

Rigid Jointed Frame Analysis

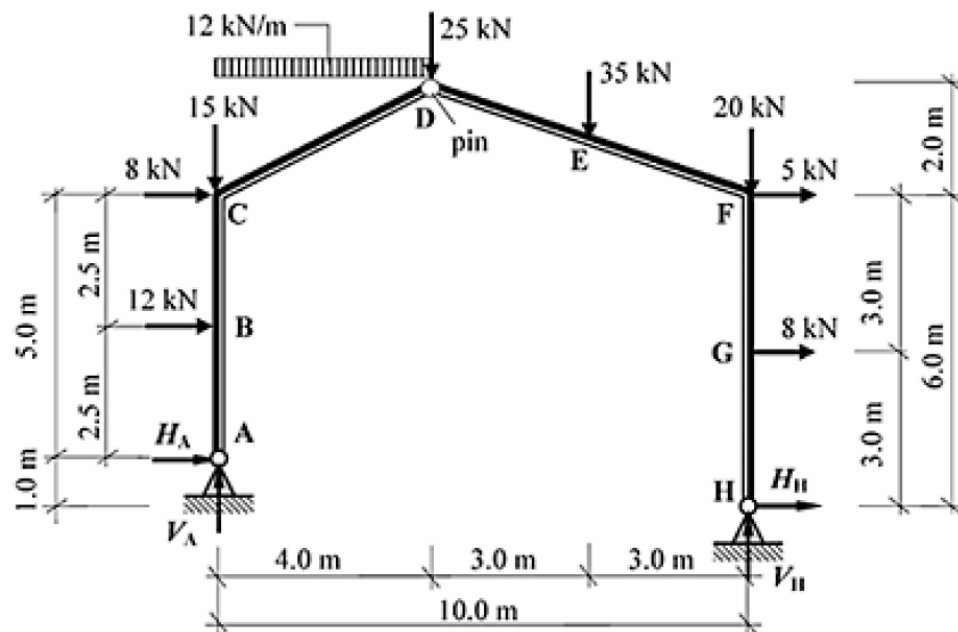
Title

Statically determinate structural analysis for a Rigid Jointed Frame

Description

A pitched-roof portal frame is pinned at supports A and H and members CD and DEF are pinned at the ridge as shown in Figure below. For the loading indicated:

- Determine the support reactions and
- Sketch the axial load, shear force and bending moment diagrams.

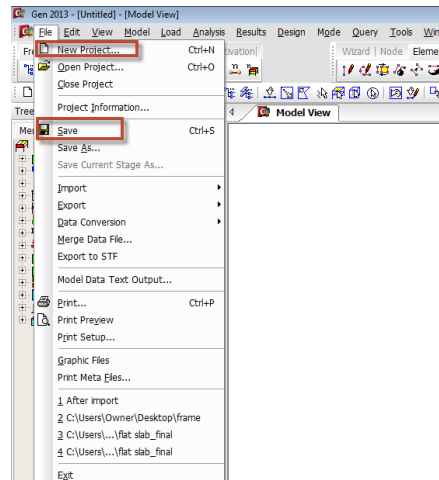


Structural geometry and analysis model

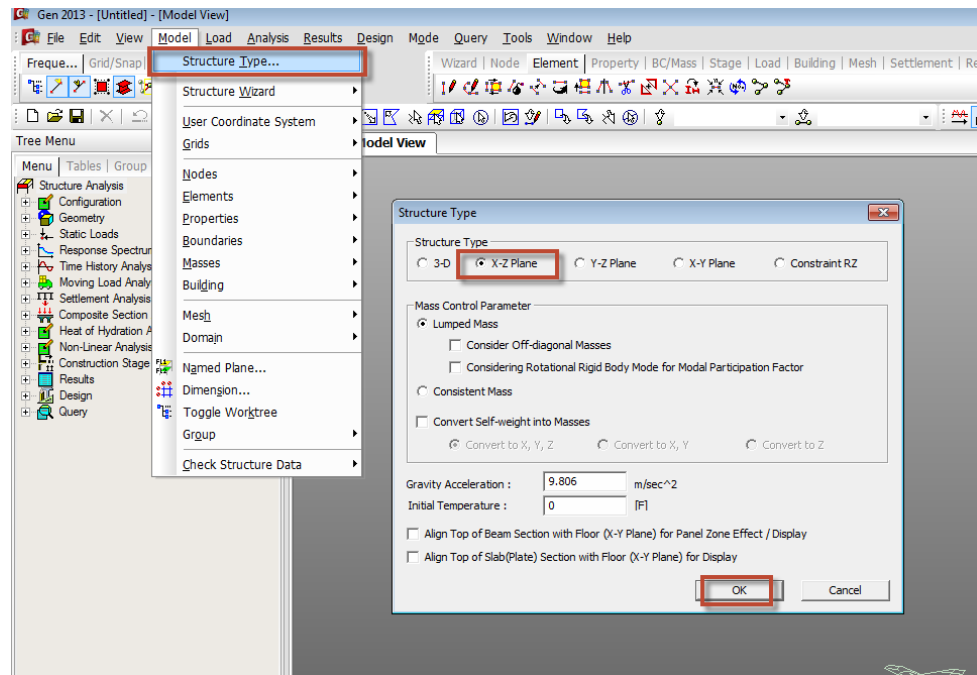
Finite Element Modelling:

- **Analysis Type:** 2-D static analysis (X-Z plane)

Step 1: Go to **File>New Project** and then go to **File>Save** to save the project with any name

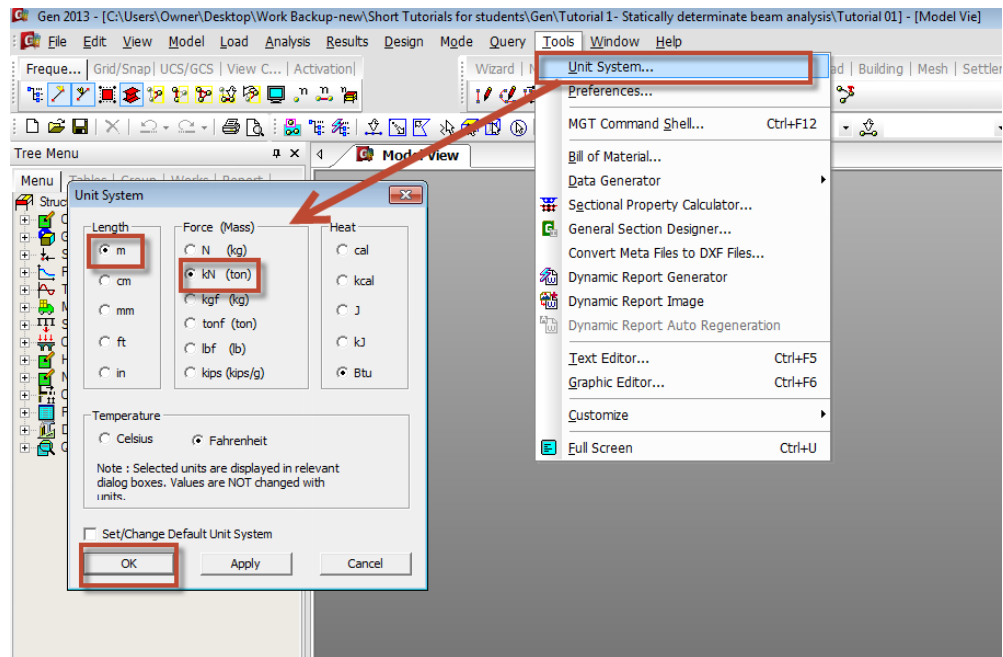




Step 2: Go to **Model>Structure Type** to set the analysis mode to 2D (X-Z plane)

