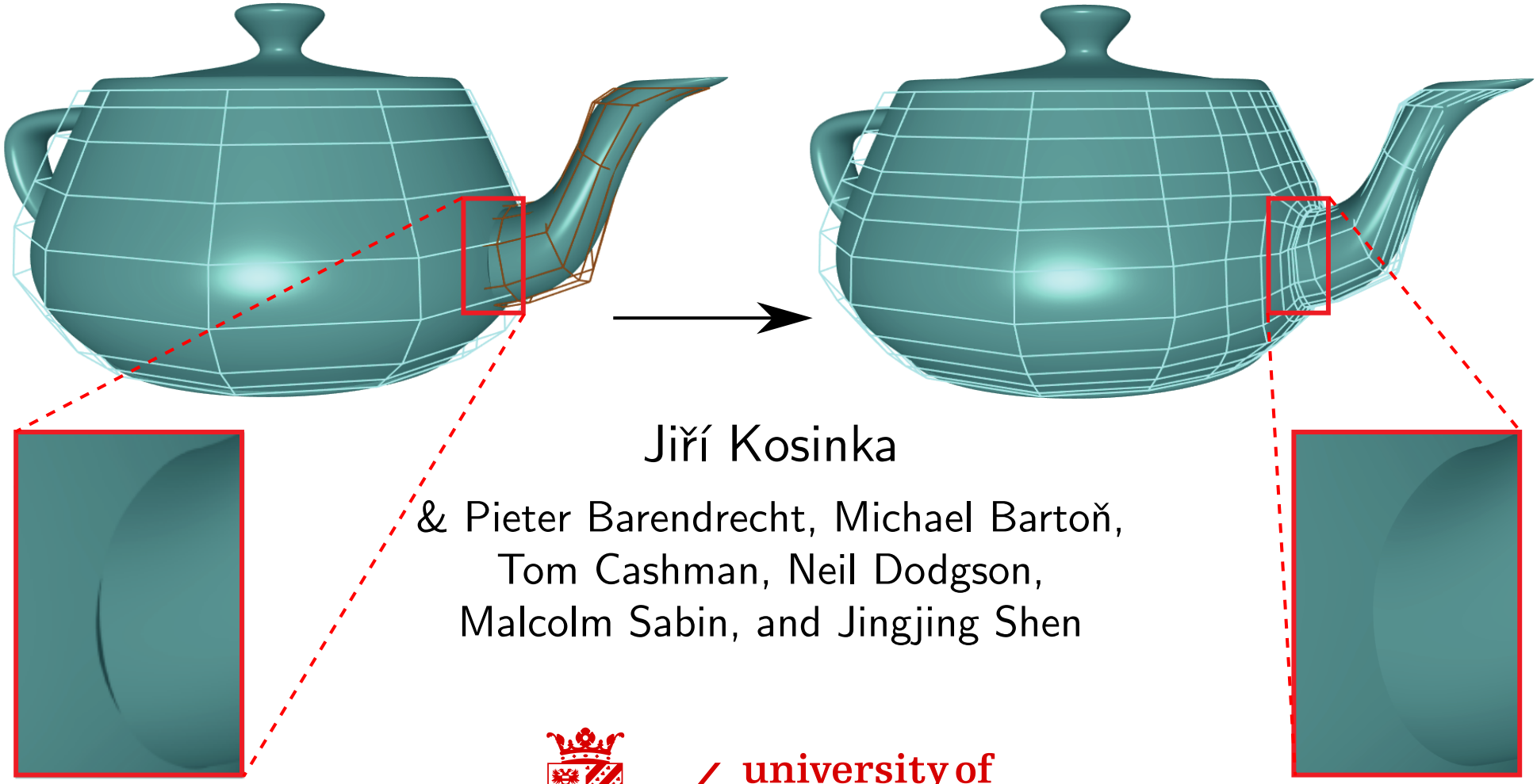
Modelling and IgA



Jiří Kosinka

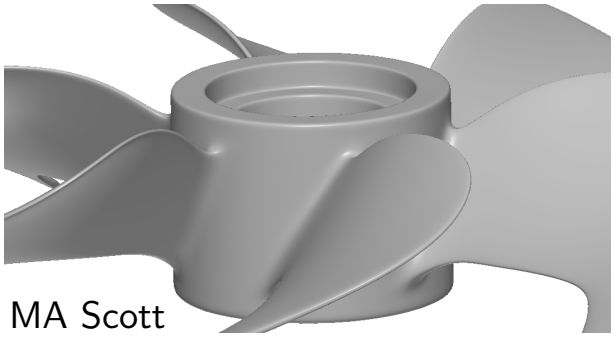
& Pieter Barendrecht, Michael Bartoň,
Tom Cashman, Neil Dodgson,
Malcolm Sabin, and Jingjing Shen

university of groningen

CAE Geometry Workshop
Cambridge, UK; 15 September 2017

Motivation

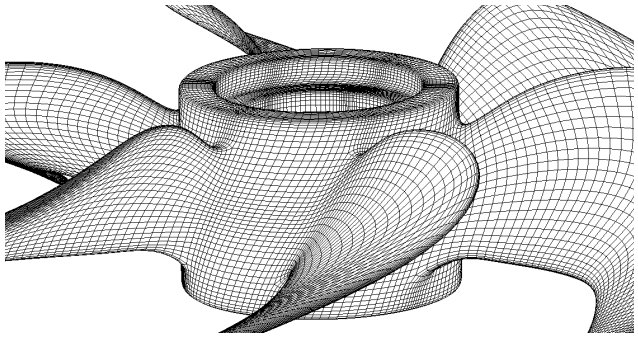
- CAD + FEM



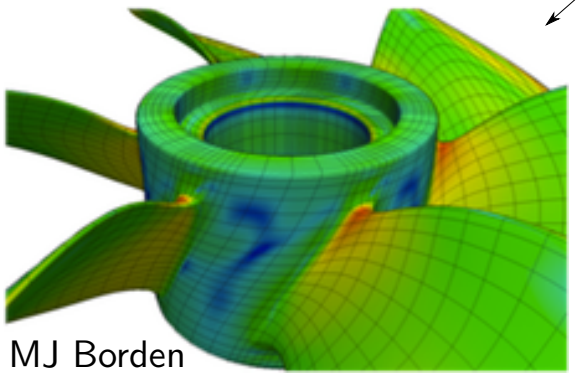
MA Scott

CAD

Mesh



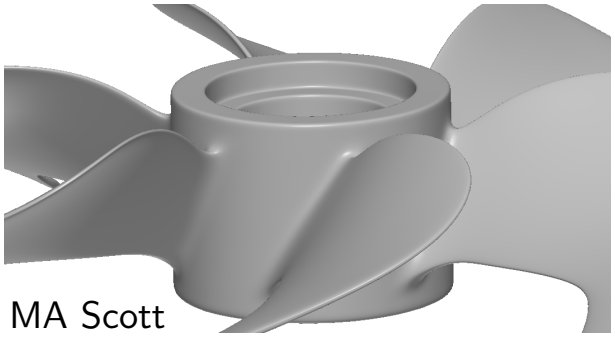
FEM



MJ Borden

Motivation

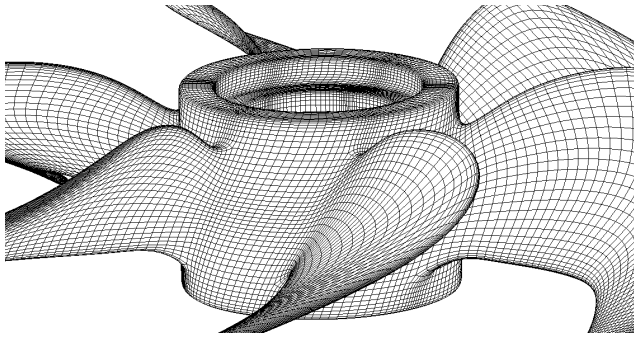
- $\int($ CAD + FEM $) = ?$



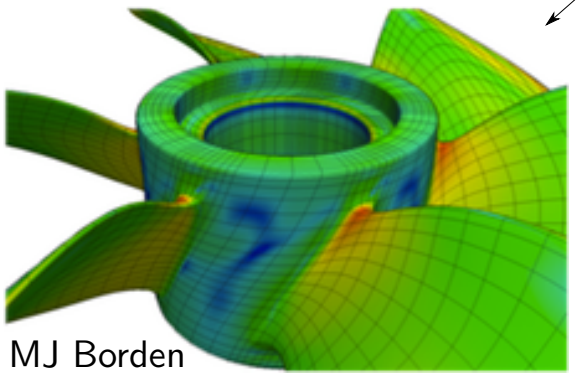
MA Scott

CAD

Mesh



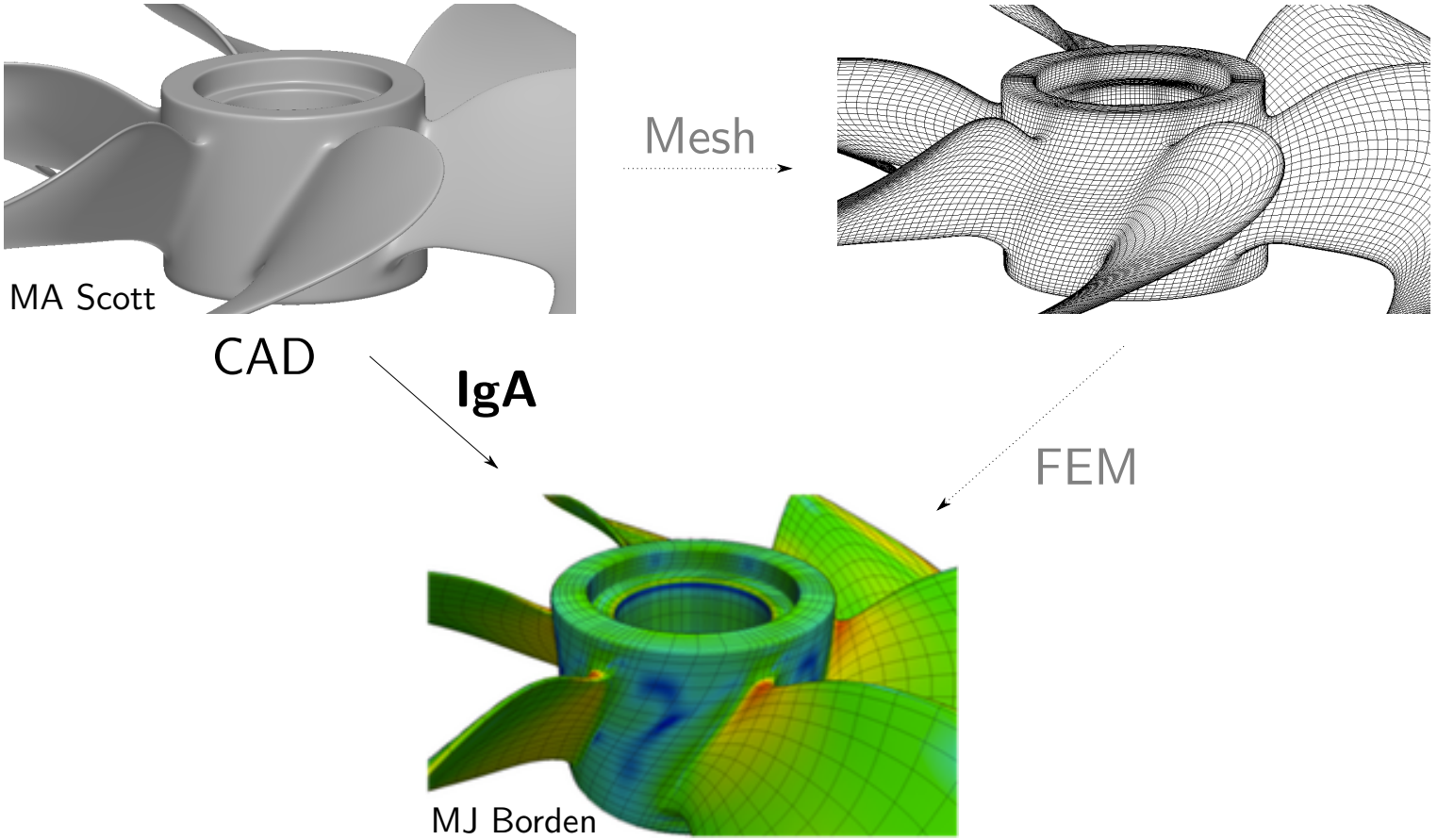
FEM



MJ Borden

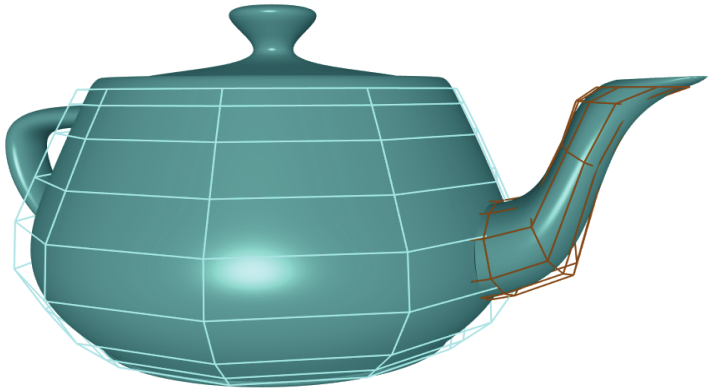
Motivation

- $\int($ CAD + FEM $) =$ **IgA**



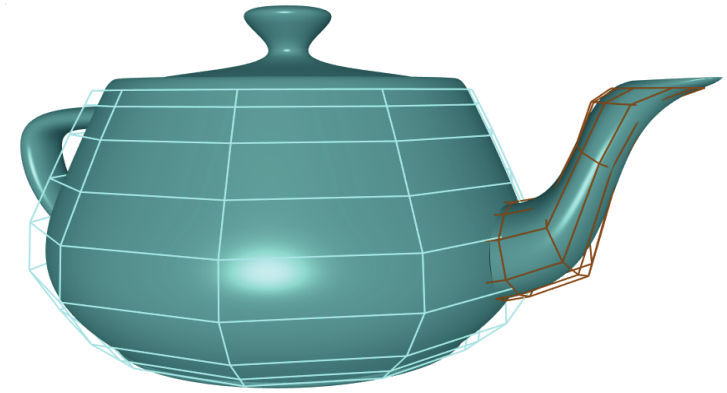
Motivation

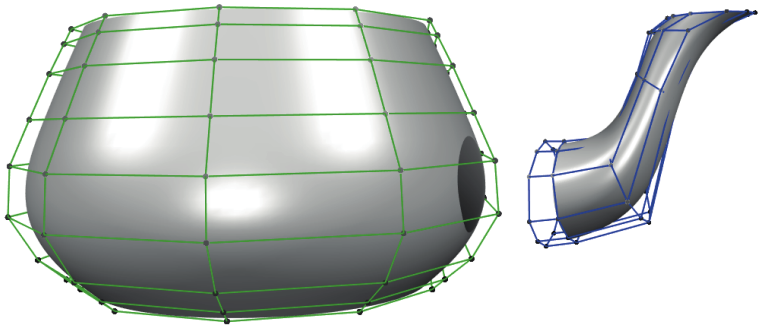
- CAD (= trimmed & stitched NURBS) geometry
-



Motivation

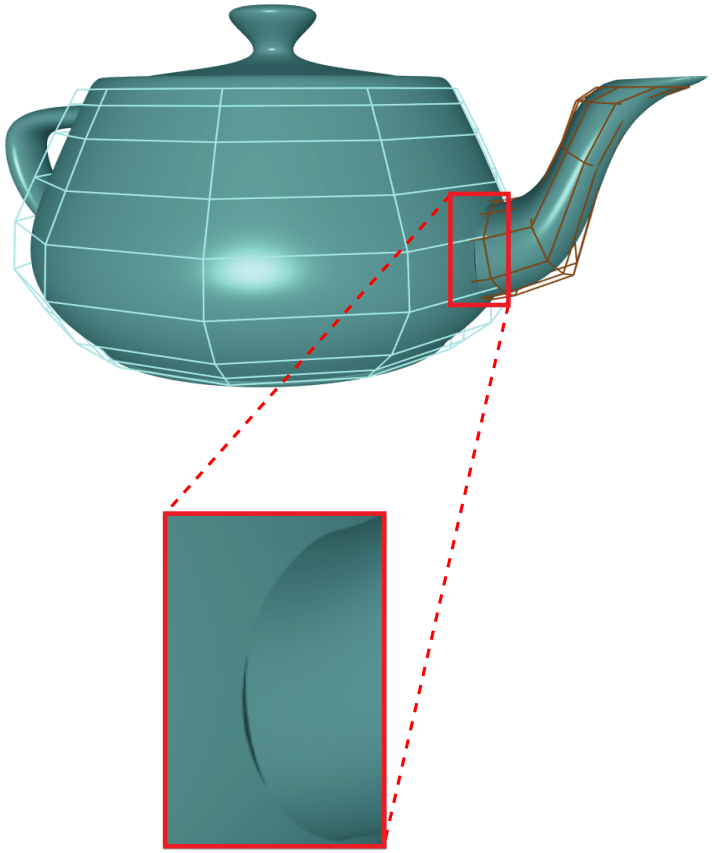
- CAD (= trimmed & stitched NURBS) geometry
-





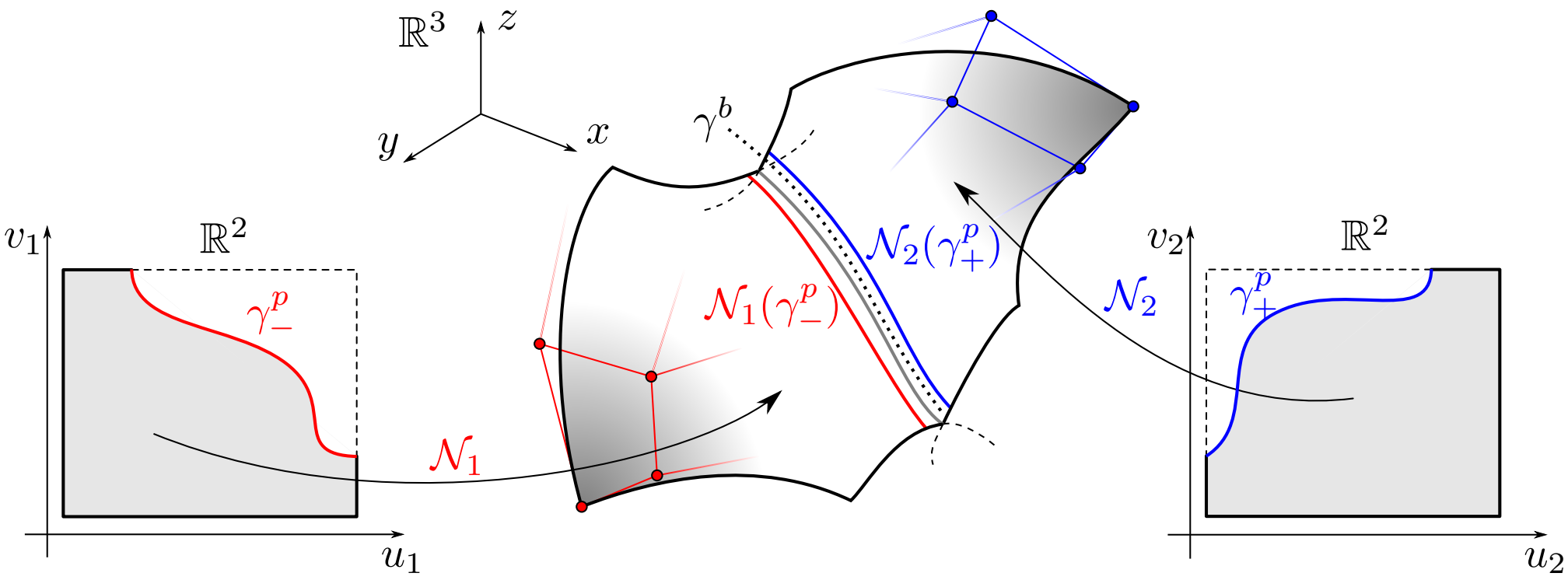
Motivation

- CAD (= trimmed & stitched NURBS) geometry: **gaps**
-



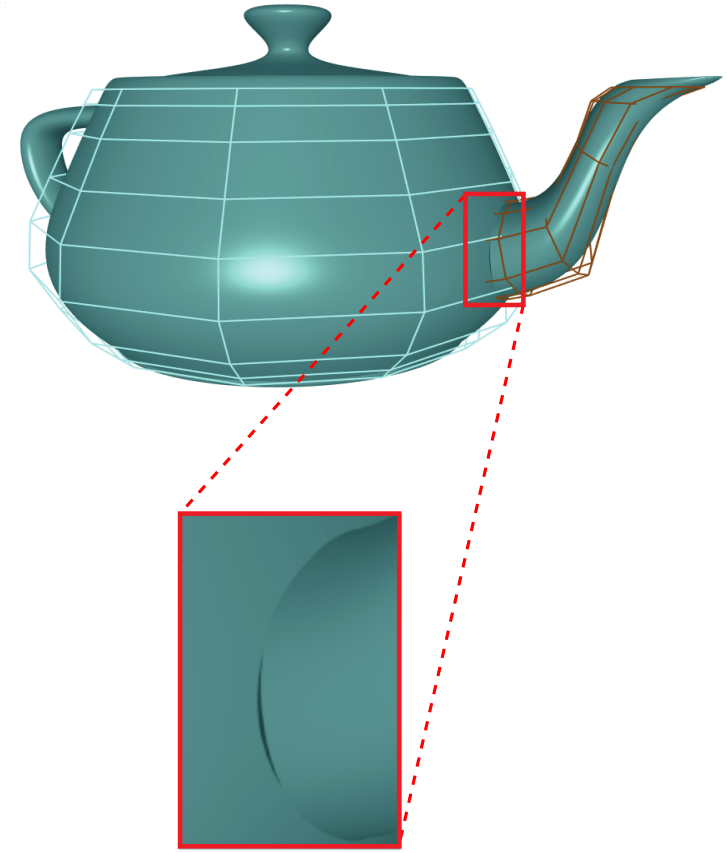
Motivation

- CAD (= trimmed & stitched NURBS) geometry: **gaps**
- Trimming curve management



