Arrhenius Plot

Summary

The **Arrhenius Plot** procedure is designed to plot data from an accelerated life test in which failure times have been recorded and percentiles estimated at a number of different temperatures. The percentiles *P* are assumed to follow an Arrhenius model, defined by

$$P = A\exp\left(-\frac{E}{kT}\right) \qquad (1)$$

where *T* is temperature in degrees Kelvin (°C + 273.15), *k* = 1/11605 (Boltzmann's constant), and *A* and *E* are two unknown parameters. Given percentiles at two or more temperatures, the procedure will fit the Arrhenius model and extrapolate the percentile to a normal operating temperature.

Sample StatFolio: *arrhenius.sgp*

Sample Data:

The file *circuits.sgd* contains data from an accelerated life test, where failures were caused by a chemical reaction within an integrated circuit. 10 items were tested at each of 5 different junction temperatures and the failure times recorded. Testing of each item stopped after 3,000 hours if the unit had not yet failed. A portion of the data, reported by Meeker and Escobar (1998), is shown below:

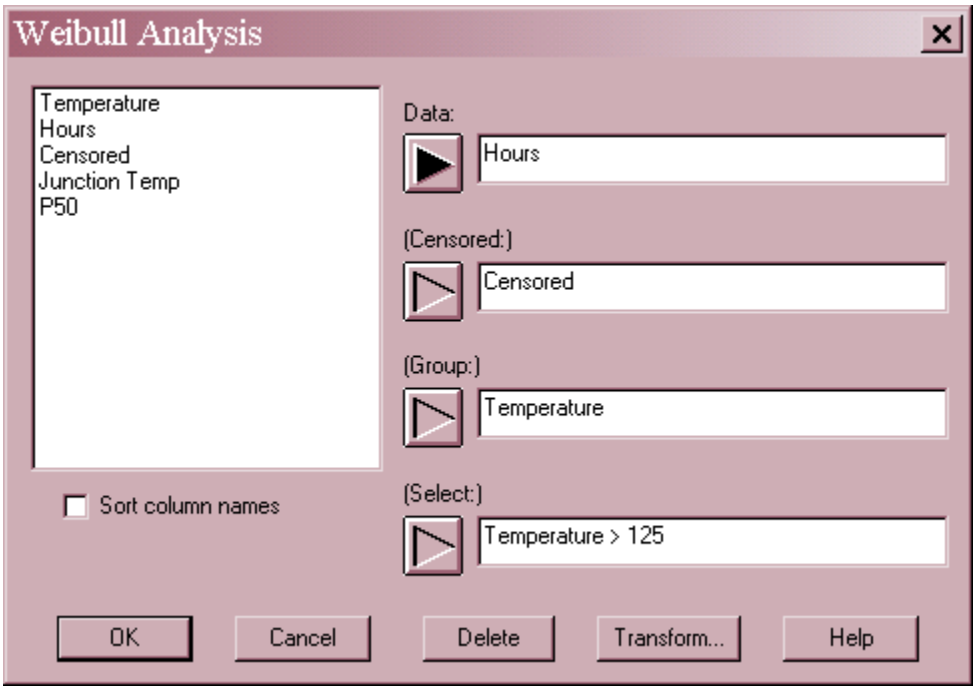
Temperature	*Hours*	*Censored*
150	2350	0
150	2560	0
150	2980	0
150	3000	1
150	3000	1
150	3000	1
150	3000	1
150	3000	1
150	3000	1
150	3000	1
175	800	0
175	1130	0

The *Temperature* ranged from 80°C to 200°C. *Hours* is the time until failure, or 3000 if the circuit did not fail. *Censored* equals 0 for an observed failure time or 1 if the unit was still operating after 3000 hours.

