

Control Chart Design

Summary

This procedure is designed to help determine the appropriate sample size and parameters for common control charts. The design is based on:

1. the magnitude of a shift away from the centerline that you wish to detect.
2. the average acceptable run length if such a shift occurs before an out-of-control signal is generated.

The procedure is most useful for setting up Phase II control charts, i.e., control charts designed to monitor real-time performance of a process once standard operating conditions have been established. Control charts for both variables and attributes are included.

Sample StatFolio: *controldesign.sgp*

Sample Data:

None.

Data Input

The data input dialog box for this procedure is shown below.

- **Type of Chart:** the type of control chart to be created. See the documentation for each of these types of charts for detailed information.
- **Process Parameters:** the established operating parameters for an in-control process. These parameters depend on the type of control chart:
 - P or nP chart* – the mean proportion of defective items.
 - C or U chart* – the mean rate in defects per unit.
 - All other charts* – the mean and standard deviation of the variable being measured.
- **Control Limits:** the type of limits on the chart.
- **Design Requirements:** the desired power or responsiveness of the chart. In designing the chart, you first select a *Target Mean* at which the responsiveness will be fixed. Typically, this value would reflect a serious shift from an in-control situation that you wished the chart to signal quickly. You then have 2 choices:

- **Desired ARL:** Select this option to have the procedure determine how many samples would need to be taken at each time period so that the average run length (number of periods until an out-of-control signal is given) is no greater than the value specified.
- **Minimize ARL for subgroup size 1:** Select this option if you can only take a single measurement at each time period. The procedure will then determine the chart parameters that give the smallest average run length. This option is only available for *Moving Average*, *EWMA*, and *CuSum* charts.
- **Hold:** Choose one of three options to set for the control chart:
 - **Control Limits at:** fix the control limits at the centerline plus and minus the specified multiple of the standard error (sigma). This value would commonly be set to 3.
 - **In-control ARL at:** fix the ARL for an in-control process. The in-control ARL for common 3-sigma X-bar and similar charts equals 370.4.
 - **False-alarm rate at:** fix the probability of obtaining a false out-of-control signal at any selected period. For common 3-sigma X-bar and similar charts, the false alarm rate equals 0.27%.

In the dialog box shown above, an X-bar chart with upper and lower 3-sigma control limits is to be created to monitor a process with a mean $\mu = 50$ and a standard deviation $\sigma = 5$. If the process shifts by one standard deviation, to either $\mu = 55$ or $\mu = 45$, the chart is to generate a signal (a point beyond the 3-sigma limits) within an average of 2 plotting periods.

Analysis Summary

The *Analysis Summary* shows the input parameters and the generated control chart:

<u>Control Chart Design</u>						
Input						
	<i>Sigma</i>	<i>Sigma</i>	<i>Process</i>	<i>Process</i>	<i>Desired</i>	<i>Target</i>
<i>Chart</i>	<i>Limits</i>	<i>Multiple</i>	<i>Mean</i>	<i>Sigma</i>	<i>ARL</i>	<i>Mean</i>
X-bar	Two-sided	3.0	50.0	5.0	2.0	55.0
Selected Chart						
	<i>Required</i>	<i>ARL at</i>	<i>In-Control</i>	<i>False</i>		
	<i>Subgroup Size</i>	<i>Target</i>	<i>ARL</i>	<i>Alarms (%)</i>		
X-bar	9	2.0	370.38	0.269993		

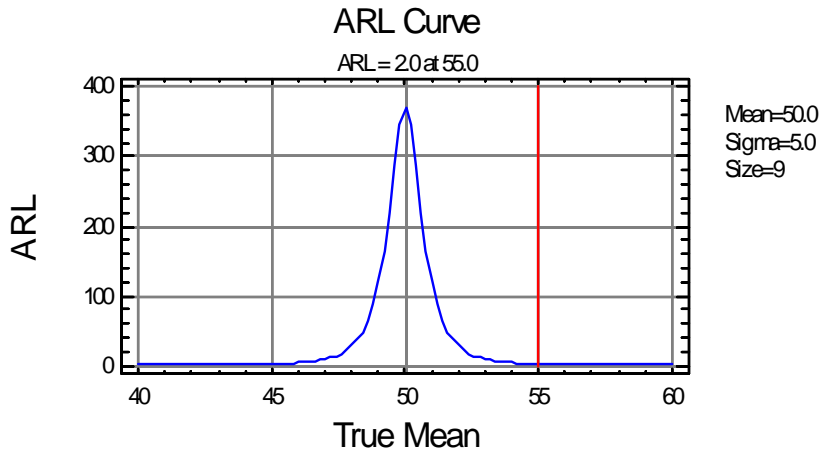
The output includes:

- **Required Subgroup Size:** the derived subgroup size (number of items to measure at each time period).
- **ARL at Target:** the average run length when the process shifts to the *Target Mean*.
- **In-Control ARL:** the average run length when the mean is at its desired value.
- **False Alarms:** the percentage of plotted points that will lie outside the control limits when the mean is at its desired value.

To meet the criteria specified in the current example, you should take a sample of $n = 9$ items at each plotting period and plot their averages.