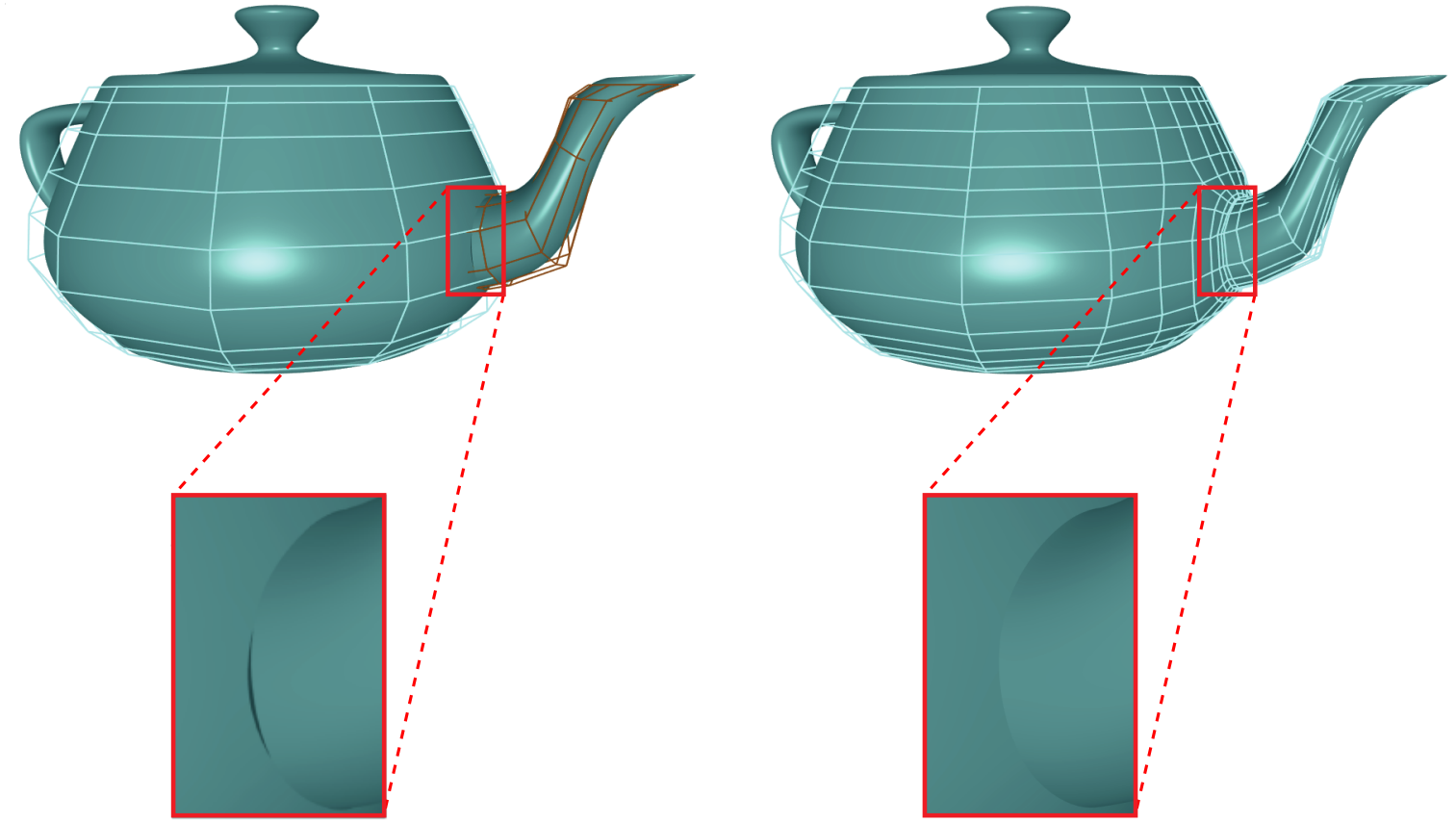
Motivation

- CAD (= trimmed & stitched NURBS) geometry: **gaps**
-



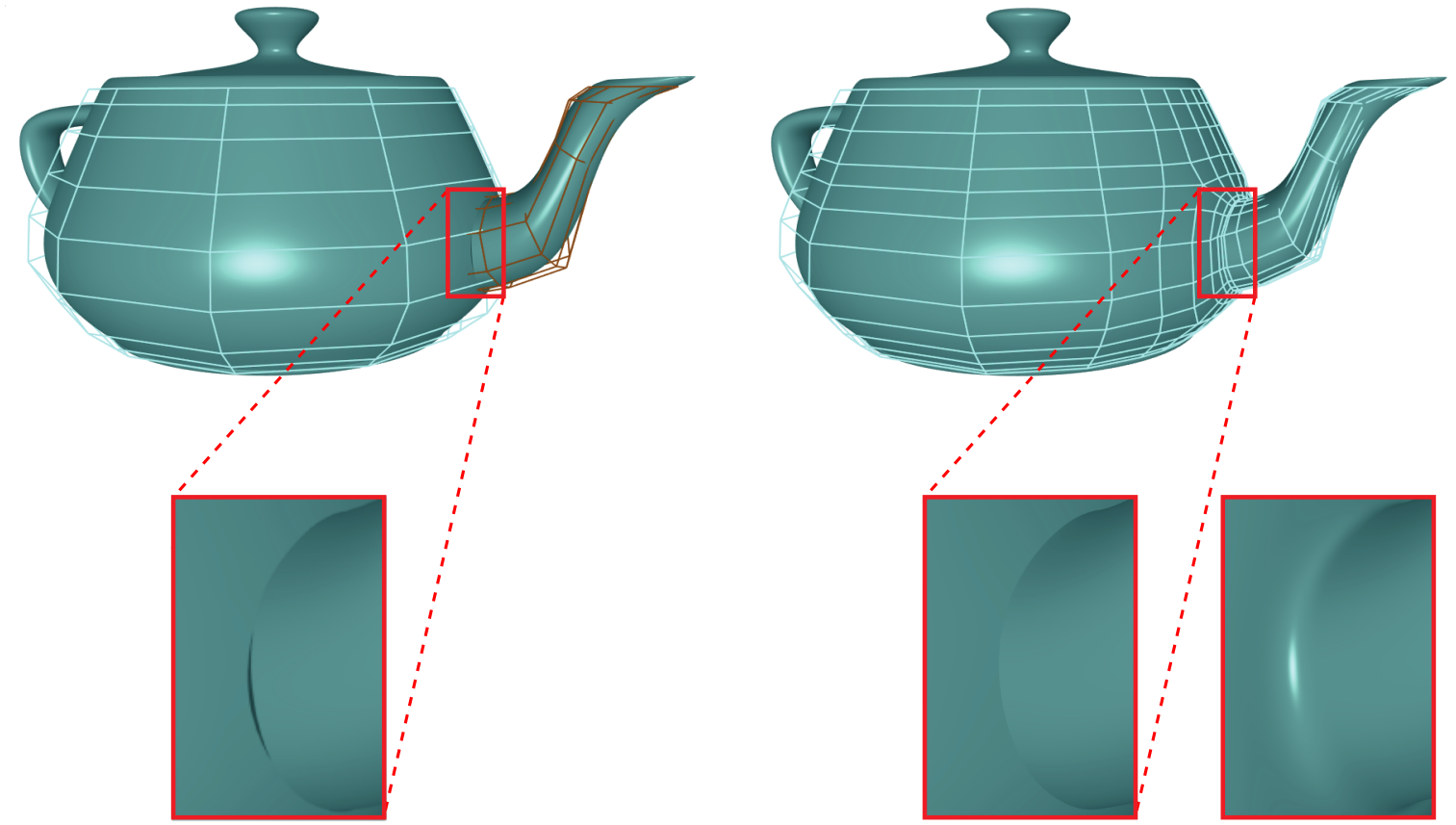
Motivation

- CAD (= trimmed & stitched NURBS) geometry: **gaps**
- $\rightarrow$ convert to **analysis-suitable CAE** geometry: **no gaps**



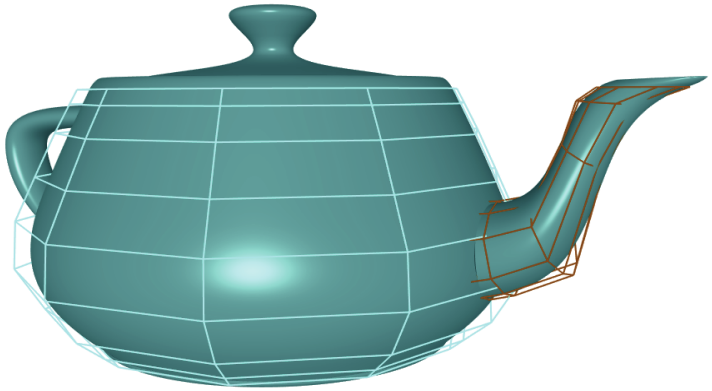
Motivation

- CAD (= trimmed & stitched NURBS) geometry: **gaps**
- $\rightarrow$ convert to **analysis-suitable CAE** geometry: **no gaps** & smooth



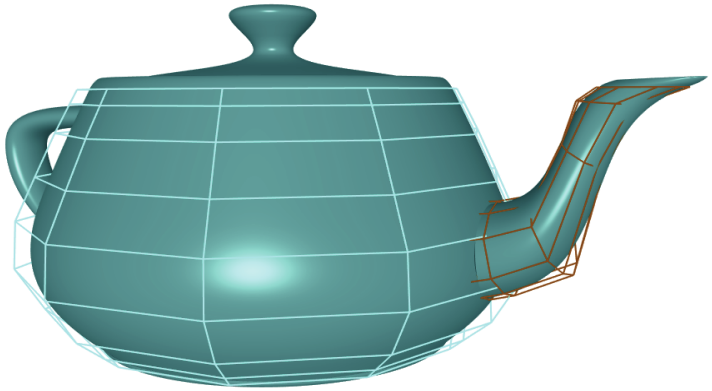
Motivation

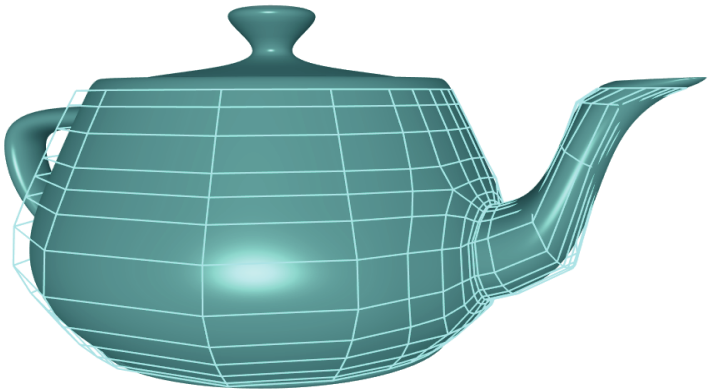
- CAD (= trimmed & stitched NURBS) geometry: **leaks**
- $\rightarrow$ convert to **analysis-suitable CAE** geometry: **no gaps**



Motivation

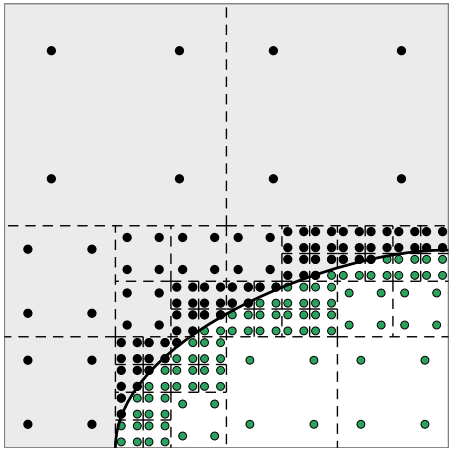
- CAD (= trimmed & stitched NURBS) geometry: **leaks**
- $\rightarrow$ convert to **analysis-suitable CAE** geometry: **watertight**





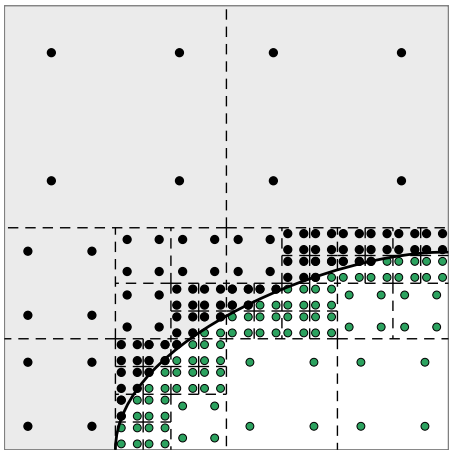
Related Work

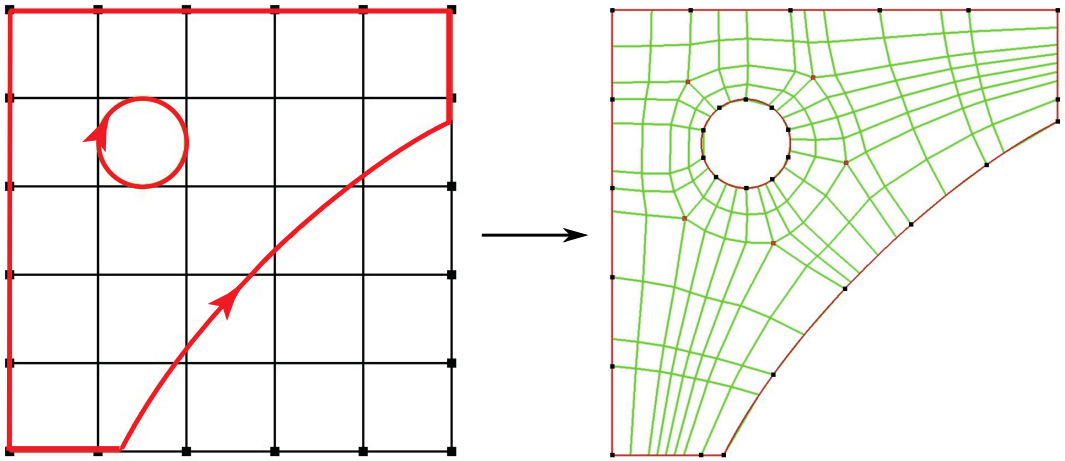
- Marussig & Hughes, *A Review of Trimming in Isogeometric Analysis*, ICES report 17-03, University of Texas, 2017
  - Local approach: analysis perspective; enhance IgA to handle trimmed geometry
  -



Related Work

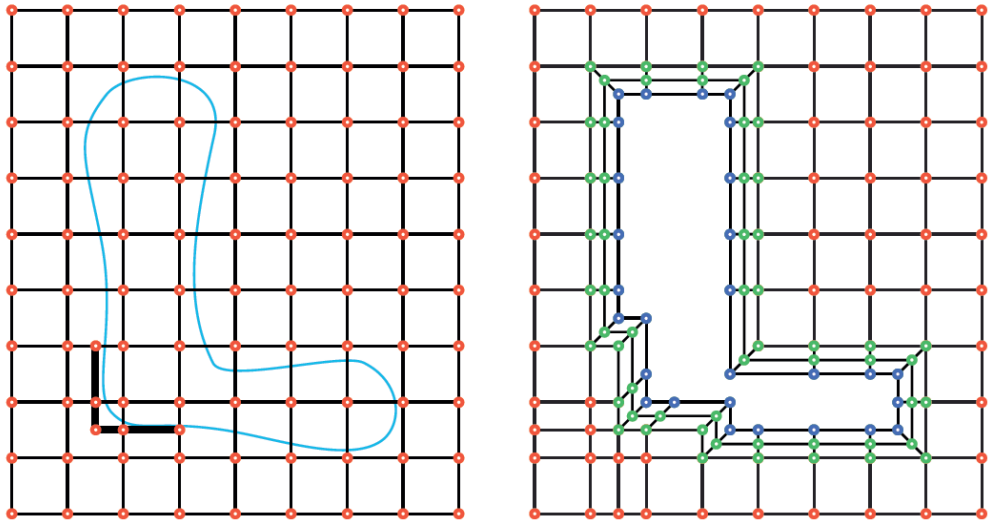
- Marussig & Hughes, *A Review of Trimming in Isogeometric Analysis*, ICES report 17-03, University of Texas, 2017
  - Local approach: analysis perspective; enhance IgA to handle trimmed geometry
  - Global appraoch: design perspective; make geometry **analysis-suitable**





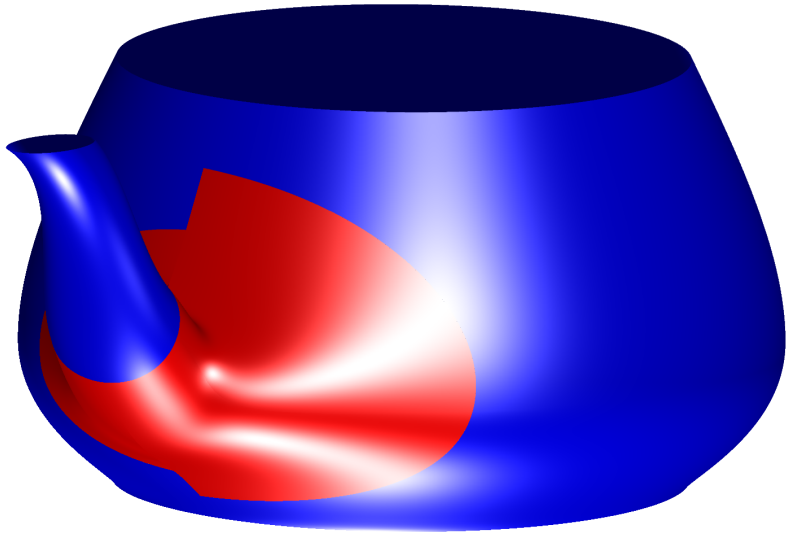
Related Work

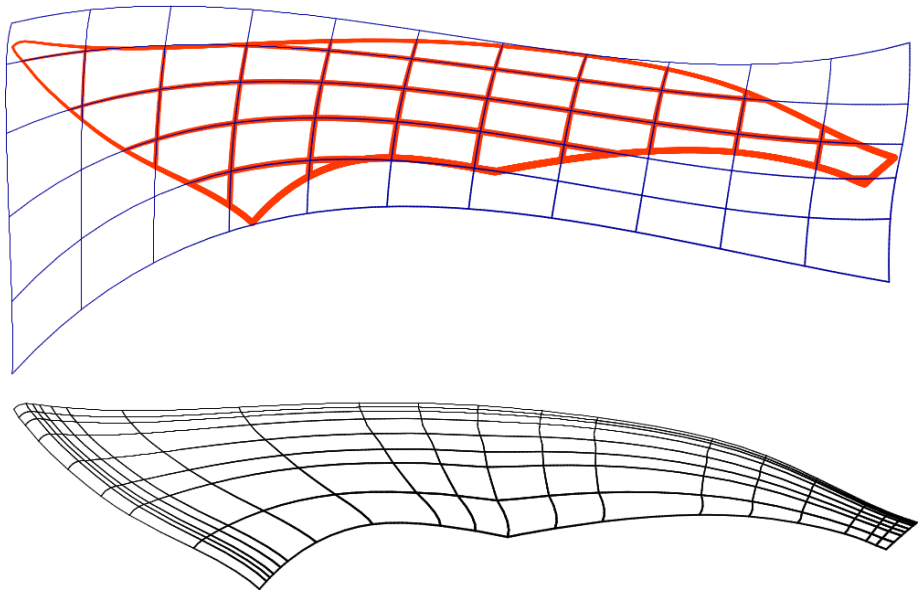
- Convert trimmed NURBS to T-splines (Sederberg et al. 2008)
  - gap-free model, approximation error controllable
  - introduces T-junctions
  - cannot avoid EVs (extraordinary vertices: valency $\neq 4$)
  - EVs (very) close to the boundary