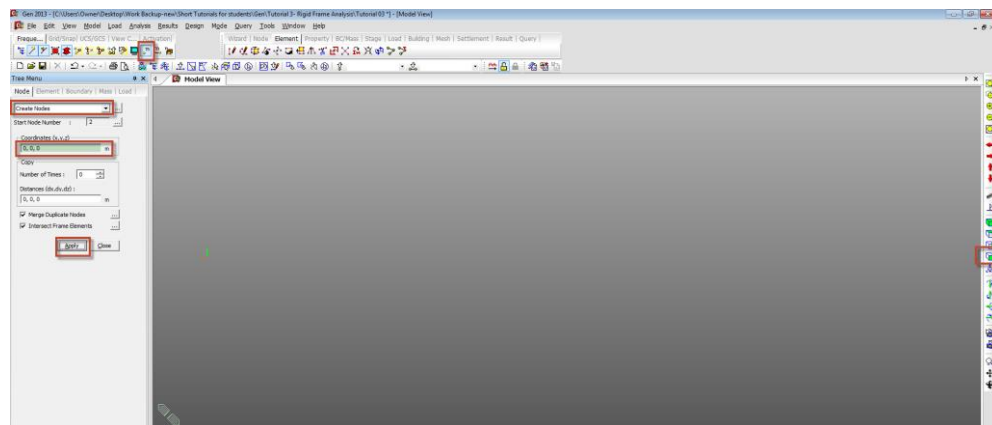



- **Unit System:** kN,m

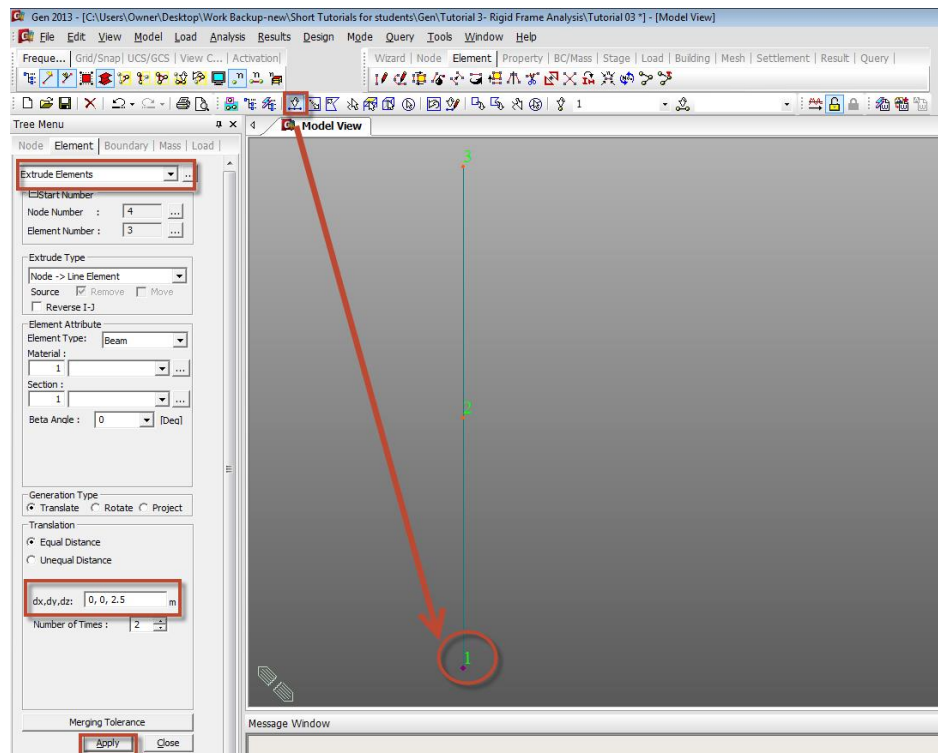
Step 3: Go to **Tools>Unit System** and change the units to kN and m.



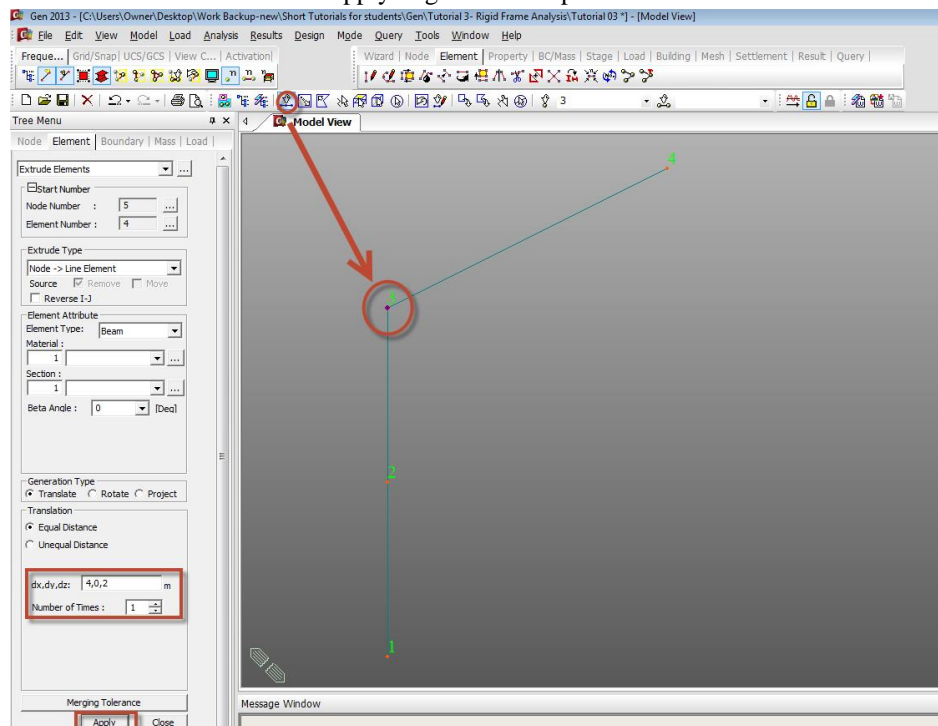
- Geometry generation:**
Step 4: Go to **Model>Nodes>Create Nodes** and type in (0,0,0) in the Coordinates box. Press Apply. Switch to the Front view by clicking on  as shown below. Switch on node number from the option . Node number 1 will be point A.



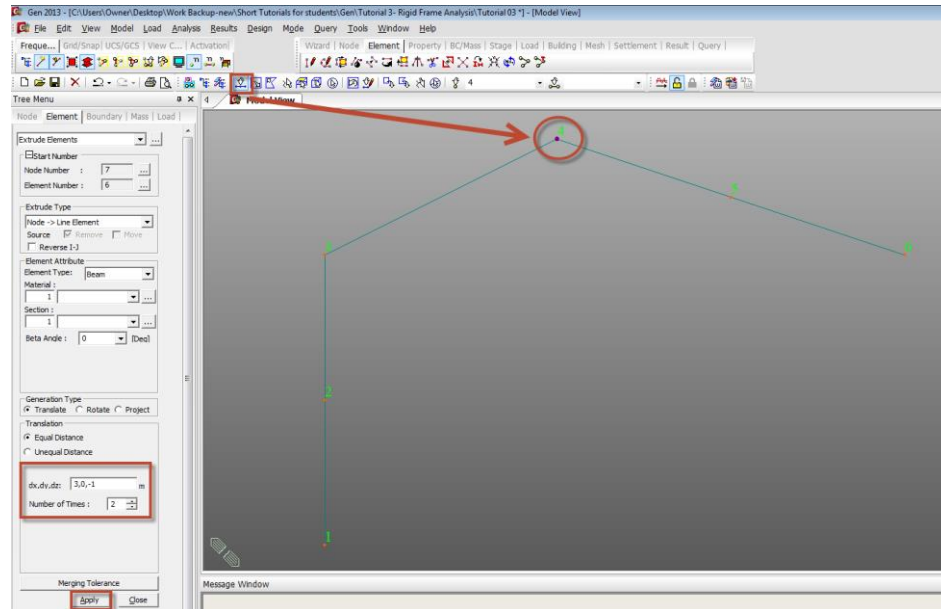
Step 5: Go to **Model>Element>Extrude Elements**. Select type as Node->Line. Use select single button  to highlight the node number 1 by clicking on it. Type in (0,0,2.5) in the Equal Distances dx,dy,dz box and enter number of times=2 and click Apply to generate the column.



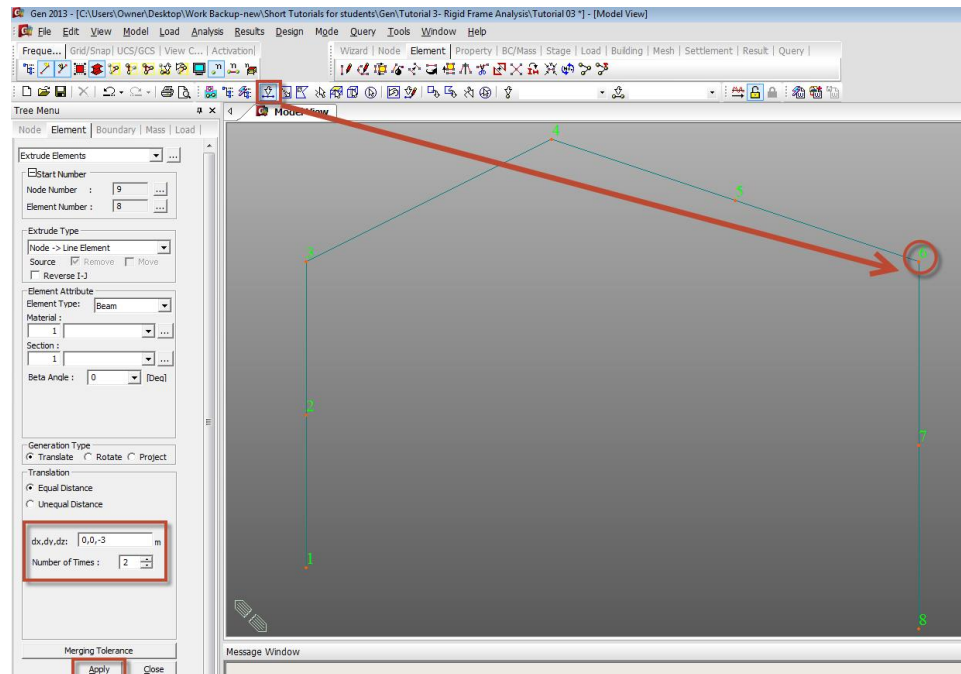
Step 6: Similar to Step 5, select this time node number 3 and enter dx,dy,dz as (4,0,2) m, number of times=1 and click Apply to generate the pitched roof.



Step 7: Select node 4 and extrude using $dx,dy,dz = (3,0,-1)$ and number of times = 2 and Apply to generate the other half of the pitched roof.