Estimating Percentiles

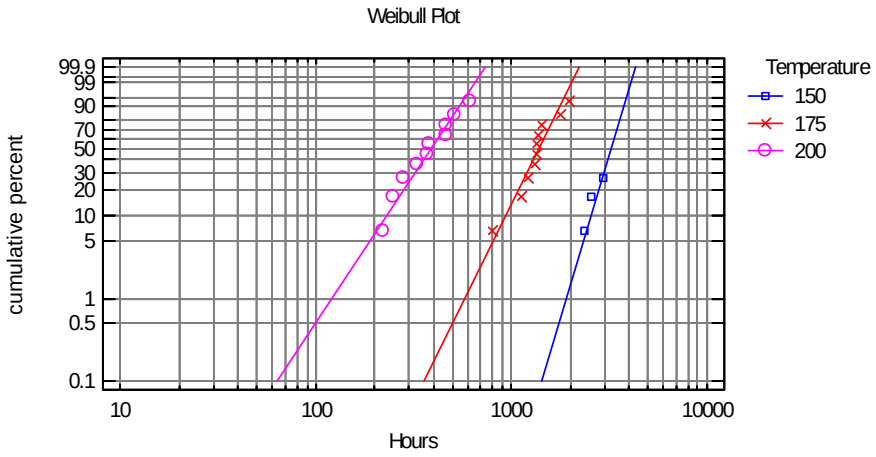
In order to create an Arrhenius plot, percentiles must be estimated for each of the temperatures at which there are observed failures. Assuming a Weibull distribution for the failure times, this can

be done using the *Weibull Analysis* procedure. The data input dialog box for that procedure is shown below:



The entry in the *Select* field, *Temperature > 125*, is used to select the temperatures at which there were failures, since no failures occurred at 125°C or below.

The Weibull plot shows that the data are well-described by Weibull distributions:



The *Critical Values* table may be used to determine the 50th percentiles at each temperature:

Critical Values for Hours

Temperature	*X*	*Lower Tail Area (<)*	*Upper Tail Area (>)*
150	3260.62	0.5	0.5
175	1379.61	0.5	0.5
200	388.615	0.5	0.5

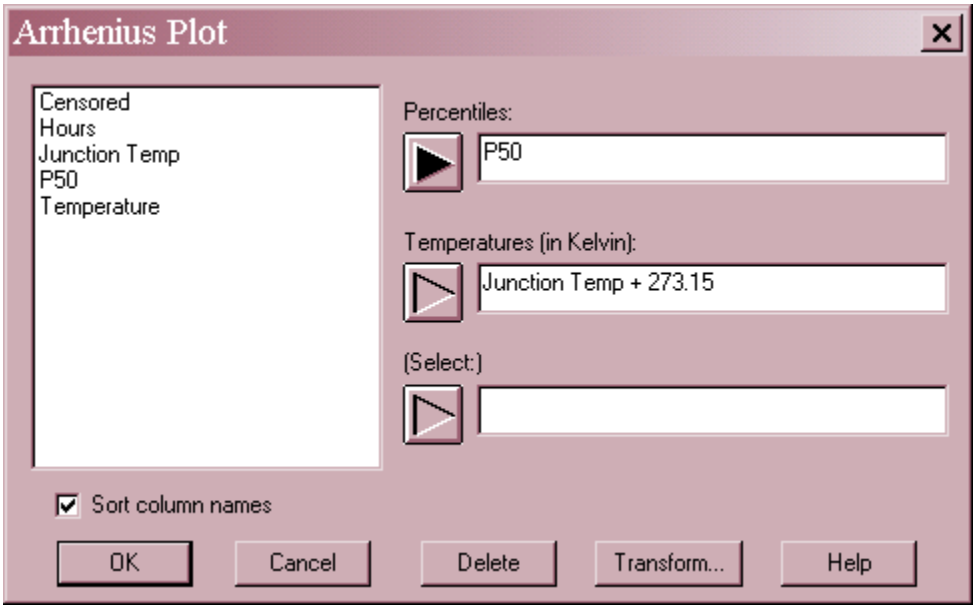
These results have been placed in two additional columns of the *arrhenius.sf3* file, as shown below:

Junction Temp	*P50*
150	3260.62
175	1379.61
200	388.615

The above table is the type of data expected by the *Arrhenius Plots* procedure.