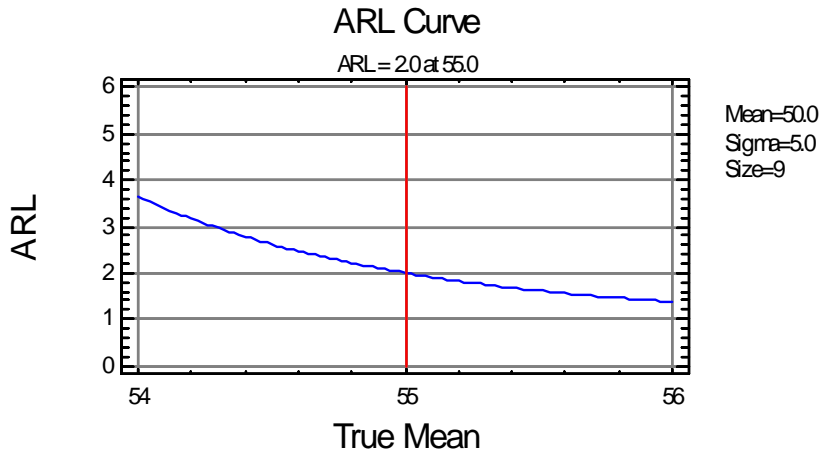
ARL Curve

The *ARL Curve* shows the average run length as a function of the shift in the process mean.



Note that the chart rises to approximately 370 at the in-control value of the mean.

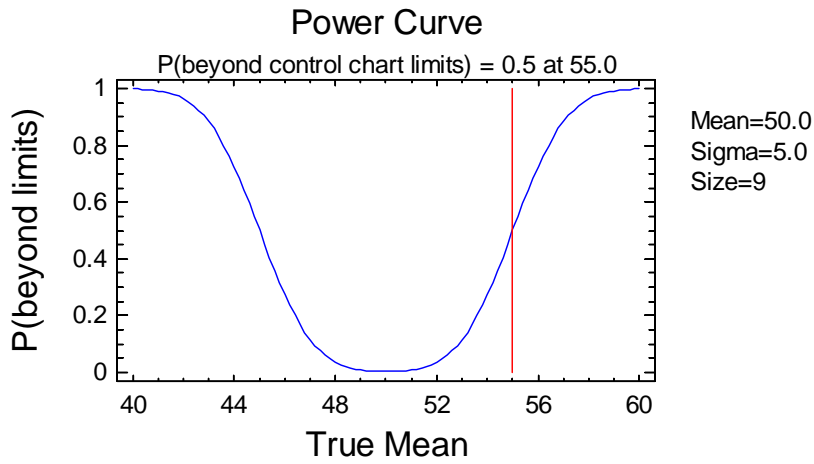
A tall vertical line is drawn at the *Target Mean*. If you rescale the graph or use the *Zoom In* feature on the right-mouse button menu, you can get a close-up of the ARL in the vicinity of the target:



Note that the ARL does indeed equal 2 at $\mu = 55$.

Power Curve

The *Power Curve* plots the probability of obtaining a point beyond the control limits at any given period.



As might be expected, there is a 50% chance of getting an out-of-control signal at any period if the mean shifts to 55 since the ARL equals 2.

Example 2: EWMA Chart

Rather than taking subgroup at each point in time, you could elect to create an individuals chart and plot an exponentially weighted moving average (EWMA). To set up this type of chart, complete the dialog box as shown below:

Control Chart Design Options

Type of Chart

- X-bar
- Moving average
- EWMA
- Cumulative sum (V-mask)
- Cumulative sum (H-K)
- P or nP chart
- C or U chart

Process Parameters

Process mean:

Process sigma:

Control Limits

- Upper and lower
- Upper only
- Lower only

Design Requirements

Target mean:

- Desired ARL:
- Minimize ARL for subgroup size

Hold

- Control limits at sigma
- In-control ARL at
- False alarm rate at %

OK Cancel Help

The request made in the above dialog box is to determine the optimal value of the EWMA smoothing parameter λ that will minimize the ARL when the process mean shifts to 55, while maintaining an in-control ARL of 370.4.

The *Analysis Summary* is shown below:

<u>Control Chart Design</u>						
Input						
	<i>Sigma</i>	<i>In-Control</i>	<i>Process</i>	<i>Process</i>	<i>Desired</i>	<i>Target</i>
<i>Chart</i>	<i>Limits</i>	<i>ARL</i>	<i>Mean</i>	<i>Sigma</i>	<i>ARL</i>	<i>Mean</i>
EWMA	Two-sided	370.4	50.0	5.0	minimize	55.0
Selected Chart						
		<i>EWMA</i>	<i>ARL at</i>	<i>Sigma</i>	<i>False</i>	
	<i>Subgroup Size</i>	<i>Parameter</i>	<i>Target</i>	<i>Multiple</i>	<i>Alarms (%)</i>	
EWMA	1	0.16	9.57483	2.81	0.269978	

The output includes:

- **Required Subgroup Size:** set to 1 for an individuals chart.
- **EWMA Parameter:** the derived value of the EWMA smoothing parameter λ . In the STATGRAPHICS EWMA control chart procedure, this is set on the *Analysis Options* dialog box.
- **ARL at Target:** the average run length when the process shifts to the *Target Mean*.
- **Sigma Multiple:** the multiple of sigma at which the control limits should be placed. In the STATGRAPHICS EWMA control chart procedure, this is also set on the *Analysis Options* dialog box.
- **False Alarms:** the percentage of plotted points that will lie outside the control limits when the mean is at its desired value.

The derived chart is an EWMA individuals chart with $\lambda = 0.16$ and control limits at 2.81 times sigma. The ARL at the target value $\mu = 55$ is about 9.57, meaning that it will take on average nearly 10 time periods to generate an out-of-control signal. While this ARL may seem large, the ARL for a corresponding individuals chart where only single data values are plotted is over 40.

Calculations

The ARL calculations are provided in the documentation for the various types of control charts.