L29)



The box with one gaming machine weighs 14 pounds, and the box containing DVDs weighs 6 pounds.

10. The length of a rectangle is 4 times the width. The perimeter of the rectangle is 45 inches. What is the area of the rectangle? (G8 M4 L29)



The width is 4.5 inches, the length is 18 inches. 4.5(18) = 81, so the area is  $81 \text{ in}^2$ .

11. Two numbers have a sum of 1,212 and a difference of 518. What are the two numbers? (G8 M4 L29)



One number is 347. 347 + 518 = 865, so the other number is 865.



12. The sum of two numbers is 361, and the difference between the two numbers is 173. What are the two numbers? (G8 M4 L29)



One number is 94. 94 + 173 = 267, so the other number is 267.

13. Marvin paid an entrance fee of \$5 plus an additional \$1.25 per game at a local arcade. Altogether, he spent \$26.25. Write and solve an equation to determine how many games Marvin played. (G8 M4 L9)



He played 17 games. If we let x represent the number of games, 1.25x + 5 = 26.25 yields 17 for x.

14. Miriam read a book with an unknown number of pages. The first week, she read five less than  $\frac{1}{3}$  of the pages. The second week, she read 171 more pages and finished the book. Write and solve an equation to determine the total number of pages in the book. (Modified, G8 M4 L1)



The total number of pages in the book is 249.

15. There are 356 eighth-grade students at Euclid's Middle School. Thirty-four more than four times the number of girls is equal to half the number of boys. How many boys are in eighth grade at Euclid's Middle School? How many girls? (G8 M4 L29)



There are 32 girls. 288 + 34 + 34 = 324, so there are 324 boys.



**Prompt:** Lulu is holding 20 coins (dimes and quarters). She is holding \$4.10. How many dimes is she holding? How many quarters?