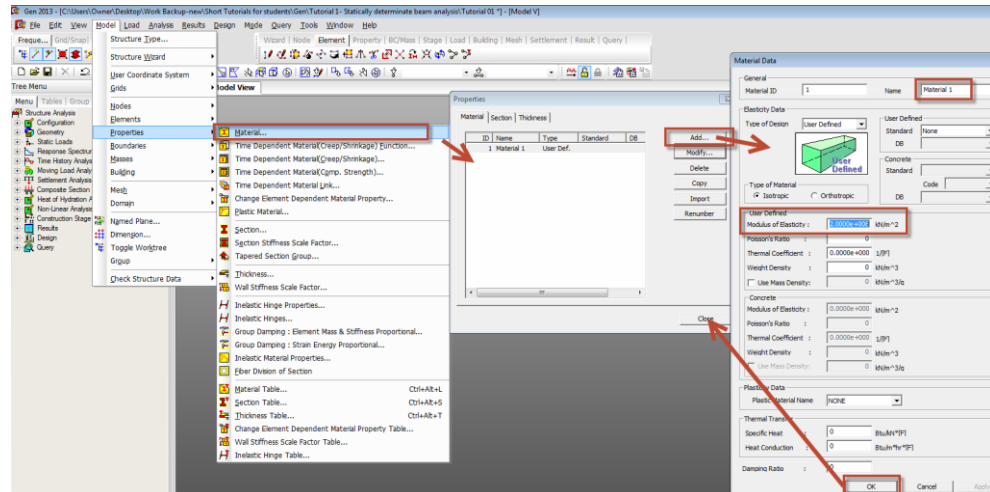


Step 8: Finally Select node 6 and extrude using $dx,dy,dz = (0,0,-3)$ and number of times=2 and click Apply and Close to generate the other column.



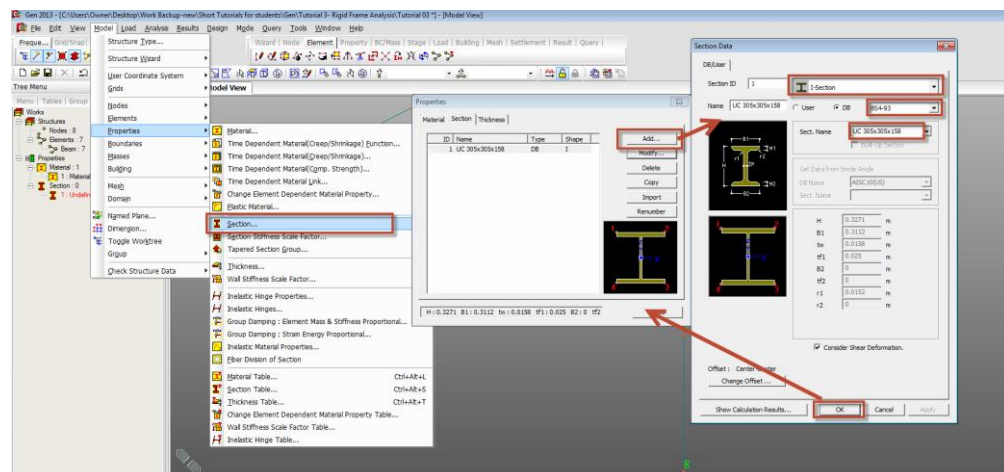
- **Material:** Modulus of elasticity, $E = 2.0 \times 10^8 \text{ kN/m}^2$

Step 9: Go to **Properties>Material Properties>Add**. Select User defined in the Type of Design and Enter $E=2.0 \times 10^8 \text{ kN/m}^2$. Enter a name for the material and click OK and Close.

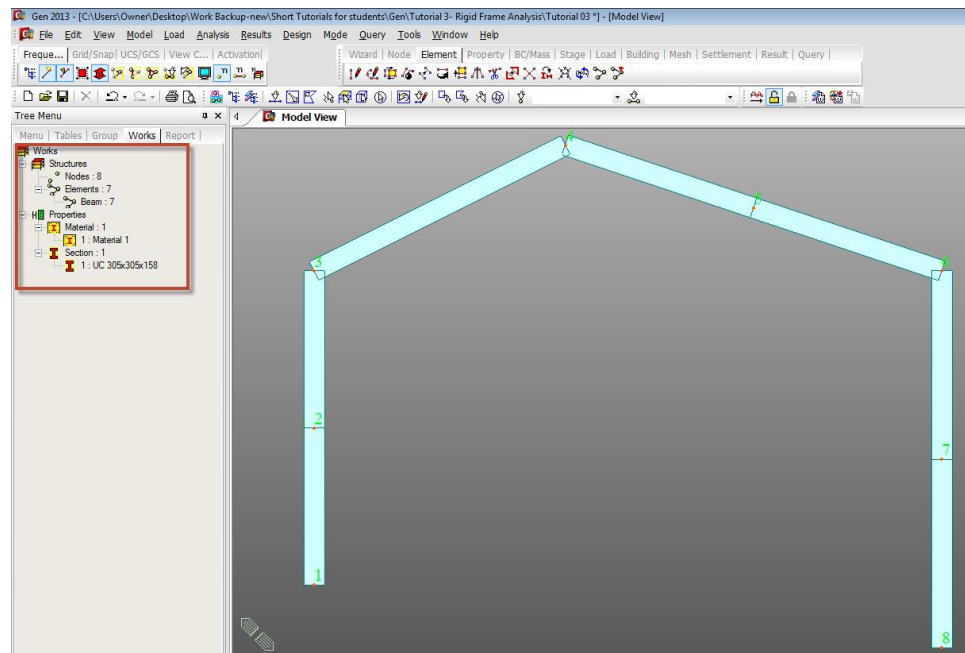


- **Section Property:** BS Column section UC 305x305x158


Step 10: Go to **Properties>Section Properties>Add**. Select DB/User tab and select I section type. Select BS4-93 from section DB and select the section UC 305x305x158 from the dropdown. Click OK and Close.

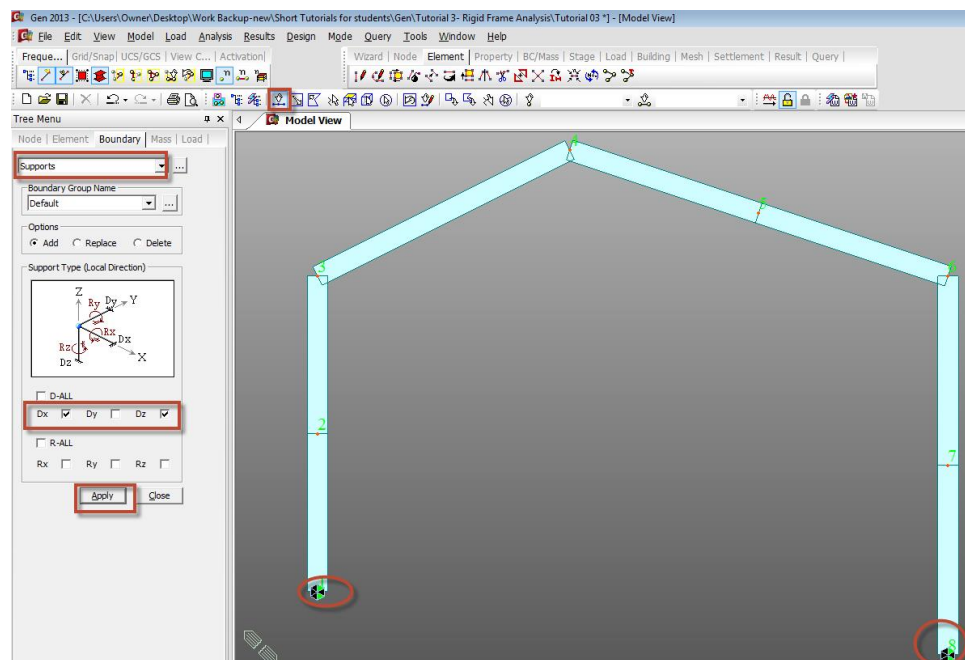


You can see the shape of the section in the model generated. Go to the Works Tree to check the information for your model.





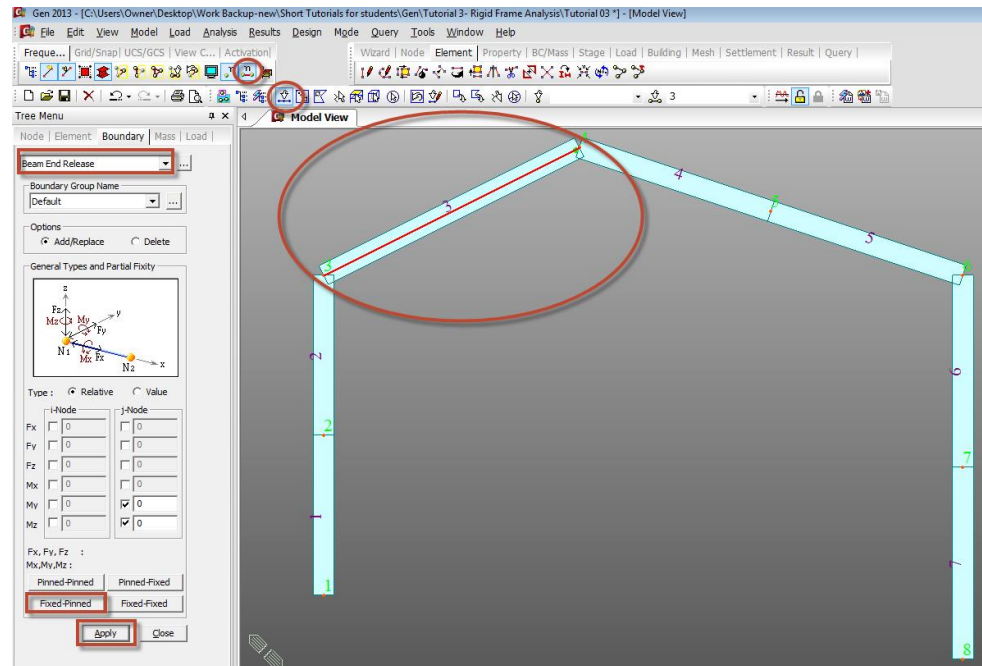
- **Boundary Condition:** Pinned Supports at A and H and a pinned connection between members CD and DEF at point D.