Related Work

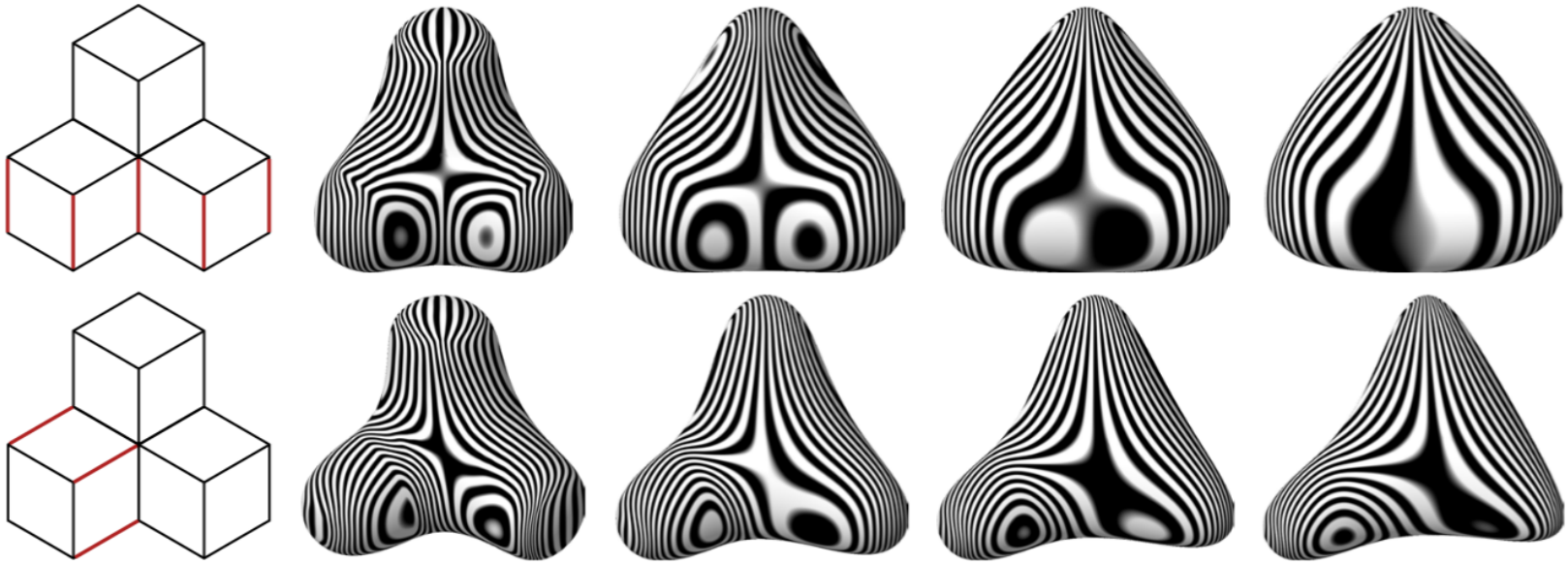
- Untrimming & composition (Elber & Kim 2014)
  - untrim by quad meshing
  - compose with original patches
  - creates high-degree surfaces
  - does not directly address EVs





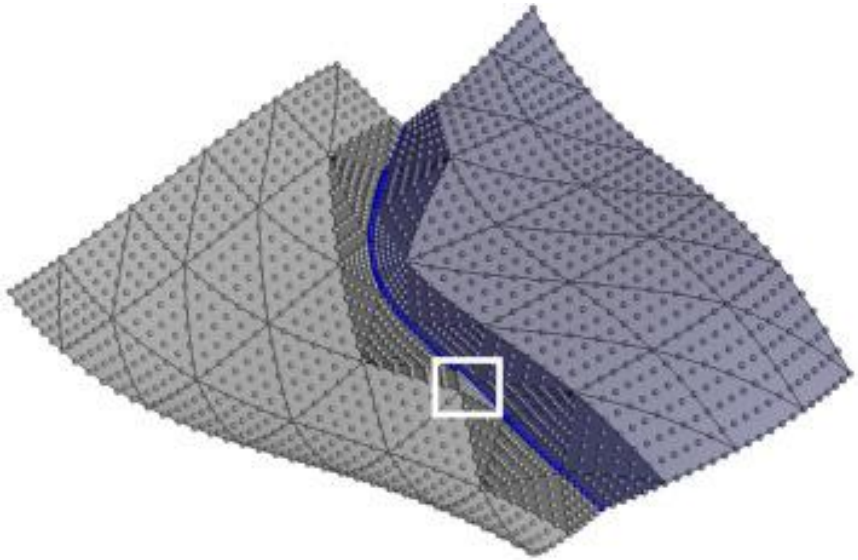
Related Work

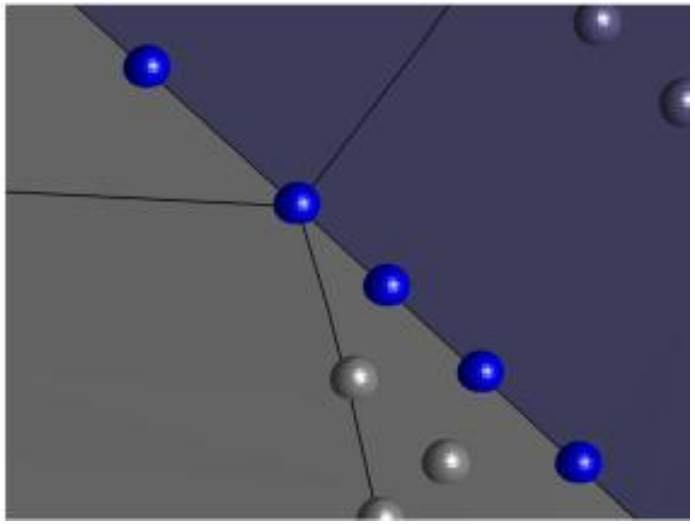
- NURBS-compatible subdivision (Cashman 2010)
  - reconciles (non-uniform) splines and subdivision
  - splines $\subset$ subdivision
  - arbitrary (odd) degrees
  - EVs supported



Related Work

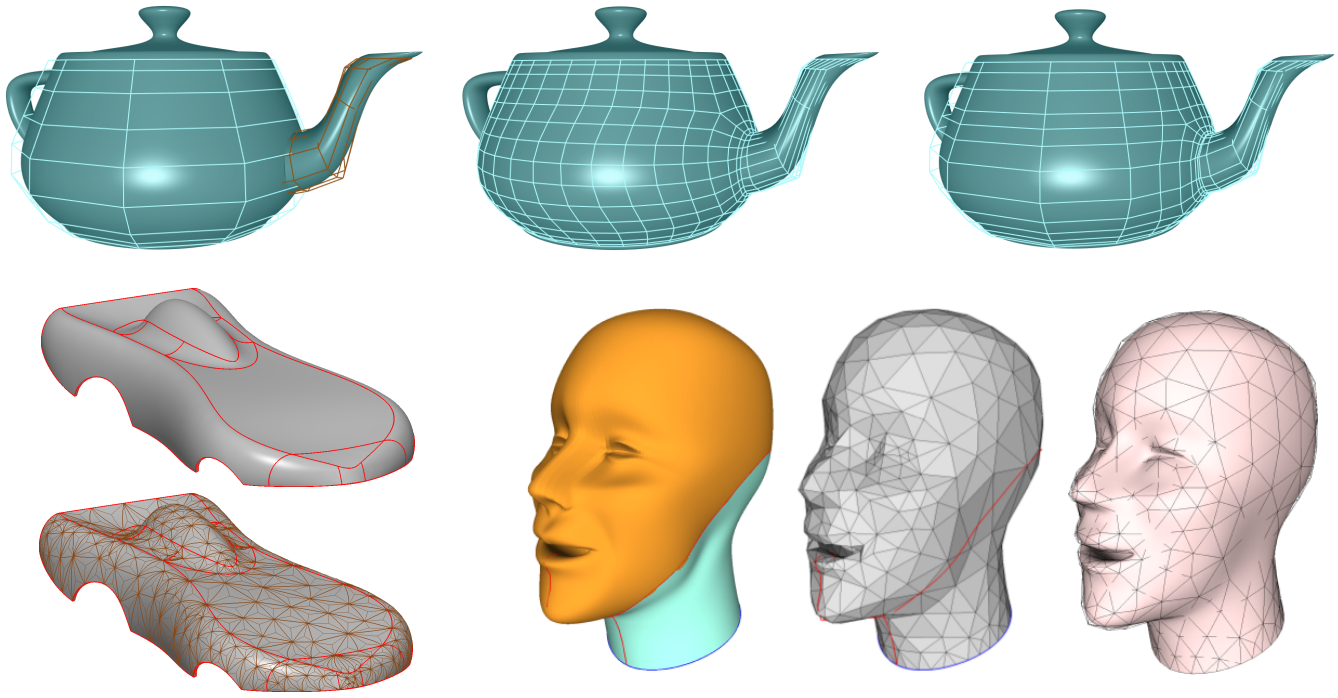
- Trimmed NURBS to Bézier triangles (Xia & Quian 2017)
  - convert a CAD model to a collection of Bézier triangles
  - cover also the volumetric case: Bézier tetrahedra
  - trimming curves become triangle edges





Our Conversion Methods

- Watertight conversion of trimmed CAD models to
  - Catmull-Clark subdivision surfaces (Shen, K, Sabin, Dodgson 2014)
  - non-uniform subdivision surfaces (Shen, K, Sabin, Dodgson 2016)
  - Loop subdivision surfaces (Shen & K 2016)
  - Clough-Tocher splines (K & Cashman 2015)



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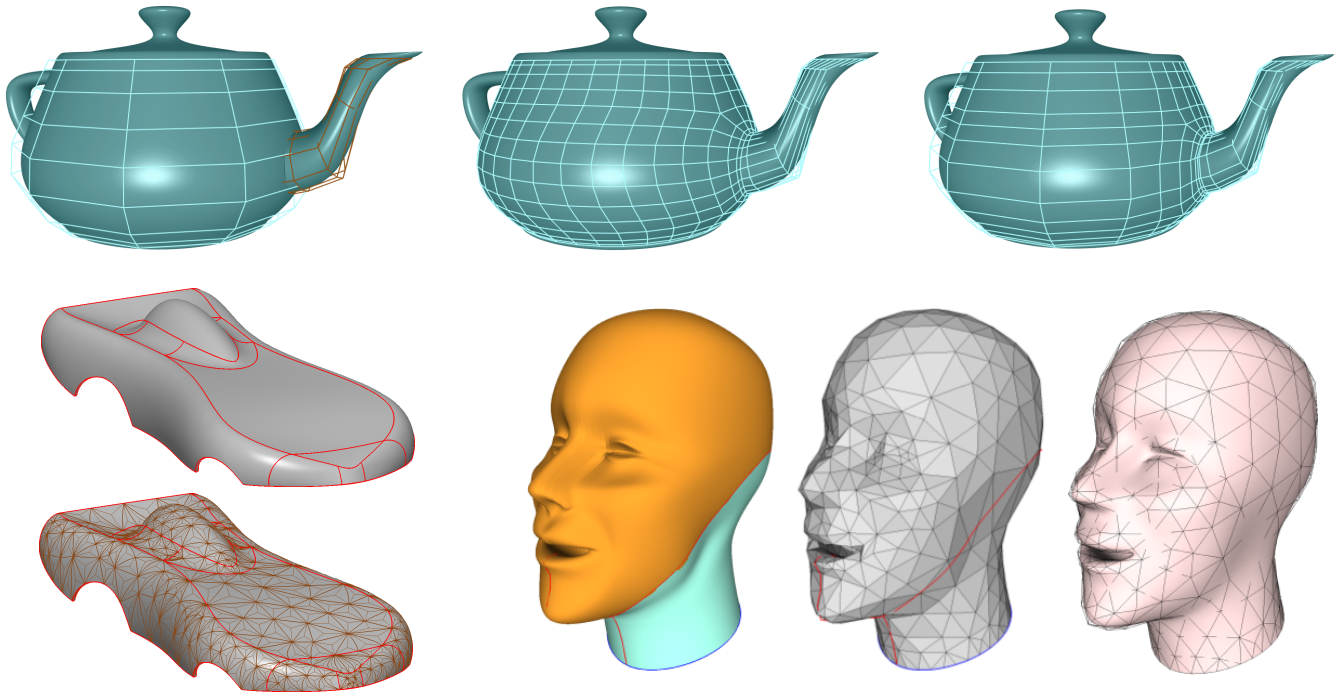
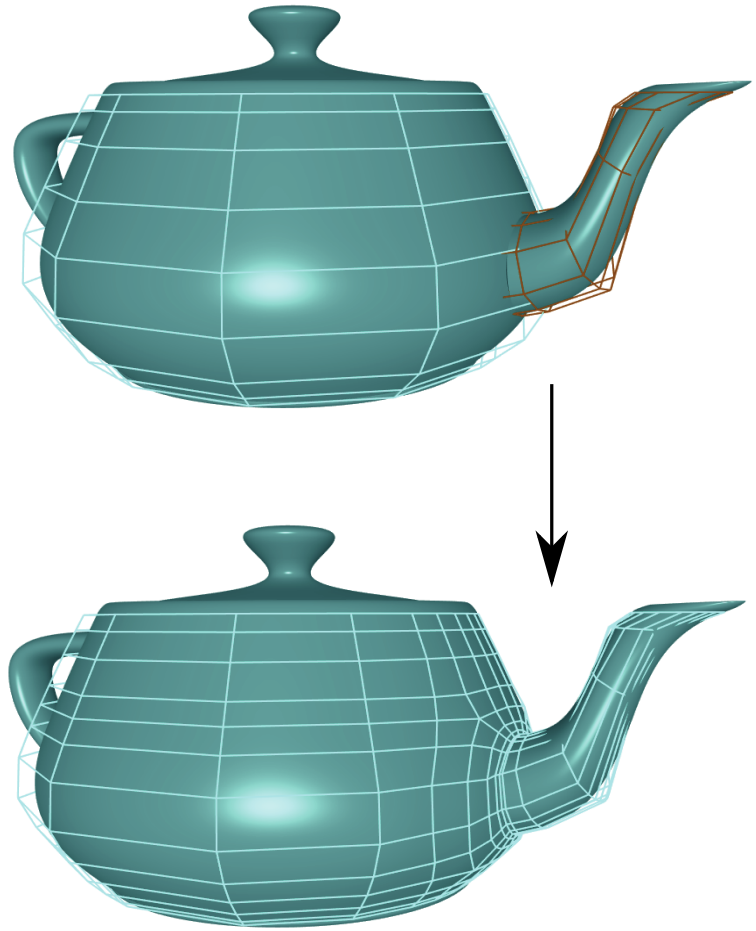
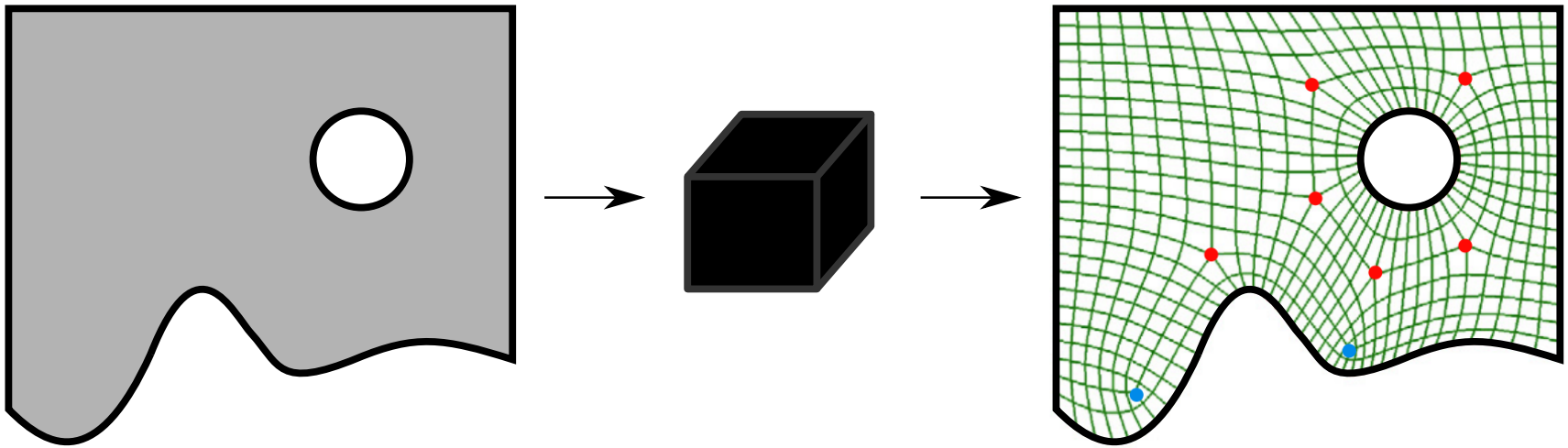


Our Conversion Process: Subdivision

- Goals
  - Good quality control mesh
  - Control over approximation error
  - Robustness
  - Continuity management
- Four stages
  - Quad-meshing
  - Knot-interval assignment
  - Computing control point positions
  - Merging meshes into a single subdivision mesh



