By Dr. Terry Wyberg

Introduction

Think about something that you are really good at doing. Imagine that you are so good at this thing that people ask you to perform this process over and over again and you do so without hesitation because you have confidence. A confidence that you can do it because you have worked hard preparing and the whole process makes sense to you. You are also flexible in performing this process in a variety of situations because you can make small changes and adapt. Adding fractions might not be what you identified as your talent but it is something that all students should be able to perform with confidence and flexibility. As teachers, we can provide students with experiences focused on sense-making so they become confident and flexible and can adapt their thinking in a variety of situations.

The goal of this paper is to unlock some of these proven methods and provide some practical ways that you can ensure your students are really good at adding fractions. You are probably doing more right than you know, and with a few of these tips, you will be able to ensure your students have the number sense they need to be flexible and confident when adding fractions.

Developing Strong Mental Images

Students need strong mental images of numbers before they can add and subtract the numbers. Look at the three different representations of the fractions ⅔ and ¼ in Figure 1.

Ask yourself this question: Which model would help you at estimating and then computing the sum:

$$\frac{2}{3} + \frac{1}{4} = ?$$

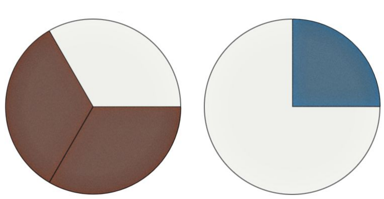
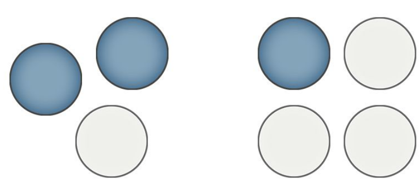
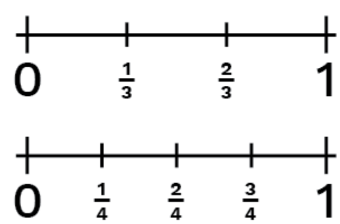
Area Model Fraction circles modeling ⅔ and ¼	**Set model** A set of tiles modeling ⅔ and ¼	**Number line** Stacked number lines modeling ⅔ and ¼
		

FIGURE 1: Three representations of ⅔ and ¼.

If you thought the area model using fraction circles, you are not alone. Most students reference an area model when they describe what they picture in their mind when they work with fractions. To estimate the sum ⅔ + ¼, students who have strong mental images of ⅔ can reason that the value is a little bit larger than ½ but exactly ⅓ less than 1. Furthermore, they can see that ¼ is smaller than ⅓ and that in turn lets them see that ⅔ + ¼ must be a little smaller than 1.

This is a critical development for students:
Number sense of fractions develops with the connections
that students make with benchmarks of ½ and 1.

Students who have strong mental images of fractions are better estimators than students who do not since most estimates are based on benchmarks and strong mental images are based on relationships with benchmarks. Mental images based on set models are typically not as helpful for students when they order or operate with fractions. For this example, reasoning with the set model would actually support the incorrect answer ⅔ + ¼ = $^3/_7$. While the set model is not as effective at supporting reasoning with benchmarks as area models, it is useful when working with equivalent fractions.

While the number line model is also an effective model for students as they represent and operate with fractions, it is tends to be a more difficult model for students as they initially learn how to work with these new numbers. The number line requires students to coordinate symbols and lengths in order to use the model, which makes it a powerful but more sophisticated tool than area models like fraction circles and fraction bars. In our example, for a student to compute ⅔ + ¼ on a number line they would first need to decide how to correctly partition the number line before representing both fractions as a single length. The planning needed for this decision is what makes using the number line a more sophisticated tool than working with area models. With area models both ⅔ + ¼ can be represented and combined before a student needs to decide to use a common size piece.

In general, students need substantial experience with various models as they make sense of fractions before they can begin operations like adding and subtracting. Students need to be able to estimate sums and differences using benchmarks which are supported by mental images before they carry out exact calculations. Area models like fraction circles help students develop number sense around fractions because they can visualize these numbers in relation to benchmarks like ½ and 1.