Step 11: Use select single  to select or highlight nodes 1 and 8 as shown in figure below. Go to **Model>Boundaries>Supports** and check on Dx and Dz and Apply.



This becomes the Pinned support at A and H. DX and DZ provide horizontal and vertical restraint simultaneously. No rotational restraint has been provided.

Step 12: Select and display element numbers by clicking on . Again use Select Single , this time to highlight or select element 3. Go to **Model>Boundaries>Beam End Release**. Select Fixed-Pinned and Apply and Close.



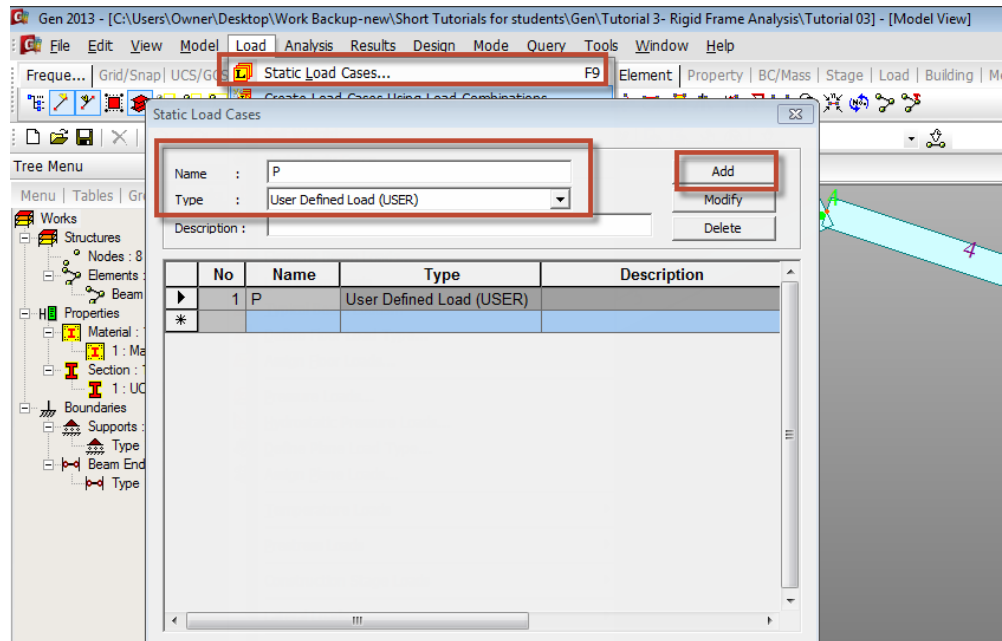
This applies a moment release or a pinned connection at point D.


- **Load Case:**

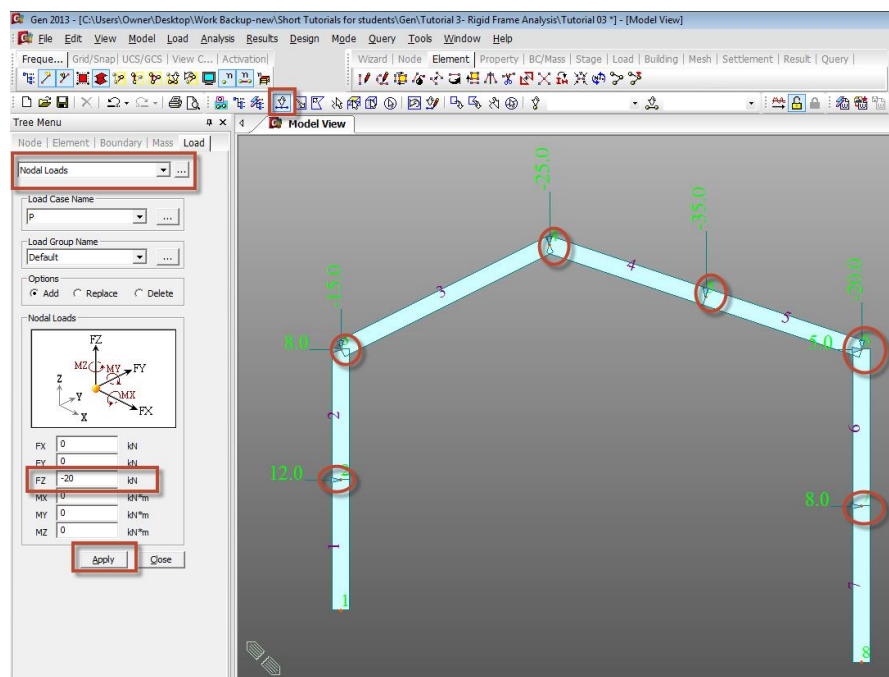
Concentrated Loads:

- Vertically downward loads of 15 kN, 25 kN, 35 kN and 20 kN at points C, D, E and F respectively applied in (-Z) direction.
- Horizontal Loads of 12kN, 8kN, 5kN and 8kN at points B, C, F and G respectively applied in (+X) direction.

Step 13: Go to **Load>Static Load Cases** and define static load case 'P'. Select Load type as User defined for both of them. Click Close after adding the load case.



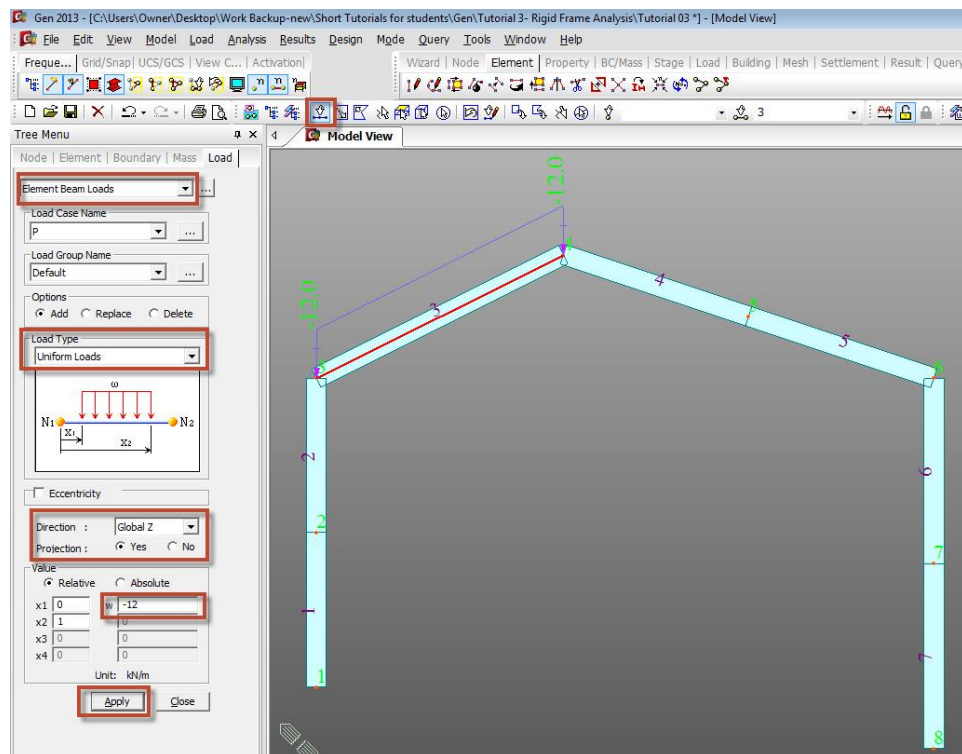
Step 13: Go to **Load>Nodal Loads** and select load case P. Select the node 2 using  and enter $F_X=12$ kN and press Apply. Select Node 3 and enter $F_X=8$ kN and $F_Z = -15$ kN and Apply. Select Node 4 and enter $F_Z=-25$ kN and Apply. Select Node 5 and enter $F_Z=-35$ kN and Apply. Select Node 6 and Apply $F_X=5$ kN and $F_Z= -20$ kN. Select node 7 and Apply $F_X=8$ kN.




You can display the values from the Works Tree by Right clicking on Nodal Loads and click Display.




Uniform Loads: A uniformly distributed load, UDL of 12 kN/m is applied vertically downward on the member CD.