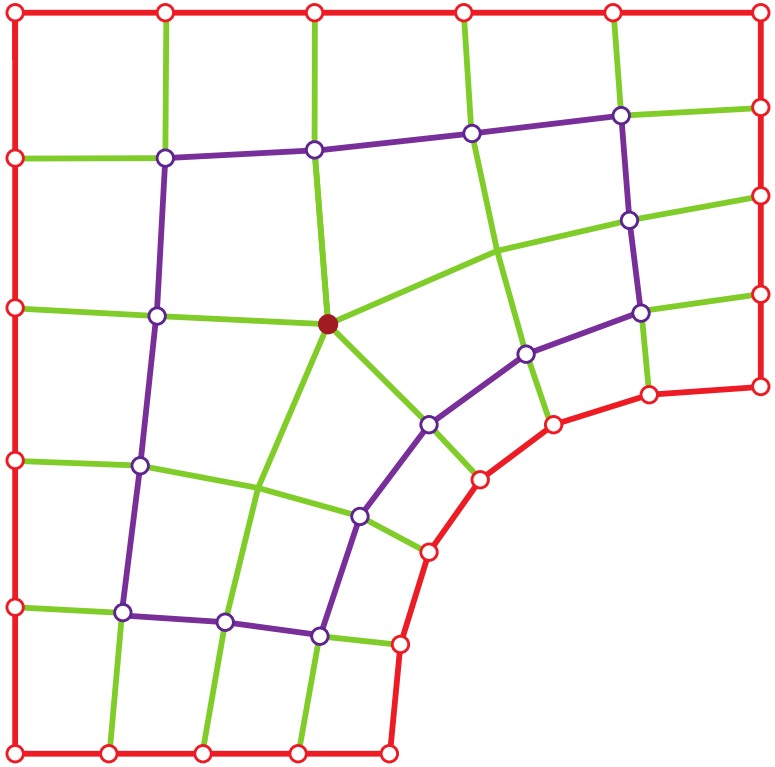
Quad Meshing



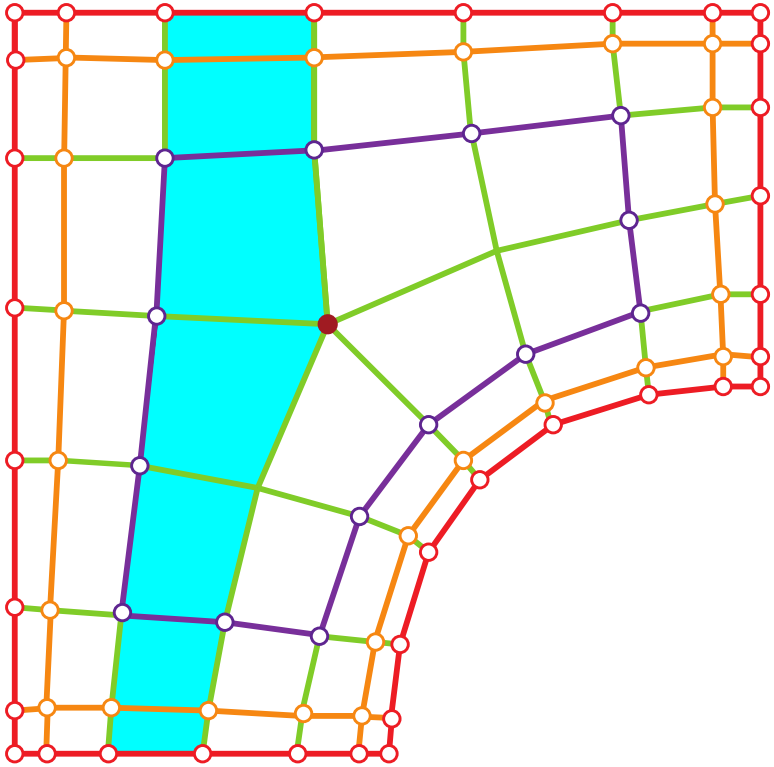
Bommes et al.: State of the Art in Quad Meshing, EG STAR 2012

Vaxman et al.: Directional field synthesis, design & processing, EG STAR 2016

Knot-interval Assignment

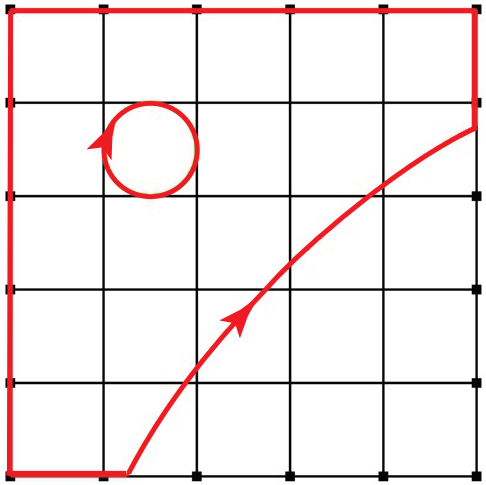


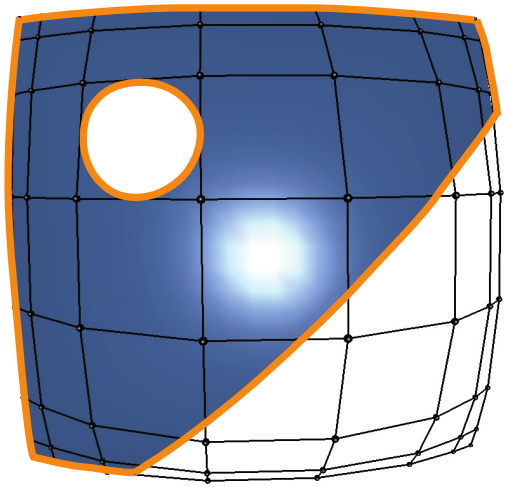
Initial quad layout



With an extra boundary layer
and knot interval per strip

Control Point Positions

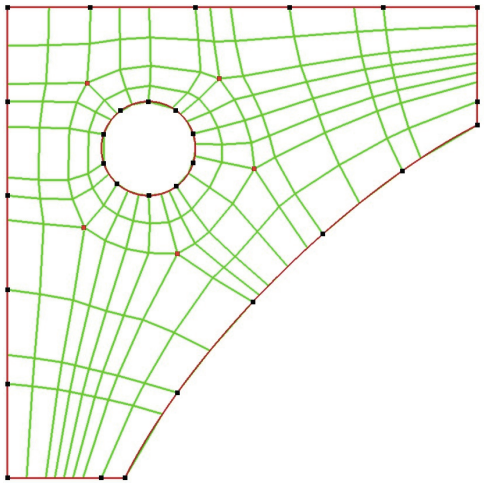


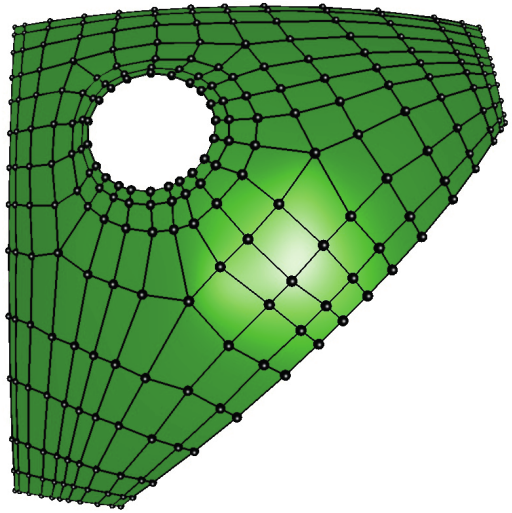


Input: trimmed NURBS patch

Domain space

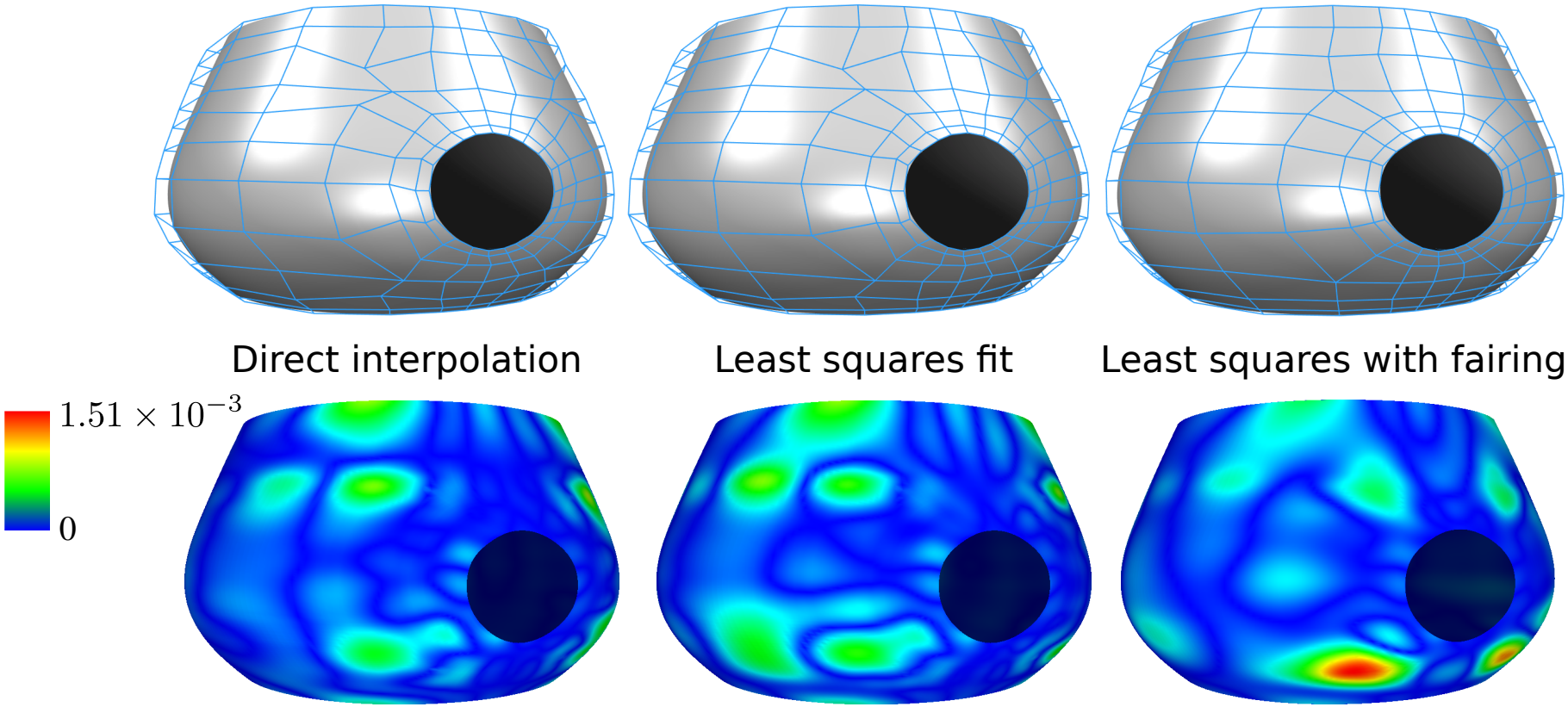
Model space





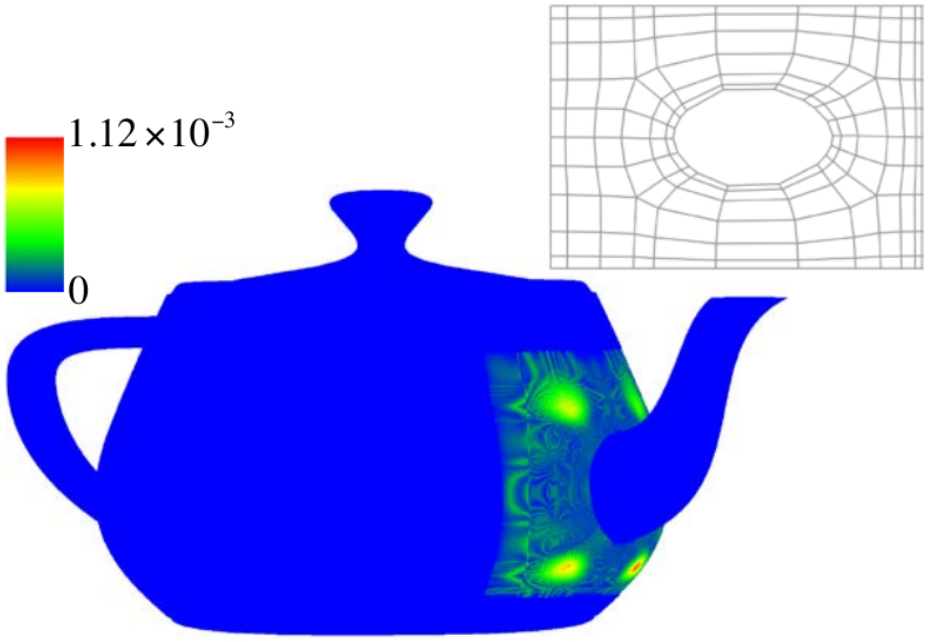
Output: subdivision surface

Control Point Positions

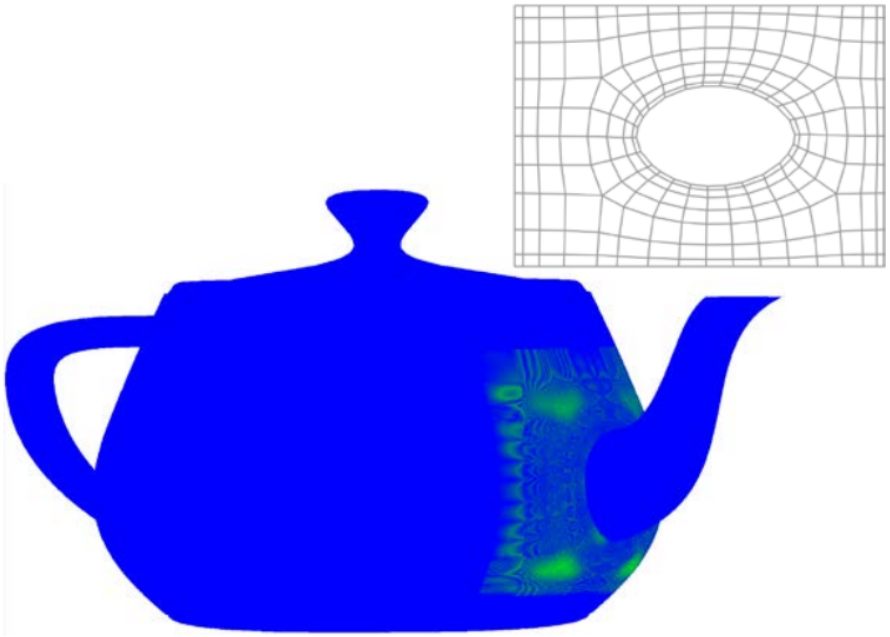


- Limit stencils for Cashman's scheme
- Boundary layer(s) fixed based on position (derivatives)
- Least squares with control points and 2×2 samples per quad
- Fairing term: thin plate energy