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Beginning Fraction Addition

As student begin to add fractions, context is critical. Let's look at a typical context you might use as you introduce fraction addition:

> *Bailey has ⅜ of a cup of flour. Seth gives Bailey ¼ of a cup of flour.*
> *Estimate the amount of flour Bailey has and then calculate the exact amount.*

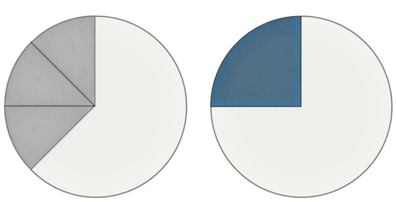
Students with strong mental images can imagine that ⅜ and ¼ are both less than half a cup of flour so together the total amount has to be less than 1 full cup.

Furthermore, ⅜ of a cup of flour needs an additional ⅛ of a cup to make ½ of cup of flour. ¼ is larger than the ⅛ of a cup so the total amount of flour is more than ½ of a cup. A good estimate for the total amount of flour is more than half a cup but less than 1 whole cup.

Students need to have strong mental images of fractions to imagine that fourths are larger than eighths.

Students can find the exact answer using fraction circles. They can represent the situation by using 3-gray pieces and 1-blue piece as illustrated in Figure 2. Most students use the pieces to explain that 2-gray pieces cover 1-blue and reason that $^{2}/_{8}$ is equivalent to ¼.

Using colors with consistent names supports students as they make sense of the common denominator algorithm for addition of two fractions. They are able to reason that the size of the piece is related to the size of the denominator, and they are able to explain that renaming a collection of pieces using one fraction requires that all the pieces be the same size.

Fraction circles	Names using color	Fraction notation
	3-gray + 1-blue	$^{3}/_{8} + ^{1}/_{4}$

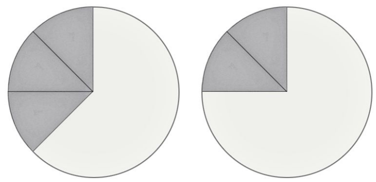
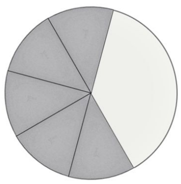
	3-grays + 2-grays	$^{3}/_{8} + {}^{2}/_{8}$
	5-grays	$^{5}/_{8}$

FIGURE 2: Using three representations to make sense of the common denominator algorithm

The language of color supports the logic of getting a common denominator when putting two fractional amounts together. The reasoning that 2 gray pieces combined with 3 gray pieces to make 5 gray pieces allows students to reason that 2-eighths combined with 3-eighths will make 5-eighths. It also makes visually clear for problems like this that ⅜ (3 grays) and ¼ (one blue) cannot be named as one fraction without first making the pieces the same size.

In order for students to add fractions flexibly and with confidence they need strong mental images so they can reason around benchmarks to estimate, and match sizes of pieces to make sense of equivalence.

Using the Number Line for Fraction Addition

We've shown how area models like fraction circles can be an ideal support for students in the initial development of addition of fractions. While the number line does not provide the same support for initial development, it is an important representation for students to be comfortable with. When students are flexible and confident with making fractions the same size when combining parts, then it is effective to introduce the number line. A gentle and effective way to use the number line model is in the context of applying the common denominator algorithm for addition for fractions. Take this example problem:

> *Shaina walks 1 ⅓ miles on Saturday and ¾ of a mile on Sunday. Estimate the total distance she walked over the weekend, then calculate the exact amount using the number line.*

Students who have strong mental images can estimate that the total amount walked will be a little more than 2 miles by picturing amounts and relating them to benchmarks. They should reason that 1 ⅓ miles is a little longer than 1 ¼ miles because ⅓ is a little larger than ¼. Effectively using benchmarks, students would reason:

1 ¼ + ¾ = 2, so

1 ⅓ + ¾ must be greater than 2.

When students use area models like fraction circles to add fractions they need to make the pieces the same size to get a common denominator to rename the sum using one fraction or mixed number. Using the fraction circles for an extended length of time allow students to develop several strategies to find a common denominator. These strategies are a critical step towards the initial use of the number line because the number line needs to be partitioned into a common size before representing each fraction together as one distance.

For this problem, students first have to decide to partition the number line into twelfths before representing either fraction as illustrated in Figure 3. Next they would have to be able to reason with the equivalencies in order to represent the problem.