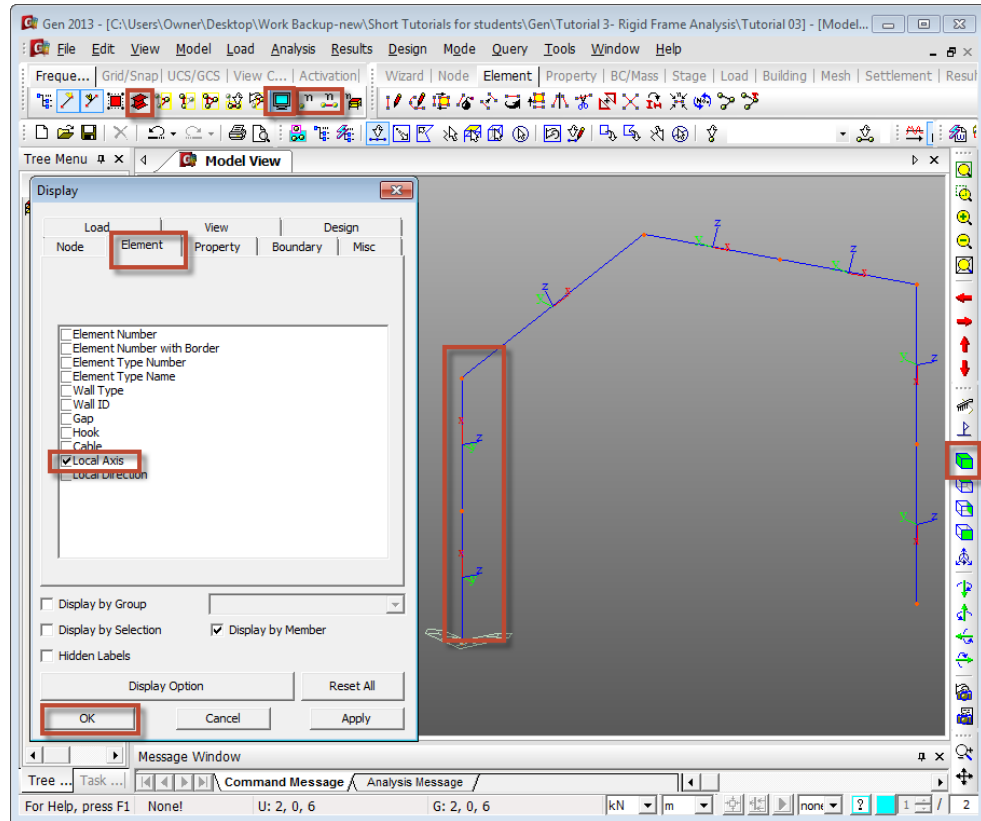
Step 14: Go to **Loads>Element Beam Loads** and select load case P. Select the element number 3 and enter $w=-12$ kN/m in the Global Z direction and select Projection as Yes and press Apply and Close.



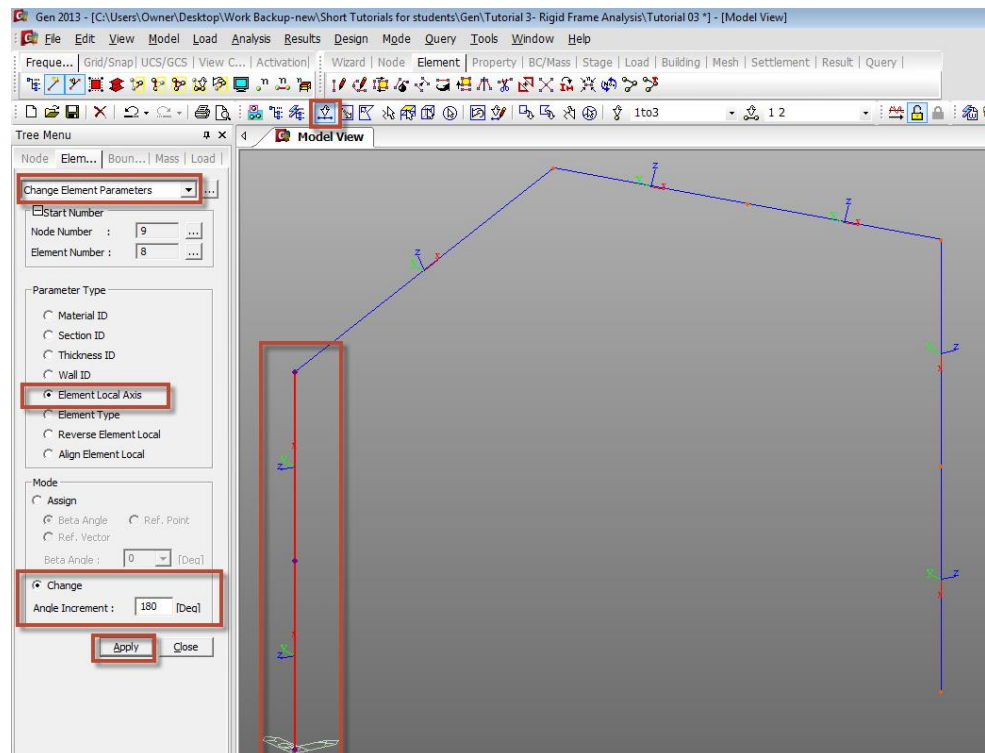
From the display, the UDL looks as if it has been applied over the length of the element 3 but internally it has been applied only on the projected length i.e. X direction length of that element since the Projection 'Yes' was selected while applying the load.


The analysis results for forces and moments in the frame are generated in the finite element software with respect to each element's local axis or centroidal axis. So before running the analysis, we must make sure that the local axes of all elements are aligned properly. Turn on the isometric view using . To display the local axis, go to **View>Display** and select Element tab and check on Local axis and click OK. You will find that the members A to C or the element numbers 1 and 2 have their local y axis inclined in the opposite direction to the rest of the model's y local axis. This may result in opposite signs for bending moments, M_y .

Check off node numbers  and element numbers . Also display only wireframe by clicking on  for clarity.




Now to align the local axis properly, go to **Model>Elements>Change Element Parameters** and go to Element Local Axis and select the 2 elements 1 and 2 that need to be changed and enter the change in beta angle (local axis angle)=180 degrees in the Change Mode.

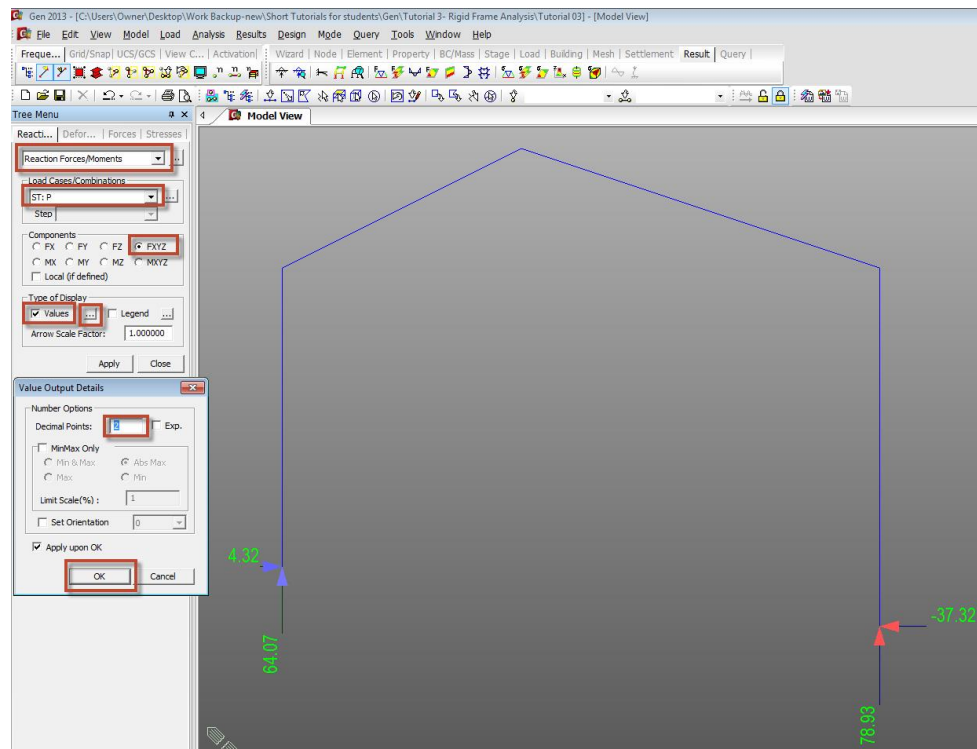


You can now undisplay the local axis by going back to **View>Display** and checking off Local Axis in the Element tab and click Apply. Switch to front view by clicking on  from right.

- **Analysis:**
Step 15: Go to **Analysis>Perform Analysis**

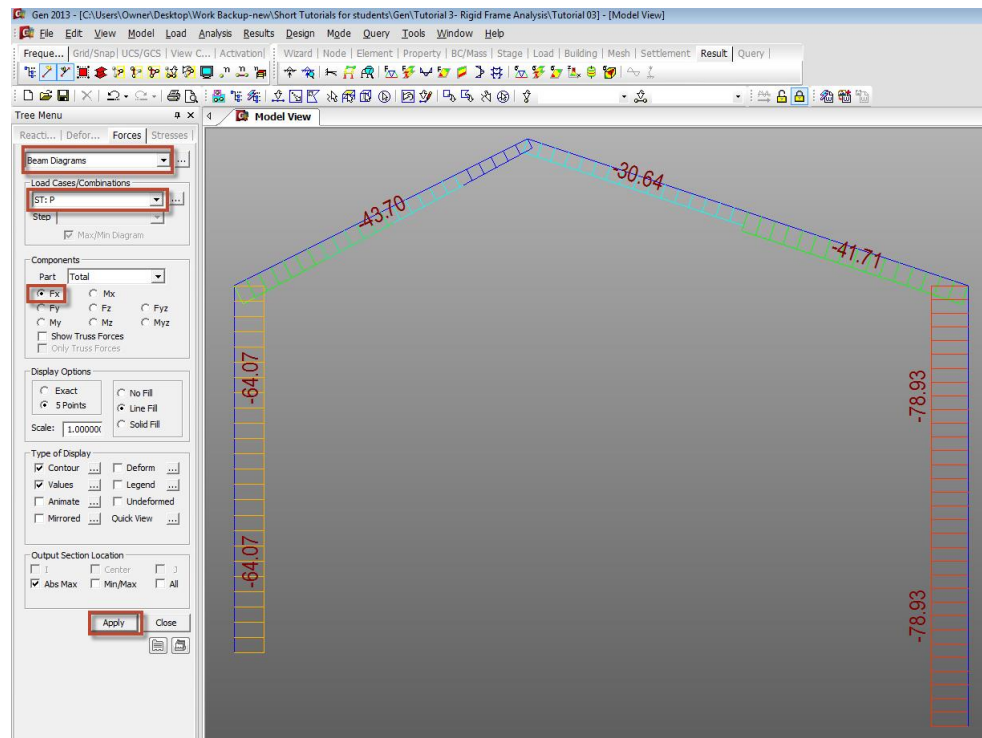
Results

- **Reaction Forces:**
Step 16: Click on **Results>Reactions>Reaction Forces/Moments** and select the load case P. Select FXYZ. Check on Values and click on the box  next to Values to change number of decimal points to 2 and click OK to see reactions graphically.