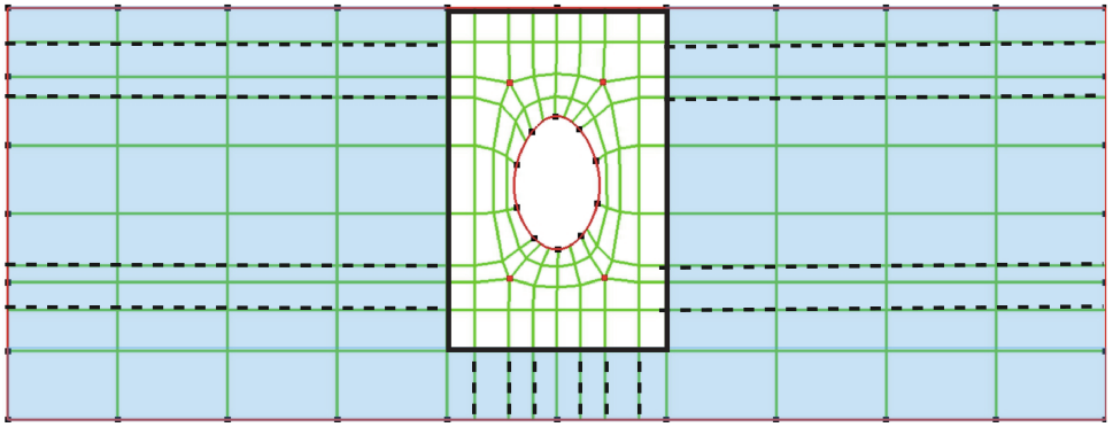
Local Approximation



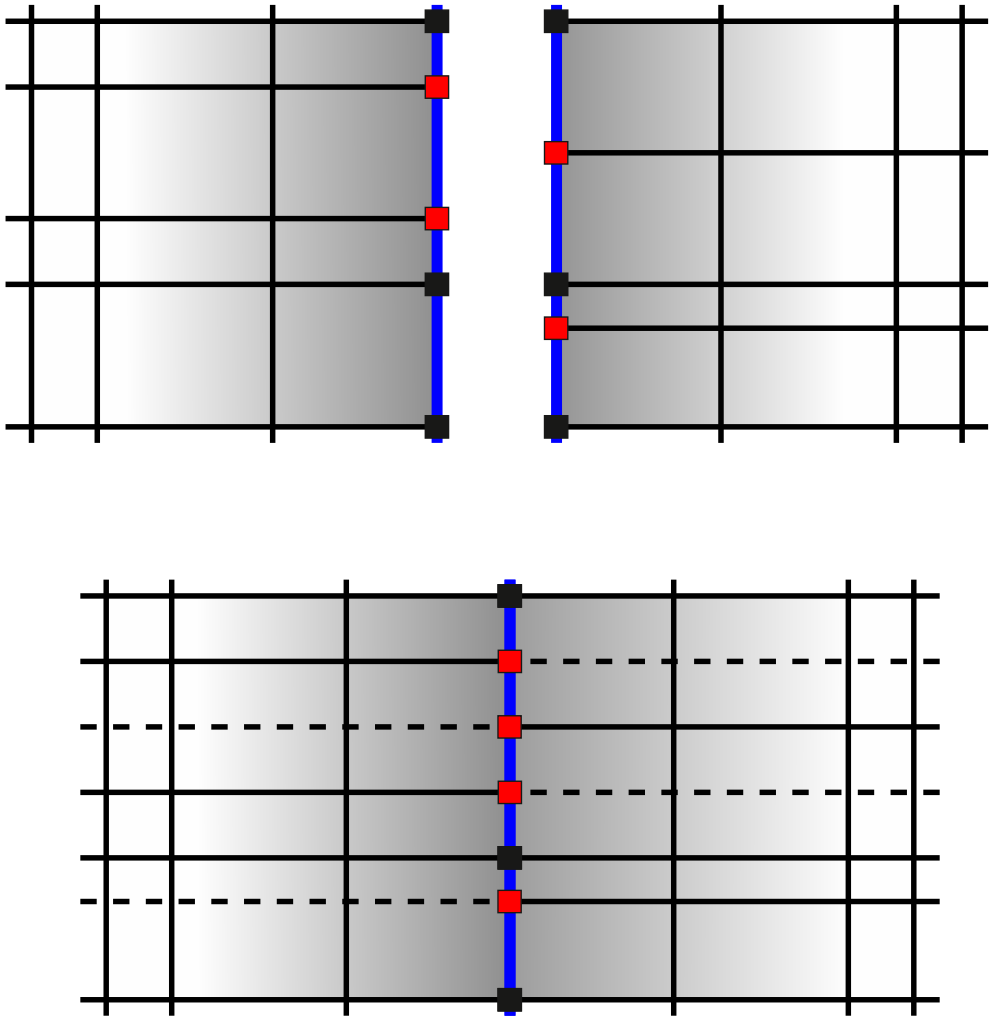
Refinement level 0



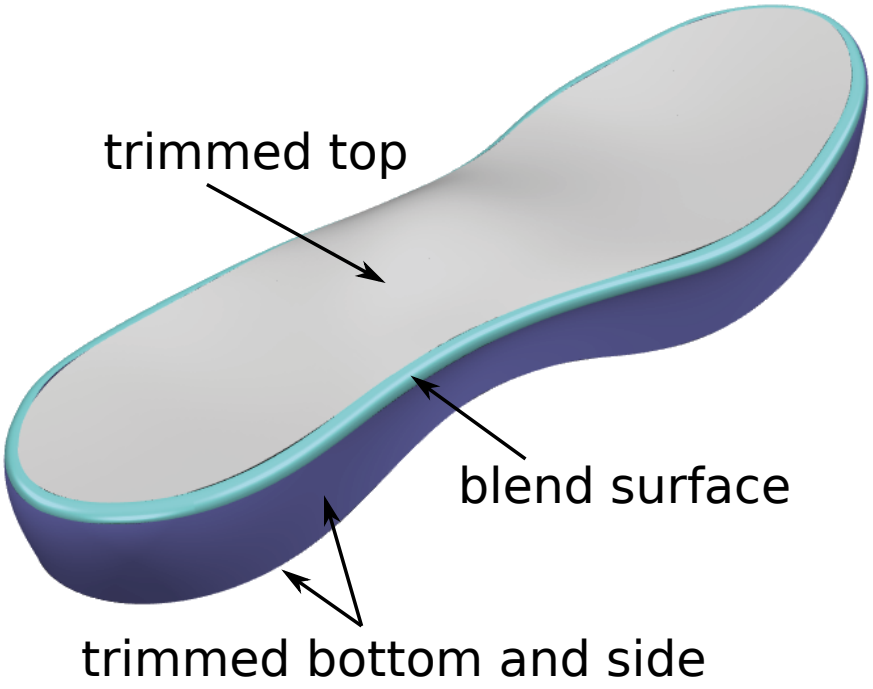
Refinement level 1

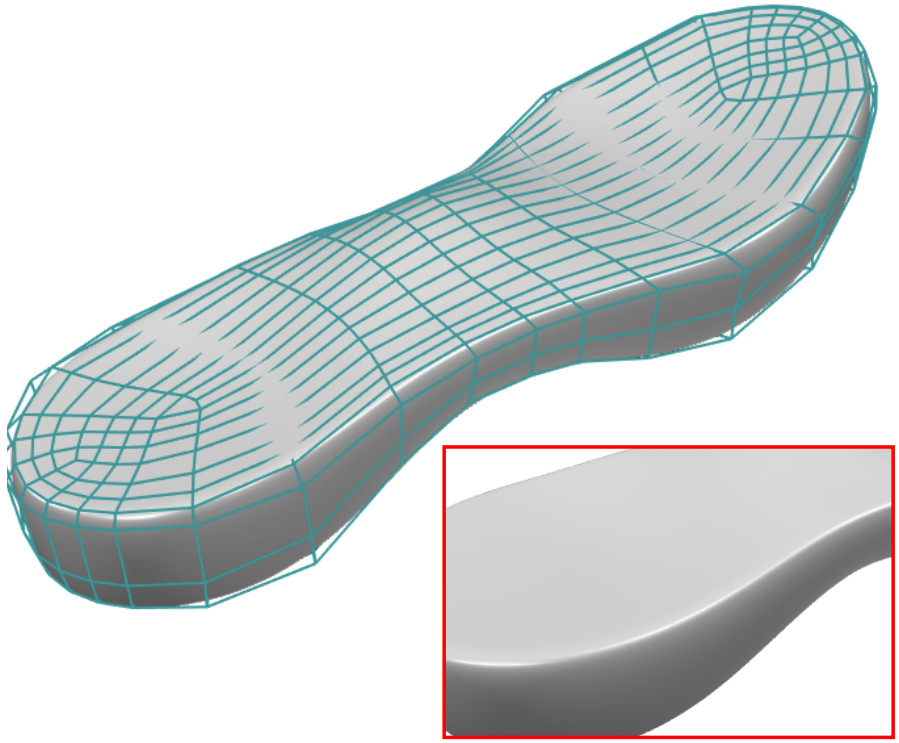


Merging Into a Single Subdivision Mesh

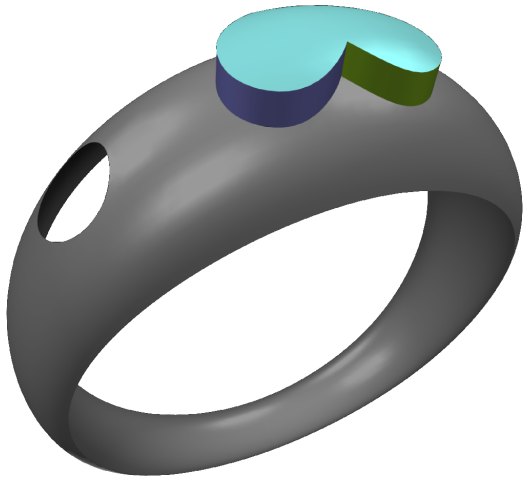


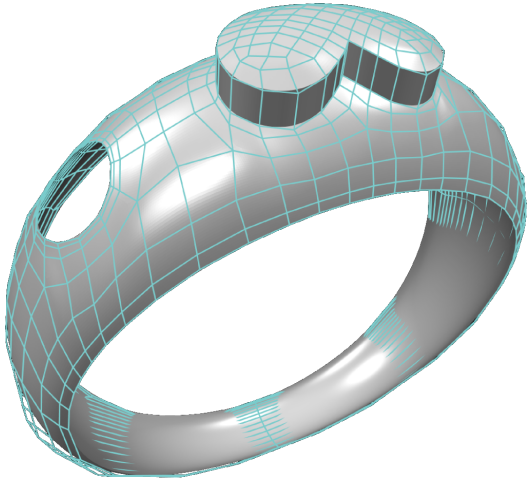
Results

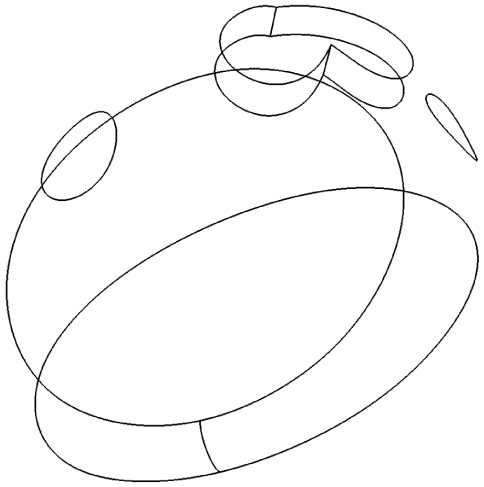


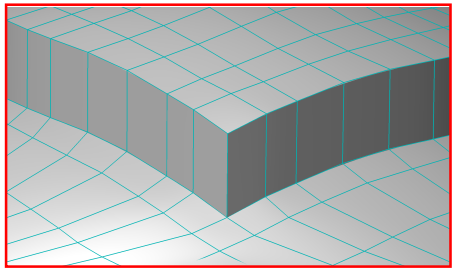


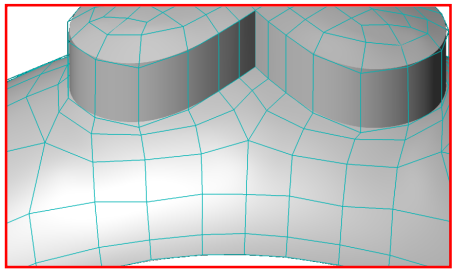
Results







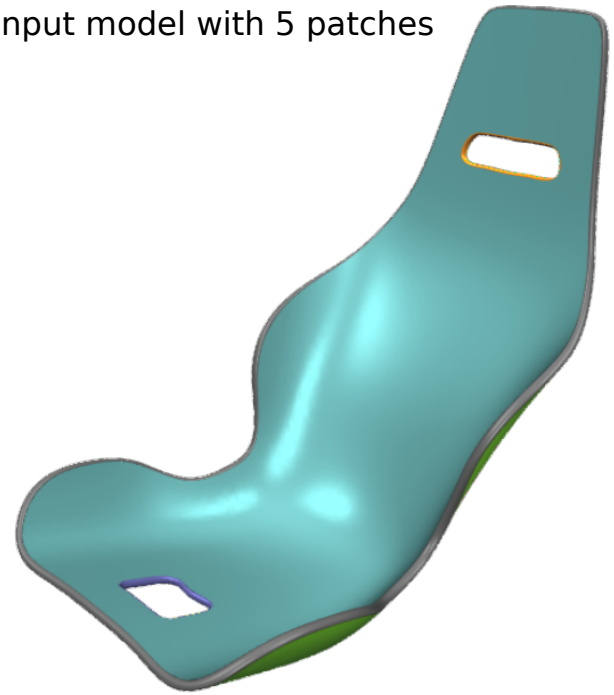


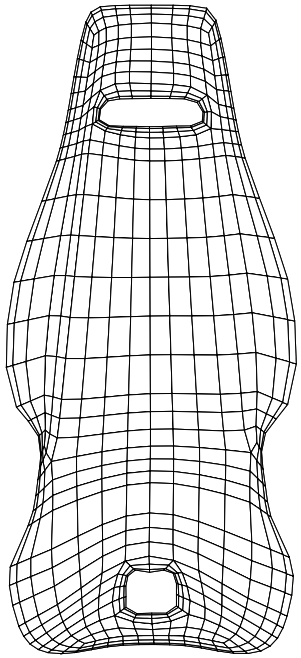


- Pixar (semi-)sharp rules for Cashman's scheme

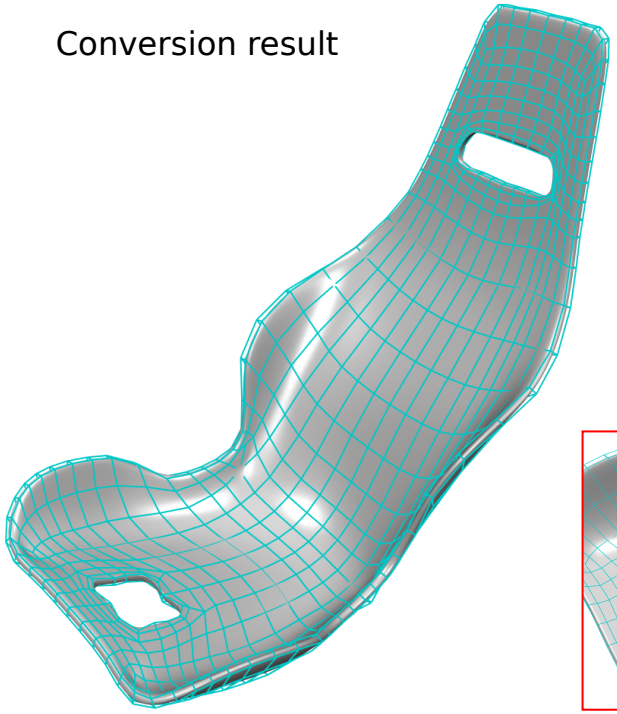
Results

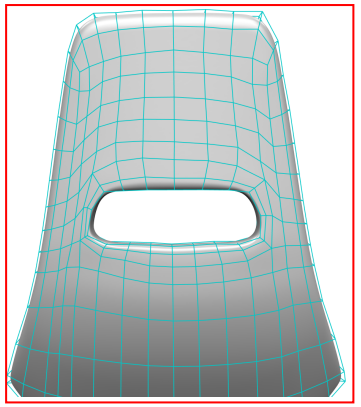
Input model with 5 patches

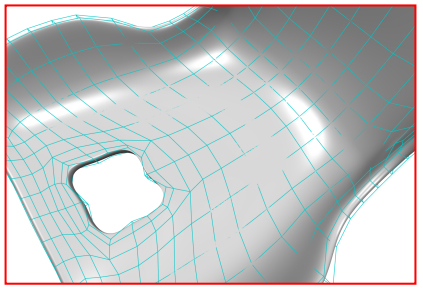




Conversion result

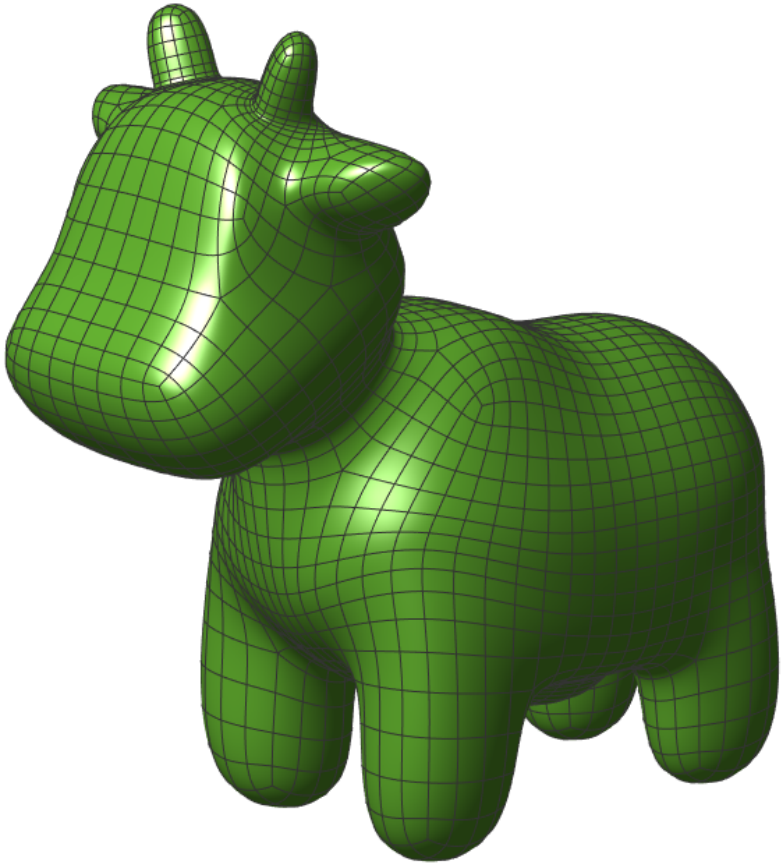






Numerical Quadrature for Subdivision Splines

- Gaussian quadrature per quad
- or Gaussian quadrature per rectangular array (Bartoň & Calo)
- But what to do at EVs?



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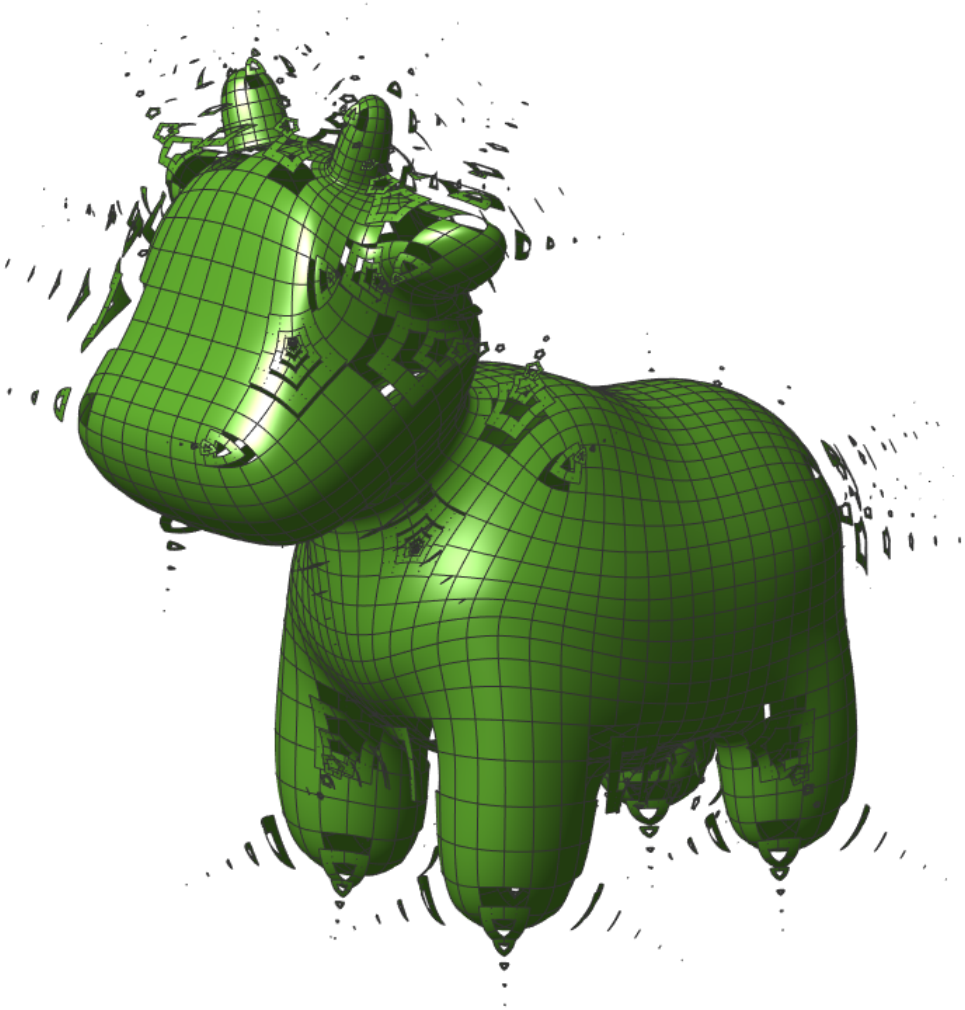

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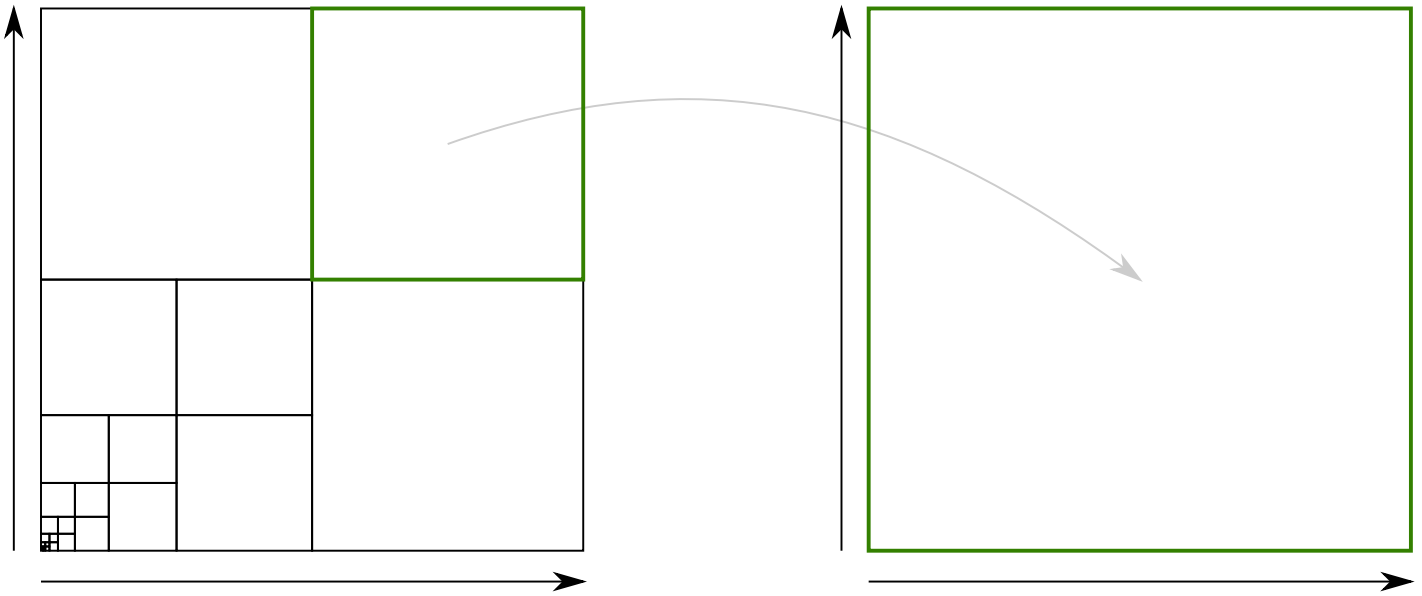