Data Input

The data input dialog box expects columns containing percentiles of the failure time distribution and the corresponding temperatures:



- **Percentiles**: numeric column containing percentiles of the failure time distribution at 2 or more temperatures.

- **Temperatures**: numeric column containing the temperatures in degrees Kelvin. If the original temperatures are in Celsius, add 273.15 to obtain temperature in Kelvin.

- **Select**: subset selection.

Analysis Summary

The *Analysis Summary* displays statistics for the fitted Arrhenius model and an extrapolation of the model to a selected temperature.

Arrhenius Plot
Percentiles: P50
Temperatures: Junction Temp + 273.15

Fitted model
P50 = 0.0000070398*exp(0.730577/k*Junction Temp + 273.15)
where k = Boltzmann's constant (8.617E-5 EV/degrees K)

Regression Statistics
Number of observations = 3
Intercept = -11.8639
Slope = 0.730577
R-squared = 97.992%

Prediction
Temperature: 353.15
Estimated percentile: 187923.0
Lower 95.0% limit: 16.8628
Upper 95.0% limit: 2.09425E9

The table shows:

- **Fitted Model** - the fitted Arrhenius model. In this case, the equation is

$$P_{50} = 7.04 \times 10^{-6} \left[\exp\left(\frac{0.7306}{kT} \right) \right] \qquad (2)$$

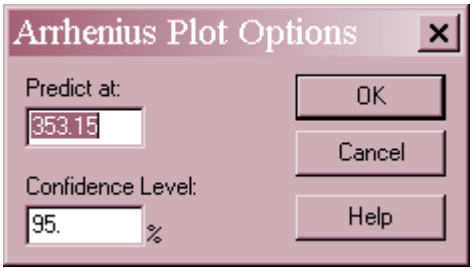
- **Regression Statistics** - the intercept, slope, and R-squared statistic from a linear regression of $log_{10}(P_{50})$ against $1/(kT)$.

- **Prediction** - the estimated percentile at the normal operating temperature, with 95% confidence limits based on the fitted regression model.

In this case, it is estimated that 50% of the items will have failed after 187,923 hours at 80°C.

Analysis Options

The *Analysis Options* dialog box specifies the normal operating temperature at which predictions are desired.

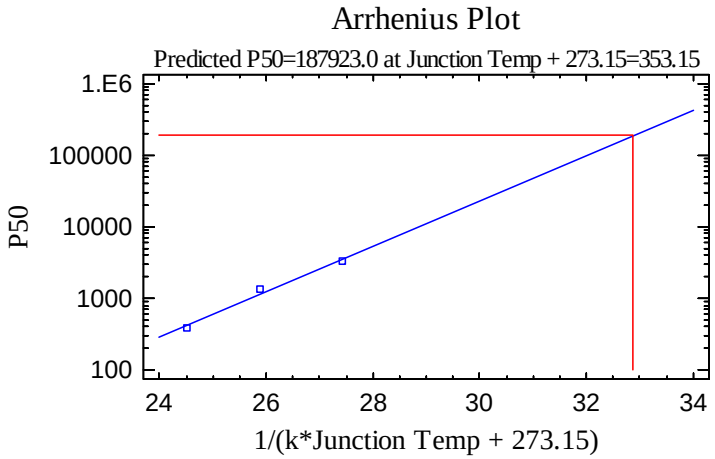


- **Predict at**: the normal operating temperature (in degrees Kelvin) at which the percentile is to be predicted. For a temperature in degrees Celsius, add 273.15. The displayed value of 353.15 corresponds to 80°C.

- **Confidence Level**: the percentage to be used for the confidence interval.

Arrhenius Plot

The Arrhenius Plot shows the data, the fitted model, and the extrapolated percentile.



If the Arrhenius model fits the data well, the points should lie close to the fitted line.