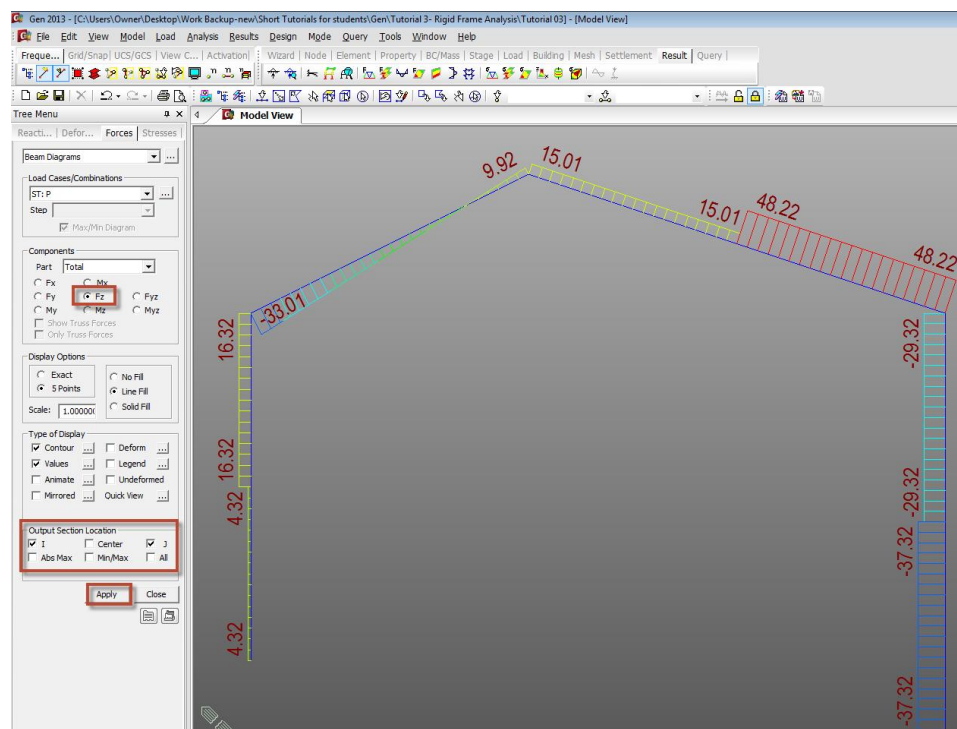


- **Beam forces/moment diagrams:**

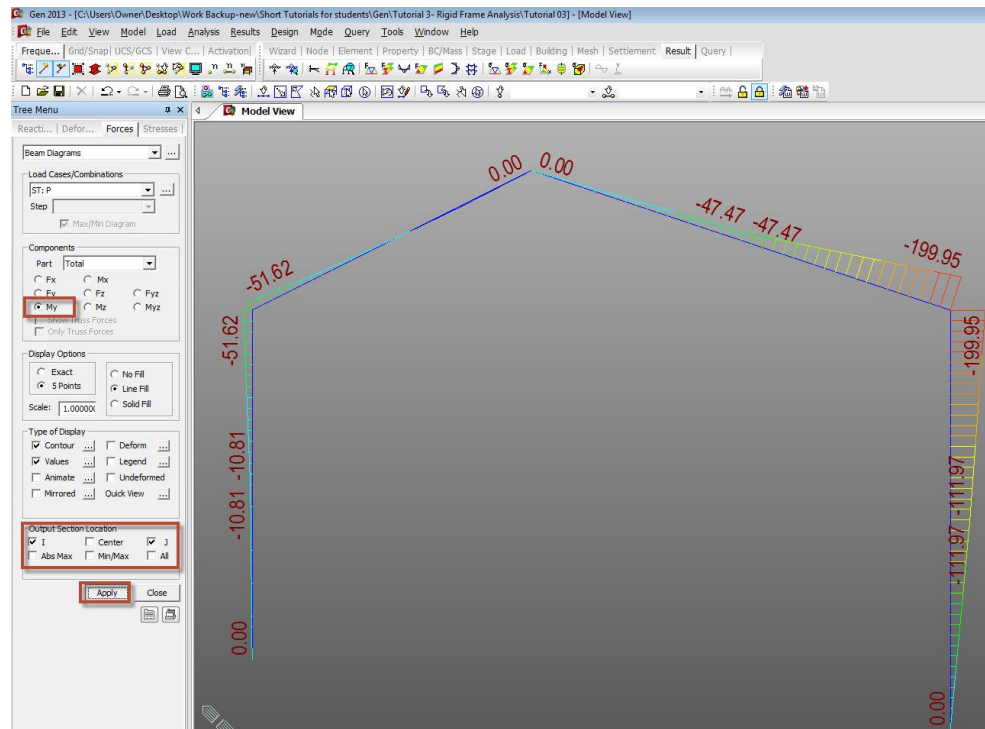
Step 17: Click on **Results>Forces>Beam Diagrams** and select the load Case P. Select Fx and Click OK to display the axial force diagram in the members. Note that the '+' sign indicates Tension and '-' indicates Compression.



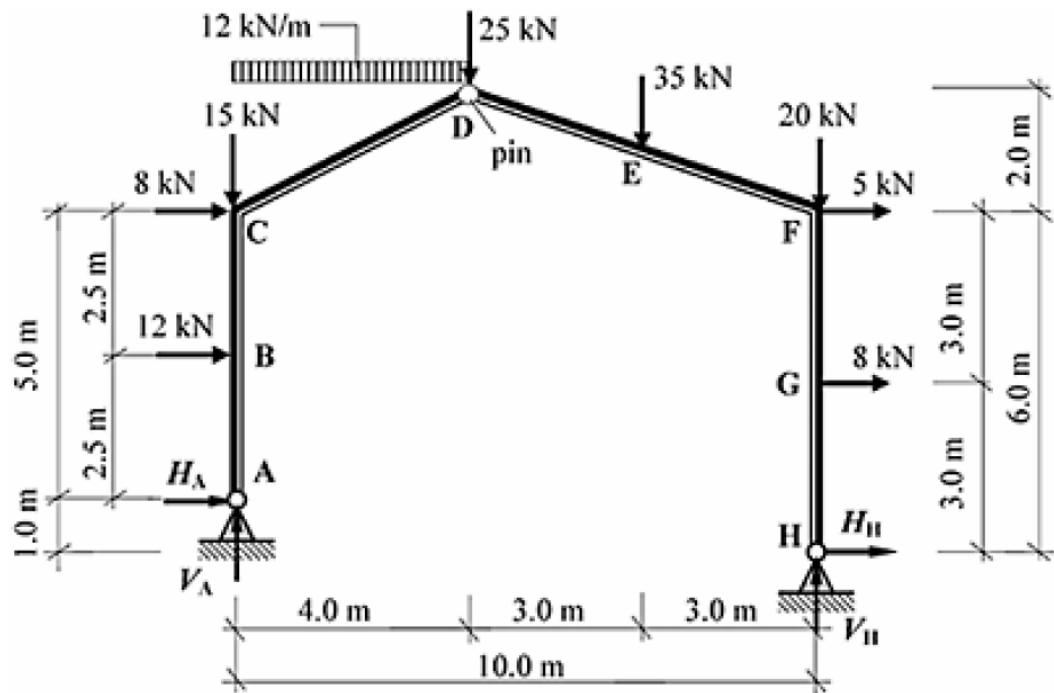
Step 18: Change to Fz to view the shear force diagram and click Apply.



Step 19: Change to My to calculate the Bending Moment Diagram.