Numerical Quadrature for Subdivision Splines

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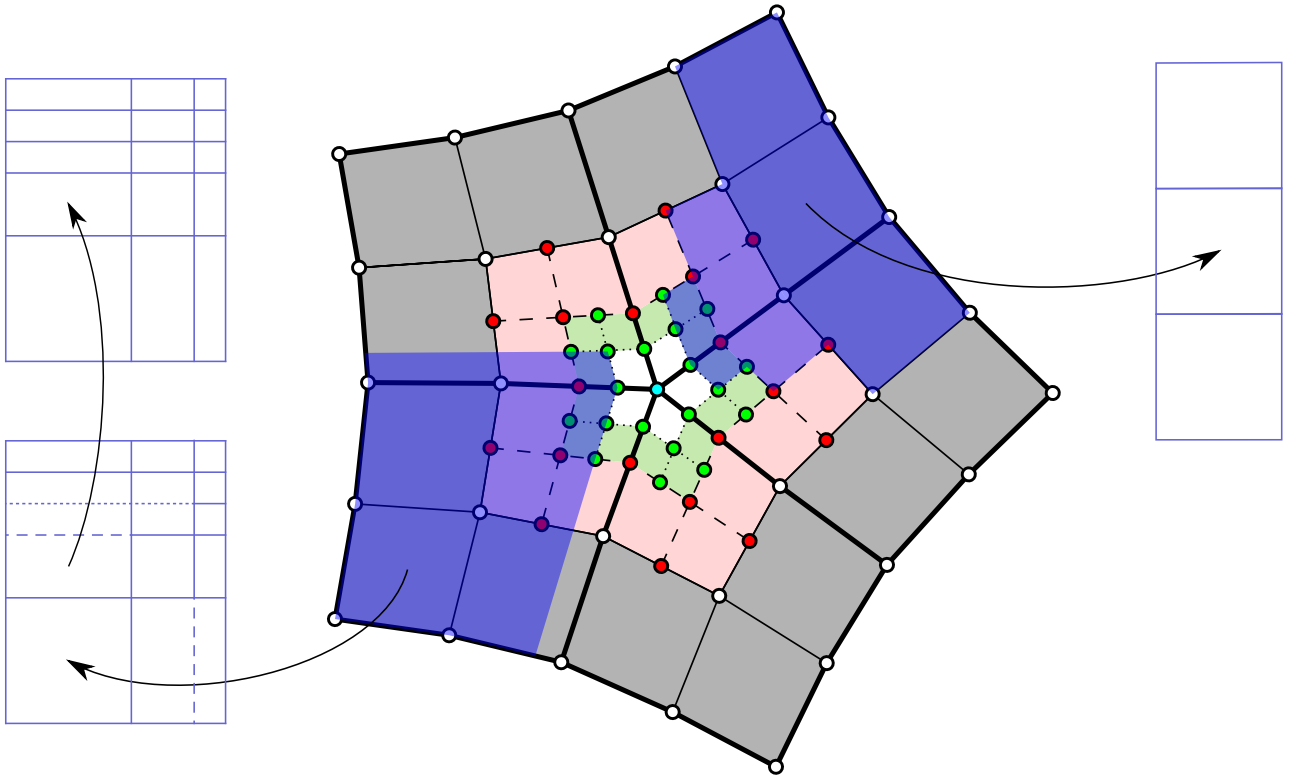
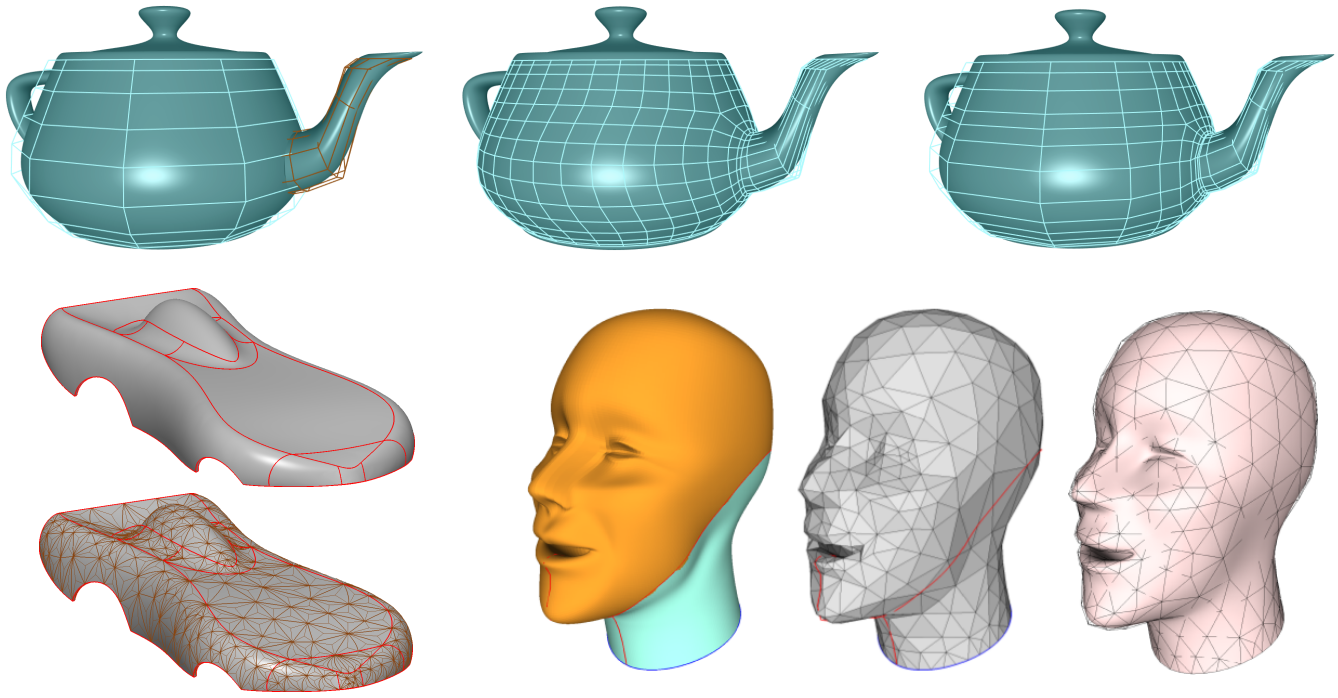


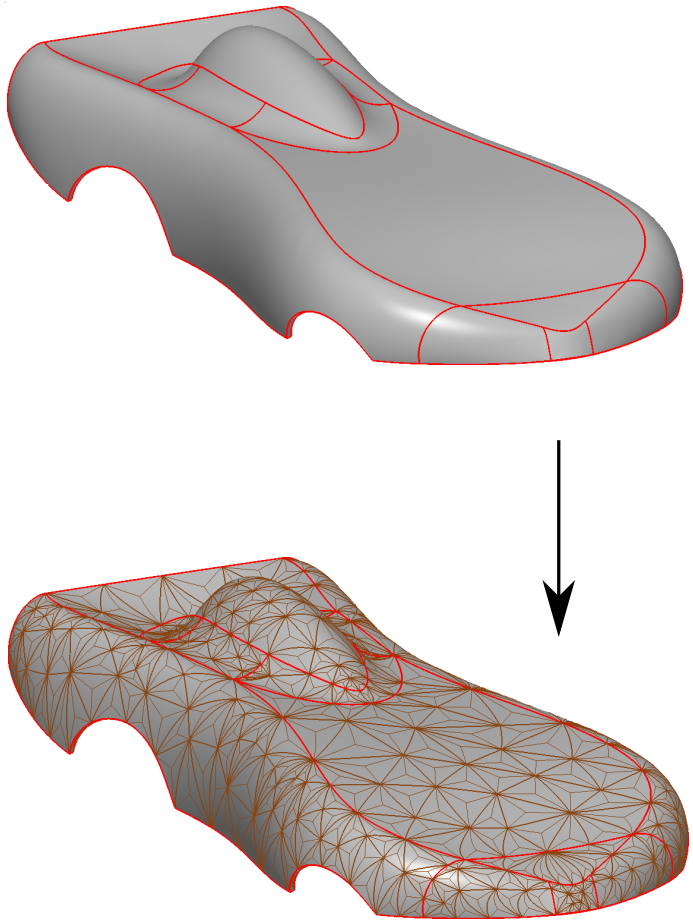
Our Conversion Methods

- Watertight conversion of trimmed CAD models to
  - Catmull-Clark subdivision surfaces (Shen, K, Sabin, Dodgson 2014)
  - **non-uniform subdivision surfaces** (Shen, K, Sabin, Dodgson 2016)
  - Loop subdivision surfaces (Shen & K 2016)
  - **Clough-Tocher splines** (K & Cashman 2015)

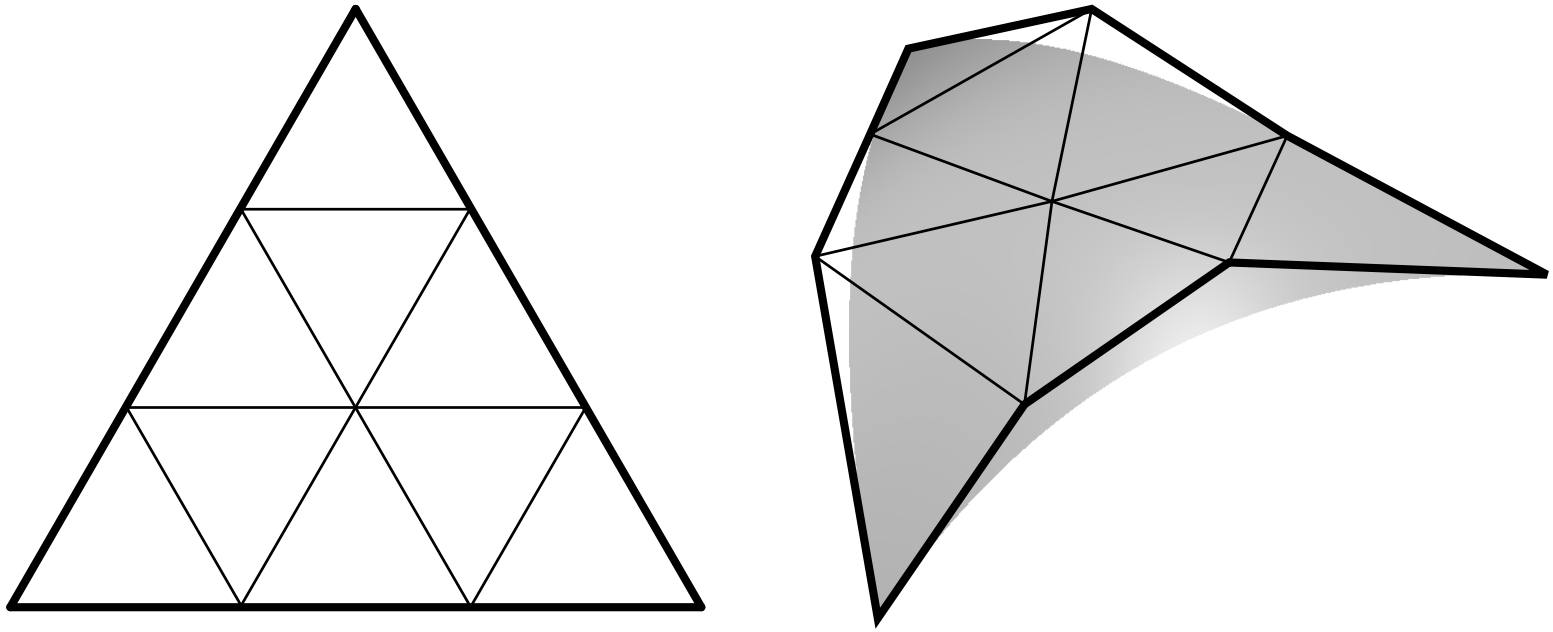


Our Conversion Process (Clough-Tocher)

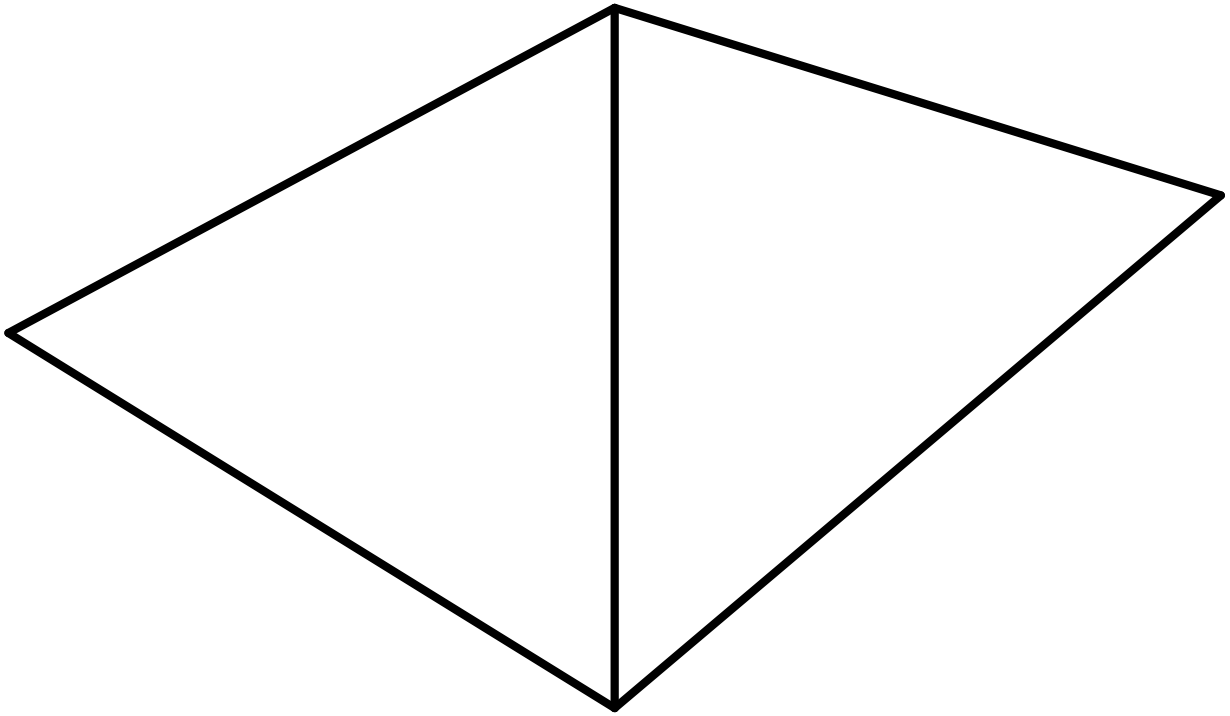
- Goals
  - Remove gaps
  - Homogeneous representation
  - Respect original (B-rep) edges
  - Allow for local adaptivity
- Three stages
  - Per-patch triangulation
  - Clough-Tocher spline
  - Trimming curve management