1 ⅓ = 1 $^{4}/_{12}$ and ¾ = $^{9}/_{12}$

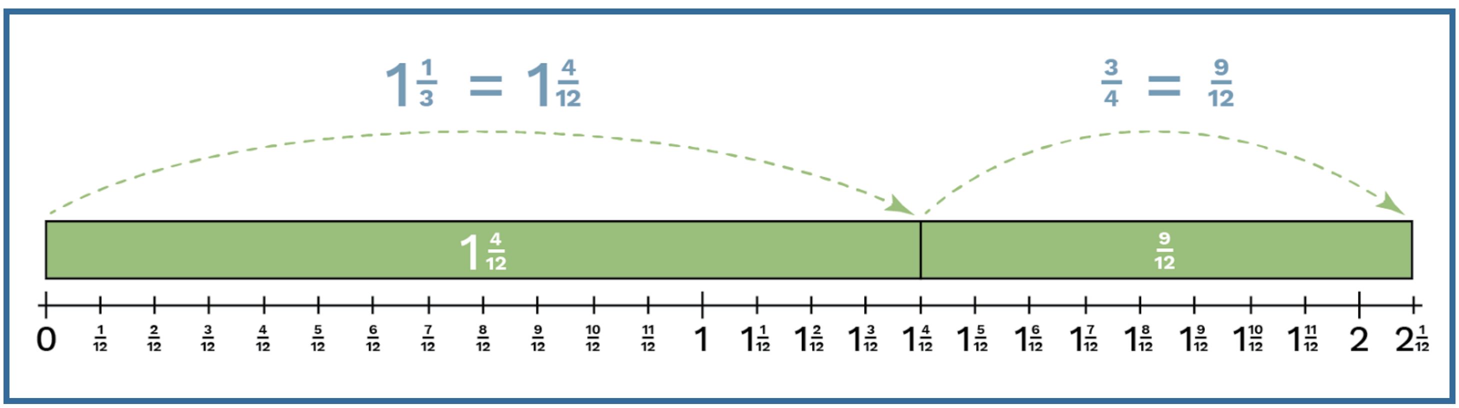


FIGURE 3: Adding 1 ⅓ plus ¾ on the Number Line

Once students have strong mental images using an area model for fraction addition, it is important to introduce the number line model. Here are some tips as your students transition to using this model:

- **Contextual problems** – Use contextual problems like the example above to help students make sense of adding lengths.
- **Benchmarks** – Spend time exploring how benchmarks ½ and 1 can be represented in different ways on the number line. For example, ½ can be seen as equivalent to $^{6}/_{12}$ by adding tick marks while fractions like $^{2}/_{4}$ can be seen as equivalent to ½ by removing tick marks.

- **Role of 0** – Focus on the role of 0 on the number line. 0 is an important benchmark because the unit on a number line is specified by the length between 0 and 1. Help your students become aware of the importance of where 0 and 1 are placed as they become comfortable with partitioning number line.
- **Multiple Representations** – Compare fractions circles to the number line. How are they different? How are they the same? For example, some students find fraction circles difficult for representing fractions greater than 1 because they have to add additional circles. Explore how the number line is simply extended to represent larger and larger fractions and can be effective for visualizing sums using a larger set of benchmark numbers.

Flexible and Confident

We know students often struggle with fraction operations and typically their first experience is with fraction addition.

> By taking the time to ensure that students have strong mental images of fractions and develop connections among fractions and common benchmarks such as 0, ½ and 1, we can dramatically improve the opportunity for students to develop the number sense that they need to succeed.

If we then thoughtfully build on this conceptual understanding with the introduction of the number line, and support their ability to flexibly move among these representations, we can ensure that students have the foundation, flexibility, and confidence they need to succeed in mathematics.

About Dr. Terry Wyberg

Dr. Terry Wyberg is a Senior Lecturer of Mathematics Education at the University of Minnesota. Dr. Wyberg is the Co-Principal Investigator on the latest Rational Number Project (RNP) grant funded by National Science Foundation that produced the second RNP curriculum. Dr. Wyberg has taught methods courses for K-12 pre-service teachers at the University of Minnesota for the past 20 years and provided large-scale professional development for in-service teachers in Minnesota rational numbers, algebra, and number sense. His publications, state, and national presentations are related to teaching and learning of rational numbers.