Hand Calculations:



(1) To determine support reactions

Taking vertical force sum:

$$\Sigma V = 0$$

$$V_A - 15.0 - (12.0 \times 4.0) - 25.0 - 35.0 - 20.0 + V_H = 0$$

$$V_A + V_H = 143 \quad \text{Equation (1)}$$

Taking horizontal force sum:

$$\Sigma H = 0$$

$$+H_A + 12 + 8.0 + 5.0 + H_H = 0$$

$$H_A + H_H = -25.0 \quad \text{Equation (2)}$$

Taking sum of moments about A:

$$\Sigma M_A = 0$$

$$(12 \times 2.5) + (8.0 \times 5.0) + (12.0 \times 4.0)(2.0) + (25.0 \times 4.0) + (35.0 \times 7.0) \\ + (20.0 \times 10.0) + (5.0 \times 5.0) + (8.0 \times 2.0) - (H_H \times 1.0) - (V_H \times 10.0) = 0$$

$$H_H + 10.0V_H = 752 \quad \text{Equation (3)}$$

Taking sum of moments about the Pin D:

$$\Sigma M_{pin} = 0$$

$$+(35.0 \times 3.0) + (20.0 \times 6.0) - (5.0 \times 2.0) - (8.0 \times 5.0) - (H_H \times 8.0) - (V_H \times 6.0) = 0$$

$$8.0H_H + 6.0V_H = 175 \quad \text{Equation (4)}$$

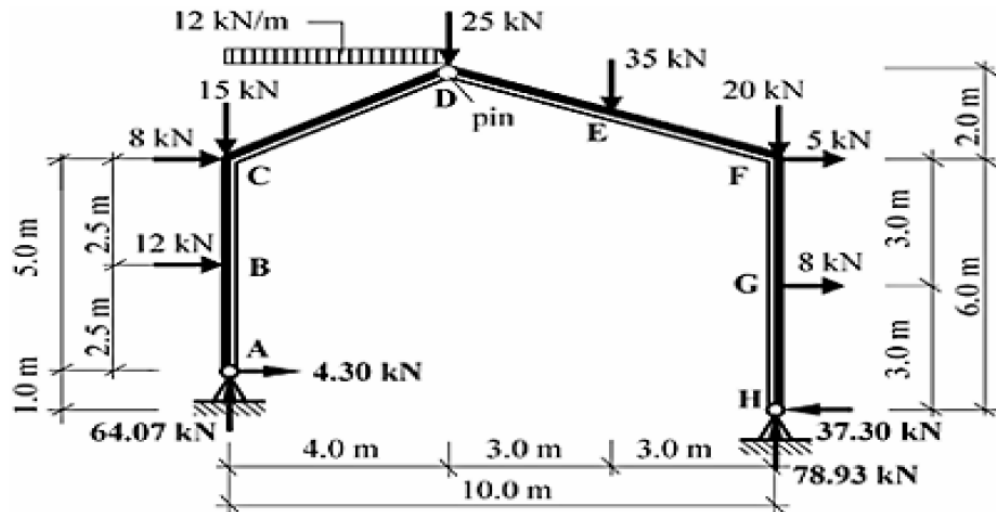
From Equations (1),(2),(3) and (4) we obtain:

$$H_A = +4.30 \text{ kN}$$

$$V_A = +64.07 \text{ kN}$$

$$H_H = -37.30 \text{ kN}$$

$$V_H = +78.93 \text{ kN}$$



(2) To determine member forces:

Assuming anticlockwise bending moments to be positive:

$$M_A = 0 \quad (\text{Pinned Support} - \text{No moments})$$

$$M_B = -(4.30 \times 2.5) = -10.75 \text{ kNm}$$

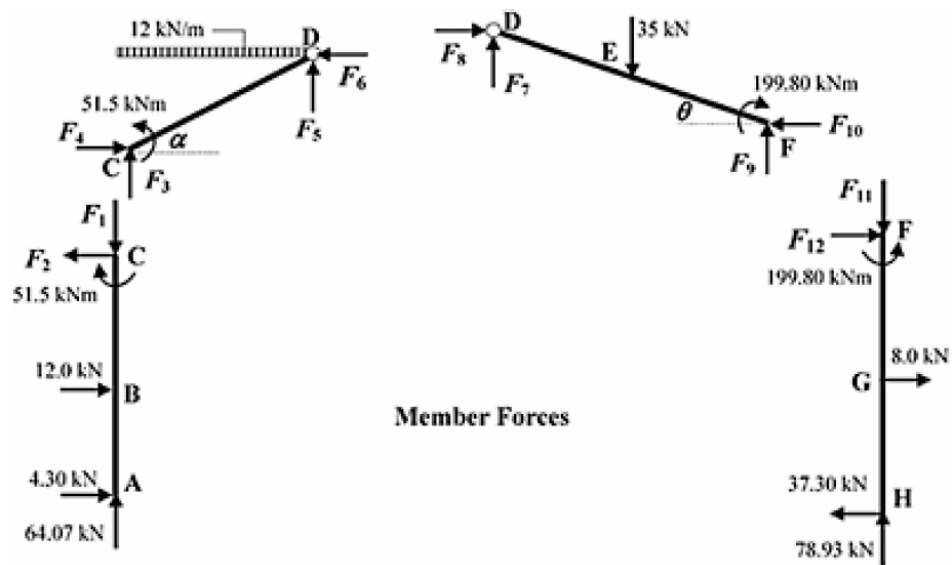
$$M_C = -(4.30 \times 5.0) - (12.0 \times 2.5) = -51.50 \text{ kNm}$$

$$M_D = 0 \quad (\text{Pin} - \text{No Moments})$$

$$M_E = -(20.0 \times 3.0) + (5.0 \times 1.0) + (8.0 \times 4.0) - (37.3 \times 7.0) + (78.93 \times 3.0) = -47.31 \text{ kNm}$$

$$M_F = +(8.0 \times 3.0) - (37.30 \times 6.0) = -199.80 \text{ kNm}$$

$$M_G = -(37.30 \times 3.0) = -111.90 \text{ kNm}$$



Member Forces

The values of the end-forces F_1 to F_{12} can be determined by considering the equilibrium of each member and joint in turn.

Consider Member ABC:

$$(\Sigma V = 0)$$

$$-F_1 + 64.07 = 0$$

$$\therefore F_1 = 64.07 \text{ kN}$$

$$(\Sigma H = 0)$$

$$+4.30 + 12.0 - F_2 = 0$$

$$\therefore F_2 = 16.30 \text{ kN}$$

Consider Joint C:

$(\Sigma V = 0)$ There is an applied vertical load at joint C = 15 kN

$$-F_1 + F_3 = -15.0$$

$$\therefore F_3 = 49.07 \text{ kN}$$

$(\Sigma H = 0)$ There is an applied horizontal load at joint C = 8 kN

$$-F_2 + F_4 = +8.0$$

$$\therefore F_4 = 24.30 \text{ kN}$$

Consider Member CD:

$$(\Sigma V = 0)$$

$$+49.07 - (12.0 \times 4.0) + F_5 = 0$$

$$\therefore F_5 = -1.07 \text{ kN}$$

$$(\Sigma H = 0)$$

$$-F_2 + F_4 = +8.0$$

$$\therefore F_6 = 24.30 \text{ kN}$$

Consider Member FGH:

$$(\Sigma V = 0)$$

$$+78.93 - F_{11} = 0$$

$$\therefore F_{11} = 78.93 \text{ kN}$$

$$(\Sigma H = 0)$$

$$-37.30 + 8.0 + F_{12} = 0$$

$$\therefore F_{12} = 29.30 \text{ kN}$$

Consider joint F:

$(\Sigma V = 0)$ There is an applied vertical load at joint F=20 kN

$$F_{11} + F_9 = -20.0$$

$$\therefore F_9 = 58.93 \text{ kN}$$

$(\Sigma H = 0)$ There is an applied horizontal load at joint F=5 kN

$$+F_{12} - F_{10} = +5.0$$

$$\therefore F_{10} = 24.30 \text{ kN}$$

Consider member DF:

$$(\Sigma V = 0)$$

$$+58.93 - 35.0 + F_7 = 0$$

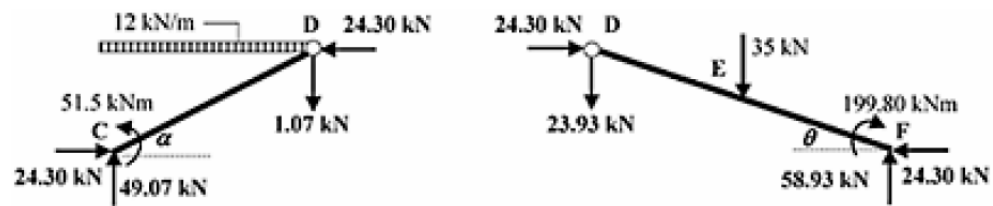
$$\therefore F_7 = 23.93 \text{ kN}$$

$$(\Sigma H = 0)$$

$$-24.30 + F_8 = 0.0$$

$$\therefore F_8 = 24.30 \text{ kN}$$

The calculated values can be checked by considering the equilibrium at joint D.



$$(\Sigma V = 0)$$

$$-1.07 - 23.93 = -25.0 \text{ kN (equal to the applied vertical load at D)}$$

$$(\Sigma H = 0)$$

$$-24.30 + 24.30 = 0.0$$

The axial force and shear force in member CD can be found from:

$$\text{Axial load} = +/-(\text{Horizontal force} \times \cos \alpha) +/-(\text{Vertical force} \times \sin \alpha)$$

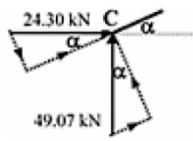
$$\text{Shear force} = +\frac{\text{Horizontal force} \times \sin \alpha}{-} + \frac{\text{Vertical force} \times \cos \alpha}{-}$$

The signs are dependent on the directions of the respective forces.

$$\alpha = \tan^{-1}(2.0/4.0) = 26.565^\circ$$

$$\cos \alpha = 0.894; \sin \alpha = 0.447$$

Member CD:

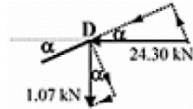


Assume axial compression to be positive.