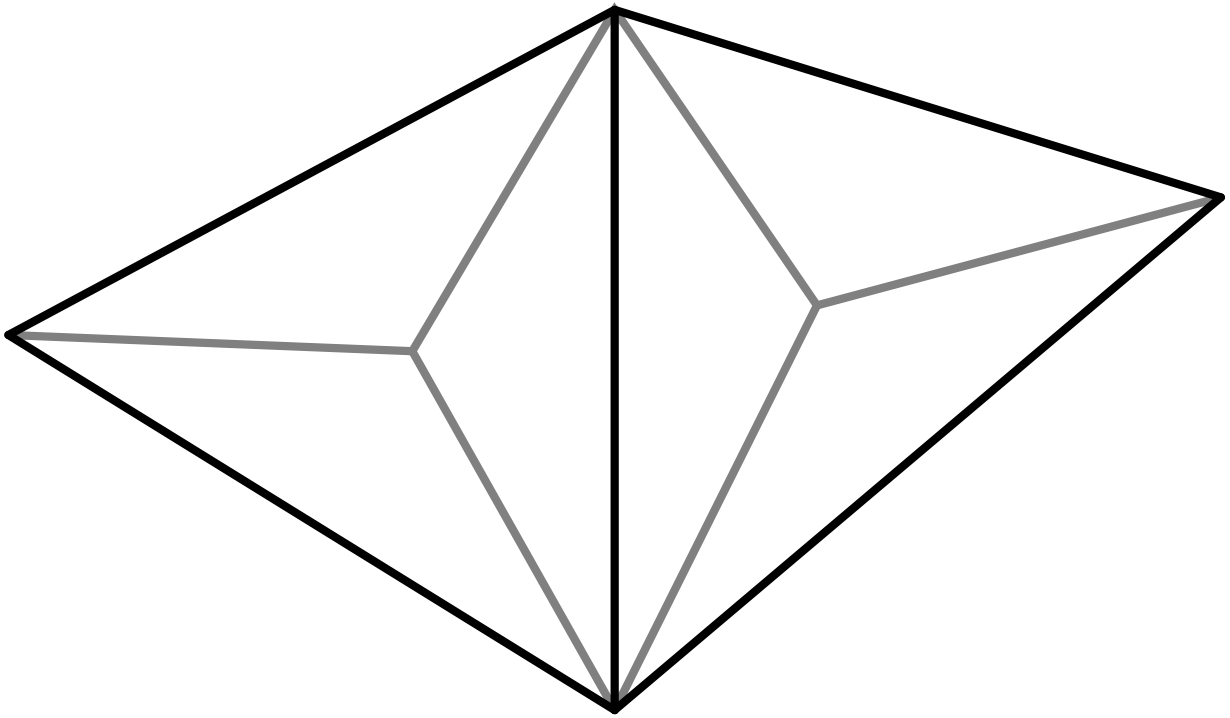
Cubic Bézier Triangles



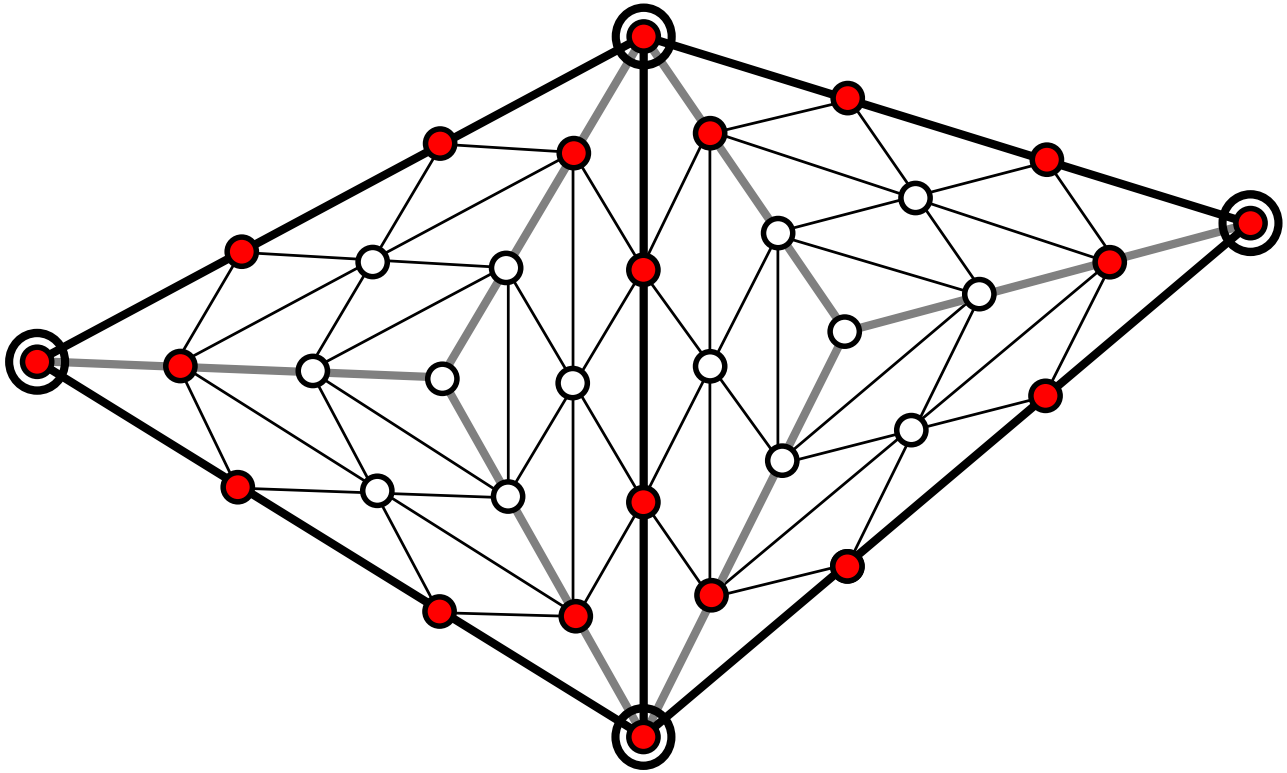
Clough-Tocher-Hsieh Split



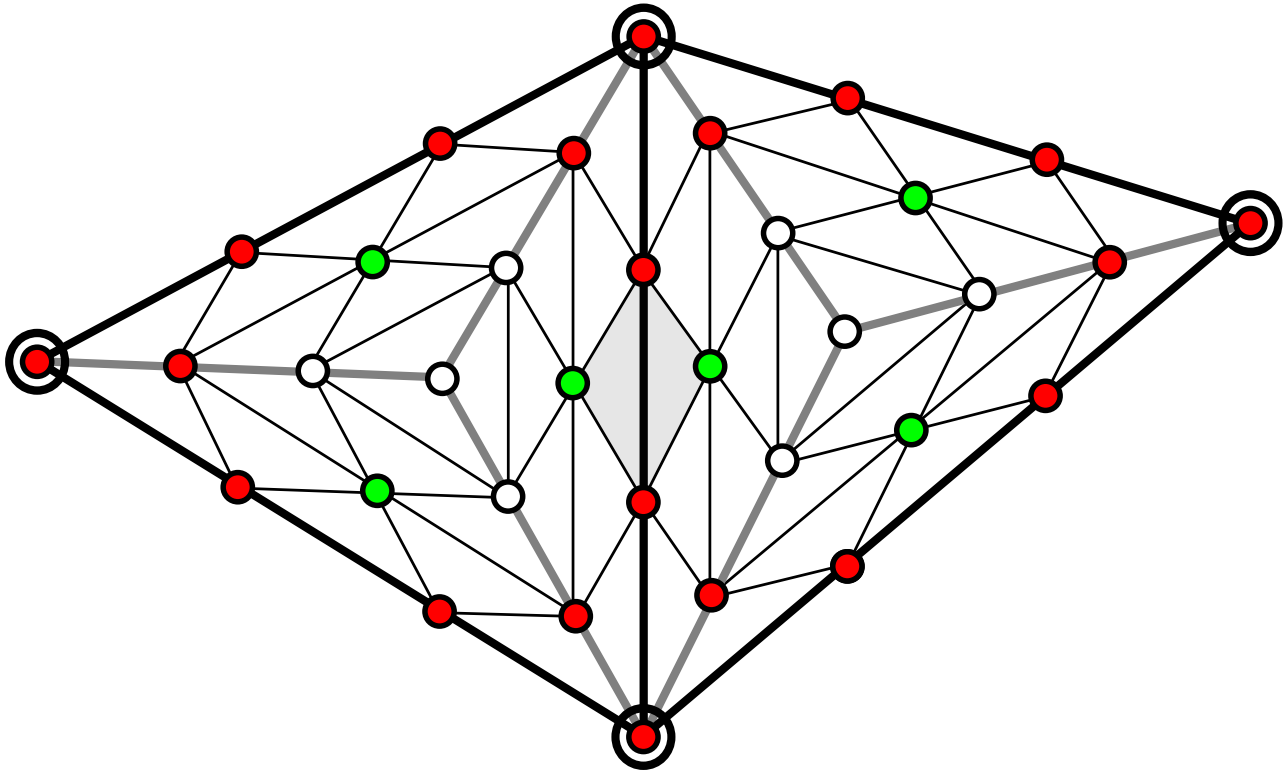
Clough-Tocher-Hsieh Split



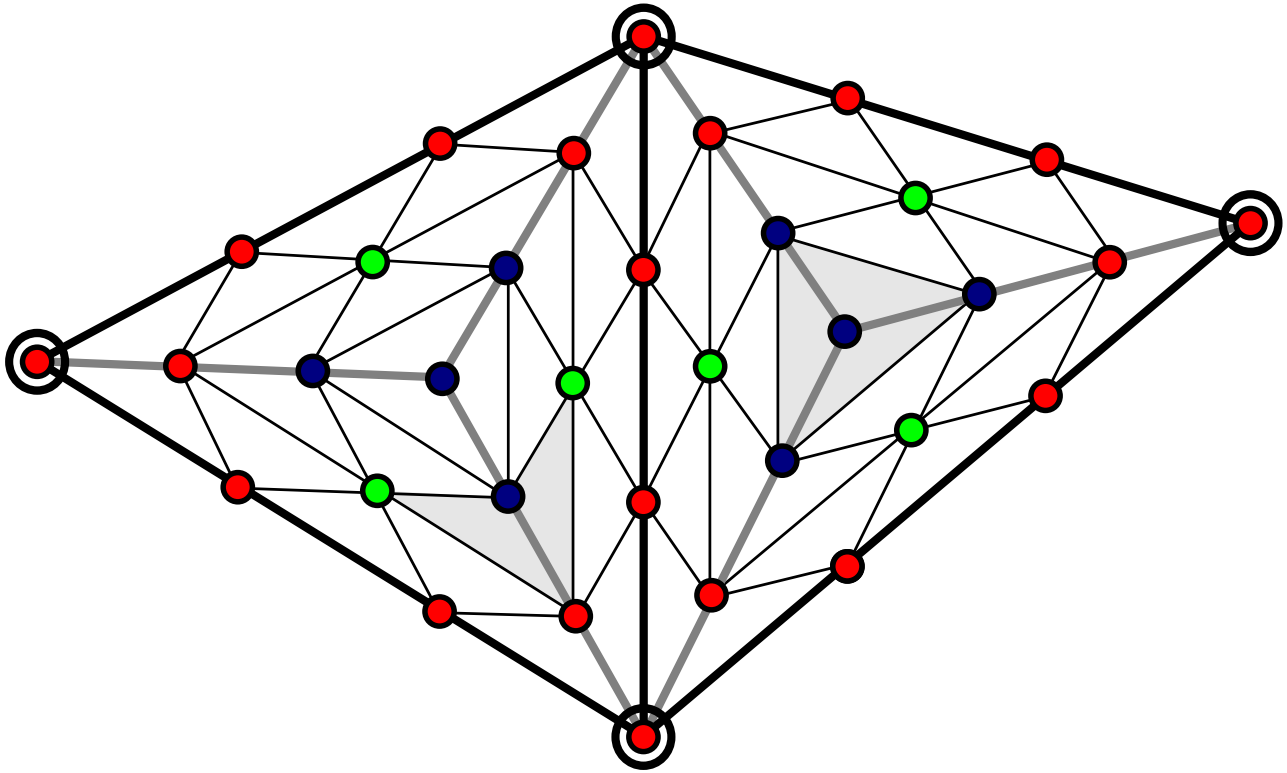
C^1 Cubic Clough-Tocher Spline Space



C^1 Cubic Clough-Tocher Spline Space



C^1 Cubic Clough-Tocher Spline Space



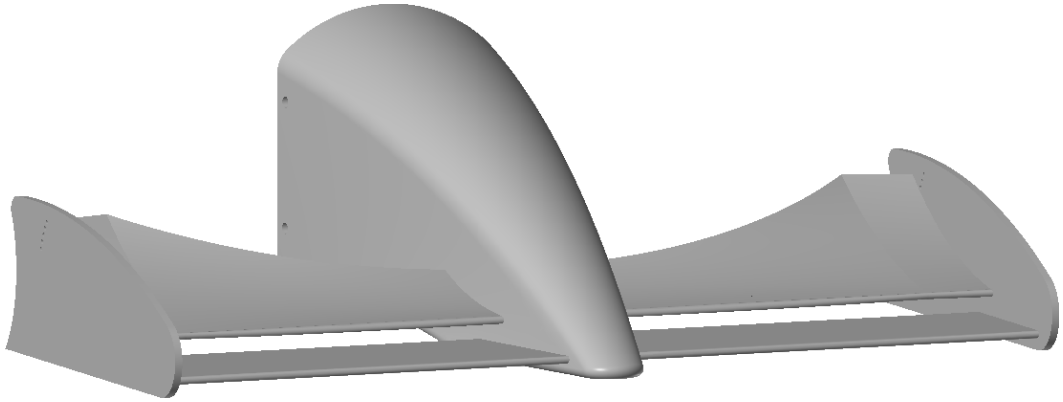
Quantitative Comparison

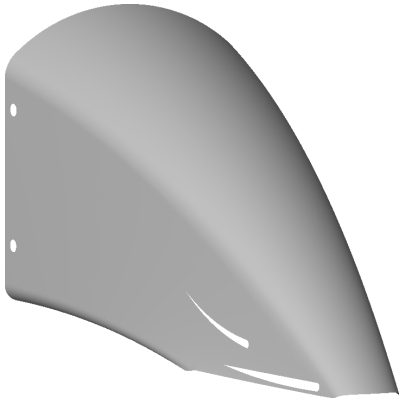
	CT_o	CT_i	Fa	FO	Ka	MG_o	MG_i
PR	2	2	2	3	3	3	3
Inv.	×	✓	✓	✓	✓	×	✓
Loc.	–	U_i	$U_i, \mathbf{f}_i, \nabla\mathbf{f}_i$	$U_i, \mathbf{f}_i, \nabla\mathbf{f}_i$	$U_i, \mathbf{f}_i, \nabla\mathbf{f}_i$	–	U_i
Sto.	11/0	11/0	11/0	11/0	11/0	11/3	11/3

- PR: Polynomial reproduction: Quadratic vs. cubic
- Inv.: Invariance to linear reparametrisations (scaling in parameter space)
- Loc.: Locality: Do we need information from adjacent triangles?
- Sto.: Storage: How much data do we need to store?

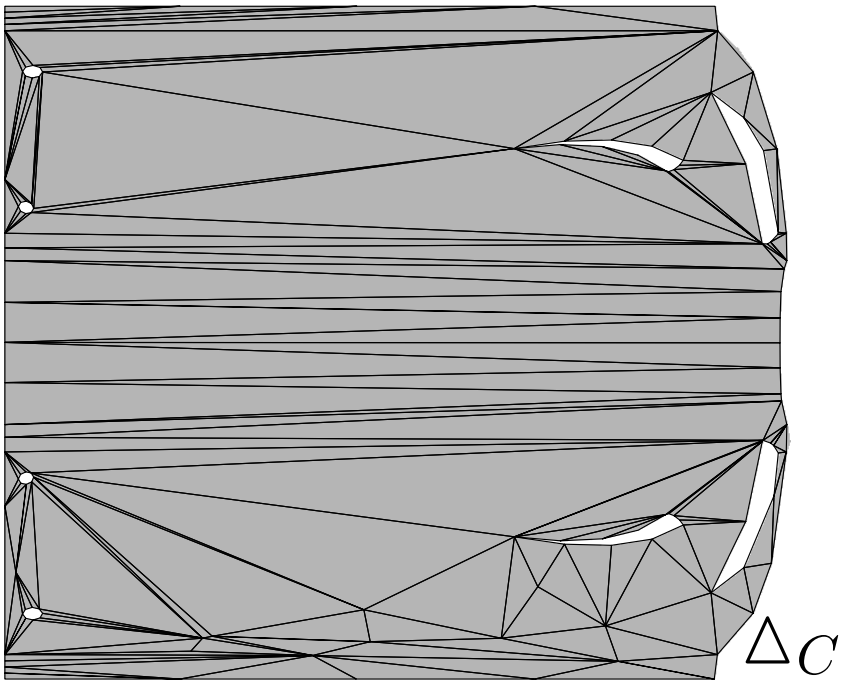
- K & Cashman 2015

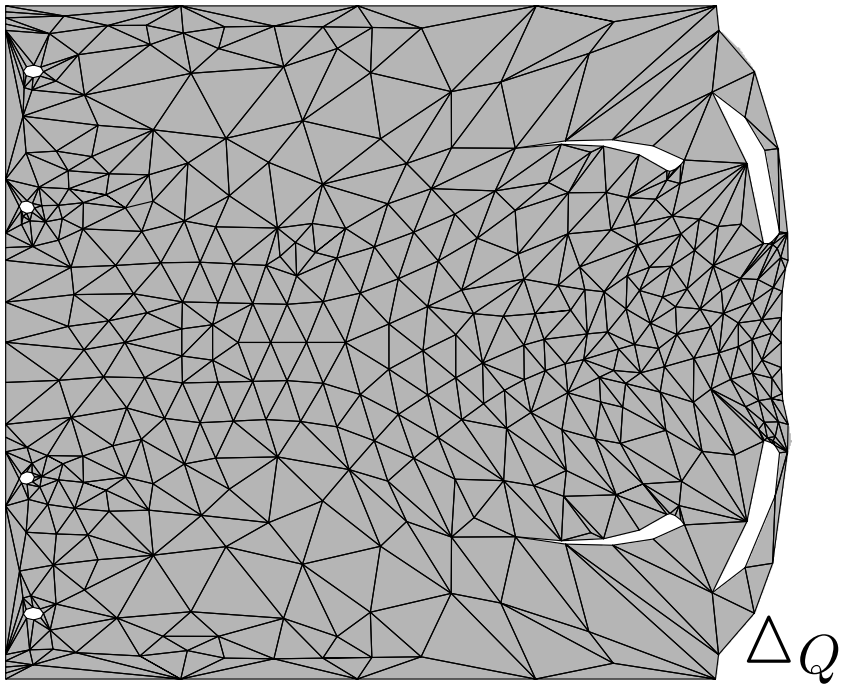
Front Car Wing





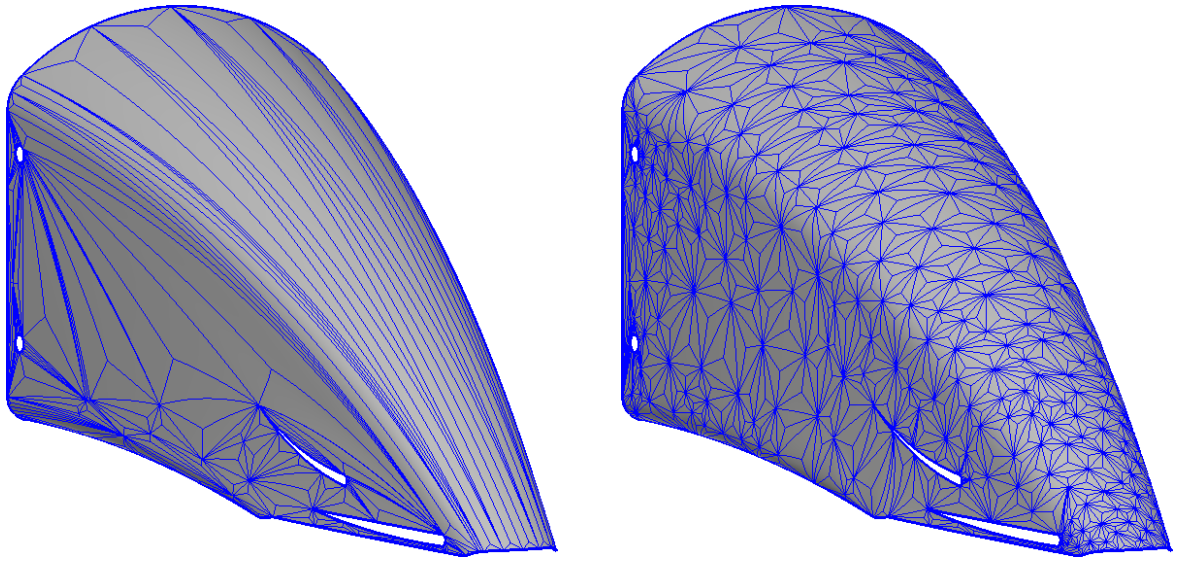
Constrained Delaunay Triangulation



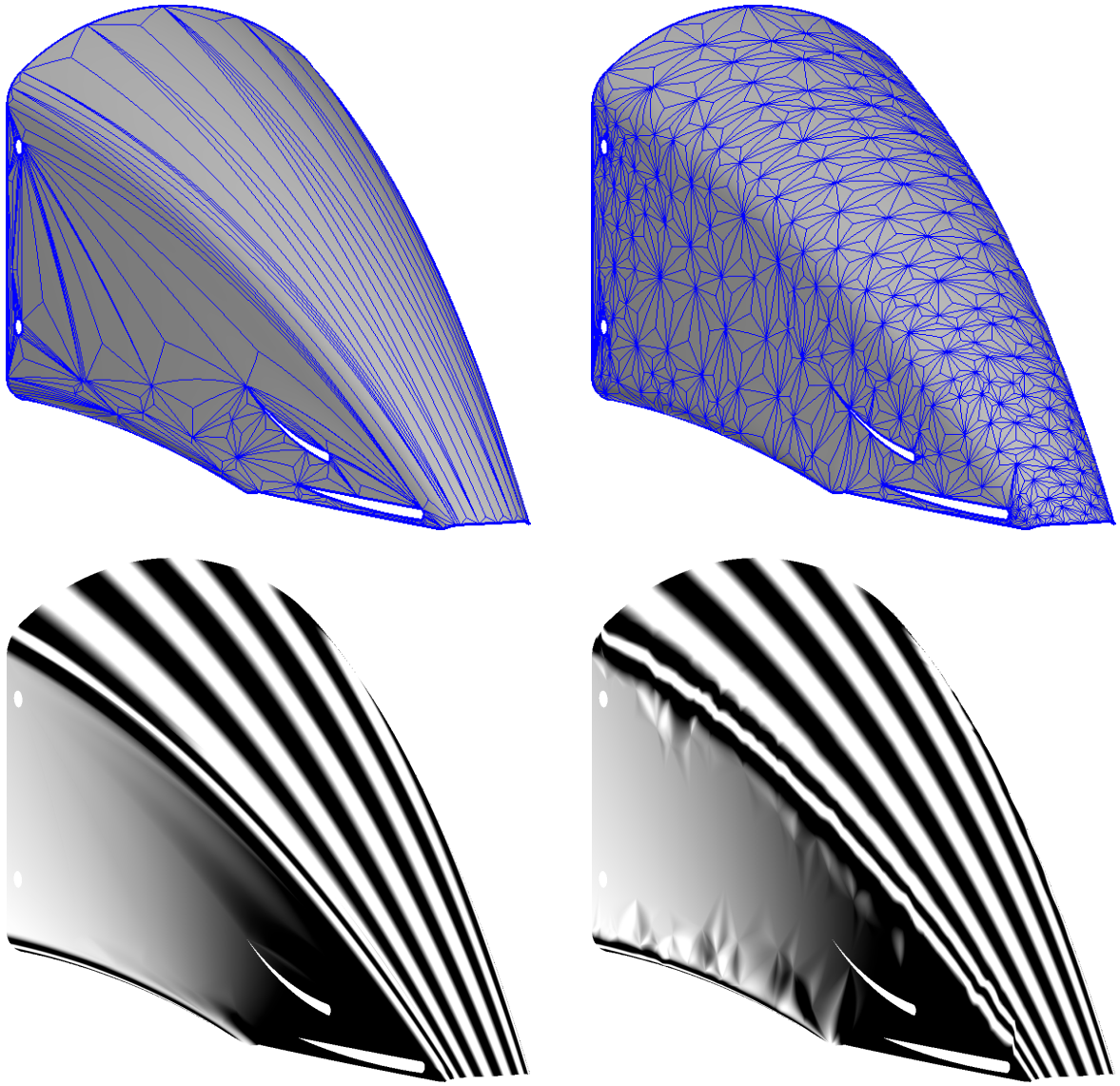


- B-rep to input data for Clouch-Tocher spline conversion
- Triangulation with parameter values and gradients
- With ($\triangle_Q$) or without ($\triangle_C$) aspect ratio control

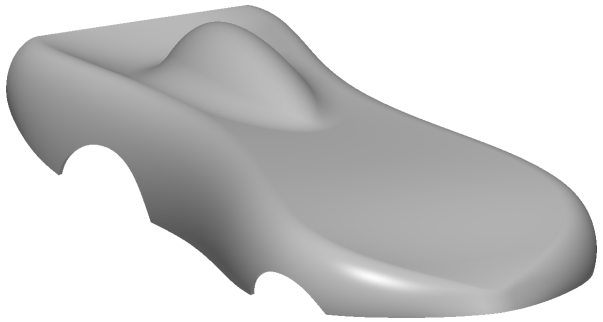
Front Car Wing



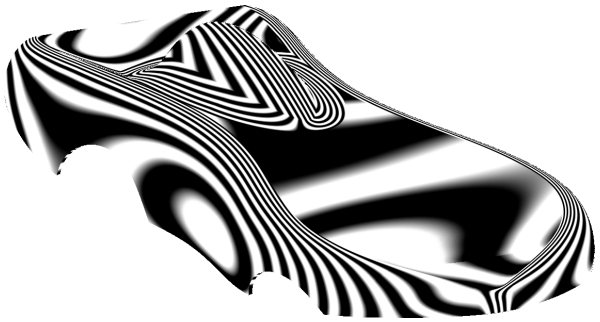
Front Car Wing



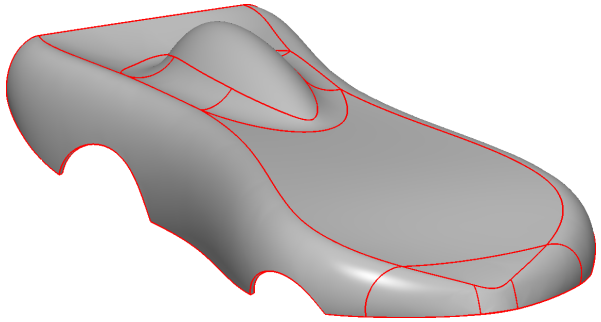
Car Model



Input model



C^{-1}



C^{-1}

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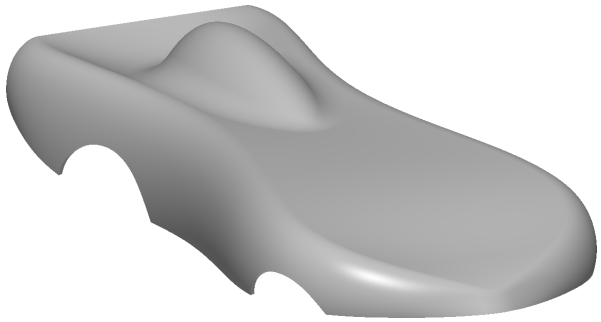
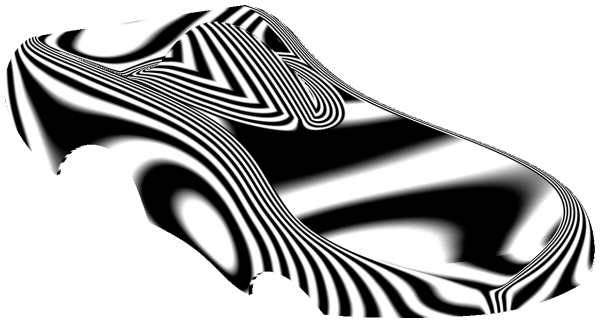
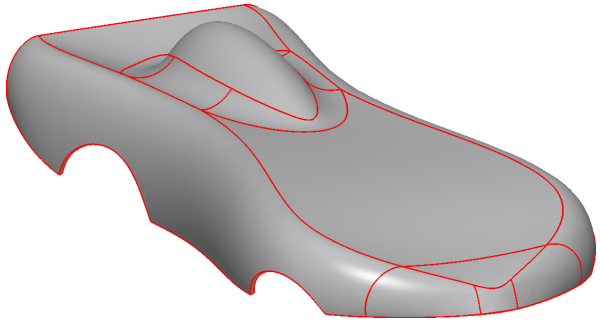


Input model



C^{-1}



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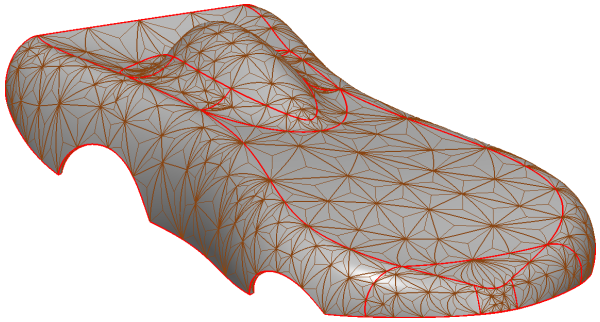
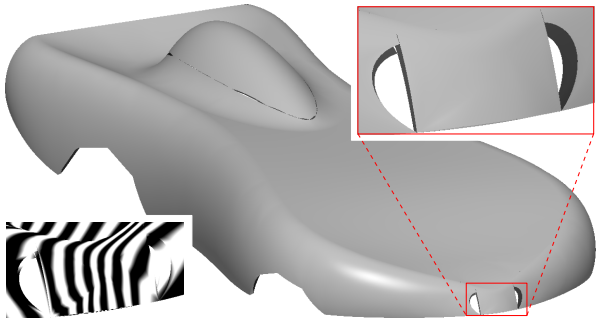


Conversion



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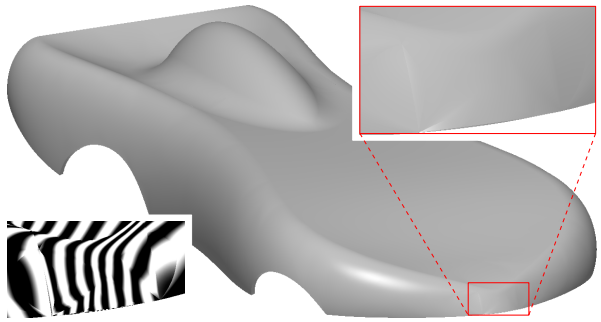
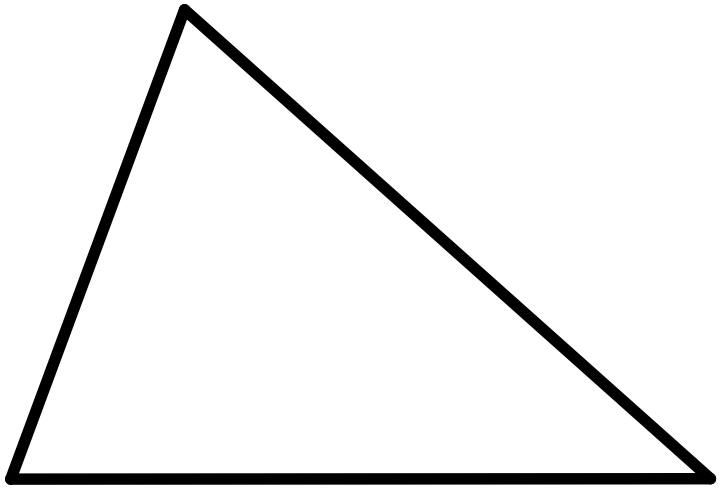
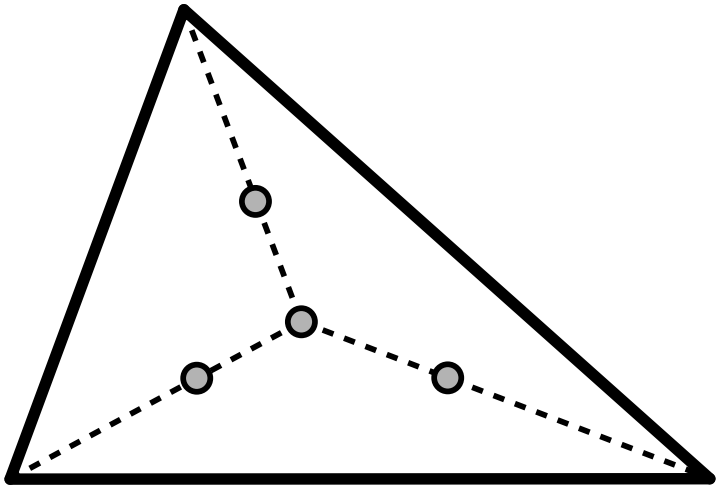


C^{0}

Quadrature for Clough-Tocher Macro-Elements

- Hammer-Stroud quadrature for cubics over triangles
- 4 quadrature points $\rightarrow$ 12 DOFs
- Cubic polynomials: 10 DOFs
-





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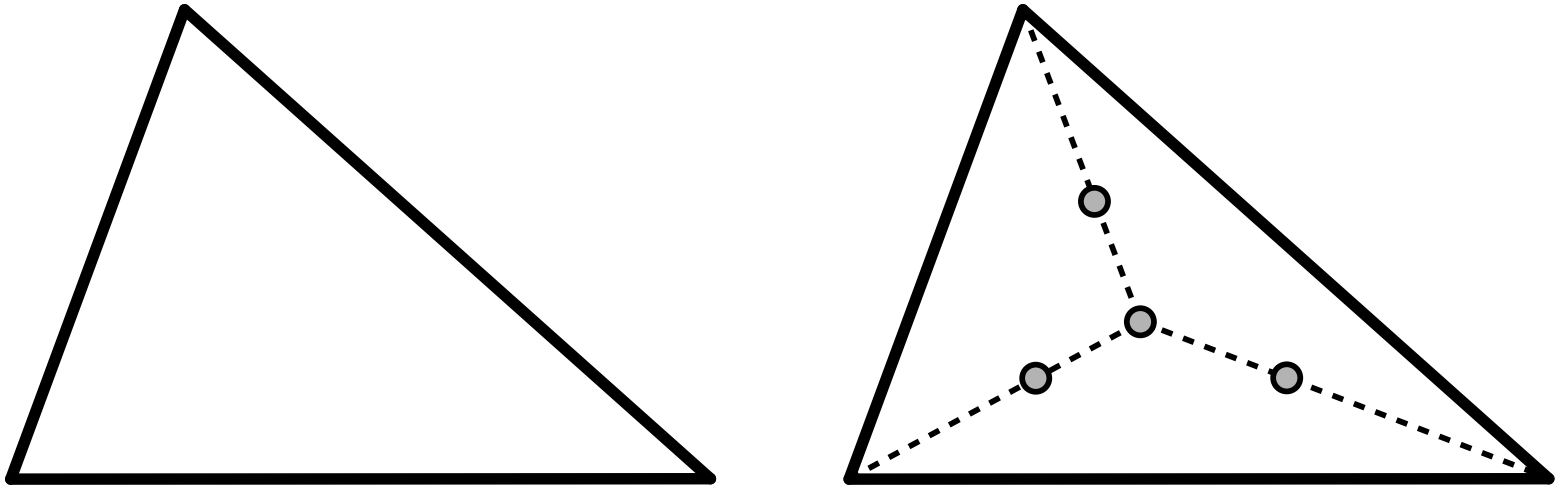

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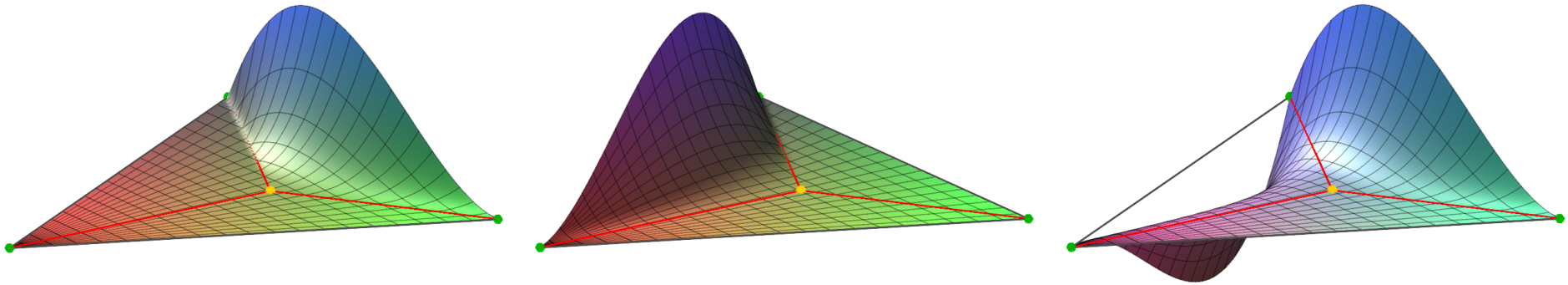

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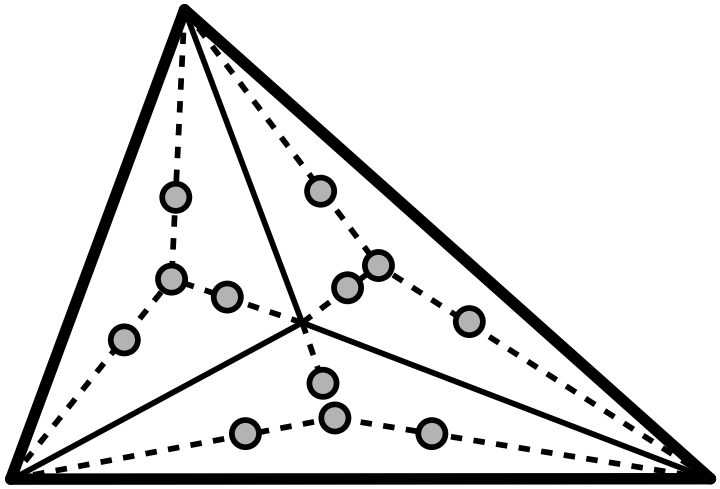
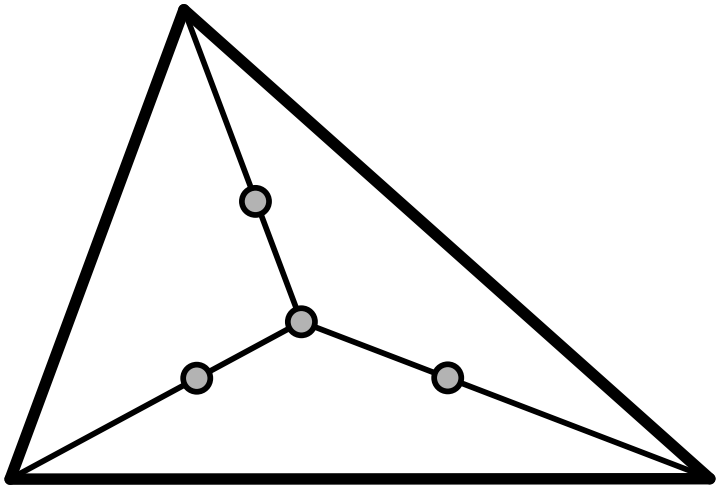
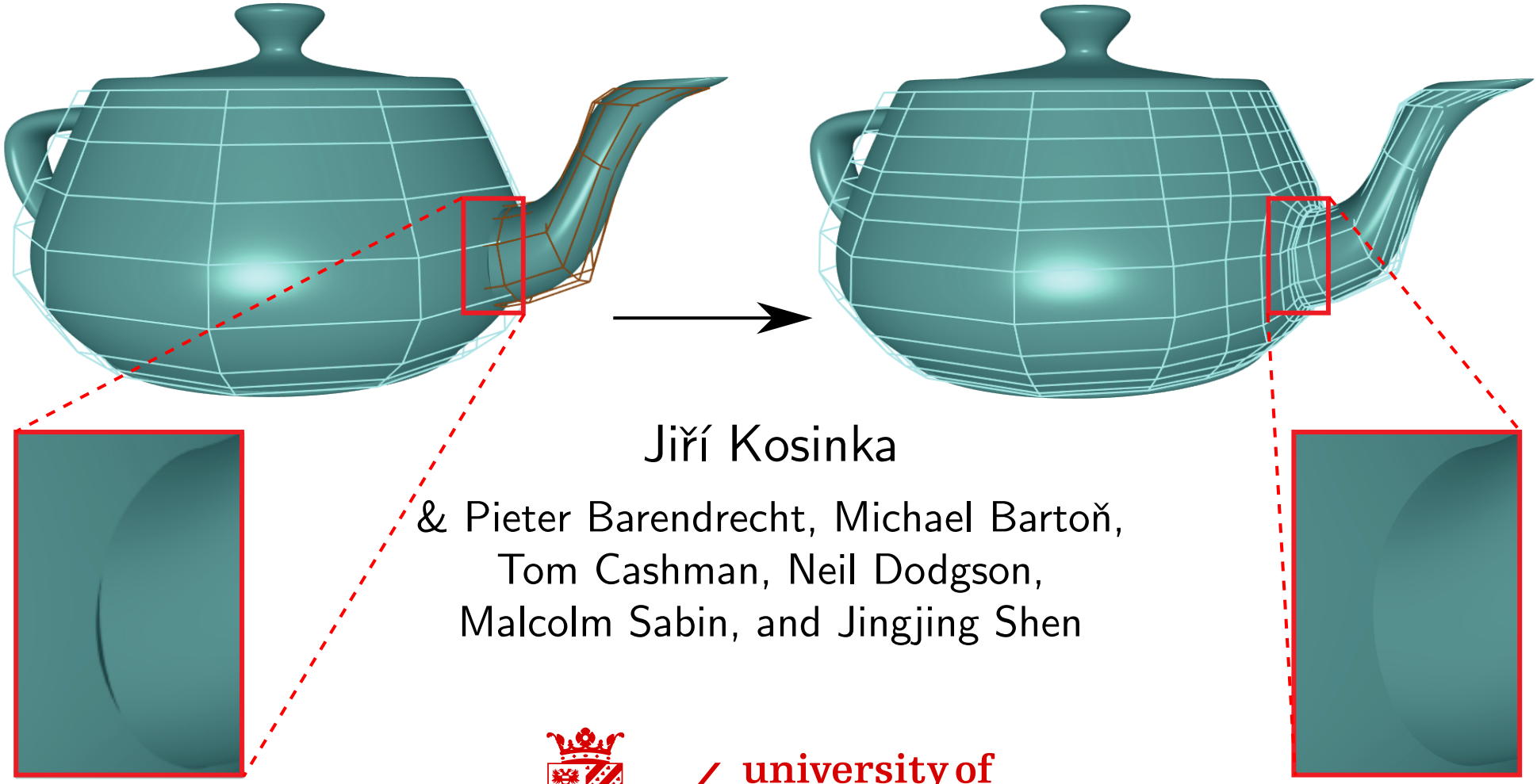


Per micro-triangle



K & Bartoň 2017

Modelling and IgA



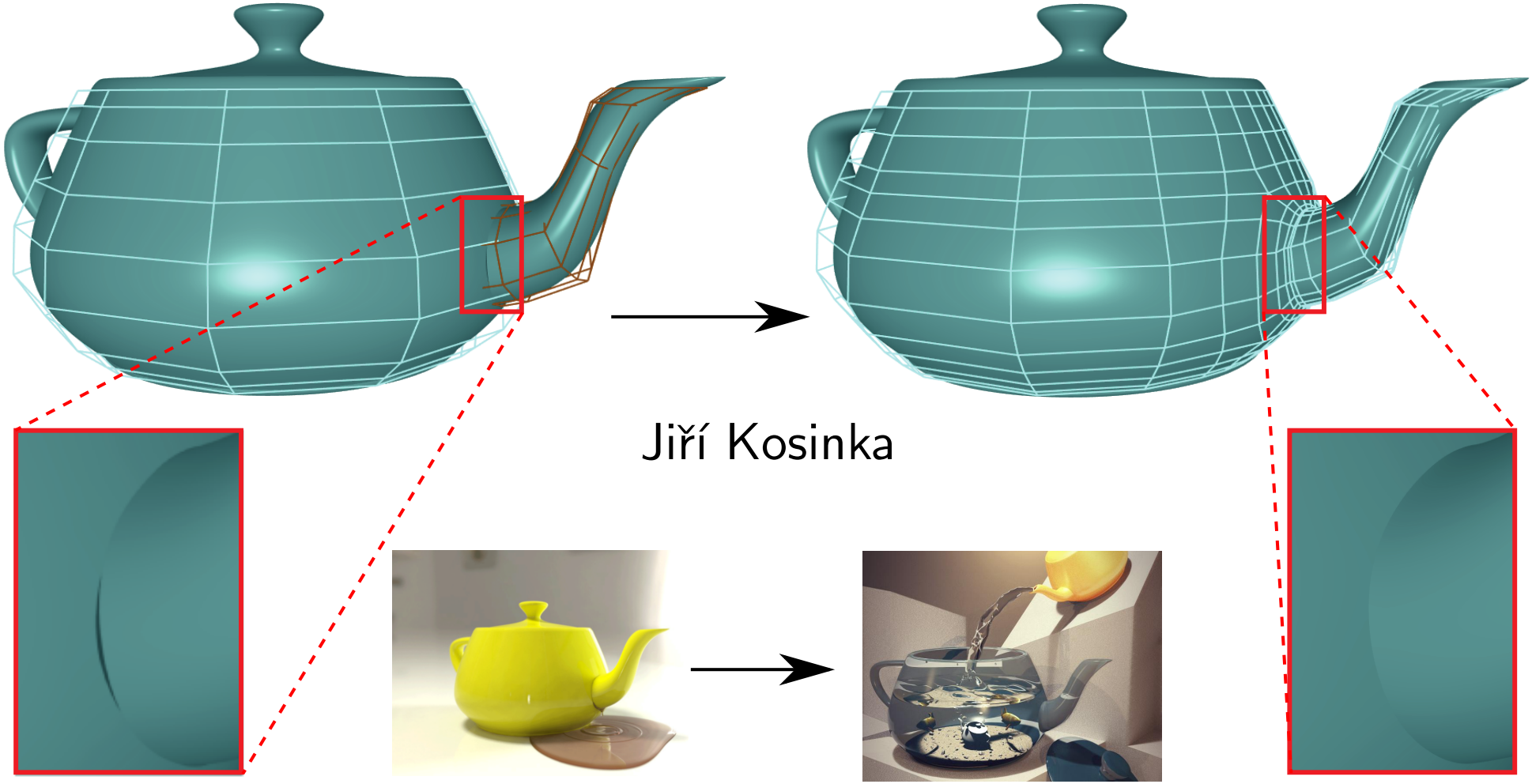
Jiří Kosinka

& Pieter Barendrecht, Michael Bartoň,
Tom Cashman, Neil Dodgson,
Malcolm Sabin, and Jingjing Shen

university of groningen

CAE Geometry Workshop
Cambridge, UK; 15 September 2017

Modelling and IgA



Jiří Kosinka

CAD geometry $\neq$ CAE geometry

CAE Geometry Workshop
Cambridge, UK; 15 September 2017

Full References

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