At joint C

$$\text{Axial force} = +(24.30 \times \cos \alpha) + (49.07 \times \sin \alpha) = +43.66 \text{ kN}$$

$$\text{Shear force} = -(24.30 \times \sin \alpha) + (49.07 \times \cos \alpha) = +33.01 \text{ kN}$$



At joint D

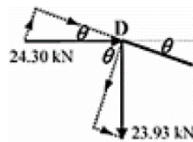
$$\text{Axial force} = +(24.30 \times \cos \alpha) + (1.07 \times \sin \alpha) = +22.20 \text{ kN}$$

$$\text{Shear force} = -(24.30 \times \sin \alpha) + (1.07 \times \cos \alpha) = -9.91 \text{ kN}$$

Member DEF:

$$\theta = \tan^{-1}(2.0/6.0) = 18.435^\circ$$

$$\cos \theta = 0.947; \sin \theta = 0.316$$

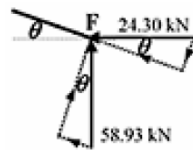


Assume axial compression to be positive.

At joint D

$$\text{Axial force} = +(24.30 \times \cos \theta) + (23.93 \times \sin \theta) = +30.57 \text{ kN}$$

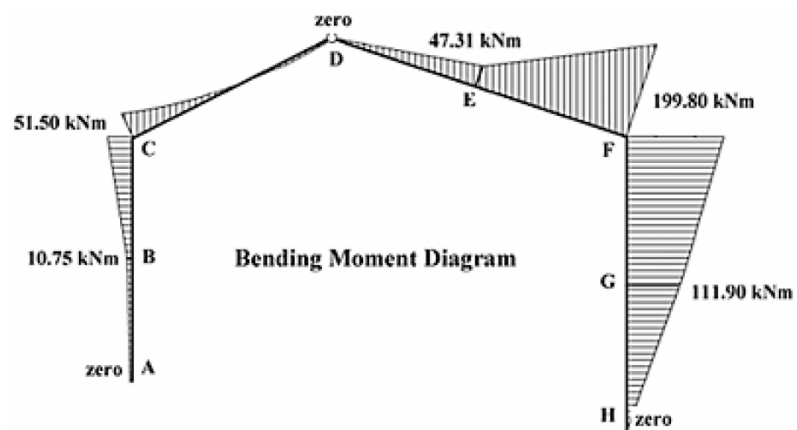
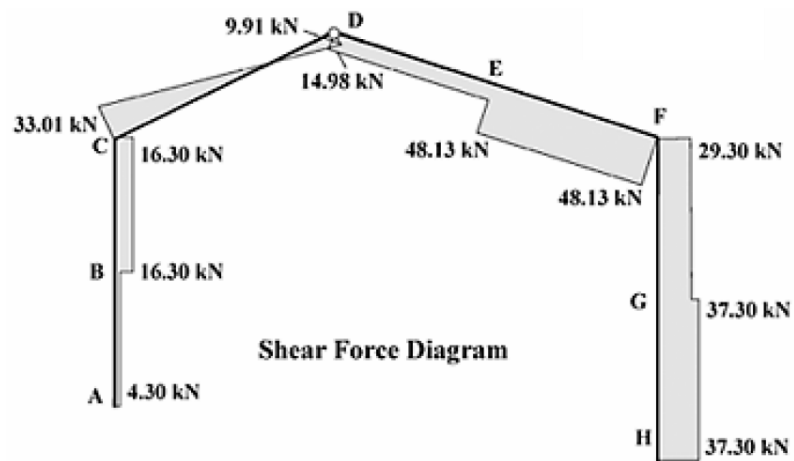
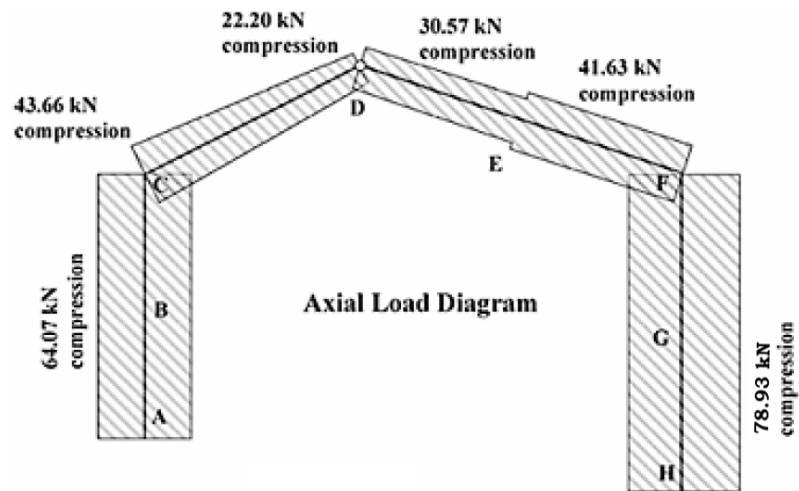
$$\text{Shear force} = +(24.30 \times \sin \theta) - (23.93 \times \cos \theta) = +14.98 \text{ kN}$$



At joint F

$$\text{Axial force} = +(24.30 \times \cos \theta) + (58.93 \times \sin \theta) = +41.63 \text{ kN}$$

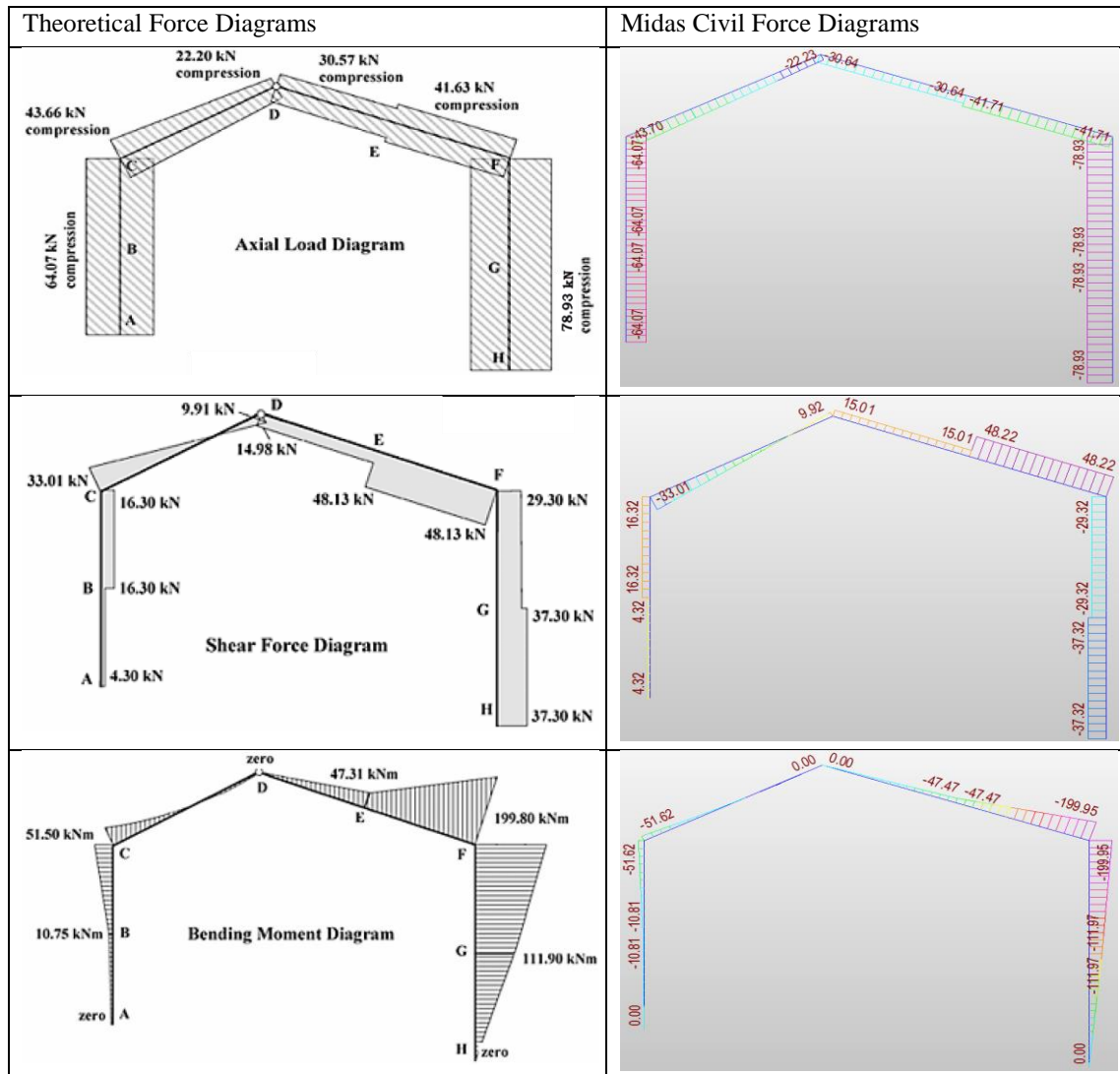
$$\text{Shear force} = -(24.30 \times \sin \theta) + (58.93 \times \cos \theta) = +48.13 \text{ kN}$$



Comparison of Results

Unit : kN,m

Reactions	Node Number	Theoretical	Midas Gen
H_A	1	4.32	4.32
V_A	1	64.07	64.07
H_H	8	-37.32	-37.32
V_H	8	78.93	78.93



Reference

William M.C. McKenzie, "Examples in Structural Analysis", 1st Edition, Taylor & Francis 2 Park Square, Milton Park, Abingdon, Oxon OX14 4RN, 2006, 5.1.2 Example 5.2, Page 361.